# Intermittent null energy condition violations during inflation and primordial gravitational waves

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Primordial null energy condition (NEC) violation would imprint a blue-tilted spectrum on gravitational wave background (GWB). However, its implications on the GWB might be far richer than expected. We present a scenario, in which after a slow-roll (NEC-preserving) inflation with Hubble parameter $H \simeq H_{inf1}$, the Universe goes through an NEC-violating period and then enters subsequent slow-roll inflation with a higher $H$ $(= H_{inf2} \gg H_{inf1})$. The resulting primordial gravitational wave spectrum is nearly flat at the cosmic microwave background band, as well as at the frequency $f \sim 1/\mathrm{yr}$ but with higher amplitude (compatible with the recent NANOGrav result). It is also highlighted that for the multistage inflation if the NEC violations happened intermittently, we might have a Great Wall–like spectrum of the stochastic GWB at the corresponding frequency band.

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## I. INTRODUCTION

The primordial gravitational wave background (GWB) [1,2] with a broad frequency-band ($10^{-18}$–$10^{10}$ Hz) carries rich information about the early Universe. It is usually thought that its detection will not only solidify our confidence in inflation, but also offer us an unparalleled probe to the physics related to the cosmological (non)singularity, in which the null energy condition (NEC) violation might play a significant role [3–13], and the UV-complete gravity theory.

The primordial gravitational waves (GWs) at the ultra-low frequency band ($10^{-18}$–$10^{-16}$ Hz) would induce the B-mode polarization in the cosmic microwave background (CMB). The search for the primordial GWs with CMB has been still in progress. The pulsar timing array (PTA) experiments focus on GWB at frequencies $f \sim 1/\mathrm{yr}$ ($\sim 10^{-8}$ Hz). Recently, based on the 12.5-yr data analysis, the NANOGrav Collaboration reported evidence for a stochastic *common-spectrum* process [14], which might be interpreted as a stochastic GWB with a spectrum tilt $-1.5 \lesssim n_T \lesssim 0.5$; see [15–18] for the implications of NANOGrav's result in inflation. The current bound on GWB at CMB band indicates a tensor-to-scalar ratio $r \lesssim 0.06$ [19]. Therefore, only if the primordial GWs have a blue-tilted spectrum, it is able to be detected by the experiments and detectors at other frequency bands.

It is well known that for inflation, if initially the GW modes sit in the Bunch-Davis state (or, e.g., [16,20]), a blue-tilted spectrum suggests that the corresponding inflation is inevitably NEC-violating, i.e., $T_{\mu\nu}n^\mu n^\nu < 0$, which corresponds to $\dot{H} > 0$, namely, super-inflation[1] [21–24]. The NEC-violating inflation may be performed stably with the Galileon theory [25,26] and the effective field theory (EFT) of inflation [27,28]. If initially the NEC is violated drastically ($\dot{H} \gg H^2$), it is also possible that our Universe is asymptotically Minkowskian and slowly expanding in infinite past [21]. In such scenarios, the hot "big bang" evolution or inflation starts after the end of the slow expansion or Genesis [29–44]. Based on the beyond-Horndeski EFT (see, e.g., [45,46] for reviews), the Genesis could be implemented without pathologies (including instabilities and superluminality) [8–13].

Inspired by current (and upcoming) experiments searching for GWB, it is significant to resurvey the imprints of NEC violation on stochastic GWB. Recently, a scenario in which the superinflation is followed by a slow-roll (NEC-preserving) inflation has been proposed in [17], which yields a large stochastic GWB with $n_T \simeq 0.9$ at the PTA

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[1]In this paper, the term superinflation is used for the accelerated expansion in which $\epsilon = -\dot{H}/H^2 < 0$, since $\ddot{a}/a = H^2(1-\epsilon)$.

 

band. But the spectrum of scalar perturbations, i.e., $P_s$, is highly blue tilted too, since $n_s - 1 \simeq n_T$. Consequently, other fields must be responsible for the density perturbation at the CMB band. However, it is possible that a slow-roll (NEC-preserving) inflation with $H = H_{inf1}$, which results in $P_s \sim H^2_{inf1}/\epsilon \sim 10^{-9}$ at the CMB band, happened before the NEC-violating phase, which is subsequently followed by a slow-roll inflation with a higher scale $H_{inf2} \gg H_{inf1}$. In Refs. [47,48], such a low-scale inflation is regarded as current accelerated expansion with $H^2_{inf1} \sim \Lambda$. The consistent joint of an NEC-preserving spacetime to an NEC-violating phase is also explored in Ref. [49]; see also [50].

In this paper, we investigate the possibility of a short NEC violation during the NEC-preserving inflation. In this scenario, the low-scale inflation prior to NEC violation is responsible for the density perturbation on large scales. We calculate the corresponding primordial GW spectrum. Specially, we highlighted that for the multistage (NEC-preserving) inflation, if the NEC violations happened intermittently, a Great Wall–like landscape of primordial GW spectrum at the full frequency band will present.

## II. OUR SCENARIO

### A. Intermittent NEC violation during inflation

In our scenario (see Fig. 1), initially the field $\phi$ (canonical scalar field) slowly rolls down a nearly flat potential, i.e., $\dot{\phi}^2 \ll V(\phi) \approx V_{inf1}$, which results in the slow-roll (NEC-preserving) inflation. In the NEC-violating phase, $\phi$ climbs up the potential rapidly so that $\dot{H} > 0$. After $\phi$ arrives at another nearly flat region of the potential but with higher energy, i.e., $V_{inf2} \gg V_{inf1}$, the slow-roll inflation restarts again.

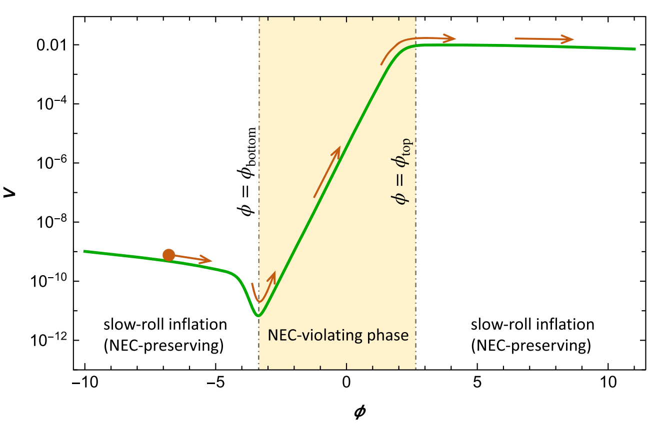


FIG. 1. A sketch of our scenario: a slow-roll (NEC-preserving) inflation occurred before the NEC-violating phase, which is subsequently followed by the slow-roll (NEC-preserving) inflation with a higher scale $H_{inf2} \gg H_{inf1}$. The potential $V(\phi)$ given by Eq. (9) is plotted with logarithmic coordinates on the vertical axis.

We present a model as follows:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - M_p^2 g_1(\phi)X/2 + g_2(\phi)X^2/4 - M_p^4 V(\phi)\right], \tag{1}$$

where $X = \nabla_\mu\phi\nabla^\mu\phi$. Here, the Galileon operator $\Box\phi = \nabla_\mu\nabla^\mu\phi$ is not required[2]; see also, e.g., [11,51,52]. The corresponding background equations are

$$3H^2M_p^2 = \frac{M_p^2}{2}g_1\dot{\phi}^2 + \frac{3}{4}g_2\dot{\phi}^4 + M_p^4 V, \tag{2}$$

$$\dot{H}M_p^2 = -\frac{M_p^2}{2}g_1\dot{\phi}^2 - \frac{1}{2}g_2\dot{\phi}^4, \tag{3}$$

$$0 = \left(g_1 + \frac{3g_2\dot{\phi}^2}{M_p^2}\right)\ddot{\phi} + 3g_1H\dot{\phi} + \frac{1}{2}g_{1,\phi}\dot{\phi}^2 + \frac{3g_2H\dot{\phi}^3}{M_p^2} + \frac{3g_{2,\phi}\dot{\phi}^4}{4M_p^2} + M_p^2V_{,\phi}, \tag{4}$$

where "$_{,\phi} = d/d\phi$." Only two of Eqs. (2)–(4) are independent.

In the NEC-preserving regimes, we require $g_1(\phi) = 1$, $g_2(\phi) = 0$ and the potential is nearly flat (see Fig. 1), so that the scalar filed $\phi$ is canonical and the slow-roll inflation ($0 < \epsilon = -\dot{H}/H^2 \ll 1$) can happen. In Fig. 1, $V_{inf1} \simeq m^2\phi^2/2$, $V_{inf2} \simeq \lambda[1 - \frac{(\phi-\phi_1)^2}{\sigma^2}]^2$ and $V_{inf2} \gg V_{inf1}$.

In the NEC-violating regime, we set $g_1(\phi) \approx -\frac{f_1 e^{2\phi}}{1+f_1 e^{2\phi}} < 0$ and $g_2(\phi) = f_2$ with $f_{1,2}$ being dimensionless constants. The coefficient of $\ddot{\phi}$ in Eq. (4), i.e., $g_1 + 3g_2\dot{\phi}^2/M_p^2$, is positive throughout so that there is no ghost instability; see Appendix. The scalar field $\phi$ will climb up the potential rapidly ($H \ll \dot{\phi} < M_p$) and arrive at the flat region $V = V_{inf2}$, as long as the condition $\frac{1}{2}g_{1,\phi}\dot{\phi}^2 + M_p^2V_{,\phi} < 0$ lasts for sufficiently long time. We require $\epsilon \ll -1$, i.e., $H^2 \ll \dot{H}$. According to Eq. (2), considering $\dot{\phi}^2 \gg V, H^2$, we have $\frac{M_p^2}{2}g_1\dot{\phi}^2 + \frac{3}{4}g_2\dot{\phi}^4 \approx 0$, which suggests $e^{2\phi} \sim \dot{\phi}^2$ for $\phi < 0$. Thus, $\dot{\phi}$ is approximately

$$\dot{\phi} \simeq \frac{1}{(t_* - t)}, \qquad t < t_*. \tag{5}$$

According to Eq. (3), we have $\dot{H} \sim \dot{\phi}^4$; hence,

[2] See, e.g., Ref. [17], for the case in which $X\Box\phi$ instead of $X^2$ is used to realize the NEC violation.

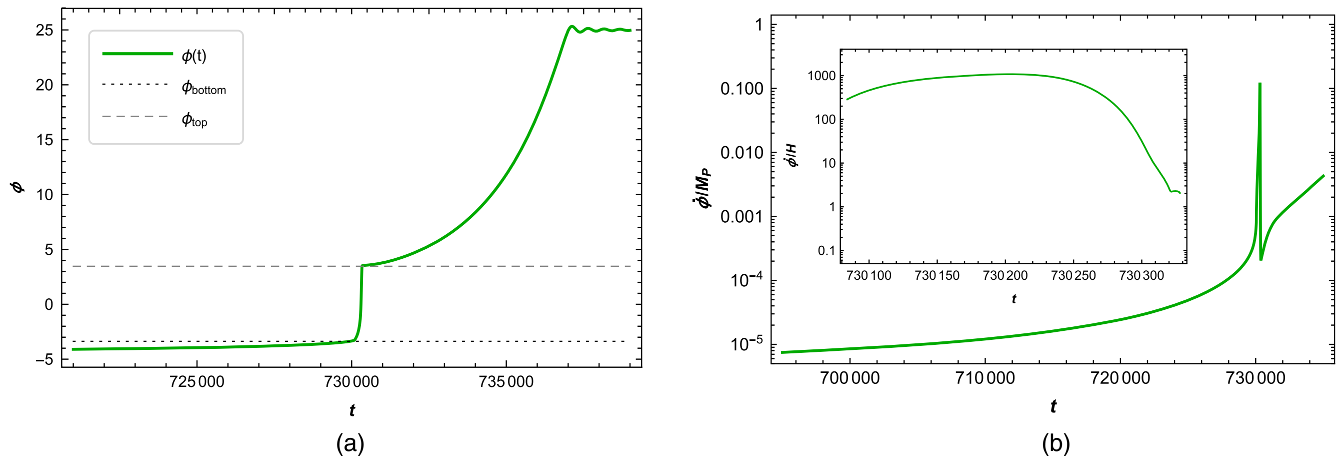


FIG. 2. Left: the evolution of $\phi$ with respect to $t$. Here, $\phi_{\rm bottom}$ and $\phi_{\rm top}$ correspond to the bottom and top of the potential in Fig. 1, respectively. Right: the evolution of $\dot{\phi}/M_p$ and $\dot{\phi}/H$. During the NEC-violating phase, which approximately corresponds to $730084 < t < 730328$, $H \ll \dot{\phi} < M_p$ is satisfied. We set $\phi(t_{\rm ini}) = -7$, $\dot{\phi}(t_{\rm ini}) = 0$, $t_{\rm ini} = 0$, $\phi_0 = 3.2$, $\phi_1 = 2$, $\phi_2 = -4$, $\phi_3 = -4.38$, $q_1 = 10$, $q_2 = 6$, $q_3 = 10$, $q_4 = 4$, $f_1 = 1$, $f_2 = 40$, $\lambda = 0.01$, $\sigma = 23$, and $m = -4.5 \times 10^{-6}$.

$$H \sim \frac{1}{(t_* - t)^3} + \text{const.} \tag{6}$$

When $t \ll t_*$, we have $H \simeq \text{const} = H_{inf1}$, which suggests that the NEC-violating phase has the chance to start after a slow-roll inflation.

As a phenomenological example, we set

$$g_1(\phi) = \frac{1}{1+e^{q_2(\phi-\phi_3)}} - \frac{f_1 e^{2\phi}}{1+f_1 e^{2\phi}} + \frac{2}{1+e^{-q_1(\phi-\phi_0)}}, \tag{7}$$

$$g_2(\phi) = \frac{f_2}{1+e^{-q_2(\phi-\phi_3)}} \frac{1}{1+e^{q_3(\phi-\phi_0)}}, \tag{8}$$

$$V(\phi) = \frac{1}{2}m^2\phi^2 \frac{1}{1+e^{q_2(\phi-\phi_2)}} + \lambda\left[1 - \frac{(\phi-\phi_1)^2}{\sigma^2}\right]^2 \frac{1}{1+e^{-q_4(\phi-\phi_1)}}, \tag{9}$$

where $\lambda, m, f_{1,2}$, and $q_{1,2,3,4}$ are positive constants. We require that $\phi_3 < \phi_2 < 0 < \phi_1 < \phi_0$. Here, for $\phi \ll \phi_3$, we have $g_1 = 1, g_2 = 0$, and $V = V_{inf1} \simeq m^2\phi^2/2$, while for $\phi \gg \phi_0$, we have $g_1 = 1$, $g_2 = 0$, and $V = V_{inf2} \simeq \lambda[1 - \frac{(\phi-\phi_1)^2}{\sigma^2}]^2$.

We solve Eqs. (3) and (4) numerically. The initial value of $H$ is set as $H_{\rm ini} \simeq H_{inf1} = 1.29 \times 10^{-5} M_p$ at $t = t_{\rm ini} = 0$, so that the "$inf1$" is responsible for the scalar perturbations on the CMB band, which indicates that $P_s \simeq \frac{1}{2M_p^2 \epsilon_{inf1}} (\frac{H_{inf1}}{2\pi})^2 \approx 2.1 \times 10^{-9}$ for $\epsilon_{inf1} = 0.001$.

We plot the evolutions of $\phi$ and $\dot{\phi}$ in Fig. 2. We can see that in the slow-roll (NEC-preserving) regimes, $\dot{\phi} \ll H$, the field $\phi$ rolls slowly. In the NEC-violating regime, $H \ll \dot{\phi} < M_p$, so that the field can rapidly climb up the potential $V = V_{inf2}$. We plot the evolutions of $H$ and $\epsilon$ in Fig. 3. During the slow-roll (NEC-preserving) phases ($0 < \epsilon \ll 1$), we have $H \simeq \text{const}$, which is intervened by an NEC-violating phase ($\dot{H} > 0$ and $\epsilon \ll -1$). Due to the NEC-violating evolution, we have

$$H_{inf2}/H_{inf1} \simeq 10^3 \gg 1. \tag{10}$$

### B. Primordial GW spectrum

In this subsection, we calculate the spectrum of primordial GWs. Generally, for the tensor perturbation $\gamma_{ij}$, we have[3]

$$S_\gamma^{(2)} = \frac{M_p^2}{8} \int d^4x a^3 \left[\dot{\gamma}_{ij}^2 - \frac{(\partial_k \gamma_{ij})^2}{a^2}\right]. \tag{11}$$

In the momentum space, we have

$$\gamma_{ij}(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{x}} \sum_{\lambda=+,\times} \hat{\gamma}_\lambda(\tau, \mathbf{k}) \epsilon_{ij}^{(\lambda)}(\mathbf{k}), \tag{12}$$

where $\hat{\gamma}_\lambda(\tau,\mathbf{k}) = \gamma_\lambda(\tau,k)a_\lambda(\mathbf{k}) + \gamma_\lambda^*(\tau,-k)a_\lambda^\dagger(-\mathbf{k})$, $\epsilon_{ij}^{(\lambda)}(\mathbf{k})$ satisfies $k_j\epsilon_{ij}^{(\lambda)}(\mathbf{k}) = 0$, $\epsilon_{ii}^{(\lambda)}(\mathbf{k}) = 0$, $\epsilon_{ij}^{(\lambda)}(\mathbf{k})\epsilon_{ij}^{*(\lambda')}(\mathbf{k}) = \delta_{\lambda\lambda'}$, and $\epsilon_{ij}^{*(\lambda)}(\mathbf{k}) = \epsilon_{ij}^{(\lambda)}(-\mathbf{k})$; $a_\lambda(\mathbf{k})$ and $a_\lambda^\dagger(\mathbf{k}')$ satisfy $[a_\lambda(\mathbf{k}), a_{\lambda'}^\dagger(\mathbf{k}')] = \delta_{\lambda\lambda'}\delta^{(3)}(\mathbf{k}-\mathbf{k}')$. The equation of motion for $\gamma_\lambda(\tau, k)$ is

[3]Here, the propagating speed of GWs is $c_T = 1$ (or see, e.g., [53–58]).

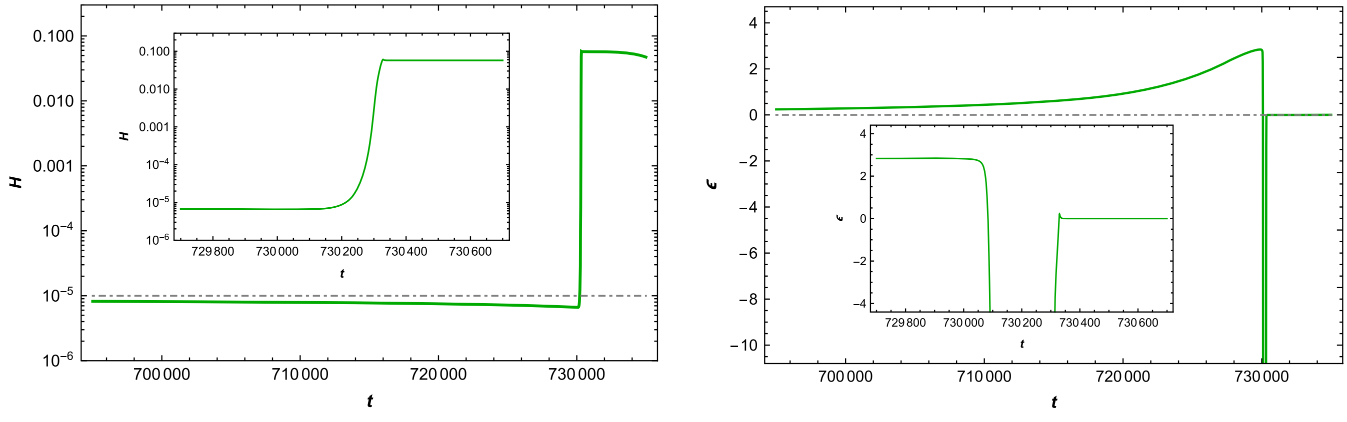


FIG. 3. Left: the evolution of $H$. Right: the evolution of $\epsilon = -\dot{H}/H^2$. We have $0 < \epsilon \ll 1$ during slow-roll inflations and $\epsilon \ll -1$ during NEC-violating superinflation.

$$\frac{d^2u_k}{d\tau^2} + \left(k^2 - \frac{a''}{a}\right)u_k = 0, \tag{13}$$

where $u_k = \gamma_\lambda(\tau, k)aM_p/2$ and $\tau = \int a^{-1}dt$.

In Sec. II A, we have shown that a model of our scenario can be constructed. Here, for simplicity, we assume that $\epsilon \approx \text{const} \ll -1$ during the NEC-violating phase, which requires some delicate design of $g_1$, $g_2$, and $V$. Around the beginning or the end of the NEC-violating phase, the detailed variation of $\epsilon$ may make some model-dependent contributions. However, for our purpose, the simplification will not make a qualitative difference.

Henceforth, we assume that the epoch of "inflation" consists of different phases with $\epsilon_j = -\dot{H}_j/H_j^2 = \frac{3}{2}(1+w_j) \simeq$ const, where $w_j$ is the state parameter. We have [59]

$$a_j(\tau) \sim (\tau_{R,j} - \tau)^{\frac{1}{\epsilon_j - 1}}, \tag{14}$$

for the $j$th phase, where $\tau_{R,j} = \tau_j - (\epsilon_j - 1)^{-1}\mathcal{H}^{-1}(\tau_j)$ and $a(\tau_j)$ is set by requiring the continuity of $a$ at the end of phase $j$ (i.e., $\tau = \tau_j$). As a result, we have

$$\frac{a_j''}{a_j} = \frac{\nu_j^2 - 1/4}{(\tau - \tau_{R,j})^2}, \tag{15}$$

where $\nu_j = \frac{3}{2}|\frac{1-w_j}{1+3w_j}|$. Regarding the phases $j$ and $j+1$ as adjacent phases, we have the solutions to Eq. (13) as

$$u_{k,j}(\tau) = \frac{\sqrt{\pi(\tau_{R,j} - \tau)}}{2}\Big\{\alpha_j H^{(1)}_{\nu_j}[k(\tau_{R,j} - \tau)] + \beta_j H^{(2)}_{\nu_j}[k(\tau_{R,j} - \tau)]\Big\}, \quad (\tau < \tau_j), \tag{16}$$

$$u_{k,j+1}(\tau) = \frac{\sqrt{\pi(\tau_{R,j+1} - \tau)}}{2}\Big\{\alpha_{j+1} H^{(1)}_{\nu_{j+1}}[k(\tau_{R,j+1} - \tau)] + \beta_{j+1} H^{(2)}_{\nu_{j+1}}[k(\tau_{R,j+1} - \tau)]\Big\}, \quad (\tau > \tau_j), \tag{17}$$

respectively, where $\alpha_{j(j+1)}$ and $\beta_{j(j+1)}$ are $k$-dependent coefficients. Using the matching conditions $u_{k,j}(\tau_{j+1}) = u_{k,j+1}(\tau_{j+1})$ and $u'_{k,j}(\tau_{j+1}) = u'_{k,j+1}(\tau_{j+1})$, we have

$$\begin{pmatrix}\alpha_{j+1}\\ \beta_{j+1}\end{pmatrix} = \mathcal{M}^{(j)}\begin{pmatrix}\alpha_j\\ \beta_j\end{pmatrix}, \quad \text{where } \mathcal{M}^{(j)} = \begin{pmatrix}\mathcal{M}^{(j)}_{11} & \mathcal{M}^{(j)}_{12}\\ \mathcal{M}^{(j)}_{21} & \mathcal{M}^{(j)}_{22}\end{pmatrix}. \tag{18}$$

See Refs. [59,60] for the matrix elements of $\mathcal{M}^{(j)}$. The information of the $1, 2\cdots j$th phases of the Universe has been encoded fully in the Bogoliubov coefficients $\alpha_{j+1}$ and $\beta_{j+1}$. We set the initial state as the Bunch-Davies vacuum (see also [61–65] for preinflationary bounce), i.e., $u_k = \frac{1}{\sqrt{2k}}e^{-ik\tau}$. Thus, $|\alpha_1| = 1$, $|\beta_1| = 0$.

In the following, we focus on the scenario in Fig. 1. Regarding $inf1$, NEC-violating and "$inf2$" phases as the $j = 1$, 2, 3th phases, respectively, we have

$$u_{k,3}(\tau) = \frac{\sqrt{\pi(\tau_{R,3} - \tau)}}{2}\Big\{\alpha_3 H^{(1)}_{3/2}[k(\tau_{R,3} - \tau)] + \beta_3 H^{(2)}_{3/2}[k(\tau_{R,3} - \tau)]\Big\}, \tag{19}$$

where $\nu_3 \simeq 3/2$ for de Sitter expansion. On super-horizon scale, we have $H^{(1)}_{3/2}(-k\tau) = -H^{(2)}_{3/2}(-k\tau) \overset{-k\tau\to 0}{\approx} -i\sqrt{2/(-\pi k^3\tau^3)}$. The resulting spectrum of primordial GWs is

$$P_T = \frac{4k^3}{\pi^2 M_p^2} \cdot \frac{|u_{k,3}|^2}{a^2} = P_{T,inf2}|\alpha_3 - \beta_3|^2, \tag{20}$$

where

$$\begin{pmatrix} \alpha_3 \\ \beta_3 \end{pmatrix} = \mathcal{M}^{(2)}\mathcal{M}^{(1)} \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix}, \tag{21}$$

$P_{T,inf2} = \frac{2H^2_{inf2}}{M_p^2\pi^2}$; $\mathcal{M}^{(1)}$ and $\mathcal{M}^{(2)}$ are given by Eq. (18). The information of $inf1$, NEC-violating and $inf2$ phases has been encoded in the Bogoliubov coefficients $\alpha_3$ and $\beta_3$.

### C. Primordial GWB at low-frequency band

It is interesting to connect $P_T$ in (20) with the observations of stochastic GWB at low-frequency bands. The BICEP/Keck+Planck bound at CMB band is $r \lesssim 0.06$ [19], which corresponds to $P_T \lesssim 10^{-10}$. The analysis result of NANOGrav 12.5-yr data [14], if regarded as the stochastic GWB (see inspired studies, e.g., [15,16,18,66–80]), suggests $\Omega_{GW} \sim 10^{-9}$ with the tilt $-1.5 \lesssim n_T \lesssim 0.5$, where

$$\Omega_{GW}(\tau_0) = \frac{k^2}{12a_0^2H_0^2} P_T(k) \left[\frac{3\Omega_m j_1(k\tau_0)}{k\tau_0} \times \sqrt{1.0 + 1.36\frac{k}{k_{\rm eq}} + 2.50\left(\frac{k}{k_{\rm eq}}\right)^2}\right]^2 \tag{22}$$

is the energy density spectrum of GWs; see, e.g., [81] (see also [82–85]). Here, $1/k_{eq}$ is the comoving Hubble scale at matter-radiation equality, and $\Omega_m = \rho_m/\rho_c$ and $\rho_c = 3H_0^2/(8\pi G)$ is the critical energy density.

According to (20), we plot $P_T$ and $\Omega_{GW}$ in Fig. 4 ($f = k/(2\pi a_0)$). We set $H_1 = 1.29 \times 10^{-5}$, which corresponds to $P_s \sim 2.1 \times 10^{-9}$ for $\epsilon_1 = 0.001$ and $H_3 \sim 10^{-2}$; $w_1 \gtrsim -1$ and $w_3 \gtrsim -1$ for the slow-roll inflations, while $w_2 \lesssim -10$ for the NEC-violating phase. We see that the yielded power spectrum of primordial GWs has a nearly flat amplitude at the CMB band and also a higher nearly flat amplitude at the PTA band. Here, the NEC-violating regime contributes the upward section of $P_T$, in which the spectrum has a blue tilt $n_T \simeq 2$ (since $\epsilon \ll -1$). Therefore, our scenario not only explains the result reported by the NANOGrav Collaboration, but also has a detectable signal $r \sim 0.01$ in the CMB.

### D. Multistage inflation with NEC violations

The multistage inflation model (see, e.g., earlier Refs. [86–90]; see also, e.g., [91] for recent study), in which a sequence of short inflations are interrupted by short periods of decelerated expansions with $w > -1/3$, is interesting, since it helps to make the EFT of inflation UV complete [92–97]. Usually, in such a scenario, a high-scale inflation is followed by a sequence of low-scale inflations. However, it might be also possible that a sequence of short inflations ($w \gtrsim -1$) are interrupted by short periods of not only decelerated expansions with $w > -1/3$ but also superinflation or Genesis with $w < -1$, so that the scales of subsequent short (NEC-preserving) inflations might be higher; see, e.g., Fig. 1.

According to Eqs. (17), (18), and (20), for a multistage scenario of inflation in which a sequence of short slow-roll inflations ($w \gtrsim -1$) are interrupted by lots of short periods of expansions with $w > -1/3$ and $w < -1$ (NEC violation), we can write the spectrum $P_T$ of primordial GWs as

$$P_T = P^{inf}_{T,l}|\alpha_l - \beta_l|^2 = \frac{2H_l^2}{M_p^2\pi^2}|\alpha_l - \beta_l|^2, \tag{23}$$

where

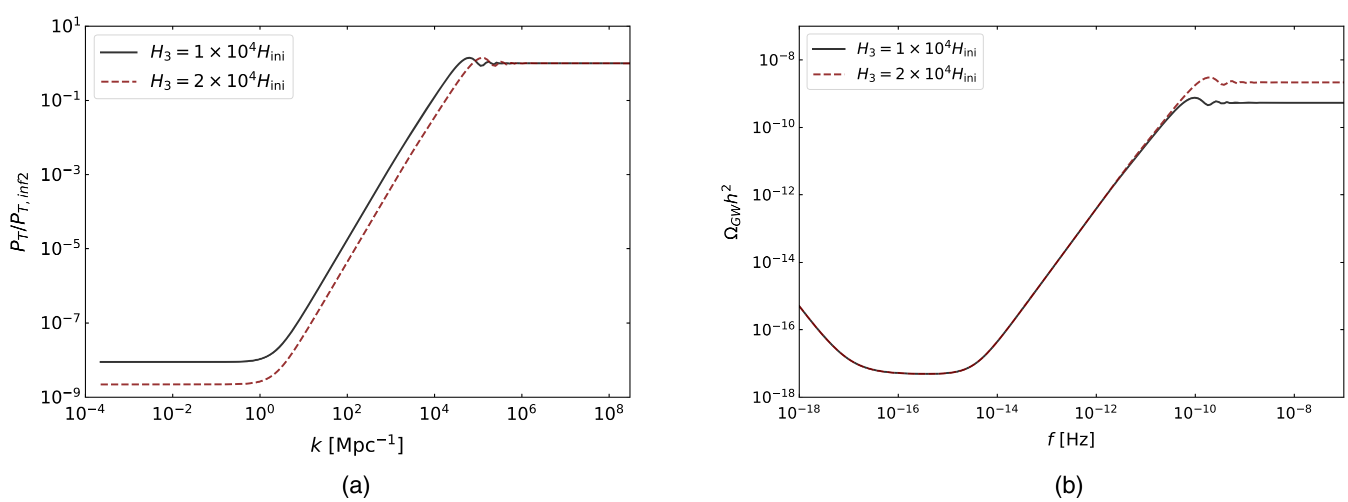


FIG. 4. Left: $P_T$. Right: $\Omega_{\rm GW}h^2$, where $h = H_0/($ km/s/Mpc$)$.

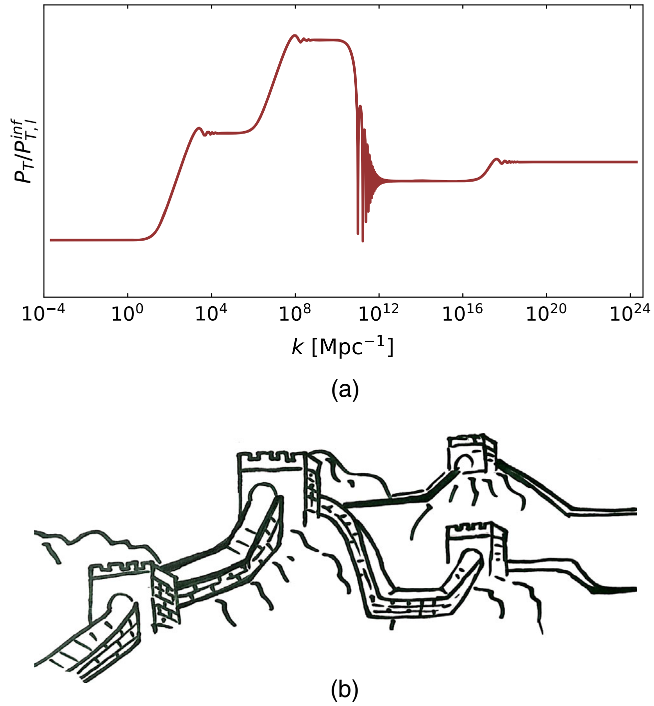


FIG. 5. The spectrum $P_T/P^{inf}_{T,l}$ of primordial GWB yielded in a multistage scenario of inflation, in which a sequence of short slow-roll inflations ($j = 1$, 3, 5, 7, 9) are interrupted by short periods of decelerated expansion ($j = 6$) and NEC-violating expansion ($j = 2$, 4, 8). We set the equation of state parameters $w_{1,3,5,7,9} \simeq -1$, $w_2 = -15$, $w_{4,8} = -10$, and $w_6 = 1/3$. The frequency band of GW spans about 28 orders. The lower panel is the Great Wall (sketched by Yu-Ze Piao). The panorama of $P_T$ looks like the Great Wall. When we climb up the Great Wall, we would see the beacon towers of different physics.

$$\begin{pmatrix} \alpha_l \\ \beta_l \end{pmatrix} = \prod_{j=1}^{l} \mathcal{M}^{(j)} \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix}, \tag{24}$$

and "$l$" labels the last short slow-roll inflations. The frequency band of stochastic GWB yielded is

$$10^{-18}\ \mathrm{Hz} \lesssim f \lesssim \exp\left(\sum_{j=1}^{l} N_j\right) 10^{-18}\ \mathrm{Hz}, \tag{25}$$

where $N_j \equiv \ln \frac{a_{j,e} H_{j,e}}{a_{j,ini} H_{j,ini}}$ is the $e$-folds number of the perturbation modes passing through the $j$th phase. Note that $N_j < 0$ for the decelerated expansion ($w > -1/3$).

According to (23), we plot $P_T/P^{inf}_{T,l}$ in Fig. 5(a) for a multistage scenario of inflation with short periods of slow-roll inflations ($j = 1$, 3, 5, 7, 9) with different $H_j$. The panorama of $P_T$ at corresponding GW frequency band looks like the Great Wall, see Fig. 5(b), in which the nearly flat roads correspond to GWB yielded by short slow-roll (NEC-preserving) inflations, the upward and downward slopes correspond to the NEC-violating expansions ($w < -1$) and decelerated expansions ($w > -1/3$), respectively. It is well known that each section of the Great Wall records a unique history.

## III. CONCLUSION

The NEC violation in the primordial Universe will bring a blue-tilted GWB. However, its implications to the GWB might be far richer than expected. We presented a scenario, in which after a slow-roll (NEC-preserving) inflation with $H \simeq H_{inf1}$ (responsible for the density perturbation on large scales), the Universe goes through an NEC-violating period, which is followed again by the slow-roll inflation but with $H_{inf2} \gg H_{inf1}$. We calculated the power spectrum of the yielded primordial GWs. As expected, the spectrum has an observable amplitude $P_T \sim H^2_{inf1}$ ($n_T \simeq 0$) at the CMB band and a higher amplitude $P_T \sim H^2_{inf2}$ ($n_T \simeq 0$) at the PTA band (compatible with recent NANOGrav result). Here, the NEC violation responsible for the upward tilt of $P_T$ played an indispensable role.

It is well known that the detection of stochastic GWB will not only solidify our confidence in inflation but also offer us a probe to the physics of the early Universe. Though the model we consider is simplified, it highlights an unexpected point that the GWB yielded in the primordial Universe might have a unique landscape. We explore the observable imprints of short NEC violations on primordial GWB. It is especially highlighted that for the multistage inflation, consisting of a sequence of short slow-roll inflations ($w \gtrsim -1$) interrupted by lots of short period of expansions with $w > -1/3$ and $w < -1$, we will have a Great Wall spectrum of stochastic GWB, which might be detectable.

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## APPENDIX: ON STABILITY OF SCALAR PERTURBATIONS

In the unitary gauge, for (1), we have

$$S^{(2)}_\zeta = \int d^4x a^3 Q_s \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial\zeta)^2}{a^2}\right], \tag{A1}$$

where

$$Q_s = \epsilon M_p^2 + \frac{g_2 \dot{\phi}^4}{H^2} = \frac{M_p^2 \dot{\phi}^2}{2H^2}\left(g_1 + 3g_2 \frac{\dot{\phi}^2}{M_p^2}\right), \tag{A2}$$

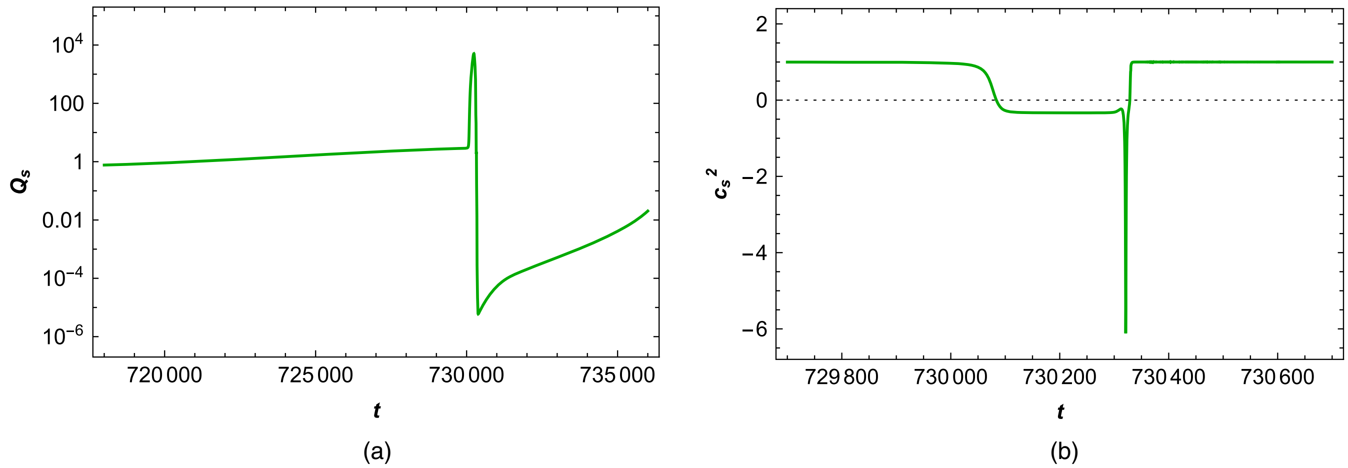


FIG. 6. Left: The evolution of $Q_s$, which is positive throughout in our model. Right: the evolution of $c_s^2$, which is negative during the NEC-violating phase (approximately corresponding to $730084 < t < 730328$).

$$c_s^2 = \frac{\epsilon M_p^2}{Q_s}. \tag{A3}$$

Around the NEC violation, though $c_s^2 < 0$ (see Fig. 6), $c_s^2 = 1$ can be set with the higher-order derivative (beyond-Horndeski) operators; see, e.g., Refs. [8,10,11,98,99] for related details. Here, $Q_s > 0$ throughout. Therefore, the ghost and gradient instabilities can be cured in the EFT regime. It is interesting to ask whether there is a danger of instabilities from the higher-derivative terms in the perspective of the UV theory. However, it is still unknown how to embed the EFT with such terms in a UV complete theory so far. The related issues require further investigation in the future.


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