One-loop graviton corrections to conformal scalars on a de Sitter background

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(Received 20 July 2020; accepted 23 April 2021; published 27 May 2021)

We exploit a recent computation of one graviton loop corrections to the self-mass [D. Glavan et al., Single graviton loop contribution to the self-mass of a massless, conformally coupled scalar on a de Sitter background, Phys. Rev. D **101**, 106016 (2020).] to quantum correct the field equation for a massless, conformally coupled scalar on a de Sitter background. With the obvious choice for the finite part of the $R^2\phi^2$ counterterm, we find that neither plane wave mode functions nor the response to a point source acquires large infrared logarithms. However, we do find a decaying logarithmic correction to the mode function and a short distance logarithmic running of the potential in addition to the power-law effect inherited from flat space.

DOI: 10.1103/PhysRevD.103.105022

I. INTRODUCTION

One of the most challenging problems of inflationary cosmology is to reliably quantify the large logarithms that come from graviton loop corrections. This is necessary in order to understand how quantum gravity affects matter in inflation. For example, graviton loop corrections to the vacuum polarization $i[{}^\mu\Pi^\nu](x;x')$ change the propagation of dynamical photons, and electromagnetic forces, through the quantum-corrected Maxwell equation,

$$\partial_\nu[\sqrt{-g}g^{\nu\rho}g^{\mu\sigma}F_{\rho\sigma}(x)] + \int d^4x'[{}^\mu\Pi^\nu](x;x')A_\nu(x') = J^\mu(x), \tag{1}$$

where $A_\mu(x)$ is the electromagnetic vector potential, $F_{\rho\sigma} \equiv \partial_\rho A_\sigma - \partial_\sigma A_\rho$ is the field strength tensor, $g_{\mu\nu}(x)$ is the background metric, and $J^\mu(x)$ is the current density. When Eq. (1) is solved on a de Sitter background using the one graviton loop correction to $i[{}^\mu\Pi^\nu](x;x')$ in the simplest gauge [1], the electric fields of plane wave photons experience a secular enhancement and the Coulomb force manifests a logarithmic running [2,3],

$$F_{0i}(t,\vec{x}) = F_{0i}^{\rm tree}(t,\vec{x})\left\{1 + \frac{2\hbar GH^2}{\pi}\ln(a) + O(\hbar^2G^2)\right\}, \tag{2}$$

$$\Phi(t,r) = \frac{Q}{4\pi ar}\left\{1 + \frac{\hbar G}{3\pi a^2r^2} + \frac{\hbar GH^2}{\pi}\ln(aHr) + O(\hbar^2G^2)\right\}, \tag{3}$$

where G is Newton's constant, $\hbar$ is the reduced Planck constant, H is the de Sitter Hubble constant, and $a(t) = e^{Ht}$ is the de Sitter scale factor. The $\hbar G/(3\pi a^2r^2)$ correction in (3) is the de Sitter analog of a well-known flat space result [4], but the order $\hbar GH^2$ logarithms in (2) and (3) are new effects due to the inflationary expansion of de Sitter. Their physical origin seems to be the tendency of redshifting real or virtual photons to acquire momentum as they scatter off the continually replenished ensemble of Hubble scale gravitons ripped out of the vacuum by inflation. Both effects grow without bound in time, and the Coulomb enhancement grows as well at large distances, leading to a breakdown of perturbation theory. This raises the fascinating possibility of significant loop corrections despite the minuscule quantum gravitational loop counting parameter $\hbar GH^2 \sim 10^{-11}$. Large logarithms have also been found for the field strengths of fermions [5–7] and gravitons [8,9], and for changes to the background geometry [10,11]. It seems inevitable that they

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occur as well in primordial perturbations, which are the principal observables of inflation [12,13].

Worries have long been expressed that the large logarithms from loops of inflationary gravitons might be artifacts of the graviton gauge or poorly chosen observables [14–19]. There are problems with invoking these arguments to deny the possibility of large logarithmic corrections [20–23], but they do highlight the importance of correctly computing the numerical coefficients. This has also been seen directly. Calculations of graviton loop corrections on a de Sitter background are so difficult that all but one of them have been made using the simplest gauge for the graviton propagator [24,25]. However, a heroic computation [26] at length produced a result for the vacuum polarization in a one-parameter family of de Sitter invariant gauges [27–29]. When this was used to solve (1) for dynamical photons, a logarithmic correction of the same form as (2) was obtained but with a different numerical coefficient [30].

Gauge dependence has long been known to afflict the effective field equations of flat space [31]. Donoghue devised a technique for purging it from exchange potentials on a flat space background [32,33]. One first computes the scattering amplitude for two particles that feel the associated force, and then solves the inverse scattering problem to reconstruct a gauge-independent potential. Applying this technique typically changes numerical coefficients but not the fact of quantum gravitational corrections. For example, Bjerrum-Bohr employed Donoghue's formalism and found that the simple gauge correction of $\frac{1}{3} \times \hbar G/\pi r^2$, which is evident in expression (3) for $H = 0$, becomes $6 \times \hbar G/\pi r^2$ in the gauge independent potential [34].

It has recently been understood how to view Donoghue's technique directly as a correction to the effective field equations, without going through the intermediate step of constructing the S-matrix [35]. This is hugely important because it can be applied even to cosmology for which the S-matrix is not an observable, if it even exists. The procedure is to write down the position space contributions to the scattering amplitude, and then remove the source and observer propagators by applying a series of identities that Donoghue derived for isolating the leading infrared phenomena [32,36]. These identities have the effect of shrinking higher-point diagrams down to two-point functions that can be viewed as corrections to the gauge-dependent one-particle-irreducible (1PI) two-point functions (such as the vacuum polarization) that appear in the linearized effective field equation. However, extending this technique to de Sitter will require considerable effort, and it is desirable from both the conceptual and the practical side to simplify the process as much as possible.

Our program consists of three parts:

(i) We first want to identify a simple system that shows large, but possibly gauge dependent, logarithms on a de Sitter background.

(ii) Then we will apply a de Sitter space adaptation of the Donoghue construction in the simple graviton gauge [24,25] to work out reliable coefficients for the large logarithms.

(iii) To explicitly demonstrate gauge independence, we plan to redo the entire analysis in a two-parameter family of generalizations to the simple gauge propagator [37].

One could perform the computation in a one-parameter family of exact generally covariant gauges [29], but that would be needlessly difficult owing to the much more complicated structure of the propagator. The graviton propagator in a two-parameter family of average generally covariant gauges has also been worked out [38], but there seems to be a topological obstacle to imposing de Sitter invariant average gauges [39]. For a discussion on older works on the graviton propagator see [29,38] and references therein.

Quantum gravitational corrections to electromagnetism are known to involve large logarithms (2) and (3) but the intricate analysis we intend would be much simpler in a scalar system. The massless, minimally coupled scalar suggests itself as a natural choice, and the one graviton loop correction to its self-mass has already been derived [40]. However, scalar plane waves are known not to acquire large logarithmic corrections [41], and the classical response to a point source is so complicated [42,43] that solving for the one-loop correction to it might be difficult.

The next most natural candidate is the *massless, conformally coupled scalar* whose one graviton loop self-mass on a de Sitter background we have recently computed [44]. Note that even though the conformal scalar is insensitive to the cosmological expansion of the conformally flat de Sitter space, the gravitons running in the loops are not conformally coupled, and thus mediate the effects of the expansion to the scalar. Previous works studying graviton loop corrections to conformal scalars [45–47] have reported a correction to the scalar mode function growing faster than the first power of the scale factor. This would constitute a huge quantum-gravitational correction, and investigating its gauge dependence would be of paramount importance. However, before embarking on the task of purging gauge dependence, we set out to check the gauge-fixed computation of [45–47] utilizing a simplified formalism, and here we report no such power-law enhancement, and no large logarithms, neither for the scalar mode function nor for the scalar point source potential, suggesting this system is not interesting for our program.

Some distinction should be drawn between the question of how quantum gravity influences matter in inflation that concerns us here and the closely related and equally important question of how quantum matter influences gravity in inflation. In the former case the issue of graviton gauge dependence appears already at leading order as the one-loop correction to the matter 1PI two-point function is built solely out of graviton propagators. On the other hand, in the latter case this issue never appears at leading order as the one-loop correction to the graviton self-energy is composed solely of matter fields.[1] Such corrections to gravity from matter loops have been studied for photons [48], as well as for minimally and conformally coupled scalars (see [49–54] and references therein).

In this paper we solve the linearized effective field equation to check for large logarithms in one-loop corrections to scalar plane waves and to the response to a point source. In Sec. II we briefly summarize our result for the self-mass [44], and use it to quantum correct the effective field equation. Sections III and IV are devoted to perturbatively solving these equations. In Sec. V we summarize our results and discuss their significance.

II. EFFECTIVE EQUATIONS OF MOTION

The tree-level Lagrangian for the system we study in four spacetime dimensions is given by

$$\mathcal{L} = \frac{R-2\Lambda}{\kappa^2}\sqrt{-g} - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{1}{12}R\phi^2\sqrt{-g}. \tag{4}$$

The first of the terms is the Einstein-Hilbert one, where $\kappa^2 = 16\pi G$ is the gravitational coupling constant, Λ is the cosmological constant, and R is the Ricci scalar; the second is the scalar kinetic term; and the third term represents the conformal coupling of the scalar to the curvature. Henceforth, we work in the natural units $\hbar = c = 1$, unless explicitly stated otherwise. The cubic and quartic interaction vertices between the scalar and the graviton are defined by expanding the metric around de Sitter space,

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + \kappa h_{\mu\nu}), \tag{5}$$

where $a(\eta) = -1/(H\eta)$ is the scale factor given in conformal time coordinate η, the constant Hubble expansion rate is denoted by H, and $h_{\mu\nu}$ is the (conformally rescaled) graviton field. Renormalizing one-loop corrections requires counterterms not already contained in (4). Apart from absorbing divergences originating from interactions [44], they also produce a finite local contribution to the one-loop effective action,

$$\Delta\mathcal{L}^{\rm loc} = \kappa^2\bigg\{-\frac{\alpha}{2}\left[\Box\phi - \frac{R\phi}{6}\right]^2\sqrt{-g} - \frac{\beta}{24}\left[\Box\phi - \frac{R\phi}{6}\right] \times \phi R\sqrt{-g} - \frac{\gamma}{24}\partial_i\phi\partial_j\phi g^{ij}\phi R\sqrt{-g} - \frac{\delta}{288}\phi^2R^2\sqrt{-g}\bigg\}. \tag{6}$$

The quantum corrections to the classical behavior of the conformal scalar in de Sitter are captured by effective field equations, which are most conveniently written for a conformally rescaled field, $\tilde{\phi} = a\phi$, since at tree level $\tilde{\phi}$ behaves as a scalar in flat space,

$$\partial^2\tilde{\phi}(x) - \int d^4x' \tilde{M}^2_R(x;x')\tilde{\phi}(x') = \tilde{J}(x). \tag{7}$$

Here $\partial^2 = -\partial_0^2 + \nabla^2$ is the flat space d'Alembertian operator, $\tilde{J} = a^3 J$ is the conformally rescaled classical source, and $\tilde{M}^2_R$ is the conformally rescaled renormalized self-mass-squared, $\tilde{M}^2_R(x;x') = (aa')^{-1}\times M^2_R(x;x')$. The retarded self-mass corresponds to the sum of the $(++)$ and $(+-)$ components that appear in the Schwinger-Keldysh formalism for nonequilibrium quantum field theory [55–62],

$$\tilde{M}^2_{\rm R}(x;x') = \tilde{M}^2_{++}(x;x') + \tilde{M}^2_{+-}(x;x'). \tag{8}$$

In Ref. [44] we reported the $(++)$ component of the renormalized one-loop self-mass, which receives contributions from diagrams in Fig. 1,

$$\begin{aligned}-i\tilde{M}^2_{++}(x;x') = {}& \kappa^2\partial^2\partial'^2\left\{\left[\frac{\ln(aa')}{96\pi^2} - \alpha\right]\frac{i\delta^4(x-x')}{aa'}\right\} + \kappa^2H^2\partial\cdot\partial'\left\{\left[\frac{19\ln(aa')}{96\pi^2} + \beta\right]i\delta^4(x-x')\right\} \\ & - \kappa^2H^2\vec{\nabla}\cdot\vec{\nabla}'\left\{\left[\frac{5\ln(aa')}{16\pi^2} + \gamma\right]i\delta^4(x-x')\right\} - \delta\kappa^2H^4a^2 i\delta^4(x-x') + \frac{\kappa^2\partial^2\partial'^2}{384\pi^4}\left(\frac{1}{aa'}\partial^2\left[\frac{\ln(\mu^2\Delta x^2_{++})}{\Delta x^2_{++}}\right]\right) \\ & - \frac{\kappa^2H^2(19\partial^4 - 18\nabla^2\partial^2)}{384\pi^4}\left[\frac{\ln(\mu^2\Delta x^2_{++})}{\Delta x^2_{++}}\right] + \frac{\kappa^2H^2\partial^2\nabla^2}{16\pi^4}\left[\frac{\frac{1}{2}\ln(\frac{1}{4}H^2\Delta x^2_{++}) + 1}{\Delta x^2_{++}}\right],\end{aligned} \tag{9}$$

where the Lorentz-invariant distance squared is

$$\Delta x^2_{++} = \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\varepsilon)^2, \tag{10}$$

[1]Strictly speaking this is true for test matter fields, while for matter fields with a classical condensate the gauge dependence issue at one loop is more complicated.

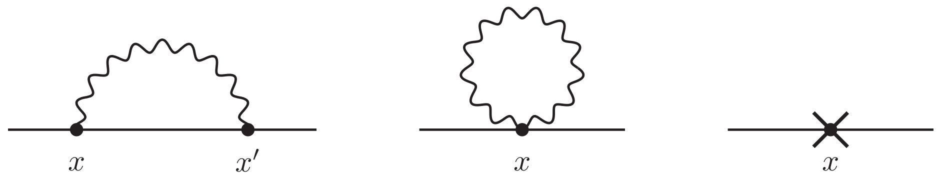

FIG. 1. One-particle-irreducible diagrams contributing to the scalar self-mass-squared at the one-loop order. The solid lines stand for the scalar and wavy ones for the graviton. The rightmost diagram stands for the counterterms.

and the physical significance of the coupling constants α, β, γ, δ can be inferred from Eq. (6). The $(+-)$ component is obtained from the $(++)$ one by (i) dropping all the local terms, (ii) substituting all Δx_{++}'s by

$$\Delta x_{+-}^2 = \|\vec{x}-\vec{x}'\|^2 - (\eta - \eta' + i\varepsilon)^2, \tag{11}$$

and (iii) appending an overall minus sign,

$$\begin{aligned} -i\tilde{M}_{+-}^2(x;x') = &-\frac{\kappa^2\partial^2\partial'^2}{384\pi^4}\left(\frac{1}{aa'}\partial^2\left[\frac{\ln(\mu^2\Delta x_{+-}^2)}{\Delta x_{+-}^2}\right]\right) \\ &+\frac{\kappa^2H^2(19\partial^4 - 18\nabla^2\partial^2)}{384\pi^4}\left[\frac{\ln(\mu^2\Delta x_{+-}^2)}{\Delta x_{+-}^2}\right] \\ &-\frac{\kappa^2H^2\partial^2\nabla^2}{16\pi^4}\left[\frac{\frac{1}{2}\ln(\frac{1}{4}H^2\Delta x_{+-}^2)+1}{\Delta x_{+-}^2}\right]. \end{aligned} \tag{12}$$

When adding (9) and (12) we make use of the two identities (that can be found in, e.g., [63]),

$$\frac{1}{\Delta x_{++}^2} - \frac{1}{\Delta x_{+-}^2} = \frac{i\pi}{2}\partial^2\theta(\Delta\eta - \|\Delta\vec{x}\|), \tag{13}$$

$$\begin{aligned} &\frac{\ln(\mu^2\Delta x_{++}^2)}{\Delta x_{++}^2} - \frac{\ln(\mu^2\Delta x_{+-}^2)}{\Delta x_{+-}^2} \\ &\quad = \frac{i\pi}{2}\partial^2\{\theta(\Delta\eta - \|\Delta\vec{x}\|)(\ln[\mu^2(\Delta\eta^2 - \|\Delta\vec{x}\|^2)] - 1)\}, \end{aligned} \tag{14}$$

where $\Delta\vec{x} = \vec{x} - \vec{x}'$ and $\Delta\eta = \eta - \eta'$ to form the retarded self-energy appearing in the effective field equations,

$$\begin{aligned} \tilde{M}_R^2(x;x') = &-\kappa^2\partial^2\partial'^2\left\{\left[\frac{\ln(aa')}{96\pi^2} - \alpha\right]\frac{\delta^4(x-x')}{aa'}\right\} - \kappa^2H^2\partial\cdot\partial'\left\{\left[\frac{19\ln(aa')}{96\pi^2} + \beta\right]\delta^4(x-x')\right\} \\ &+\kappa^2H^2\vec{\nabla}\cdot\vec{\nabla}'\left\{\left[\frac{5\ln(aa')}{16\pi^2} + \gamma\right]\delta^4(x-x')\right\} + \delta\kappa^2H^4a^2\delta^4(x-x') \\ &-\frac{\kappa^2\partial^2\partial'^2}{768\pi^3}\left\{\frac{1}{aa'}\partial^4[\theta(\Delta\eta - \|\Delta\vec{x}\|)(\ln[\mu^2(\Delta\eta^2 - \|\Delta\vec{x}\|^2)] - 1)]\right\} \\ &+\frac{\kappa^2H^2(19\partial^2 - 18\nabla^2)\partial^4}{768\pi^3}\{\theta(\Delta\eta - \|\Delta\vec{x}\|)(\ln[\mu^2(\Delta\eta^2 - \|\Delta\vec{x}\|^2)] - 1)\} \\ &-\frac{\kappa^2H^2\partial^4\nabla^2}{64\pi^3}\left\{\theta(\Delta\eta - \|\Delta\vec{x}\|)\left(\ln\left[\frac{1}{4}H^2(\Delta\eta^2 - \|\Delta\vec{x}\|^2)\right] + 1\right)\right\}. \end{aligned} \tag{15}$$

The first four terms containing a delta function we refer to as *local terms*, while the remaining three terms have support away from coincidence, and we refer to them as *nonlocal terms*.

The two physical systems we are interested in are the dynamical scalar where $\tilde{J}(x) = 0$, and the point source $\tilde{J}(x) = \delta^3(\vec{x})$. Quantum effects will modify the classical behavior. We have the self-mass-squared computed at one loop, so it only makes sense to compute the first correction to the scalar mode function,

$$\tilde{J}(\eta,\vec{x}) = 0 \Rightarrow \tilde{\phi}(\eta,\vec{x}) = [u_0(\eta,k) + \kappa^2u_1(\eta,k) + \mathcal{O}(\kappa^4)]e^{i\vec{k}\cdot\vec{x}}, \tag{16}$$

$$\begin{aligned} \tilde{J}(\eta,\vec{x}) = \delta^3(\vec{x}) \Rightarrow \tilde{\phi}(\eta,\vec{x}) \\ = \frac{-1}{4\pi\|\vec{x}\|}[1 + \kappa^2\Phi_1(\eta,\|\vec{x}\|) + \mathcal{O}(\kappa^4)], \end{aligned} \tag{17}$$

where $u_0(\eta,k) = e^{-ik\eta}$ is the tree-level mode function of the monochromatic conformally rescaled field. Solving for the quantum corrections amounts to solving

$$-\kappa^2(\partial_0^2 + k^2)u_1(\eta,k) = e^{-i\vec{k}\cdot\vec{x}}\int d^4x'\tilde{M}_R^2(x;x')e^{-ik\eta' + i\vec{k}\cdot\vec{x}'}, \tag{18}$$

$$\kappa^2 \partial^2 \left[\frac{\Phi_1(\eta, \|\vec{x}\|)}{\|\vec{x}\|}\right] = \int d^4x' \tilde{M}_R^2(x;x') \frac{1}{\|\vec{x}'\|}. \tag{19}$$

We solve these two equations in the two following sections, using the one-loop retarded self-mass from Eq. (15).

III. DYNAMICAL SCALAR

In this section we solve Eq. (18) to determine the one-loop graviton correction to the conformal scalar mode function at late times for which $a \to \infty$. It is convenient to split the source on the right-hand side into seven pieces,

$$-(\partial_0^2 + k^2) u_1(\eta, k) = \sum_{n=1}^{7} I_n(\eta, k), \tag{20}$$

where each of them corresponds to one term in the retarded one-loop self-mass (15),

$$I_1 = -\int d^4x' \partial^2 \partial'^2 \left\{ \left[\frac{\ln(aa')}{96\pi^2} - \alpha\right] \frac{\delta^4(x-x')}{aa'} \right\} e^{-ik\eta' - i\vec{k}\cdot(\vec{x}-\vec{x}')}, \tag{21}$$

$$I_2 = -\int d^4x' H^2 \partial \cdot \partial' \left\{ \left[\frac{19\ln(aa')}{96\pi^2} + \beta\right] \delta^4(x-x') \right\} e^{-ik\eta' - i\vec{k}\cdot(\vec{x}-\vec{x}')}, \tag{22}$$

$$I_3 = \int d^4x' H^2 \vec{\nabla} \cdot \vec{\nabla}' \left\{ \left[\frac{5\ln(aa')}{16\pi^2} + \gamma\right] \delta^4(x-x') \right\} e^{-ik\eta' - i\vec{k}\cdot(\vec{x}-\vec{x}')}, \tag{23}$$

$$I_4 = \int d^4x' \delta H^4 a^2 \delta^4(x-x') e^{-ik\eta' - i\vec{k}\cdot(\vec{x}-\vec{x}')}, \tag{24}$$

$$I_5 = -\frac{1}{768\pi^3} \int d^4x' \partial^2 \partial'^2 \left\{ \frac{1}{aa'} \partial^4 [\theta(\Delta\eta - \|\Delta\vec{x}\|) \times (\ln[\mu^2(\Delta\eta^2 - \|\Delta\vec{x}\|^2)] - 1)] \right\} e^{-ik\eta' - i\vec{k}\cdot(\vec{x}-\vec{x}')}, \tag{25}$$

$$I_6 = \frac{1}{768\pi^3} \int d^4x' H^2 (19\partial^2 - 18\nabla^2) \partial^2 \partial'^2 \{\theta(\Delta\eta - \|\Delta\vec{x}\|) \times (\ln[\mu^2(\Delta\eta^2 - \|\Delta\vec{x}\|^2)] - 1)\} e^{-ik\eta' - i\vec{k}\cdot(\vec{x}-\vec{x}')}, \tag{26}$$

$$I_7 = -\frac{1}{64\pi^3} \int d^4x' H^2 \nabla^2 \partial^2 \partial'^2 \left\{ \theta(\Delta\eta - \|\Delta\vec{x}\|) \times \left(\ln\left[\frac{1}{4} H^2 (\Delta\eta^2 - \|\Delta\vec{x}\|^2)\right] + 1 \right) \right\} e^{-ik\eta' - i\vec{k}\cdot(\vec{x}-\vec{x}')}. \tag{27}$$

Note that in the last two sources for convenience we have turned one ∂^2 into ∂'^2, as it acts on a function of relative coordinates only. The first four sources, descending from the local terms in the self-mass, are straightforward to evaluate:

$$I_1 = 0, \tag{28}$$

$$I_2 = \frac{19}{48\pi^2} (ikH^3 a) \times u_0(\eta, k), \tag{29}$$

$$I_3 = \left[\frac{5\ln(a)}{8\pi^2} + \gamma\right] H^2 k^2 \times u_0(\eta, k), \tag{30}$$

$$I_4 = \delta H^4 a^2 \times u_0(\eta, k). \tag{31}$$

The remaining three sources, corresponding to nonlocal terms in the self-mass, can only produce terms of the form of initial state corrections that decay in time. This is seen by integrating by parts ∂'^2 onto the classical mode function, which annihilates it. The only contributions then come from the surface terms evaluated at the initial time surface, which decay at late times,

$$I_5 = I_6 = I_7 = 0. \tag{32}$$

The contributions from the initial time surface that we have dropped one should be able to absorb into initial state corrections, in a manner analogous to what was done in Ref. [64], and are thus not dynamical effects we are interested in. They can be evaluated as was done in, e.g., [63].

The three nonvanishing sources (29)–(31) are all proportional to u_0, so it makes sense to look for the late time solution for u_1 in the form

$$u_1(\eta, k) = H^2 f(\eta, k) \times u_0(\eta, k), \tag{33}$$

so that $f(\eta, k)$ satisfies

$$\partial_0(\partial_0 - 2ik) f(\eta, k) = -\delta H^2 a^2 - \frac{19ikH}{48\pi^2} a - \frac{5k^2}{8\pi^2} \ln(a) - \gamma k^2. \tag{34}$$

Integrating once produces

$$(\partial_0 - 2ik)f(\eta,k) = -\delta Ha - \frac{19ik}{48\pi^2}\ln(a) + \frac{5k^2\ln(a)}{8\pi^2 Ha} + \left(\frac{5}{8\pi^2}+\gamma\right)\frac{k^2}{Ha} + C(k), \tag{35}$$

where $C(k)$ is an integration constant dependent on initial conditions. Inverting this first order differential equation is now straightforward,

$$f(\eta,k) \overset{a\to\infty}{\longrightarrow} -\delta\ln(a) + \bar{C}(k) + \frac{ik}{H}\left(\frac{19}{48\pi^2}+2\delta\right)\frac{\ln(a)}{a} + \mathcal{O}(1/a), \tag{36}$$

where

$$u(\eta,k) = u_0(\eta,k)\times[1+(\kappa H)^2 f(\eta,k)]. \tag{37}$$

The first and the third terms in (36) contain logarithms and represent unambiguous dynamical effects from graviton loops in de Sitter, and these are the corrections we are interested in. The second term in (36), on the other hand, does not represent a dynamical correction, but rather can be absorbed into perturbative non-Gaussian corrections of the initial state, much as in Ref. [64].

IV. POINT SOURCE

This section is devoted to solving Eq. (19) for the one-loop graviton correction to the scalar point source potential. We are interested in obtaining the solution at late times for which $a\to\infty$, after releasing the point source to interact with inflationary gravitons at the initial time $\eta_0 = -1/H$. We are interested in dynamical corrections, which propagate within the future light cone of the source which—from the point of view of a late time local observer—encompasses both sub-Hubble, and super-Hubble distances away from the point source, as illustrated in Fig. 2.

First, the source on the right-hand side of (19) needs to be computed, and we split it into seven parts,

$$\partial^2\left[\frac{\Phi_1(\eta,\|\vec{x}\|)}{\|\vec{x}\|}\right] = \sum_{n=1}^{7} K_n, \tag{38}$$

according to the seven terms in the retarded self-mass (15),

$$K_1 = -\int d^4x'\partial^2\partial'^2\left\{\left[\frac{\ln(aa')}{96\pi^2}-\alpha\right]\frac{\delta^4(x-x')}{aa'}\right\}\frac{1}{\|\vec{x}'\|}, \tag{39}$$

$$K_2 = -\int d^4x' H^2\partial\cdot\partial'\left\{\left[\frac{19\ln(aa')}{96\pi^2}+\beta\right]\delta^4(x-x')\right\}\frac{1}{\|\vec{x}'\|}, \tag{40}$$

$$K_3 = \int d^4x' H^2\vec{\nabla}\cdot\vec{\nabla}'\left\{\left[\frac{5\ln(aa')}{16\pi^2}+\gamma\right]\delta^4(x-x')\right\}\frac{1}{\|\vec{x}'\|}, \tag{41}$$

$$K_4 = \int d^4x'\delta H^4(a')^2\delta^4(x-x')\frac{1}{\|\vec{x}'\|}, \tag{42}$$

$$K_5 = \frac{-1}{768\pi^3}\int d^4x'\partial^2\partial'^2\left\{\frac{1}{aa'}\partial^4[\theta(\Delta\eta-\|\Delta\vec{x}\|)\times(\ln[\mu^2(\Delta\eta^2-\|\Delta\vec{x}\|^2)]-1)]\right\}\frac{1}{\|\vec{x}'\|}, \tag{43}$$

$$K_6 = \frac{1}{768\pi^3}\int d^4x' H^2\partial^4(19\partial'^2-18\nabla'^2)\{\theta(\Delta\eta-\|\Delta\vec{x}\|)\times(\ln[\mu^2(\Delta\eta^2-\|\Delta\vec{x}\|^2)]-1)\}\frac{1}{\|\vec{x}'\|}, \tag{44}$$

$$K_7 = \frac{-1}{64\pi^3}\int d^4x' H^2\partial^4\nabla'^2\left\{\theta(\Delta\eta-\|\Delta\vec{x}\|)\times\left(\ln\left[\frac{1}{4}H^2(\Delta\eta^2-\|\Delta\vec{x}\|^2)\right]+1\right)\right\}\frac{1}{\|\vec{x}'\|}. \tag{45}$$

In the last two integrals we have used that the derivatives act on a function of relative coordinates only to change some of them into primed ones for later convenience. Evaluating the first four source integrals is straightforward,

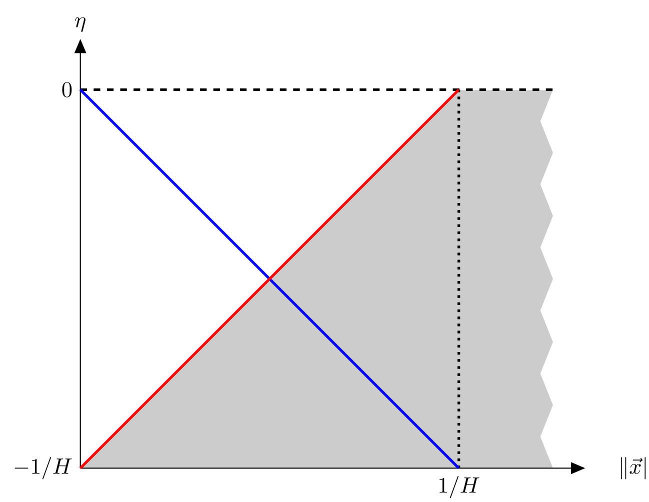

FIG. 2. Conformal diagram of the cosmological patch of de Sitter space. The system is released at time $\eta_0 = -1/H$, with a scalar point source at the origin $\vec{x} = 0$. The asymptotic future corresponds to the $\eta = 0$ slice. The red line denotes the light cone of the point source given by $(\eta-\eta_0) - \|\vec{x}\| = 0$, while the blue line denotes the Hubble distance from the source given by $aH\|\vec{x}\| = 1$, which coincides with the past particle horizon of a distant future observer at the origin. We are interested in the effects within the light cone (nonshaded region) which capture the dynamical effects of graviton loops.

$$K_1 = 4\pi\partial^2\left\{\frac{\delta^3(\vec{x})}{a^2}\left[\frac{\ln(a)}{48\pi^2} - \alpha\right]\right\}, \tag{46}$$

$$K_2 = -4\pi\delta^3(\vec{x})H^2\left[\frac{19\ln(a)}{48\pi^2} + \beta\right], \tag{47}$$

$$K_3 = 4\pi\delta^3(\vec{x})H^2\left[\frac{5\ln(a)}{8\pi^2} + \gamma\right], \tag{48}$$

$$K_4 = \frac{\delta H^4 a^2}{\|\vec{x}\|}. \tag{49}$$

For the remaining three sources it proves best to first take all the unprimed derivatives out of the integral, then to integrate by parts the remaining primed derivatives onto the classical point source potential, and to use the classical equation of motion

$$\nabla^2\frac{1}{\|\vec{x}\|} = \partial^2\frac{1}{\|\vec{x}\|} = -4\pi\delta^3(\vec{x}). \tag{50}$$

This procedure is exact for integrating ∇^2 by parts, while for ∂'^2 we drop the surface terms from the initial time surface, which decay at late times and can be absorbed into non-Gaussian corrections of the initial state [64] (the integrals corresponding to the terms we drop were computed in, e.g., [3]). The delta function allows us to integrate over the spatial coordinates, leaving single temporal integrals,

$$K_5 = \frac{\partial^2}{192\pi^2}\frac{1}{a}\partial^4\int_{-1/H}^{\eta} d\eta'\frac{1}{a'}\theta(\Delta\eta - \|\vec{x}\|)\{\ln[\mu^2(\Delta\eta^2 - \|\vec{x}\|^2)] - 1\}, \tag{51}$$

$$K_6 = -\frac{H^2\partial^4}{192\pi^2}\int_{-1/H}^{\eta} d\eta'\theta(\Delta\eta - \|\vec{x}\|)\{\ln[\mu^2(\Delta\eta^2 - \|\vec{x}\|^2)] - 1\}, \tag{52}$$

$$K_7 = \frac{H^2\partial^4}{16\pi^2}\int_{-1/H}^{\eta} d\eta'\theta(\Delta\eta - \|\vec{x}\|)\left\{\ln\left[\frac{1}{4}H^2(\Delta\eta^2 - \|\vec{x}\|^2)\right] + 1\right\}, \tag{53}$$

which are all elementary, and evaluate to

$$\begin{aligned} K_5 = \frac{\partial^2}{192\pi^2}\frac{1}{a}\partial^4\Bigg\{\theta(\Delta\eta_0 - \|\vec{x}\|)\Bigg[&H(\Delta\eta_0^2 - \|\vec{x}\|^2)\left(\frac{1}{2}\ln[\mu^2(\Delta\eta_0^2 - \|\vec{x}\|^2)] - 1\right) \\ &+ \frac{1}{a}(-2\|\vec{x}\|\ln(2\mu\|\vec{x}\|) - 3(\Delta\eta_0 - \|\vec{x}\|) + (\Delta\eta_0 - \|\vec{x}\|)\ln[\mu(\Delta\eta_0 - \|\vec{x}\|)] \\ &+ (\Delta\eta_0 + \|\vec{x}\|)\ln[\mu(\Delta\eta_0 + \|\vec{x}\|)])\Bigg]\Bigg\}, \end{aligned} \tag{54}$$

$$\begin{aligned} K_6 = -\frac{H^2\partial^4}{192\pi^2}\{&\theta(\Delta\eta_0 - \|\vec{x}\|)[-2\|\vec{x}\|\ln(2\mu\|\vec{x}\|) - 3(\Delta\eta_0 - \|\vec{x}\|) \\ &+ (\Delta\eta_0 - \|\vec{x}\|)\ln[\mu(\Delta\eta_0 - \|\vec{x}\|)] + (\Delta\eta_0 + \|\vec{x}\|)\ln[\mu(\Delta\eta_0 + \|\vec{x}\|)]]\}, \end{aligned} \tag{55}$$

$$\begin{aligned} K_7 = \frac{H^2\partial^4}{16\pi^2}\Bigg\{&\theta(\Delta\eta_0 - \|\vec{x}\|)\Bigg[-2\|\vec{x}\|\ln(H\|\vec{x}\|) - (\Delta\eta_0 - \|\vec{x}\|) \\ &+ (\Delta\eta_0 - \|\vec{x}\|)\ln\left[\frac{1}{2}H(\Delta\eta_0 - \|\vec{x}\|)\right] + (\Delta\eta_0 + \|\vec{x}\|)\ln\left[\frac{1}{2}H(\Delta\eta_0 + \|\vec{x}\|)\right]\Bigg]\Bigg\}. \end{aligned} \tag{56}$$

The final step in evaluating these is to act with all the external derivatives, except for the one ∂^2, which is useful to keep as is, since it allows us to invert the equation of motion (38) by simply dropping it. However, we must not forget that this ∂^2 still acts on a function, and it annihilates its homogeneous solutions, which yields rather simple results,

$$K_5 = \frac{\partial^2}{48\pi^2}\left[\frac{\theta(\Delta\eta_0 - \|\vec{x}\|)}{\|\vec{x}\|} \times \frac{1}{(a\|\vec{x}\|)^2}\right], \tag{57}$$

$$K_6 = \frac{H^2\partial^2}{48\pi^2}\left[\frac{\theta(\Delta\eta_0 - \|\vec{x}\|)}{\|\vec{x}\|} \times \ln(2\mu\|\vec{x}\|)\right], \tag{58}$$

$$K_7 = \frac{H^2\partial^2}{4\pi^2}\left[\frac{\theta(\Delta\eta_0 - \|\vec{x}\|)}{\|\vec{x}\|}\times(-\ln(H\|\vec{x}\|)-1)\right]. \tag{59}$$

In the expression above we did not bother to keep the terms with support only on the light cone, or outside of it, as in the late time limit the entire region of physical interest is within the light cone of the point source released to interact at $\eta_0 = -1/H$, as depicted in Fig. 2. In what follows we drop the theta function from the three sources above and explicitly focus on corrections inside the light cone.

Inverting Eq. (38) for sources (46)–(49) and (57)–(59) we just computed yields the correction to the point source potential we are after. This is trivial for sources K_1 and K_5–K_7, as it simply involves dropping the overall ∂^2 from the sources. Inverting sources K_2–K_4 is only slightly more involved. It is facilitated by noting the following two identities for d'Alembertian operators acting on spherically symmetric functions:

$$\partial^2\left[\frac{f(\eta\mp\|\vec{x}\|)}{\|\vec{x}\|}\right] = -4\pi\delta^3(\vec{x})f(\eta), \tag{60}$$

$$\partial^2\left[\frac{f(\eta)}{\|\vec{x}\|}\right] = -4\pi\delta^3(\vec{x})f(\eta) - \frac{\partial_0^2 f(\eta)}{\|\vec{x}\|}. \tag{61}$$

These are easily proven by specializing the d'Alembertian operator to functions of just η and $\|\vec{x}\|$ and then factorizing it,

$$\partial^2 = -\frac{1}{\|\vec{x}\|}\left[\partial_0 - \frac{\partial}{\partial\|\vec{x}\|}\right]\left[\partial_0 + \frac{\partial}{\partial\|\vec{x}\|}\right]\|\vec{x}\|. \tag{62}$$

The inversion for sources K_2–K_4 involves two particular identities,

$$\partial^2\left\{\frac{1}{\|\vec{x}\|}\ln[H(\|\vec{x}\|-\eta)]\right\} = 4\pi\delta^3(\vec{x})\ln(a), \tag{63}$$

$$\partial^2\left\{\frac{\ln(a)}{\|\vec{x}\|} + \frac{1}{\|\vec{x}\|}\ln[H(\|\vec{x}\|-\eta)]\right\} = -\frac{H^2a^2}{\|\vec{x}\|}. \tag{64}$$

This determines the graviton one-loop correction to the point-source potential at late times,

$$\begin{aligned}\Phi_1(\eta,\|\vec{x}\|) &= 4\pi\left[\frac{\ln(a)}{48\pi^2} - \alpha\right](a\|\vec{x}\|)\delta^3(a\vec{x}) + \frac{1}{48\pi^2(a\|\vec{x}\|)^2} \\ &\quad + \frac{H^2}{48\pi^2}\left[-48\pi^2\delta\ln(1+aH\|\vec{x}\|) + 11\ln\left(\frac{1}{a}+H\|\vec{x}\|\right)\right. \\ &\quad \left.+\ln(2\mu\|\vec{x}\|) - 12\ln(H\|\vec{x}\|) - 12 + 48\pi^2(\beta-\gamma)\right].\end{aligned} \tag{65}$$

We have determined this one-loop graviton correction to the point source potential up to homogeneous terms. However, these necessarily take the form of surface terms from the initial time surface, and thus can be absorbed into perturbative non-Gaussian initial state corrections [64]. Our result captures the dynamical effects generated by interactions that do not depend on the choice of the initial state.

V. DISCUSSION AND CONCLUSIONS

In this work we have investigated graviton loop corrections to a massless, conformally coupled scalar on a de Sitter background, with a particular emphasis on large logarithms whose gauge dependence could be the object of further study. Our main results are the plane wave scalar mode function (36) and the exchange potential (65). We discuss each in turn.

Dynamical scalar corrections. The late-time limit of a plane wave is

$$\begin{aligned}\phi(\eta,\vec{x}) = \phi_0(\eta,\vec{x})\left\{1+\hbar GH^2\left[-16\pi\delta\ln(a)\right.\right. \\ \left.\left.+\frac{ik}{H}\left(32\pi\delta+\frac{19}{3\pi}\right)\frac{\ln(a)}{a}+\text{const}\right]\right\},\end{aligned} \tag{66}$$

where $\phi_0(\eta,\vec{x}) = e^{-ik\eta+i\vec{k}\cdot\vec{x}}/a$ is the tree-level contribution, G is Newton's constant, and we have restored the reduced Planck constant $\hbar$. The large logarithm in (66) vanishes if we choose the $R^2\phi^2$ counterterm $\delta = 0$. The decaying logarithm $\ln(a)/a$ comes from the local part of the retarded self-mass-squared (15), while the constant contribution originates from both the local and the nonlocal parts. The constant contribution also depends on the choice of the initial state and for that reason cannot be fixed. The decaying logarithm does cause the time derivative of the conformally rescaled field to grow relative to its classical value, and that might be significant [65].

We should also comment on the work of Boran, Kahya, and Park who studied the same system [45–47]. Their result for the self-mass was given in Refs. [45,46], while their solution for scalar plane waves appears in Eqs. (44) and (56) of Ref. [47]. Their leading one-loop corrections are of order $a\ln(a)$ and a, and are claimed to originate from the nonlocal contributions. In contrast, the only nonlocal contributions we find come from the lower limits of temporal integrations and fall off at late time. They also claim a $\ln(a)$ enhancement from the local part of the self-mass (6) as we do, but they get it from the coupling constant γ (their $-\Delta c_4$), whereas ours comes from δ (related to their Δc_3). We are unable to account for these discrepancies but it might be relevant to note that they employed a cumbersome de Sitter invariant representation in which surface terms must be handled with great care [66]. Fröb also reported a problem with the flat space correspondence limit of their result [67].

Point source corrections. At late times the one-loop corrected exchange potential is given by Eqs. (17) and (65),

$$\begin{aligned}\phi(\eta,\vec{x}) &= \frac{-1}{4\pi ar}\Bigg\{1+\frac{\hbar G}{3\pi(ar)^2}+\frac{4\hbar G}{3}(\ln(a)-48\pi^2\alpha)(ar)\delta^3(a\vec{x}) \\ &\quad+\frac{\hbar GH^2}{3\pi}\Bigg[-48\pi^2\delta\ln(1+aHr)+11\ln\Big(\frac{1+aHr}{aHr}\Big) \\ &\quad-\ln\Big(\frac{\hbar H}{2\mu}\Big)-12+48\pi^2(\beta-\gamma)\Bigg]\Bigg\},\end{aligned} \tag{67}$$

where $r\equiv\|\vec{x}\|$. This result captures corrections from graviton loops inside the light cone of the point source, as depicted by the white region in Fig. 2. Note that the constant terms in the last line of the result above contain a part that is logarithmically dependent on the arbitrary renormalization scale μ. This term can be reinterpreted as a logarithmic running of the coupling constants $\beta-\gamma$ from Eq. (3), and could be used to cancel all the constant terms.[2]

There are two interesting regimes of (67)—the sub-Hubble regime of $ar\ll 1/H$ and the super-Hubble regime of $ar\gg 1/H$. In the sub-Hubble regime the potential reduces to

$$\begin{aligned}\phi(t,\vec{x}) &\xrightarrow{aHr\ll 1} 16\pi\hbar G\Big[\alpha-\frac{\ln(a)}{48\pi^2}\Big]\delta^3(a\vec{x}) \\ &\quad-\frac{1}{4\pi ar}\Bigg\{1+\frac{\hbar G}{3\pi a^2r^2} \\ &\quad+\frac{\hbar GH^2}{3\pi}[-11\ln(aHr)+\text{irrelevant}]\Bigg\}.\end{aligned} \tag{68}$$

[2]The running of $\beta-\gamma$ with μ is determined by the beta function, $\beta_{\beta-\gamma}=1/(48\pi^2)$.

The delta function contribution arises from the first term in (6), and the secular correction $\propto\ln(a)$ acts as a dynamical screening of α. The flat space limit $a\to 1$ and $H\to 0$ is captured by the terms in the first line of (68), which contains only conformally rescaled flat space corrections. The second line in (68) is of a purely de Sitter origin and contains a large logarithm and a constant term. The logarithm can be seen as a logarithmic anti-screening of the source. However, its effect is small compared with the conformally rescaled flat space correction.

In the super-Hubble regime the potential (67) reduces to

$$\phi(\eta,\vec{x}) \xrightarrow{aHr\gg 1}\frac{-1}{4\pi ar}\{1+16\pi\hbar GH^2[-\delta\ln(aHr)+\text{irrelevant}]\}. \tag{69}$$

The large logarithm can be eliminated by choosing $\delta=0$, which also eliminates the large logarithm in the scalar plain wave. It therefore seems that the massless, conformal scalar is not a good venue for studying the gauge dependence of large logarithms from inflationary gravitons.

ACKNOWLEDGMENTS

This work was partially supported by the Fonds de la Recherche Scientifique–FNRS under Grant No. IISN 4.4517.08–Theory of fundamental interactions; by Taiwan MOST Grant No. 108-2112-M-006-004; by the D-ITP consortium, a program of the Netherlands Organization for Scientific Research (NWO) that is funded by the Dutch Ministry of Education, Culture and Science (OCW); by NSF Grants No. PHY-1806218 and No. PHY-1912484; and by the Institute for Fundamental Theory at the University of Florida.

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