# Comparison of cosmic string and superstring models to NANOGrav 12.5-year results

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We compare the spectrum of the stochastic gravitational wave background produced in several models of cosmic strings with the common-spectrum process recently reported by NANOGrav. We discuss theoretical uncertainties in computing such a background, and show that despite such uncertainties, cosmic strings remain a good explanation for the potential signal, but the consequences for cosmic string parameters depend on the model. Superstrings could also explain the signal, but only in a restricted parameter space where their network behavior is effectively identical to that of ordinary cosmic strings.

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## I. INTRODUCTION

The NANOGrav Collaboration has recently reported some evidence of a stochastic signal in their 12.5-year data set on pulsar timing [1]. Their observation of 45 pulsars indicates the presence of a common-spectrum "red noise" process. It is unclear whether one can consider these results as a first hint of a gravitational wave background in this frequency band. In particular, the data so far show only weak evidence of a quadrupole (Hellings-Downs [2]) spatial correlation, so the NANOGrav collaboration has not claimed a detection of a gravitational wave signal yet. Further analysis is required in order to confirm this as a first observation of an stochastic gravitational wave background (SGWB). It is, however, tantalizing to consider this data seriously and ask ourselves about its possible implications for astrophysics and cosmology.

Our current understanding of galaxy evolution and merging history leads us to the idea that there should be a large number of supermassive black hole binaries (SMBHB) throughout the universe. This incoherent sum of all such SMBHB will in turn produce an SGWB. The predicted spectrum of this type of source in the nanohertz frequency band has been estimated to be close to the current limits of the Pulsar Timing Array (PTA) observatories [3]. Moreover, the frequency dependence of the spectrum is well known: the energy density in such waves is given by a power law of the form $\Omega^{\mathrm{GW}}_{\mathrm{SMBHB}} \sim f^{2/3}$. All this makes them the most likely candidate to explain a potential signal at these frequencies.

There are, however, other potential sources of gravitational waves at these frequencies which are associated with cosmological processes in the primordial universe. One of the most natural and promising sources is the stochastic background of gravitational waves created by a network of cosmic strings. Cosmic strings are effectively one-dimensional topological defects that may have been produced by a phase transition in the early universe [4,5]. We will be interested here in the simple case of Abelian-Higgs strings, or superstrings, with no couplings to any massless particle other than the graviton. Such a string network is described by a single quantity that parametrizes the characteristic energy scale of the universe at the time of string formation. This energy scale specifies the energy per unit length of the string as well as its tension, $\mu$. Since we are interested in gravitational effects, we will be most interested in the combination $G\mu$, where $G$ is Newton's constant. We will work in units where $c = 1$, so that $G\mu$ is dimensionless.

The equality between the energy per unit length of the strings and their tension implies that the dynamics of these strings are relativistic. Putting all of these facts together, one can immediately see why cosmic strings are good candidates for gravitational waves: they are cosmologically large relics that store very high energy densities associated with the early universe, and they move relativistically

 

under their own tension. This explains why an accurate computation of the SGWB from strings has been pursued for a long time in the cosmic string community [6–26].

Because string models are described by a single parameter related to the universe's energy at their time of formation, an observation of the SGWB from strings would indicate the existence of new physics at the string scale. However, the apparent simplicity of the single-parameter model is deceiving when it comes to detecting strings. The dynamics of the cosmic string network are complicated, making it difficult to obtain detailed descriptions of the necessary ingredients to compute the SGWB. One has to resort to large scale simulations to be able to establish basic facts needed in this calculation, like the number density of cosmic string loops throughout the history of the universe, or the typical power spectrum of such loops.[1] These are questions that one would have to answer in any model that produces a stochastic background: how many emitters are there, and how do they emit? Knowing this, we can estimate the combined effect of all sources.

Comparisons of some cosmic string models' predictions with the NANOGrav data have recently been made in [27–30]. We will focus here on how theoretical uncertainties in the typical power spectrum of a cosmic string loop impact the amplitude and slope of the SGWB signal in the NANOGrav window, and therefore how the most-likely $\mu$ (and the associated confidence intervals) changes due to this uncertainty. We do not suggest that a confirmed cosmic string detection would resolve this theoretical uncertainty, as that requires a better understanding of cosmic string networks and evolution.

## II. THE SGWB FROM COSMIC STRINGS

The basic idea behind the string SGWB computation is simple: for any given observational frequency, collect the contributions from all the different strings throughout the history of the universe that emit waves with the appropriate frequency such that they are observed at the observational frequency today. It is customary to present this information by calculating the critical density fraction of energy in gravitational waves per logarithmic frequency today,

$$\Omega_{\rm GW}(\ln f) = \frac{8\pi G}{3H_0^2} f \rho_{\rm GW}(t_0, f), \tag{1}$$

where $H_0$ is the Hubble parameter today, and $\rho_{\rm GW}$ denotes the energy density in gravitational waves per unit frequency.

[1]The string network contains both loops and long, horizon-spanning strings, but the contribution of long strings to the SGWB is subdominant for all $\mu$. We consider only the SGWB due to loops.

The calculation of the energy density has been described in detail in [22], and a summary can be found in the Appendix. Each loop radiates in discrete multiples $n$ of its fundamental oscillation frequency $2/l$, where $l$ is the invariant loop length, given by the loop energy divided by $\mu$. We write the power from loop $i$ in harmonic $n$ as $P_n^{(i)} G\mu^2$, so $P_n^{(i)}$ is dimensionless. We write the total radiation power $\Gamma^{(i)} G\mu^2$, where $\Gamma^{(i)} = \sum_{n=1}^{\infty} P_n^{(i)}$. For our purposes here, we will neglect differences in $\Gamma^{(i)}$ between loops and just write $\Gamma$.

The three main ingredients we need to compute the string SGWB are:

(i) A cosmological model.
(ii) The number density of non-self-intersecting loops as a function of length at any moment in time.
(iii) The average power spectrum of gravitational waves from non-self-intersecting loops in the network, $P_n$.

We consider a standard cosmological history, and take the loop number density described in [22] based on the simulations reported in [31]. This leaves the average power spectrum of non-self-intersecting loops, $P_n$. This is probably the quantity in the calculation with the highest uncertainty at this moment, since it depends not only on the gravitational radiation spectrum of non-self-intersecting loops at formation, but also on their evolution. This is a challenging problem, since one needs to follow the change in shape of a representative set of non-self-intersecting loops throughout their lifetimes; in other words, one needs to account for gravitational backreaction. Lacking this information, one can either take an ansatz for backreaction, or model the power spectrum in some theoretically motivated way which should hold true, in general, even after accounting for backreaction.

An early attempt to take gravitational backreaction into account was done in [32]. There, the authors implemented a toy model for backreaction on a large set of non-self-intersecting loops obtained from the simulations described in [31]. The idea behind this toy model was to simulate backreaction by smoothing structures on the loops at different time scales. The results of this procedure indicated that the distribution of values of the total power was peaked around $\Gamma \sim 50$, which we will take as the $\Gamma$ for all SGWB we study. We use $P_n^{\rm BOS}$ (after the authors' initials) to indicate the average power spectrum computed by this work, and will use it as one of the models we study in the following section. It is quite smooth, and has a long tail describing the emission of a substantial amount of power at the high-frequency modes of the string. This can be traced to the presence of cusps in the final stages of the evolution of these smoothed loops. The SGWB spectra arising from this model were discussed in [22].

Cusps are moments of the loop's oscillation when a point on the loop formally reaches the speed of light [33]. Cusp formation leads to the loop emitting a significant amount of radiation, which is beamed in the direction of motion of the

cusp [8].[2] Accumulating radiation from many such events forms a stochastic background whose power spectrum has a long tail, of the form $P_n^{\rm cusp} \propto n^{-4/3}$ [8]. Because cusps are thought to be generic features of loops, a common model of the power spectrum is one where low modes, which describe the shape of the loop, are less important than high modes. If we focus on these high-mode contributions to gravitational waves, then we can use a model where the spectrum is simply given by $P_n^{\rm cusp}$. We will choose a constant of proportionality so that $\sum_{n=1}^{\infty} P_n^{\rm cusp} = \Gamma$. This is the second model of $P_n$ we consider when discussing a possible string SGWB.

Another characteristic feature on realistic loops are kinks: points along the string where there is a discontinuity in its tangent vector. These occur every time two segments of string intersect one another and exchange partners. Kinks move at the speed of light along the string, emitting a fan of radiation whose spectrum at high mode number emission goes as $P_n^{\rm kink} \propto n^{-5/3}$ [35]. As with cusps, we can consider a model with only kink radiation. This is our third model.

A fourth and final model takes the reverse approach: instead of focusing on the high-harmonic tail, we consider a spectrum consisting only of the fundamental mode,

$$P_n^{\rm mono} = \begin{cases} \Gamma & \text{if } n = 1 \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

Like the pure-cusp and pure-kink spectra, this is not a realistic assumption, but it serves as a limiting case for strings which radiate primarily in low harmonics.

The real average spectrum should be calculated from a realistic distribution of non-self-intersecting loops obtained from a scaling simulation *and* evolved under their own gravity. This can be done using linearized gravity, since the force that affects each loop's shape depends on $G\mu$, which in our case is always very small. This idea was first developed in [36], and has recently been advanced both analytically [37–40] and numerically [41]. The results from these papers indicate that cusps and kinks are smoothed over time. Some of the effects of backreaction are captured by the smoothing procedure of [32], but there are cases where this approach is not so accurate. The specific results of the long-term effect of backreaction on loops produced by a scaling string network are therefore still unclear. Thus we will show the gravitational wave amplitudes and spectral slopes to be expected for all four models and compare them with the NANOGrav observations.

## III. COMPARISON WITH NANOGRAV 12.5-YEAR DATA

The NANOGrav collaboration presents their data using the characteristic strain of the form

$$h_c(f) = A\left(\frac{f}{f_{\rm yr}}\right)^{\alpha} = A\left(\frac{f}{f_{\rm yr}}\right)^{(3-\gamma)/2}, \tag{3}$$

where $f_{\rm yr} = 1/\text{year}$, $A$ is the strain amplitude, and $\gamma$ is the spectral index. The energy density in gravitational waves can be obtained from this characteristic strain using the relation

$$\Omega_{\rm GW}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f). \tag{4}$$

NANOGrav also reports likelihoods in the parameter space of $(\gamma, A)$, which we will use to construct confidence regions to use for our analysis of the effect of different $P_n$.

For a given $G\mu$ and $P_n$, we can compute the energy density in gravitational waves with Eq. (1). From this, we approximate the spectral index and amplitude using the two lowest frequencies seen in NANOGrav, $f_1 = 1/(12.5\ \text{yr})$ and $f_2 = 2f_1$. This process provides a good fit to compare to the 5-frequency contours because the two lowest frequencies in NANOGrav are much better determined in comparison to the third through fifth lowest frequencies. Our method is to calculate

$$\gamma = 5 - \frac{\ln(\Omega_{\rm GW}(f_2)/\Omega_{\rm GW}(f_1))}{\ln(2)}, \tag{5a}$$

$$A = \sqrt{\frac{3H_0^2\Omega(f_1)f_{\rm yr}^{3-\gamma}}{2\pi^2 f_1^{5-\gamma}}}, \tag{5b}$$

for each $\Omega_{\rm GW}$.

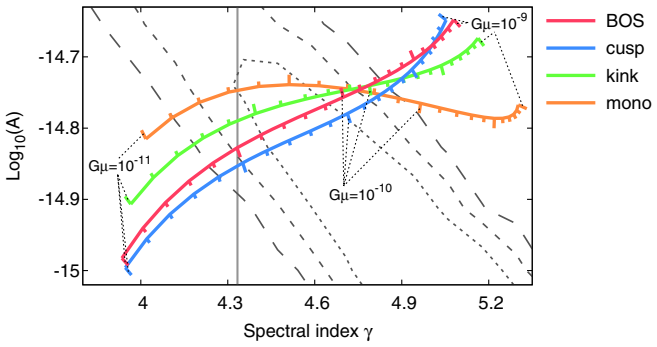


FIG. 1. The amplitude vs spectral index for various cosmic string tensions, $G\mu$, for four models of the average power spectrum, $P_n$. The tic marks show steps of 0.1 in $\log G\mu$, from $-9$ to $-11$, with large tics every 0.5. The short, medium, and long dashes show the $1\sigma$, $1.5\sigma$, and $2\sigma$ contours (i.e., they enclose 68%, 86%, and 95% of the likelihood, respectively) made from the NANOGrav 12.5-yr 5-frequency chain data [42]. We follow the process outlined in the PTA GWB Analysis tutorial [43] and then extract the contour control points directly from the resulting graphical object [44]. We use 50 bins in each direction to produce higher-resolution contours than are seen in Fig. 1 of [1]. The vertical gray line shows the spectral index to be expected from SMBHB.

[2]Cusp bursts can be sources of transient events in gravitational wave detectors. See the discussion in [12,34,35].

TABLE I. The approximate values of $\log G\mu$ falling within the $1\sigma$, $1.5\sigma$, and $2\sigma$ confidence intervals of NANOGrav for the four $P_n$ models.

| $P_n$ model | $1\sigma$ range | $1.5\sigma$ range | $2\sigma$ range |
|---|---|---|---|
| BOS | $(-10.08, -10.40)$ | $(-9.92, -10.52)$ | $(-9.77, -10.62)$ |
| cusp | $(-10.02, -10.39)$ | $(-9.85, -10.50)$ | $(-9.72, -10.60)$ |
| kink | $(-10.24, -10.52)$ | $(-10.08, -10.64)$ | $(-9.93, -10.74)$ |
| mono | $(-10.45, -10.67)$ | $(-10.27, -10.80)$ | $(-10.08, -10.90)$ |

Figure 1 shows the curves one obtains in the $(\gamma, A)$ plane for $G\mu \in [10^{-9}, 10^{-11}]$ for our four models of $P_n$. All of the models have been normalized so that the total power is given by $\Gamma = 50$. We report the approximate ranges of $\log G\mu$, which predict values within the $1\sigma$, $1.5\sigma$, and $2\sigma$ confidence range, in Table I.

The important general result of Fig. 1 is that the $(\gamma, A)$ parameters predicted by the different spectra are quite similar over the range we investigate. This means that the theoretical uncertainty in the average gravitational wave power spectrum from loops will not greatly affect the conclusions obtained from identifying the NANOGrav result with the SGWB from cosmic strings. In other words, assuming the actual spectrum of realistic loops is somewhere close to the models we study here, we can infer that the constraints on $G\mu$ are quite similar to the ones obtained from this figure. Of course, future data and analysis will likely reduce uncertainties, shrinking the range of the significance contours, and allowing us to pin down the most likely value of $G\mu$. Our ability to do that will depend on reducing our uncertainty in the loop power spectrum. This is the job of simulation, and a detection consistent with any of the above curves should not be considered evidence for that $P_n$ being the true power spectrum of loops in nature.

## IV. RELATIONSHIP TO PREVIOUS WORK

### A. Upper bounds

The authors of [23], including two of us, derived bounds on the possible values of $G\mu$ from nonobservation of a SGWB. We concentrated on the BOS model. Using results from the Parkes PTA [3,45], we gave a limit of $G\mu < 1.5 \times 10^{-11}$, and using the NANOGrav 9-year results 46]], we gave $G\mu < 4.0 \times 10^{-11}$. However, referring to Fig. 1, we see that the best fit $G\mu$ is about $5.6 \times 10^{-11}$, 40% larger than the limit based on NANOGrav and about four times the limit based on Parkes.

There are two reasons for this discrepancy. First, all pulsar timing arrays include models of individual pulsar noise. If not treated correctly, this modeling can absorb the effects of the SGWB, leading to incorrect upper bounds. This is discussed in detail in [47]. Reference [1] compares the NANOGrav red noise process detection with their previously given upper limits.

Second, pulsar timing is dependent on the solar system ephemeris, which tells us how to remove the earth's motion through the solar system from the observed data. We do not know this ephemeris to the accuracy necessary, and thus ephemeris uncertainty is an additional source of error in gravitational wave measurements. In particular, if one allows the observations to influence the choice of ephemeris, one may thereby absorb some gravitational wave power and infer incorrect limits. See [48] for more detailed discussion.

### B. Other cosmic string SGWB results

Other recent papers [27–30] have interpreted the NANOGrav 12.5-year data as a cosmic string signal. We discuss the similarities and differences between their approaches and ours here, and comment generally on agreements between those approaches.

The majority of the sources mentioned [27,28,30] employ the velocity-dependent one-scale (VOS) model for generating the cosmic string SGWB. This model has an additional parameter: the loop size at formation as a fraction of horizon size, $\alpha$, which [27] sets to 0.1 and which [28,30] allow to vary over some range.[3] Reference [29] follows the same approach as this paper. All of the aforementioned use a cusp power spectrum in creating their SGWB, and so we can only make meaningful comparisons between their results and our cusp results.

The VOS model and the one we use here are in near-exact agreement when VOS takes $\alpha = 0.1$ and one corrects for the overall energy loss into kinetic energy of the loops [49]. We would therefore expect close agreement between our results and those of [27], and between our results and those of [28,30] for $\alpha = 0.1$.[4] Reference [29] considers metastable cosmic strings, characterized by a parameter $\kappa$; we would expect their results to match ours in the limit $\kappa \to \infty$, i.e., when the decay rate of cosmic strings due to monopole–antimonopole pair production goes to zero and the strings decay only via gravitational waves.

[3]When not exploring the effect of varying $\alpha$, taking $\alpha = 0.1$ for the VOS model is a typical one, based on simulations of string networks.

[4]Note that [28,30] conclude that values of $\alpha < 0.1$ produce better fits to the NANOGrav data.

There is one additional concern in comparing different results in the $\gamma - \log A$ plane. Suppose two different approaches generate identical SGWB, so they predict the same $\Omega_{\rm GW}$ at some common reference frequency $f_{\rm ref}$, but they use different approaches to determine $\gamma$. When they extrapolate the amplitude from $f_{\rm ref}$ to $f_{\rm yr}$ to report $A$ [see Eq. (5b)], the resulting $A$ will be different. The difference in the reported logarithmic amplitude is

$$\Delta(\log(A)) = \log(A_1/A_2) = \frac{1}{2}(\gamma_2 - \gamma_1)\log\left(\frac{f_{\rm ref}}{f_{\rm yr}}\right). \quad (6)$$

Taking this effect into account, [27] draws very similar conclusions to ours as to the bounds on $G\mu$, as does [28] for the $\alpha = 0.1$ case. Reference [30] does not display their results for $\alpha = 0.1$, and so we cannot make a direct comparison. Reference [29] does not display a comparison to their results with a stable string SGWB, but their bounds on $G\mu$ as $\kappa$ increases seem to be converging towards results consistent with ours (e.g., the point with $G\mu = 10^{-10}$ and the largest $\kappa$ is on the edge of the $1\sigma$ contour).

## V. COSMIC SUPERSTRINGS

Until now, we have been discussing the gravitational spectrum produced by a network of cosmic strings that exchange partners whenever they intersect. This is the expected interaction of strings that appear as topological defects in field theory (e.g., in the Abelian-Higgs model [50,51]). There are, however, other scenarios where a network of cosmologically interesting stringlike objects is produced. In particular, many cosmological models of superstring theory suggest the production of fundamental strings, which are then stretched to cosmological size by an expanding universe [52–55]. Once stretched, these fundamental strings have similar dynamics to their classical counterparts, except for the crucial aspect that their intercommutation is different. This is due to the fact that their interactions are quantum mechanical in origin, and also because the strings in these models may move in a space with additional dimensions. Both these effects may significantly reduce their chance to intercommute. That is, the strings sometimes pass through one another, rather than splitting and rejoining to form sharply-angled kinks. This issue has been studied in [56], where the conclusion was that the probability $p$ of reconnection could be as low as $10^{-3}$.

A decrease in the intercommutation probability should have an effect on the macroscopic properties of the network. There has been some debate in the literature about how this lower probability would modify the overall density of the strings [53,57,58]. This is important to the calculation of the SGWB, since the density of loops has a direct impact on the size of $\Omega_{\rm GW}$. Large scale simulations would be necessary to establish the precise modifications

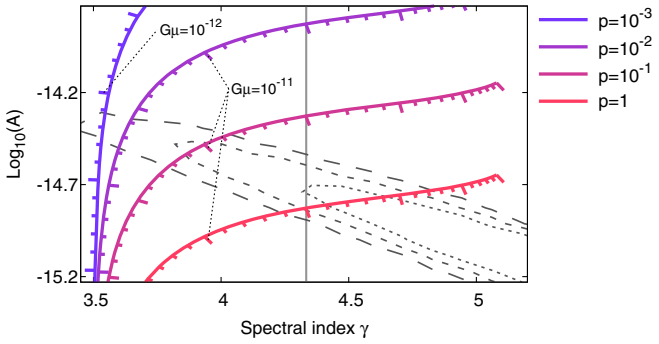


FIG. 2. The amplitude vs spectral index for various superstring tensions, $G\mu$, for the BOS model of the average power spectrum. The intercommutation probabilities go from 1 to $10^{-3}$ in powers of ten, with lower $p$ moving the curve out of NANOGrav's significance region (c.f. Fig. 1), represented by the dashed grey lines. The other models of $P_n$ return similar results. The vertical gray line shows the spectral index to be expected from SMBHB.

that this reduced probability will bring to the final scaling distribution of loops presented earlier, but they have not yet been done. Here, we will assume that the effect of reducing $p$ is to increase the loop number density by factor $1/p$, without changing the properties of the loops, so that

$$\Omega_{\rm GW} \propto \frac{1}{p}. \quad (7)$$

Lowering the intercommutation probability increases the amplitude of gravitational waves without changing the slope, and so we may estimate the range of $p$ which is compatible with the current NANOGrav data. The upward displacement of the curves in the $(\gamma, A)$ plane quickly moves them away from the $1\sigma$ region, as shown in Fig. 2, in agreement with the result of [27].

However, the current likelihood data will never completely exclude a superstring network at the $2\sigma$ level. The $1/p$ enhancement means that for small $p$ we are interested in a smaller $G\mu$. This puts us in the low-$f$ region in the cosmic string background spectrum [22], where $\Omega_{\rm GW}$ rises with frequency as $f^{3/2}$, giving $\gamma = 7/2$. For any small $p$, there will be some $G\mu$ giving the $A$ that lies in the $2\sigma$ region at the left of Fig. 2. While we only show superstrings using the BOS model of $P_n$, the $3/2$ rising slope does not depend on $P_n$, and so this effect is generally true.

Despite this, the NANOGrav data as currently given is most consistent with $p \approx 1$. As a consequence, superstrings are likely to explain the potential signal only if their network properties are very similar to those of cosmic strings.

A counterargument to this claim is the idea that strings are wiggly, and so each string crossing has multiple potential intersection events, increasing the chance that strings intercommute and thus depressing the $1/p$ enhancement to the energy density. A specific example of such an

argument can be made using the results of [58], which found the energy density to have very little enhancement down to $p \approx 0.1$, after which it follows $\Omega_{\rm GW} \propto p^{-0.6}$. This would relax the bounds on $p$ somewhat, allowing superstrings down to $p \sim 10^{-2}$ to fall at the edge of the $1.5\sigma$ region, roughly where $p = 10^{-1}$ lies in Fig. 2.

Our conclusions about superstring viability change slightly if improved statistics moves the confidence interval contours towards the left, towards the predicted SMBHB signal's vertical line at $\gamma = 13/3$. There, an enhancement to the amplitude due to $p \sim 0.1$ would cause the string curves presented to overlap with the SMBHB signal around $G\mu \lesssim 10^{-10.5}$. It may therefore be necessary to distinguish a superstring SGWB from a supermassive black hole binary SGWB. Because we expect the number of cosmic strings or superstrings that contribute to the SGWB to be large in the frequency band seen by NANOGrav, this could be accomplished by studying anisotropies in the reported signal, which we would not expect if strings are the source.

## VI. CONCLUSIONS

Regardless of the model chosen to represent the average power spectrum of a cosmic string loop, the potential signal reported by NANOGrav could be a cosmic string stochastic gravitational wave background. Thus, as long as these models are close to the true average power spectrum, a confirmation of a cosmic-string signal would predict the existence of a network of strings with a tension in the range of $G\mu \approx [10^{-10.0}, 10^{-10.7}]$. Such $G\mu$ values are low enough that we would not expect such strings to be visible in the cosmic microwave background [59] or to produce gravitational wave bursts that can be seen in interferometers [60][5] or pulsar timing arrays [62].

Superstrings are less favorable as an explanation for the signal. They would either have to have very similar network properties to cosmic strings, due to $p \approx 1$, or would have to be rescued by changes to the confidence interval contours.

If the signal is indeed from cosmic strings, then we can expect to see other parts of the SGWB in future gravitational wave telescopes. The values of $G\mu$ we consider are too low for LIGO/VIRGO to observe the SGWB [61],[6] but LISA, the Einstein Telescope, or the BBO are sensitive in the correct frequency and amplitude range. In LISA, for example, we could measure the section of the SGWB which contains information about cosmological history, particularly the effect of changing degrees of freedom [22,49,64,65], as shown in Fig. 3. Such a measurement could be used to quantify deviations from the standard model and thus probe new physics.

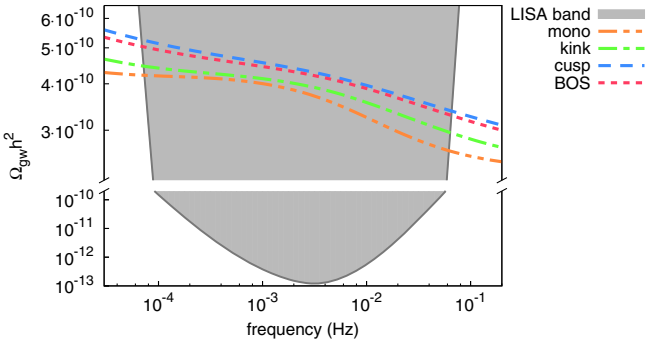


FIG. 3. The energy density vs frequency of the SGWB for the four $P_n$ models we consider, as seen in the LISA band. All curves are for $G\mu = 10^{-10}$, but the range of tensions which fit the NANOGrav data produce similar results. The decline in the lines, and its variation, are direct consequences of changing degrees of freedom in the universe's past, and so LISA could measure deviations from a standard cosmological model for such curves [65].

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## APPENDIX: COMPUTING THE GRAVITATIONAL WAVE ENERGY DENSITY

Fundamental to Eq. (1) is $\rho_{\rm GW}$, the energy density in gravitational waves per unit frequency. It can be written

$$\rho_{\rm GW}(t,f) = G\mu^2 \sum_{n=1}^{\infty} C_n P_n, \tag{A1}$$

where $P_n$ describes the average gravitational wave spectrum of the cosmic string loops in the network and

$$\begin{aligned} C_n &= \int_0^{t_0} \frac{dt}{(1+z)^5} \frac{2n}{f^2} \mathsf{n}(l,t) \\ &= \frac{2n}{f^2} \int_0^{\infty} \frac{dz}{H(z)(1+z)^6} \mathsf{n}\left(\frac{2n}{(1+z)f}, t(z)\right), \end{aligned} \tag{A2}$$

where $\mathsf{n}(l,t)$ is the loop number density, $H(z)$ is the Hubble parameter and $t(z)$ the age of the universe at redshift $z$. We consider a standard cosmological history, so $H(z)$ and $t(z)$ are given by the usual expressions in terms of the components of the universe, $\Omega_r$, $\Omega_m$, and $\Omega_\Lambda$, as well as

[5]See also the results presented for model A in [61].

[6]Here we only consider model A in [61]. See [63] for a critical discussion of the viability of other models presented in this reference.

the number of degrees of freedom at each moment in time (see [22] for a detailed explanation of these functions).

Finding the form of $\mathsf{n}(l,t)$, is equivalent to finding the distribution of non-self-intersecting loops at all times in the history of the universe. This sounds like a challenging problem since it will be impossible to simulate the evolution of the network for such a wide range of time scales. Luckily for us the evolution of a cosmic string network has a *scaling solution*, where the energy density of the string remains a small fraction of the background energy density of the universe. This is an important property of the model since it makes cosmological string networks compatible with observations. There is a more important aspect of this scaling solution for our calculation: in a scaling solution, the form of the loop distribution satisfies

$$\mathsf{n}(l,t) = t^{-4}\mathsf{n}(x), \tag{A3}$$

where $x = l/t$ is the ratio of the loop size to the age of the universe at some particular time, and $\mathsf{n}(x)$ is the number of loops per unit $x$ in a volume $t^3$. The scaling solution simplifies the problem, reducing it to finding $\mathsf{n}(x)$. Finding this scaling solution from numerical simulations presents a big challenge, since one has to run for extremely long periods of time before reaching a true scaling solution for the loop distribution.[7] Here, we use the results of the Nambu-Goto simulations presented in [31] and analyzed in [20], which allow us to write the distribution for loops as

$$\mathsf{n}_r(l,t) = \frac{0.18}{t^{3/2}(l+\Gamma G\mu t)^{5/2}} \tag{A4}$$

for loops existing in the radiation era. Some of these loops will survive until the matter era, when they will contribute to the number of loops as

$$\mathsf{n}_{rm}(l,t) = \frac{0.18(2\sqrt{\Omega_r})^{3/2}}{(l+\Gamma G\mu t)^{5/2}}(1+z)^3. \tag{A5}$$

Finally, loops produced in the matter era contribute as

$$\mathsf{n}_m(l,t) = \frac{0.27 - 0.45(l/t)^{0.31}}{t^2(l+\Gamma G\mu t)^2}, \tag{A6}$$

for $l < 0.18t$, but these make no significant contribution to the gravitational wave spectrum today.

We note that these expressions depend on the parameter $\Gamma$, which describes the average total power of gravitational radiation emitted by the population of non-self-intersecting loops. The emission of energy into gravitational waves reduces the length of the loop according to

$$l = l_0 - \Gamma G\mu(t - t_0), \tag{A7}$$

which is why the previous expressions depend on $\Gamma$.

The final ingredient, $P_n$, is discussed in the main text.

[7] See, for example, [66] for a discussion of the existence of transient solutions early in a simulation.


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