

Relation between general relativity and a class of Hořava gravity theories

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Violations of Lorentz (and specifically boost) invariance can make gravity renormalizable in the ultraviolet, as initially noted by Hořava, but are increasingly constrained in the infrared. At low energies, Hořava gravity is characterized by three dimensionless couplings, α , β and λ , which vanish in the general relativistic limit. Solar system and gravitational wave experiments bound two of these couplings (α and β) to tiny values, but the third remains relatively unconstrained ($0 \leq \lambda \lesssim 0.01-0.1$). Moreover, demanding that (slowly moving) black-hole solutions are regular away from the central singularity requires α and β to vanish *exactly*. Although a canonical constraint analysis shows that the class of khronometric theories resulting from these constraints ($\alpha = \beta = 0$ and $\lambda \neq 0$) cannot be equivalent to General Relativity, even in vacuum, previous calculations of the dynamics of the solar system, binary pulsars and gravitational-wave generation show perfect agreement with general relativity. Here, we analyze spherical collapse and compute black-hole quasinormal modes, and find again that they behave *exactly* as in general relativity, as far as *observational* predictions are concerned. Nevertheless, we find that spherical collapse leads to the formation of a regular *universal* horizon, i.e., a causal boundary for signals of arbitrary propagation speeds, inside the usual event horizon for matter and tensor gravitons. Our analysis also confirms that the additional scalar degree of freedom present alongside the spin-2 graviton of general relativity remains strongly coupled at low energies, even on curved backgrounds. These puzzling results suggest that any further bounds on Hořava gravity will probably come from cosmology.

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I. INTRODUCTION

Lorentz symmetry is one of the cornerstone of our understanding of theoretical physics, and has been tested to exquisite precision in particle physics experiments [1–4]. Bounds on Lorentz violations (LVs) in gravity are however much weaker [5–7]. This is particularly interesting because violations of boost symmetry in gravity may allow for constructing a theory of quantum gravity that is power counting (or even perturbatively) renormalizable in the ultraviolet [8,9]. This proposal, initially put forward by Hořava [8], may still pass particle physics tests of Lorentz symmetry if a mechanism is included to prevent “percolation” of large LVs from gravity to matter. Among such putative mechanisms are renormalization group flows (whereby Lorentz invariance may be recovered, at least in matter, in the infrared) [10–13], accidental symmetries allowing for different degrees of LVs in gravity and matter [14], or the suppression of LVs in matter via a large energy scale [15].

The infrared limit of Hořava gravity, also known as khronometric theory, is characterized by three dimensionless coupling parameters α , β and λ , in terms of which the theory’s action is [8,16,17]

$$S = \frac{1-\beta}{16\pi G} \int dT d^3x N \sqrt{\gamma} \left(\mathcal{K}_{ij} \mathcal{K}^{ij} - \frac{1+\lambda}{1-\beta} \mathcal{K}^2 + \frac{1}{1-\beta} {}^{(3)}R + \frac{\alpha}{1-\beta} a_i a^i \right) + S_{\text{matter}}[g_{\mu\nu}, \Psi], \quad (1)$$

in units where $c = 1$ (used throughout this article), and where the bare gravitational constant G is related to the one measured on Earth and in the solar system (G_N) by

$$G_N = \frac{G}{1-\alpha/2}. \quad (2)$$

The action is written in terms of a *preferred* spacetime foliation described by $T = \text{const}$, and the metric has been decomposed in the $3+1$ form¹

$$ds^2 = N^2 dT^2 - \gamma_{ij} (dx^i + N^i dT)(dx^j + N^j dT), \quad (3)$$

where we recognize a lapse function N , a shift three-vector N_i and the spatial three-metric γ_{ij} . Also defined in terms of

¹From now on, Latin indices will run only over space directions, while Greek indices will also include time.

this decomposition are the other quantities appearing in the action, e.g., the determinant of the three-metric γ ; the extrinsic curvature of the foliation,

$$\mathcal{K}_{ij} = -\frac{1}{2N}(\partial_T \gamma_{ij} - D_i N_j - D_j N_i), \quad (4)$$

where the covariant derivative D_i is defined with respect to γ_{ij} ; the three-dimensional Ricci scalar ${}^{(3)}R$; $\mathcal{K} = \mathcal{K}^{ij}\gamma_{ij}$; and $a_i = \partial_i \ln N$. With Ψ we refer here to standard matter fields, which couple to the full four-dimensional metric $g_{\mu\nu}$. This action can be obtained from that of *nonprojectable Hořava Gravity* [16] by neglecting operators with more than two derivatives, relevant only at high energies. Note that by introducing a preferred foliation, Lorentz symmetry is broken at the local level. The action is invariant under foliation-preserving diffeomorphisms ($T \rightarrow \tilde{T}(T)$, $x^i \rightarrow \tilde{x}^i(x, T)$) but *not* under full four-dimensional diffeomorphisms.

The same action can be recast in covariant form by promoting the coordinate T to a (timelike) scalar field (the “khronon”) and defining a unit-norm, timelike “aether” vector field orthogonal to the hypersurfaces of $T = \text{const}$,

$$u_\mu = \frac{\nabla_\mu T}{\sqrt{\nabla^\alpha T \nabla_\alpha T}}, \quad (5)$$

where we assume a $+---$ metric signature (as in the following). This allows for writing the action as [18]

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} [R + \lambda(\nabla_\mu u^\mu)^2 + \beta \nabla_\mu u^\nu \nabla_\nu u^\mu + \alpha a_\mu a^\mu] + S_{\text{matter}}[g_{\mu\nu}, \Psi], \quad (6)$$

where $a^\mu \equiv u^\nu \nabla_\nu u^\mu$. Here, LVs are made apparent by the fact that the vector field \mathbf{u} is timelike, i.e., according to the definition (5),

$$u_\mu u^\mu = 1. \quad (7)$$

Although still weaker than in matter, LVs in gravity are becoming increasingly constrained, especially by gravitational wave (GW) experiments. Bounds on the propagation speed of GWs from GW170817 constrain $|\beta| \lesssim 10^{-15}$ [19,20], which paired with bounds from solar system experiments also allows for constraining $|\alpha| \lesssim 10^{-7}$ (with λ left unconstrained), or $|\alpha| \lesssim 0.25 \times 10^{-4}$ and $\lambda \approx \alpha/(1-2\alpha)$ [21–25]. Measurements of the abundance of primordial elements produced by big bang nucleosynthesis (BBN) constrain $\lambda \lesssim 0.1$ [26–28], with $\lambda \geq 0$ required to ensure absence of ghosts [16,20]. These bounds therefore seem to suggest that α and β should be tiny, while λ could still be sizeable. Indeed, an additional theoretical constraint—namely that black holes moving slowly relative to the preferred foliation remain regular except for their central singularity—would require α and β to vanish *exactly* [25].

We will refer to the theory with $\alpha = \beta = 0$ and $\lambda \neq 0$ as *minimal Hořava gravity* (mHG) in the following. Remarkably, all noncosmological *observables* that have been computed in Hořava gravity reduce to their GR counterparts in the mHG case. For instance, the dynamics in the solar system (i.e., at first post-Newtonian order) exactly matches that of GR [23,24]. GWs also propagate exactly at the speed of light [23]. Moreover, static spherically symmetric black holes are described by the Schwarzschild metric [29], and so are those moving slowly relative to the preferred foliation [25]. The same applies to stars, for which both static spherically symmetric solutions and ones describing slowly moving bodies are characterized by the same (GR) geometry [30]. Note that for both stars and black holes the khronon configuration is non-trivial, but does not backreact on the geometry in mHG. This is quite surprising—because objects at rest and in motion are expected to be described by the same metric only in a Lorentz-symmetric theory such as GR, and not (*a priori*) in a theory with LVs—and has implications also for the dynamics of binaries of compact objects and for GW generation.

Indeed, since the geometry of slowly moving stars and black holes is the same as in GR, the “sensitivities”—which parametrize violations of the strong equivalence principle at the leading post-Newtonian (PN) order [27,28,31]—can be shown to vanish exactly in mHG [25,30]. Therefore, no dipole GW emission from binaries of compact objects is expected in mHG [25,30], unlike for generic α , β (where this effect was used to test the theory with binary pulsars [27,28]).

A possible caveat regarding these experimental bounds is that the khronon becomes strongly coupled around the Minkowski and Robertson-Walker geometries in the mHG limit, since the scalar field T becomes non-propagating (i.e., its speed diverges) when $\alpha, \beta \rightarrow 0$ and $\lambda \neq 0$ [23,32]. We stress that strong coupling does not mean that the theory is not viable, but simply that the linearized calculations on the simple backgrounds mentioned above may provide incorrect results. However, since the strong coupling affects the khronon and not the tensor sector, the linear calculation of the speed of GWs (used to compare to the GW170817 observations) is expected to provide trustworthy results.

As for the PN calculations of the solar system dynamics and GW generation, it should be noted that (i) the PN scheme is an expansion in powers of $1/c$, and it thus includes nonlinear terms; and (ii) the Newtonian/PN dynamics is strongly coupled in GR as well, and yet it gives meaningful results. Indeed, at leading (Newtonian) order the gravitational field does not propagate in GR (i.e., the equation describing it is elliptic), and propagation only appears at higher PN orders [21,33]. We therefore expect the results from a PN expansion of the field equations to remain valid also in the mHG limit.

Given this wealth of (noncosmological) observables for which mHG provides the same predictions as GR, it is natural to wonder whether mHG and GR may be equivalent, at least in some regimes. Obviously, a full equivalence between GR and mHG can be excluded, since the cosmological expansion history is different in the two theories (a fact that is used to constrain λ with BBN data [27,28]), but it may hold in more specific settings. For instance Refs. [34,35], based on a constraint analysis of mHG, claimed that the theory may be equivalent to GR in vacuum and under asymptotically flat boundary conditions. While suggestive in the light of the ‘‘coincidences’’ presented above, this conclusion disagrees with that of Ref. [36], which solved the (tertiary) constraint equation of mHG and showed explicitly that the theory cannot be equivalent to GR unless $N = 0$ (in which case the metric is degenerate).

In this work, we will therefore attempt to identify (noncosmological) astrophysical observables for which mHG may differ from GR, focusing on fully nonperturbative calculations, or on ones that involve perturbations over backgrounds different from the Minkowski and Robertson-Walker geometries (on which the khronon is strongly coupled). In more detail, in Sec. II we will review linear perturbations of mHG on flat space. We will then study the non-linear dynamics of spherically symmetric collapse (in Sec. III), showing that a universal horizon (i.e., a boundary for signals of arbitrary speeds) [37,38] naturally forms inside the usual horizon for tensor gravitons and matter. Nevertheless, the collapse is completely indistinguishable from GR as far as observable quantities are concerned. In Sec. IV we will then derive the equations for linear metric perturbations over static spherically symmetric black holes, and show that they also coincide with the GR ones, when focusing on the tensor modes. The scalar mode remains instead strongly coupled (like in flat space), but decouples from the tensor sector. Our conclusions are finally presented in Sec. V.

II. KHROMETRIC THEORY AROUND FLAT SPACE

The dynamics of the action (6) is described in terms of the metric $g_{\mu\nu}$ and the æther vector \mathbf{u} . The latter is constrained to be unit-norm and timelike [cf. Eq. (5)] and hypersurface orthogonal, i.e., it must have, from the Fröbenius theorem, zero vorticity

$$u_{[\mu}\nabla_{\nu}u_{\sigma]} = 0. \quad (8)$$

Since the theory breaks boost invariance at the local level, it should propagate additional degrees of freedom besides the usual spin-2 graviton field $h_{\mu\nu}$ of GR. Indeed, a generic four-dimensional vector \mathbf{u} contains four degrees of freedom—which can be arranged into a three-dimensional divergence-less vector and two scalars. However, the unit

norm (7) and vorticity (8) conditions eliminate three of these degrees of freedom, leaving a single scalar behind (corresponding obviously to the khronon scalar field T defining the preferred foliation).

This can be seen directly at the level of the action by perturbing both the metric and æther around flat space:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad u^\mu = (1, \mathbf{0}) + v^\mu. \quad (9)$$

Replacing this into the action (6), we first go to momentum space—where $\partial_i \equiv i\omega$ and $D_i \equiv iq_i$, with ω and q_i the frequency and three-momentum respectively—and perform a 3 + 1 decomposition adapted to the foliation orthogonal to the background æther $(1, \mathbf{0})$, i.e., we decompose the metric perturbation and æther as

$$h_{\mu\nu} = \begin{pmatrix} h_{00} & h_{0i} \\ h_{0i} & h_{ij} \end{pmatrix}, \quad v_\mu = (v_0, v_i). \quad (10)$$

We then split the various quantities in modes that transform as scalars, vectors and tensors under rotations

$$h_{ij} = \zeta_{ij} + \frac{q_i}{q}X_j + \frac{q_j}{q}X_i + \frac{q_iq_j}{q^2}s_1 + \left(\delta_{ij} - \frac{q_iq_j}{q^2}\right)s_2, \quad (11)$$

$$v_i = Y_i + \frac{q_i}{q}s_3, \quad (12)$$

$$h_{0i} = Z_i + \frac{q_i}{q}s_4, \quad (13)$$

where X_i , Y_i and Z_i are divergenceless vectors—i.e., $D_iX^i = D_iY^i = D_iZ^i = 0$; s_i , v_0 and h_{00} are scalars; and ζ_{ij} is a transverse-traceless tensor—thus satisfying $D_i\zeta^{ij} = D_j\zeta^{ij} = \zeta_i^i = 0$.

The two constraints (7) and (8) kill three of these degrees of freedom, as previously mentioned. At the linear level, they impose

$$h_{00} + 2v_0 = 0, \quad \epsilon_{ijk}q^jY^k = 0, \quad (14)$$

where the second of these conditions is satisfied by setting $Y^k = 0$. Once these conditions are enforced, the momentum space Lagrangian for the perturbations, retaining only quadratic terms, becomes

$$\begin{aligned} \mathcal{L} = & -\frac{1}{8}(\lambda + \beta)\omega^2s_1^2 - \frac{1}{2}(1 + \lambda)\omega^2s_1^2 - \lambda q\omega s_1s_3 \\ & + \frac{1}{4}(-q^2 - (1 + \beta + 2\lambda)\omega^2)s_1^2 - \frac{1}{2}(\lambda + \beta)q\omega s_1s_3 \\ & + \frac{1}{2}(-(\lambda + \beta)q^2 - \alpha\omega^2)s_3^2 + q\omega s_1s_4 - \alpha\omega^2s_3s_4 \end{aligned} \quad (15)$$

$$\begin{aligned}
& -\frac{1}{2}\alpha\omega^2 s_4^2 + \frac{1}{8}(q^2 + (1-\beta)\omega^2)\zeta_{ab}\zeta^{ab} - \frac{1}{2}q\omega X^a Z_a \\
& + \frac{1}{4}(1-\beta)\omega^2 X_a X^a - \alpha q\omega s_4 v_0 - \frac{1}{2}\alpha q^2 v_0^2 - \alpha q\omega s_3 v_0 \\
& + \frac{1}{4}((1-\beta)q^2 - 2\alpha\omega^2)Z_a Z^a + q^2 s_1 v_0, \quad (16)
\end{aligned}$$

where we have omitted a global factor of G .

We are left with the task of choosing a suitable gauge. Since the action (6) from which we started is covariant (being related to the “unitary gauge” action (1) by a Stuckelberg transformation), we need to choose four gauge conditions. These can be given as the requirement that two scalars and one of the three-dimensional divergenceless vectors vanish. We choose $s_1 = v_0 = X^a = 0$, and replacing these conditions (as well as those that follow from the equations of motion of these fields) back in the action, we obtain

$$\begin{aligned}
\mathcal{L} = & \frac{1-\beta}{8}\zeta_{ab}(\omega^2 - c_2^2 q^2)\zeta^{ab} \\
& + \frac{(\beta-1)(\lambda+\beta)(2+2\lambda+\beta)}{4(1+\lambda)^2}\tilde{s}(\omega^2 - c_0^2 q^2)\tilde{s}, \quad (17)
\end{aligned}$$

where we have also rescaled the remaining scalar as $\tilde{s} = \frac{q}{\omega}s_3$. This is the Lagrangian of two modes propagating with speeds [23,39]

$$c_2^2 = \frac{1}{1-\beta}, \quad (18)$$

$$c_0^2 = \frac{(\lambda+\beta)(2-\alpha)}{\alpha(1-\beta)(2+3\lambda+\beta)}. \quad (19)$$

As previously mentioned, we find an extra propagating scalar field with velocity c_0 , besides the usual transverse-traceless graviton with velocity c_2 . Both propagation velocities can be different from the speed of light, although the coincident observation of GW170817 and GRB 170817A constrains c_2 to match c to within about 10^{-15} (which in turn bounds $|\beta| \lesssim 10^{-15}$). As for the scalar mode, cosmic ray observations require $c_0 \gtrsim 1$, because otherwise ultrahigh energy particles would lose energy to the khronon in a Cherenkov-like cascade [40].

Superluminality is of course not surprising, since the theory is not boost-invariant, and thus $c = 1$ is not a universal maximum speed. However, although in general the scalar velocity (19) is finite, it diverges in the mHG limit, if $\lambda \neq 0$. This is a signal that the linearized expansion breaks down for the dynamics of the scalar field, which is then out of reach of perturbative techniques, while the tensor mode remains healthy. The same conclusion is achieved by performing an identical expansion around FRW space-times or around any maximally symmetric spacetime [41]. Note also that this potentially problematic

behavior of the scalar field only appears at low energies. At higher energies the action must be extended by operators with higher number of derivatives, which deform the dispersion relations and lead to a healthy propagating scalar mode.

In light of this “strong-coupling problem” for the khronon on flat space, we pursue in the following two distinct calculations in mHG, namely spherically symmetric gravitational collapse and linear perturbations over spherically symmetric static black hole spacetimes. We will aim to assess whether the khronon dynamics remains strongly coupled when nonlinearities are included in the equations of motion, or when spacetimes more general than Minkowski space are considered.

III. SPHERICAL COLLAPSE

Unlike in GR, Birkhoff’s theorem does not hold in khronometric theories, and vacuum spherically symmetric solutions (even when one imposes that they are static and asymptotically flat) are not unique [37,38,42–44]. In more detail, in a given khronometric theory, there exists a two-parameter family of static, spherically symmetric and asymptotically flat vacuum solutions. One of the parameters characterizing these solutions is (like in GR) their mass, while the second parameter regulates the radial tilt of the æther near spatial infinity [37,38,42–44]. In particular, for a given mass, only a specific value of this second parameter yields solutions that are regular everywhere except for the central $r = 0$ curvature singularity. These are the solutions that are expected to form in gravitational collapse [45] and which are usually referred to as “black holes” in the literature [37,38,42].

Although the geometry of these black holes is similar to that of the Schwarzschild solution of GR (with which it actually coincides exactly in the mHG limit), the existence of the khronon mode has profound implications for their causal structure. As shown in Sec. II, at low energies Hořava gravity propagates both spin-2 and spin-0 gravitons, whose speeds are generally different from c (i.e., the limiting speed for matter modes). As a result, different causal boundaries exist for spin-0, spin-2 and matter modes, i.e., black holes in khronometric theories present spin-0, spin-2 and matter horizons at (generally) distinct locations.

Even more worryingly, when terms of higher order in the (spatial) derivatives are included in the infrared action (1), Hořava gravity predicts that the dispersion relations (for both the gravitons and matter) will take the form $\omega^2 = c_i^2 q^2 + Aq^4 + Bq^6$, with A and B constant coefficients and c_i the species infrared phase velocity. Therefore, the group velocity $d\omega/dq$ of all species will diverge in the ultraviolet limit, which questions whether it makes sense to talk about event horizons at all. The problem is even more evident in mHG, where the spin-0 propagation speed diverges already in the infrared limit (cf. Sec. II).

However, an unavoidable requirement for any physical modes is that they propagate in the future, as defined by the preferred foliation. Therefore, the topology of the hypersurfaces of constant khronon plays a crucial role in defining the spacetime's causal structure. In the infrared black hole solutions of [37,38], there exists indeed a hypersurface of $T = \text{const}$ that is also a hypersurface of constant radius. Once inside this hypersurface—which was called the “universal horizon” in [37,38]—no modes can escape, even if they propagate at infinite speeds, simply because they need to move in the future direction defined by the background preferred foliation. Note that in the special case of mHG, this universal horizon coincides with the spin-0 horizon, since the khronon propagation speed diverges already in the infrared limit.

Despite their attractive features, as mentioned above, black holes are not the only static and spherically symmetric solutions of khronometric theory. Indeed, generic values of the æther tilt parameter yield solutions that are singular at the spin-0 horizon. In particular, if the tilt parameter is such that the æther does not present any radial component at spatial infinity, that component vanishes throughout the entire spacetime (i.e., the æther is always parallel to the timelike Killing vector), and the resulting solutions describe the exterior spacetime of static spherically symmetric stars (whose matter “covers” the singularity at the spin-0 horizon) [43,44].

For concreteness, let us examine the special case of mHG, where these static and spherically symmetric vacuum solutions can be obtained analytically and read [29]

$$ds^2 = f(r)dt^2 - \frac{B(r)^2}{f(r)}dr^2 - r^2d\Omega^2, \quad (20)$$

$$u_\alpha dx^\alpha = \frac{1+f(r)A(r)^2}{2A(r)}dt + \frac{B(r)}{2A(r)}\left[\frac{1}{f(r)} - A(r)^2\right]dr, \quad (21)$$

where

$$f(r) = 1 - \frac{2G_N M}{r}, \quad B(r) = 1, \quad (22)$$

$$A(r) = \frac{1}{f} \left(-\frac{r_\text{æ}^2}{r^2} + \sqrt{f + \frac{r_\text{æ}^4}{r^4}} \right). \quad (23)$$

The two parameters characterizing each solution are the mass M and the “radial tilt” $r_\text{æ}$. These solutions are singular at the universal horizon (which in mHG coincides with the spin-0 horizon, as mentioned above) unless $r_\text{æ} = 3^{3/4}G_N M/2$ [29]. The latter value describes instead a black hole with a regular universal horizon (located at areal radius $r_U = 3G_N M/2$), while $r_\text{æ} = 0$ describes a static æther $\mathbf{u} \propto \partial_t$.

Note that Eqs. (20)–(23), whatever the value of $r_\text{æ}$, yield $\nabla_\mu u^\mu = 0$. If we now express the class of solutions given by Eqs. (20)–(23) in the unitary gauge (where the khronon is

used as the time coordinate T), the unit-norm, future directed æther vector \mathbf{u} becomes orthogonal to the preferred foliation $T = \text{const}$, which therefore presents $\mathcal{K} = \nabla_\mu u^\mu = 0$. Therefore, Eqs. (20)–(23) yield the Schwarzschild geometry foliated in maximal (preferred) slices $\mathcal{K} = 0$, which have long been studied in the context of numerical relativity [46–51].

In order to ascertain which solution, in the class described by Eqs. (20)–(23), is produced as the end-point of gravitational collapse, let us consider the equations of motion for time dependent configurations, which can be obtained by varying the action (1). Variation with respect to the shift yields the momentum constraint $\mathcal{H}^i = 0$. Variation with respect to the lapse yields an equation $\mathcal{H} = 0$ that reduces to the GR energy constraint when $\alpha, \beta, \lambda \rightarrow 0$, but which is *not a priori* a constraint equation in khronometric theory. In fact, for generic α, β, λ the resulting equation is *not* a constraint, but corresponds to the khronon's evolution equation in the covariant formalism of action (6). Finally, by varying with respect to γ_{ij} one obtains the evolution equations $\mathcal{E}^{ij} = 0$. We describe matter by a perfect fluid, whose stress-energy tensor

$$\mathcal{T}^{\mu\nu} = (\rho + p)U^\mu U^\nu - pg^{\mu\nu}, \quad (24)$$

(where p, ρ and \mathbf{U} are the fluid's pressure, energy density and four-velocity) is covariantly conserved ($\nabla_\mu \mathcal{T}^{\mu\nu} = 0$), since in Eqs. (1) and (6) matter couples only to the four-dimensional metric $g_{\mu\nu}$. The explicit form of $\mathcal{H}, \mathcal{H}^i$ and \mathcal{E}^{ij} is given in the Appendix A (see also [24]) for generic khronometric theories, from which the mHG equations can be obtained by setting $\alpha = \beta = 0$.

To simplify the algebra, let us choose spatial coordinates on the preferred slices such that $N^i = 0$. Unlike in GR, however, the lapse is not a gauge field, i.e., we have already chosen our time coordinate to be the khronon when writing the action (1), and thus no further conditions can be imposed on N . The most generic ansatz that we can write in spherical symmetry is therefore

$$\gamma_{ij}dx^i dx^j = A(T, R)dR^2 + R^2B(T, R)d\Omega^2, \quad (25)$$

$$N = Z(T, R). \quad (26)$$

For the matter, we assume that ρ and p are also functions of T and R alone, and that the three-velocity is radial, i.e., only $U^R(T, R)$ and $U^T(T, R)$ are nonzero (and related by the normalization condition $U_\alpha U^\alpha = 1$). Note that the class of metrics given by Eqs. (20)–(23) can be easily put in the form of Eq. (25) by performing first a coordinate transformation $t = T + H(r)$ (to go to the unitary gauge) and then a further (time dependent) coordinate transformation $r = RB(T, R)^{1/2}$ to eliminate the shift and render the metric diagonal. More explicitly, at large radii the solution given by Eqs. (20)–(23) yields

$$Z = 1 - \frac{G_N M}{R} + \frac{(2k_1 - 1)G_N^2 M^2}{2R^2} + \frac{(-2k_1^2 + 2k_1 + 2k_2 - 1)G_N^3 M^3}{2R^3} + \frac{-8G_N M T r_{\text{e}}^2 + 4r_{\text{e}}^4 + G_N^4 M^4 [8k_1^3 - 12k_1^2 + 8k_2 - 4k_1(4k_2 - 3) - 5]}{8R^4} + \mathcal{O}\left(\frac{1}{R}\right)^5, \quad (27)$$

$$A = 1 + \frac{2G_N M}{R} - \frac{2[(k_1 + k_2 - 2)G_N^2 M^2]}{R^2} + \frac{4T r_{\text{e}}^2 + G_N^3 M^3 (2k_1^2 - 8k_1 - 6k_2 + 8)}{R^3} + \mathcal{O}\left(\frac{1}{R}\right)^4, \quad (28)$$

$$B = 1 + \frac{2k_1 G_N M}{R} + \frac{(k_1^2 + 2k_2)G_N^2 M^2}{R^2} + \frac{2k_1 k_2 G_N^3 M^3 - 2T r_{\text{e}}^2}{R^3} + \mathcal{O}\left(\frac{1}{R}\right)^4, \quad (29)$$

where k_1 and k_2 are free parameters entering in the coordinate transformation.² Note that even though Z , A and B are time dependent, the dependence on time appears at subleading order in $1/R$. Moreover, the trace of the extrinsic curvature vanishes, as it did in the original foliation given by Eqs. (20)–(23), since $\mathcal{K} = \nabla_\mu u^\mu$ is a scalar under four-dimensional diffeomorphisms.

In order to have only first order equations for our system, let us then introduce $\mathcal{K}_A \equiv \mathcal{K}_R^R = -\partial_T A / 2AZ$, $\mathcal{K}_B \equiv \mathcal{K}_\theta^\theta = \mathcal{K}_\varphi^\varphi = -\partial_T B / 2BZ$, $D_Z \equiv \partial_R \log Z$, $D_A \equiv \partial_R \log A$ and $D_B \equiv \partial_R \log B$. With these variables, $\mathcal{K} = -\partial_T \ln \sqrt{\gamma} / N = \mathcal{K}_A + 2\mathcal{K}_B$.

With our ansatz, the nontrivial field equations are $\mathcal{H} = \mathcal{H}^R = \mathcal{E}^{RR} = \mathcal{E}^{\theta\theta} = 0$. As mentioned above, $\mathcal{H} = 0$ becomes the energy constraint in the GR limit, but is not generically a constraint in khronometric theory. To check whether $\mathcal{H} = 0$ is a constraint in mHG, let us consider the time derivative of \mathcal{H} . By using the equations of motions to simplify the expressions, one obtains

$$\partial_T \mathcal{H} = -\frac{\lambda}{RAZ} [(2 + RD)\partial_R \mathcal{K} + r\partial_R^2 \mathcal{K}], \quad (30)$$

where $D = 2D_Z - D_A/2 + D_B$. Equation (30) vanishes either in the GR limit $\lambda = 0$, or when the quantity within brackets is zero.

Barring the case $\lambda = 0$, one therefore has to solve $\partial_T \mathcal{H} = 0$ (which follows from the original field equation $\mathcal{H} = 0$) at each time T . Actually, the generic solution to $\partial_T \mathcal{H} = 0$ is simply

$$\partial_R \mathcal{K} = C(T)^2 \frac{A^{1/2}}{R^2 B Z^2}, \quad (31)$$

²Indeed, our ansatz (25) does not completely fix the gauge, as it is invariant under a time-independent redefinition of the radius. This residual gauge freedom arises because choosing $N_i = 0$ does not completely fix the spatial coordinates on $T = \text{constant}$ hypersurfaces, but merely ensures that *once* a set of spatial coordinates is chosen on some initial $T = \text{constant}$ hypersurface, then the spatial coordinates are fixed in the whole spacetime. The choice of coordinate on the initial slice, however, is arbitrary.

where $C(T)$ is an integration constant. In a gravitational collapse, e.g., of a star, one requires regularity at the center of the coordinates to obtain a physically meaningful solution [50,51]. Necessary conditions for regularity are that A, Z, B are finite (and nonvanishing) at $R = 0$, and that \mathcal{K} and its radial derivative also remain finite at the center. From Eq. (31), it is therefore clear that the only way to impose regularity at $r = 0$ is to set $C(T) = 0$ for any T , i.e., the extrinsic curvature \mathcal{K} must be constant on any given spatial foliation, i.e., $\mathcal{K}(T, R) = k(T)$. Note that a spatially constant trace was also expected from the Hamiltonian analysis of [35,52].

If $k(T) \neq 0$, by exploiting time-reparametrization invariance one can set $k(T) = 1$. The evolution equations $\mathcal{E}^{RR} = \mathcal{E}^{\theta\theta} = 0$ and the momentum constraint $\mathcal{H}^R = 0$ then take the same form as in GR, whereas the equation $\mathcal{H} = 0$ [which is now a *bona fide* Hamiltonian constraint since $C(T) = 0$] contains a term proportional to λ . However, as noted e.g., by [35], if asymptotically flat boundary conditions are assumed, at spatial infinity one must necessarily have $\mathcal{K} = k(T) = 0$ at all times [cf. also Eqs. (27)–(29)]. Boundary conditions at spatial infinity that are not necessarily flat, but which are time-independent, will yield also $\mathcal{K} = k(T) = 0$ at all times (see e.g., [53] for an example of one such GR collapse solution).

Similarly, outgoing boundary conditions at infinity also imply $\mathcal{K} = k(T) = 0$ at all times. This can be seen by noting that if one imposes $N \approx 1 + A_N \exp[i\omega_N(T - R)]/R$ and $\sqrt{\gamma} \approx 1 + A_\gamma \exp[i\omega_\gamma(T - R)]/R$ (with A_N, A_γ, ω_N , and ω_γ free coefficients), then at large R we find

$$\partial_R \mathcal{K} \simeq -\frac{A_\gamma \omega_\gamma^2}{R} \exp[i\omega_\gamma(T - R)]. \quad (32)$$

Requiring that $\partial_R \mathcal{K} = 0$ implies $A_\gamma = 0$ and thus $\mathcal{K} = 0$.

Finally, let us note that if $\mathcal{K} = 0$ at all times, the spherical collapse equations and the constraints become identical to the GR ones, written in the maximal slicing gauge $\mathcal{K} = 0$ [and in our zero-shift ansatz (25)]. Since $\mathcal{K} = \nabla_\mu u^\mu$ (with \mathbf{u} the unit-norm future-directed vector orthogonal to the foliation) is a scalar under four-dimensional

diffeomorphisms, one can then transform the spherical collapse equations of mHG into those of GR with maximal time-slicing $\mathcal{K} = 0$, *but* more general spatial coordinates (i.e., ones yielding general nonvanishing shift).³

The maximal time-slicing gauge has been extensively used in GR to study gravitational collapse, as it allows for penetrating the black hole horizon [46–51]. One can therefore utilize the results of GR simulations (either performed in the maximal slicing gauge, or transformed to that gauge *a posteriori*) to gain insight on spherical collapse in mHG.

Indeed, GR collapse simulations in the maximal time-slicing found that there exists a “limiting slice”, i.e., a limiting hypersurface that the maximal slices approach at late times [46–51]. In more detail, the slicing that arises in these simulations outside the collapsing sphere turns out to be described by the unit-norm future-directed vector \mathbf{u} given by Eq. (21), with the parameter r_{e} asymptotically approaching the critical value $3^{3/4}G_N M/2$. The limiting slice is therefore defined by areal radius $r = 3G_N M/2$ [46–51].

While in GR the foliation of the spacetime in time slices has no physical meaning (as it is merely a coordinate effect), the slicing has instead an important physical meaning in mHG, since we are using the unitary gauge, where the time coordinate coincides with the khronon scalar field. Indeed, the appearance of the limiting slice $r = 3G_N M/2$ in the GR maximal-slicing collapse simulations corresponds to the formation of a universal horizon in mHG.⁴ This can be understood because in spherical symmetry the universal horizon is, by definition, the outermost hypersurface $r = \text{const}$ that is also orthogonal to \mathbf{u} (or equivalently, the outermost hypersurface $r = \text{const}$ that is also a hypersurface of constant khronon $T = \text{const}$). We can therefore conclude that spherical collapse in mHG produces “regular” black holes, i.e., ones described by Eqs. (20)–(23). In particular, no singularity forms at the spin-0/universal horizon.

We stress, however, that an analysis of the principal part of the fully non-linear spherical collapse equations in generic khronometric theories, which we present in Appendix B, shows that the characteristic speed of the

scalar mode diverges in the mHG limit. This suggests that the effect of the khronon on spherical collapse in mHG may vanish simply because it satisfies an elliptic equation. It is therefore unclear if gravitational collapse will be the same as in GR when the assumption of spherical symmetry is relaxed. To partially tackle this problem, as well as to assess if moving away from flat space can fix the strong coupling of the khronon reviewed in Sec. II, in the next section we will consider linear, but otherwise generic, perturbations of black holes in mHG.

IV. QUASINORMAL MODES

Linear gravitational perturbations of black hole spacetimes in GR have been studied for decades, since the seminal work by Regge, Wheeler and Zerilli for the Schwarzschild geometry [57,58] and by Teukolsky for the Kerr one [59]. The frequency spectrum of these perturbations, once ingoing/outgoing boundary conditions are imposed at the event horizon/far from the black hole, is discrete and consists of complex frequencies. Since the imaginary part of the latter is such that the spectrum modes are exponentially damped (thus pointing, in particular, to linear stability of the Schwarzschild and Kerr solutions, at least for nonextremal spins), these modes are usually referred to as quasinormal modes (QNMs).

Interestingly, because the Kerr geometry can only depend on two “hairs” [60–63] (mass and spin⁵), the QNMs frequencies are found to only depend on the same two quantities. This observation has long prompted suggestions to use QNM observations to test the no-hair theorem and thus GR [65,66], a proposal that the LIGO/Virgo collaboration is starting to tentatively apply to real data [67–70], even though really constraining tests will probably have to wait for future detectors [71].

In order to compute QNM frequencies in mHG, let us start from the equations of motion in vacuum derived from the covariant action (6). From variations of the metric, one obtains

$$E_{\mu\nu} = G_{\mu\nu} - \lambda T_{\mu\nu}^{\text{kh}} = 0, \quad (33)$$

where $G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu}R/2$ and $T_{\mu\nu}^{\text{kh}}$ contains the contribution from the khronon:

$$\begin{aligned} T_{\mu\nu}^{\text{kh}} = & u_{\mu}u_{\nu}u^{\sigma}\nabla_{\sigma}\nabla_{\rho}u^{\rho} - g_{\mu\nu}\nabla_{\rho}u^{\rho} + \frac{1}{2}g_{\mu\nu}(\nabla_{\sigma}u^{\sigma})^2 \\ & + 2(g^{\rho}_{(\mu}u_{\nu)}) - u_{(\mu}u_{\nu)}u^{\rho})\nabla_{\rho}\nabla_{\sigma}u^{\sigma}. \end{aligned} \quad (34)$$

Variation of the khronon field T yields a scalar equation that is equivalent to the covariant conservation of $T_{\mu\nu}^{\text{kh}}$, already a consequence of (33) [18]. As such, it does not

³Note that we are changing the spatial coordinates to reinstate the shift, but *not* the time coordinate, which still coincides with the khronon field (unitary gauge).

⁴The correspondence between the appearance of a limiting foliation in GR in the maximal time slicing gauge and the formation of a universal horizon was also noticed in Ref. [54] for the case of Cuscuton theories. The latter are indeed equivalent to mHG if their scalar potential is quadratic [55,56] (even though the equivalence is subtle when it comes to the hypersurface orthogonality condition (8), as a result of which suitable boundary conditions are required to obtain an exact equivalence between the two theories [55]). However, Ref. [54] worked in the decoupling limit (i.e., neglecting the backreaction of the Lorenz-violating field on the geometry) and with zero potential (in which case the theory is not equivalent to mHG).

⁵The electric charge is believed to be zero or extremely small for astrophysical black holes [64].

need to be independently enforced if all of the ten components of Eq. (33) are satisfied. However, we show it here for completeness:

$$\kappa \equiv \lambda \nabla_\mu \left[\frac{(g^{\mu\nu} - u^\mu u^\nu)(\nabla_\nu \nabla_\sigma u^\sigma)}{\sqrt{\nabla_\alpha T \nabla^\alpha T}} \right] = 0. \quad (35)$$

Let us now perturb the metric and khronon fields around a curved background geometry, characterized by the pair $\{\bar{g}_{\mu\nu}, \bar{u}_\mu\}$:

$$\begin{aligned} g_{\mu\nu} &= \bar{g}_{\mu\nu} + \epsilon h_{\mu\nu} + \mathcal{O}(\epsilon^2), \\ u_\mu &= \bar{u}_\mu + \epsilon v_\mu + \mathcal{O}(\epsilon^2), \end{aligned} \quad (36)$$

where ϵ is a perturbative parameter (which sets the amplitude of the perturbations of the metric and æther/khronon, which needs to be small for the linear theory to be a good approximation). In the following, to keep the analysis more general, we will simply assume a spherically symmetric and static background, given by the ansatz of Eqs. (20)–(21). To restrict to a black hole background, one can then simply assume the validity of Eqs. (22)–(23), with $r_\varkappa = 3^{3/4} G_N M / 2$.

Inserting Eq. (36) into Eq. (33) and expanding to linear order in ϵ , we obtain the equations of motion for the perturbations in covariant form,

$$\bar{E}_{\mu\nu} + \epsilon \delta E_{\mu\nu} + \mathcal{O}(\epsilon^2) = 0, \quad (37)$$

where $\bar{E}_{\mu\nu} = 0$ is automatic from the choice of background. From now on we will drop the $\mathcal{O}(\epsilon^2)$ symbol everywhere for notational clarity. Note that the æther field enters Eq. (34) both with upper and lower indices. This implies that even if we set $v_\mu = 0$, we do not trivially recover the same equations for the perturbation as in GR, since there are still non-negligible contributions to $\delta E_{\mu\nu}$ coming from $u^\mu \approx \bar{u}^\mu + \epsilon v^\mu - \epsilon \bar{u}_\nu h^{\mu\nu}$. Note that this signals that the gravitational perturbations “feel” the presence of a background violating Lorentz invariance through the presence of the preferred foliation.

Since the background $\{\bar{g}_{\mu\nu}, \bar{u}_\mu\}$ is spherically symmetric, it is convenient to expand the perturbations in spin-weighted spherical harmonics. Using the standard Regge-Wheeler gauge [57] for the metric perturbations and performing a Fourier transform in the time coordinate (exploiting the staticity of the background), we obtain

$$h_{\mu\nu} = e^{-i\omega t} (h_{\mu\nu}^{\text{even}} + h_{\mu\nu}^{\text{odd}} \sin \theta \partial_\theta) P_\ell(\cos \theta), \quad (38)$$

where $P_\ell(x)$ is the ℓ th Legendre polynomial, with ℓ the angular momentum eigenvalue, and

$$h_{\mu\nu}^{\text{even}} = \begin{pmatrix} f(r)H_0^\ell(r) & H_1^\ell(r) & 0 & 0 \\ H_1^\ell(r) & \frac{H_2^\ell(r)}{f(r)} & 0 & 0 \\ 0 & 0 & r^2 K^\ell(r) & 0 \\ 0 & 0 & 0 & r^2 K^\ell(r) \sin^2 \theta \end{pmatrix}, \quad (39)$$

$$h_{\mu\nu}^{\text{odd}} = \begin{pmatrix} 0 & 0 & 0 & h_0^\ell(r) \\ 0 & 0 & 0 & h_1^\ell(r) \\ 0 & 0 & 0 & 0 \\ h_0^\ell(r) & h_1^\ell(r) & 0 & 0 \end{pmatrix}. \quad (40)$$

Here, without loss of generality (thanks to spherical symmetry), we have set the azimuthal number $m = 0$. The functions $H_0(r), H_1(r), H_2(r), K(r), h_0(r)$ and $h_1(r)$, where we have dropped the index ℓ to keep the notation compact, characterize the radial profile of the degrees of freedom of the metric perturbations. The perturbation of the æther v_μ depends on that of the khronon field T . If we make this explicit in the equations, the expressions quickly become very cumbersome. Instead, and equivalently, we choose to write a generic æther vector perturbation

$$v_\mu = (\phi_1^\ell(r), \phi_2^\ell(r), 2u_t \phi_3^\ell(r) \partial_\theta, 0) P_\ell(\cos \theta) e^{-i\omega t}, \quad (41)$$

where the factor of u_t is chosen for convenience, since it makes the resulting equations simpler. Imposing here the unit-norm and hypersurface-orthogonality conditions [Eqs. (7) and (8)] expanded at linear order in ϵ allows one to eliminate two of the three free functions appearing in Eq. (41).

Focusing first on the odd part of Eq. (37), we find that there are only three potentially independent equations, corresponding to $\delta E_{t\theta}, \delta E_{r\theta}$ and $\delta E_{\theta\phi}$. Notice that the perturbation of the æther, arising from the perturbation of a scalar field, has no odd contribution and therefore does not appear in the odd sector.

The function $h_0(r)$ can be algebraically solved from the system and, after defining $Q(r) \equiv f(r)h_1(r)/r$, we find that one of the remaining equations implies the other. We are thus left with a single independent equation of the Regge-Wheeler form [57]

$$\frac{d^2 Q}{dr_*^2} + [\omega^2 - V_{\text{odd}}(r)]Q = 0, \quad (42)$$

where we have introduced the tortoise coordinate in the usual way, i.e., $dr/dr_* = f(r)$. The effective potential $V_{\text{odd}}(r)$ reads

$$V_{\text{odd}} = \frac{\Lambda f(r)}{r^2} + \frac{2f(r)[f(r) - 1]}{r^2} - \frac{f'(r)f(r)}{r}, \quad (43)$$

where $\Lambda = \ell(\ell + 1)$. Replacing the black hole metric [Eq. (22)], this reduces *exactly* to the potential found in GR for the same Schwarzschild solution. We thus conclude that no differences from GR arise in the equations for odd perturbations, nor in the QNM frequencies in this parity sector.

In the even sector, the manipulation of the equations gets more complicated as they now involve the khronon perturbations as well. The full derivation of the equations presented below is shown in Appendix C, as it is rather lengthy and not particularly enlightening. In summary, the system is reduced to two second-order equations, for $\phi_3(r)$ and for an additional variable $\Psi(r)$ defined in Appendix C. These two modes represent the perturbations of the khronon and metric, respectively. The equation for Ψ decouples and reads

$$\frac{d^2\Psi}{dr_*^2} + [\omega^2 - V_{\text{even}}(r)]\Psi = 0, \quad (44)$$

with the potential

$$V_{\text{even}} = \frac{f}{r^2(1 + \Lambda - 3f)^2} [(1 + \Lambda)(\Lambda(\Lambda - 2) + 3) - 3f[(1 + \Lambda)^2 + 3f(f - 1 - \Lambda)]]. \quad (45)$$

This is a wave equation (in Fourier space) and again it *agrees exactly with the GR result* [58], once specialized to the black hole background [Eq. (22)]. Therefore, the even QNM frequencies for the metric perturbations coincide with their GR counterparts.

The equation for $\phi_3(r)$, however, remains coupled to $\Psi(r)$, which enters as a source

$$\phi_3''(r) + W_1(r)\phi_3'(r) + W_0(r)\phi_3(r) = j(r), \quad (46)$$

with

$$j(r) = U_1(r)\Psi'(r) + U_0(r)\Psi(r) \quad (47)$$

and

$$W_1(r) = \frac{-4A^4f^2 + 2A^2(5f + 3) - 4}{r(A^2f + 1)^2} + \frac{\omega(2i - 2iA^2f)}{A^2f^2 + f}, \quad (48)$$

$$W_0(r) = \frac{i(3f + 1)\omega(A^2f - 1)^3}{f^2r(A^2f + 1)^3} - \frac{\omega^2(A^2f - 1)^2}{f^2(A^2f + 1)^2} - \frac{4A^2\Lambda}{r^2(A^2f + 1)^2}. \quad (49)$$

The $U_i(r)$ are (very complicated) functions of the geometry, the frequency ω and the angular momentum ℓ , and explicit expressions for them are given in the Supplemental

Material [72] as *Mathematica* [73] files. We have confirmed that this is a general result by looking at the eigensystem of the generalized linear problem, when all equations are taken together: There is no (linear) change of variables which decouples the system into two independent differential equations.

Taking a closer look at our result, it may seem that the khronon field, which was strongly coupled around maximally symmetric spaces (cf. Sec. II), is now propagating, since Eq. (46) has a potential W_0 including ω^2 , which seems to indicate a finite propagating speed. However, this is just an illusion due to a poor choice of variables, since the equation also contains terms proportional to $\phi_3'(r)$. Performing a change of variables $\phi_3(r) = g(r)\phi(r)$ and choosing $g(r)$ to cancel all terms proportional to $\phi'(r)$, we get

$$g(r) = C_1 e^{\int_1^r dz l(z)}, \quad (50)$$

$$l(r) = \frac{i\omega A^2f - 1}{f A^2f + 1} + \frac{2 + 2A^4f^2 - A^2(3 + 5f)}{r(A^2f + 1)^2}, \quad (51)$$

where C_1 is an integration constant. The equation thus becomes

$$\phi''(r) - V_\phi(r)\phi(r) = j(r). \quad (52)$$

The new potential $V_\phi(r)$ has no term proportional to ω^2 , whose contribution has been cancelled by that coming from $g(r)$. This corresponds to a field propagating with infinite speed (so that $c_\phi^{-2} = 0$, with c_ϕ the propagation speed). This is analog to the situation in flat space. Therefore, we conclude that the khronon field remains strongly coupled also around black hole geometries [and actually around any static spherically symmetric solution of the class described by Eqs. (22)–(23)].

This result can also be confirmed by looking at the position of the spin-0 horizon for radial khronon modes ($\ell = 0$). In more detail, if one restores time derivatives in Eq. (46) by replacing $\omega \rightarrow i\partial_t$ and takes the eikonal limit—thus keeping only the highest radial and time derivatives of $\phi_3(r)$ —we find that Eq. (46) can be rewritten as

$$\mathbf{g}^{AB}\partial_A\partial_B\phi_3(r) \approx 0, \quad (53)$$

where the indices A, B run on $\{t, r\}$ and the (inverse) effective metric \mathbf{g} is given by

$$\mathbf{g}^{AB} = \begin{pmatrix} 1 - A^2f & f(A^2f - 1)(1 + A^2f) \\ f(A^2f - 1)(1 + A^2f) & f^2(1 + A^2f)^2 \end{pmatrix}. \quad (54)$$

By computing the null cones of this effective metric, one finds that the apparent horizon for the khronon

modes—which coincides with the event horizon, since the metric is static—lies at the outermost radius at which $\mathbf{g}^{rr} = 0$ (or equivalently $\mathbf{g}_{tt} = 0$). Since that location corresponds to $1 + A^2(r)f(r) = 0$, one can conclude that the spin-0 horizon coincides with the universal horizon, which we recall is the causal boundary for signals of infinite speed. This again signals that the khronon field remains strongly coupled even on spherically symmetric and static black hole spacetimes. We stress however that the khronon does not couple directly to matter (and in particular to GW detectors) at tree level.

V. CONCLUSIONS

In this work, we have explored the possibility of finding astrophysical signatures of theories of gravity that violate Lorentz symmetry, focusing on the case of Hořava gravity. Because of existing experimental and theoretical constraints, we have enforced $\alpha = \beta = 0$ in the action (1) that describes the low energy limit of Hořava gravity, obtaining a theory (for which we coined the name *minimal Hořava gravity*) depending on only one dimensionless coupling parameter λ , on which experimental bounds are relatively loose. We have focused on two phenomena that explore both the linear (around curved spacetime) and nonlinear dynamics of mHG, i.e., gravitational collapse of spherically symmetric matter configurations and the dynamics of the QNMs of the black holes produced by the collapse.

We have found that spherical collapse proceeds exactly as in GR as far as the spacetime metric is concerned, but that the khronon field undergoes a nontrivial dynamical evolution. In more detail, we have found that the hypersurfaces of constant khronon follow the same evolution as the maximal time slices $K = 0$ of GR, which are known to asymptote to a limiting slice (corresponding to areal radius $r = 3G_N M/2$) in spherical collapse [46–51]. Although in the case of GR the appearance of this limiting slice is just a coordinate effect (since the choice of foliation has only practical but not physical meaning), the foliation has a physical meaning in mHG (where diffeomorphism invariance is broken). We interpret the appearance of the aforementioned limiting slice as the formation of a universal horizon (i.e., a causal boundary for signals of arbitrary speed). This is a hallmark of black holes in Hořava gravity [37,38], and we thus conclude that collapse in mHG leads to the formation of a black hole (as opposed to other vacuum solutions of the theory).

We have also studied QNMs around this family of Lorentz-violating black holes, which are described by

the Schwarzschild metric, but which present a non-trivial khronon configuration. By using the standard Regge-Wheeler gauge, we have shown that the metric perturbations, both in the even and odd sectors, satisfy the same (linearized) equations as in GR. We have also found that the extra scalar mode of the theory, i.e., the perturbation of the khronon field, remains strongly coupled also around static and spherically symmetric spacetimes.

We therefore conclude that no (classical) observable deviations from GR arise in either spherical collapse or in the spectrum of black hole QNMs, at least if the khronon (which undergoes a nontrivial dynamics) does not couple directly to matter. While direct coupling of the khronon to matter is certainly possible, this would produce large violations of Lorentz invariance in the matter sector, which are tightly constrained by experiments. Another possibility to further test mHG may be provided by cosmology (which already places mild constraints on mHG via e.g., BBN) and in general by spacetimes which are not asymptotically flat.

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APPENDIX A: EQUATIONS OF MOTION IN THE UNITARY GAUGE

The variation of the action (1) with respect to the lapse N yields

$$\mathcal{H} \equiv {}^{(3)}R - (1 - \beta)\mathcal{K}_{ij}\mathcal{K}^{ij} + (1 + \lambda)\mathcal{K}^2 - \alpha a_i a^i - 2\alpha D_i a^i - 8\pi G_N(2 - \alpha)N^2 T^{00} = 0; \quad (\text{A1})$$

on the other hand, a variation with respect to the shift N_i gives

$$\mathcal{H}^i \equiv D_j \left(\mathcal{K}^{ij} - \frac{1 + \lambda}{1 - \beta} \gamma^{ij} \mathcal{K} \right) + 4\pi G_N N \frac{2 - \alpha}{1 - \beta} (T^{0i} + N^i T^{00}) = 0; \quad (\text{A2})$$

finally, variation with respect to the metric γ_{ij} gives

$$\begin{aligned}
\mathcal{G}^{ij} \equiv & {}^{(3)}R^{ij} - \frac{1}{2}{}^{(3)}R\gamma^{ij} - \frac{1}{N}D_T[(1-\beta)\mathcal{K}^{ij} - (1+\lambda)\gamma^{ij}\mathcal{K}] + \frac{2}{N}D_k[N^i((1-\beta)\mathcal{K}^{jk} - (1+\lambda)\mathcal{K}\gamma^{jk})] \\
& - \frac{1}{2}\gamma^{ij}[(1-\beta)\mathcal{K}^{kl}\mathcal{K}_{kl} + (1+\lambda)\mathcal{K}^2] + 2(1-\beta)\mathcal{K}^{ik}\mathcal{K}^j_k - \frac{1}{N}(D^iD^jN - \gamma^{ij}D_kD^kN) \\
& + \alpha\left(a^ia^j - \frac{1}{2}\gamma^{ij}a^2\right) - (1+\beta+2\lambda)\mathcal{K}^{ij}\mathcal{K} - 4\pi G_N(2-\alpha)(T^{ij} - N^iN^jT^{00}) = 0,
\end{aligned} \tag{A3}$$

where D_i is the covariant derivative compatible with γ_{ij} and $D_T \equiv \partial_T - N_k D^k$.

APPENDIX B: CHARACTERISTIC SPEEDS OF KHROMETRIC THEORY IN SPHERICAL SYMMETRY

In this Appendix, we write the evolution equations for the metric and khronon [Eqs. (A3) and (A1)] for the ansatz (25) in generic khrometric theories and in spherical symmetry, and compute their characteristic speeds. We also refer the reader to [45,74] for more details on spherical collapse in generic khrometric theories.

By combining with the momentum constraint (A2) and by introducing the variables $X \equiv \partial_T\sqrt{A}/Z$, $Y \equiv \partial_T\sqrt{B}/Z$, $A_R \equiv \partial_R\sqrt{A}$ and $B_R \equiv \partial_R\sqrt{B}$, Eqs. (A3) and (A1) can be put in the first order form

$$\partial_T \mathbf{u} + \mathbf{M} \cdot \partial_R \mathbf{u} = \mathbf{S} \tag{B1}$$

$$\partial_R^2 \sqrt{Z} = S_Z \tag{B2}$$

where $\mathbf{u} = (X, Y, B_R, A_R)$, \mathbf{M} is the characteristic matrix

$$\mathbf{M} = \begin{pmatrix}
(\beta + \lambda)\sqrt{ZAB}k_1 & 2(\lambda + 1)\sqrt{ZBA}k_1 & \frac{2(\alpha-2)(\lambda+1)Z}{\alpha(\beta-1)(\beta+3\lambda+2)\sqrt{AB}} & 0 \\
(\beta + \lambda)\sqrt{AZB}k_2 & 2(\lambda + 1)\sqrt{ZBA}k_2 & -\frac{(\alpha-2)(\beta+\lambda)Z}{\alpha(\beta-1)(\beta+3\lambda+2)A} & 0 \\
0 & -Z & 0 & 0 \\
-Z & 0 & 0 & 0
\end{pmatrix}$$

while \mathbf{S} and S_Z are complicated source terms that depend on Z , $\partial_R Z$, A , B , X , Y , B_R , A_R and the matter variables.

The characteristic matrix has four eigenvalues

$$\dot{R} = 0 \tag{B3}$$

$$\dot{R} = \sqrt{ABZ}(k_1(\beta + \lambda)\sqrt{B} + 2(\lambda + 1)k_2\sqrt{A}) \tag{B4}$$

$$\dot{R} = \pm c_0 \frac{Z}{\sqrt{A}} \tag{B5}$$

where k_1 and k_2 are functions of T and R , which can be chosen arbitrarily (as they regulate how the momentum constraint is linearly combined with the evolution equations), and c_0 is the propagation speed for the spin-0 modes in Minkowski space [cf. Eq. (19)], This means that the subsystem X, Y, B_R, A_R is strongly hyperbolic if $k_1 = k_2 \neq 0$ and if c_0 is real and finite, while Eq. (B2) can be solved as an ordinary differential equation at each time step (provided

that suitable boundary conditions are imposed on it). Note however, as stressed in the main text, that c_0 diverges in the mHG limit, signaling a strong-coupling problem.

APPENDIX C: THE LINEARIZED FIELD EQUATIONS FOR THE EVEN-PARITY SECTOR

Let us start from the trace-reversed system

$$\tilde{E}_{\mu\nu} \equiv E_{\mu\nu} - \frac{1}{2}g_{\mu\nu}E_{\alpha\beta}g^{\alpha\beta} = 0, \tag{C1}$$

and use it to compute the linearized equations $\delta\tilde{E}_{\mu\nu}$. In order to simplify them, we make use of the background field equations, and of the unit-norm and hypersurface-orthogonality constraints (7)–(8), in order to get rid of ϕ_1 , ϕ_2 and their derivatives.

In more detail, the seven nontrivial linearized equations have the following structure:

$$\begin{aligned}
\delta\tilde{E}_{tt} \propto & \lambda [C_K^u K + C_{H_0}^u H_0 + C_{H_1}^u H_1 + C_{H_2}^u H_2 + C_{\phi_3}^u \phi_3 + C_{K'}^u K' + C_{H_0'}^u H_0' + C_{H_1'}^u H_1' + C_{H_2'}^u H_2' + C_{\phi_3'}^u \phi_3' \\
& + C_{K''}^u K'' + C_{H_0''}^u H_0'' + C_{H_1''}^u H_1'' + C_{H_2''}^u H_2'' + C_{\phi_3''}^u \phi_3'' + C_{\phi_3^{(3)}}^u \phi_3^{(3)}] + \frac{(3f+1)i\omega H_1}{2r} - \frac{\omega^2 H_2}{2} \\
& + \frac{1}{2} H_0'' f^2 + \frac{(f-1)K'f}{2r} + \frac{(f+3)H_0'f}{4r} + i\omega H_1' f - \frac{(f-1)H_2'f}{4r} - \frac{\Lambda H_0'f}{2r^2} - K\omega^2 = 0,
\end{aligned} \tag{C2a}$$

$$\begin{aligned}
\delta\tilde{E}_{tr} \propto & \lambda [C_K^r K + C_{H_0}^r H_0 + C_{H_1}^r H_1 + C_{H_2}^r H_2 + C_{\phi_3}^r \phi_3 + C_{K'}^r K' + C_{H_0'}^r H_0' + C_{H_1'}^r H_1' + C_{H_2'}^r H_2' + C_{\phi_3'}^r \phi_3' \\
& + C_{K''}^r K'' + C_{H_0''}^r H_0'' + C_{H_1''}^r H_1'' + C_{H_2''}^r H_2'' + C_{\phi_3''}^r \phi_3'' + C_{\phi_3^{(3)}}^r \phi_3^{(3)}] \\
& - \frac{i(3f-1)K\omega}{2fr} - \frac{H_1\Lambda}{2r^2} + \frac{iH_2\omega}{r} - i\omega K' = 0,
\end{aligned} \tag{C2b}$$

$$\begin{aligned}
\delta\tilde{E}_{rr} \propto & \lambda [C_K^{rr} K + C_{H_0}^{rr} H_0 + C_{H_1}^{rr} H_1 + C_{H_2}^{rr} H_2 + C_{\phi_3}^{rr} \phi_3 + C_{K'}^{rr} K' + C_{H_0'}^{rr} H_0' + C_{H_1'}^{rr} H_1' + C_{H_2'}^{rr} H_2' + C_{\phi_3'}^{rr} \phi_3' \\
& + C_{K''}^{rr} K'' + C_{H_0''}^{rr} H_0'' + C_{H_1''}^{rr} H_1'' + C_{H_2''}^{rr} H_2'' + C_{\phi_3''}^{rr} \phi_3'' + C_{\phi_3^{(3)}}^{rr} \phi_3^{(3)}] \\
H_2 \left(\frac{\omega^2}{2f^2} - \frac{\Lambda}{2fr^2} \right) & + \frac{i(f-1)H_1\omega}{2f^2r} + \frac{3(f-1)H_0'}{4fr} - \frac{(3f+1)H_2'}{4fr} - \frac{i\omega H_1'}{f} + \frac{(3f+1)K'}{2fr} - \frac{H_0''}{2} + K'' = 0,
\end{aligned} \tag{C2c}$$

$$\begin{aligned}
\delta\tilde{E}_{\theta\theta} \propto & \frac{\delta\tilde{E}_{\phi\phi}}{\sin^2\theta} \propto \lambda [C_K^{\theta\theta} K + C_{H_0}^{\theta\theta} H_0 + C_{H_1}^{\theta\theta} H_1 + C_{H_2}^{\theta\theta} H_2 + C_{\phi_3}^{\theta\theta} \phi_3 + C_{K'}^{\theta\theta} K' + C_{H_0'}^{\theta\theta} H_0' + C_{H_1'}^{\theta\theta} H_1' \\
& + C_{H_2'}^{\theta\theta} H_2' + C_{\phi_3'}^{\theta\theta} \phi_3' + C_{K''}^{\theta\theta} K'' + C_{H_0''}^{\theta\theta} H_0'' + C_{H_1''}^{\theta\theta} H_1'' + C_{H_2''}^{\theta\theta} H_2'' + C_{\phi_3''}^{\theta\theta} \phi_3'' + C_{\phi_3^{(3)}}^{\theta\theta} \phi_3^{(3)}] \\
& - frH_0' - frH_2' + fr^2K'' + (3fr+r)K' + K \left(\frac{r^2\omega^2}{f} - \Lambda + 2 \right) \\
& + \frac{H_0\Lambda}{2} + H_2 \left(-\frac{\Lambda}{2} - 2 \right) - 2iH_1r\omega = 0,
\end{aligned} \tag{C2d}$$

$$\begin{aligned}
\delta\tilde{E}_{t\theta} \propto & \lambda \left[H_0 \left(\frac{(3f+1)(A^2f+1)(A^2f-1)^3}{32A^4f^2r} + \frac{i\omega(A^4f^2-1)^2}{32A^4f^2} \right) \right. \\
& + H_1 \left(\frac{(A^2f+1)^2(3A^4f^3 + (A^2-12)A^2f^2 + (3-4A^2)f+1)}{16A^4f^2r} + \frac{i\omega(A^2f+1)(A^2f-1)^3}{16A^4f^2} \right) \\
& + H_2 \left(\frac{(3f+1)(A^2f+1)(A^2f-1)^3}{32A^4f^2r} + \frac{i\omega(A^2f+1)^2(A^4f^2-6A^2f+1)}{32A^4f^2} \right) \\
& + \frac{(-A^8f^4 - 6A^6f^3 + 6A^2f+1)H_0'}{32A^4f} + \frac{(A^4f^2-1)K'}{4A^2} \\
& + \phi_3 \left(\frac{i(3f+1)\omega(A^2f+1)(A^2f-1)^3}{8A^4f^2r} - \frac{\omega^2(A^4f^2-1)^2}{8A^4f^2} - \frac{\Lambda(A^2f+1)^2}{2A^2r^2} \right) \\
& + \phi_3' \left(-\frac{(2A^4f^2 - A^2(5f+3) + 2)(A^2f+1)^2}{4A^4r} - \frac{i\omega(A^2f-1)(A^2f+1)^3}{4A^4f} \right) \\
& - \frac{(A^2f+1)^4H_1'}{16A^4f} - \frac{(A^2f-1)(A^2f+1)^3H_2'}{32A^4f} - \frac{iK\omega(A^2f+1)^2}{4A^2f} + \frac{(A^2f+1)^4\phi_3''}{8A^4} \left. \right] \\
& + \frac{(f-1)H_1}{2r} - \frac{fH_1'}{2} - \frac{1}{2}iH_2\omega - \frac{1}{2}iK\omega = 0,
\end{aligned} \tag{C2e}$$

$$\begin{aligned}
\delta\tilde{E}_{r\theta} \propto \lambda & \left[H_0 \left(-\frac{(3f+1)(A^2f-1)^4}{32A^4f^3r} - \frac{i\omega(A^2f+1)(A^2f-1)^3}{32A^4f^3} \right) \right. \\
& + H_1 \left(\frac{-3A^8f^5 - (A^2-12)A^6f^4 + 4A^6f^3 - 12A^2f^2 + (3-4A^2)f + 1}{16A^4f^3r} - \frac{i\omega(A^2f-1)^4}{16A^4f^3} \right) \\
& + H_2 \left(-\frac{(3f+1)(A^2f-1)^4}{32A^4f^3r} - \frac{i\omega(A^8f^4 - 6A^6f^3 + 6A^2f - 1)}{32A^4f^3} \right) + \frac{(A^4f^2-1)^2H'_2}{32A^4f^2} \\
& + \frac{(A^2f-1)(A^2f+1)^3H'_1}{16A^4f^2} + \frac{(A^2f-1)^2(A^4f^2 + 6A^2f + 1)H'_0}{32A^4f^2} + \frac{iK\omega(A^4f^2-1)}{4A^2f^2} \\
& + \phi_3 \left(\frac{\Lambda(A^4f^2-1)}{2A^2fr^2} - \frac{i(3f+1)\omega(A^2f-1)^4}{8A^4f^3r} + \frac{\omega^2(A^2f+1)(A^2f-1)^3}{8A^4f^3} \right) \\
& + \phi'_3 \left(\frac{2A^8f^4 - A^6f^2(5f+3) + A^2(5f+3) - 2}{4A^4fr} + \frac{i\omega(A^4f^2-1)^2}{4A^4f^2} \right) - \frac{(A^2f-1)^2K'}{4A^2f} \\
& \left. - \frac{(A^2f-1)(A^2f+1)^3\phi''_3}{8A^4f} \right] + \frac{(3f-1)H_0}{4fr} - \frac{(f+1)H_2}{4fr} - \frac{iH_1\omega}{2f} - \frac{H'_0}{2} + \frac{K'}{2} = 0, \tag{C2f}
\end{aligned}$$

$$\delta\tilde{E}_{\theta\phi} \propto H_0 - H_2 = 0. \tag{C2g}$$

The explicit expressions for the coefficients C_k^{ij} are given in the Supplemental Material [72] as *Mathematica* [73] files.

Let us notice that these seven equations contain only five independent variables H_0, H_1, H_2, K and ϕ_3 . This seems to imply that the system may be overdetermined. This turns out not to be the case, since some of these equations are redundant due to the Bianchi identity.

In more detail, from diffeomorphism invariance of the covariant gravitational action (6) (without the matter contribution) one obtains the generalized Bianchi identity [25]

$$\nabla_\mu E^{\mu\nu} = -\frac{\kappa}{2} \sqrt{\nabla_\alpha T \nabla^\alpha T} u^\nu. \tag{C3}$$

Taking linear combinations to cancel out the explicit dependence on T and κ and performing trivial manipulations, one can then write the identity

$$\nabla_\nu (E^{\mu\nu} u^\alpha - E^{\alpha\nu} u^\mu) = E^{\mu\nu} \nabla_\nu u^\alpha - E^{\alpha\nu} \nabla_\nu u^\mu, \tag{C4}$$

which can be used to show that two of the seven equations can be eliminated from the system without loss of generality. This can also be seen by direct manipulation of the equations of motion, as we will now show.

From $\delta\tilde{E}_{\theta\phi} = 0$, we obtain

$$H_2(r) = H_0(r), \tag{C5}$$

which allows us to get rid of H_2 completely. The structure of the remaining equations is then the following: on the one hand, the equations $\delta\tilde{E}_{tt}, \delta\tilde{E}_{tr}, \delta\tilde{E}_{rr}, \delta\tilde{E}_{\theta\theta}$ contain up to second derivatives of the metric perturbations and up to

third derivatives of $\phi_3(r)$ ⁶; on the other hand, in $\delta\tilde{E}_{t\theta}$ and $\delta\tilde{E}_{r\theta}$ one can find up to first derivatives of the metric perturbations and up to second derivatives of $\phi_3(r)$. Thus, from $\delta\tilde{E}_{t\theta} = 0$ and $\delta\tilde{E}_{r\theta} = 0$ we can solve algebraically for $H'_1(r)$ and $\phi''_3(r)$. This defines an equation for the scalar field, which we denote by $F_\phi = 0$. The next step is to use the expressions for $H'_1(r)$ and $\phi''_3(r)$ (and their derivatives) to eliminate H'_1, H''_1, ϕ''_3 and $\phi_3^{(3)}$ from the rest of the equations. By doing so, we obtain $\tilde{E}_{rr} \propto \tilde{E}_{tt}$.

We then solve $\delta E_{tt} = 0, \delta E_{rt} = 0$ and $\delta E_{\theta\theta} = 0$ and get algebraic expressions for $H''_0(r), K''(r)$ and $H_1(r)$, which take the schematic form

$$F_0 \equiv H''_0 - d_1 H'_0 + d_2 K' + d_3 H_0 + d_4 K = 0, \tag{C6}$$

$$F_K \equiv K'' - d_5 H'_0 + d_6 K' + d_7 H_0 + d_8 K = 0, \tag{C7}$$

$$H_1 - d_9 H'_0 + d_{10} K' + d_{11} H_0 + d_{12} K = 0, \tag{C8}$$

where the d_i are functions of r, ω and Λ . We have checked that the derivative of Eq. (C8) coincides with the previous analytic solution that we had found for H'_1 , so Eq. (C8) is redundant. We are thus left with three independent equations, corresponding to $F_\phi = 0, F_0 = 0$ and $F_K = 0$, which depend only on three variables ϕ_3, H_0 and K , with the scalar field present only in F_ϕ . Moreover, both F_0 and F_K are independent of λ , and F_ϕ contains only first derivatives of H_0 and K .

⁶Third radial derivatives appear after imposing the hypersurface-orthogonality condition, Eq. (8).

In the GR limit, $\lambda \rightarrow 0$, the dependence on ϕ_3 also disappears from F_ϕ . In that case, compatibility of the system would require that one of the equations is redundant. Note that this must be the case since we know that in the GR limit

the energy constraint is reinstated, cf. Eq. (30). Since F_ϕ reduces to a first order equation in the GR limit, it can be used, upon substitution into the other equations, to reduce the whole system to two first-order equations relating H_0 and K :

$$-\frac{(f^2(-\Lambda) + f(\Lambda^2 - 2\Lambda + 14r^2\omega^2) + \Lambda - 2\Lambda r^2\omega^2 - 6r^2\omega^2)}{fr(f\Lambda - \Lambda + 4r^2\omega^2)}H_0 + \frac{(f^2(\Lambda^2 - 2\Lambda + 9r^2\omega^2) - 2f(2\Lambda + 1)r^2\omega^2 + 4r^4\omega^4 + r^2\omega^2)}{f^2r(f\Lambda - \Lambda + 4r^2\omega^2)}K + H'_0 = 0, \quad (\text{C9a})$$

$$\frac{H_0(-2f\Lambda + \Lambda^2 + 4r^2\omega^2)}{-f\Lambda r - 4r^3\omega^2 + \Lambda r} + \frac{K(f(\Lambda^2 - 2\Lambda + 6r^2\omega^2) - 2(\Lambda + 1)r^2\omega^2)}{fr(f\Lambda - \Lambda + 4r^2\omega^2)} + K' = 0. \quad (\text{C9b})$$

Since Eqs. (C6)–(C7) can be shown to be independent of λ , Eq. (C9) also holds in the general case, as can be checked by direct substitution.

A last simplification occurs by introducing the same variable transformation as in [58,75], given by

$$K = \frac{\Lambda(1 + \Lambda) - 3(2 + \Lambda)f + 6f^2}{2r(1 + \Lambda - 3f)}\Psi + f\Psi', \quad (\text{C10})$$

$$H_0 = -\frac{1 + \Lambda - 3\Lambda f + 3f^2}{2(1 + \Lambda - 3f)}\Psi' + \left[\frac{1 + \Lambda - 3f}{6r} + \frac{(\Lambda - 2)^2(1 + \Lambda)}{3r(1 + \Lambda - 3f)^2} - \frac{r\omega^2}{f} \right]\Psi. \quad (\text{C11})$$

After performing this transformation, Eq. (C9) reduces to the simple equation

$$\frac{d^2\Psi}{dr_*^2} + [\omega^2 - V_{\text{even}}(r)]\Psi = 0, \quad (\text{C12})$$

with potential

$$V_{\text{even}} = \frac{f}{r^2(1 + \Lambda - 3f)^2} [(1 + \Lambda)(\Lambda(\Lambda - 2) + 3) - 3f[(1 + \Lambda)^2 + 3f(f - 1 - \Lambda)]]. \quad (\text{C13})$$

However, in the general case of nonvanishing λ , the third equation $F_\phi = 0$ remains independent and serves as the equation of motion for the scalar field:

$$\phi_3''(r) + W_1(r)\phi_3'(r) + W_0(r)\phi_3(r) = j(r), \quad (\text{C14})$$

with

$$j(r) = U_1(r)\Psi'(r) + U_0(r)\Psi(r). \quad (\text{C15})$$

The explicit forms of the functions $W_i(r)$ are

$$W_1(r) = \frac{-4A^4f^2 + 2A^2(5f + 3) - 4}{r(A^2f + 1)^2} + \frac{\omega(2i - 2iA^2f)}{A^2f^2 + f}, \quad (\text{C16})$$

$$W_0(r) = \frac{i(3f + 1)\omega(A^2f - 1)^3}{f^2r(A^2f + 1)^3} - \frac{\omega^2(A^2f - 1)^2}{f^2(A^2f + 1)^2} - \frac{4A^2\Lambda}{r^2(A^2f + 1)^2}, \quad (\text{C17})$$

while $U_0(r)$ and $U_1(r)$ are included in the Supplemental Material [72] as *Mathematica* [73] files.

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