Study on pure annihilation type $B \to V\gamma$ decays

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We investigate the pure annihilation type radiative B meson decays $B^0 \to \phi\gamma$ and $B_s \to \rho^0(\omega)\gamma$ in the soft-collinear effective theory. We consider three types of contributions to the decay amplitudes, including the direct annihilation topology, the contribution from the electromagnetic penguin operator, and the contribution of the neutral vector meson mixing. The numerical analysis shows that the decay amplitudes are dominated by the $\omega - \phi$ mixing effect in the $B^0 \to \phi\gamma$ and $B_s \to \omega\gamma$ modes. The corresponding decay branching ratios are enhanced about 3 orders of magnitude relative to the pure annihilation type contribution in these two decay channels. The decay rate of $B_s \to \rho^0\gamma$ is much smaller than that of $B_s \to \omega\gamma$ because of the smaller $\rho^0 - \phi$ mixing. The predicted branching ratios $\langle B(B^0 \to \phi\gamma)\rangle = (3.96^{+1.67}_{-1.45}) \times 10^{-9}$ and $\langle B(B_s \to \omega\gamma)\rangle = (1.99^{+0.81}_{-0.70}) \times 10^{-7}$ are to be tested by the Belle-II and LHCb experiments.

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I. INTRODUCTION

The exclusive radiative B decay modes $B \to V\gamma$ are very interesting and valuable probes of flavor physics, since they provide an excellent platform to constrain standard model parameters, to test new physics models, and to understand QCD factorization of the decay amplitudes [1]. Most $B \to V\gamma$ decays occur via the flavor-changing neutral-current transitions $b \to s\gamma$ or $b \to d\gamma$, and the quark-level transition amplitudes are now approaching next-to-next-to-leading-order accuracy [2,3]. It is more profound to evaluate the exclusive decay modes $B \to V\gamma$, based on the effective theory with the expansion in the inverse powers of the b quark mass. At leading power in $1/m_b$, the QCD factorization of $B \to V\gamma$ decays has been established up to next-to-leading order in α_s [4–11]. The leading power factorization formula was confirmed in a more elegant way with soft-collinear effective theory (SCET) [12]. The exclusive $B \to V\gamma$ decays have also been investigated in the alternative approach of perturbative QCD factorization based on k_T factorization [13].

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In modern accelerators with high luminosity, more accurate data have been accumulated; therefore, besides the leading power contributions, we must consider power corrections on the theoretical side to improve the theoretical precision. Among the power suppressed corrections, the weak annihilation diagrams are of great importance, as they might be mediated by tree operators, and they play an important role in the determination of the time-dependent CP asymmetry in $B \to V\gamma$ (see Refs. [14–17]) as well as isospin asymmetries [18]. There exists a special type of radiative decays where the decay amplitude contains only annihilation type diagrams, including $B^0 \to \phi\gamma$ and $B_s \to \rho^0(\omega)\gamma$ decays. Relatively less attention is paid to them due to their tiny branching ratios [19]. The $B^0 \to \phi\gamma$ decay is mediated by penguin annihilation topology, with a very small Wilson coefficient. In addition, this decay mode is suppressed by Λ/m_b, since the emitted vector meson must be transversely polarized. In naive factorization, its branching ratio is estimated to be at the order of 10^{-13}, and QCD corrections can enhance the result to about 10^{-12}. In Ref. [20], it was found that the electromagnetic penguin operator $O_{7\gamma}$ contribution through $B^0 \to \gamma\gamma^*$ with the virtual photon connecting to the ϕ meson can increase the branching ratio for $B^0 \to \phi\gamma$ to the order of 10^{-11}. The predicted branching ratio within the framework of the perturbative QCD factorization approach is also at this order [21]. For the $B_s \to \rho^0(\omega)\gamma$ mode, the contribution from the electromagnetic penguin operator is also of great importance, and the branching ratio is at the order of $10^{-10} - 10^{-9}$ [20].

On the other hand, the radiative decays mediated by tensor transition form factors have much larger branching ratios. The measured branching fractions of $B^0 \to \rho^0(\omega)\gamma$ and $B_s \to \phi\gamma$ read [22]

$$\begin{aligned} B(B^0 \to \rho^0\gamma) &= (8.6 \pm 1.5) \times 10^{-7}, \\ B(B^0 \to \omega\gamma) &= (4.4^{+1.8}_{-1.6}) \times 10^{-7}, \\ B(B_s \to \phi\gamma) &= (3.4 \pm 0.4) \times 10^{-5}. \end{aligned} \tag{1}$$

They are at least 4 orders larger than the predicted $B^0 \to \phi\gamma$ and $B_s \to \rho^0(\omega)\gamma$ decays. Such a large discrepancy might lead to a large contribution to $B^0 \to \phi\gamma$ and $B_s \to \rho^0(\omega)\gamma$ decays through the mixing between the neutral vector mesons ω, ρ^0, and ϕ. The $\omega - \phi$ mixing effect is regarded to be large in many B meson and D meson decay modes [23–25]. Thus, it is valuable to investigate the contribution of this effect in purely annihilation type $B \to V\gamma$ decays, which may be the dominant contribution. If the branching ratios of pure annihilation type B decays can be significantly enhanced by the neutral meson mixing, the Super-B factory and LHCb might have a chance to find the signals of these processes.

This paper is arranged as follows: In the next section, we will present the factorization formulas of $B^0 \to \phi\gamma$ and $B_s \to \rho^0(\omega)\gamma$ decays, including the leading power contribution, and the contributions from the annihilation topology, the electromagnetic penguin operator, and the $\omega - \phi$ mixing effect. Numerical analysis will be presented in Sec. III. The last section contains closing remarks.

II. THEORETICAL OVERVIEW OF PURE ANNIHILATION TYPE RADIATIVE $B(B_s)$ DECAYS

The effective Hamiltonian for $b \to D\gamma$ transitions, with $D = s, d$, reads

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10} C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \right] + \text{H.c.}, \tag{2}$$

where $\lambda_p = V^*_{pD} V_{pb}$ and V_{ij} are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $O_i^{(p)}(\mu)$ are the relevant operators, and $C_i(\mu)$ are the corresponding Wilson coefficients, which are shown in Refs. [26–28].

The $B \to V\gamma$ decays contain several kinds of momentum modes, for which it is convenient to work in the light-cone coordinate system, where the collinear momentum of the vector meson p can be expressed as $p = (n \cdot p, \bar{n} \cdot p, p_\perp) \sim (\lambda^2, 1, \lambda) m_b$, with the null vector n and $\bar{n}$ satisfying $n \cdot \bar{n} = 2$. The anticollinear photon momentum q scales as $(1, \lambda^2, \lambda) m_b$. In addition, the momentum of soft quark inside the B meson and intermediate hard-collinear quark or gluon can be expressed as $(\lambda, \lambda, \lambda) m_b$ and $(\lambda, 1, \sqrt{\lambda}) m_b$, respectively. All these modes are necessary to correctly reproduce the infrared behavior of full QCD. SCET provides a more transparent language of the factorization of multiscale problems than the diagrammatic approach. In SCET, the fields with a typical momentum mode have definite power counting rules. The power behaviors of the fields appear at $B \to V\gamma$ decays are as follows:

$$\begin{aligned} &\xi_c \sim \lambda, \quad A_c^\mu \sim (\lambda^2, 1, \lambda), \quad \xi_{hc} \sim \lambda^{1/2}, \quad A_{hc}^\mu \sim (\lambda, 1, \lambda^{1/2}), \\ &\xi_{\bar{c}} \sim \lambda, \quad A_{\bar{c}}^\mu \sim (1, \lambda^2, \lambda), \quad q_s \sim \lambda^{3/2}, \quad A_s^\mu \sim (\lambda, \lambda, \lambda), \quad h_v \sim \lambda^{3/2}. \end{aligned} \tag{3}$$

Since SCET contains two kinds of collinear fields, i.e., hard-collinear and collinear fields, an intermediate effective theory, called SCET_I, is introduced that contains soft, collinear, and hard-collinear fields. The final effective theory, called SCET_II, contains only soft and collinear fields. To obtain the amplitudes of radiative decays, one needs to do a two-step matching from QCD $\to \text{SCET}_\text{I} \to \text{SCET}_\text{II}$. The matching procedure of leading power amplitude has been performed in Ref. [12]. In the following, we give a brief review in order that we can conveniently express the contribution of the neutral meson mixing.

In the first step, the hard scale m_b is integrated out by matching the operators Q_i in the weak Hamiltonian onto a set of operators in SCET_I. Merely considering the operators contributing at leading power, the matching takes the form

$$\mathcal{H}_{\text{eff}} \to C^A Q^A + C^B \otimes Q^B. \tag{4}$$

The $\otimes$ denotes a convolution over space-time or momentum fractions. The momentum-space Wilson coefficients depend only on quantities at the hard scale m_b. The specific form of the operators $Q^{(i)}$ is written by

$$\begin{aligned} Q^A &= (\bar{\xi} W_{hc})(s\bar{n}) \mathcal{A}^{em}_{\underline{hc\perp}}(tn)(1-\gamma_5) h_v, \\ Q^B &= (\bar{\xi} W_{hc})(s\bar{n}) \mathcal{A}^{em}_{\underline{hc\perp}}(tn) \not{\mathcal{A}}_{hc\perp}(r\bar{n})(1+\gamma_5) h_v. \end{aligned} \tag{5}$$

The definition of SCET_I building block $\mathcal{A}_{hc}$ and Wilson line W_{hc} has been given in Ref. [12]. The B-type operators are actually power suppressed in SCET_I but contribute at the same order as the A-type operator upon the transition to SCET_II.

The matrix element of the operator Q^A is proportional to the SCET form factor $\zeta_{V_\perp}$, i.e.,

$$\langle V_\perp(\varepsilon_1) | \bar{\xi} \Gamma h_v | \bar{B}_v \rangle = 2E \zeta_{V\perp}(E) \text{Tr} \left[\frac{\not{n}\not{\bar{n}}}{4} \not{\varepsilon}^*_{1\perp} \Gamma \frac{1+\not{v}}{2} \gamma_5 \right]. \tag{6}$$

The operators Q^B can be further matched onto four-quark operators in $\mathrm{SCET_{II}}$ through time-ordered product with $\mathrm{SCET_I}$ Lagrangian [29]

$$\int d^4x \langle V_\perp \gamma | T\{\mathcal{L}^{(1)}_{\xi q}(x), Q^B(0)\} | \bar{B}_v \rangle = \int ds \int dt \tilde{J}_\perp(s,t) O^B(s,t). \quad (7)$$

with

$$O^B(s,t) = [\bar{\xi} W_c](s\bar{n})(1+\gamma_5)\mathcal{A}^{\mathrm{em}}_{\bar{c}\perp} \frac{\bar{\not{n}}}{2}[W_c^\dagger \xi](0)[\bar{q}_s Y_s](tn)(1-\gamma_5) \times \frac{\not{n}}{2} h_v(0). \quad (8)$$

When matching the operator Q^B onto $\mathrm{SCET_{II}}$, the hard-collinear virtuality $m_b\Lambda$ is integrated out, and the matching coefficient gives rise to the jet function

$$J^B_\perp(\omega,u) = \int dt e^{-i\omega t} \int ds e^{-2iEus} \tilde{J}^B_\perp(s,t). \quad (9)$$

The final low-energy theory $\mathrm{SCET_{II}}$ contains only soft and collinear fields. At leading power, the factorization theorem is proved in an elegant way with SCET, since the soft and collinear fields decouple. The soft fields are restricted to the B-meson light-cone distribution amplitude (LCDA) and collinear ones to the vector meson LCDA defined as

$$\langle 0|[\bar{q}_s Y_s](tn)\frac{\not{n}}{2}\Gamma Y_s^\dagger h_v(0)|\bar{B}_v\rangle = -\frac{iF(\mu)\sqrt{m_B}}{2}\mathrm{tr}\left[\frac{\not{n}}{2}\Gamma\frac{1+\not{v}}{2}\gamma_5\right]\int_0^\infty d\omega e^{-i\omega tn\cdot v}\phi_B^+(\omega,\mu),$$
$$\langle V(p)|[\bar{\xi} W_c](s\bar{n})\Gamma\frac{\bar{\not{n}}}{2}W_c^\dagger \xi(0)|0\rangle = \frac{if_V \bar{n}\cdot p}{4}\mathrm{tr}\left[\frac{\not{n}\bar{\not{n}}}{4}\not{\epsilon}^*\Gamma\frac{\bar{\not{n}}}{2}\right]\int_0^1 d\omega e^{ius\bar{n}\cdot p}\phi_V(u,\mu). \quad (10)$$

The final factorization formula is then written by

$$\langle V\gamma|\mathcal{H}_{\mathrm{eff}}|\bar{B}\rangle|_{\mathrm{LP}} = 2m_B\left[C^A\zeta_{V_\perp} + \frac{\sqrt{m_B}F(\mu)f_{V_\perp}}{4}(C^B\otimes J_\perp)\otimes\phi_\perp^V\otimes\phi_+^B\right]. \quad (11)$$

Up to the order of α_s, the explicit expression of hard functions $C^A(\mu)$ and $C^B(\mu)$ have been given in Ref. [12] [here we use C^B instead of $C_1^B(\mu)$]. The leading power contribution is dominant in the decays $B^0 \to \rho^0(\omega)\gamma$ and $B_s \to \phi\gamma$, which will be employed in our evaluation of the contribution from mixing of neutral vector mesons.

A. Contribution from weak annihilation

Now we are ready to investigate the weak annihilation contribution to the purely annihilation type operators. The weak annihilation diagrams are shown in Fig. 1, all of which are mediated by the four-quark operators. In order to produce a transversely polarized vector meson, the four-quark operators must be matched to the $\mathrm{SCET_I}$ operators which are suppressed by $1/m_b$. Among the four diagrams in Fig. 1, diagram (c) is dominant, because it is enhanced by a hard-collinear propagator compared with the other diagrams. Therefore, we neglect diagrams (a), (b), and (d), which are highly suppressed in our calculation. Only considering the contribution of the leading two-particle Fock state of the vector meson, the physical $\mathrm{SCET_I}$ operator, which can contribute to purely annihilation type decays, is written by

$$Q_1 = [\bar{\chi}_{hc}(t\bar{n})(1+\gamma_5)\gamma_\perp^\mu \eta_{hc} + \mathrm{H.c.}]\bar{\xi}_{\overline{hc}} W_{\overline{hc}}(s\bar{n})\gamma_\mu^\perp(1-\gamma_5)h_v, \quad (12)$$

with

$$\eta_{hc} = -\frac{1}{in\cdot D_{hc}} i\not{D}_{hc\perp}\frac{\not{n}}{2}\chi_{hc}. \quad (13)$$

Although this operator is suppressed relative to the leading power $\mathrm{SCET_I}$ four-quark operators, they share the same matching coefficients, since the relevant QCD diagrams in the matching procedure are the same. Therefore, the hard function at one-loop level can be extracted from the effective Wilson coefficients in the QCD factorization approach of nonleptonic B decays [28]. Similar to the nonleptonic B decays, the hard function is also convoluted with the vector meson LCDAs defined below:

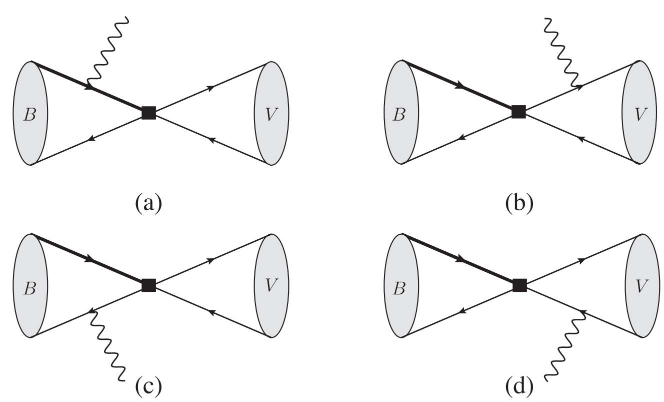

FIG. 1. Weak annihilation diagrams for $B^0 \to \phi\gamma$ decay.

$$
\begin{aligned}
&\langle V(p,\epsilon_1)|[\bar{\chi}(s\bar{n})\gamma_\perp^\mu\eta+\mathrm{H.c.}]|0\rangle \\
&\quad = f_V m_V \varepsilon_{1\perp}^{*\mu}\int du e^{ius\bar{n}\cdot p} g_\perp^{(v)}(u), \\
&\langle V(p,\epsilon_1)|[\bar{\chi}(s\bar{n})\gamma_\perp^\mu\gamma_5\eta+\mathrm{H.c.}]|0\rangle \\
&\quad = \frac{i}{4} f_V m_V \epsilon_\perp^{\mu\nu}\varepsilon_{1\perp\nu}^*\int du e^{ius\bar{n}\cdot p} g_\perp^{(a)\prime}(u), \qquad (14)
\end{aligned}
$$

with $\epsilon_{\mu\nu}^\perp = \epsilon_{\mu\nu\bar{n}n}/2$. As the hard-collinear part decouples from the soft and collinear parts, the factorization for the annihilation diagram also holds. We take $B^0 \to \phi\gamma$ decay as an example; the matrix element can be factorized as

$$
\begin{aligned}
&\langle \phi(\varepsilon_1)\gamma(\varepsilon_2)|\mathcal{H}_{\mathrm{eff}}|\bar{B}\rangle|_{\mathrm{anni}} \\
&\quad = -\frac{G_F}{\sqrt{2}} V_{tb}V_{td}^*\left(\alpha_3 - \frac{1}{2}\alpha_{3EW}\right)\frac{f_\phi m_\phi}{E_\gamma} \\
&\quad\quad \times \langle \gamma(\varepsilon_2)|C_{\mathrm{FF}}\bar{\xi}\not\varepsilon_1^*(1-\gamma_5)h_v|\bar{B}\rangle, \qquad (15)
\end{aligned}
$$

where the anticollinear vector meson LCDA has been convoluted with the hard function, and the effective Wilson coefficients are written by [19]

$$
\begin{aligned}
\alpha_3 &= C_3 + \frac{C_4}{N_c} + C_5 + \frac{C_6}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C}\frac{f_V^\perp}{f_V}(C_4V_1 + C_6V_2), \\
\alpha_{3EW} &= C_7 + \frac{C_8}{N_c} + C_9 + \frac{C_{10}}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C}\frac{f_V^\perp}{f_V}(C_8V_2 + C_{10}V_1)
\end{aligned} \qquad (16)
$$

with the vertex correction term

$$
\begin{aligned}
V_1 &= \int_0^1 du T_1(u)\left[\frac{1}{4}g_\perp^{(a)\prime}(u) - g_\perp^{(v)}(u)\right], \\
V_2 &= \int_0^1 du T_2(u)\left[\frac{1}{4}g_\perp^{(a)\prime}(u) + g_\perp^{(v)}(u)\right],
\end{aligned}
$$

where [30]

$$
\begin{aligned}
T_1(u) &= 12\ln\frac{m_b}{\mu} - 18 + g(u), \\
T_2(u) &= -12\ln\frac{m_b}{\mu} + 6 - g(\bar{u}), \\
g(u) &= \frac{4-6u}{\bar{u}}\ln u - 3i\pi + \Bigg(2\mathrm{Li}_2(u) - \ln^2 u \\
&\quad + \frac{2\ln u}{\bar{u}} - (3+2\pi i)\ln u - [u\to\bar{u}]\Bigg). \qquad (17)
\end{aligned}
$$

The hard function C_{FF} arises from matching the weak current $\bar{u}\gamma_{\mu\perp}(1-\gamma_5)b$ onto the corresponding SCET current. The remaining $B\to\gamma$ transition matrix element containing soft and collinear fields can be parameterized by

$$
\begin{aligned}
&\langle \gamma(\varepsilon_2,p)|C_{\mathrm{FF}}\bar{\xi}\gamma_\mu(1-\gamma_5)h_v|\bar{B}_v\rangle \\
&\quad = E_\gamma \varepsilon_{2\nu}^*(g_\perp^{\mu\nu}F_A + i\epsilon_\perp^{\mu\nu}F_V). \qquad (18)
\end{aligned}
$$

The $B\to\gamma$ transition form factors $F_{V,A}$ also present in the $B\to\gamma\ell\nu$ decay, which have been extensively studied [31–42]. At leading power $F_A = F_V$ due to the left-handedness of the weak interaction current and helicity conservation of the quark-gluon interaction in the high-energy limit, and this symmetry relation is broken by power suppressed local contributions. At leading power both the hard function and the jet function have been calculated up to two-loop level and next-to-leading logarithmic resummation has been performed. The power suppressed symmetry-breaking local contribution and symmetry-conserving high-twist contribution and resolved photon contribution are also considered. Utilizing the result of Ref. [42] in our calculation, the transition amplitude of $B^0\to\phi\gamma$ and $B_s\to\rho^0(\omega)\gamma$ is then written by

$$
\begin{aligned}
A(B\to\phi\gamma)|_{\mathrm{anni}} &= -\frac{G_F}{\sqrt{2}}\lambda_t\left(\alpha_3 - \frac{1}{2}\alpha_{3EW}\right) e f_\phi m_\phi (F_A g_{\mu\nu}^\perp + iF_V\epsilon_{\mu\nu}^\perp)\epsilon_2^{*\mu}\epsilon_1^{*\nu}, \\
\sqrt{2}A(B_s\to\rho^0\gamma)|_{\mathrm{anni}} &= \frac{G_F}{\sqrt{2}}\left(\lambda_u\alpha_2 - \frac{3}{2}\lambda_t\alpha_{3EW}\right) e f_\rho m_\rho (F_A g_{\mu\nu}^\perp + iF_V\epsilon_{\mu\nu}^\perp)\epsilon_2^{*\mu}\epsilon_1^{*\nu}, \\
\sqrt{2}A(B_s\to\omega\gamma)|_{\mathrm{anni}} &= \frac{G_F}{\sqrt{2}}\left(\lambda_u\alpha_2 - \lambda_t\alpha_3 - \frac{1}{2}\lambda_t\alpha_{3EW}\right) e f_\omega m_\omega (F_A g_{\mu\nu}^\perp + iF_V\epsilon_{\mu\nu}^\perp)\epsilon_2^{*\mu}\epsilon_1^{*\nu}, \qquad (19)
\end{aligned}
$$

where

$$
\alpha_2 = C_2 + \frac{C_1}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C}\frac{f_V^\perp}{f_V}C_2V_1. \qquad (20)
$$

B. Contribution from electromagnetic penguin operator

The annihilation diagram is power suppressed, because leading power four-quark operators cannot contribute to a transversely polarized vector meson. However, if the power suppressed operator $\bar{\chi}\gamma_\perp\eta \sim \lambda^{3/2}$ is replaced by a photon field $\mathcal{A}_{\bar{c}\perp}^{em} \sim \lambda^{1/2}$, this will lead to a large enhancement factor m_b/Λ. Furthermore, for the pure annihilation type decays such as the $B^0\to\phi\gamma$, the rather small color suppressed penguin operator Wilson coefficient will also be replaced by $C_{7\gamma}$, at the cost of an electromagnetic coupling constant α_{em}. The leading-order Feynman diagrams of

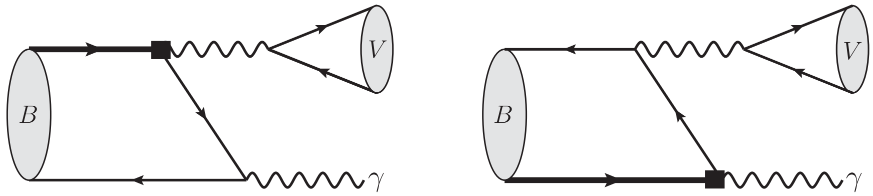

FIG. 2. Production of a vector meson via electromagnetic penguin operator.

the electromagnetic penguin operator contribution are plotted in Fig. 2. They are corresponding to the matrix element

$$A^{\mathrm{LO}}_{\mathrm{EMP}} = -\frac{G_F}{\sqrt{2}}\lambda_t \int d^4x \langle \phi(p,\epsilon_1^*)\gamma(q,\epsilon_2^*)| \times \mathrm{T}\{Q_q e\bar{q}\not{A}q(x), C_{7\gamma}O_{7\gamma}(0)\}|\bar{B}(v)\rangle + [p \leftrightarrow q]. \tag{21}$$

To evaluate this amplitude, one must have knowledge of the matrix element of $O_{7\gamma}$. When the photon field is sandwiched between the vector meson state and the vacuum, the matrix element reads [43]

$$\langle V|e\mathcal{A}^{\mathrm{em}}_{\bar{c}\perp\mu}|0\rangle = -\frac{2}{3} i a_V \frac{e^2 f_V}{m_V}\varepsilon^*_{\perp\mu} \tag{22}$$

with $a_\rho = 3/2$, $a_\omega = 1/2$, and $a_\phi = -1/2$. Taking advantage of the above matrix element, the leading-order result can be obtained, which has been given in Ref. [20].

In this work, we make the improvement by taking the QCD correction of the Wilson coefficients of $O_{7\gamma}$ operator into account. The complete $\mathcal{O}(\alpha_s)$ corrections including the contribution from four-quark operators and chromomagnetic operator are accomplished in Ref. [44]. Similar to the decay amplitude of $B^0 \to \gamma\gamma$ [45] and $B^0 \to \gamma\ell\ell$ mode [46], the leading power contribution of the electromagnetic dipole operator to $B^0 \to \phi\gamma$ and $B_s \to \rho^0(\omega)\gamma$ decay can be expressed as

$$A_{\mathrm{EMP}} = i\frac{4G_F}{\sqrt{2}}\lambda_t \frac{\alpha_{\mathrm{em}}}{4\pi}\left(-\frac{2}{3}ia_V\right)\frac{ef_V}{m_V}\varepsilon_1^{*\alpha}(p)\varepsilon_2^{*\beta}(q)[g^{\perp}_{\alpha\beta} + i\epsilon^{\perp}_{\alpha\beta}]Q_q f_{B_q} m_{B_q} E_\gamma \bar{m}_b V_7^{\mathrm{eff}}(0) \times \left[\frac{V_7^{\mathrm{eff}}(q^2)}{V_7^{\mathrm{eff}}(0)}\frac{m_{B_q}}{2E_\gamma}\int_0^\infty \frac{d\omega}{\omega} J(2E_\gamma, 0, \omega)\phi_+^B(\omega) + \int_0^\infty \frac{d\omega}{\omega - m_V^2/m_{B_q}} J(2E_\gamma, m_V^2, \omega)\phi_+^B(\omega)\right]. \tag{23}$$

The explicit expression of effective Wilson coefficient $V_7^{\mathrm{eff}}(q^2)$ and the jet function $J(2E_\gamma, q^2, \omega)$ at one-loop level are given in Ref. [46], including the factorization scale dependence obtained from renormalization group evolution. In our numerical analysis, the factorization scale is chosen at an intermediate scale $\mu_{hc} \sim \sqrt{E_\gamma \omega}$.

C. Contribution from mixing of neutral vector mesons

The mixing of the flavor-SU(3) singlet and octet states of vector mesons to form mass eigenstates is of fundamental importance in hadronic physics. It is commonly accepted that the vector meson states satisfy the "ideal" mixing, close to the value that would lead to the complete decoupling of the light u and d quarks from the heavier s quark in the resultant mass eigenstates ω and ϕ. Actually, ω and ϕ are not pure states with definite isospin given by

$$|\omega_I\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle), \qquad |\phi_I\rangle = |\bar{s}s\rangle. \tag{24}$$

The mass eigenstates ω and ϕ deviate from the "ideal" states ω_I and ϕ_I through a mixing matrix

$$\begin{pmatrix} |\omega\rangle \\ |\phi\rangle \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix}\begin{pmatrix} |\omega_I\rangle \\ |\phi_I\rangle \end{pmatrix}, \tag{25}$$

where the mixing angle δ can be determined from the experimental data or by model calculation. The isospin triplet ρ^0 can also mix with ω and ϕ through electromagnetic interactions; however, the mixing angle is about one order smaller than the $\omega - \phi$ mixing, since the isospin breaking is much smaller than the flavor SU(3) breaking effect. Thus, we do not take this isospin-breaking mixing effect into account in our analysis. After considering the mixing between the ω and ϕ meson, the $B^0 \to \phi\gamma$ and $B_s \to \omega\gamma$ decays can be expressed in terms of the decay amplitude with the ideal mixing meson final state, i.e.,

$$A(B^0 \to \phi\gamma)|_{\mathrm{mixing}} = -\sin\delta A(B \to \omega_I\gamma),$$
$$A(B_s \to \omega\gamma)|_{\mathrm{mixing}} = \sin\delta A(B_s \to \phi_I\gamma). \tag{26}$$

To show that the $\omega - \phi$ meson mixing will dominate the $B^0 \to \phi\gamma$ and $B_s \to \omega\gamma$ decays, we estimate the relative size of different contributions to these decay modes.

TABLE I. Estimation of the relative size from different contributions to $B^0 \to \phi\gamma$.

Contributions	Suppression	Enhancement	Typical value
$\frac{\lvert A(B\to\phi\gamma)\rvert_{\mathrm{anni}}}{\lvert A(B\to\rho^0\gamma)\rvert_{\mathrm{LP}}}$	$\frac{\alpha_3 - 1/2\alpha_{3EW}}{C_{7\gamma}} \times \frac{m_V}{m_b}$	$\cdots$	0.004
$\frac{\lvert A(B\to\phi\gamma)\rvert_{\mathrm{EMP}}}{\lvert A(B\to\rho^0\gamma)\rvert_{\mathrm{LP}}}$	$\alpha_{\mathrm{em}} \times a_\phi$	$\frac{m_b}{m_V}$	0.01
$\frac{\lvert A(B\to\phi\gamma)\rvert_{\mathrm{mixing}}}{\lvert A(B\to\rho^0\gamma)\rvert_{\mathrm{LP}}}$	$\sin\delta$	$\cdots$	0.06

For convenience, we investigate the proportion of the absolute value of the amplitude of each contribution with respect to the absolute value of the leading power amplitude in $B \to \rho^0\gamma$ decay in Table I. From this table, we can see that the mixing effect can increase the branching ratio from annihilation topology in QCD factorization approach over 2 orders of magnitude and is one order larger than the contribution from electromagnetic operators. For $B_s \to \rho^0(\omega)\gamma$ decays, the tree operators with a large Wilson coefficient can contribute, while they are suppressed by a small suppression factor from CKM matrix elements, i.e., $|V_{ub}V_{us}^*/V_{tb}V_{ts}^*|$; therefore, the relative size of different types of contribution in $B_s \to \rho^0(\omega)\gamma$ decays is similar to $B \to \phi\gamma$.

III. NUMERICAL ANALYSIS

A. Input parameters

The decay amplitudes for the $B^0 \to \phi\gamma$ and $B_s \to \rho^0(\omega)\gamma$ decays have been obtained in the previous section; they will be utilized to predict the branching ratios of these decay modes. First, we specify the input parameters which will be used in the numerical calculation. Among various parameters, the mixing angle δ is of unique importance, because it will provide the major source of uncertainties in our calculations. The mixing angle has been discussed in many phenomenological methods such as the framework of the hidden local symmetry Lagrangian [47,48], the chiral perturbation theory [49,50], the light front quark model [51] and the Nambu-Jona-Lasinio model [52,53], etc., with the obtained values varying at the interval about 3°–5° (most of the studies prefer [3°, 4°]). In this work, we adopt the value of mixing angle as $\delta = 3.5° \pm 0.5°$.

To arrive at the result of the decay amplitudes from the $\omega - \phi$ mixing, the leading power contribution of $B^0 \to \omega\gamma$ and $B_s \to \phi\gamma$ is necessary. The basic nonperturbative inputs in these amplitudes are the soft form factor $\zeta_\perp^{\mathrm{BV}}$ and the light-cone distribution amplitude of B meson and ρ, ω, ϕ meson. The soft factors $\zeta_\perp^{\mathrm{BV}}$ defined in terms of the matrix element of $\mathrm{SCET_I}$ operators have been calculated using SCET sum rules. The complete next-to-leading-order corrections to the correlation function as well as the power suppressed higher twist contribution have been calculated in Ref. [54]. The result is adopted as $\zeta_\perp^{\mathrm{BV}} = 0.33 \pm 0.10$. For the $B_s \to \phi$ transition, the result is $\zeta_\perp^{\mathrm{B_sV}} = 0.35 \pm 0.10$, allowing a small SU(3) breaking effect.

For the leading twist two-particle B-meson distribution amplitude, we will employ the following three-parameter model:

$$\phi_B^+(\omega) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} U\left(\beta - \alpha, 3 - \alpha, \frac{\omega}{\omega_0}\right), \tag{27}$$

where $U(\alpha, \gamma, x)$ is the confluent hypergeometric function of the second kind. A special case is the exponential model when $\alpha = \beta$:

$$\phi_B^+(\omega) = \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0}. \tag{28}$$

To estimate the error from the models, we will let $\alpha - \beta$ vary at the region $-0.5 < \alpha - \beta < 0.5$, and then we employ two models with $\alpha = 2.0$, $\beta = 1.5$ and $\alpha = 1.5$, $\beta = 2.0$. The parameter ω_0 is closely related to the first inverse moment $1/\lambda_B$, whose determination has been discussed extensively in the context of exclusive B-meson decays (see [54–57] for more discussions). Here we will employ the result from a recent study [58], i.e., $\lambda_B = 0.383 \pm 0.153$ GeV and $\lambda_{B_s} = 0.438 \pm 0.150$ GeV. For the light vector meson, the leading twist LCDAs can be expanded in terms of Gegenbauer polynomials due to the behavior of scale evolution, i.e.,

$$\begin{aligned} \phi_V(x) &= 6x(1-x)[1 + a_V C_2^{3/2}(2x-1)], \\ \phi_{V_\perp}(x) &= 6x(1-x)[1 + a_{V_\perp} C_2^{3/2}(2x-1)]. \end{aligned} \tag{29}$$

For the power suppressed vector meson LCDAs, ignoring the three-parton LCDA, we have the following expression [59]:

$$\begin{aligned} \frac{1}{4} g_\perp^{(a)\prime}(u) - g_\perp^{(v)}(u) &= -\int_0^u \frac{\phi_V(v)}{\bar{v}}, \\ \frac{1}{4} g_\perp^{(a)\prime}(u) + g_\perp^{(v)}(u) &= \int_u^1 \frac{\phi_V(v)}{v}. \end{aligned} \tag{30}$$

In the annihilation topology, the $B_{(s)} \to \gamma$ transition form factors $F_{V,A}$ are required. We employ the computing formulas of $F_{V,A}$ from a recent study [42], and the results indicate that the SU(3) breaking effect is negligible. For fixed $\lambda_{B_{(s)}}$, we obtain $F_V^{B_{(s)}} = 0.20^{+0.06}_{-0.04}$ and $F_A^{B_{(s)}} = 0.18^{+0.06}_{-0.04}$, where the uncertainties arise from the various parameters in $F_{V,A}$ except for $\lambda_{B_{(s)}}$. Besides the theoretical inputs discussed in the above, the values of the other parameters are presented in Table II. The first logarithmic moment $\sigma_{B_{(s)}}^{(1)}$ comes from the convolution between the logarithmic term in the jet function and the B-meson LCDA in Eq. (23) [46].

TABLE II. Input parameters.

τ_{B^0}	1.52 ps	G_F	1.116637×10^{-5}
τ_{B_s}	1.51 ps	λ	0.22650
f_B	0.192	$\bar{\rho}$	0.141
f_{B_s}	0.230	A	0.790
$\sigma_B^{(1)}$	1.63 ± 0.15	$\bar{\eta}$	0.357
$\sigma_{B_s}^{(1)}$	1.49 ± 0.15		
$f_\rho(1\ \mathrm{GeV})$	0.216 ± 0.003	$a_{2\rho}(1\ \mathrm{GeV})$	0.15 ± 0.07
$f_\omega(1\ \mathrm{GeV})$	0.187 ± 0.005	$a_{2\omega}(1\ \mathrm{GeV})$	0.15 ± 0.07
$f_\phi(1\ \mathrm{GeV})$	0.215 ± 0.005	$a_{2\phi}(1\ \mathrm{GeV})$	0.18 ± 0.08
$f_{\rho\perp}(1\ \mathrm{GeV})$	0.165 ± 0.009	$a_{2\rho\perp}(1\ \mathrm{GeV})$	0.14 ± 0.06
$f_{\omega\perp}(1\ \mathrm{GeV})$	0.151 ± 0.009	$a_{2\omega\perp}(1\ \mathrm{GeV})$	0.14 ± 0.06
$f_{\phi\perp}(1\ \mathrm{GeV})$	0.186 ± 0.009	$a_{2\phi\perp}(1\ \mathrm{GeV})$	0.14 ± 0.07

B. Phenomenological predictions

Collecting all the contributions to the factorization amplitudes calculated in the previous section together, we arrive at the final expression of the decay amplitudes for the pure annihilation type $B \to V\gamma$ decays:

$$\begin{aligned} A(B^0 \to \phi\gamma) &= \cos\delta[A(B^0 \to \phi_I\gamma)|_{\mathrm{anni}} + A(B^0 \to \phi_I\gamma)|_{\mathrm{EMP}}] \\ &\quad - \sin\delta A(B \to \omega_I\gamma), \\ A(B_s \to \omega\gamma) &= \cos\delta[A(B_s \to \omega_I\gamma)|_{\mathrm{anni}} + A(B_s \to \omega_I\gamma)|_{\mathrm{EMP}}] \\ &\quad + \sin\delta A(B_s \to \phi_I\gamma), \\ A(B_s \to \rho^0\gamma) &= A(B_s \to \rho^0\gamma)|_{\mathrm{anni}} + A(B_s \to \rho^0\gamma)|_{\mathrm{EMP}}. \end{aligned} \tag{31}$$

The results for the phenomenological observables in pure annihilation type decays are then studied. As the decay rates are relatively small and the observables such as CP asymmetry are hard to detect, we concentrate on the CP-averaged branching ratios defined below:

$$\langle B(B^0 \to V\gamma)\rangle = \frac{B(\bar{B}^0 \to V\gamma) + B(B^0 \to V\gamma)}{2}, \tag{32}$$

where the specific expression of the branching ratio is give by

$$B(\bar{B}^0 \to V\gamma) = \frac{\tau_B}{16\pi m_B}\left(1 - \frac{m_V^2}{m_B^2}\right)|A(\bar{B}^0 \to V\gamma)|^2. \tag{33}$$

To illustrate the contribution from various sources, we first present the results of each individual contribution in Table III. In the contribution from mixing of neutral vector mesons, we consider only the leading power contribution to the $B^0 \to \omega_I\gamma$ and $B_s \to \phi_I\gamma$ amplitudes, because the mixing angle is already a small quantity. The QCD factorization result of the pure annihilation contribution is consistent with the result in Ref. [19]. The contribution from electromagnetic penguin operator is a bit larger than our previous predictions in Ref. [20], as the leading logarithm resummation of effective Wilson coefficient $C_{7\mathrm{eff}}$ is employed and some parameters are updated. Our results indicate that the branching ratio of $B^0 \to \phi\gamma$ purely from the $\phi - \omega$ mixing is 3 orders larger than that from the annihilation topology and also about 2 orders larger than that from the electromagnetic penguin contribution in the decays. Apparently, this result is consistent with our rough estimation. Taking advantage of the central values in Table I, the total branching ratio of $B^0 \to \phi\gamma$ is obtained as 3.96×10^{-9}, which has the chance to be measured in Belle-II with an ultimate integrated luminosity of 50 ab^{-1}. The $B_s \to \omega\gamma$ decay with the branching ratio 1.99×10^{-7} can also be searched for at the LHCb data.

We define the following ratios of CP-averaged branching fractions, which can highlight the importance of the vector meson mixing effects:

$$R_{\rho\omega} = \frac{\langle B(B_s^0 \to \rho^0\gamma)\rangle}{\langle B(B_s^0 \to \omega^0\gamma)\rangle}, \qquad R_{\rho\phi} = \frac{\langle B(B_s^0 \to \rho^0\gamma)\rangle}{\langle B(B^0 \to \phi\gamma)\rangle}. \tag{34}$$

Naively considering the first ratio, $R_{\rho\omega} \sim 1$, since the Feynman diagrams of the annihilation topology are the same for $B_s \to \rho^0\gamma$ and $B_s \to \omega\gamma$ decays; furthermore, the contribution from the electromagnetic penguin will even enhance this ratio to 10, as $|a_\rho/a_\omega| = 3$. The second ratio $R_{\rho\phi}$ is expected to be large for the CKM enhancement from the ratio $|V_{tb}V_{ts}^*/V_{tb}V_{td}^*|^2$. After the $\phi - \omega$ mixing effect is taken into account, the values of these ratios are dramatically changed. Our result shows that $R_{\rho\omega} \simeq 0.03$, which confirms the dominance of meson mixing effects, and $R_{\rho\phi} \simeq 1.3$, which indicates that the contribution from $\omega - \phi$ mixing is larger than the electromagnetic penguin amplitude by a factor of $|V_{tb}V_{ts}^*/V_{tb}V_{td}^*|$ approximately. The predicted values of these ratios are expected to be tested in the future experiments.

Now we investigate the theoretical uncertainties. Inspecting the distinct sources of the yielding theory uncertainties as collected in the following formula, we have

TABLE III. Branching fractions of different contributions.

Channels	A_{anni} only	A_{EMP} only	A_{mixing} only	Total
$\langle B(B^0 \to \phi\gamma)\rangle$	2.96×10^{-12}	1.91×10^{-11}	3.59×10^{-9}	3.96×10^{-9}
$\langle B(B_s^0 \to \omega^0\gamma)\rangle$	5.13×10^{-11}	3.67×10^{-10}	1.79×10^{-7}	1.99×10^{-7}
$\langle B(B_s^0 \to \rho^0\gamma)\rangle$	3.61×10^{-11}	4.50×10^{-9}	$\cdots$	5.22×10^{-9}

$$\begin{aligned}\langle B(B^0 \to \phi\gamma)\rangle &= 3.96^{+1.19}_{-1.03}|_{\zeta\perp}{}^{+1.15}_{-1.00}|_{\delta}{}^{+0.02}_{-0.02}|_{f_V,\phi_V}{}^{+0.23}_{-0.09}|_{\lambda_B}{}^{+0.06}_{-0.15}|_{\phi_B}{}^{+0.01}_{-0.01}|_{F_V}{}^{+0.01}_{-0.01}|_{F_A} \times 10^{-9},\\ \langle B(B_s \to \omega\gamma)\rangle &= 1.99^{+0.56}_{-0.49}|_{\zeta\perp}{}^{+0.57}_{-0.50}|_{\delta}{}^{+0.01}_{-0.01}|_{f_V,\phi_V}{}^{+0.12}_{-0.06}|_{\lambda_B}{}^{+0.00}_{-0.02}|_{\phi_B}{}^{+0.01}_{-0.00}|_{F_V}{}^{+0.01}_{-0.00}|_{F_A} \times 10^{-7},\\ \langle B(B_s \to \rho^0\gamma)\rangle &= 5.22^{+0.15}_{-0.14}|_{f_V,\phi_V}{}^{+2.57}_{-1.05}|_{\lambda_B}{}^{+1.54}_{-1.64}|_{\phi_B}{}^{+0.04}_{-0.04}|_{\sigma^{(1)}_{B_s}}{}^{+0.12}_{-0.08}|_{F_V}{}^{+0.12}_{-0.08}|_{F_A} \times 10^{-9}. \end{aligned} \tag{35}$$

Among various sources of uncertainties in Eq. (35), ϕ_V stands for the Gegenbauer moments in the light meson LCDAs, ϕ_B denotes the shape of the B meson, i.e., the parameters α and β, and the uncertainties from $F_{V,A}$ do not contain the part that arises from $\lambda_{B_{(s)}}$. The uncertainty from $\sigma^{(1)}_{B_{(s)}}$ is not included in the $B \to \phi\gamma$ and $B_s \to \omega\gamma$ decays, because it is negligible compared with the other uncertainties. It is obvious that the soft form factors which play the dominant role in the $B \to \omega\gamma$ and $B_s \to \phi\gamma$ decays provide an important source of uncertainties. The mixing angle between ϕ and ω mesons is another major source of uncertainty as expected. As the decay amplitudes of $B \to \phi\gamma$ and $B_s \to \omega\gamma$ are very sensitive to the vector meson mixing effect, these channels can serve as a good platform to determine the mixing angle; i.e., the mixing angle between ω and ϕ meson can be determined by

$$\sin\delta \simeq \sqrt{\frac{\langle B(B^0 \to \phi\gamma)\rangle}{\langle B(B^0 \to \omega\gamma)\rangle}} \quad \text{or} \quad \sqrt{\frac{\langle B(B^0_s \to \omega\gamma)\rangle}{\langle B(B^0_s \to \phi\gamma)\rangle}}, \tag{36}$$

if the related decay modes are measured. For the $B(B_s \to \rho^0\gamma)$ decay which is dominated by the electromagnetic penguin operator, the major source of uncertainty is from the shape and the first inverse moment of the LCDA of the B_s meson. Therefore, it is of great importance to improve the study of $B_{(s)}$ meson LCDA. A recent effort is the introduction of the quasiparton distribution amplitude of B meson [60] so that it can be calculated by lattice QCD simulation.

IV. CLOSING REMARKS

The pure annihilation type radiative B meson decays, including $B^0 \to \phi\gamma$ and $B_s \to \rho^0(\omega)\gamma$ decays, are very rare in the standard model, which make them very sensitive to the new physics signals beyond the standard model. We reviewed factorization of $B \to V\gamma$ decays at leading power using SCET and derived the factorization formula for annihilation topology. The electromagnetic penguin contribution to the pure annihilation radiative decays, which is power enhanced, is also revisited with leading logarithm resummation of the effective Wilson coefficients taken into account. As the major subject of this work, we studied the contribution of the neutral vector meson $\omega - \phi$ mixing to the decay amplitudes. Although the mixing angle of the $\phi - \omega$ is only a few percent, this contribution owns larger Wilson coefficients as well as power enhancement compared with annihilation topology. A rough estimate indicates that the contribution from $\phi - \omega$ mixing is dominant in the pure annihilation radiative decays. The numerical calculation shows that the branching ratio of $B^0 \to \phi\gamma$ purely from the $\phi - \omega$ mixing is 3 orders larger than that from the annihilation topology and also 2 orders larger than that from the electromagnetic penguin contribution in the decays. The similar hierarchy between the different contributions holds for $B^0 \to \omega\gamma$. The decay rate of $B_s \to \rho^0\gamma$ is much smaller than that of $B_s \to \omega\gamma$, for the suppressed mixing effect is not considered. The new defined ratios $R_{\rho\omega} \simeq 0.03$ and $R_{\rho\phi} \simeq 1.3$ further highlight the importance of the mixing effect. The predicted branching ratios of $B^0 \to \phi\gamma$ and $B_s \to \rho^0(\omega)\gamma$ decays are given below:

$$\begin{aligned} B(B^0 \to \phi\gamma) &= 3.96^{+1.67}_{-1.45} \times 10^{-9},\\ B(B_s \to \omega\gamma) &= 1.99^{+0.81}_{-0.70} \times 10^{-7},\\ B(B_s \to \rho^0\gamma) &= 5.22^{+3.00}_{-1.96} \times 10^{-9}. \end{aligned} \tag{37}$$

These results are to be tested by the Belle-II and LHCb experiments.

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