Dictionary for the type II nongeometric flux compactifications

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(Received 18 May 2020; accepted 22 February 2021; published 15 April 2021)

We study the T-dual completion of the four-dimensional $\mathcal{N}=1$ type II effective potentials in the presence of (non)geometric fluxes. First, we invoke a cohomology version of the T-dual transformations among the various moduli, axions, and fluxes appearing in the type IIA and type IIB effective supergravities. This leads to some useful observations about a significant mixing of the standard NS-NS fluxes with the (non)geometric fluxes on the mirror side. Further, using our T-duality rules, we establish an explicit mapping among the F terms, D terms, tadpole conditions, and Bianchi identities of the two theories. Second, we propose what we call a set of "axionic flux polynomials," which depend on all of the axionic moduli and the fluxes. This subsequently helps to present the two scalar potentials in a concise and manifestly T-dual form, which can be directly utilized for various phenomenological purposes, as we illustrate in a couple of examples.

DOI: 10.1103/PhysRevD.103.086009

I. INTRODUCTION

The study of four-dimensional (4D) effective potentials arising from type II flux compactifications has been one of the most active research areas and it has received a tremendous amount of attention for more than a decade, especially in the context of moduli stabilization [1–7]. In this regard, nongeometric flux compactification has emerged as an interesting playground for model builders [8–22]. The existence of nongeometric fluxes is rooted in a successive application of T duality on the three-form H flux of the type II supergravities, where a chain with geometric and nongeometric fluxes appears in the following manner [23]:

$$H_{ijk} \to \omega_{ij}{}^{k} \to Q_i{}^{jk} \to R^{ijk}. \qquad (1.1)$$

In addition, S-duality invariance of the type IIB superstring compactifications requires an additional flux, the so-called P flux, which is S dual to the nongeometric Q flux [24–29]. Generically, such fluxes can appear as parameters in the four-dimensional effective theories, and subsequently can help in developing a suitable scalar potential for the various moduli and the axions. A consistent incorporation of various such fluxes makes the compactification background richer and more flexible for model building. In this regard, continuous progress has been made regarding various phenomenological aspects, such as moduli stabilization [9,22,30–33], constructing de Sitter vacua [10,11,16,17,19,34], and the realization of the minimal aspects of inflationary cosmology [18,20,35,36]. Moreover, interesting connections among the toolkits of superstring flux compactifications, gauged supergravities, and double field theory (DFT) via nongeometric fluxes have provided a platform for approaching phenomenology-based goals from these three directions [8,14,15,23,30,37–48].

In the conventional approach of studying four-dimensional type II effective theories in a nongeometric flux compactification framework, most studies have centered around toroidal examples, in particular, with a $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ orientifold. A simple justification for this lies in the relatively simpler structure needed to perform explicit computations, which have led toroidal setups to serve as promising toolkits in studying concrete examples. However, some interesting recent studies in Refs. [20,32,34,34,36,49,50] regarding formal developments as well as applications towards moduli stabilization, searching de Sitter vacua, and building inflationary models have increased the interest in setups beyond toroidal examples, e.g., in compactifications using Calabi-Yau (CY) threefolds. As the explicit form of the metric for a CY threefold is not known, when studying the ten-dimensional origin of the 4D effective scalar potential one should represent it in a framework where one can bypass the need to know the Calabi-Yau metric. In this regard, the close connections between the symplectic geometry and effective potentials of type II supergravities [51–53] are crucial. For example, in the context of type IIB orientifolds in the presence of the standard NS-NS three-form flux (H_3) and Ramond-Ramond (RR) three-form flux (F_3), the two scalar potentials—one arising from the F-term contributions and

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the other from the dimensional reduction of the ten-dimensional kinetic pieces—can be matched via merely using the period matrices and without the need of knowing the CY metric [53,54]. Similarly, an extensive study of the effective actions in the symplectic formulation was done for both the type IIA and type IIB flux compactifications in the presence of standard fluxes using Calabi-Yau threefolds and their orientifolds [55–57].

In the context of nongeometric flux compactifications, there has been great effort in studying the 4D effective potentials derived from the Kähler and super potentials [9,16–19,58–61], while their ten-dimensional origin was later explored via DFT [43,62,63] and in supergravity theories [44–46,58,59,61,64–66]. In this regard, the symplectic approach of Refs. [53,54] for the standard type IIB flux compactification with the H_3/F_3 fluxes was recently generalized by taking several iterative steps, i.e., by including the nongeometric Q flux [67] and subsequently providing its S-dual completion by adding the nongeometric P flux [68]. In the meantime, a very robust analysis was performed by considering the DFT reduction on the CY threefolds, and subsequently the generic $\mathcal{N} = 2$ results were used to derive the $\mathcal{N} = 1$ type IIB effective potential with nongeometric fluxes [63]. An explicit connection between this DFT reduction formulation and the direct symplectic approaches of computing the scalar potential using the superpotential was presented in Ref. [67] for type IIB and in Ref. [69] for type IIA nongeometric scenarios.

Motivation and goals—The crucial importance of the nongeometric flux compactification scenarios can be illustrated by the fact that, generically speaking, one can stabilize all moduli by the tree-level effects; for example, this also includes the Kähler moduli in the type IIB framework which, in conventional flux compactifications, are protected by the so-called "no-scale structure." However, the complexity of introducing many flux parameters not only facilitates the possibility of obtaining easier samplings to fit the values, but also backreacts on the overall strategy itself in the sense that it creates some inevitable challenges, which can sometimes make the situation even worse. For example, the four-dimensional scalar potentials realized in concrete models, such as those obtained using the type IIA/IIB setups with $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ toroidal orientifolds, are very often so huge that it becomes difficult to analytically solve the extremization conditions, and one has to either look for a simplified ansatz by switching off certain flux components at a time, or opt for some highly involved numerical analysis [16,18,19,24–26]. In our opinion, this obstacle can be overcome if one can find some concise formulation of the scalar potential. Usually the convention is to start with a flux superpotential with several terms, and so it is natural to anticipate that the numerical computation will result in a complicated scalar potential with no guaranteed hierarchy among the various terms, and thus it would be hard to do anything analytically at that level. Along these lines, we aim to provide a concise and concrete formulation of the scalar potentials of the two theories with a sense of distinctness between the axionic and saxionic sectors, along with a manifestation of the T duality between them.[1] The details of our goals are as follows:

(1) The T-dual completions of type II effective theories obtained by including the (non)geometric fluxes were studied in the toroidal context in Refs. [10,11,30, 71–73]; however, a concrete connection between the (non)geometric ingredients of the two theories is still missing in the beyond-toroidal case. Although a couple of interesting efforts have been initiated along these lines [57,74], albeit without a full understanding of the T duality at the level of NS-NS nongeometric flux components and the two scalar potentials; we attempt to fill this gap.

(2) We present a cohomology version of the T-duality rules between the type IIA and type IIB theories, which subsequently enables us to read off the T-dual ingredients of one theory from those of the other and vice-versa.. This includes fluxes, moduli, axions, F/D terms, tadpole cancellation conditions, and the NS-NS Bianchi identities.

(3) To extend out understanding of the T-dual mapping from the level of the flux superpotential and D terms to the level of the total scalar potential, we invoke some interesting flux combinations with axions, which we call "axionic flux polynomials," which are useful for writing down the full scalar potential in a few lines! Recalling the difficulty in moduli stabilization and subsequent phenomenology given that the toroidal model has around 2000 terms, it is remarkable that the form of the generic scalar potential for the two theories can be so compact.

(4) With the above step, we present the generic formulation of the type IIA and type IIB scalar potentials, which can be explicitly written for a particular compactification by merely knowing (some of) the topological data (such as Hodge numbers and intersection numbers) of the compactifying (CY) threefolds and their mirrors.

(5) We collect the T-duality rules for the fluxes, moduli, scalar potentials, and Bianchi identities in a concise dictionary in the form of six tables, which present a one-to-one mapping between the various ingredients of the type IIA and type IIB theories.

The article is organized as follows. In Sec. II we provide the basic ingredients for the nongeometric type II flux

[1] In this article we consider type II compactifications using nonrigid Calabi-Yau threefolds. The study of scalar potentials arising in rigid Calabi-Yau compactifications can be found in Ref. [70].

compactifications in some detail. Section III is devoted to invoking the cohomology version of the T-duality rules and checking the consistency of the F/D terms, tadpoles conditions, and Bianchi identities. In Sec. IV we present axionic flux polynomials and a concise form of the scalar potentials for the two theories, which are manifestly T dual to each other. Section V presents the illustration of the scalar potential formulation for two particular examples using toroidal orientifolds, which subsequently also ensures the T-duality checks. Section VI includes a summary and outlook. In the Appendix we provide a T-dual dictionary in the form of six tables, namely, Tables VII–XII, which can be used to read off the relevant T-dual details of the two type II theories.

II. NONGEOMETRIC FLUX COMPACTIFICATIONS: PRELIMINARIES

In this section we review the relevant pieces of information regarding the type IIA and type IIB orientifold setups with the presence of (non)geometric fluxes, in addition to the usual NS-NS and RR fluxes. In this regard, we also revisit several standard techniques for setting up a consistent notation in order to fix any possible conflicts in conventions, signs, or factors.

Considering the bosonic sector of $\mathcal{N}=1$ supergravity theory with one gravity multiplet, a set of complex scalars $\varphi^{\mathcal{A}}$, and a set of vectors A^α, the effective action can be given as [55]

$$S^{(4)}=-\int_{M_4}\left(-\frac{1}{2}R*1+K_{\mathcal{A}\bar{\mathcal{B}}}d\varphi^{\mathcal{A}}\wedge *d\bar{\varphi}^{\bar{\mathcal{B}}}+V*1\right)+\frac{1}{2}(\mathrm{Re}f_g)_{\alpha\beta}F^\alpha\wedge *F^\beta+\frac{1}{2}(\mathrm{Im}f_g)_{\alpha\beta}F^\alpha\wedge F^\beta, \qquad (2.1)$$

where $*$ is the four-dimensional Hodge star and $F^\alpha=dA^\alpha$. There are three main ingredients—namely, the Kähler potential (K), the superpotential (W), and the holomorphic gauge kinetic function (f_g)—for determining the four-dimensional scalar potential (V) appearing in the above generic action. In fact, the total scalar potential can be simply expressed as a sum of F-term and D-term contributions as

$$V\equiv V_F+V_D, \qquad (2.2)$$

where

$$V_F=e^K(K^{\mathcal{A}\bar{\mathcal{B}}}D_{\mathcal{A}}W\bar{D}_{\bar{\mathcal{B}}}\bar{W}-3|W|^2),$$
$$V_D=\frac{1}{2}(\mathrm{Re}f_g)^{\alpha\beta}D_\alpha D_\beta.$$

Note that the sum in the piece V_F is over "all" of the moduli, the covariant derivative is defined through the relation $D_{\mathcal{A}}W=d_{\mathcal{A}}W+W\partial_{\mathcal{A}}K$, and D_α is the D term for the $U(1)$ gauge group corresponding to A^α,

$$D_\alpha=(\partial_{\mathcal{A}}K)(\mathcal{T}_\alpha)^{\mathcal{A}}{}_{\mathcal{B}}\varphi^{\mathcal{B}}+\zeta_\alpha, \qquad (2.3)$$

where $\mathcal{T}_\alpha$ is the generator of the gauge group and ζ_α denotes the Fayet-Iliopoulos term. Now we come to the two specific $\mathcal{N}=1$ supergravities, namely, type IIA and type IIB, including various fluxes.

A. Nongeometric type IIA setup

We consider type IIA superstring theory compactified on an orientifold of a Calabi-Yau threefold X_3. The orientifold is constructed via modding out the CY with a discrete symmetry $\mathcal{O}$ which includes the world-sheet parity Ω_p combined with the space-time fermion number in the left-moving sector $(-1)^{F_L}$. In addition, $\mathcal{O}$ can act nontrivially on the Calabi-Yau manifold so that altogether one has

$$\mathcal{O}=\Omega_p(-1)^{F_L}\sigma, \qquad (2.4)$$

where σ is an involutive symmetry (i.e., $\sigma^2=1$) of the internal CY and acts trivially on the four flat dimensions. The massless states in the four-dimensional effective theory are in one-to-one correspondence with various involutively even/odd harmonic forms, and hence they generate the equivariant cohomology groups $H^{p,q}_\pm(X_3)$. To begin with, following the conventions of [55] we consider the representations for the various involutively even and odd harmonic forms as given in Table I. Here the dimensionalities of the bases μ_α and $\tilde{\mu}^\alpha$ are counted by the Hodge number $h^{(1,1)}_+(X_3)$, while those of the bases ν_a and $\tilde{\nu}^a$ are counted by $h^{(1,1)}_-(X_3)$. Moreover, the indices $\hat{k}$ and λ involved in the even/odd three-forms are such that summing over the same gives the total number of real harmonic three-forms, which is $2(h^{2,1}(X_3)+1)$.

The various field ingredients can be expanded in appropriate bases of the equivariant cohomologies. In order to preserve $\mathcal{N}=1$ supersymmetry, one needs the involution σ to be antiholomorphic, isometric, and acting on the Kähler form J,

$$\sigma^*(\mathrm{J})=-\mathrm{J}, \qquad (2.5)$$

which generically results in the presence of $O6$ planes. Given that the Kähler form J and the NS-NS two-form potential B_2 are odd under the involution, the same can be expanded in the odd two-form basis ν_a as

$$\mathrm{J}=t^a\nu_a, \qquad \mathrm{B}_2=-b^a\nu_a, \qquad (2.6)$$

where t^a denotes the string-frame two-cycle volume, while b^a denotes axionic moduli. This leads to the

TABLE I. Representation of various forms and their bases.

Cohomology group	$H^{(1,1)}_+$	$H^{(1,1)}_-$	$H^{(2,2)}_+$	$H^{(2,2)}_-$	$H^{(3)}_+$	$H^{(3)}_-$
Basis	μ_α	ν_a	$\tilde{\nu}^a$	$\tilde{\mu}^\alpha$	$(\alpha_{\hat{k}},\beta^\lambda)$	$(\alpha_\lambda,\beta^{\hat{k}})$

following complexified Kähler class J_c defining the chiral coordinates T^a:

$$\mathrm{J}_c = \mathrm{B}_2 + i\mathrm{J} = -\mathrm{T}^a \nu_a, \quad \text{where } \mathrm{T}^a = b^a - i t^a. \tag{2.7}$$

Similarly, the nowhere-vanishing holomorphic three-form (Ω_3) of the Calabi Yau threefold can be expanded in the three-form basis using a prepotential $\mathcal{G}^{(q)}$ of the quaternion sector in the $\mathcal{N}=2$ theory as follows:

$$\Omega_3 = \mathcal{Z}^K \alpha_K - \mathcal{G}^{(q)}_K \beta^K. \tag{2.8}$$

Now, the compatibility of the orientifold involution σ with the Calabi-Yau condition $(J \wedge J \wedge J) \propto (\Omega_3 \wedge \bar{\Omega}_3)$ demands the following condition:

$$\sigma^*(\Omega_3) = e^{2i\theta}\bar{\Omega}_3 \quad \Rightarrow \quad \mathrm{Im}(e^{-i\theta}\mathcal{Z}^K) = 0, \quad \mathrm{Re}(e^{-i\theta}\mathcal{G}^{(q)}_K) = 0. \tag{2.9}$$

In addition, note that only one of these equations is relevant due to the scale invariance of Ω_3 which is defined only up to a complex rescaling, and here we simply set θ in Eq. (2.9) to zero, which leads to $\sigma^*(\Omega_3) = \bar{\Omega}_3$ and subsequently the following relations:

$$\mathrm{Im}\mathcal{Z}^{\hat{k}} = 0, \qquad \mathrm{Re}\mathcal{G}^{(q)}_{\hat{k}} = 0, \qquad \mathrm{Re}\mathcal{Z}^{\lambda} = 0, \qquad \mathrm{Im}\mathcal{G}^{(q)}_{\lambda} = 0. \tag{2.10}$$

1. Kähler potential

The Kähler potential consists of two pieces and can be written as [55]

$$K_{\mathrm{IIA}} \equiv K^{(k)} + K^{(q)}. \tag{2.11}$$

Let us first consider the $K^{(k)}$ part which encodes the information about the moduli space of the Kähler moduli, and can be computed from a prepotential of the following type [75,76]:

$$\mathcal{G}^{(k)} = -\frac{\kappa_{abc}\mathrm{T}^a\mathrm{T}^b\mathrm{T}^c}{6\mathrm{T}^0} + \frac{1}{2}p_{ab}\mathrm{T}^a\mathrm{T}^b + p_a\mathrm{T}^a\mathrm{T}^0 - \frac{i}{2}p_0(\mathrm{T}^0)^2 + \cdots, \tag{2.12}$$

where we have ignored the nonperturbative effects by assuming the large-volume limit. Here we have introduced $\mathrm{T}^0 = 1$ as the parameter analogous to the complex structure homogeneous parameter on the mirror side. In addition, κ_{abc} denotes the classical triple intersection number determining the volume of the Calabi-Yau threefold in terms of the two-cycle volume as $\mathcal{V} = \frac{1}{6}\kappa_{abc}t^a t^b t^c$, while the pieces with p_{ab}, p_a, and p_0 correspond to the curvature corrections arising from different orders in the α' series. Although their origin from the ten-dimensional perspective is yet to be understood, the mirror symmetry arguments suggest that the three quantities p_{ab}, p_a, and p_0 are real numbers and can be defined as [77,78]

$$p_{ab} = \frac{1}{2}\int_{\mathrm{CY}} \hat{D}_a \wedge \hat{D}_b \wedge \hat{D}_b, \qquad p_a = \frac{1}{24}\int_{\mathrm{CY}} c_2(\mathrm{CY}) \wedge \hat{D}_a, \qquad p_0 = -\frac{\zeta(3)\chi(\mathrm{CY})}{8\pi^3}, \tag{2.13}$$

where $\hat{D}_a$, $c_2(\mathrm{CY})$, and $\chi(\mathrm{CY})$ denote the dual to the divisor class, the second Chern class, and the Euler characteristic of the Calabi-Yau threefold, respectively. Subsequently, the Kähler potential is given as

$$K^{(k)} \equiv -\ln[-i(\bar{T}^A\mathcal{G}^{(k)}_A - T^A\bar{\mathcal{G}}^{(k)}_A)] = -\ln(8\mathcal{V} + 2p_0) = -\ln\left(-\frac{i}{6}\kappa_{abc}(\mathrm{T}^a - \bar{\mathrm{T}}^a)(\mathrm{T}^b - \bar{\mathrm{T}}^b)(\mathrm{T}^c - \bar{\mathrm{T}}^c) + 2p_0\right). \tag{2.14}$$

The second piece $K^{(q)}$ encodes the information from the moduli space of the complex structure deformations, and to express it we start by defining a compensator field C,

$$C \equiv e^{-\varphi}e^{\frac{1}{2}K^{(cs)}_{\mathrm{IIA}} - \frac{1}{2}K^{(k)}} = e^{-D_{4\mathrm{d}}}e^{\frac{1}{2}K^{(cs)}_{\mathrm{IIA}}}, \tag{2.15}$$

where the ten-dimensional dilaton φ is related to the four-dimensional dilaton $D_{4\mathrm{d}}$ as

$$e^{D_{4\mathrm{d}}} \equiv \sqrt{8}e^{\varphi + \frac{1}{2}K_k} = \frac{e^{\varphi}}{\sqrt{\mathcal{V} + \frac{p_0}{4}}}. \tag{2.16}$$

With our normalizations, the piece $K^{(cs)}_{\mathrm{IIA}}$ can be determined from the prepotential $\mathcal{G}^{(q)}$ as

$$K^{(cs)}_{\mathrm{IIA}} = -\ln\left(-\frac{i}{8}\int_{X_3}\Omega \wedge \bar{\Omega}\right) = -\ln\left[\frac{1}{4}(\mathrm{Re}(\mathcal{Z}^{\hat{k}})\mathrm{Im}(\mathcal{G}^{(q)}_{\hat{k}}) - \mathrm{Im}(\mathcal{Z}^{\lambda})\mathrm{Re}(\mathcal{G}^{(q)}_{\lambda}))\right]. \tag{2.17}$$

Now, using the compensator C, we consider the following expansion of three-form:

$$C\Omega = \mathrm{Re}(C\mathcal{Z}^{\hat{k}})\alpha_{\hat{k}} + i\mathrm{Im}(C\mathcal{Z}^{\lambda})\alpha_{\lambda} - i\mathrm{Im}(C\mathcal{G}^{(q)}_{\hat{k}})\beta^{\hat{k}} - \mathrm{Re}(C\mathcal{G}^{(q)}_{\lambda})\beta^{\lambda}, \tag{2.18}$$

where we have used the compensated orientifold constraints given in Eq. (2.10),

$$\mathrm{Im}(C\mathcal{Z}^{\hat{k}}) = \mathrm{Re}(C\mathcal{G}^{(q)}_{\hat{k}}) = \mathrm{Re}(CZ^{\lambda}) = \mathrm{Im}(C\mathcal{G}^{(q)}_{\lambda}) = 0. \tag{2.19}$$

Using an expansion of the RR three-form that is even under the involution,

$$\mathrm{C}_3 = \xi^{\hat{k}}\alpha_{\hat{k}} - \xi_\lambda \beta^\lambda, \tag{2.20}$$

we define a complexified three-form Ω_c as

$$\begin{aligned}\Omega_c &= \mathrm{C}_3 + i\mathrm{Re}(C\Omega)\\ &= (\xi^{\hat{k}} + i\mathrm{Re}(C\mathcal{Z}^{\hat{k}}))\alpha_{\hat{k}} - (\xi_\lambda + i\mathrm{Re}(C\mathcal{G}_\lambda))\beta^\lambda\\ &\equiv \mathrm{N}^{\hat{k}}\alpha_{\hat{k}} - \mathrm{U}_\lambda\beta^\lambda.\end{aligned} \tag{2.21}$$

Here the lowest components of the $\mathcal{N}=1$ chiral superfields $\mathrm{N}^{\hat{k}}$ and U_λ are defined as follows:

$$\begin{aligned}\mathrm{N}^{\hat{k}} &\equiv \int_{X_3}\Omega_c \wedge \beta^{\hat{k}} = \xi^{\hat{k}} + i\mathrm{Re}(C\mathcal{Z}^{\hat{k}}),\\ \mathrm{U}_\lambda &\equiv \int_{X_3}\Omega_c\wedge\alpha_\lambda = \xi_\lambda + i\mathrm{Re}(C\mathcal{G}_\lambda^{(q)}).\end{aligned} \tag{2.22}$$

Now, using these pieces of information, the second part of the Kähler potential, namely, the $K^{(q)}$ piece, can be written as

$$K^{(q)} \equiv -2\ln\left[\frac{1}{4}\int_{X_3}\mathrm{Re}(C\Omega)\wedge *\mathrm{Re}(C\Omega)\right] = 4D_{4\mathrm{d}}, \tag{2.23}$$

where in the second step we have utilized the following identity:

$$\begin{aligned}&\int_{X_3}\mathrm{Re}(C\Omega)\wedge *\mathrm{Re}(C\Omega)\\ &\quad= \mathrm{Re}(C\mathcal{Z}^{\hat{k}})\mathrm{Im}(C\mathcal{G}_{\hat{k}}^{(q)}) - \mathrm{Im}(C\mathcal{Z}^\lambda)\mathrm{Re}(C\mathcal{G}_\lambda^{(q)})\\ &\quad= 4e^{-2D_{4\mathrm{d}}}.\end{aligned} \tag{2.24}$$

The above identity can be derived using the definitions of the four-dimensional dilaton $D_{4\mathrm{d}}$ through Eq. (2.16) and the $K_{\mathrm{IIA}}^{(cs)}$ given in Eq. (2.17). Moreover, the Kähler potential part $K^{(q)}$ can be further rewritten with explicit dependence on a set of special coordinates defined as

$$\mathrm{Re}(C\mathcal{Z}^0) = y^0, \quad \mathrm{Re}(C\mathcal{Z}^k) = y^k, \quad \mathrm{Im}(C\mathcal{Z}^\lambda) = y^\lambda. \tag{2.25}$$

For the explicit form of the prepotential $\mathcal{G}^{(q)}$ for the quaternion case we consider the generic expression

$$\begin{aligned}\mathcal{G}^{(q)}(\mathcal{Y}) = \frac{k_{ABC}\mathcal{Y}^A\mathcal{Y}^B\mathcal{Y}^C}{6\mathcal{Y}^0} + \frac{1}{2}\tilde{p}_{AB}\mathcal{Y}^A\mathcal{Y}^B + \tilde{p}_A\mathcal{Y}^A\mathcal{Y}^0\\ + \frac{i}{2}\tilde{p}_0(\mathcal{Y}^0)^2,\end{aligned} \tag{2.26}$$

which subsequently gives the following derivatives:

$$\begin{aligned}\partial_{\mathcal{Y}^0}\mathcal{G}^{(q)} &= -\frac{k_{ABC}\mathcal{Y}^A\mathcal{Y}^B\mathcal{Y}^C}{6(\mathcal{Y}^0)^2} + \tilde{p}_A\mathcal{Y}^A + ip_0\mathcal{Y}^0,\\ \partial_{\mathcal{Y}^A}\mathcal{G}^{(q)} &= \frac{1}{2}\frac{k_{ABC}\mathcal{Y}^B\mathcal{Y}^C}{\mathcal{Y}^0} + \tilde{p}_{AB}\mathcal{Y}^B + \tilde{p}_A\mathcal{Y}^0.\end{aligned} \tag{2.27}$$

Now, considering the identification of coordinates $\mathcal{Y}^0 = y^0$ and $\mathcal{Y}^A = \{y^k, iy^\lambda\}$, the prepotential $\mathcal{G}^{(q)}$ takes the form

$$\begin{aligned}\mathcal{G}^{(q)}(y^0, y^k, iy^\lambda) = -\frac{i}{6y^0}k_{\lambda\rho\kappa}y^\lambda y^\rho y^\kappa + \frac{i}{2y^0}\hat{k}_{\lambda km}y^\lambda y^k y^m\\ + i\tilde{p}_{k\lambda}y^k y^\lambda + i\tilde{p}_\lambda y^\lambda y^0 + \frac{i}{2}\tilde{p}_0(y^0)^2,\end{aligned} \tag{2.28}$$

and along with this we have the following expressions:

$$\begin{aligned}\mathrm{Im}(C\mathcal{G}_0^{(q)}) &= \frac{1}{6(y^0)^2}k_{\lambda\rho\kappa}y^\lambda y^\rho y^\kappa - \frac{1}{2(y^0)^2}\hat{k}_{\lambda km}y^\lambda y^k y^m\\ &\quad + \tilde{p}_\lambda y^\lambda + \tilde{p}_0 y^0,\\ \mathrm{Im}(C\mathcal{G}_k^{(q)}) &= \frac{1}{y^0}\hat{k}_{\lambda km}y^\lambda y^m + \tilde{p}_{k\lambda}y^\lambda + \tilde{p}_k y^0,\\ \mathrm{Re}(C\mathcal{G}_\lambda^{(q)}) &= -\frac{1}{2y^0}k_{\lambda\rho\kappa}y^\rho y^\kappa + \frac{1}{2y^0}\hat{k}_{\lambda km}y^k y^m + \tilde{p}_{k\lambda}y^k\\ &\quad + \tilde{p}_\lambda y^0.\end{aligned} \tag{2.29}$$

Further, we define a new set of special nonhomogeneous coordinates $z^0 = (y^0)^{-1}$, $z^k = y^k/y^0$ and $z^\lambda = y^\lambda/y^0$, and subsequently the prepotential in Eq. (2.28) simplifies as

$$\mathcal{G}^{(q)}(z^0, z^k, z^\lambda) = (z^0)^{-2}g^{(q)}(z^k, z^\lambda), \tag{2.30}$$

where $g^{(q)}(z^k, z^\lambda)$ in the special coordinates is given as

$$\begin{aligned}g^{(q)}(z^k, z^\lambda) = -\frac{i}{6}k_{\lambda\rho\kappa}z^\lambda z^\rho z^\kappa + \frac{i}{2}\hat{k}_{\lambda km}z^\lambda z^k z^m\\ + i\tilde{p}_{k\lambda}z^k z^\lambda + ip_\lambda z^\lambda + \frac{i}{2}\tilde{p}_0.\end{aligned} \tag{2.31}$$

In addition, one has the useful relations

$$\begin{aligned}\mathrm{Im}(C\mathcal{G}_0^{(q)}) &= (z^0)^{-1}\left(\frac{1}{6}k_{\lambda\rho\kappa}z^\lambda z^\rho z^\kappa - \frac{1}{2}\hat{k}_{\lambda km}z^\lambda z^k z^m\right.\\ &\quad\left. + \tilde{p}_\lambda z^\lambda + \tilde{p}_0\right),\\ \mathrm{Im}(C\mathcal{G}_k^{(q)}) &= (z^0)^{-1}(\hat{k}_{\lambda km}z^\lambda z^m + \tilde{p}_{k\lambda}z^\lambda),\\ \mathrm{Re}(C\mathcal{G}_\lambda^{(q)}) &= (z^0)^{-1}\left(-\frac{1}{2}k_{\lambda\rho\kappa}z^\rho z^\kappa + \frac{1}{2}\hat{k}_{\lambda km}z^k z^m\right.\\ &\quad\left. + \tilde{p}_{k\lambda}z^k + \tilde{p}_\lambda\right),\end{aligned} \tag{2.32}$$

which give the following explicit forms for the chiral variables:

$$\begin{aligned} \mathrm{T}^a &= b^a - i t^a, \\ \mathrm{N}^0 &= \xi^0 + i(z^0)^{-1}, \\ \mathrm{N}^k &= \xi^k + i(z^0)^{-1} z^k, \\ \mathrm{U}_\lambda &= \xi_\lambda - i(z^0)^{-1}\left(\frac{1}{2}k_{\lambda\rho\kappa}z^\rho z^\kappa - \frac{1}{2}\hat{k}_{\lambda k m}z^k z^m - \tilde{p}_{k\lambda}z^k - \tilde{p}_\lambda\right). \end{aligned} \tag{2.33}$$

Moreover, we find that $K^{(q)}$ simplifies to the following form:

$$\begin{aligned} K^{(q)} &\equiv 4D_{4d} \\ &= -2\ln\left[\frac{1}{4}(\mathrm{Re}(C\mathcal{Z}^{\hat{k}})\mathrm{Im}(C\mathcal{G}^{(q)}_{\hat{k}}) - \mathrm{Im}(C\mathcal{Z}^\lambda)\mathrm{Re}(C\mathcal{G}^{(q)}_\lambda))\right] \\ &= -4\ln(\mathrm{z}^0)^{-1} - 2\ln\left(\frac{1}{6}k_{\lambda\rho\kappa}z^\lambda z^\rho z^\kappa + \frac{\tilde{p}_0}{4}\right), \end{aligned} \tag{2.34}$$

where the various moduli z^0, z^k, z^λ implicitly depend on the variables N^0, N^k, and U_λ. Subsequently, the full Kähler potential can be collected as

$$\begin{aligned} K_{\mathrm{IIA}} = &-\ln\left(\frac{4}{3}\kappa_{abc}t^a t^b t^c + 2p_0\right) - 4\ln(\mathrm{z}^0)^{-1} \\ &- 2\ln\left(\frac{1}{6}k_{\lambda\rho\kappa}z^\lambda z^\rho z^\kappa + \frac{\tilde{p}_0}{4}\right), \end{aligned} \tag{2.35}$$

which can be thought of as a real function of the complexified moduli T^a, N^0, N^k, and U_λ. For the latter purpose, we also define $\mathcal{U} = \frac{1}{6}k_{\lambda\rho\kappa}z^\lambda z^\rho z^\kappa$ for the complex structure side, which is an analogous quantity to the overall volume $\mathcal{V}$ of the CY threefold, and subsequently the Kähler potential can also be written as

$$\begin{aligned} K_{\mathrm{IIA}} = &-\ln(8\mathcal{V} + 2p_0) - 4\ln(z^0)^{-1} \\ &- 2\ln\left(\mathcal{U} + \frac{\tilde{p}_0}{4}\right). \end{aligned} \tag{2.36}$$

Here we would like to convey to the reader that the forms and notations are being put in place while keeping the mirror symmetry arguments in mind, which will be illustrated/manifested after considering the type IIB side later on.

2. Flux superpotential

To get the generalized version of Gukov-Vafa-Witten flux superpotential [79], we need to define the twisted differential operator [23],

$$\mathrm{D} = \mathrm{d} - \mathrm{H}\wedge . - w \triangleleft . - \mathrm{Q} \triangleright . - \mathrm{R}\bullet . \tag{2.37}$$

The actions of the operators $\triangleleft$, $\triangleright$, and $\bullet$ on a p-form changes it into a $(p+1)$-, $(p-1)$-, or $(p-3)$-form, respectively, and the various flux actions can be given as [9]

$$\begin{aligned} &\mathrm{H}\wedge\alpha_{\hat{k}} = \mathrm{H}_{\hat{k}}\Phi_6, \quad \mathrm{H}\wedge\beta^\lambda = -\mathrm{H}^\lambda\Phi_6, \quad \mathrm{H}\wedge\alpha_\lambda = 0 = \mathrm{H}\wedge\beta^{\hat{k}}; \\ &w\triangleleft\alpha_{\hat{k}} = w_{a\hat{k}}\tilde{\nu}^a, \quad w\triangleleft\beta^\lambda = -w_a{}^\lambda\tilde{\nu}^a, \quad w\triangleleft\alpha_\lambda = \hat{w}_{\alpha\lambda}\tilde{\mu}^\alpha, \quad w\triangleleft\beta^{\hat{k}} = -\hat{w}_\alpha{}^{\hat{k}}\tilde{\mu}^\alpha; \\ &\mathrm{Q}\triangleright\alpha_{\hat{k}} = \mathrm{Q}^a{}_{\hat{k}}\nu_a, \quad \mathrm{Q}\triangleright\beta^\lambda = -\mathrm{Q}^{a\lambda}\nu_a, \quad \mathrm{Q}\triangleright\alpha_\lambda = \hat{\mathrm{Q}}^\alpha{}_\lambda\mu_\alpha, \quad \mathrm{Q}\triangleright\beta^{\hat{k}} = -\hat{\mathrm{Q}}^{\alpha\hat{k}}\mu_\alpha; \\ &\mathrm{R}\bullet\alpha_{\hat{k}} = \mathrm{R}_{\hat{k}}\mathbf{1}, \quad \mathrm{R}\bullet\beta^\lambda = -\mathrm{R}^\lambda\mathbf{1}, \quad \mathrm{R}\bullet\alpha_\lambda = 0 = \mathrm{R}\bullet\beta^{\hat{k}}; \\ &\mathrm{H}\wedge\mathbf{1} \equiv \mathrm{H} \equiv -\mathrm{H}^\lambda\alpha_\lambda - \mathrm{H}_{\hat{k}}\beta^{\hat{k}}; \\ &w\triangleleft\nu_a = w_a{}^\lambda\alpha_\lambda + w_{a\hat{k}}\beta^{\hat{k}}, \quad w\triangleleft\mu_\alpha = \hat{w}_\alpha{}^{\hat{k}}\alpha_{\hat{k}} + \hat{w}_{\alpha\lambda}\beta^\lambda; \\ &\mathrm{Q}\triangleright\tilde{\nu}^a = -\mathrm{Q}^{a\lambda}\alpha_\lambda - \mathrm{Q}^a{}_{\hat{k}}\beta^{\hat{k}}, \quad \mathrm{Q}\triangleright\tilde{\mu}^\alpha = -\hat{\mathrm{Q}}^{\alpha\hat{k}}\alpha_{\hat{k}} - \hat{\mathrm{Q}}^\alpha{}_\lambda\beta^\lambda; \\ &\mathrm{R}\bullet\mathbf{\Phi}_6 = \mathrm{R}^\lambda\alpha_\lambda + \mathrm{R}_{\hat{k}}\beta^{\hat{k}}. \end{aligned} \tag{2.38}$$

Further, we take the following expansion for the multiform RR fluxes F_{RR}:

$$\begin{aligned} F_{\mathrm{RR}} &\equiv F_0 + F_2 + F_4 + F_6 \\ &= m^0\mathbf{1} + m^a\nu_a + e_a\tilde{\nu}^a + e_0\Phi_6. \end{aligned} \tag{2.39}$$

Now we consider the Kähler form expansion $\mathrm{J}_c = -\mathrm{T}^a\nu_a$ to obtain the following multiform Π_{J_c}, which is analogous to the period vectors on the mirror side:

$$\Pi_{\mathrm{J}_c} = \begin{pmatrix} 1 \\ -\mathrm{T}^a\nu_a \\ (\frac{1}{2}\kappa_{abc}\mathrm{T}^a\mathrm{T}^b - \mathrm{p}_{ab}\mathrm{T}^b - p_a)\tilde{\nu}^c \\ -(\frac{1}{3!}\kappa_{abc}\mathrm{T}^a\mathrm{T}^b\mathrm{T}^c + p_a\mathrm{T}^a + ip_0)\Phi_6 \end{pmatrix}. \tag{2.40}$$

Note that usually in the absence of any α' corrections and the prepotential quantities such as p_{ab}, p_a, and p_0, we usually denote Π_{J_c} as

$$\Pi_{J_c} \equiv e^{J_c} = \mathbf{1} + J_c + \frac{1}{2} J_c \wedge J_c + \frac{1}{3!} J_c \wedge J_c \wedge J_c$$
$$= \mathbf{1} - T^a \nu_a + \frac{1}{2} \kappa_{abc} T^a T^b \tilde{\nu}^c - \frac{1}{3!} \kappa_{abc} T^a T^b T^c \Phi_6, \qquad (2.41)$$

which gets modified after including the α' corrections. Now, the generalized flux superpotential with contributions from the NS-NS and RR fluxes can be given as [9,30, 55–57,74]

$$W_{\rm IIA} \equiv W^{\rm R}_{\rm IIA} + W^{\rm NS}_{\rm IIA}$$
$$\coloneqq -\frac{1}{\sqrt{2}} \int_{X_3} \langle F_{\rm RR} + D\Omega_c, \Pi_{J_c} \rangle, \qquad (2.42)$$

where we have introduced a normalization factor of $\sqrt{2}$. Here the antisymmetric multiforms are defined through the following Mukai pairings:

$$\langle \Gamma, \Delta \rangle_{\rm even} = \Gamma_0 \wedge \Delta_6 - \Gamma_2 \wedge \Delta_4 + \Gamma_4 \wedge \Delta_2 - \Gamma_6 \wedge \Delta_0,$$
$$\langle \Gamma, \Delta \rangle_{\rm odd} = -\Gamma_1 \wedge \Delta_5 + \Gamma_3 \wedge \Delta_3 - \Gamma_5 \wedge \Delta_1, \qquad (2.43)$$

where Γ and Δ denote some even/odd multiforms. Now, utilizing the flux actions of various NS-NS and RR fluxes on various cohomology bases as given in Eqs. (2.38) and (2.39), the superpotential takes the form

$$\begin{aligned}\sqrt{2} W_{\rm IIA} = & \left[\bar{e}_0 + T^a \bar{e}_a + \frac{1}{2}\kappa_{abc} T^a T^b m^c + \frac{1}{6}\kappa_{abc} T^a T^b T^c m^0 - i p_0 m^0\right] \\ & - N^0 \left[\bar{H}_0 + T^a \bar{w}_{a0} + \frac{1}{2}\kappa_{abc} T^b T^c Q^a{}_0 + \frac{1}{6}\kappa_{abc} T^a T^b T^c R_0 - i p_0 R_0\right] \\ & - N^k \left[\bar{H}_k + T^a \bar{w}_{ak} + \frac{1}{2}\kappa_{abc} T^b T^c Q^a{}_k + \frac{1}{6}\kappa_{abc} T^a T^b T^c R_k - i p_0 R_k\right] \\ & - U_\lambda \left[\bar{H}^\lambda + T^a \bar{w}_a{}^\lambda + \frac{1}{2}\kappa_{abc} T^b T^c Q^{a\lambda} + \frac{1}{6}\kappa_{abc} T^a T^b T^c R^\lambda - i p_0 R^\lambda\right],\end{aligned} \qquad (2.44)$$

where we have introduced a shifted version of the flux parameters to absorb the effects from p_{ab} and p_a in the following manner:

$$\begin{aligned}\bar{e}_0 &= e_0 - p_a m^a, \quad & \bar{e}_a &= e_a - p_{ab} m^b + p_a m^0, \\ \bar{H}_0 &= H_0 - p_a Q^a{}_0, \quad & \bar{w}_{a0} &= w_{a0} - p_{ab} Q^b{}_0 + p_a R_0, \\ \bar{H}_k &= H_k - p_a Q^a{}_k, \quad & \bar{w}_{ak} &= w_{ak} - p_{ab} Q^b{}_k + p_a R_k, \\ \bar{H}^\lambda &= H^\lambda - p_a Q^{a\lambda}, \quad & \bar{w}_a{}^\lambda &= w_a{}^\lambda - p_{ab} Q^{b\lambda} + p_a R^\lambda.\end{aligned} \qquad (2.45)$$

Thus, we note that considering the α'-corrected prepotential of the form (2.12) consistent with the mirror symmetry arguments generically results in some rational shifts via p_{ab} and p_a for some of the conventional flux components. This was already observed for the case without nongeometric flux [75]. Usually, one does not care about the quantities p_{ab} and p_a as it is only p_0 that appears in the Kähler potential (and not p_{ab} and p_a); however, in that case, while studying phenomenology one should be careful with strictly considering the integral fluxes and using mirror symmetric arguments at the same time. In addition, we also note that the analogous prepotential for the quaternionic sector given in Eq. (2.30) leads to a slight modification in the variable U_λ, as does its mirror-symmetric counterpart on the type IIB side, as we will see later.

Utilizing the generic form of the Kähler potential (2.35) and the superpotential (2.44), the F-term contribution to the four-dimensional scalar potential $V^F_{\rm IIA}$ can be computed using Eq. (2.2), where the sum is taken over all of the T^a, N^0, N^k, and U_λ moduli.

3. Gauge kinetic couplings and the D-term effects

Let us quickly recall the D-term contribution to the scalar potential by mostly following the ideas proposed in Refs. [60,63,65]. Keeping in mind that four-dimensional vectors can generically descend from the reduction on the three-form potential C_3 while the dual four-form gauge fields can arise from the reduction on the five-form potential C_5, let us consider the following expansions of C_3 and C_5:

$$C_3 = \xi^{\hat{k}} \alpha_{\hat{k}} - \xi_\lambda \beta^\lambda + A^\alpha \mu_\alpha, \qquad C_5 = A_\alpha \tilde{\mu}^\alpha. \qquad (2.46)$$

Now considering a pair $(\gamma^\alpha, \gamma_\alpha)$ to ensure the 4D gauge transformations of the quantities (A^α, A_α), we have the following transformations:

$$A^\alpha \to A^\alpha + d\gamma^\alpha, \qquad A_\alpha \to A_\alpha + d\gamma_\alpha. \qquad (2.47)$$

Subsequently, by considering the twisted differential D given in Eq. (2.76) we find the following transformation of the RR forms:

$$
\begin{aligned}
\mathrm{C}_{\mathrm{RR}} &\equiv \mathrm{C}_1+\mathrm{C}_3+\mathrm{C}_5 \\
&= \xi^{\hat{k}}\alpha_{\hat{k}}-\xi_\lambda\beta^\lambda+A^\alpha\mu_\alpha+A_\alpha\tilde{\mu}^\alpha \\
&\to \mathrm{C}_{\mathrm{RR}}+\mathrm{D}(\gamma^\alpha\mu_\alpha+\gamma_\alpha\tilde{\mu}^\alpha) \\
&= (\xi^{\hat{k}}-\gamma^\alpha\hat{w}_\alpha{}^{\hat{k}}+\gamma_\alpha\hat{\mathrm{Q}}^{\alpha\hat{k}})\alpha_{\hat{k}}-(\xi_\lambda+\gamma^\alpha\hat{w}_{\alpha\lambda}-\gamma_\alpha\hat{\mathrm{Q}}^\alpha{}_\lambda)\beta^\lambda \\
&\quad +A^\alpha\mu_\alpha+A_\alpha\tilde{\mu}^\alpha, \qquad (2.48)
\end{aligned}
$$

where we have used the flux actions given in Eq. (2.38). Now the transformation given in Eq. (2.48) shows that the axions $\xi^{\hat{k}}$ and ξ_λ are not invariant under the gauge transformation, and this leads to the following shifts in the $\mathcal{N}=1$ coordinate $\mathrm{N}^{\hat{k}}$ and U_λ:

$$\delta\mathrm{N}^{\hat{k}}=-\gamma^\alpha\hat{w}_\alpha{}^{\hat{k}}+\gamma_\alpha\hat{\mathrm{Q}}^{\alpha\hat{k}}, \quad \delta\mathrm{U}_\lambda=\gamma^\alpha\hat{w}_{\alpha\lambda}-\gamma_\alpha\hat{\mathrm{Q}}^\alpha{}_\lambda. \qquad (2.49)$$

In particular, this implies that if we define two types of fields

$$\Xi_{\hat{k}}=e^{i\mathrm{N}^{\hat{k}}}, \qquad \Xi^\lambda=e^{i\mathrm{U}_\lambda}, \qquad (2.50)$$

then these fields $\Xi_{\hat{k}}$ and Ξ^λ are electrically charged under the gauge group $U(1)_\alpha$ with charges $(-\hat{w}_\alpha{}^{\hat{k}})$ and $(\hat{w}_{\alpha\lambda})$, respectively, and they are magnetically charged with charges $(\hat{\mathrm{Q}}^{\alpha\hat{k}})$ and $(-\hat{\mathrm{Q}}^\alpha{}_\lambda)$, respectively. Now using the type IIA Kähler potential given in Eq. (2.35) and the variables in Eq. (2.33), we derive the following Kähler derivatives:

$$
\begin{aligned}
K_{\mathrm{N}^0}&=\frac{i}{2(z^0)^{-1}}\left(1-\frac{\hat{k}_{\lambda km}z^\lambda z^k z^m}{2(\mathcal{U}+\frac{\tilde{p}_0}{4})}+\frac{3\tilde{p}_0}{4(\mathcal{U}+\frac{\tilde{p}_0}{4})}\right),\\
K_{\mathrm{N}^k}&=\frac{i\hat{k}_{\lambda km}z^\lambda z^m}{2(z^0)^{-1}(\mathcal{U}+\frac{\tilde{p}_0}{4})}, \quad K_{\mathrm{U}_\lambda}=-\frac{iz^\lambda}{2(z^0)^{-1}(\mathcal{U}+\frac{\tilde{p}_0}{4})}. \qquad (2.51)
\end{aligned}
$$

Subsequently, one can compute the following two D terms:

$$
\begin{aligned}
D_\alpha&=-i[(\partial_{\mathrm{N}^{\hat{k}}}K)\hat{w}_\alpha{}^{\hat{k}}-(\partial_{\mathrm{U}_\lambda}K)\hat{w}_{\alpha\lambda}],\\
D^\alpha&=i[(\partial_{\mathrm{N}^{\hat{k}}}K)\hat{\mathrm{Q}}^{\alpha\hat{k}}-(\partial_{\mathrm{U}_\lambda}K)\hat{\mathrm{Q}}^\alpha{}_\lambda]. \qquad (2.52)
\end{aligned}
$$

In addition, the gauge kinetic functions follow from the prepotential derivatives $\mathcal{G}^{(k)}_{\alpha\beta}$ for the T moduli written out by considering the even sector, which is written as

$$(f_g^{\mathrm{ele}})_{\alpha\beta}=-\frac{i}{2}(\hat{\kappa}_{a\alpha\beta}\mathrm{T}^a-p_{\alpha\beta}), \qquad (2.53)$$

where we also observe the presence of parameters $p_{\alpha\beta}$ which, however, will not appear in the "real" part and hence in the gauge kinetic couplings given as $\mathrm{Re}(f_g^{\mathrm{ele}})_{\alpha\beta}=-\frac{1}{2}\hat{\kappa}_{a\alpha\beta}t^a$. This leads to the following D-term contributions to the four-dimensional scalar potential:

$$V^D_{\mathrm{IIA}}=\frac{1}{2}D_\alpha[\mathrm{Re}(f_g^{\mathrm{ele}})_{\alpha\beta}]^{-1}D_\beta+\frac{1}{2}D^\alpha[\mathrm{Re}(f_g^{\mathrm{mag}})^{\alpha\beta}]^{-1}D^\beta, \qquad (2.54)$$

where the explicit expressions of the D terms given in Eq. (2.52) are

$$
\begin{aligned}
D_\alpha&=\frac{(z^0)^{-1}e^{\frac{K_q}{2}}}{2}\left[\left(\mathcal{U}+\tilde{p}_0-\frac{1}{2}\hat{k}_{\lambda km}z^\lambda z^k z^m\right)\hat{w}_\alpha{}^0\right.\\
&\quad\left.+\hat{k}_{\lambda km}z^\lambda z^m\hat{w}_\alpha{}^k+z^\lambda\hat{w}_{\alpha\lambda}\right],\\
D^\alpha&=-\frac{(z^0)^{-1}e^{\frac{K_q}{2}}}{2}\left[\left(\mathcal{U}+\tilde{p}_0-\frac{1}{2}\hat{k}_{\lambda km}z^\lambda z^k z^m\right)\hat{\mathrm{Q}}^{\alpha 0}\right.\\
&\quad\left.+\hat{k}_{\lambda km}z^\lambda z^m\hat{\mathrm{Q}}^{\alpha k}+z^\lambda\hat{\mathrm{Q}}^\alpha{}_\lambda\right]. \qquad (2.55)
\end{aligned}
$$

Here $e^{\frac{K(q)}{2}}=(z^0)^2/(\mathcal{U}+\frac{\tilde{p}_0}{4})$, and also note that $\mathrm{Re}(f_g^{\mathrm{ele}})>0$ and $\mathrm{Re}(f_g^{\mathrm{ele}})>0$ as these are related to moduli space metrics which are positive definite, and can be shown to be $V^D_{\mathrm{IIA}}\geq 0$.

4. Tadpole cancellation conditions and Bianchi identities

Generically, there are tadpole terms present due to the presence of $O6$ planes, and these can be canceled by either imposing a set of flux constraints or adding counterterms that can arise from the presence of local sources such as (stacks of) $D6$-branes. These effects equivalently provide the following contributions in the effective potential to compensate the tadpole terms [30]:

$$V^{\mathrm{tad}}_{\mathrm{IIA}}=\frac{1}{2}e^{K_q}\int_{X_3}\langle[\mathrm{Im}\Omega_c],\mathrm{D}F_{\mathrm{RR}}\rangle, \qquad (2.56)$$

where the three-form $\mathrm{D}F_{\mathrm{RR}}$ can be expanded as [69]

$$
\begin{aligned}
\mathrm{D}F_{\mathrm{RR}}&=-(\mathrm{H}^\lambda m_0-\omega_a{}^\lambda m^a+Q^{a\lambda}e_a-R^\lambda e_0)\alpha_\lambda\\
&\quad-(\mathrm{H}_{\hat{k}}m_0-\omega_{a\hat{k}}m^a+Q^a{}_{\hat{k}}e_a-R_{\hat{k}}e_0)\beta^{\hat{k}}. \qquad (2.57)
\end{aligned}
$$

Subsequently, Eq. (2.56) simplifies to the following form:

$$
\begin{aligned}
V^{\mathrm{tad}}_{\mathrm{IIA}}&=\frac{1}{2}e^{K_q}[(\mathrm{ImN}^{\hat{k}})(\mathrm{H}_{\hat{k}}m_0-\omega_{a\hat{k}}m^a+Q^a{}_{\hat{k}}e_a-R_{\hat{k}}e_0)\\
&\quad+(\mathrm{ImU}_\lambda)(\mathrm{H}^\lambda m_0-\omega_a{}^\lambda m^a+Q^{a\lambda}e_a-R^\lambda e_0)]. \qquad (2.58)
\end{aligned}
$$

In the four-dimensional type IIA effective theory, the dynamics of various moduli is determined by the total scalar potential given as a sum of the F-term and D-term contributions,

$$V^{\mathrm{tot}}_{\mathrm{IIA}}=V^F_{\mathrm{IIA}}+V^D_{\mathrm{IIA}}, \qquad (2.59)$$

where the various fluxes appearing in the scalar potential must satisfy the full set of NS-NS Bianchi identities and RR tadpole cancellation conditions.

B. Nongeometric type IIB setup

In this subsection we present the relevant details about the nongeometric type IIB orientifold setup. The allowed orientifold projections can be classified by their action $\mathcal{O}$ on the Kähler form J and the holomorphic three-form Ω_3 of the Calabi-Yau metric, which are given explicitly as [56]

$$\mathcal{O}=\begin{cases}\Omega_p\sigma & :\sigma^*(J)=J,\ \ \sigma^*(\Omega_3)=\Omega_3,\\ (-)^{F_L}\Omega_p\sigma & :\sigma^*(J)=J,\ \ \sigma^*(\Omega_3)=-\Omega_3.\end{cases} \tag{2.60}$$

Note that Ω_p is the world-sheet parity, F_L is the left-moving space-time fermion number, and σ is a holomorphic, isometric involution. The first choice leads to an orientifold with $O5/O9$ planes, whereas the second choice leads to $O3/O7$ planes.

As in the type IIA case, we denote the bases of even/odd two-forms as (μ_α,ν_a) and four-forms as $(\tilde{\mu}_\alpha,\tilde{\nu}_a)$ where $\alpha\in h^{1,1}_+(X_3)$, $a\in h^{1,1}_-(X_3)$.[2] However, for the type IIB setups we denote the bases for the even/odd cohomologies $H^3_\pm(X_3)$ of three-forms as symplectic pairs (a_K,b^J) and $(\mathcal{A}_\Lambda,\mathcal{B}^\Delta)$, respectively, where we fix their normalization as

$$\int_X a_K\wedge b^J=\delta_K{}^J,\qquad \int_X \mathcal{A}_\Lambda\wedge\mathcal{B}^\Delta=\delta_\Lambda{}^\Delta. \tag{2.61}$$

Here, for the orientifold choice with $O3/O7$ planes the indices are distributed in the even/odd sector as $K\in\{1,\ldots,h^{2,1}_+(X_3)\}$ and $\Lambda\in\{0,\ldots,h^{2,1}_-(X_3)\}$, while for $O5/O9$ planes one has $K\in\{0,\ldots,h^{2,1}_+(X_3)\}$ and $\Lambda\in\{1,\ldots,h^{2,1}_-(X_3)\}$. In this article, we only focus on the orientifold involutions leading to the $O3/O7$ planes.

The various field ingredients can be expanded in appropriate bases of the equivariant cohomologies. For example, the Kähler form J, the two-forms B_2 and C_2, and the RR four-form C_4 can be expanded as

$$\begin{aligned}&J=t^\alpha\mu_\alpha,\quad B_2=-b^a\nu_a,\\ &C_2=-c^a\nu_a,\quad C_4=c_\alpha\tilde{\mu}^\alpha+D_2^\alpha\wedge\mu_\alpha+V^K\wedge a_K-V_K\wedge b^K.\end{aligned} \tag{2.62}$$

Note that t^α are string-frame two-cycle volume moduli, while b^a, c^a, and c_α are various axions. Further, (V^K,V_K) forms a dual pair of space-time one-forms and D_2^α is a space-time two-form dual to the scalar field c_α. Also, since σ^* reflects the holomorphic three-form Ω_3, we have $h^{2,1}_-(X)$ number of complex structure moduli appearing as complex scalars.

[2] For an explicit construction of type IIB toroidal/CY orientifold setups with $h^{1,1}_-(X_3)\neq 0$, see Refs. [80–85].

1. Kähler potential

The generic form of the type IIB Kähler potential can be written as a sum of two pieces motivated from their underlying $\mathcal{N}=2$ special Kähler and quaternionic structure [56],

$$K_{\rm IIB}=K^{(\rm c.s.)}+K^{(Q)}, \tag{2.63}$$

where the $K^{(\rm c.s.)}$ piece depends mainly on the complex structure moduli, while the $K^{(Q)}$ part depends on the volume of the Calabi-Yau threefold and the dilaton. To compute the $K^{(\rm c.s.)}$ piece, we consider the involutively odd holomorphic three-form $\Omega_3\equiv\mathcal{X}^\Lambda\mathcal{A}_\Lambda-\mathcal{F}_\Lambda\mathcal{B}^\Lambda$ which can be written using a prepotential of the following form [77,86]:

$$\begin{aligned}\mathcal{F}^{(\rm c.s.)}=&-\frac{l_{ijk}\mathcal{X}^i\mathcal{X}^j\mathcal{X}^k}{6\mathcal{X}^0}+\frac{1}{2}\tilde{p}_{ij}\mathcal{X}^i\mathcal{X}^j+\tilde{p}_i\mathcal{X}^i\mathcal{X}^0\\ &-\frac{i}{2}\tilde{p}_0(\mathcal{X}^0)^2+i(\mathcal{X}^0)^2\mathcal{F}_{\rm inst.}(U^i),\end{aligned} \tag{2.64}$$

where the l_{ijk}'s are the classical triple intersection numbers on the mirror (Calabi-Yau) threefold and we have defined the inhomogeneous coordinates (U^i) as $U^i=\frac{\mathcal{X}^i}{\mathcal{X}^0}$ via further setting $\mathcal{X}^0=1$. Further, the quantities $\tilde{p}_{ij}$, $\tilde{p}_i$, and $\tilde{p}_0$ are real numbers, and moreover $\tilde{p}_0$ is related to the perturbative $(\alpha')^3$ corrections on the mirror type IIA side (as we have argued before) and so is proportional to the Euler characteristic of the mirror Calabi-Yau threefold. In general, $f(U^i)$ has an infinite series of nonperturbative contributions denoted as $\mathcal{F}_{\rm inst.}(U^i)$; however, assuming the large complex structure limit, we will ignore such corrections in the current work. The derivatives of the prepotential needed to explicitly determine the Kähler and the superpotential terms are given as

$$\begin{aligned}\mathcal{F}_0^{(\rm c.s.)}&=\frac{1}{6}l_{ijk}U^iU^jU^k+\tilde{p}_iU^i-i\tilde{p}_0,\\ \mathcal{F}_i^{(\rm c.s.)}&=-\frac{1}{2}l_{ijk}U^jU^k+\tilde{p}_{ij}U^iU^j+\tilde{p}_i.\end{aligned} \tag{2.65}$$

Subsequently, the components of the holomorphic three-form Ω_3 can be explicitly rewritten as period vectors in terms of complex structure moduli U^i,

$$\Pi_{\Omega_3}=\begin{pmatrix}\mathcal{A}_0\\ U^i\mathcal{A}_i\\ (\frac{1}{2}l_{ijk}U^jU^k-\tilde{p}_{ij}U^iU^j-\tilde{p}_i)\mathcal{B}^i\\ -(\frac{1}{6}l_{ijk}U^iU^jU^k+\tilde{p}_iU^i-i\tilde{p}_0)\mathcal{B}^0\end{pmatrix}. \tag{2.66}$$

Now the complex-structure-moduli-dependent part of the Kähler potential can be simply given as

$$\begin{aligned} K^{(\mathrm{c.s.})} &\equiv -\ln\left(-i\int_X \Pi_{\Omega_3}\wedge\bar{\Pi}_{\Omega_3}\right)\\ &= -\ln[-i(\bar{\mathcal{X}}^\Lambda\mathcal{F}_\Lambda^{(\mathrm{c.s.})}-\mathcal{X}^\Lambda\bar{\mathcal{F}}_\Lambda^{(\mathrm{c.s.})})]\\ &= -\ln\left(\frac{4}{3}l_{ijk}u^iu^ju^k+2\tilde{p}_0\right)\\ &= -\ln\left[-i\frac{l_{ijk}}{6}(U^i-\bar{U}^i)(U^j-\bar{U}^j)(U^k-\bar{U}^k)+2\tilde{p}_0\right], \end{aligned} \tag{2.67}$$

where we have used saxions/axions of the complex structure moduli via defining U^i as $U^i = v^i - iu^i$. For the Kähler potential piece $K^{(Q)}$ which arises from the quaternion sector, we consider the Kähler form expansion $\mathcal{J} = T^A\mu_A$, where μ_A denotes the (1,1)-form before orientifolding, and subsequently one can follow a similar approach as was taken for the mirror type IIA case by considering a prepotential of the following form [87]:

$$\begin{aligned} \mathcal{F}^{(q)} &= \ell_{ABC}\frac{T^AT^BT^C}{6T^0}+\frac{1}{2}p_{AB}T^AT^B+p_AT^AT^0\\ &\quad+\frac{1}{2}ip_0(T^0)^2, \end{aligned} \tag{2.68}$$

where by assuming the large-volume limit we neglect the nonperturbative effects from the world-sheet instanton correction [88]. Now we define a multiform ρ using the periods of the prepotential in the following manner:

$$\rho = 1+T^A\mu_A-\mathcal{F}_A^{(q)}\tilde{\mu}^A+(2\mathcal{F}^{(q)}-t^A\mathcal{F}_A^{(q)})\Phi_6. \tag{2.69}$$

Now, unlike the type IIA case, one can use a compensator field $\mathrm{C} = e^{-\phi}$ which does not depend on the volume, and by using the RR potential as $C_{\mathrm{RR}} = C_0 + C_2 + C_4$ we consider a complex multiform of even degree defined as [57]

$$\begin{aligned} \Phi_c^{\mathrm{even}} &\equiv e^{B_2}\wedge C_{\mathrm{RR}}^{(0)}+i\mathrm{Re}(\mathrm{C}\rho)\\ &\equiv S\mathbf{1}-G^a\nu_a+T_\alpha\tilde{\mu}^\alpha, \end{aligned} \tag{2.70}$$

where the explicit forms for the chiral coordinates in Eq. (2.70) are given as

$$\begin{aligned} S &= C_0^{(0)}+ie^{-\phi}=c_0+is,\\ G^a &= c^a+Sb^a,\\ T_\alpha &= c_\alpha+\hat{\ell}_{\alpha ab}b^ac^b+\frac{1}{2}c_0\hat{\ell}_{\alpha ab}b^ab^b\\ &\quad-is\left[\frac{1}{2}\ell_{\alpha\beta\gamma}t^\beta t^\gamma-\frac{1}{2}\hat{\ell}_{\alpha ab}b^ab^b-p_{\alpha a}b^a-p_\alpha\right], \end{aligned} \tag{2.71}$$

where we have rewritten the dilaton as $e^{-\phi} = s$ and $\{\ell_{\alpha\beta\gamma}, \hat{\ell}_{\alpha ab}\}$ represents the set of triple intersection numbers which survive under the orientifold action [67]. It is worth noting that there is a shift in the coordinates T_α due to the presence of $p_{\alpha a}$ and p_α in the prepotential $\mathcal{F}^{(q)}$, while the other variables remain the same. Now the Kähler potential can be computed in the following steps [87]:

$$\begin{aligned} K^{(Q)} &= -2\ln\left[i\int_{\mathrm{CY}}\langle \mathrm{C}\rho,\mathrm{C}\bar{\rho}\rangle\right]\\ &= -2\ln[|\mathrm{C}|^2(2(\mathcal{F}^{(q)}-\bar{\mathcal{F}}^{(q)})-(\mathcal{F}_\alpha^{(q)}+\bar{\mathcal{F}}_\alpha^{(q)})(T^\alpha-\bar{T}^\alpha))]\\ &= -4\ln s-2\ln\left(\mathcal{V}+\frac{p_0}{4}\right), \end{aligned} \tag{2.72}$$

where the overall internal volume of the CY threefold is written as $\mathcal{V} = \frac{1}{6}\ell_{\alpha\beta\gamma}t^\alpha t^\beta t^\gamma$ using the string-frame two-cycle volume moduli. Further, the string-frame $\mathcal{V}$ can be identified with the Einstein-frame volume $\mathcal{V}_E$ via $\mathcal{V}_E = s^{3/2}\mathcal{V}$. Note that this α' correction in the Kähler potential has been used to naturally realize the LARGE volume scenarios [2]. To summarize, the full type IIB Kähler potential can be given by

$$\begin{aligned} K_{\mathrm{IIB}} &= -\ln\left(\frac{4}{3}l_{ijk}u^iu^ju^k+2\tilde{p}_0\right)-4\ln s\\ &\quad-2\ln\left(\frac{1}{6}\ell_{\alpha\beta\gamma}t^\alpha t^\beta t^\gamma+\frac{p_0}{4}\right). \end{aligned} \tag{2.73}$$

Further, in order to compute the Kähler metric and its inverse for the scalar potential computations, one needs to rewrite the dilaton (s), the two-cycle volume moduli (t^α), and the complex structure saxion moduli (u^i) in terms of the correct variables S, T_α, G^a, and U^i which in the string frame are defined as

$$\begin{aligned} U^i &= v^i-iu^i,\\ S &= c_0+is,\\ G^a &= (c^a+c_0b^a)+isb^a,\\ T_\alpha &= \hat{c}_\alpha-is\left[\frac{1}{2}\ell_{\alpha\beta\gamma}t^\beta t^\gamma-\frac{1}{2}\hat{\ell}_{\alpha ab}b^ab^b-p_{\alpha a}b^a-p_\alpha\right], \end{aligned} \tag{2.74}$$

where $\hat{c}_\alpha$ represents the axionic combination $\hat{c}_\alpha = c_\alpha + \hat{\ell}_{\alpha ab}b^ac^b + \frac{1}{2}c_0\hat{\ell}_{\alpha ab}b^ab^b$.

2. *Flux superpotential*

It is important to note that in a given setup, all flux components will not be generically allowed under the full orientifold action $\mathcal{O} = \Omega_p(-)^{F_L}\sigma$. For example, only the geometric flux ω and nongeometric flux R remain invariant under $\Omega_p(-)^{F_L}$, while the standard fluxes (F, H) and nongeometric flux (Q) are anti-invariant [32,60].

Therefore, under the full orientifold action, we can only have the following flux components:

$$F_3 \equiv (F_\Lambda, F^\Lambda), \qquad H_3 \equiv (H_\Lambda, H^\Lambda),$$
$$\omega \equiv ({\omega_a}^\Lambda, \omega_{a\Lambda}, {\hat\omega_\alpha}{}^K, \hat\omega_{\alpha K}),$$
$$Q \equiv (Q^{aK}, {Q^a}_K, \hat Q^{\alpha\Lambda}, {\hat Q^\alpha}{}_\Lambda), \qquad R \equiv (R_K, R^K). \tag{2.75}$$

In order to keep the type IIB case distinct from the type IIA case, we define a new twisted differential $\mathcal{D}$ involving the actions from all of the NS-NS (non)geometric fluxes as [60]

$$\mathcal{D} = d - H \wedge . - \omega \triangleleft . - Q \triangleright . - R \bullet . \tag{2.76}$$

The action of the operator $\triangleleft$, $\triangleright$, and $\bullet$ on a p-form changes it into a $(p+1)$-, $(p-1)$-, or $(p-3)$-form, respectively, and we have the following flux actions [60]:

$$H \wedge \mathcal{A}_\Lambda = -H_\Lambda \Phi_6, \qquad H \wedge \mathcal{B}^\Lambda = -H^\Lambda \Phi_6,$$
$$H \wedge a_K = 0, \qquad H \wedge b^K = 0, \qquad H \wedge \mathbf{1} = H = -H^\Lambda \mathcal{A}_\Lambda + H_\Lambda \mathcal{B}^\Lambda, \tag{2.77}$$

$$\omega \triangleleft \mathcal{A}_\Lambda = -\omega_{b\Lambda}\tilde\nu^a, \qquad \omega \triangleleft \mathcal{B}^\Lambda = -{\omega_b}^\Lambda \tilde\nu^a, \qquad \omega \triangleleft \nu_a = {\omega_a}^\Lambda \mathcal{A}_\Lambda - \omega_{a\Lambda}\mathcal{B}^\Lambda,$$
$$\omega \triangleleft a_K = -\hat\omega_{\beta K}\tilde\mu^\alpha, \qquad \omega \triangleleft b^K = -{\hat\omega_\beta}{}^K \tilde\mu^\alpha, \qquad \omega \triangleleft \mu_\alpha = {\hat\omega_\alpha}{}^K a_K - \hat\omega_{\alpha K} b^K,$$

$$Q \triangleright \mathcal{A}_\Lambda = -\hat Q^\alpha_\Lambda \mu_\beta, \qquad Q \triangleright \mathcal{B}^\Lambda = -\hat Q^{\alpha\Lambda}\mu_\beta, \qquad Q \triangleright \tilde\mu^\alpha = -\hat Q^{\alpha\Lambda}\mathcal{A}_\Lambda + {\hat Q^\alpha}{}_\Lambda \mathcal{B}^\Lambda,$$
$$Q \triangleright a_K = -Q^a_K \nu_b, \qquad Q \triangleright b^K = -Q^{aK}\nu_b, \qquad Q \triangleright \tilde\nu^a = -Q^{aK} a_K + {Q^a}_K b^K,$$

$$R \bullet \mathcal{A}_\Lambda = 0, \qquad R \bullet \mathcal{B}^\Lambda = 0, \qquad R \bullet a_K = -R_K \mathbf{1}, \qquad R \bullet b^K = -R^K \mathbf{1},$$
$$R \bullet \Phi_6 = R^K a_K - R_K b^K.$$

Using the flux actions given in Eq. (2.77) for the NS-NS fluxes and the expansion of the RR flux F_3 as $F_{\rm RR} = -F^\Lambda \mathcal{A}_\Lambda + F_\Lambda \mathcal{B}^\Lambda$, one obtains the following generic form for the flux superpotential [24,26,30,32]:

$$W_{\rm IIB} \equiv W_{\rm R}^{\rm IIB} + W_{\rm NS}^{\rm IIB} = -\frac{1}{\sqrt{2}}\int_{X_3} [F_{\rm RR} + \mathcal{D}\Phi_c^{\rm even}] \wedge \Pi_{\Omega_3}$$
$$= \frac{1}{\sqrt{2}}(F_\Lambda - SH_\Lambda - G^a \omega_{a\Lambda} - T_\alpha {\hat Q^\alpha}{}_\Lambda)\mathcal{X}^\Lambda - \frac{1}{\sqrt{2}}(F^\Lambda - SH^\Lambda - G^a {\omega_a}^\Lambda - T_\alpha \hat Q^{\alpha\Lambda})\mathcal{F}_\Lambda. \tag{2.78}$$

Subsequently, using Eq. (2.65) leads to the following explicit form of the type IIB generalized flux superpotential:

$$\begin{aligned}\sqrt{2}W_{\rm IIB} = &\left[\bar F_0 + U^i \bar F_i + \frac{1}{2} l_{ijk} U^i U^j F^k - \frac{1}{6} l_{ijk} U^i U^j U^k F^0 - i\tilde p_0 F^0\right]\\ &- S\left[\bar H_0 + U^i \bar H_i + \frac{1}{2} l_{ijk} U^i U^j H^k - \frac{1}{6} l_{ijk} U^i U^j U^k H^0 - i\tilde p_0 H^0\right]\\ &- G^a\left[\bar\omega_{a0} + U^i \bar\omega_{ai} + \frac{1}{2} l_{ijk} U^i U^j {\omega_a}^k - \frac{1}{6} l_{ijk} U^i U^j U^k {\omega_a}^0 - i\tilde p_0 {\omega_a}^0\right]\\ &- T_\alpha\left[{\bar{\hat Q}^\alpha}{}_0 + U^i {\bar{\hat Q}^\alpha}{}_i + \frac{1}{2} l_{ijk} U^i U^j \hat Q^{\alpha k} - \frac{1}{6} l_{ijk} U^i U^j U^k \hat Q^{\alpha 0} - i\tilde p_0 \hat Q^{\alpha 0}\right],\end{aligned} \tag{2.79}$$

where because of the α' corrections on the mirror side, the complex structure sector is modified to induce rational shifts in the usual flux components, given as

$$\begin{aligned}
&\bar{F}_0 = F_0 - \tilde{p}_i F^i, \quad \bar{F}_i = F_i - \tilde{p}_{ij} F^j - \tilde{p}_i F^0, \\
&\bar{H}_0 = H_0 - \tilde{p}_i H^i, \quad \bar{H}_i = H_i - \tilde{p}_{ij} H^i - \tilde{p}_i H^0, \\
&\bar{\omega}_{a0} = \omega_{a0} - \tilde{p}_i \omega_a{}^i, \quad \bar{\omega}_{ai} = \omega_{ai} - \tilde{p}_{ij} \omega_a{}^j - \tilde{p}_i \omega_a{}^0, \\
&\bar{\hat{Q}}^\alpha{}_0 = \hat{Q}^\alpha{}_0 - \tilde{p}_i \hat{Q}^{\alpha i}, \quad \bar{\hat{Q}}^\alpha{}_i = \hat{Q}^\alpha{}_i - \tilde{p}_{ij} \hat{Q}^{\alpha j} - \tilde{p}_i \hat{Q}^{\alpha 0}. \qquad (2.80)
\end{aligned}$$

3. Gauge kinetic couplings and the D-term effects

In the presence of a nontrivial sector of even (2,1) cohomology, i.e., for $h_+^{2,1}(X) \neq 0$, there are D-term contributions to the four-dimensional scalar potential. Following the strategy of Ref. [60], the same can be determined by considering the following gauge transformations of the RR potentials $C_{\rm RR} = C_0 + C_2 + C_4$:

$$\begin{aligned}
C_{\rm RR} &\to C_{\rm RR} + \mathcal{D}(\gamma^K a_K - \gamma_K b^K) \\
&\supset (C_0 + R_K \gamma^K - R^K \gamma_K) - (c^a + Q^a{}_K \gamma^K - Q^{aK} \gamma_K) \nu_a \\
&\quad + (c_\alpha + \hat{\omega}_{\alpha K} \gamma^K - \hat{\omega}_\alpha{}^K \gamma_K) \tilde{\mu}^\alpha. \qquad (2.81)
\end{aligned}$$

These lead to the following flux-dependent shifts in the variables S, G^a, and T_α induced by the respective shifts in the axionic components c_0, c^a, and c_α:

$$\begin{aligned}
&\delta S = R_K \gamma^K - R^K \gamma_K, \qquad \delta G^a = Q^a{}_K \gamma^K - Q^{aK} \gamma_K, \\
&\delta T_\alpha = \hat{\omega}_{\alpha K} \gamma^K - \hat{\omega}_\alpha{}^K \gamma_K. \qquad (2.82)
\end{aligned}$$

This leads to the following two D terms being generated by the gauge transformations:

$$\begin{aligned}
D_K &= i[R_K(\partial_S K) + Q^a{}_K(\partial_a K) + \hat{\omega}_{\alpha K}(\partial^\alpha K)], \\
D^K &= -i[R^K(\partial_S K) + Q^{aK}(\partial_a K) + \hat{\omega}_\alpha{}^K(\partial^\alpha K)]. \qquad (2.83)
\end{aligned}$$

Now, using the Kähler potential in Eq. (2.73) and the variables given in Eq. (2.74), the Kähler derivatives can be given as

$$\begin{aligned}
&K_S = \frac{i}{2s}\left(1 - \frac{\hat{\ell}_{\alpha ab} t^\alpha b^a b^b}{2(\mathcal{V} + \frac{p_0}{4})} + \frac{3 p_0}{4(\mathcal{V} + \frac{p_0}{4})}\right) = -K_{\bar{S}}, \\
&K_{G^a} = \frac{i \hat{\ell}_{\alpha ab} t^\alpha b^b}{2s(\mathcal{V} + \frac{p_0}{4})} = -K_{\bar{G}^a}, \qquad K_{T_\alpha} = -\frac{i t^\alpha}{2s(\mathcal{V} + \frac{p_0}{4})} = -K_{\bar{T}_\alpha}, \\
&\qquad (2.84)
\end{aligned}$$

which gives the following two explicit D terms:

$$\begin{aligned}
D_K = -\frac{s e^{\frac{K(Q)}{2}}}{2}\Big[&R_K\Big(\mathcal{V} + p_0 - \frac{1}{2}\hat{\ell}_{\alpha ab} t^\alpha b^a b^b\Big) \\
&+ Q^a{}_K \hat{\ell}_{\alpha ac} t^\alpha b^c - t^\alpha \hat{\omega}_{\alpha K}\Big], \\
D^K = \frac{s e^{\frac{K_{(Q)}}{2}}}{2}\Big[&R^K\Big(\mathcal{V} + p_0 - \frac{1}{2}\hat{\ell}_{\alpha ab} t^\alpha b^a b^b\Big) \\
&+ Q^{aK} \hat{\ell}_{\alpha ac} t^\alpha b^c - t^\alpha \hat{\omega}_\alpha{}^K\Big]. \qquad (2.85)
\end{aligned}$$

Using these results in the D-term expression given in Eq. (2.85) leads to the following contributions in the four-dimensional scalar potential [9]:

$$V_{\rm IIB}^D = \frac{1}{2} D_J [{\rm Re}(f_{JK})]^{-1} D_K + \frac{1}{2} D^J [{\rm Re}(f^{JK})]^{-1} D^K. \qquad (2.86)$$

Here the gauge kinetic couplings for the electric and magnetic components can be computed from the orientifold even sector of the holomorphic three-form. For that we consider the holomorphic three-form of the $\mathcal{N} = 2$ theory, and after the imposition of the orientifold involution it can be split into the even/odd sectors,

$$\begin{aligned}
\Omega_3 &= \Omega_3^{\rm odd} + \Omega_3^{\rm even} \\
&= \mathcal{X}^\Lambda \mathcal{A}_\Lambda - \mathcal{F}_\Lambda \mathcal{B}^\Lambda + \mathcal{X}^K a_K - \mathcal{F}_K b^K, \qquad (2.87)
\end{aligned}$$

which leads to the following electric gauge kinetic coupling from the even sector [56]:

$$f_{JK} = -\frac{i}{2} \mathcal{F}_{JK}\big|_{\text{evaluated at } \mathcal{X}^K = 0}. \qquad (2.88)$$

For the case of compactifications using rigid Calabi Yau threefold and the cases of frozen complex structure moduli, the gauge coupling f_{KJ} is just a constant [9], which otherwise can generically depend on the complex structure moduli U^i. Moreover, using mirror arguments and the prepotential, one can show that

$$f_{JK} = -\frac{i}{2}(\hat{l}_{iJK} U^i - \tilde{p}_{JK}). \qquad (2.89)$$

Here we recall that the index i runs in odd (2,1) cohomology which counts the number of complex structure moduli U^i, while the indices J and K run in the even (2,1) cohomology. Given that the $\tilde{p}_{JK}$'s are real quantities, they will not appear in the real gauge kinetic couplings, which is denoted as ${\rm Re}(f_{JK}) = -\frac{1}{2}\hat{l}_{iJK} u^i = -\frac{1}{2}\hat{l}_{JK}$, and similarly for the magnetic couplings we have ${\rm Re}(f^{JK}) = -\frac{1}{2}\hat{l}^{JK}$. Also note that both of the gauge couplings are positive, and this ensures the positive definiteness of the D-term contribution to the scalar potential, $V_{\rm IIB}^D \geq 0$.

4. Tadpole cancellation conditions and Bianchi identities

Generically, there are tadpole terms present due to the presence of $O3/O7$ planes, and these can be canceled by either imposing a set of flux constraints or adding the counterterms that can arise from the presence of local sources such as (stacks of) $D3/D7$-branes. These effects equivalently provide the following contributions in the effective potential:

$$V_{\rm IIB}^{\rm tad} = \frac{1}{2} e^{K(Q)} \int_{X_3} \langle [{\rm Im}\Phi_c^{\rm even}], DF_{\rm RR}\rangle, \qquad (2.90)$$

where the multiform $DF_{\rm RR}$ can be expanded using the flux actions in the generalized twisted differential operator, given as [30,61,63,67,68]

$$\begin{aligned} DF_{\rm RR} = {} & (F_\Lambda H^\Lambda - F^\Lambda H_\Lambda)\Phi_6 + (F_\Lambda \omega_a{}^\Lambda - F^\Lambda \omega_{a\Lambda})\tilde{\nu}^a \\ & + (F_\Lambda \hat{Q}^{\alpha\Lambda} - F^\Lambda \hat{Q}^\alpha{}_\Lambda)\mu_\alpha. \end{aligned}$$

In addition, using the definition of $\Phi_c^{\rm even}$ given in Eq. (2.70), the tadpole term given in Eq. (2.90) simplifies to the following form:

$$\begin{aligned} V_{\rm IIB}^{\rm tad} = {} & \frac{1}{2} e^{K(Q)} [(F_\Lambda H^\Lambda - F^\Lambda H_\Lambda)[{\rm Im} S] \\ & + (F_\Lambda \omega_a{}^\Lambda - F^\Lambda \omega_{a\Lambda})[{\rm Im} G^a] \\ & + (F_\Lambda \hat{Q}^{\alpha\Lambda} - F^\Lambda \hat{Q}^\alpha{}_\Lambda)[{\rm Im} T_\alpha]]. \qquad (2.91) \end{aligned}$$

The moduli dynamics of the 4D effective theory is determined by the total scalar potential given as a sum of F- and D-term contributions,

$$V_{\rm IIB}^{\rm tot} = V_{\rm IIB}^F + V_{\rm IIB}^D, \qquad (2.92)$$

where the various fluxes appearing in the scalar potential must satisfy the full set of NS-NS Bianchi identities and RR tadpole cancellation conditions.

III. ACTION OF THE T-DUALITY TRANSFORMATIONS

In this section we invoke the T-duality rules in the cohomology formulation by taking some iterative steps. We know that in the fluxless case, the mirror symmetry is present and hence type IIA and type IIB ingredients can be mapped to each other. After including the fluxes, this T duality is destroyed or restored if appropriate fluxes are included. So our plan is to first look for the T-duality rules among the various moduli and axions in the fluxless case, and then look at the superpotentials and D terms to invoke the mapping between the various components of the type IIA and type IIB fluxes.

Looking at the two Kähler potentials given in Eqs. (2.35) and (2.73), we observe that they are exchanged under a combined action of the following set of transformations:

$$\begin{aligned} & ({\rm z}^0)^{-1} \leftrightarrow s, \qquad t^a \leftrightarrow u^i, \qquad z^\lambda \leftrightarrow t^\alpha, \\ & k_{\lambda\rho\mu} \leftrightarrow \ell_{\alpha\beta\gamma}, \qquad \hat{k}_{\lambda mn} \leftrightarrow \hat{\ell}_{\alpha ab}, \qquad \kappa_{abc} \leftrightarrow l_{ijk}, \qquad \hat{\kappa}_{a\alpha\beta} \leftrightarrow \hat{l}_{iJK}, \\ & p_{ab} \leftrightarrow \tilde{p}_{ij}, \qquad p_a \leftrightarrow \tilde{p}_i, \qquad p_0 \leftrightarrow \tilde{p}_0, \qquad \tilde{p}_{k\lambda} \leftrightarrow p_{a\alpha}, \qquad \tilde{p}_\lambda \leftrightarrow p_\alpha. \end{aligned} \qquad (3.1)$$

In the above mapping, the quantities on the left side of the equivalence belong to the type IIA theory while the respective ones on the right side belong to the type IIB theory. Moreover, it is easy to observe that the complexified variables of type IIA given in Eq. (2.33) and those of type IIB in Eq. (2.74) are exchanged with the mapping details given in Table II.

A. F-term contributions

Let us begin by summarizing the various flux components that contribute to the effective four-dimensional potential via the F-term contributions.

TABLE II. T-duality transformations for various type IIA and type IIB moduli.

IIA	${\rm N}^0$	${\rm N}^k$	${\rm U}_\lambda$	${\rm T}^a$	$\frac{1}{z^0}$	z^k	z^λ	b^a	t^a	ξ^0	ξ^k	ξ_λ
IIB	S	G^a	T_α	U^i	s	b^a	t^α	v^i	u^i	c_0	$c^a + c_0 b^a$	$c_\alpha + \hat{\ell}_{\alpha ab} c^a b^b + \frac{1}{2} c_0 \hat{\ell}_{\alpha ab} b^a b^b$

TABLE III. T-duality transformations among the NS-NS fluxes appearing in the F-term effects.

IIA	${\rm H}_0$	${\rm H}_k$	${\rm H}^\lambda$	w_{a0}	w_{ak}	$w_a{}^\lambda$	${\rm Q}^a{}_0$	${\rm Q}^a{}_k$	${\rm Q}^{a\lambda}$	${\rm R}_0$	${\rm R}_k$	${\rm R}^\lambda$
IIB	H_0	ω_{a0}	$\hat{Q}^\alpha{}_0$	H_i	ω_{ai}	$\hat{Q}^\alpha{}_i$	H^i	$\omega_a{}^i$	$\hat{Q}^{\alpha i}$	$-H^0$	$-\omega_a{}^0$	$-\hat{Q}^{\alpha 0}$

Type IIA:

$$\text{RR flux} \equiv (\mathbf{F_6}: e_0, \mathbf{F_4}: e_a, \mathbf{F_2}: m^a, \mathbf{F_0}: m_0),$$
$$\text{NS flux} \equiv (\mathrm{H}_0, \mathrm{H}_k, \mathrm{H}^\lambda, \quad w_{a0}, w_{ak}, w_a{}^\lambda, \quad \mathrm{Q}^a{}_0, \mathrm{Q}^a{}_i, \mathrm{Q}^{a\lambda}, \quad \mathrm{R}_0, \mathrm{R}_i, \mathrm{R}^\lambda).$$

Type IIB:

$$\text{RR flux} \equiv (\mathbf{F_3}: F_0, F_i, F^i, F^0),$$
$$\text{NS flux} \equiv (H_0, H_i, H^i, H^0, \quad \omega_{a0}, \omega_{ai}, \omega_a{}^i, \omega_a{}^0, \quad \hat{Q}^\alpha{}_0, \hat{Q}^\alpha{}_i, \hat{Q}^{\alpha 0}, \hat{Q}^{\alpha i}). \tag{3.2}$$

Now it is interesting thing to observe that the explicit expressions of the type IIA and type IIB superpotentials as given in Eqs. (2.44) and (2.79), respectively, are exchanged under a combined action of a set of T-duality transformations for the fluxes given in Tables III and IV.

B. D-term contributions

In the string frame, the D terms in both the type IIA and type IIB theories are as follows.

IIA:

$$D_\alpha = \frac{(z^0)^{-1} e^{\frac{K^{(q)}}{2}}}{2}\left[\left(\mathcal{U} + \tilde{p}_0 - \frac{1}{2}\hat{k}_{\lambda km} z^\lambda z^k z^m\right)\hat{w}_\alpha{}^0 + \hat{k}_{\lambda km} z^\lambda z^m \hat{w}_\alpha{}^k + z^\lambda \hat{w}_{\alpha\lambda}\right],$$

$$D^\alpha = -\frac{(z^0)^{-1} e^{\frac{K^{(q)}}{2}}}{2}\left[\left(\mathcal{U} + \tilde{p}_0 - \frac{1}{2}\hat{k}_{\lambda km} z^\lambda z^k z^m\right)\hat{\mathrm{Q}}^{\alpha 0} + \hat{k}_{\lambda km} z^\lambda z^m \hat{\mathrm{Q}}^{\alpha k} + z^\lambda \hat{\mathrm{Q}}^\alpha{}_\lambda\right].$$

IIB:

$$D_K = -\frac{s e^{\frac{K^{(Q)}}{2}}}{2}\left[R_K\left(\mathcal{V} + p_0 - \frac{1}{2}\hat{\ell}_{\alpha ab} t^\alpha b^a b^b\right) + Q^a{}_K \hat{\ell}_{\alpha ac} t^\alpha b^c - t^\alpha \hat{\omega}_{\alpha K}\right],$$

$$D^K = \frac{s e^{\frac{K^{(Q)}}{2}}}{2}\left[R^K\left(\mathcal{V} + p_0 - \frac{1}{2}\hat{\ell}_{\alpha ab} t^\alpha b^a b^b\right) + Q^{aK} \hat{\ell}_{\alpha ac} t^\alpha b^c - t^\alpha \hat{\omega}_\alpha{}^K\right]. \tag{3.3}$$

Recalling that $\tilde{p}_0 \leftrightarrow p_0$ and $\mathcal{V} \leftrightarrow \mathcal{U}$ under the mirror symmetry, and subsequently after using the T-duality transformation listed for the moduli and the axions given in Table II, we find the T-duality transformation of D-term fluxes as presented in Table V.

C. Tadpole conditions

Now we compare the various tadpole terms generated in the type IIA and type IIB theories, which can also be compensated by appropriately adding the local effects from various D_p-brane and O_p planes. In particular, in this work the tadpoles on the type IIA side can be compensated by the $D6/O6$ effects, while the tadpoles on the type IIB side can be compensated by $D3/O3$ and $D7/O7$ effects. These are given as

$$V_{\mathrm{IIA}}^{\mathrm{tad}} = \frac{1}{2} e^{K^{(q)}}[(\mathrm{Im}\mathrm{N}^{\hat{k}})(\mathrm{H}_{\hat{k}} m_0 - \omega_{a\hat{k}} m^a + Q^a{}_{\hat{k}} e_a - R_{\hat{k}} e_0) + (\mathrm{Im}\mathrm{U}_\lambda)(\mathrm{H}^\lambda m_0 - \omega_a{}^\lambda m^a + Q^{a\lambda} e_a - R^\lambda e_0)],$$

$$V_{\mathrm{IIB}}^{\mathrm{tad}} = \frac{1}{2} e^{K^{(Q)}}[(F_\Lambda H^\Lambda - F^\Lambda H_\Lambda)[\mathrm{Im}S] + (F_\Lambda \omega_a{}^\Lambda - F^\Lambda \omega_{a\Lambda})\mathrm{Im}(G^a) + (F_\Lambda \hat{Q}^{\alpha\Lambda} - F^\Lambda \hat{Q}^\alpha{}_\Lambda)\mathrm{Im}(T_\alpha)]. \tag{3.4}$$

Now, given that $K^{(q)} \leftrightarrow K^{(Q)}$, $\mathrm{N}^0 \leftrightarrow S$, $\mathrm{N}^k \leftrightarrow G^a$, and $\mathrm{U}_\lambda \leftrightarrow T_\alpha$ under the explicit T-duality rules, it is simple to observe that the type IIA and type IIB tadpole terms are exchanged under the T-dual flux transformations given in Tables III and IV.

D. Bianchi identities

As we have already established the exchange symmetry of the F and D terms, we now check how our T-duality rules are applied to the flux constraints in the Bianchi identities of the two sides. This is necessary to prove the claim for the exchange symmetry between the actual effective potentials of the two type II theories, in the sense that if some pieces are

TABLE IV. T-duality transformations among the RR-flux components.

IIA	e_0	e_a	m^a	m^0
IIB	F_0	F_i	F^i	$-F^0$

TABLE V. T-duality transformations among the NS-NS fluxes appearing in the D-term effects.

IIA	$\hat{\mathrm{Q}}^\alpha{}_\lambda$	$\hat{w}_{\alpha\lambda}$	$\hat{\mathrm{Q}}^{\alpha k}$	$\hat{w}_\alpha{}^k$	$\hat{\mathrm{Q}}^{\alpha 0}$	$\hat{w}_\alpha{}^0$
IIB	$\hat{\omega}_\alpha{}^K$	$\hat{\omega}_{\alpha K}$	$-Q^{aK}$	$-Q^a{}_K$	$-R^K$	$-R_K$

killed by the Bianchi identities on one side then that should also be the case on the mirror dual side.

1. Five classes of Bianchi identities for type IIA

Using the flux actions given in Eq. (2.38), the following five classes of NS-NS Bianchi identities are obtained by demanding the nilpotency of the twisted differential operator D as defined in Eq. (2.37) by imposing $\mathrm{D}^2 = 0$ on the various harmonic forms:

$$
\begin{aligned}
&(\mathbf{I}).\ \mathrm{H}^\lambda \hat{w}_{\alpha\lambda} = \mathrm{H}_{\hat{k}} \hat{w}_\alpha{}^{\hat{k}}.\\
&(\mathbf{II}).\ \mathrm{H}^\lambda \hat{\mathrm{Q}}^\alpha{}_\lambda = \mathrm{H}_{\hat{k}} \hat{\mathrm{Q}}^{\alpha\hat{k}}, \qquad w_a{}^\lambda \hat{w}_{\alpha\lambda} = w_{a\hat{k}} \hat{w}_\alpha{}^{\hat{k}}.\\
&(\mathbf{III}).\ \hat{\mathrm{Q}}^\alpha{}_\lambda w_a{}^\lambda = w_{a\hat{k}} \hat{\mathrm{Q}}^{\alpha\hat{k}}, \qquad \mathrm{Q}^a{}_{\hat{k}} \hat{w}_\alpha{}^{\hat{k}} = \mathrm{Q}^{a\lambda} \hat{w}_{\alpha\lambda},\\
&\qquad \hat{w}_{\alpha\lambda} \hat{\mathrm{Q}}^{\alpha\hat{k}} = \hat{\mathrm{Q}}^\alpha{}_\lambda \hat{w}_\alpha{}^{\hat{k}}, \qquad \hat{w}_{\alpha\lambda} \hat{\mathrm{Q}}^\alpha{}_\rho = \hat{\mathrm{Q}}^\alpha{}_\lambda \hat{w}_{\alpha\rho},\\
&\qquad \hat{w}_\alpha{}^{\hat{k}} \hat{\mathrm{Q}}^{\alpha\hat{k}'} = \hat{\mathrm{Q}}^{\alpha\hat{k}} \hat{w}_\alpha{}^{\hat{k}'},\\
&\qquad \mathrm{H}_{[\hat{k}} \mathrm{R}_{\hat{k}']} + \mathrm{Q}^a{}_{[\hat{k}} w_{a\hat{k}']} = 0, \qquad \mathrm{H}^{[\lambda} \mathrm{R}^{\rho]} + \mathrm{Q}^{a[\lambda} w_a{}^{\rho]} = 0,\\
&\qquad \mathrm{R}^\lambda \mathrm{H}_{\hat{k}} - \mathrm{H}^\lambda \mathrm{R}_{\hat{k}} + w_a{}^\lambda \mathrm{Q}^a{}_{\hat{k}} - \mathrm{Q}^{a\lambda} w_{a\hat{k}} = 0.\\
&(\mathbf{IV}).\ \mathrm{R}^\lambda \hat{w}_{\alpha\lambda} = \mathrm{R}_{\hat{k}} \hat{w}_\alpha{}^{\hat{k}}, \qquad \mathrm{Q}^{a\lambda} \hat{\mathrm{Q}}^\alpha{}_\lambda = \mathrm{Q}^a{}_{\hat{k}} \hat{\mathrm{Q}}^{\alpha\hat{k}}.\\
&(\mathbf{V}).\ \mathrm{R}^\lambda \hat{\mathrm{Q}}^\alpha{}_\lambda = \mathrm{R}_{\hat{k}} \hat{\mathrm{Q}}^{\alpha\hat{k}}.
\end{aligned} \tag{3.5}
$$

These identities suggest that if one considers the antiholomorphic involution such that the even (1,1) cohomology is trivial, which is very often the case, then there will be no D terms and the only Bianchi identities to worry about are

$$
\begin{aligned}
&\mathrm{R}^\lambda \mathrm{H}_{\hat{k}} - \mathrm{H}^\lambda \mathrm{R}_{\hat{k}} + w_a{}^\lambda \mathrm{Q}^a{}_{\hat{k}} - \mathrm{Q}^{a\lambda} w_{a\hat{k}} = 0,\\
&\mathrm{H}_{[\hat{k}} \mathrm{R}_{\hat{k}']} + \mathrm{Q}^a{}_{[\hat{k}} w_{a\hat{k}']} = 0, \quad \mathrm{H}^{[\lambda} \mathrm{R}^{\rho]} + \mathrm{Q}^{a[\lambda} w_a{}^{\rho]} = 0.
\end{aligned} \tag{3.6}
$$

2. Five classes of Bianchi identities for type IIB

Similarly, using the flux actions given in Eq. (2.77), the following five classes of NS-NS Bianchi identities are obtained by imposing $\mathcal{D}^2 = 0$ on the various harmonic forms [60]:

$$
\begin{aligned}
&(\mathbf{I}).\ H_\Lambda \omega_a{}^\Lambda = H^\Lambda \omega_{\Lambda a}.\\
&(\mathbf{II}).\ H^\Lambda \hat{Q}_\Lambda{}^\alpha = H_\Lambda \hat{Q}^{\alpha\Lambda}, \qquad \omega_a{}^\Lambda \omega_{b\Lambda} = \omega_b{}^\Lambda \omega_{a\Lambda},\\
&\qquad \hat{\omega}_\alpha{}^K \hat{\omega}_{\beta K} = \hat{\omega}_\beta{}^K \hat{\omega}_{\alpha K}.\\
&(\mathbf{III}).\ \omega_{a\Lambda} \hat{Q}^{\alpha\Lambda} = \omega_a{}^\Lambda \hat{Q}^\alpha{}_\Lambda, \qquad Q^{aK} \hat{\omega}_{\alpha K} = Q^a{}_K \hat{\omega}_\alpha^K,\\
&\qquad H_\Lambda R_K + \omega_{a\Lambda} Q^a{}_K + \hat{Q}^\alpha{}_\Lambda \hat{\omega}_{\alpha K} = 0,\\
&\qquad H^\Lambda R_K + \omega_a{}^\Lambda Q^a{}_K + \hat{Q}^{\alpha\Lambda} \hat{\omega}_{\alpha K} = 0,\\
&\qquad H_\Lambda R^K + \omega_{a\Lambda} Q^{aK} + \hat{Q}^\alpha{}_\Lambda \hat{\omega}_\alpha{}^K = 0,\\
&\qquad H^\Lambda R^K + \omega_a{}^\Lambda Q^{aK} + \hat{Q}^{\alpha\Lambda} \hat{\omega}_\alpha{}^K = 0.\\
&(\mathbf{IV}).\ R^K \hat{\omega}_{\alpha K} = R_K \hat{\omega}_\alpha{}^K, \qquad \hat{Q}^{\alpha\Lambda} \hat{Q}^\beta{}_\Lambda = \hat{Q}^{\beta\Lambda} \hat{Q}^\alpha{}_\Lambda,\\
&\qquad Q^{aK} Q^b{}_K = Q^{bK} Q^a{}_K.\\
&(\mathbf{V}).\ R_K Q^{aK} - R^K Q^a{}_K = 0.
\end{aligned} \tag{3.7}
$$

The above set of type IIB Bianchi identities suggests that if one chooses the holomorphic involution such that the even (2,1) cohomology is trivial, then only the following Bianchi identities remain nontrivial:

$$
\begin{aligned}
&H_\Lambda \omega_a{}^\Lambda = H^\Lambda \omega_{\Lambda a}, \qquad H^\Lambda \hat{Q}_\Lambda{}^\alpha = H_\Lambda \hat{Q}^{\alpha\Lambda},\\
&\omega_a{}^\Lambda \omega_{b\Lambda} = \omega_b{}^\Lambda \omega_{a\Lambda}, \qquad \omega_{a\Lambda} \hat{Q}^{\alpha\Lambda} = \omega_a{}^\Lambda \hat{Q}^\alpha{}_\Lambda,\\
&\hat{Q}^{\alpha\Lambda} \hat{Q}^\beta{}_\Lambda = \hat{Q}^{\beta\Lambda} \hat{Q}^\alpha{}_\Lambda.
\end{aligned} \tag{3.8}
$$

In such a situation, there will be no D term generated as all of the fluxes with $\{J, K\} \in h^{2,1}_+$ indices are projected out. Moreover, if the holomorphic involution is chosen to result in a trivial odd (1,1) cohomology, which corresponds to the situation of the absence of odd moduli G^a and is an often studied case, then there are only two types of Bianchi identities to worry about:

$$
H^\Lambda \hat{Q}_\Lambda{}^\alpha = H_\Lambda \hat{Q}^{\alpha\Lambda}, \qquad \hat{Q}^{\alpha\Lambda} \hat{Q}^\beta{}_\Lambda = \hat{Q}^{\beta\Lambda} \hat{Q}^\alpha{}_\Lambda.
$$

Using the T-duality transformations among the various NS-NS fluxes as listed in Tables III and V, we find that indeed the 14 Bianchi identities on the type IIA side are precisely mapped onto the 14 Bianchi identities on the type IIB side, and vice versa. However, there is a rather significant mixing across the five classes of identities on the two sides. For example, the identity $H^\Lambda \hat{Q}_\Lambda{}^\alpha = H_\Lambda \hat{Q}^{\alpha\Lambda}$ corresponding to class $(\mathbf{II})$ on the type IIB side produces the identity $(\mathrm{R}^\lambda \mathrm{H}_0 - \mathrm{H}^\lambda \mathrm{R}_0 + w_a{}^\lambda \mathrm{Q}^a{}_0 - \mathrm{Q}^{a\lambda} w_{a0}) = 0$, which corresponds to class $(\mathbf{III})$ on the type IIA side. To illustrate these features, we present a one-to-one correspondence among all of the identities in Table XI of the Appendix.

IV. EXCHANGING THE SCALAR POTENTIALS UNDER T DUALITY

In this section our first goal is to present a new set of axionic flux polynomials for both the type IIA and type IIB theories which would include all of the axionic fields appearing in those respective theories, and without having any saxions involved. These will be subsequently used to present the two scalar potentials completely in terms of these axionic flux polynomials and the moduli space metrics on the two theories.

A. Axionic flux polynomials

1. Type IIA

A careful look at the type IIA superpotential given in Eq. (2.44) and the D terms given in Eq. (2.55) suggests defining some axionic flux combinations—which we call “axionic flux polynomials”—that can be useful for rewriting the generic complicated scalar potential with explicit dependence on the saxions/axions within a few lines.

These axionic flux polynomials can be given by the following expressions:

$$f_0 = \mathbb{G}_0 - \xi^{\hat{k}}\mathcal{H}_{\hat{k}} - \xi_\lambda \mathcal{H}^\lambda,$$
$$f_a = \mathbb{G}_a - \xi^{\hat{k}}\mho_{a\hat{k}} - \xi_\lambda \mho_a{}^\lambda,$$
$$f^a = \mathbb{G}^a - \xi^{\hat{k}}\mathcal{Q}^a{}_{\hat{k}} - \xi_\lambda \mathcal{Q}^{a\lambda},$$
$$f^0 = \mathbb{G}^0 - \xi^{\hat{k}}\mathcal{R}_{\hat{k}} - \xi_\lambda \mathcal{R}^\lambda, \tag{4.1}$$

$$h_0 = \mathcal{H}_0 + \mathcal{H}_k z^k + \frac{1}{2}\hat{k}_{\lambda mn} z^m z^n \mathcal{H}^\lambda,$$
$$h_a = \mho_{a0} + \mho_{ak} z^k + \frac{1}{2}\hat{k}_{\lambda mn} z^m z^n \mho_a{}^\lambda,$$
$$h^a = \mathcal{Q}^a{}_0 + \mathcal{Q}^a{}_k z^k + \frac{1}{2}\hat{k}_{\lambda mn} z^m z^n \mathcal{Q}^{a\lambda},$$
$$h^0 = \mathcal{R}_0 + \mathcal{R}_k z^k + \frac{1}{2}\hat{k}_{\lambda mn} z^m z^n \mathcal{R}^\lambda,$$

$$h_{k0} = \mathcal{H}_k + \hat{k}_{\lambda kn} z^n \mathcal{H}^\lambda, \qquad h_{ak} = \mho_{ak} + \hat{k}_{\lambda kn} z^n \mho_a{}^\lambda,$$
$$h^a{}_k = \mathcal{Q}^a{}_k + \hat{k}_{\lambda kn} z^n \mathcal{Q}^{a\lambda}, \qquad h_k{}^0 = \mathcal{R}_k + \hat{k}_{\lambda kn} z^n \mathcal{R}^\lambda,$$

$$h^\lambda{}_0 = \mathcal{H}^\lambda, \qquad h_a{}^\lambda = \mho_a{}^\lambda, \qquad h^{a\lambda} = \mathcal{Q}^{a\lambda}, \qquad h^{\lambda 0} = \mathcal{R}^\lambda,$$
$$\hat{h}_{\alpha\lambda} = \hat{\mho}_{\alpha\lambda}, \qquad \hat{h}^\alpha{}_\lambda = \hat{\mathcal{Q}}^\alpha{}_\lambda, \qquad \hat{h}_\alpha{}^0 = \hat{\mho}_\alpha{}^0, \qquad \hat{h}^{\alpha 0} = \hat{\mathcal{Q}}^{\alpha 0},$$

where the intermediate axionic flux polynomials appearing in Eq. (4.1) are given as

$$\mathbb{G}_0 = \bar{e}_0 + b^a \bar{e}_a + \frac{1}{2}\kappa_{abc} b^a b^b m^c + \frac{1}{6}\kappa_{abc} b^a b^b b^c m_0,$$
$$\mathbb{G}_a = \bar{e}_a + \kappa_{abc} b^b m^c + \frac{1}{2}\kappa_{abc} b^b b^c m_0,$$
$$\mathbb{G}^a = m^a + m_0 b^a,$$
$$\mathbb{G}^0 = m_0,$$

$$\mathcal{H}_{\hat{k}} = \bar{\mathrm{H}}_{\hat{k}} + \bar{w}_{a\hat{k}} b^a + \frac{1}{2}\kappa_{abc} b^b b^c \mathrm{Q}^a{}_{\hat{k}} + \frac{1}{6}\kappa_{abc} b^a b^b b^c \mathrm{R}_{\hat{k}},$$
$$\mho_{a\hat{k}} = \bar{w}_{a\hat{k}} + \kappa_{abc} b^b \mathrm{Q}^c{}_{\hat{k}} + \frac{1}{2}\kappa_{abc} b^b b^c \mathrm{R}_{\hat{k}},$$
$$\mathcal{Q}^a{}_{\hat{k}} = \mathrm{Q}^a{}_{\hat{k}} + b^a \mathrm{R}_{\hat{k}},$$
$$\mathcal{R}_{\hat{k}} = \mathrm{R}_{\hat{k}}, \tag{4.2}$$

$$\mathcal{H}^\lambda = \bar{\mathrm{H}}^\lambda + \bar{w}_a{}^\lambda b^a + \frac{1}{2}\kappa_{abc} b^b b^c \mathrm{Q}^{a\lambda} + \frac{1}{6}\kappa_{abc} b^a b^b b^c \mathrm{R}^\lambda,$$
$$\mho_a{}^\lambda = \bar{w}_a{}^\lambda + \kappa_{abc} b^b \mathrm{Q}^{c\lambda} + \frac{1}{2}\kappa_{abc} b^b b^c \mathrm{R}^\lambda,$$
$$\mathcal{Q}^{a\lambda} = \mathrm{Q}^{a\lambda} + b^a \mathrm{R}^\lambda,$$
$$\mathcal{R}^\lambda = \mathrm{R}^\lambda,$$

$$\hat{\mho}_{\alpha\lambda} = \hat{w}_{\alpha\lambda} + \hat{k}_{\lambda km} z^m \hat{w}_\alpha{}^k - \frac{1}{2}\hat{k}_{\lambda km} z^k z^m \hat{w}_\alpha{}^0,$$
$$\hat{\mho}_\alpha{}^k = \hat{w}_\alpha{}^k - z^k \hat{w}_\alpha{}^0,$$
$$\hat{\mho}_\alpha{}^0 = \hat{w}_\alpha{}^0,$$
$$\hat{\mathcal{Q}}^\alpha{}_\lambda = \hat{\mathrm{Q}}^\alpha{}_\lambda + \hat{k}_{\lambda km} z^m \hat{\mathrm{Q}}^{\alpha k} - \frac{1}{2}\hat{k}_{\lambda km} z^\lambda z^k z^m \hat{\mathcal{Q}}^{\alpha 0},$$
$$\hat{\mathcal{Q}}^{\alpha k} = \hat{\mathrm{Q}}^{\alpha k} - z^k \hat{\mathrm{Q}}^{\alpha 0},$$
$$\hat{\mathcal{Q}}^{\alpha 0} = \hat{\mathrm{Q}}^{\alpha 0}.$$

Here we have utilized the shifted fluxes as defined in Eq. (2.45) due to the inclusion of α' corrections in the Kähler-moduli-dependent prepotential. The (partial) appearance of the type IIA axionic flux polynomials in Eq. (4.2) was seen before in Refs. [58,63,69]. In addition, the generalized RR flux polynomials defined as $\mathbb{G}_0$, $\mathbb{G}_a$, $\mathbb{G}^a$, $\mathbb{G}^0$ were used in Refs. [75,89–92] in the absence of (non) geometric flux.

2. Type IIB

Similarly, a careful look at the type IIB superpotential given in Eq. (2.79) and the D terms given in Eq. (2.85) suggests defining the following axionic flux polynomials, which are in direct one-to-one correspondence with the T-dual fluxes on the type IIA side, as we will see in a moment:

$$f_0 = \mathbb{F}_0 + v^i \mathbb{F}_i + \frac{1}{2} l_{ijk} v^j v^k \mathbb{F}^i - \frac{1}{6} l_{ijk} v^i v^j v^k \mathbb{F}^0,$$
$$f_i = \mathbb{F}_i + l_{ijk} v^j \mathbb{F}^k - \frac{1}{2} l_{ijk} v^j v^k \mathbb{F}^0,$$
$$f^i = \mathbb{F}^i - v^i \mathbb{F}^0,$$
$$f^0 = -\mathbb{F}^0, \tag{4.3}$$

$$h_0 = \mathbb{H}_0 + v^i \mathbb{H}_i + \frac{1}{2} l_{ijk} v^j v^k \mathbb{H}^i - \frac{1}{6} l_{ijk} v^i v^j v^k \mathbb{H}^0,$$
$$h_i = \mathbb{H}_i + l_{ijk} v^j \mathbb{H}^k - \frac{1}{2} l_{ijk} v^j v^k \mathbb{H}^0,$$
$$h^i = \mathbb{H}^i - v^i \mathbb{H}^0,$$
$$h^0 = -\mathbb{H}^0,$$

$$h_{a0} = \mho_{a0} + v^i \mho_{ai} + \frac{1}{2} l_{ijk} v^j v^k \mho_a{}^i - \frac{1}{6} l_{ijk} v^i v^j v^k \mho_a{}^0,$$
$$h_{ai} = \mho_{ai} + l_{ijk} v^j \mho_a{}^k - \frac{1}{2} l_{ijk} v^j v^k \mho_a{}^0,$$
$$h_a{}^i = \mho_a{}^i - v^i \mho_a{}^0,$$
$$h_a{}^0 = -\mho_a{}^0,$$

$$h^{\alpha}{}_0 = \hat{\mathbb{Q}}^{\alpha}{}_0 + v^i \hat{\mathbb{Q}}^{\alpha}{}_i + \frac{1}{2} l_{ijk} v^j v^k \hat{\mathbb{Q}}^{\alpha i} - \frac{1}{6} l_{ijk} v^i v^j v^k \hat{\mathbb{Q}}^{\alpha 0},$$
$$h^{\alpha}{}_i = \hat{\mathbb{Q}}^{\alpha}{}_i + l_{ijk} v^j \hat{\mathbb{Q}}^{\alpha k} - \frac{1}{2} l_{ijk} v^j v^k \hat{\mathbb{Q}}^{\alpha 0},$$
$$h^{\alpha i} = \hat{\mathbb{Q}}^{\alpha i} - v^i \hat{\mathbb{Q}}^{\alpha 0},$$
$$h^{\alpha 0} = -\hat{\mathbb{Q}}^{\alpha 0},$$

$$\hat{h}_{\alpha K} = \hat{\mho}_{\alpha K},$$
$$\hat{h}_{\alpha}{}^K = \hat{\mho}_{\alpha}{}^K,$$
$$\hat{h}_K{}^0 = -\mathbb{R}_K,$$
$$\hat{h}^{K0} = -\mathbb{R}^K.$$

The intermediate flux polynomials appearing in Eq. (4.3) are given as follows.

F-term flues:

$$\mathbb{F}_\Lambda = \bar{F}_\Lambda - \bar{\omega}_{a\Lambda} c^a - \bar{\hat{Q}}^{\alpha}{}_\Lambda (c_\alpha + \hat{\ell}_{\alpha ab} c^a b^b) - c_0 \mathbb{H}_\Lambda,$$
$$\mathbb{F}^\Lambda = F^\Lambda - \omega_a{}^\Lambda c^a - \hat{Q}^{\alpha\Lambda} (c_\alpha + \hat{\ell}_{\alpha ab} c^a b^b) - c_0 \mathbb{H}^\Lambda,$$
$$\mathbb{H}_\Lambda = \bar{H}_\Lambda + \bar{\omega}_{a\Lambda} b^a + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b \bar{\hat{Q}}^{\alpha}{}_\Lambda,$$
$$\mho_{a\Lambda} = \bar{\omega}_{a\Lambda} + \bar{\hat{Q}}^{\alpha}{}_\Lambda \hat{\ell}_{\alpha ab} b^b,$$
$$\hat{\mathbb{Q}}^{\alpha}{}_\Lambda = \bar{\hat{Q}}^{\alpha}{}_\Lambda, \tag{4.4}$$

$$\mathbb{H}^\Lambda = H^\Lambda + \omega_a{}^\Lambda b^a + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b \hat{Q}^{\alpha\Lambda},$$
$$\mho_a{}^\Lambda = \omega_a{}^\Lambda + \hat{Q}^{\alpha\Lambda} \hat{\ell}_{\alpha ab} b^b,$$
$$\hat{\mathbb{Q}}^{\alpha\Lambda} = \hat{Q}^{\alpha\Lambda};$$

D-term fluxes:

$$\hat{\mho}_{\alpha K} = \hat{\omega}_{\alpha K} - Q^a{}_K \hat{\ell}_{\alpha ab} b^b + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b R_K,$$
$$\mathbb{Q}^a{}_K = Q^a{}_K + R_K b^a,$$
$$\mathbb{R}_K = R_K,$$

$$\hat{\mho}_\alpha{}^K = \hat{\omega}_\alpha{}^K - Q^{aK} \hat{\ell}_{\alpha ab} b^b + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b R^K,$$
$$\mathbb{Q}^{aK} = Q^{aK} + R^K b^a,$$
$$\mathbb{R}^K = R^K.$$

Note that we have utilized the shifted fluxes with bars in some places, which are defined in Eq. (2.80). Recall that the axionic flux polynomials in Eq. (4.4) have been invoked as some peculiar flux combinations called *new generalized axionic flux polynomials* by considering a deep investigation of the flux superpotential and the D terms in the type IIB setting [64,65]. Moreover, it is interesting to note that these flux polynomials are also useful in the sense that they collectively satisfy the generic Bianchi identities as presented in Table XII.

It is worth recalling that all of the axionic flux polynomials given in Eqs. (4.1)–(4.2) for type IIA and Eqs. (4.3)–(4.4) for type IIB involve fluxes and all of the axions without having any dependence on the saxionic moduli. It is a tedious but straightforward computation to show that under the T-duality transformations, the various axionic flux polynomials are exchanged as presented in Table VI.

In order to prove that the axionic flux polynomials transform under T duality as per the rules given in Table VI, one can use the following type IIB to type IIA transformations at the intermediate stage of computation:

$$\mathbb{H}_0 \to \bar{\mathrm{H}}_0 + \bar{\mathrm{H}}_k z^k + \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n \bar{\mathrm{H}}^\lambda,$$
$$\mho_{a0} \to \bar{\mathrm{H}}_k + \bar{\mathrm{H}}^\lambda \hat{k}_{\lambda kn} z^n,$$
$$\hat{\mathbb{Q}}^{\alpha}{}_0 \to \bar{\mathrm{H}}^\lambda,$$

$$\mathbb{H}_i \to \bar{w}_{a0} + \bar{w}_{ak} z^k + \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n \bar{w}_a{}^\lambda,$$
$$\mho_{ai} \to \bar{w}_{ak} + \bar{w}_a{}^\lambda \hat{k}_{\lambda kn} z^n,$$
$$\hat{\mathbb{Q}}^{\alpha}{}_i \to \bar{w}_a{}^\lambda,$$

$$\mathbb{H}^i \to \mathrm{Q}^a{}_0 + \mathrm{Q}^a{}_k z^k + \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n \mathrm{Q}^{a\lambda},$$
$$\mho_a{}^i \to \mathrm{Q}^a{}_k + \mathrm{Q}^{a\lambda} \hat{k}_{\lambda kn} z^n,$$
$$\hat{\mathbb{Q}}^{\alpha i} \to \mathrm{Q}^{a\lambda},$$

$$\mathbb{H}^0 \to -\mathrm{R}_0 - \mathrm{R}_k z^k - \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n \mathrm{R}^\lambda,$$
$$\mho_a{}^0 \to -\mathrm{R}_k - \mathrm{R}^\lambda \hat{k}_{\lambda kn} z^n,$$
$$\hat{\mathbb{Q}}^{\alpha 0} \to -\mathrm{R}^\lambda,$$

$$\mathbb{F}_0 \to \bar{e}_0 - (\xi^{\hat{k}} \bar{\mathrm{H}}_{\hat{k}} + \xi_\lambda \bar{\mathrm{H}}^\lambda),$$
$$\mathbb{F}_i \to \bar{e}_a - (\xi^{\hat{k}} \bar{w}_{a\hat{k}} + \xi_\lambda \bar{w}_a{}^\lambda), \tag{4.5}$$

$$\mathbb{F}^i \to m^a - (\xi^{\hat{k}} \mathrm{Q}^a{}_{\hat{k}} + \xi_\lambda \mathrm{Q}^{a\lambda}),$$
$$\mathbb{F}^0 \to -m^0 + (\xi^{\hat{k}} \mathrm{R}_{\hat{k}} + \xi_\lambda \mathrm{R}^\lambda).$$

TABLE VI. Axionic flux polynomials under T duality.

IIA	f_0	f_a	f^a	f^0	h_0	h_k	h^k	h^0	h_{k0}	h_{ak}	$h^a{}_k$	$h_k{}^0$	$h^\lambda{}_0$	$h_a{}^\lambda$	$h^{a\lambda}$	$h^{\lambda 0}$
IIB	f_0	f_i	f^i	f^0	h_0	h_i	h^i	h^0	h_{a0}	h_{ai}	$h_a{}^i$	$h_a{}^0$	$h^\alpha{}_0$	$h^\alpha{}_i$	$h^{\alpha i}$	$h^{\alpha 0}$

The transformations for the D-term flux polynomials are given as

$$\mho_{\alpha K} \to \hat{w}_{\alpha\lambda} + \hat{w}_\alpha{}^k \hat{k}_{\lambda k m} z^m - \frac{1}{2}\hat{k}_{\lambda mn} z^m z^n \hat{w}_\alpha{}^0,$$
$$\mathbb{R}_K \to -w_\alpha{}^0, \tag{4.6}$$
$$\mho_\alpha{}^K \to \hat{\mathrm{Q}}^\alpha{}_\lambda + \hat{\mathrm{Q}}^{\alpha k}\hat{k}_{\lambda k m} z^m - \frac{1}{2}\hat{k}_{\lambda mn} z^m z^n \hat{\mathrm{Q}}^{\alpha 0},$$
$$\mathbb{R}^K \to -\hat{\mathrm{Q}}^\alpha{}_0.$$

Note that fluxes with a bar on top are the shifted fluxes, as defined in Eqs. (2.45) and (2.80).

B. Scalar potentials

For the scalar potential computations we mainly need to focus on rewriting the F-term contributions arising from the type IIA and type IIB superpotentials as presented in Eqs. (2.44) and (2.79), respectively. Also, for our scalar potential computations we will ignore the effects of all of the p_0's which depend on the Euler characteristics of the CY three-form and its mirror, as this creates unnecessary complexities in the various expressions in the respective scalar potentials, making it hard to enjoy the simple observations and their possibly easy utilities. However, we will continue to consider the prepotential terms with coefficients p_{ab}, p_a, $\tilde{p}_{ij}$, etc., which are linear and quadratic in the chiral variables (involving the saxions of the Kähler and complex-structure moduli), and so may remain relevant in some regime of the moduli space even after imposing the large-volume and large-complex-structure limit. In this limit, we can estimate the pieces with $\chi(\mathrm{CY})$ as

$$\mathcal{V} \gg \frac{p_0}{4} = -\frac{\zeta[3]\chi(\mathrm{CY})}{32\pi^3} \propto 10^{-3}\chi(\mathrm{CY}),$$
$$\mathcal{U} \gg \frac{\tilde{p}_0}{4} = -\frac{\zeta[3]\chi(\tilde{\mathrm{CY}})}{32\pi^3} \propto 10^{-3}\chi(\tilde{\mathrm{CY}}). \tag{4.7}$$

Therefore, for a trustworthy model building within a valid effective field theory description where one anyway demands $\mathcal{V} \gg 1$ and $\mathcal{U} \gg 1$, the above assumption we make is quite automatically justified, and it is very likely that the correction with p_0's will not be effective up to quite large value of the Euler characteristics of the CY and its mirror. Moreover, p_0 appears at $(\alpha')^3$ order in type IIA, and we keep corrections until $(\alpha')^2$ through p_{ab} and p_a, and therefore our assumption should be fairly justified. Given that all moduli should be present in the generic nongeometric scalar potential, it is natural to expect that all of them (at least the saxionic ones) are dynamically fixed; otherwise, the $(\alpha')^3$ effects with $\chi(\mathrm{CY})$ may become relevant at some subleading order.

1. Type IIB

With some tedious but conceptually straightforward computations using the axionic flux polynomials given in Eqs. (4.3)–(4.4) and following the strategy of Refs. [64,67,68], the total scalar potential generated as a sum of the F-term and D-term contributions for the type IIB orientifold compactifications (in the string frame) can be written as

$$V_{\mathrm{IIB}}^{\mathrm{toal}} \equiv V_{\mathrm{IIB}}^F + V_{\mathrm{IIB}}^D = V_{\mathrm{IIB}}^{\mathrm{RR}} + V_{\mathrm{IIB}}^{\mathrm{NS}} + V_{\mathrm{IIB}}^{\mathrm{loc}} + V_{\mathrm{IIB}}^D, \tag{4.8}$$

where the four pieces are given as follows:

$$\begin{aligned}
V_{\mathrm{IIB}}^{\mathrm{RR}} &= \frac{e^{4\phi}}{4\mathcal{V}^2\mathcal{U}}[f_0^2 + \mathcal{U} f^i \mathcal{G}_{ij} f^j + \mathcal{U} f_i \mathcal{G}^{ij} f_j + \mathcal{U}^2 (f^0)^2],\\
V_{\mathrm{IIB}}^{\mathrm{NS}} &= \frac{e^{2\phi}}{4\mathcal{V}^2\mathcal{U}}\Big[h_0^2 + \mathcal{U} h^i \mathcal{G}_{ij} h^j + \mathcal{U} h_i \mathcal{G}^{ij} h_j + \mathcal{U}^2 (h^0)^2\\
&\quad + \mathcal{V}\mathcal{G}^{ab}\Big(h_{a0}h_{b0} + \frac{l_i l_j}{4} h_a{}^i h_b{}^j + h_{ai}h_{bj}u^i u^j + \mathcal{U}^2 h_a{}^0 h_b{}^0 - \frac{l_i}{2} h_a{}^i h_{b0} - \frac{l_i}{2} h_{a0} h_b{}^i - \mathcal{U} u^i h_a{}^0 h_{bi} - \mathcal{U} u^i h_b{}^0 h_{ai}\Big)\\
&\quad + \mathcal{V}\mathcal{G}_{\alpha\beta}\Big(h^\alpha{}_0 h^\beta{}_0 + \frac{l_i l_j}{4} h^{\alpha i} h^{\beta j} + u^i u^j h^\alpha{}_i h^\beta{}_j + \mathcal{U}^2 h^{\alpha 0} h^{\beta 0} - \frac{l_i}{2} h^\alpha{}_0 h^{\beta i} - \frac{l_i}{2} h^{\alpha i} h^\beta{}_0 - \mathcal{U} u^i h^{\alpha 0} h^\beta{}_i - \mathcal{U} u^i h^\alpha{}_i h^{\beta 0}\Big)\\
&\quad + \frac{\ell_\alpha \ell_\beta}{4}\Big(\mathcal{U} h^{\alpha i}\mathcal{G}_{ij} h^{\beta j} + \mathcal{U} h^\alpha{}_i \mathcal{G}^{ij} h^\beta{}_j + \mathcal{U} u^i h^{\alpha 0} h_i{}^\beta + \mathcal{U} u^i h^\alpha{}_i h^{\beta 0} - u^i u^j h^\alpha{}_i h^\beta{}_j + \frac{l_i}{2} h^\alpha{}_0 h^{\beta i} + \frac{l_i}{2} h^{\alpha i} h^\beta{}_0 - \frac{l_i l_j}{4} h^{\alpha i} h^{\beta j}\Big)\\
&\quad - 2 \times \frac{\ell_\alpha}{2}\Big(\mathcal{U} h^i \mathcal{G}_{ij} h^{\alpha j} + \mathcal{U} h_i \mathcal{G}^{ij} h^\alpha{}_j + \mathcal{U} u^i h^0 h^\alpha{}_i + \mathcal{U} u^i h_i h^{\alpha 0} - u^i u^j h_i h^\alpha{}_j + \frac{l_i}{2} h^i h^\alpha{}_0 + \frac{l_i}{2} h_0 h^{\alpha i} - \frac{l_i l_j}{4} h^i h^{\alpha j}\Big)\Big],\\
V_{\mathrm{IIB}}^{\mathrm{loc}} &= \frac{e^{3\phi}}{2\mathcal{V}^2}\Big[(f^0 h_0 - f^i h_i + f_i h^i - f_0 h^0) - (f^0 h^\alpha{}_0 - f^i h^\alpha{}_i + f_i h^{\alpha i} - f_0 h^{\alpha 0})\frac{\ell_\alpha}{2}\Big],\\
V_{\mathrm{IIB}}^D &= \frac{e^{2\phi}}{4\mathcal{V}^2}[(\mathcal{V}\hat{h}_J{}^0 - t^\alpha \hat{h}_{\alpha J})\mathcal{G}^{JK}(\mathcal{V}\hat{h}_K{}^0 - t^\beta \hat{h}_{\beta K}) + (\mathcal{V}\hat{h}^{J0} - t^\alpha \hat{h}_\alpha{}^J)\mathcal{G}_{JK}(\mathcal{V}\hat{h}^{K0} - t^\beta \hat{h}_\beta{}^K)].
\end{aligned} \tag{4.9}$$

Here, using $\mathcal{V} = \frac{1}{6}\ell_{\alpha\beta\gamma}t^\alpha t^\beta t^\gamma, \mathcal{U} = \frac{1}{6}l_{ijk}u^i u^j u^k$, etc. as shorthand notations, we have the following form of the moduli space metrics:

$$\mathcal{G}_{ij} = \frac{l_i l_j - 4\mathcal{U} l_{ij}}{4\mathcal{U}}, \qquad \mathcal{G}^{ij} = \frac{2u^i u^j - 4\mathcal{U} l^{ij}}{4\mathcal{U}}, \qquad \mathcal{G}_{JK} = -\hat{l}_{JK}, \qquad \mathcal{G}^{JK} = -\hat{l}^{JK},$$
$$\mathcal{G}_{\alpha\beta} = \frac{\ell_\alpha \ell_\beta - 4\mathcal{V}\ell_{\alpha\beta}}{4\mathcal{V}}, \qquad \mathcal{G}^{\alpha\beta} = \frac{2t^\alpha t^\beta - 4\mathcal{V}\ell^{\alpha\beta}}{4\mathcal{V}}, \qquad \mathcal{G}^{ab} = -\hat{\ell}^{ab}, \qquad \mathcal{G}_{ab} = -\hat{\ell}_{ab}. \tag{4.10}$$

2. Type IIA

Although it is equally tedious to compute the scalar potential from the flux superpotential for the type IIA case, one can show that by using our axionic flux polynomials given in Eqs. (4.1)–(4.2) and following the strategy of Ref. [69] the total scalar potential for the type IIA orientifold compactifications (in the string frame) can be written as

$$V_{\rm IIA}^{\rm tot} \equiv V_{\rm IIA}^F + V_{\rm IIA}^D = V_{\rm IIA}^{\rm RR} + V_{\rm IIA}^{\rm NS} + V_{\rm IIA}^{\rm loc} + V_{\rm IIA}^D, \tag{4.11}$$

where the four pieces are given explicitly as follows:

$$\begin{aligned}
V_{\rm IIA}^{\rm RR} &= \frac{e^{4D_{4d}}}{4\mathcal{V}}[f_0^2 + \mathcal{V} f^a \tilde{\mathcal{G}}_{ab} f^b + \mathcal{V} f_a \tilde{\mathcal{G}}^{ab} f_b + \mathcal{V}^2 (f^0)^2],\\
V_{\rm IIA}^{\rm NS} &= \frac{e^{2D_{4d}}}{4\mathcal{U}\mathcal{V}}\Big[h_0^2 + \mathcal{V} h^a \tilde{\mathcal{G}}_{ab} h^b + \mathcal{V} h_a \tilde{\mathcal{G}}^{ab} h_b + \mathcal{V}^2 (h^0)^2\\
&\quad + \mathcal{U}\tilde{\mathcal{G}}^{ij}\Big(h_{i0}h_{j0} + \frac{\kappa_a\kappa_b}{4} h_i{}^a h_j{}^b + h_{ai}h_{bj}t^a t^b + \mathcal{V}^2 h_i{}^0 h_j{}^0 - \frac{\kappa_a}{2} h^a{}_i h_{j0} - \frac{\kappa_a}{2} h_{i0} h^a{}_j - \mathcal{V} t^a h_i{}^0 h_{aj} - \mathcal{V} t^a h_{ai} h_j{}^0\Big)\\
&\quad + \mathcal{U}\tilde{\mathcal{G}}_{\lambda\rho}\Big(h^\lambda{}_0 h^\rho{}_0 + \frac{\kappa_a\kappa_b}{4} h^{\lambda a} h^{\rho b} + t^a t^b h_a{}^\lambda h_b{}^\rho + \mathcal{V}^2 h^{\lambda 0} h^{\rho 0} - \frac{\kappa_a}{2} h^\lambda{}_0 h^{\rho a} - \frac{\kappa_a}{2} h^{\lambda a} h^\rho{}_0 - \mathcal{V} t^a h^{\lambda 0} h_a{}^\rho - \mathcal{V} t^a h_a{}^\lambda h^{\rho 0}\Big)\\
&\quad + \frac{k_\lambda k_\rho}{4}\Big(\mathcal{V} h^{a\lambda}\tilde{\mathcal{G}}_{ab} h^{b\rho} + \mathcal{V} h_a{}^\lambda \tilde{\mathcal{G}}^{ab} h_b{}^\rho + \mathcal{V} t^a h^{\lambda 0} h_a{}^\rho + \mathcal{V} t^a h_a{}^\lambda h^{\rho 0} - t^a t^b h_a{}^\lambda h_b{}^\rho + \frac{\kappa_a}{2} h^\lambda{}_0 h^{a\rho} + \frac{\kappa_a}{2} h^{a\lambda} h^\beta{}_0 - \frac{\kappa_a\kappa_b}{4} h^{a\lambda} h^{b\rho}\Big)\\
&\quad - 2\times\frac{k_\lambda}{2}\Big(\mathcal{V} h^a \tilde{\mathcal{G}}_{ab} h^{b\lambda} + \mathcal{V} h_a \tilde{\mathcal{G}}^{ab} h_b{}^\lambda + \mathcal{V} t^a h^0 h_a{}^\lambda + \mathcal{V} t^a h_a h^{\lambda 0} - t^a t^b h_a h_b{}^\lambda + \frac{\kappa_a}{2} h^a h_0{}^\lambda + \frac{\kappa_a}{2} h_0 h^{a\lambda} - \frac{\kappa_a\kappa_b}{4} h^a h^{b\lambda}\Big)\Big],\\
V_{\rm IIA}^{\rm loc} &= \frac{e^{3D_{4d}}}{2\sqrt{\mathcal{U}}}\Big[(f^0 h_0 - f^a h_a + f_a h^a - f_0 h^0) - (f^0 h^\lambda{}_0 - f^a h^\lambda{}_a + f_a h^{\lambda a} - f_0 h^{\lambda 0})\frac{k_\lambda}{2}\Big],\\
V_{\rm IIA}^D &= \frac{e^{2D_{4d}}}{4\mathcal{U}}[(\mathcal{U}\hat{h}_\alpha{}^0 + z^\lambda \hat{h}_{\alpha\lambda})\tilde{\mathcal{G}}^{\alpha\beta}(\mathcal{U}\hat{h}_\beta{}^0 + z^\rho \hat{h}_{\beta\rho}) + (\mathcal{U}\hat{h}^{\alpha 0} + z^\lambda \hat{h}^\alpha{}_\lambda)\tilde{\mathcal{G}}_{\alpha\beta}(\mathcal{U}\hat{h}^{\beta 0} + z^\rho \hat{h}^\beta{}_\rho)],
\end{aligned} \tag{4.12}$$

where

$$\tilde{\mathcal{G}}_{ab} = \frac{\kappa_a\kappa_b - 4\mathcal{V}\kappa_{ab}}{4\mathcal{V}}, \qquad \tilde{\mathcal{G}}^{ab} = \frac{2t^a t^b - 4\mathcal{V}\kappa^{ab}}{4\mathcal{V}}, \qquad \tilde{\mathcal{G}}^{\alpha\beta} = -\hat{\kappa}^{\alpha\beta}, \qquad \tilde{\mathcal{G}}_{\alpha\beta} = -\hat{\kappa}_{\alpha\beta},$$
$$\tilde{\mathcal{G}}_{\lambda\rho} = \frac{k_\lambda k_\rho - 4\mathcal{U} k_{\lambda\rho}}{4\mathcal{U}}, \qquad \tilde{\mathcal{G}}^{\lambda\rho} = \frac{2z^\lambda z^\rho - 4\mathcal{U} k^{\lambda\rho}}{4\mathcal{U}}, \qquad \tilde{\mathcal{G}}^{jk} = -\hat{k}^{jk}, \qquad \tilde{\mathcal{G}}_{jk} = -\hat{k}_{jk}. \tag{4.13}$$

Note that we have $\mathcal{V} = \frac{1}{6}\kappa_{abc}t^a t^b t^c$, $\mathcal{U} = \frac{1}{6}k_{\lambda\rho\gamma}z^\lambda u^j u^k$ for the type IIA case, and we have also used $e^{K_q} = e^{4D_{4d}} = \frac{(z^0)^4}{\mathcal{U}^2}$ from Eq. (2.34) to restore the popular factor of $e^{4D_{4d}}$ in the RR sector and $e^{2D_{4d}}$ in the NS-NS sector and the D-term contributions, along with a factor of $e^{3D_{4d}}$ in the local piece.

V. APPLICATIONS

In this section we illustrate the utilities of our scalar potential formulation by considering two explicit toroidal examples. All we need to know is the orientifold even/odd Hodge numbers and some of the topological quantities such as nonvanishing triple intersection numbers, etc., and the rest will subsequently follow from our formulation. Therefore, it can be considered as a direct way of computing the scalar potential with explicit dependence on the saxionic and axionic moduli.

A. Type IIA on a $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ orientifold

Considering the untwisted sector with the nongeometric type IIA setup having the standard involution (e.g., see Refs. [58,69] for details), we can begin to extract information from our formulation for this model by starting with the following input:

$$h^{1,1}_{-} = 3, \qquad h^{1,1}_{+} = 0, \qquad h^{2,1} = 3. \tag{5.1}$$

The Hodge numbers show that there are three U_λ moduli and three T^a moduli along with a single N^0 modulus. There are no N^k moduli present as the even (1,1) cohomology is trivial. Subsequently, it turns out that all of the fluxes with index k are absent. There are four components for both the H_3 flux (namely, H_0 and H^λ) and the nongeometric R flux, which are denoted as R_0 and R^λ for $\lambda\in\{1,2,3\}$. In addition, there are 12 flux components for both the geometric (w) flux and the nongeometric (Q) flux, denoted as $\{w_{a0}, w_a{}^\lambda\}$ and $\{\mathrm{Q}^a{}_\lambda, \mathrm{Q}^{a\lambda}\}$ for $\alpha\in\{1,2,3\}$ and $\lambda\in\{1,2,3\}$. On the RR side, there are eight flux components in total: one from each of the F_0 and F_6 fluxes denoted as m_0 and e_0, and three from each of the F_2 and F_4 fluxes denoted as m^a, e_a for $a\in\{1,2,3\}$. In addition, we also note that there will be no D terms generated in the scalar potential as the even (1,1) cohomology is trivial, which projects out all of the relevant D-term fluxes. Having the above orientifold-related ingredients in hand, one can directly read off the scalar potential pieces from our generic formula in two steps.

(1) Step 1: Work out all of the axionic flux polynomials.
(2) Step 2: Work out the moduli-space metric.

1. Step 1

The following eight types of NS-NS axionic flux polynomials are trivial in this model:

$$h_k = 0, \qquad h_{ak} = 0, \qquad h_a{}^k = 0, \qquad h_k{}^0 = 0,$$
$$\hat{h}_\alpha{}^0 = 0, \qquad \hat{h}_{\alpha\lambda} = 0, \qquad \hat{h}_{\alpha 0} = 0, \qquad \hat{h}^\alpha{}_\lambda = 0, \tag{5.2}$$

where one can anticipate from the trivial cohomology indices that such fluxes are absent. Further, using Eq. (4.1), the eight classes of nonzero NS-NS axionic flux polynomials can be explicitly written out in terms of the 32 flux combinations, along with eight flux polynomials coming from the RR sector given in the following manner:

$$f_0 = \mathbb{G}_0 - \xi^0\mathcal{H}_0 - \xi_\lambda\mathcal{H}^\lambda, \qquad f_a = \mathbb{G}_a - \xi^0\mho_{a0} - \xi_\lambda\mho_a{}^\lambda,$$
$$f^a = \mathbb{G}^a - \xi^0\mathcal{Q}^a{}_0 - \xi_\lambda\mathcal{Q}^{a\lambda}, \qquad f^0 = \mathbb{G}^0 - \xi^0\mathcal{R}_0 - \xi_\lambda\mathcal{R}^\lambda,$$
$$h_0 = \mathcal{H}_0, \qquad h_a = \mho_{a0}, \qquad h^a = \mathcal{Q}^a{}_0, \qquad h^0 = \mathcal{R}_0,$$
$$h^\lambda{}_0 = \mathcal{H}^\lambda, \qquad h_a{}^\lambda = \mho_a{}^\lambda, \qquad h^{a\lambda} = \mathcal{Q}^{a\lambda}, \qquad h^{\lambda 0} = \mathcal{R}^\lambda, \tag{5.3}$$

where the axionic flux polynomials in Eq. (5.3) are given as

$$\mathcal{H}_0 = \mathrm{H}_0 + w_{10}b^1 + w_{20}b^2 + w_{30}b^3 + b^1b^2\mathrm{Q}^3{}_0 + b^2b^3\mathrm{Q}^1{}_0 + b^3b^1\mathrm{Q}^2{}_0 + b^1b^2b^3\mathrm{R}_0,$$
$$\mho_{10} = w_{10} + b^2\mathrm{Q}^3{}_0 + \mathrm{b}^3\mathrm{Q}^2{}_0 + b^2b^3\mathrm{R}_0, \qquad \mathcal{Q}^1{}_0 = \mathrm{Q}^1{}_0 + b^1\mathrm{R}_0,$$
$$\mho_{20} = w_{20} + b^1\mathrm{Q}^3{}_0 + b^3\mathrm{Q}^1{}_0 + b^1b^3\mathrm{R}_0, \qquad \mathcal{Q}^2{}_0 = \mathrm{Q}^2{}_0 + b^2\mathrm{R}_0,$$
$$\mho_{30} = w_{30} + b^1\mathrm{Q}^2{}_0 + b^2\mathrm{Q}^1{}_0 + b^1b^2\mathrm{R}_0, \qquad \mathcal{Q}^3{}_0 = \mathrm{Q}^3{}_0 + b^3\mathrm{R}_0, \qquad \mathcal{R}_0 = \mathrm{R}_0,$$

$$\mathcal{H}^\lambda = \mathrm{H}^\lambda + w_1{}^\lambda b^1 + w_2{}^\lambda b^2 + w_3{}^\lambda b^3 + b^1b^2\mathrm{Q}^{3\lambda} + b^2b^3\mathrm{Q}^{1\lambda} + b^3b^1\mathrm{Q}^{2\lambda} + b^1b^2b^3\mathrm{R}^\lambda,$$
$$\mho_1{}^\lambda = w_1{}^\lambda + b^2\mathrm{Q}^{3\lambda} + b^3\mathrm{Q}^{2\lambda} + b^2b^3\mathrm{R}^\lambda, \qquad \mathcal{Q}^{1\lambda} = \mathrm{Q}^{1\lambda} + b^1\mathrm{R}^\lambda,$$
$$\mho_2{}^\lambda = w_2{}^\lambda + b^1\mathrm{Q}^{3\lambda} + b^3\mathrm{Q}^{1\lambda} + b^1b^3\mathrm{R}^\lambda, \qquad \mathcal{Q}^{2\lambda} = \mathrm{Q}^{2\lambda} + b^2\mathrm{R}^\lambda,$$
$$\mho_3{}^\lambda = w_3{}^\lambda + b^1\mathrm{Q}^{2\lambda} + b^2\mathrm{Q}^{1\lambda} + b^1b^2\mathrm{R}^\lambda, \qquad \mathcal{Q}^{3\lambda} = \mathrm{Q}^{3\lambda} + b^3\mathrm{R}^\lambda, \qquad \mathcal{R}^\lambda = \mathrm{R}^\lambda,$$

$$\mathbb{G}_0 = e_0 + b^1e_1 + b^2e_2 + b^3e_3 + b^1b^2m^3 + b^2b^3m^1 + b^3b^1m^2 + b^1b^2b^3m_0,$$
$$\mathbb{G}_1 = e_1 + b^2m^3 + b^3m^2 + b^2b^3m_0, \qquad \mathbb{G}^1 = m^1 + m_0b^1,$$
$$\mathbb{G}_2 = e_2 + b^1m^3 + b^3m^1 + b^1b^3m_0, \qquad \mathbb{G}^2 = m^2 + m_0b^2,$$
$$\mathbb{G}_3 = e_3 + b^1m^2 + b^2m^1 + b^1b^2m_0, \qquad \mathbb{G}^3 = m^3 + m_0b^3, \qquad \mathbb{G}^0 = m_0.$$

In simplifying the axionic flux polynomials we have used the fact that the only nonzero intersection number that survives in the Kähler moduli part of the prepotential is $\kappa_{123} = 1$. The same thing happens on the complex structure moduli side:

$$\kappa_{123} = 1, \qquad \hat{\kappa}_{a\alpha\beta} = 0, \qquad k_{123} = 1, \qquad \hat{k}_{\lambda mn} = 0. \tag{5.4}$$

2. Step 2

In order to fully know the scalar potential, we now only need to know the moduli-space metrics to supplement the axionic flux polynomials, which are given as

$$\kappa_{ab} = \begin{pmatrix} 0 & t^3 & t^2 \\ t^3 & 0 & t^1 \\ t^2 & t^1 & 0 \end{pmatrix}, \qquad -4\mathcal{V}\kappa^{ab} = \begin{pmatrix} 2(t^1)^2 & -2t^1t^2 & -2t^1t^3 \\ -2t^1t^2 & 2(t^2)^2 & -2t^2t^3 \\ -2t^1t^3 & -2t^2t^3 & 2(t^3)^2 \end{pmatrix},$$
$$\mathcal{V}\tilde{\mathcal{G}}^{ab} = \begin{pmatrix} (t^1)^2 & 0 & 0 \\ 0 & (t^2)^2 & 0 \\ 0 & 0 & (t^3)^2 \end{pmatrix}, \qquad \mathcal{U}\tilde{\mathcal{G}}^{\lambda\rho} = \begin{pmatrix} (z^1)^2 & 0 & 0 \\ 0 & (z^2)^2 & 0 \\ 0 & 0 & (z^3)^2 \end{pmatrix}.$$

In addition, we also have the following useful shorthand notations:

$$\begin{aligned} &\mathcal{V} = t^1t^2t^3, \qquad \kappa_1 = 2t^2t^3, \qquad \kappa_2 = 2t^1t^3, \qquad \kappa_3 = 2t^1t^2, \\ &\mathcal{U} = z^1z^2z^3, \qquad k_1 = 2z^2z^3, \qquad k_2 = 2z^1z^3, \qquad k_3 = 2z^1z^2. \end{aligned} \tag{5.5}$$

To verify our scalar potential formulation, first we compute it from the flux superpotential as given in Eq. (2.44), which results in 2422 terms. Subsequently, we show that our collection of pieces gives the same result after using the simplified axionic flux polynomials and the moduli-space metrics as presented above. These scalar potential pieces are given as

$$\begin{aligned}
V_{\rm IIA}^{\rm RR} =& \frac{e^{4D_{4d}}}{4\mathcal{V}}[f_0^2 + \mathcal{V}f^a\tilde{\mathcal{G}}_{ab}f^b + \mathcal{V}f_a\tilde{\mathcal{G}}^{ab}f_b + \mathcal{V}^2(f^0)^2], \\
V_{\rm IIA}^{\rm NS1} =& \frac{e^{2D_{4d}}}{4\mathcal{U}\mathcal{V}}[h_0^2 + \mathcal{V}h^a\tilde{\mathcal{G}}_{ab}h^b + \mathcal{V}h_a\tilde{\mathcal{G}}^{ab}h_b + \mathcal{V}^2(h^0)^2], \\
V_{\rm IIA}^{\rm NS2} =& \frac{e^{2D_{4d}}}{4\mathcal{U}\mathcal{V}}\Bigg[\mathcal{U}\tilde{\mathcal{G}}_{\lambda\rho}\Bigg(h^\lambda{}_0 h^\rho{}_0 + \frac{\kappa_a\kappa_b}{4}h^{\lambda a}h^{\rho b} + t^a t^b h_a{}^\lambda h_b{}^\rho + \mathcal{V}^2 h^{\lambda 0}h^{\rho 0} - \frac{\kappa_a}{2}h^\lambda{}_0 h^{\rho a} - \frac{\kappa_a}{2}h^{\lambda a}h^\rho{}_0 - \mathcal{V}t^a h^{\lambda 0}h_a{}^\rho - \mathcal{V}t^a h_a{}^\lambda h^{\rho 0}\Bigg) \\
&+ \frac{k_\lambda k_\rho}{4}\Bigg(\mathcal{V}h^{a\lambda}\tilde{\mathcal{G}}_{ab}h^{b\rho} + \mathcal{V}h_a{}^\lambda\tilde{\mathcal{G}}^{ab}h_b{}^\rho + \mathcal{V}t^a h^{\lambda 0}h_a{}^\rho + \mathcal{V}t^a h_a{}^\lambda h^{\rho 0} - t^a t^b h_a{}^\lambda h_b{}^\rho + \frac{\kappa_a}{2}h^\lambda{}_0 h^{a\rho} + \frac{\kappa_a}{2}h^{a\lambda}h^\beta{}_0 - \frac{\kappa_a\kappa_b}{4}h^{a\lambda}h^{b\rho}\Bigg)\Bigg], \\
V_{\rm IIA}^{\rm NS3} =& \frac{e^{2D_{4d}}}{4\mathcal{U}\mathcal{V}}\Bigg[-2\times\frac{k_\lambda}{2}\Bigg(\mathcal{V}h^a\tilde{\mathcal{G}}_{ab}h^{b\lambda} + \mathcal{V}{\rm h}_a\tilde{\mathcal{G}}^{ab}h_b{}^\lambda + \mathcal{V}t^a h^0 h_a{}^\lambda + \mathcal{V}t^a h_a h^{\lambda 0} - t^a t^b h_a h_b{}^\lambda + \frac{\kappa_a}{2}h^a h_0{}^\lambda + \frac{\kappa_a}{2}h_0 h^{a\lambda} - \frac{\kappa_a\kappa_b}{4}h^a h^{b\lambda}\Bigg)\Bigg], \\
V_{\rm IIA}^{\rm loc} =& \frac{e^{3D_{4d}}}{2\sqrt{\mathcal{U}}}\Bigg[(f^0h_0 - f^a h_a + f_a h^a - f_0 h^0) - (f^0 h^\lambda{}_0 - f^a h^\lambda{}_a + f_a h^{\lambda a} - f_0 h^{\lambda 0})\frac{k_\lambda}{2}\Bigg].
\end{aligned} \tag{5.6}$$

To appreciate the numerics, we mention that the above pieces of the scalar potential match the following splitting of 2422 terms computed from the superpotential:

$$\begin{aligned} &\#(V_{\rm IIB}^{\rm RR}) = 1630, \qquad \#(V_{\rm IIB}^{\rm NS1}) = 76, \qquad \#(V_{\rm IIB}^{\rm NS2}) = 408, \\ &\#(V_{\rm IIB}^{\rm NS3}) = 180, \qquad \#(V_{\rm IIB}^{\rm loc}) = 128. \end{aligned} \tag{5.7}$$

B. Type IIB on a $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ orientifold

Considering the untwisted sector with the standard involution for the nongeometric type IIB setup (e.g., see Refs. [58,61,67,68] for details), we can start with the following input:

$$h^{1,1}_+ = 3, \qquad h^{1,1}_- = 0, \qquad h^{2,1}_+ = 0, \qquad h^{2,1}_- = 3. \tag{5.8}$$

The Hodge numbers show that there are three T_α moduli and three U^i moduli along with the universal axio-dilaton S in this setup. There are no odd-moduli G^a present in this setup as the odd (1,1) cohomology is trivial. It turns out that the geometric flux ω and nongeometric R flux do not survive the orientifold projection in this setup, and the only allowed NS-NS fluxes are the three-form H_3 flux and nongeometric Q flux. There are eight components for the H_3 flux and 24 components for the Q flux, denoted as H_Λ, H^Λ, $\hat{Q}^\alpha{}_\Lambda$, $\hat{Q}^{\alpha\Lambda}$ for $\alpha \in \{1,2,3\}$ and $\Lambda \in \{0,1,2,3\}$. On the RR side, there are eight flux components for the three-form F_3 flux. In addition, there are no D terms generated in the scalar potential as the even (2,1) cohomology is trivial, which projects out all of the D-term fluxes. Now we repeat the two steps followed for the type IIA case.

1. Step 1

It turns out that the following eight NS-NS axionic flux polynomials are trivial in this model:

$$h_a = 0, \qquad h_{ai} = 0, \qquad h_a{}^i = 0, \qquad h_a{}^0 = 0,$$
$$\hat{h}_{\alpha K} = 0, \qquad \hat{h}_\alpha{}^K = 0, \qquad \hat{h}_{K0} = 0, \qquad \hat{h}^{K0} = 0, \tag{5.9}$$

where one can anticipate from the trivial cohomology indices that such fluxes are absent. Further, using Eq. (4.3), the eight classes of nonzero NS-NS axionic flux polynomials can be explicitly written out in terms of the 32 flux combinations as

$$\begin{aligned}
&h_0 = H_0 + v^1 H_1 + v^2 H_2 + v^3 H_3 + v^1 v^2 H^3 + v^2 v^3 H^1 + v^3 v^1 H^2 - v^1 v^2 v^3 H^0,\\
&h_1 = H_1 + v^2 H^3 + v^3 H^2 - v^2 v^3 H^0, \qquad h^1 = H^1 - v^1 H^0,\\
&h_2 = H_2 + v^1 H^3 + v^3 H^1 - v^1 v^3 H^0, \qquad h^2 = H^2 - v^2 H^0,\\
&h_3 = H_3 + v^1 H^2 + v^2 H^1 - v^1 v^2 H^0, \qquad h^3 = H^3 - v^3 H^0, \qquad h^0 = -H^0,\\
&h^\alpha{}_0 = \hat{Q}^\alpha{}_0 + v^1 \hat{Q}^\alpha{}_1 + v^2 \hat{Q}^\alpha{}_2 + v^3 \hat{Q}^\alpha{}_3 + v^1 v^2 \hat{Q}^{\alpha 3} + v^2 v^3 \hat{Q}^{\alpha 1} + v^3 v^1 \hat{Q}^{\alpha 2} - v^1 v^2 v^3 \hat{Q}^{\alpha 0},\\
&h^\alpha{}_1 = \hat{Q}^\alpha{}_1 + v^2 \hat{Q}^{\alpha 3} + v^3 \hat{Q}^{\alpha 2} - v^2 v^3 \hat{Q}^{\alpha 0}, \qquad h^{\alpha 1} = \hat{Q}^{\alpha 1} - v^1 \hat{Q}^{\alpha 0},\\
&h^\alpha{}_2 = \hat{Q}^\alpha{}_2 + v^1 \hat{Q}^{\alpha 3} + v^3 \hat{Q}^{\alpha 1} - v^1 v^3 \hat{Q}^{\alpha 0}, \qquad h^{\alpha 2} = \hat{Q}^{\alpha 2} - v^2 \hat{Q}^{\alpha 0},\\
&h^\alpha{}_3 = \hat{Q}^\alpha{}_3 + v^2 \hat{Q}^{\alpha 1} + v^1 \hat{Q}^{\alpha 2} - v^1 v^2 \hat{Q}^{\alpha 0}, \qquad h^{\alpha 3} = \hat{Q}^{\alpha 3} - v^3 \hat{Q}^{\alpha 0}, \qquad h^{\alpha 0} = -\hat{Q}^{\alpha 0}.
\end{aligned} \tag{5.10}$$

In addition, there are eight axionic flux polynomials which also involve the RR axions c_0 and c_α along with the complex structure axions v^i, which are given as

$$\begin{aligned}
&f_0 = \mathbb{F}_0 + v^1 \mathbb{F}_1 + v^2 \mathbb{F}_2 + v^3 \mathbb{F}_3 + v^1 v^2 \mathbb{F}^3 + v^2 v^3 \mathbb{F}^1 + v^3 v^1 \mathbb{F}^2 - v^1 v^2 v^3 \mathbb{F}^0,\\
&f_1 = \mathbb{F}_1 + v^2 \mathbb{F}^3 + v^3 \mathbb{F}^2 - v^2 v^3 \mathbb{F}^0, \qquad f^1 = \mathbb{F}^1 - v^1 \mathbb{F}^0,\\
&f_2 = \mathbb{F}_2 + v^1 \mathbb{F}^3 + v^3 \mathbb{F}^1 - v^1 v^3 \mathbb{F}^0, \qquad f^2 = \mathbb{F}^2 - v^2 \mathbb{F}^0,\\
&f_3 = \mathbb{F}_3 + v^1 \mathbb{F}^2 + v^2 \mathbb{F}^1 - v^1 v^2 \mathbb{F}^0, \qquad f^3 = \mathbb{F}^3 - v^3 \mathbb{F}^0, \qquad f^0 = -\mathbb{F}^0,
\end{aligned} \tag{5.11}$$

$$\begin{aligned}
\mathbb{F}_0 &= F_0 - \hat{Q}^\alpha{}_0 c_\alpha - c_0 H_0, \qquad \mathbb{F}_i = F_i - \hat{Q}^\alpha{}_i c_\alpha - c_0 H_i\\
\mathbb{F}^0 &= F^0 - \hat{Q}^{\alpha 0} c_\alpha - c_0 H^0, \qquad \mathbb{F}^i = F^i - \hat{Q}^{\alpha i} c_\alpha - c_0 H^i.
\end{aligned}$$

Here we have used the fact that the only nonzero intersection numbers are given as

$$l_{123} = 1, \qquad \hat{l}_{iJK} = 0, \qquad \ell_{123} = 1, \qquad \hat{\ell}_{\alpha ab} = 0, \tag{5.12}$$

which result in the following useful shorthand notations:

$$\begin{aligned}
&\mathcal{V} = t^1 t^2 t^3, \qquad \ell_1 = 2t^2 t^3, \qquad \ell_2 = 2t^1 t^3, \qquad \ell_3 = 2t^1 t^2,\\
&\mathcal{U} = u^1 u^2 u^3, \qquad l_1 = 2u^2 u^3, \qquad l_2 = 2u^1 u^3, \qquad l_3 = 2u^1 u^2.
\end{aligned} \tag{5.13}$$

2. Step 2

In order to fully know the scalar potential, we now only need to know the moduli-space metrics to supplement the axionic flux polynomials, which are given as

$$\mathcal{V}\mathcal{G}^{\alpha\beta} = \begin{pmatrix} (t^1)^2 & 0 & 0 \\ 0 & (t^2)^2 & 0 \\ 0 & 0 & (t^3)^2 \end{pmatrix}, \qquad \mathcal{U}\mathcal{G}^{ij} = \begin{pmatrix} (u^1)^2 & 0 & 0 \\ 0 & (u^2)^2 & 0 \\ 0 & 0 & (u^3)^2 \end{pmatrix}.$$

To verify the scalar potential formulation, we first compute it using the flux superpotential as given in Eq. (2.79), which results in 2422 terms, and subsequently we confirm that our following collection of pieces gives the same result after using the simplified axionic flux polynomials and moduli-space metrics:

$$
\begin{aligned}
V_{\rm IIB}^{\rm RR} &= \frac{e^{4\phi}}{4\mathcal{V}^2\mathcal{U}}[f_0^2 + \mathcal{U} f^i \mathcal{G}_{ij} f^j + \mathcal{U} f_i \mathcal{G}^{ij} f_j + \mathcal{U}^2 (f^0)^2],\\
V_{\rm IIB}^{\rm NS1} &= \frac{e^{2\phi}}{4\mathcal{V}^2\mathcal{U}}[h_0^2 + \mathcal{U} h^i \mathcal{G}_{ij} h^j + \mathcal{U} h_i \mathcal{G}^{ij} h_j + \mathcal{U}^2 (h^0)^2],\\
V_{\rm IIB}^{\rm NS2} &= \frac{e^{2\phi}}{4\mathcal{V}^2\mathcal{U}}\left[\mathcal{V}\mathcal{G}_{\alpha\beta}\left(h^\alpha{}_0 h^\beta{}_0 + \frac{l_i l_j}{4} h^{\alpha i} h^{\beta j} + u^i u^j h^\alpha{}_i h^\beta{}_j + \mathcal{U}^2 h^{\alpha 0} h^{\beta 0} - \frac{l_i}{2} h^\alpha{}_0 h^{\beta i} - \frac{l_i}{2} h^{\alpha i} h^\beta{}_0 - \mathcal{U} u^i h^{\alpha 0} h^\beta{}_i - \mathcal{U} u^i h^\alpha{}_i h^{\beta 0}\right)\right.\\
&\quad \left. + \frac{\ell_\alpha \ell_\beta}{4}\left(\mathcal{U} h^{\alpha i}\mathcal{G}_{ij} h^{\beta j} + \mathcal{U} h^\alpha{}_i \mathcal{G}^{ij} h^\beta{}_j + \mathcal{U} u^i h^{\alpha 0} h_i{}^\beta + \mathcal{U} u^i h^\alpha{}_i h^{\beta 0} - u^i u^j h^\alpha{}_i h^\beta{}_j + \frac{l_i}{2} h^\alpha{}_0 h^{\beta i} + \frac{l_i}{2} h^{\alpha i} h^\beta{}_0 - \frac{l_i l_j}{4} h^{\alpha i} h^{\beta j}\right)\right],\\
V_{\rm IIB}^{\rm NS3} &= \frac{e^{2\phi}}{4\mathcal{V}^2\mathcal{U}}\left[-2\times\frac{\ell_\alpha}{2}\left(\mathcal{U} h^i \mathcal{G}_{ij} h^{\alpha j} + \mathcal{U} h_i \mathcal{G}^{ij} h^\alpha{}_j + \mathcal{U} u^i h^0 h^\alpha{}_i + \mathcal{U} u^i h_i h^{\alpha 0} - u^i u^j h_i h^\alpha{}_j + \frac{l_i}{2} h^i h^\alpha{}_0 + \frac{l_i}{2} h_0 h^{\alpha i} - \frac{l_i l_j}{4} h^i h^{\alpha j}\right)\right],\\
V_{\rm IIB}^{\rm loc} &= \frac{e^{3\phi}}{2\mathcal{V}^2}\left[(f^0 h_0 - f^i h_i + f_i h^i - f_0 h^0) - (f^0 h^\alpha{}_0 - f^i h^\alpha{}_i + f_i h^{\alpha i} - f_0 h^{\alpha 0})\frac{\ell_\alpha}{2}\right].
\end{aligned}
\tag{5.14}
$$

These match the following splitting of 2422 terms computed from the superpotential:

$$
\#(V_{\rm IIB}^{\rm RR}) = 1630, \qquad \#(V_{\rm IIB}^{\rm NS1}) = 76, \qquad \#(V_{\rm IIB}^{\rm NS2}) = 408,
$$
$$
\#(V_{\rm IIB}^{\rm NS3}) = 180, \qquad \#(V_{\rm IIB}^{\rm loc}) = 128. \tag{5.15}
$$

Thus, we have explicitly verified our generic type IIA potential in Eq. (4.12) and type IIB potential in Eq. (4.9) for the $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ orientifold setups, in which there are no D terms present while the F-term contribution results in precisely the same number (2242) of terms in the scalar potential as it could be found by their respective flux superpotential computations! It is needless to say that there is a perfect match for the two scalar potentials under our T-duality transformation for this canonical T-dual pair of models.

It is quite impressive to have written thousands of terms in just a few lines and kept the information about the saxionic and axionic parts distinct! These generic toroidal type IIA and IIB setups have been found to be interesting in several numerical approaches [10,11,16,17,21], and our formulation certainly opens up the possibilities for making attempts towards nonsupersymmetric moduli stabilization in an analytic approach.

VI. SUMMARY AND CONCLUSIONS

In this article we have studied the T-dual completion of the four-dimensional type IIA and type IIB effective supergravity theories with the presence of (non)geometric fluxes. In order to establish a single consistent convention and notation by fixing signs, factors, etc., we first revisited the relevant ingredients for the type IIA and type IIB setups in some detail.

Considering an iterative approach, we have invoked the T-duality transformations among the various standard and (non)geometric fluxes of the two theories. This connection has been explicitly known for fluxes written in the non-cohomology formulation, mostly applicable to the toroidal examples [10,11,30,71–73] but not in the cohomology formulation which could be directly promoted for the beyond toroidal cases such as with using CY compactifications. Given that in the absence of fluxes mirror symmetry exchanges the two theories, we first considered the Kähler potential with explicit computations including α' corrections on the compactifying threefold and its mirror. This helped us to rederive the T-duality rules for the moduli, axions, and chiral variables on the two sides [55,57,74]. Subsequently, in the second step we investigated the fluxes in the superpotential where the moduli have explicit polynomial dependence through the chiral variables, and utilizing the T-duality rules for the chiral variables fixed in the fluxless scenario we derived the explicit transformations for the various fluxes on the two sides. This leads to some very interesting and nontrivial mixing among the (non)geometric fluxes with the standard fluxes, as we present in Table VII. We repeated the same step for the D-term contributions to derive the T-dual connection among the relevant fluxes appearing in the scalar potentials through the D-term contributions. These are also presented in Table VII.

A genuine effective potential should be the one obtained after taking care of the tadpole conditions and NS-NS Bianchi identities, which generically have the potential to nullify some terms in the respective scalar potentials and hence can influence the *effectiveness* of scalar potential pieces governing the moduli dynamics. Therefore, in order to confirm the mapping one has to ensure that the T-duality rules invoked for the fluxes and moduli in the earlier steps are compatible with these constraints. We found that this is indeed the case, we confirmed a one-to-one mapping among all of the Bianchi identities of the two theories. The explicit details are presented in Tables XI and XII. It is worth noting that there is a rather nontrivial mixing among the flux identities in the sense that, e.g., a "HQ-type" identity on the type IIB side gets mapped onto a "$(\mathrm{HR} + w\mathrm{Q})$-type" identity on the type IIA side. Nevertheless, the full set of constraints do have a perfect one-to-one correspondence under T duality.

As the superpotentials can be directly useful only for supersymmetric stabilization, we have extended our studies to the level of scalar potentials to deepen our understanding of the T-dual picture in terms of explicit dependence on the saxions/axions, where it can be directly used for non-supersymmetric moduli stabilization and other phenomenological purposes. In this regard, we first invoked what we call "axionic flux polynomials" from the superpotentials and D terms of the two theories. These axionic flux polynomials include all of the axions and fluxes but do not include any saxions, which helps us to rewrite the scalar potential in a concise form while (more importantly) keeping the saxionic/axionic dependence distinct and explicit. These relevant details are presented in Tables VIII–X. We have demonstrated how our scalar potential formulation can be used to read off the scalar potentials by applying the same for two explicit toroidal orientifolds.

There are many reasons for reformulating the scalar potential. First, it is concise in the sense that the generic scalar potential can be written in a few lines, making it possible to make attempts for model-independent moduli stabilization. This step is quite nontrivial in itself as we recall that a toroidal $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold gives more than 2000 terms arising from the flux superpotential in both the type IIA and type IIB 4D theories, and it is hard even to analytically solve the extremization conditions. The second reason is to make the exchange of the two potentials manifest under the T-duality transformations. As scalar potentials are the starting point or building blocks for moduli stabilization, there can be several possible applications of our one-to-one proposed formulation. For example, this enables one to translate any useful findings in one setup into its T-dual picture. In this regard, we note that there are several well-known de Sitter no-go theorems on the type IIA side, and subsequently there should be T-dual counterparts on the type IIB side, which of course have not received due attention. We have performed a detailed study along these lines in a companion work [93], which illustrated the direct use of the concise pieces of information presented in this work.

ACKNOWLEDGMENTS

I am grateful to Fernando Quevedo for his kind support and encouragements throughout. I would like to thank Gerardo Aldazabal, Xin Gao, Mariana Graña, Fernando Marchesano, Fabio Riccioni, Andreas Schachner, Wieland Staessens, Rui Sun, Thomas Van Riet, and Timm Wrase for useful discussions and communications.

APPENDIX: *T*-DUAL DICTIONARY FOR TYPE II NONGEOMETRIC SETUPS

In this appendix, we present six tables representing the T-dual exchange of the various ingredients of type IIA and type IIB theories. This should serve as what we call a useful "dictionary" for phenomenological model building using (non) geometric fluxes.

TABLE VII. *T*-duality transformations among the various fluxes, moduli, and axions.

	Type IIA with $D6/O6$	Type IIB with $D3/O3$ and $D7/O7$
F-term fluxes	$\mathrm{H}_0,\ \mathrm{H}_k,\ \mathrm{H}^\lambda,$ $w_{a0},\ w_{ak},\ w_a{}^\lambda,$ $\mathrm{Q}^a{}_0,\ \mathrm{Q}^a{}_k,\ \mathrm{Q}^{a\lambda},$ $\mathrm{R}_0,\ \mathrm{R}_k,\ \mathrm{R}^\lambda,$ $e_0,\ e_a,\ m^a,\ m_0.$	$H_0,\ \omega_{a0},\ \hat{Q}^\alpha{}_0,$ $H_i,\ \omega_{ai},\ \hat{Q}^\alpha{}_i,$ $H^i,\ \omega_a{}^i,\ \hat{Q}^{\alpha i},$ $-H^0,\ -\omega_a{}^0,\ -\hat{Q}^{\alpha 0},$ $F_0,\ F_i,\ F^i,\ -F^0.$
D-term fluxes	$\hat{w}_\alpha{}^0,\ \hat{w}_\alpha{}^k,\ \hat{w}_{\alpha\lambda},$ $\hat{\mathrm{Q}}^{\alpha 0},\ \hat{\mathrm{Q}}^{\alpha k},\ \hat{\mathrm{Q}}^\alpha{}_\lambda.$	$-R_K,\ -Q^a{}_K,\ \hat{\omega}_{\alpha K},$ $-R^K,\ -Q^{aK},\ \hat{\omega}_\alpha{}^K.$
Complex moduli	$\mathrm{N}^0,\ \mathrm{N}^k,\ \mathrm{U}_\lambda,\ \mathrm{T}^a.$ $\mathrm{T}^a = b^a - i t^a,$ $\mathrm{N}^0 = \xi^0 + i (z^0)^{-1},$ $\mathrm{N}^k = \xi^k + i (z^0)^{-1} z^k,$ $\mathrm{U}_\lambda = -\frac{i}{2 z^0}(k_{\lambda\rho\kappa} z^\rho z^\kappa - \hat{k}_{\lambda k m} z^k z^m) + \xi_\lambda.$	$S,\ G^a,\ T_\alpha,\ U^i.$ $U^i = v^i - i u^i,$ $S = c_0 + i s,$ $G^a = (c^a + c_0 b^a) + i s b^a,$ $T_\alpha = -\frac{is}{2}(\ell_{\alpha\beta\gamma} t^\beta t^\gamma - \hat{\ell}_{\alpha a b} b^a b^b) + (c_\alpha + \hat{\ell}_{\alpha a b} c^a b^b + \frac{1}{2} c_0 \hat{\ell}_{\alpha a b} b^a b^b).$
Axions	$z^k,\ b^a,\ \xi^0,\ \xi^k,\ \xi_\lambda.$	$b^a,\ v^i,\ c_0,\ c^a + c_0 b^a,\ c_\alpha + \hat{\ell}_{\alpha a b} c^a b^b + \frac{1}{2} c_0 \hat{\ell}_{\alpha a b} b^a b^b.$
Saxions	$(z^0)^{-1},\ z^\lambda,\ t^a,\ \mathcal{V},\ \mathcal{U},$	$s \equiv e^{-\phi},\ t^\alpha,\ u^i\ \mathcal{U},\ \mathcal{V},$
Intersections	$k_{\lambda\rho\mu},\ \hat{k}_{\lambda m n},\ \kappa_{abc},\ \hat{\kappa}_{a\alpha\beta}.$	$\ell_{\alpha\beta\gamma},\ \hat{\ell}_{\alpha a b},\ l_{ijk},\ \hat{l}_{iJK}.$

1. *T*-dual dictionary for type II nongeometric setups

TABLE VIII. Axionic flux polynomials for the type IIA side.

	Type IIA axionic flux polynomials
f_0	$\mathbb{G}_0 - \xi^{\hat{k}}\mathcal{H}_{\hat{k}} - \xi_\lambda \mathcal{H}^\lambda$
f_a	$\mathbb{G}_a - \xi^{\hat{k}}\mho_{a\hat{k}} - \xi_\lambda \mho_a{}^\lambda$
f^a	$\mathbb{G}^a - \xi^{\hat{k}}\mathcal{Q}^a{}_{\hat{k}} - \xi_\lambda \mathcal{Q}^{a\lambda}$
f^0	$\mathbb{G}^0 - \xi^{\hat{k}}\mathcal{R}_{\hat{k}} - \xi_\lambda \mathcal{R}^\lambda$
h_0	$\mathcal{H}_0 + \mathcal{H}_k \mathrm{z}^k + \frac{1}{2}\hat{k}_{\lambda mn}\mathrm{z}^m \mathrm{z}^n \mathcal{H}^\lambda$
h_a	$\mho_{a0} + \mho_{ak} z^k + \frac{1}{2}\hat{k}_{\lambda mn} z^m z^n \mho_a{}^\lambda$
h^a	$\mathcal{Q}^a{}_0 + \mathcal{Q}^a{}_k z^k + \frac{1}{2}\hat{k}_{\lambda mn} z^m z^n \mathcal{Q}^{a\lambda}$
h^0	$\mathcal{R}_0 + \mathcal{R}_k z^k + \frac{1}{2}\hat{k}_{\lambda mn} z^m z^n \mathcal{R}^\lambda$
h_{k0}	$\mathcal{H}_k + \hat{k}_{\lambda kn} z^n \mathcal{H}^\lambda$
h_{ak}	$\mho_{ak} + \hat{k}_{\lambda kn} z^n \mho_a{}^\lambda$
$h^a{}_k$	$\mathcal{Q}^a{}_k + \hat{k}_{\lambda kn} z^n \mathcal{Q}^{a\lambda}$
$h_k{}^0$	$\mathcal{R}_k + \hat{k}_{\lambda kn} z^n \mathcal{R}^\lambda$
$h^\lambda{}_0$	$\mathcal{H}^\lambda$
$h_a{}^\lambda$	$\mho_a{}^\lambda$
$h^{a\lambda}$	$\mathcal{Q}^{a\lambda}$
$h^{\lambda 0}$	$\mathcal{R}^\lambda$
F-term fluxes	$\mathbb{G}_0 = \bar{e}_0 + b^a \bar{e}_a + \frac{1}{2}\kappa_{abc} b^a b^b m^c + \frac{1}{6}\kappa_{abc} b^a b^b b^c m_0,$ $\mathbb{G}_a = \bar{e}_a + \kappa_{abc} b^b m^c + \frac{1}{2}\kappa_{abc} b^b b^c m_0,$ $\mathbb{G}^a = m^a + m_0 b^a,$ $\mathbb{G}^0 = m_0,$ $\mathcal{H}_{\hat{k}} = \bar{\mathrm{H}}_{\hat{k}} + \bar{w}_{a\hat{k}} b^a + \frac{1}{2}\kappa_{abc} b^b b^c \mathrm{Q}^a{}_{\hat{k}} + \frac{1}{6}\kappa_{abc} b^a b^b b^c \mathrm{R}_{\hat{k}},$ $\mathcal{H}^\lambda = \bar{\mathrm{H}}^\lambda + \bar{w}_a{}^\lambda b^a + \frac{1}{2}\kappa_{abc} b^b b^c \mathrm{Q}^{a\lambda} + \frac{1}{6}\kappa_{abc} b^a b^b b^c \mathrm{R}^\lambda,$ $\mho_{a\hat{k}} = \bar{w}_{a\hat{k}} + \kappa_{abc} b^b \mathrm{Q}^c{}_{\hat{k}} + \frac{1}{2}\kappa_{abc} b^b b^c \mathrm{R}_{\hat{k}},$ $\mho_a{}^\lambda = \bar{w}_a{}^\lambda + \kappa_{abc} b^b \mathrm{Q}^{c\lambda} + \frac{1}{2}\kappa_{abc} b^b b^c \mathrm{R}^\lambda,$ $\mathcal{Q}^a{}_{\hat{k}} = \mathrm{Q}^a{}_{\hat{k}} + b^a \mathrm{R}_{\hat{k}},\ \mathcal{Q}^{a\lambda} = \mathrm{Q}^{a\lambda} + b^a \mathrm{R}^\lambda,$ $\mathcal{R}_{\hat{k}} = \mathrm{R}_{\hat{k}},\ \mathcal{R}^\lambda = \mathrm{R}^\lambda.$
D-term fluxes	$\hat{h}_{\alpha\lambda} \equiv \hat{\mho}_{\alpha\lambda} = \hat{w}_{\alpha\lambda} + \hat{k}_{\lambda km} z^m \hat{w}_\alpha{}^k - \frac{1}{2}\hat{k}_{\lambda km} z^k z^m \hat{w}_\alpha{}^0$ $\hat{h}_\alpha{}^k \equiv \hat{\mho}_\alpha{}^k = \hat{w}_\alpha{}^k - z^k \hat{w}_\alpha{}^0, \qquad \hat{h}_\alpha{}^0 \equiv \hat{\mho}_\alpha{}^0 = \hat{w}_\alpha{}^0,$ $\hat{h}^\alpha{}_\lambda \equiv \hat{\mathcal{Q}}^\alpha{}_\lambda = \hat{\mathrm{Q}}^\alpha{}_\lambda + \hat{k}_{\lambda km} z^m \hat{\mathrm{Q}}^{\alpha k} - \frac{1}{2}\hat{k}_{\lambda km} z^\lambda z^k z^m \hat{\mathcal{Q}}^{\alpha 0},$ $\hat{h}^{\alpha k} \equiv \hat{\mathcal{Q}}^{\alpha k} = \hat{\mathrm{Q}}^{\alpha k} - z^k \hat{\mathrm{Q}}^{\alpha 0}, \qquad \hat{h}^{\alpha 0} \equiv \hat{\mathcal{Q}}^{\alpha 0} = \hat{\mathrm{Q}}^{\alpha 0}.$

TABLE IX. Type IIB axionic flux polynomials with their dual type IIA counterparts.

	Type IIB axionic flux polynomials	Dual type IIA flux polynomials
f_0	$\mathbb{F}_0 + v^i \mathbb{F}_i + \frac{1}{2} l_{ijk} v^j v^k \mathbb{F}^i - \frac{1}{6} l_{ijk} v^i v^j v^k \mathbb{F}^0$	f_0
f_i	$\mathbb{F}_i + l_{ijk} v^j \mathbb{F}^k - \frac{1}{2} l_{ijk} v^j v^k \mathbb{F}^0$	f_a
f^i	$\mathbb{F}^i - v^i \mathbb{F}^0$	f^a
f^0	$-\mathbb{F}^0$	f^0
h_0	$\mathbb{H}_0 + v^i \mathbb{H}_i + \frac{1}{2} l_{ijk} v^j v^k \mathbb{H}^i - \frac{1}{6} l_{ijk} v^i v^j v^k \mathbb{H}^0$	h_0
h_i	$\mathbb{H}_i + l_{ijk} v^j \mathbb{H}^k - \frac{1}{2} l_{ijk} v^j v^k \mathbb{H}^0$	h_a
h^i	$\mathbb{H}^i - v^i \mathbb{H}^0$	h^a
h^0	$-\mathbb{H}^0$	h^0
h_{a0}	$\mho_{a0} + v^i \mho_{ai} + \frac{1}{2} l_{ijk} v^j v^k \mho_a{}^i - \frac{1}{6} l_{ijk} v^i v^j v^k \mho_a{}^0$	h_{k0}
h_{ai}	$\mho_{ai} + l_{ijk} v^j \mho_a{}^k - \frac{1}{2} l_{ijk} v^j v^k \mho_a{}^0$	h_{ak}
$h_a{}^i$	$\mho_a{}^i - v^i \mho_a{}^0$	$h^a{}_k$
$h_a{}^0$	$-\mho_a{}^0$	$h_k{}^0$
$h^\alpha{}_0$	$\hat{\mathbb{Q}}_0{}^\alpha + v^i \hat{\mathbb{Q}}_i{}^\alpha + \frac{1}{2} l_{ijk} v^j v^k \hat{\mathbb{Q}}^{\alpha i} - \frac{1}{6} l_{ijk} v^i v^j v^k \hat{\mathbb{Q}}^{\alpha 0}$	$h^\lambda{}_0$
$h^\alpha{}_i$	$\hat{\mathbb{Q}}_i{}^\alpha + l_{ijk} v^j \hat{\mathbb{Q}}^{\alpha k} - \frac{1}{2} l_{ijk} v^j v^k \hat{\mathbb{Q}}^{\alpha 0}$	$h_a{}^\lambda$
$h^{\alpha i}$	$\hat{\mathbb{Q}}^{\alpha i} - v^i \hat{\mathbb{Q}}^{\alpha 0}$	$h^{a\lambda}$
$h^{\alpha 0}$	$-\hat{\mathbb{Q}}^{\alpha 0}$	$h^{\lambda 0}$
F-term fluxes	$\mathbb{F}_\Lambda = \bar{F}_\Lambda - \bar{\omega}_{a\Lambda} c^a - \bar{\hat{Q}}^\alpha{}_\Lambda (c_\alpha + \hat{\ell}_{\alpha ab} c^a b^b) - c_0 \mathbb{H}_\Lambda$	
	$\mathbb{F}^\Lambda = F^\Lambda - \omega_a{}^\Lambda c^a - \hat{Q}^{\alpha\Lambda} (c_\alpha + \hat{\ell}_{\alpha ab} c^a b^b) - c_0 \mathbb{H}^\Lambda$	
	$\mathbb{H}_\Lambda = \bar{H}_\Lambda + \bar{\omega}_{a\Lambda} b^a + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b \bar{\hat{Q}}^\alpha{}_\Lambda$	
	$\mathbb{H}^\Lambda = H^\Lambda + \omega_a{}^\Lambda b^a + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b \hat{Q}^{\alpha\Lambda}$	
	$\mho_{a\Lambda} = \bar{\omega}_{a\Lambda} + \bar{\hat{Q}}^\alpha{}_\Lambda \hat{\ell}_{\alpha ab} b^b$	
	$\mho_a{}^\Lambda = \omega_a{}^\Lambda + \hat{Q}^{\alpha\Lambda} \hat{\ell}_{\alpha ab} b^b$	
	$\hat{\mathbb{Q}}^\alpha{}_\Lambda = \overline{\hat{Q}^\alpha}{}_\Lambda,\ \hat{\mathbb{Q}}^{\alpha\Lambda} = \hat{Q}^{\alpha\Lambda}$	
D-term fluxes	$\hat{h}_{\alpha K} \equiv \hat{\mho}_{\alpha K} = \hat{\omega}_{\alpha K} - Q^a{}_K \hat{\ell}_{\alpha ab} b^b + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b R_K$	$\hat{h}_{\alpha\lambda}$
	$\hat{h}_\alpha{}^K \equiv \hat{\mho}_\alpha{}^K = \hat{\omega}_\alpha{}^K - Q^{aK} \hat{\ell}_{\alpha ab} b^b + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b R^K$	$\hat{h}^\alpha{}_\lambda$
	$h^a{}_K \equiv \mathbb{Q}^a{}_K = -Q^a{}_K + R_K b^a,\ h^{aK} \equiv \mathbb{Q}^{aK} = -Q^{aK} + R^K b^a$	$\hat{h}_\alpha{}^k,\ \hat{h}^{\alpha k}$
	$\hat{h}_K{}^0 \equiv -\mathbb{R}_K = -R_K,\ \hat{h}^{K0} \equiv -\mathbb{R}^K = -R^K$	$\hat{h}_\alpha{}^0,\ \hat{h}^{\alpha 0}$

2. One-to-one exchange of the scalar potentials under *T* duality

TABLE X. Scalar potentials for type IIA and IIB theories.

IIA	$\begin{aligned} V^{\rm tot}_{\rm IIA} = {} & \frac{e^{4D}}{4\mathcal{V}}[f_0^2 + \mathcal{V} f^a \tilde{\mathcal{G}}_{ab} f^b + \mathcal{V} f_a \tilde{\mathcal{G}}^{ab} f_b + \mathcal{V}^2 (f^0)^2] + \frac{e^{2D}}{4\mathcal{U}\mathcal{V}}\Big[h_0^2 + \mathcal{V} h^a \tilde{\mathcal{G}}_{ab} h^b \\ & + \mathcal{V} h_a \tilde{\mathcal{G}}^{ab} h_b + \mathcal{V}^2 (h^0)^2 + \mathcal{U}\tilde{\mathcal{G}}^{ij}\Big(h_{i0}h_{j0} + \frac{\kappa_a\kappa_b}{4} h_i{}^a h_j{}^b + h_{ai}h_{bj}t^a t^b + \mathcal{V}^2 h_i{}^0 h_j{}^0 \\ & - \frac{\kappa_a}{2} h^a{}_i h_{j0} - \frac{\kappa_a}{2} h_{i0} h^a{}_j - \mathcal{V} t^a h_i{}^0 h_{aj} - \mathcal{V} t^a h_{ai} h_j{}^0\Big) + \mathcal{U}\tilde{\mathcal{G}}_{\lambda\rho}\Big(h^\lambda{}_0 h^\rho{}_0 + \frac{\kappa_a\kappa_b}{4} h^{\lambda a} h^{\rho b} \\ & + t^a t^b h_a{}^\lambda h_b{}^\rho + \mathcal{V}^2 h^{\lambda 0} h^{\rho 0} - \frac{\kappa_a}{2} h^\lambda{}_0 h^{\rho a} - \frac{\kappa_a}{2} h^{\lambda a} h^\rho{}_0 - \mathcal{V} t^a h^{\lambda 0} h_a{}^\rho - \mathcal{V} t^a h_a{}^\lambda h^{\rho 0}\Big) \\ & + \frac{k_\lambda k_\rho}{4}\Big(\mathcal{V} h^{a\lambda}\tilde{\mathcal{G}}_{ab} h^{b\rho} + \mathcal{V} h_a{}^\lambda \tilde{\mathcal{G}}^{ab} h_b{}^\rho + \mathcal{V} t^a h^{\lambda 0} h_a{}^\rho + \mathcal{V} t^a h_a{}^\lambda h^{\rho 0} - t^a t^b h_a{}^\lambda h_b{}^\rho \\ & + \frac{\kappa_a}{2} h^\lambda{}_0 h^{a\rho} + \frac{\kappa_a}{2} h^{a\lambda} h^\beta{}_0 - \frac{\kappa_a\kappa_b}{4} h^{a\lambda} h^{b\rho}\Big) - 2\times\frac{k_\lambda}{2}\Big(\mathcal{V} h^a \tilde{\mathcal{G}}_{ab} h^{b\lambda} + \mathcal{V} h_a \tilde{\mathcal{G}}^{ab} h_b{}^\lambda \\ & + \mathcal{V} t^a h^0 h_a{}^\lambda + \mathcal{V} t^a h_a h^{\lambda 0} - t^a t^b h_a h_b{}^\lambda + \frac{\kappa_a}{2} h^a h_0{}^\lambda + \frac{\kappa_a}{2} h_0 h^{a\lambda} - \frac{\kappa_a\kappa_b}{4} h^a h^{b\lambda}\Big) \\ & + [(\mathcal{U}\hat{h}_\alpha{}^0 + z^\lambda \hat{h}_{\alpha\lambda})\mathcal{V}\tilde{\mathcal{G}}^{\alpha\beta}(\mathcal{U}\hat{h}_\beta{}^0 + z^\rho \hat{h}_{\beta\rho}) + (\mathcal{U}\hat{h}^{\alpha 0} + z^\lambda \hat{h}^\alpha{}_\lambda)\mathcal{V}\tilde{\mathcal{G}}_{\alpha\beta}(\mathcal{U} h^{\beta 0} + z^\rho \hat{h}^\beta{}_\rho)]\Big] \\ & + \frac{e^{3D}}{2\sqrt{\mathcal{U}}}\Big[(f^0 h_0 - f^a h_a + f_a h^a - f_0 h^0) - (f^0 h^\lambda{}_0 - f^a h^\lambda{}_a + f_a h^{\lambda a} - f_0 h^{\lambda 0})\frac{k_\lambda}{2}\Big]. \end{aligned}$ $\tilde{\mathcal{G}}_{ab} = \frac{\kappa_a\kappa_b - 4\mathcal{V}\kappa_{ab}}{4\mathcal{V}}, \quad \tilde{\mathcal{G}}^{ab} = \frac{2t^a t^b - 4\mathcal{V}\kappa^{ab}}{4\mathcal{V}}, \quad \tilde{\mathcal{G}}_{\alpha\beta} = -\hat{\kappa}_{\alpha\beta}, \quad \tilde{\mathcal{G}}^{\alpha\beta} = -\hat{\kappa}^{\alpha\beta},$ $\tilde{\mathcal{G}}_{\lambda\rho} = \frac{k_\lambda k_\rho - 4\mathcal{U} k_{\lambda\rho}}{4\mathcal{U}}, \quad \tilde{\mathcal{G}}^{\lambda\rho} = \frac{2z^\lambda z^\rho - 4\mathcal{U} k^{\lambda\rho}}{4\mathcal{U}}, \quad \tilde{\mathcal{G}}_{jk} = -\hat{k}_{jk}, \quad \tilde{\mathcal{G}}^{jk} = -\hat{k}^{jk}.$
IIB	$\begin{aligned} V^{\rm tot}_{\rm IIB} = {} & \frac{e^{4\phi}}{4\mathcal{V}^2\mathcal{U}}[f_0^2 + \mathcal{U} f^i \mathcal{G}_{ij} f^j + \mathcal{U} f_i \mathcal{G}^{ij} f_j + \mathcal{U}^2 (f^0)^2] + \frac{e^{2\phi}}{4\mathcal{V}^2\mathcal{U}}\Big[h_0^2 + \mathcal{U} h^i \mathcal{G}_{ij} h^j \\ & + \mathcal{U} h_i \mathcal{G}^{ij} h_j + \mathcal{U}^2 (h^0)^2 + \mathcal{V}\mathcal{G}^{ab}\Big(h_{a0}h_{b0} + \frac{l_i l_j}{4} h_a{}^i h_b{}^j + h_{ai}h_{bj}u^i u^j + \mathcal{U}^2 h_a{}^0 h_b{}^0 \\ & - \frac{l_i}{2} h_a{}^i h_{b0} - \frac{l_i}{2} h_{a0} h_b{}^i - \mathcal{U} u^i h_a{}^0 h_{bi} - \mathcal{U} u^i h_b{}^0 h_{ai}\Big) + \mathcal{V}\mathcal{G}_{\alpha\beta}\Big(h^\alpha{}_0 h^\beta{}_0 + \frac{l_i l_j}{4} h^{\alpha i} h^{\beta j} \\ & + u^i u^j h^\alpha{}_i h^\beta{}_j + \mathcal{U}^2 h^{\alpha 0} h^{\beta 0} - \frac{l_i}{2} h^\alpha{}_0 h^{\beta i} - \frac{l_i}{2} h^{\alpha i} h^\beta{}_0 - \mathcal{U} u^i h^{\alpha 0} h^\beta{}_i - \mathcal{U} u^i h^\alpha{}_i h^{\beta 0}\Big) \\ & + \frac{\ell_\alpha \ell_\beta}{4}\Big(\mathcal{U} h^{\alpha i}\mathcal{G}_{ij} h^{\beta j} + \mathcal{U} h^\alpha{}_i \mathcal{G}^{ij} h^\beta{}_j + \mathcal{U} u^i h^{\alpha 0} h_i{}^\beta + \mathcal{U} u^i h^\alpha{}_i h^{\beta 0} - u^i u^j h^\alpha{}_i h^\beta{}_j \\ & + \frac{l_i}{2} h^\alpha{}_0 h^{\beta i} + \frac{l_i}{2} h^{\alpha i} h^\beta{}_0 - \frac{l_i l_j}{4} h^{\alpha i} h^{\beta j}\Big) - 2\times\frac{\ell_\alpha}{2}\Big(\mathcal{U} h^i \mathcal{G}_{ij} h^{\alpha j} + \mathcal{U} h_i \mathcal{G}^{ij} h^\alpha{}_j \\ & + \mathcal{U} u^i h^0 h^\alpha{}_i + \mathcal{U} u^i h_i h^{\alpha 0} - u^i u^j h_i h^\alpha{}_j + \frac{l_i}{2} h^i h^\alpha{}_0 + \frac{l_i}{2} h_0 h^{\alpha i} - \frac{l_i l_j}{4} h^i h^{\alpha j}\Big) \\ & + [(\mathcal{V}\hat{h}_J{}^0 - t^\alpha \hat{h}_{\alpha J})\mathcal{U}\mathcal{G}^{JK}(\mathcal{V}\hat{h}_K{}^0 - t^\beta \hat{h}_{\beta K}) + (\mathcal{V}\hat{h}^{J0} - t^\alpha \hat{h}_\alpha{}^J)\mathcal{U}\mathcal{G}_{JK}(\mathcal{V}\hat{h}^{K0} - t^\beta \hat{h}_\beta{}^K)]\Big] \\ & + \frac{e^{3\phi}}{2\mathcal{V}^2}\Big[(f^0 h_0 - f^i h_i + f_i h^i - f_0 h^0) - (f^0 h^\alpha{}_0 - f^i h^\alpha{}_i + f_i h^{\alpha i} - f_0 h^{\alpha 0})\frac{\ell_\alpha}{2}\Big]. \end{aligned}$ $\mathcal{G}_{\alpha\beta} = \frac{\ell_\alpha \ell_\beta - 4\mathcal{V}\ell_{\alpha\beta}}{4\mathcal{V}}, \quad \mathcal{G}^{\alpha\beta} = \frac{2t^\alpha t^\beta - 4\mathcal{V}\ell^{\alpha\beta}}{4\mathcal{V}}, \quad \mathcal{G}_{ab} = -\hat{\ell}_{ab}, \quad \mathcal{G}^{ab} = -\hat{\ell}^{ab},$ $\mathcal{G}_{ij} = \frac{l_i l_j - 4\mathcal{U} l_{ij}}{4\mathcal{U}}, \quad \mathcal{G}^{ij} = \frac{2u^i u^j - 4\mathcal{U} l^{ij}}{4\mathcal{U}}, \quad \mathcal{G}^{JK} = -\hat{l}^{JK}, \quad \mathcal{G}^{JK} = -\hat{l}^{JK}.$

3. One-to-one exchange of the Bianchi identities under T duality

TABLE XI. One-to-one correspondence between the Bianchi identities (BIs) under the T-dual flux transformations. Here we consider $\Lambda = \{0, i\}$ on the type IIB side and $\hat{k} = \{0, k\}$ on the type IIA side.

BIs	Type IIB with $D3/O3$ and $D7/O7$	Type IIA with $D6/O6$
(1)	$H_\Lambda \omega_a{}^\Lambda = H^\Lambda \omega_{\Lambda a}$	$\mathrm{H}_{[0}\mathrm{R}_{k]} + \mathrm{Q}^a{}_{[0} w_{ak]} = 0$
(2)	$H^\Lambda \hat{Q}_\Lambda{}^\alpha = H_\Lambda \hat{Q}^{\alpha\Lambda}$	$\mathrm{R}^\lambda \mathrm{H}_0 - \mathrm{H}^\lambda \mathrm{R}_0 + w_a{}^\lambda \mathrm{Q}^a{}_0 - \mathrm{Q}^{a\lambda} w_{a0} = 0$
(3)	$\omega_a{}^\Lambda \omega_{b\Lambda} = \omega_b{}^\Lambda \omega_{a\Lambda}$	$\mathrm{H}_{[k}\mathrm{R}_{k']} + \mathrm{Q}^a{}_{[k} w_{ak']} = 0$
(4)	$\hat\omega_\alpha{}^K \hat\omega_{\beta K} = \hat\omega_\beta{}^K \hat\omega_{\alpha K}$	$\hat{w}_{\alpha\lambda}\hat{\mathrm{Q}}^\alpha{}_\rho = \hat{\mathrm{Q}}^\alpha{}_\lambda \hat{w}_{\alpha\rho}$
(5)	$\omega_{a\Lambda}\hat{Q}^{\alpha\Lambda} = \omega_a{}^\Lambda \hat{Q}^\alpha{}_\Lambda$	$\mathrm{R}^\lambda \mathrm{H}_k - \mathrm{H}^\lambda \mathrm{R}_k + w_a{}^\lambda \mathrm{Q}^a{}_k - \mathrm{Q}^{a\lambda} w_{ak} = 0$
(6)	$Q^{aK}\hat\omega_{\alpha K} = Q^a{}_K \hat\omega_\alpha^K$	$\hat{w}_{\alpha\lambda}\hat{\mathrm{Q}}^{\alpha k} = \hat{\mathrm{Q}}^\alpha{}_\lambda \hat{w}_\alpha{}^k$
(7)	$H_0 R_K + \omega_{a0} Q^a{}_K + \hat{Q}^\alpha{}_0 \hat\omega_{\alpha K} = 0$	$\mathrm{H}^\lambda \hat{w}_{\alpha\lambda} = \mathrm{H}_{\hat k}\hat{w}_\alpha{}^{\hat k}$
	$H_i R_K + \omega_{ai} Q^a{}_K + \hat{Q}^\alpha{}_i \hat\omega_{\alpha K} = 0$	$w_a{}^\lambda \hat{w}_{\alpha\lambda} = w_{a\hat k}\hat{w}_\alpha{}^{\hat k}$
(8)	$H^0 R_K + \omega_a{}^0 Q^a{}_K + \hat{Q}^{\alpha 0}\hat\omega_{\alpha K} = 0$	$\mathrm{R}^\lambda \hat{w}_{\alpha\lambda} = \mathrm{R}_{\hat k}\hat{w}_\alpha{}^{\hat k}$
	$H^i R_K + \omega_a{}^i Q^a{}_K + \hat{Q}^{\alpha i}\hat\omega_{\alpha K} = 0$	$\mathrm{Q}^a{}_{\hat k}\hat{w}_\alpha{}^{\hat k} = \mathrm{Q}^{a\lambda}\hat{w}_{\alpha\lambda}$
(9)	$H_0 R^K + \omega_{a0}Q^{aK} + \hat{Q}^\alpha{}_0 \hat\omega_\alpha{}^K = 0$	$\mathrm{H}^\lambda \hat{\mathrm{Q}}^\alpha{}_\lambda = \mathrm{H}_{\hat k}\hat{\mathrm{Q}}^{\alpha\hat k}$
	$H_i R^K + \omega_{ai}Q^{aK} + \hat{Q}^\alpha{}_i\hat\omega_\alpha{}^K = 0$	$\hat{\mathrm{Q}}^\alpha{}_\lambda w_a{}^\lambda = w_{a\hat k}\hat{\mathrm{Q}}^{\alpha\hat k}$
(10)	$H^0 R^K + \omega_a{}^0 Q^{aK} + \hat{Q}^{\alpha 0}\hat\omega_\alpha{}^K = 0$	$\mathrm{R}^\lambda\hat{\mathrm{Q}}^\alpha{}_\lambda = \mathrm{R}_{\hat k}\hat{\mathrm{Q}}^{\alpha\hat k}$
	$H^i R^K + \omega_a{}^i Q^{aK} + \hat{Q}^{\alpha i}\hat\omega_\alpha{}^K = 0$	$\mathrm{Q}^{a\lambda}\hat{\mathrm{Q}}^\alpha{}_\lambda = \mathrm{Q}^a{}_{\hat k}\hat{\mathrm{Q}}^{\alpha\hat k}$
(11)	$\hat{Q}^{\alpha\Lambda}\hat{Q}^\beta{}_\Lambda = \hat{Q}^{\beta\Lambda}\hat{Q}^\alpha{}_\Lambda$	$\mathrm{H}^{[\lambda}\mathrm{R}^{\rho]} + \mathrm{Q}^{a[\lambda} w_a{}^{\rho]} = 0$
(12)	$Q^{aK}Q^b{}_K = Q^{bK}Q^a{}_K$	$\hat{w}_\alpha{}^k \hat{\mathrm{Q}}^{\alpha k'} = \hat{\mathrm{Q}}^{\alpha k}\hat{w}_\alpha{}^{k'}$
(13)	$R^K\hat\omega_{\alpha K} = R_K \hat\omega_\alpha{}^K$	$\hat{w}_{\alpha\lambda}\hat{\mathrm{Q}}^{\alpha 0} = \hat{\mathrm{Q}}^\alpha{}_\lambda \hat{w}_\alpha{}^0$
(14)	$R_K Q^{aK} = R^K Q^a{}_K$	$\hat{w}_\alpha{}^0\hat{\mathrm{Q}}^{\alpha k} = \hat{\mathrm{Q}}^{\alpha 0}\hat{w}_\alpha{}^k$

4. One-to-one exchange of the Bianchi identities with flux polynomials having b^a axions

TABLE XII. One-to-one correspondence between the Bianchi identities with generalized flux polynomials having the NS-NS b^a axions as presented in Eq. (4.4) for type IIB and in Eq. (4.2) for type IIA. Here we consider $\Lambda = \{0, i\}$ on the type IIB side and $\hat{k} = \{0, k\}$ on the type IIA side.

BIs	Type IIB with $D3/O3$ and $D7/O7$	Type IIA with $D6/O6$
(1)	$\mathbb{H}_\Lambda \mho_a{}^\Lambda = \mathbb{H}^\Lambda \mho_{a\Lambda}$	$\mathcal{H}_{[0}\mathcal{R}_{k]} + \mathcal{Q}^a{}_{[0} w_{ak]} = 0$
(2)	$\mathbb{H}^\Lambda \hat{\mathbb{Q}}_\Lambda{}^\alpha = \mathbb{H}_\Lambda \hat{\mathbb{Q}}^{\alpha\Lambda}$	$\mathcal{R}^\lambda \mathcal{H}_0 - \mathcal{H}^\lambda \mathcal{R}_0 + \mho_a{}^\lambda \mathcal{Q}^a{}_0 - \mathcal{Q}^{a\lambda}\mho_{a0} = 0$
(3)	$\mho_a{}^\Lambda \mho_{b\Lambda} = \mho_b{}^\Lambda \mho_{a\Lambda}$	$\mathcal{H}_{[k}\mathcal{R}_{k']} + \mathcal{Q}^a{}_{[k}\mho_{ak']} = 0$
(4)	$\hat\mho_\alpha{}^K \hat\mho_{\beta K} = \hat\mho_\beta{}^K \hat\mho_{\alpha K}$	$\hat\mho_{\alpha\lambda}\hat{\mathcal{Q}}^\alpha{}_\rho = \hat{\mathcal{Q}}^\alpha{}_\lambda \hat\mho_{\alpha\rho}$
(5)	$\mho_{a\Lambda}\hat{\mathbb{Q}}^{\alpha\Lambda} = \mho_a{}^\Lambda \hat{\mathbb{Q}}^\alpha{}_\Lambda$	$\mathrm{R}^\lambda \mathcal{H}_k - \mathcal{H}^\lambda \mathcal{R}_k + \mho_a{}^\lambda \mathcal{Q}^a{}_k - \mathcal{Q}^{a\lambda}\mho_{ak} = 0$
(6)	$\mathbb{Q}^{aK}\hat\mho_{\alpha K} = \mathbb{Q}^a{}_K \hat\mho_\alpha^K$	$\hat\mho_{\alpha\lambda}\hat{\mathcal{Q}}^{\alpha k} = \hat{\mathcal{Q}}^\alpha{}_\lambda \hat\mho_\alpha{}^k$
(7)	$\mathbb{H}_0 \mathbb{R}_K + \mho_{a0}\mathbb{Q}^a{}_K + \hat{\mathbb{Q}}^\alpha{}_0 \hat\mho_{\alpha K} = 0$	$\mathcal{H}^\lambda \hat\mho_{\alpha\lambda} = \mathcal{H}_{\hat k}\hat\mho_\alpha{}^{\hat k}$
	$\mathbb{H}_i \mathbb{R}_K + \mho_{ai}\mathbb{Q}^a{}_K + \hat{\mathbb{Q}}^\alpha{}_i \hat\mho_{\alpha K} = 0$	$\mho_a{}^\lambda \hat\mho_{\alpha\lambda} = \mho_{a\hat k}\hat\mho_\alpha{}^{\hat k}$
(8)	$\mathbb{H}^0 \mathbb{R}_K + \mho_a{}^0 \mathbb{Q}^a{}_K + \hat{\mathbb{Q}}^{\alpha 0}\hat\mho_{\alpha K} = 0$	$\mathcal{R}^\lambda \hat\mho_{\alpha\lambda} = \mathcal{R}_{\hat k}\hat\mho_\alpha{}^{\hat k}$
	$\mathbb{H}^i \mathbb{R}_K + \mho_a{}^i \mathbb{Q}^a{}_K + \hat{\mathbb{Q}}^{\alpha i}\hat\mho_{\alpha K} = 0$	$\mathcal{Q}^a{}_{\hat k}\hat\mho_\alpha{}^{\hat k} = \mathcal{Q}^{a\lambda}\hat\mho_{\alpha\lambda}$
(9)	$\mathbb{H}_0 \mathbb{R}^K + \mho_{a0}\mathbb{Q}^{aK} + \hat{\mathbb{Q}}^\alpha{}_0 \hat\mho_\alpha{}^K = 0$	$\mathrm{H}^\lambda \hat{\mathcal{Q}}^\alpha{}_\lambda = \mathrm{H}_{\hat k}\hat{\mathcal{Q}}^{\alpha\hat k}$
	$\mathbb{H}_i \mathbb{R}^K + \mho_{ai}\mathbb{Q}^{aK} + \hat{\mathbb{Q}}^\alpha{}_i \hat\mho_\alpha{}^K = 0$	$\hat{\mathcal{Q}}^\alpha{}_\lambda \mho_a{}^\lambda = \mho_{a\hat k}\hat{\mathcal{Q}}^{\alpha\hat k}$
(10)	$\mathbb{H}^0 \mathbb{R}^K + \mho_a{}^0 \mathbb{Q}^{aK} + \hat{\mathbb{Q}}^{\alpha 0}\hat\mho_\alpha{}^K = 0$	$\mathcal{R}^\lambda \hat{\mathcal{Q}}^\alpha{}_\lambda = \mathcal{R}_{\hat k}\hat{\mathcal{Q}}^{\alpha\hat k}$
	$\mathbb{H}^i \mathbb{R}^K + \mho_a{}^i \mathbb{Q}^{aK} + \hat{\mathbb{Q}}^{\alpha i}\hat\mho_\alpha{}^K = 0$	$\mathcal{Q}^{a\lambda}\hat{\mathcal{Q}}^\alpha{}_\lambda = \mathcal{Q}^a{}_{\hat k}\hat{\mathcal{Q}}^{\alpha\hat k}$
(11)	$\hat{\mathbb{Q}}^{\alpha\Lambda}\hat{\mathbb{Q}}^\beta{}_\Lambda = \hat{\mathbb{Q}}^{\beta\Lambda}\hat{\mathbb{Q}}^\alpha{}_\Lambda$	$\mathcal{H}^{[\lambda}\mathrm{R}^{\rho]} + \mathcal{Q}^{a[\lambda}\mho_a{}^{\rho]} = 0$
(12)	$\mathbb{Q}^{aK}\mathbb{Q}^b{}_K = \mathbb{Q}^{bK}\mathbb{Q}^a{}_K$	$\hat\mho_\alpha{}^k \hat{\mathcal{Q}}^{\alpha k'} = \hat{\mathcal{Q}}^{\alpha k}\hat\mho_\alpha{}^{k'}$
(13)	$\mathbb{R}^K \hat\mho_{\alpha K} = \mathbb{R}_K \hat\mho_\alpha{}^K$	$\hat\mho_{\alpha\lambda}\hat{\mathcal{Q}}^{\alpha 0} = \hat{\mathcal{Q}}^\alpha{}_\lambda \hat\mho_\alpha{}^0$
(14)	$\mathbb{R}_K \mathbb{Q}^{aK} = \mathbb{R}^K \mathbb{Q}^a{}_K$	$\hat\mho_\alpha{}^0 \hat{\mathcal{Q}}^{\alpha k} = \hat{\mathcal{Q}}^{\alpha 0}\hat\mho_\alpha{}^k$

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