Anisotropic dark matter stars

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The properties of exotic stars are investigated. In particular, we study objects made entirely of dark matter, and we take into account intrinsic anisotropies which have been ignored so far. We obtain semianalytical solutions to the structure equations, where all quantities of interest are expressed in terms of the energy density, while the latter is computed numerically. We show that those solutions (i) are well behaved within general relativity *and* (ii) are capable of describing realistic astrophysical configurations. A direct comparison with their isotropic counterparts with the same radius reveals that the latter are slightly more massive.

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I. INTRODUCTION

Dark matter (DM) has certainly been one of the greatest mysteries of physics. Important evidence of its existence came from the analysis of rotation curves of spiral galaxies by Rubin *et al.* in the 1970s [1–3]. DM is thought to be a kind of matter that does not interact electromagnetically and therefore cannot be seen, which is why it is called *dark*. However, it interacts gravitationally. In the case of spiral galaxies, it causes their rotation curves to be significantly higher than one would expect by measuring only the gravitational field of luminous matter.

It also has a fundamental role in the formation of galaxies and large-scale structures in the Universe [4–7]. Actually it is believed that when baryonic matter decoupled from radiation at redshift $z \sim 1100$, the DM gravitational potential wells were already formed and rapidly attracted baryonic matter, what has speeded up the structure formation mechanism [8,9] so that we can see the large-scale structures we see today.

Moreover, according to the standard cosmological model matched with observational data coming from temperature fluctuations in the cosmic microwave background radiation, DM makes up roughly 25% of the matter density of the entire Universe composition [10].

We still have not detected DM particles with experimental apparatus despite the efforts [11–14]. So far we have only detected its gravitational effects when pointing telescopes to the sky. In this latter regard, gravitational lensing has been fundamental [15–17].

At least a portion of DM may be in the form of massive compact halo objects [18–20]. Those are massive baryonic matter objects that emit low or no electromagnetic radiation and habit galactic halos, and an example of them would be neutron stars. They can also bend light, causing gravitational microlensing effects, that have been detected for some time [21–25].

It should also be noted that DM gravitational effects could be understood as purely geometrical effects of extended gravity theories [26]. Rotation curves [27–29] and even structure formation [30–34] have been explained through the extended gravity channel.

Here, in the present article, based on some of the several studies that empirically prove DM existence [35], we will stick to the standard approach, considering DM exists and is nonbaryonic.

The Bose-Einstein condensate is a possibility in the DM particle scenario [36–40], and it was recently shown that could exist in space by the Cold Atom Laboratory orbiting Earth on board the International Space Station [41].

The weakly interacting massive particles (so-called WIMPs) [42–44] are among the most well motivated DM particle candidates. WIMPS interact through a feeble new force and gravity as predicted by supersymmetry among other theories [45,46]. If they were in thermal equilibrium in the early Universe they annihilated with one another so that a predictable number of them remain today [47].

There may exist DM stars (DMSs) [48,49] powered by WIMP DM annihilation [50,51]. In regions of high DM density, such as the Galactic Center, the capture and

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annihilation of WIMP DM by stars has the potential to significantly alter their evolution [52–55]. In Ref. [56] it was shown that WIMPs accreted onto neutron stars may provide a mechanism to seed strangelets in compact objects for WIMP masses above a few GeV. This effect may trigger a conversion of most of the star into a strange star. Recall that neutron stars are pulsars, high-density stars with large rotation frequency rates located in the core of supernovae remnants [57,58]. Some models predicted that strange stars could form inside these stars due to the conversion of neutrons into their constituent quarks [59]. Due to a matter of stability, a portion ($\sim 1/3$) of these quarks is converted to strange quarks and the resulting matter is known as strange quark matter.

Neutron stars are expected to efficiently capture WIMPs due to their strong gravitational field. The annihilation of DM in the center of these stars could lead to detectable effects on their surface temperature, especially if they are in the center of our Galaxy [60].

In [61], Kurita and Nakano investigated the collapse of clusters of WIMPs in the core of Sun-like stars and the consequent possible formation of mini-black-holes, which would generate gravitational wave emission.

The aforementioned Bose-Einstein condensate has also been considered as the DM modeling for stars. In this regard, one can consult e.g., [62–69]. In particular, in [68,69], DMSs were investigated in the Starobinsky model of gravity [70]. It has been shown in [69] that DMSs have smaller radius and are slightly more massive in Starobinsky gravity.

In the present article we will assume a boson star as our model for a DMS. A wide variety of boson stars have been proposed and investigated in the literature [71–75] (for some recent references on this subject, one can check [76–79]). Our DMS will be modeled from the equation of state (EOS) proposed in [75] (check also [80]). It is interesting to mention that some proposals for detecting boson stars were reported in [81–86].

The environment inside DMSs is expected to be extremely dense, especially when neutron starlike objects are under consideration. Under such conditions of extreme density, anisotropy is expected to appear [87–90].

Anisotropy in neutron stars has been investigated in the literature. The hydrostatic features were first approached in [91], where it was shown that deviations from isotropy would entail changes in the star maximum mass. This approach was extended in [92] to also cover the problem of stability under radial and nonradial pulsations. The effects of anisotropy on slowly rotating neutron stars was studied in [93]. In [94], anisotropic neutron stars were also considered in the framework of Starobinsky gravity. Further studies of anisotropic neutron stars can be seen in Refs. [95–98].

To the best knowledge of the present authors, anisotropy has not yet been considered in DMSs. Such an investigation is the main goal of the present article. The plan of our work is the following: In the next section we will briefly summarize the structure equations describing hydrostatic equilibrium of anisotropic stars. In Sec. III we will present the exact analytical solution and we will show that it is well behaved and realistic within general relativity. Finally, we will finish our work in Sec. IV with the concluding remarks.

II. RELATIVISTIC STARS WITH ANISOTROPIC MATTER

Within general relativity the starting point is Einstein's field equations

$$\mathcal{G}_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{1}$$

In (1), $\mathcal{G}_{\mu\nu}$ is the Einstein tensor, $\mathcal{R}_{\mu\nu}$ is the Ricci tensor, $\mathcal{R}$ is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, we set Newton's constant G and the speed of light, c, to 1, while for anisotropic matter the stress-energy tensor $T_{\mu\nu}$ has the form

$$T^{\mu}_{\nu} = \mathrm{Diag}(-\rho, p_r, p_t, p_t), \tag{2}$$

with ρ being the energy density, p_r the radial pressure and p_t the tangential pressure.

In order to find interior solutions describing hydrostatic equilibrium of relativistic stars, we integrate the structure equations including the presence of a nonvanishing anisotropic factor [99,100]:

$$m'(r) = 4\pi r^2 \rho(r), \tag{3}$$

$$\nu'(r) = 2\frac{m(r) + 4\pi r^3 p_r(r)}{r^2[1 - 2m(r)/r]}, \tag{4}$$

$$p_r'(r) = -[\rho(r) + p_r(r)]\frac{m(r) + 4\pi r^3 p_r(r)}{r^2[1 - 2m(r)/r]} + \frac{2\Delta}{r}, \tag{5}$$

where $m(r)$ and $\nu(r)$ are the components of the metric tensor assuming static, spherically symmetric solutions in Schwarzschild-like coordinates, (t, r, θ, ϕ),

$$ds^2 = -e^{\nu}dt^2 + \frac{1}{1 - 2m(r)/r}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{6}$$

and $\Delta \equiv p_t - p_r$ is the anisotropic factor. All quantities depend on the radial coordinate r only, and a prime denotes differentiation with respect to r. Clearly, setting $\Delta = 0$ we recover the usual Tolman-Oppenheimer-Volkoff equations [101,102] for isotropic matter.

Moreover we impose at the center of the star, $r = 0$, the following initial conditions:

$$m(0) = 0, \tag{7}$$

$$\rho(0) = \rho_c, \tag{8}$$

with ρ_c being the central energy density. Upon matching with the exterior vacuum solution ($T_{\mu\nu} = 0$, Schwarzschild geometry) at the surface of the star, $r = R$, the following boundary conditions must be satisfied:

$$\rho(R) = 0, \tag{9}$$

$$m(R) = M, \tag{10}$$

$$e^{\nu(R)} = 1 - \frac{2M}{R}, \tag{11}$$

with R being the radius of the star, and M being its mass.

III. ANISOTROPIC DARK MATTER STARS

Boson stars are self-gravitating clumps made of either spin-zero fields called scalar boson stars [72] or vector bosons called Proca stars [103,104]. The maximum mass for scalar boson stars in noninteracting systems was found in [105,106], while in [75,107] it was pointed out that self-interactions can cause significant changes.

A complex scalar field Φ minimally coupled to gravity is described by the Einstein-Klein-Gordon action [108]

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi} + \mathcal{L}_M\right), \tag{12}$$

$$\mathcal{L}_M = -g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi^* - V(|\Phi|) \tag{13}$$

where g is the metric determinant, $\mathcal{L}_M$ is the matter Lagrangian and V is the self-interaction scalar potential.

For static spherically symmetric solutions we make for the scalar field the ansatz [108]

$$\Phi(r,t) = \phi(r)\exp(-i\omega t), \tag{14}$$

where the oscillation frequency ω is a real parameter.

Although the scalar field itself depends on time, its stress-energy tensor is time independent and the Einstein field equations take the usual form for a fluid, for which the energy density is computed to be [109,110]

$$\rho = \omega^2 e^{-\nu}\phi^2 + e^{-\lambda}\phi'^2 + V(\phi), \tag{15}$$

while the radial and tangential pressures are found to be [109,110]

$$p_r = \omega^2 e^{-\nu}\phi^2 + e^{-\lambda}\phi'^2 - V(\phi), \tag{16}$$

$$p_t = \omega^2 e^{-\nu}\phi^2 - e^{-\lambda}\phi'^2 - V(\phi). \tag{17}$$

Clearly, a boson star is anisotropic since the two pressures are different. Under certain conditions, however, the anisotropy may be ignored and the system can be treated as an isotropic object. A concrete model of the form

$$V(|\Phi|) = m^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4, \tag{18}$$

with m being the mass of the scalar field and λ being the self-interaction coupling constant, was studied e.g., in [80], in which the authors considered the following EOS [75]:

$$p_r = \frac{\rho_0}{3}\left(\sqrt{1+\frac{\rho}{\rho_0}} - 1\right)^2, \tag{19}$$

where ρ_0 is a constant given by

$$\rho_0 = \frac{m^4}{3\lambda}. \tag{20}$$

This EOS describes the boson stars that are approximately isotropic provided that the condition

$$\frac{\lambda}{4\pi} \gg m^2 \tag{21}$$

holds [80].

In the two extreme limits we recover the well-known results

$$p_r \approx \frac{\rho^2}{12\rho_0}, \qquad \rho \ll \rho_0, \tag{22}$$

for diluted stars [39], and

$$p_r \approx \frac{\rho}{3}, \qquad \rho \gg \rho_0 \tag{23}$$

in the ultrarelativistic limit.

In the first extreme limit, any model, irrespective of the form of the potential, will be described by the same polytropic EOS, with index $n = 1$ and $\gamma = 2$. In the present work we propose to investigate the properties of relativistic stars made of anisotropic exotic matter characterized by the polytropic EOS

$$p_r = K\rho^2, \qquad K = z/B \tag{24}$$

where z is a dimensionless number while B has dimension of pressure and it is of the order of the energy density of neutron stars and quark stars, $B \simeq (150\ \mathrm{MeV})^4$.

In the case of stars with anisotropic matter there are five unknown quantities in total and only three differential equations. Therefore, we are free to impose two conditions.

The first is the adopted EOS, while for the other, one option would be to assume a certain profile for the anisotropic factor. Since for boson stars it must be negative, in the following we shall consider the ansatz

$$\Delta(r) = -(r/r_0)^2\rho(r) \tag{25}$$

with r_0 being a length scale, which ensures that Δ has the right dimensions, it is manifestly negative, and it vanishes both at the origin and at the surface of the star.

Thus, all quantities may be expressed in terms of ρ. In particular, the radial pressure is immediately computed making use of the EOS, while the mass function is computed using the tt component of the field equations, and it is given by

$$m(r) = 4\pi \int_0^r dx x^2 \rho(x) \tag{26}$$

although the energy density must be computed numerically.

The temporal metric component ν is computed making use of the radial field equation as follows:

$$\nu(r) = \log(1 - 2M/R) - 2\int_R^r dx \frac{m(x) + 4\pi x^3 p_r(x)}{x^2[1 - 2m(x)/x]}. \tag{27}$$

Next we shall investigate the behavior as well as the viability of the solutions we just found.

Causality, stability and energy conditions

The radial and tangential speeds of sound defined by

$$c_r^2 \equiv \frac{dp_r}{d\rho}, \tag{28}$$

$$c_t^2 \equiv \frac{dp_t}{d\rho} \tag{29}$$

should take values in the interval $0 < c_{r,t}^2 < 1$ throughout the stars, so that causality is not violated.

Moreover, Bondi suggested that for a stable Newtonian sphere the radial adiabatic index defined by

$$\Gamma \equiv c_r^2\left(1 + \frac{\rho}{p_r}\right) \tag{30}$$

should be larger than $4/3$ [111]. In fact, using the definition and the EOS it is easy to verify that

$$\Gamma(r) = c_r^2 + 2 = 2(1 + K\rho(r)), \tag{31}$$

and therefore clearly $\Gamma \geq 2$, irrespective of the central value of energy density/radial pressure. In particular, the relativistic adiabatic index is a monotonically decreasing function of r throughout the star, taking at the origin and at the surface of the star the following values:

$$\Gamma_c \equiv \Gamma(r = 0) = 2(1 + K\rho_c), \tag{32}$$

$$\Gamma_s \equiv \Gamma(r = R) = 2. \tag{33}$$

Finally, the solutions obtained here should be able to describe realistic astrophysical configurations. Therefore, as a further check we investigate if the energy conditions are fulfilled or not. To that end, the conditions [112–116]

$$\rho \geq 0, \tag{34}$$

$$\rho + p_{r,t} \geq 0, \tag{35}$$

$$\rho - p_{r,t} \geq 0, \tag{36}$$

$$E_+ \equiv \rho + p_r + 2p_t \geq 0, \tag{37}$$

$$E_- \equiv \rho - p_r - 2p_t \geq 0 \tag{38}$$

are investigated.

Our main numerical results are summarized in Figs. 1–6 below, assuming the following numerical values for z, B, r_0 and ρ_c:

$$B = 2 \times 10^{-80}\ m_{\rm pl}^4, \tag{39}$$

$$\rho_c = 25B, \tag{40}$$

$$z = 0.01, \tag{41}$$

$$r_0 = 100\ \text{km}, \tag{42}$$

with $m_{\rm pl} = 1.22 \times 10^{19}$ GeV being the Planck mass corresponding to a star with the following properties:

$$R = 9.77\ \text{km}, \tag{43}$$

$$M = 1.44\ M_\odot, \tag{44}$$

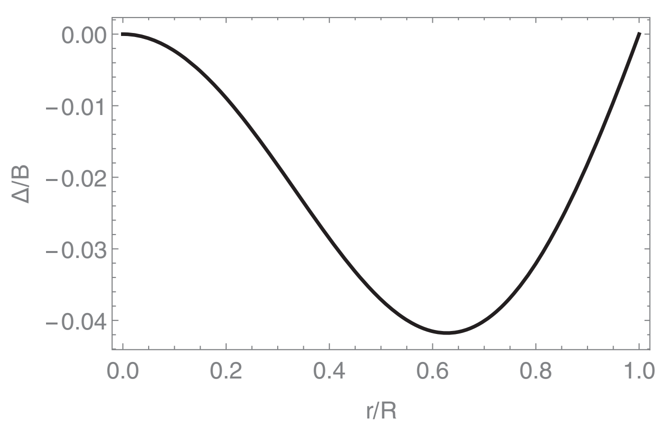

FIG. 1. Normalized anisotropic factor, Δ/B, versus normalized radial coordinate r/R.

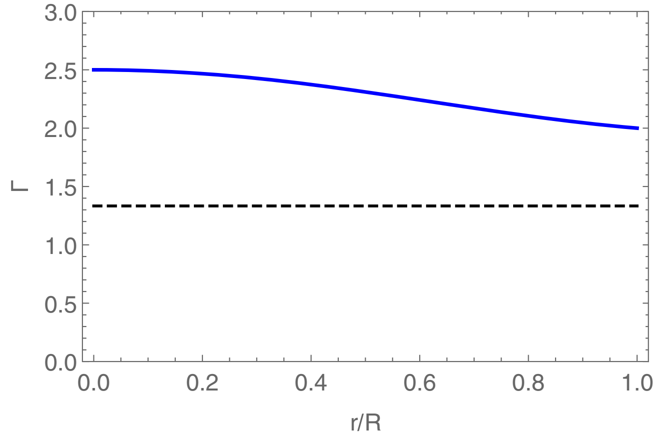

FIG. 2. Relativistic adiabatic index, Γ, versus normalized radial coordinate r/R. The horizontal line corresponds to the Newtonian limit of $4/3$.

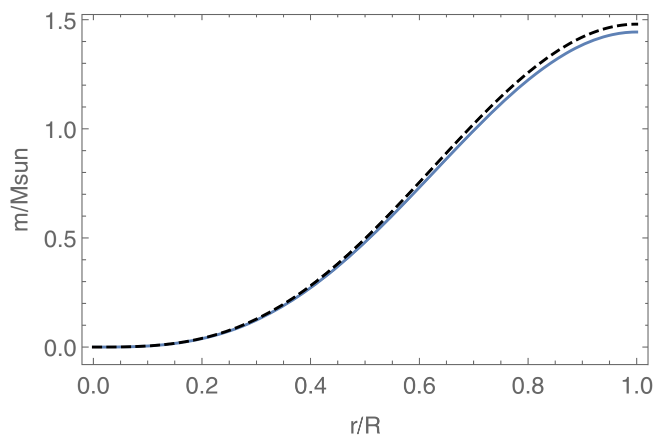

FIG. 3. Mass function (in solar masses) versus normalized radial coordinate r/R. The dashed curve corresponds to isotropic stars.

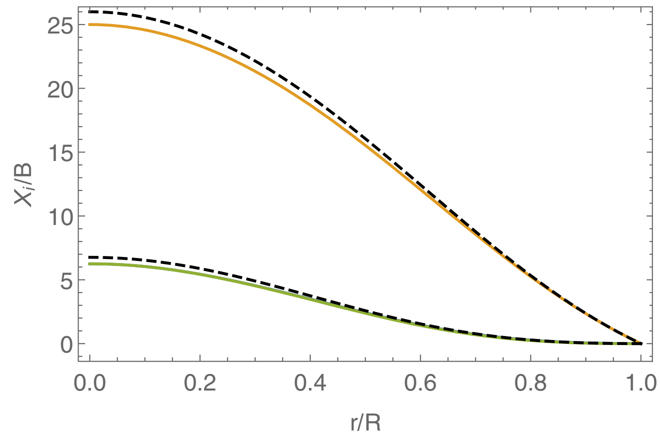

FIG. 5. Energy density ρ/B (orange curve) radial pressure p_r/B (blue curve) and tangential pressure p_t/B (green curve) versus r/R. The dashed curves correspond to isotropic stars, where energy density lies above pressure.

$$C = 0.22, \tag{45}$$

where $C = M/R$ is the compactness factor of the star.

In particular, Fig. 1 shows the normalized anisotropic factor $\Delta(r)/B$ versus r/R. It vanishes both at the center and at the surface of the star, and it is negative throughout the object. The relativistic adiabatic index, Γ, versus r/R is shown in Fig. 2, where the Newtonian limit of $4/3$ is shown as well.

In Figs. 3–6 a comparison is made between stars with anisotropic matter (solid curves) and their isotropic counterparts (dashed curves) with the same EOS and the same radius. In particular, in Fig. 3 we show the mass functions versus r/R, while Fig. 4 shows the two metric potentials versus r/R. Finally, Fig. 5 shows normalized energy density and pressures versus r/R, while in Fig. 6 we show

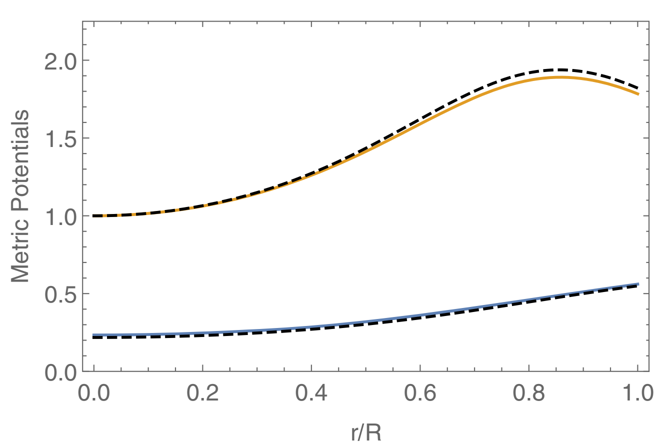

FIG. 4. The two metric components, e^{ν} (lower curve) and $1/(1-2m/r)$ (upper curve) versus normalized radial coordinate r/R. The dashed curves correspond to isotropic stars.

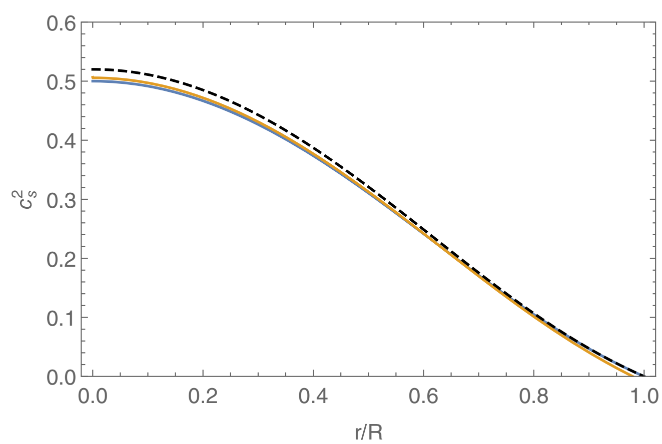

FIG. 6. Radial (blue curve) and tangential (orange curve) sound speeds, c_r^2, c_t^2, versus normalized radial coordinate r/R. The dashed curve corresponds to isotropic stars.

the speeds of sound, both radial (in blue) and tangential (in orange), versus r/R.

Clearly, causality is not violated as both sound speeds take values in the range (0,1) throughout the star. Moreover, the condition $\Gamma > 4/3$ is satisfied as well. Finally, since both pressures are positive and lower than the energy density, all energy conditions are fulfilled.

IV. CONCLUSIONS

In summary, in the present work we have studied exotic stars with anisotropic matter within general relativity. We have investigated in detail the properties of dark matter-type configurations, taking into account the presence of anisotropies. Semianalytic solutions have been obtained, where all quantities of interest, such as mass function, anisotropic factor, relativistic index, speed of sound etc., may be expressed in terms of energy density, which has been computed numerically. Causality, stability criteria and energy conditions have also been discussed. It has been found that the solutions obtained here are well-behaved solutions capable of describing realistic astrophysical configurations. Finally, a direct comparison with their isotropic counterparts has been made as well.

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