Unruh effect of detectors with quantized center of mass

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The Unruh effect is the prediction that particle detectors accelerated through the vacuum get excited by the apparent presence of radiation quanta—a fundamental quantum phenomenon in the presence of acceleration. Prior treatments of the Unruh effect, that presume a classically prescribed trajectory, do not account for the quantum dynamics of the center of mass. Here, we study more realistic detectors whose center of mass is a quantized degree of freedom being accelerated by an external classical field. We investigate the detector's recoil due to the emission of Unruh quanta. Vice versa, we also study the recoil's impact on the emission of Unruh quanta and the excitation of the detector. We find that the recoil due to the emission of Unruh quanta may be a relevant experimental signature of the Unruh effect.

I. INTRODUCTION

An idealized particle detector—a two-level system—accelerated through the vacuum of a quantum field can become excited [1–6]. In the case of uniform acceleration, the state of the detector assumes a thermal form with an apparent temperature proportional to the acceleration. This is the Unruh effect, and the particle detector is eponymously referred to as the Unruh–DeWitt (UDW) detector [1,2]. In conventional lore, the energy and momentum required to excite the detector, along with the subsequent emission of a photon, come from an unspecified external agent which enforces a prescribed classical uniformly accelerated trajectory for the detector. These idealized treatments of the Unruh effect neither describe the accelerating agent nor the recoil of the detector due to photon emission. Both are important, however, to the question of the viability of an experimental observation of the Unruh effect. For example, real experiments cannot sustain uniform acceleration indefinitely, while detector recoil makes it unrealistic to consider uniform acceleration. In fact, the recoil could itself be an avenue towards the detection of the Unruh effect. Importantly, in this case the sensitivities required to experimentally resolve the recoil are likely to be comparable to the quantum fluctuations of the detector's center-of-mass motion. This means that the recoil needs to be calculated within a full quantum mechanical treatment of the detector's center-of-mass degree of freedom. From this perspective, the handful of works studying detector recoil from the Unruh effect [7–10] remain incomplete.

Here, we dynamically account for the acceleration of the detector, and self-consistently treat the recoil of the detector center of mass. To this end, the detector's acceleration is described by coupling the detector to an external classical accelerating field [7], and its center-of-mass degree of freedom is treated quantum mechanically [11]. Within this framework, we study the vacuum excitation process for the internal and center-of-mass degrees of freedom of the detector—a process we term the *massive Unruh effect*, in reference to the detector mass being assumed finite. By letting the detector mass go to infinity, the behavior of classical detector trajectories can be recovered. We also compute the recoil pattern and its relationship to the pattern of the emission of Unruh quanta. Both patterns assume characteristic anisotropic forms. The peak emission probability is proportional to the acceleration, which allows us to informally talk of a "temperature" for the massive Unruh effect, in analogy with the conventional Unruh effect.

II. UNRUH EFFECT WITH INFINITE MASS DETECTOR

We briefly recall the Unruh effect for a detector accelerated on a prescribed trajectory through a scalar quantum field. The detector-field system is described by the free Hamiltonian ($\hbar = 1$ henceforth),

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$$\hat{H}_0 = \Omega |e\rangle\langle e| + \int d^3k\, ck\, \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}}, \qquad (1)$$

where Ω denotes the energy gap of the detector's ground ($|g\rangle$) and excited ($|e\rangle$) states, and $\hat{a}_{\mathbf{k}}^\dagger$ ($\hat{a}_{\mathbf{k}}$) is the creation (annihilation) operator of the scalar quantum field mode of momentum $\mathbf{k}$. Following Unruh and DeWitt [1,2], and in analogy with realistic models of light-matter interaction in electrodynamics [12], we assume that the detector's internal state couples to the field through the interaction Hamiltonian,

$$\hat{H}_{\text{int}} = q\hat{\mu}(t) \otimes \hat{\phi}(\mathbf{x}(t)). \qquad (2)$$

Here, q is the coupling strength, $\hat{\mu} = |e\rangle\langle g| + |g\rangle\langle e|$ is the detector's "monopole" moment, and $\hat{\phi}(\mathbf{x}(\tau))$ is the scalar field operator along the detector's trajectory. We limit ourselves to the regime of nonrelativistic detector velocity, which allows us to identify the detector's proper time τ with the coordinate time t.

The crux of the Unruh effect is that the detector can be excited by accelerating it through the quantum vacuum of the scalar field. Given the structure of the interaction, the detector can be excited only if the quantum field is simultaneously excited to (at least) the single particle quantum state, $\hat{a}_{\mathbf{k}}^\dagger|0\rangle$. That is, contrary to resonance effects such as absorption, the Unruh effect is the result of counterrotating wave terms in $\hat{H}_{\text{int}}$ [13]. To elucidate this, we compute the probability that the initial state, $|\psi_i\rangle = |g\rangle \otimes |0\rangle$—the joint ground state of the system—transitions to the final state, $|\psi_f\rangle = |e\rangle \otimes \hat{a}_{\mathbf{k}}^\dagger|0\rangle$, where both the detector and the field are excited. Working in the interaction picture defined by the free Hamiltonian, in which,

$$\hat{\mu}(t) = e^{i\Omega t}|e\rangle\langle g| + \text{H.c.}$$

$$\hat{\phi}(\mathbf{x}, t) = \int \frac{d^3k}{(2\pi)^{3/2}} \sqrt{\frac{c^2}{2k}}[e^{-ickt+i\mathbf{k}\cdot\mathbf{x}}\hat{a}_{\mathbf{k}} + \text{H.c.}],$$

the probability amplitude (to first order in perturbation theory) for the aforementioned excitation process is

$$\mathcal{A}_{\text{U}}(\mathbf{k}) = \langle\psi_f| \int_{-\infty}^\infty dt\, \hat{H}_{\text{int}}(t)|\psi_i\rangle = \frac{qc}{2\pi\sqrt{4\pi k}}\mathcal{I},$$

where

$$\mathcal{I} = \int_{-\infty}^\infty dt\, e^{it(ck+\Omega)-i\mathbf{k}\cdot\mathbf{x}(t)}. \qquad (3)$$

We now define the probabilities for the detector to get excited and a field quantum of momentum $\mathbf{k}$ to be emitted,

$$P_{\text{U}}(\mathbf{k}) = |\mathcal{A}_{\text{U}}(\mathbf{k})|^2;$$

and the excitation of the detector, irrespective of the momentum of the emitted photon,

$$P_{\text{U}} = \int d^3k|\mathcal{A}_{\text{U}}(\mathbf{k})|^2.$$

When the detector is in inertial motion, $\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{v}t$, the excitation amplitude is zero:

$$\begin{aligned}
\mathcal{A}_{\text{U}} &= \frac{qc}{2\pi\sqrt{4\pi k}} \int_{-\infty}^\infty dt\, e^{it(ck+\Omega-\mathbf{k}\cdot\mathbf{v})-i\mathbf{k}\cdot\mathbf{x}_0} \\
&= \frac{qc}{\sqrt{4\pi k}} e^{-i\mathbf{k}\cdot\mathbf{x}_0}\delta(\Omega + ck - \mathbf{k}\cdot\mathbf{v}) \\
&= 0,
\end{aligned}$$

essentially because $\Omega + ck - \mathbf{k}\cdot\mathbf{v} \neq 0$, owing to the fact the energy gap is positive ($\Omega > 0$), the photon momentum is positive ($k > 0$), and the detector speed is less than that of light ($|\mathbf{v}| < c$). That is, in inertial motion through the vacuum, the detector does not get excited.

In contrast, consider the detector in noninertial motion with a uniform acceleration a for time T along the z direction, i.e.,

$$\mathbf{a}(t) = a\Theta(t)\Theta(T - t)\mathbf{e}_z,$$

where Θ is the Heaviside step function. Assuming that the detector's initial position coincides with the origin of the coordinate system, its trajectory is

$$\mathbf{x}(t) = \left[\frac{at^2}{2}\Theta(t)\Theta(T - t) + \frac{aT}{2}(2t - T)\Theta(t - T)\right]\mathbf{e}_z. \qquad (4)$$

In the following, we restrict the time duration T so that the velocity developed in that time with an acceleration a is well within the nonrelativistic regime; in particular, we will always take $|\mathbf{v}(T)| = aT \lesssim 0.01c$. Within this nonrelativistic regime, and for the spatial trajectory in Eq. (4), we obtain for the time integral given in Eq. (3) [here we define $\omega = \Omega + ck$, $\omega' = \omega - ak_zT$, and, k_z the z component of the momentum of the emitted Unruh photon, $\mathbf{k} = (k_x, k_y, k_z) = (k\sin(\theta)\cos(\phi), k\sin(\theta)\sin(\phi), k\cos(\theta))$],

$$\begin{aligned}
\mathcal{I} &= \frac{1}{i\omega} + \pi\delta(\omega) + e^{iT(\omega'+ak_zT/2)}\left(\frac{i}{\omega'} + \pi\delta(\omega')\right) \\
&\quad + \sqrt{\frac{\pi e^{\frac{i\omega^2}{ak_z}}}{2iak_z}}\left[\text{Erf}\left(\frac{i\omega}{\sqrt{2iak_z}}\right) - \text{Erf}\left(\frac{i\omega'}{\sqrt{2iak_z}}\right)\right].
\end{aligned}$$

Since both the energy gap of the detector and the absolute value of the momentum of the emitted photon are strictly positive ($\Omega > 0$ and $k > 0$), the delta distribution $\delta(\omega)$ can be omitted. Similarly, since the detector's velocity is strictly

smaller than the speed of light, we find $\omega' > 0$, and so $\delta(\omega')$ can also be omitted. Thus,

$$\mathcal{I} = \frac{1}{i\omega} - \frac{e^{iT(\omega'+ak_zT/2)}}{i\omega'} + \sqrt{\frac{\pi e^{i\omega^2/ak_z}}{2iak_z}}\mathrm{Erf}\left(\frac{i\omega}{\sqrt{2iak_z}}\right)$$
$$- \sqrt{\frac{\pi e^{i\omega^2/ak_z}}{2iak_z}}\mathrm{Erf}\left(\frac{i\omega'}{\sqrt{2iak_z}}\right); \quad (5)$$

which is in general nonzero in contrast to the case of inertial motion. Therefore, the total excitation probability, defined by

$$P_{\mathrm{U}} = \frac{q^2c^2}{8\pi^2}\int_{-1}^{1}dz\int_{0}^{\infty}dk k|\mathcal{I}|^2,$$

where $z = \cos(\theta) \in [-1, 1]$ can be nonzero. Similarly, the excitation probability density for the detector to be excited while emitting a photon of momentum k irrespective of direction becomes

$$P_{\mathrm{U}}(k) = \frac{q^2c^2k}{8\pi^2}\int_{-1}^{1}dz|\mathcal{I}|^2.$$

The symmetry of the problem along the z axis means that the photon emission is azimuthally symmetric, so that it is useful to consider the probability density,

$$P_{\mathrm{U}}(k, z) = \frac{q^2c^2k}{8\pi^2}|\mathcal{I}|^2,$$

corresponding to a photon of momentum k emitted along the polar angle, $\theta = \cos^{-1}z$.

Figure 1(a) shows the probability $P_{\mathrm{U}}(k)$ for the excitation of the UDW detector by the emission of an Unruh quantum of momentum k; by momentum conservation, this is equivalent to the angle-integrated probability of finding an Unruh quantum of momentum k. Note that for the case we consider here, where the detector is not eternally accelerated, the emission is not isotropic [see Fig. 1(b)], precluding complete analogy with blackbody radiation. In fact, radiation is preferentially emitted along/against the direction of acceleration. (For realistic scenarios involving charges or atoms as UDW detectors, the correct vacuum would not be that of the scalar quantum field—which is what we consider here—but the vacuum of the full vector electromagnetic field; the vectorial character of the latter is expected to produce radiation transverse to the acceleration, in analogy with classical synchrotron radiation [14].) However there is one aspect of blackbody radiation that is reflected in Fig. 1(a): as the energy gap gets smaller the peak of the emission shifts to lower momenta, an observation that can be put in correspondence with Wien's displacement law for blackbody radiation (i.e., $k_{\mathrm{peak}} \propto$ temperature) if we associate a temperature
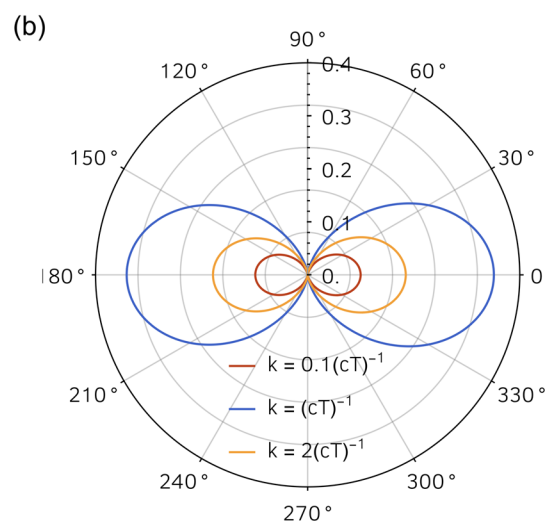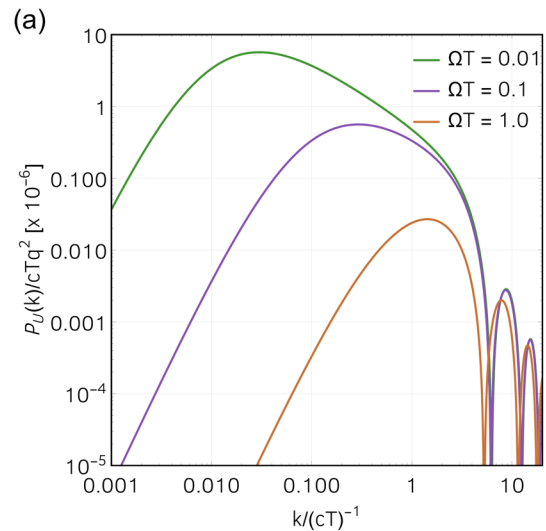

FIG. 1. Emission probability for an accelerated UDW detector. (a) The total (i.e., angle-integrated) probability $P_{\mathrm{U}}(k)$ for the emission of an Unruh quantum with momentum k, for various values of the energy gap Ω. (b) Angle-resolved emission probability $P_{\mathrm{U}}(k, \theta)$ (in units of $10^{-6}cTq^2$), for energy gap $\Omega = 0.2/T$. Both plots are for acceleration, $a = 8 \times 10^{-3}(c/T)$. Note that the oscillations in (a) arise from accelerating the detector for a time interval that is compact.

proportional to the energy gap of the detector. (Note that one can always assign a temperature for a two-level system whenever its density matrix is diagonal: the ratio of the diagonal elements can be compared to that of the canonical thermal state, and so an effective temperature can be defined.) In this sense, we may formally associate a temperature to the Unruh process, even in the case where the detector is accelerated only for a finite time interval. Finally, in the nonrelativistic regime, when the emitted photon momentum is "small," we have that $\omega - \omega' = aTk_z \ll 1$ (clearly, small means $k_z \ll 1$, which, in the dimensioned units of Fig. 1(a) reads $k_z \ll (cT)^{-1}$); explicitly expanding the amplitude integral in Eq. (5) in the small

parameter $\omega - \omega' \propto a$, one can also show that $P_U \propto a$. Thus we are able to formally establish that the temperature is proportional to the acceleration even in the case where the detector is accelerated only for a finite duration. (For uniform acceleration, the emitted Planckian radiation satisfies Wien's displacement law [15].)

III. UNRUH EFFECT WITH FINITE MASS DETECTOR

Experimentally producing the large accelerations required to observe the Unruh effect call for low mass UDW detectors. Any such detector will experience significant recoil once an Unruh quantum is emitted. Once the possibility of recoil is admitted, it becomes unphysical to consider an externally prescribed acceleration, even for a finite time interval (especially if we are interested in measuring the random recoil over an ensemble of multiple emission events). We also envision the possibility of inferring the Unruh effect through a direct experimental measurement of the recoil; the required measurement sensitivities are expected to be comparable to the quantum fluctuations of the detector center-of-mass degree of freedom. For these reasons, we must treat the detector's center of mass in a full quantum framework, and, self-consistently incorporate the external agency that accelerates the system.

To this end, we consider a massive detector with a quantized center-of-mass degree of freedom which couples to a quantum scalar field $\hat{\phi}$ [4,11]. The detector's center of mass is coupled to a classical "electric" field $\mathbf{E}$, which allows us to dynamically model the acceleration of the detector. This scenario is modeled by the Hamiltonian,

$$\hat{H} = \frac{\hat{\mathbf{p}}^2}{2M} - q\mathbf{E} \cdot \hat{\mathbf{x}} + \Omega|e\rangle\langle e| + \int d^3k\, ck\, \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}}$$
$$+ q\int d^3x\, \hat{\mathcal{P}}(\mathbf{x}) \otimes \hat{\mu} \otimes \hat{\phi}(\mathbf{x}),$$

with M the mass of the detector, and $\hat{\mathcal{P}}(\mathbf{x}) = |\mathbf{x}\rangle\langle\mathbf{x}|$ is the projector onto the center-of-mass position eigenstates. In the interaction picture, the Hamiltonian reads

$$H_{\text{int}}(t) = q\int d^3x\, \hat{\mathcal{P}}(\mathbf{x}, t) \otimes \hat{\mu}(t) \otimes \hat{\phi}(\mathbf{x}, t), \quad (6)$$

with $\hat{\mathcal{P}}(\mathbf{x}, t) = |\mathbf{x}(t)\rangle\langle\mathbf{x}(t)|$. In order to model a scenario comparable to the situation considered in the previous section, we assume an electric field,

$$\mathbf{E}(t) = \mathcal{E}\Theta(t)\Theta(T - t)\mathbf{e}_z,$$

with a nonzero strength $\mathcal{E}$ in the time interval $t \in [0, T]$, and zero elsewhere. It models the detector's center of mass being uniformly accelerated in that interval, while it evolves freely for $t \notin [0, T]$. Note that this is different

from the theoretical contrivance of switching the coupling between the detector and the quantum vacuum (i.e., effectively a time-varying coupling q), which can produce excitation of the detector due to the time-energy uncertainty principle [16].

Before we can study the vacuum excitation process for the massive detector, we need the time evolved operator $\hat{\mathcal{P}}(\mathbf{x}, t) = |\mathbf{x}(t)\rangle\langle\mathbf{x}(t)|$. To this end, we write the Heisenberg equation for the detector's position and momentum,

$$\dot{\hat{\mathbf{x}}}(t) = \frac{\hat{\mathbf{p}}(t)}{M}, \quad \dot{\hat{\mathbf{p}}}(t) = q\mathcal{E}(t)\mathbf{e}_z,$$

which produces the time-dependent position operator,

$$\hat{\mathbf{x}}(t) = \hat{\mathbf{x}}(0) + \hat{\mathbf{p}}(0)t/M + f(t)\mathbf{e}_z,$$

where

$$f(t) = \frac{a}{2}[t^2\Theta(t)\Theta(T - t) + T(2t - T)\Theta(t - T)],$$

with $a = q\mathcal{E}/M$ being the uniform acceleration due to the electric field. We note that $f(t)$ is of the same form as the z component of the classical trajectory which we prescribed in Eq. (4) for the UDW detector with classical center of mass. Since the position and momentum operators coincide at time $t = 0$ between the Heisenberg and Schrödinger pictures, we have that $\hat{x}(0)\psi(\mathbf{x}) = \mathbf{x}\psi(\mathbf{x})$ and $\hat{\mathbf{p}}(0)\psi(\mathbf{x}) = -i\nabla\psi(\mathbf{x})$. Next, to find the time-dependent position eigenfunction $\psi_\xi(\mathbf{x}, t) = \langle\mathbf{x}|\psi_\xi(t)\rangle$ for a given position eigenvalue ξ, we solve the Schrödinger equation,

$$\left(\mathbf{x} + f(t)\mathbf{e}_z - \frac{it}{M}\nabla\right)\psi_\xi(\mathbf{x}, t) = \xi\psi_\xi(\mathbf{x}, t),$$

with the initial condition $|\psi_\xi(0)\rangle = |\xi\rangle$, and enforcing the normalization condition,

$$\int d^3x\, \psi_\xi^*(\mathbf{x}, t)\psi_{\xi'}(\mathbf{x}, t) = \delta^{(3)}(\xi - \xi').$$

The required wave function is

$$|\psi_\xi(t)\rangle = \int \frac{d^3p}{(2\pi)^{3/2}} \exp\left[it\frac{p^2}{2M} - i\mathbf{p}\cdot\xi + ip_z f(t)\right]|\mathbf{p}\rangle.$$

Putting all this together, we obtain the time evolved projection operator,

$$\hat{\mathcal{P}}(\mathbf{x}, t) = \int \frac{d^3p\, d^3q}{(2\pi)^3} \exp\left[it\frac{q^2 - p^2}{2M} - i(\mathbf{q} - \mathbf{p})\cdot\mathbf{x}\right.$$
$$\left. + i(q_z - p_z)f(t)\right]|\mathbf{q}\rangle\langle\mathbf{p}|,$$

that fully determines the interaction Hamiltonian in Eq. (6).

A. Transition amplitude, probability and probability densities

We are now equipped to study what we refer to as the *massive Unruh effect*, that is, the excitation process both of a UDW detector with a finite mass, initially in the ground state of its internal degree of freedom coupled to a scalar quantum field initially in its vacuum state, and accelerated by an external electric field. That is, we consider initial and final states of the form,

$$|\psi_i\rangle = |\varphi\rangle \otimes |g\rangle \otimes |0\rangle,$$

$$|\psi_f\rangle = |\mathbf{r}\rangle \otimes |e\rangle \otimes \hat{a}_{\mathbf{k}}^\dagger |0\rangle,$$

where $|\varphi\rangle = \int d^3 p \, \tilde{\varphi}(\mathbf{p})|\mathbf{p}\rangle$ is the initial center-of-mass state, $|\mathbf{p}\rangle$ the center-of-mass momentum eigenstates, $\tilde{\varphi}(\mathbf{p})$ the initial center-of-mass wave function in the momentum representation, and $\mathbf{r}$ the detector's recoil momentum.

The transition amplitude for the process where the detector gets excited, its center-of-mass recoils with momentum $\mathbf{r}$, and emits an Unruh quantum of momentum $\mathbf{k}$, is (up to a phase factor)

$$\mathcal{A}_{\mathrm{M}} = \frac{qc}{2\pi\sqrt{4\pi k}} \tilde{\varphi}(\mathbf{r} + \mathbf{k}) \mathcal{J}(\mathbf{r}),$$

where we define

$$\mathcal{J}(\mathbf{r}) = \int_{-\infty}^{\infty} dt \exp\left[it\left(\frac{k^2}{2M} - \frac{\mathbf{r} \cdot \mathbf{k}}{M} + ck + \Omega\right) - ik_z f(t)\right].$$

The corresponding transition probability density is

$$P_{\mathrm{M}}(\mathbf{k}, \mathbf{r}) = |\mathcal{A}_{\mathrm{M}}|^2 = \frac{q^2 c^2}{(2\pi)^2 4\pi k} |\tilde{\varphi}(\mathbf{r} + \mathbf{k}) \mathcal{J}(\mathbf{r})|^2.$$

To study the recoil of the detector, we consider the excitation probability density as a function of the recoil momentum $\mathbf{r}$ (and irrespective of the momentum of the emitted photon),

$$P_{\mathrm{M}}(\mathbf{r}) = \int d^3 k \frac{q^2 c^2}{(2\pi)^2 4\pi k} |\tilde{\varphi}(\mathbf{r} + \mathbf{k}) \mathcal{J}(\mathbf{r})|^2;$$

and the total excitation probability for the massive Unruh process,

$$P_{\mathrm{M}} = \int d^3 k \int d^3 p \frac{q^2 c^2}{(2\pi)^2 4\pi k} |\tilde{\varphi}(\mathbf{p}) \mathcal{J}(\mathbf{p} - \mathbf{k})|^2.$$

To resolve the angular dependence of the emission and recoil, we write $\mathbf{k} = (k_x, k_y, k_z) = (k \sin(\theta) \cos(\phi), k \sin(\theta) \sin(\phi), k \cos(\theta))$, with ϕ the azimuthal angle and with θ the polar angle, that is, the angle between the

momentum of the emitted photon and the direction of the electric field lines (and therefore, of the acceleration); as before, we also define $z = \cos(\theta)$ (in the following we will refer to both z and θ as the polar angle of the emitted photon). The excitation probability density for the process to happen while emitting an Unruh quantum of momentum k in magnitude, irrespective of direction, is

$$P_{\mathrm{M}}(k) = \int_{-1}^{1} dz \int_{0}^{2\pi} d\phi \int d^3 k \int d^3 p \frac{q^2 c^2 k}{(2\pi)^2 4\pi}$$
$$\times |\tilde{\varphi}(\mathbf{p}) \mathcal{J}(\mathbf{p} - \mathbf{k})|^2. \tag{7}$$

Similarly, we define the excitation probability density as a function of both the magnitude k and polar angle z of the emitted photon:

$$P_{\mathrm{M}}(k, z) = \int_{0}^{2\pi} d\phi \int d^3 p \frac{q^2 c^2 k}{(2\pi)^2 4\pi} |\tilde{\varphi}(\mathbf{p}) \mathcal{J}(\mathbf{p} - \mathbf{k})|^2. \tag{8}$$

All the above expressions depend on the time integral $\mathcal{J}$, which can be explicitly evaluated,

$$\mathcal{J}(\mathbf{p} - \mathbf{k}) = \frac{1}{i\omega_{\mathrm{M}}} - \frac{e^{iT(\omega'_{\mathrm{M}} + ak_z T/2)}}{i\omega'_{\mathrm{M}}}$$
$$+ \sqrt{\frac{\pi e^{i\omega_{\mathrm{M}}^2/ak_z}}{2iak_z}} \mathrm{Erf}\left(\frac{i\omega_{\mathrm{M}}}{\sqrt{2iak_z}}\right)$$
$$- \sqrt{\frac{\pi e^{i\omega_{\mathrm{M}}'^2/ak_z}}{2iak_z}} \mathrm{Erf}\left(\frac{i\omega'_{\mathrm{M}}}{\sqrt{2iak_z}}\right) \tag{9}$$

in terms of $\omega_{\mathrm{M}} = \omega + \frac{k^2}{2M} - \frac{\mathbf{p} \cdot \mathbf{k}}{M}$ and $\omega'_{\mathrm{M}} = \omega_{\mathrm{M}} - ak_z T$. In writing the above expression, we have omitted two terms each involving delta distributions $\delta(\omega_{\mathrm{M}})$ and $\delta(\omega'_{\mathrm{M}})$, which is justified for the following reasons. First, let us write $\mathbf{p} \cdot \mathbf{k} = pk \cos(\kappa)$, with κ the angle between $\mathbf{p}$ and $\mathbf{k}$, and define $\omega_0 = \omega + \frac{k^2}{2M}$ for brevity. The delta distribution $\delta(\omega_{\mathrm{M}})$ then peaks only for $\cos(\kappa) = \frac{M\omega_0}{pk}$. Furthermore, the delta distribution $\delta(\omega'_{\mathrm{M}})$ peaks only for $\cos(\kappa) = \frac{M}{pk}(\omega_0 - aTk_z)$. But since $\cos(\kappa) \in [-1, 1]$, a necessary condition for $\delta(\omega_{\mathrm{M}})$ to peak is $p \geq Mc$, which translates to saying that the initial virtual center-of-mass velocities would have to be superluminal, which is ruled out. Similarly, a necessary condition for $\delta(\omega'_{\mathrm{M}})$ to peak is $p + MaT \geq Mc$, which would require the virtual center-of-mass velocities to be superluminal by the end of the accelerated phase. Physically, the delta distributions $\delta(\omega_{\mathrm{M}})$ and $\delta(\omega'_{\mathrm{M}})$ have their origin in the virtual inertial motion of the detector, respectively for the times $t < 0$ and $t > T$ during which the electric field is switched off. Inertial virtual motion (just like inertial real motion) should not

cause excitation of the detector and the field, which is reflected in the vanishing of these delta distributions.

B. The Unruh effect as a limiting case of the massive Unruh effect

Before proceeding further, let us see how to recover the conventional Unruh effect (of Sec. II)—i.e., a UDW detector with a prescribed classical trajectory—from the "massive Unruh effect" studied above.

In order to recover the traditional Unruh effect for a detector experiencing a uniform acceleration a, let us consider the limit of infinite detector mass—so that the center-of-mass wave function delocalizes infinitely slowly and so it effectively behave classically. A classical particle of charge q and mass M in a constant electric field $\mathcal{E}$ experiences an acceleration $a = q\mathcal{E}/M$. Defining $M = m\gamma$ and $\mathcal{E} = \varepsilon\gamma$ and letting $\gamma \to \infty$ allows us to keep the acceleration a experienced by the detector constant, while considering the infinite mass limit:

$$\lim_{\gamma \to \infty} P_{\mathrm{M}} = P_{\mathrm{U}}.$$

The above equation holds true for all the probabilities defined above; in this sense, the massive Unruh effect subsumes the conventional (infinite mass) Unruh effect.

C. Example of a Gaussian center-of-mass wave packet

In order to apply the formalism developed in Sec. III A for the massive Unruh effect, we need to specify an initial wave function, $\tilde{\varphi}(\mathbf{p})$, for the detector's center of mass. We consider a Gaussian initial center-of-mass (momentum) wave packet of the form

$$\tilde{\varphi}(\mathbf{p}) = \left(\frac{L^2}{2\pi}\right)^{3/4} e^{-p^2 L^2/4},$$

assumed to exist at time $t = 0$, so that when the electric field is switched on, the detector's center of mass in position space is localized at the origin with width L.

Since we work in the nonrelativistic regime, it is necessary to choose the parameters L, M, T and a in a way that ensures that the detector's virtual center-of-mass velocities are much less than the speed of light. The initial momentum of the detector in the z direction, i.e., parallel to the electric field, is Gaussian distributed around $p_z = 0$, with a standard deviation of $\sqrt{2}/L$. Initial momenta which are σ standard deviations away from the mean then correspond to initial virtual center-of-mass speed, $|v_z(0)| = 2\sigma \cdot \sqrt{2}/(LM)$. If we require that these tails of the wavepacket, after having the accelerating field $\mathcal{E}$ turned on for a time T, be still less than 1% the speed of light, then we need to maintain an electric field that satisfies $v_z(T) = |v_z(0)| + aT \lesssim 0.01c$. In the following, we choose electric field strengths weak enough that

this is satisfied for virtual velocities that are $\sigma = 3.5$ standard deviations from the mean velocity. This means that the center-of-mass velocity is a small fraction of the speed of light, i.e., $v/c = p/(Mc) \ll 1$; thus $|\mathcal{J}(\mathbf{p} - \mathbf{k})|^2$ [from Eq. (9)] can be approximated around $\mathbf{p}/(Mc) = 0$ by using only its first few Taylor coefficients (for $n = 0, 1, 2, \ldots$):

$$J_n = \frac{1}{n!}\left[\frac{\partial^n}{\partial\omega_{\mathrm{M}}^n}|\mathcal{J}(\mathbf{p} - \mathbf{k})|^2\right]_{\omega_{\mathrm{M}}=\omega}.$$
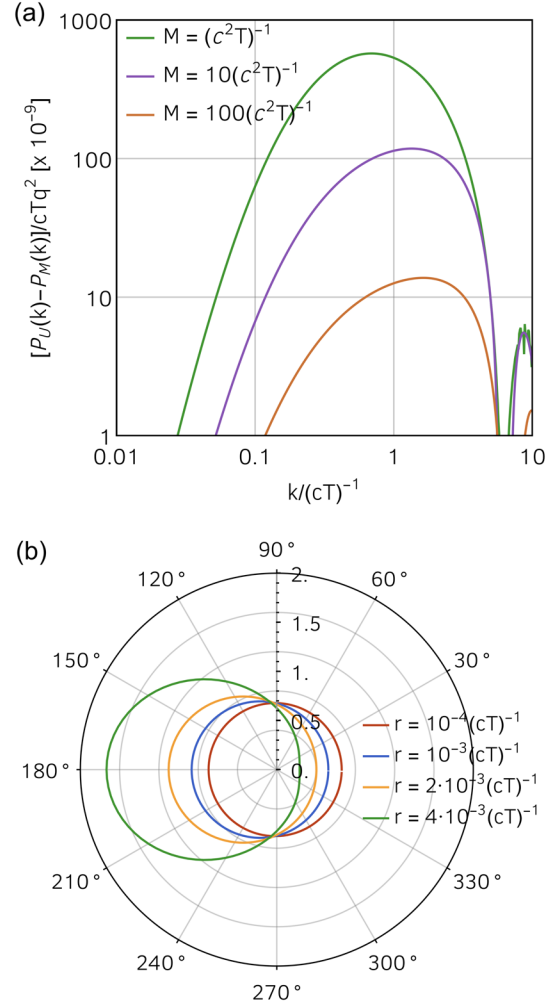
FIG. 2. Emission and recoil probability for a massive UDW detector. (a) Plot shows the difference of the emission probabilities between the cases where the detector center of mass is infinitely heavy [i.e., $P_{\mathrm{U}}(k)$] and the case where the detector center of mass has a finite mass [i.e., $P_{\mathrm{M}}(k)$]. (b) Recoil probability density $P_{\mathrm{M}}(r, \zeta)$ [in units of $10^{-6}(cTq^2)^{-1}$] as a function of recoil momentum angle ζ, for a few values of the magnitude of the recoil momentum magnitude r. The detector mass is $M = 10/(c^2 T)$, while in both panels, its internal energy gap is $\Omega = 0.2/T$, acceleration $a = 8 \times 10^{-3}(c/T)$, and the center-of-mass wavepacket has an initial width, $L = 100(cT)$.

Expressing J_n in terms of the variables k and z, we then obtain for the basic excitation probability densities introduced above [valid to $\mathcal{O}((LMc)^{-4})$]:

$$P_{\mathrm{M}}(k,z) \approx \frac{q^2 c^2 k}{8\pi^2}\left[J_0(k,z) + \frac{k^2 J_2(k,z)}{(ML)^2}\right],$$

$$P_{\mathrm{M}}(r,\zeta) \approx \frac{L^3 q^2 c^2}{2(2\pi)^{5/2}} \int_0^\infty dk \int_{-1}^1 dz\, k e^{-(r^2+k^2+2rk\zeta)L^2/2}$$
$$\times \left[J_0(k,z) + \frac{(rk\zeta+k^2)^2}{M^2}J_2(k,z)\right].$$

Here, r is the magnitude of the recoil momentum and $\zeta = \mathbf{r}\cdot\mathbf{k}/(rk)$ is the cosine of the angle between the recoil momentum ($\mathbf{r}$) and Unruh photon momentum ($\mathbf{k}$).

Figure 2(a) depicts the difference, $P_{\mathrm{U}}(k) - P_{\mathrm{M}}(k)$, between the angle-integrated emission probabilities of the conventional infinite mass UDW detector and a UDW detector of finite mass, as a function of the momentum k of the Unruh quantum. As the mass increases, the finite mass case approaches the standard case (i.e., $P_{\mathrm{U}} - P_{\mathrm{M}} \to 0$) as expected.

Although the total emission probability, and hence the emitted flux of Unruh quanta, is seen to decrease with mass, each individual emission causes greater recoil of the detector center of mass for a lower mass detector. This is shown in Fig. 2(b), where the angular recoil probability is plotted as a function of the magnitude of the recoil momentum. Recoil with larger momenta happen in the direction opposite to the acceleration (0° in the polar plot)—a consequence of momentum conservation.

IV. CONCLUSION

We analyzed the behavior of UDW detectors which possess a quantum mechanical center of mass and that are finitely accelerated through the vacuum of a scalar quantum field. We found a characteristic interplay between the acceleration-induced excitation of the UDW detector, the anisotropic patterns of the flux of the emitted Unruh quanta and the corresponding quantum recoil. This makes the quantum recoil a potentially experimentally relevant signature of the Unruh effect.

In practice, trapped and accelerated (sub)atomic particles with internal states can act as UDW detectors. The ability to precisely measure forces that act on them may then provide a route to a direct detection of the Unruh effect on a single particle. Indeed, electron bunches in storage rings have long been suspected of being probes of the Unruh effect [17,18]. However, space charge effects and other systematics have prevented a decisive measurement of the Unruh effect; nor does the accelerator environment offer an ideal venue to study more fundamental predictions of the Unruh effect [19,20].

Although any realistic experiment along these lines will have to consider the coupling of the internal degree of freedom to the electromagnetic vacuum, the scalar field scenario considered here allows a first glimpse of what to expect. Further studies along these directions are in progress.

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