Self-consistent light-front quark model analysis of $B \to D\ell\nu_\ell$ transition form factors

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We investigate the transition form factors $f_+(q^2)$ and $f_-(q^2)$ [or $f_0(q^2)$] for the exclusive semileptonic $B \to D\ell\nu_\ell$ ($\ell = e$, μ, τ) decays in the standard light-front quark model based on the light-front quantization. The common belief is that while $f_+(q^2)$ can be obtained without involving any treacherous contributions such as the zero mode and the instantaneous contribution, $f_-(q^2)$ receives those treacherous contributions since it involves at least two components of the current, e.g., (J^+, J^-) or $(J^+, \mathbf{J}_\perp)$. Contrary to the common belief, we show in the Drell-Yan ($q^+ = 0$) frame that $f_-(q^2)$ obtained from (J^+, J^-) gives identical result to $f_-(q^2)$ obtained from $(J^+, \mathbf{J}_\perp)$ without involving such treacherous contributions in the standard light-front quark model. In our numerical calculations, we obtain the form factors and branching ratios for $B \to D\ell\nu_\ell$ ($\ell = e$, μ, τ) and compare with the experimental data as well as other theoretical model predictions. Our results for $\mathrm{Br}(B \to D\ell\nu_\ell)$ show reasonable agreement with the experimental data except for the semitauonic $B^0 \to D^-\tau\nu_\tau$ decay. The ratio $\mathscr{R}(D) = \frac{\mathrm{Br}(B\to D\tau\nu_\tau)}{\mathrm{Br}(B\to D\ell'\nu_{\ell'})}$ ($\ell' = e, \mu$) is also estimated and compared with the experimental data as well as other theoretical predictions.

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I. INTRODUCTION

The semileptonic $B \to D\ell\nu_\ell$ ($\ell = e$, μ, τ) decays have attracted a lot of attention in extracting the exclusive Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cb}|$. Especially, the substantial difference for the ratio $\mathscr{R}(D) = \mathrm{Br}(B \to D\tau\nu_\tau)/\mathrm{Br}(B \to D\ell'\nu_{\ell'})$ $\quad(\ell' = e, \quad \mu)$ between the experimental data and the standard model (SM) predictions generated a great excitement in testing the SM and searching for new physics beyond the SM. The experimental data, $\mathcal{R}^{\mathrm{exp}}(D) = 0.440(58)(42)$ measured from *BABAR* [1,2] and $\mathcal{R}^{\mathrm{exp}}(D) = 0.375(64)(26)$ from Belle [3], have shown an excess over the SM prediction $\mathcal{R}^{\mathrm{SM}}(D) = 0.299(3)$ [4]. Many theoretical efforts have been made in resolving the issue of $\mathscr{R}(D)$ anomaly and searching for new physics beyond the SM [5–9].

We note that the $B \to D\ell\nu_\ell$ decays involve two transition form factors (TFFs), i.e., the vector form factor $f_+(q^2)$ and the scalar form factor $f_0(q^2)$. The analysis of both TFFs $f_{+,0}(q^2)$ for $B \to D$ transitions can be found in various theoretical approaches such as the lattice QCD [5,6], the light cone sum rule (LCSR) [9–12], and the light-front quark model (LFQM) [13]. While $\mathrm{Br}(B \to D\ell\nu_\ell)$ for the light lepton decay modes ($\ell = e, \mu$) needs only $f_+(q^2)$, $\mathrm{Br}(B \to D\tau\nu_\tau)$ for the heavy τ decay mode receives contributions from both $f_+(q^2)$ and $f_0(q^2)$. The ratio $\mathcal{R}(D)$ is in particular quite sensitive to the scalar form factor. This leads us to speculate that the scalar contribution is the main source of $\mathcal{R}(D)$ anomaly and thus the new physics effect beyond the SM. However, since the predictions of $f_0(q^2)$ as well as $f_+(q^2)$ are quite different for different theoretical approaches within the SM, it is very important to obtain the reliable and self-consistent results for the TFFs before drawing any sound conclusion from the $\mathcal{R}(D)$ anomaly.

The purpose of this paper is to present the self-consistent descriptions of the $B \to D\ell\nu_\ell$ TFFs in the standard LFQM based on the LF quantization [14]. There have been many previous LFQM analyses for the semi-leptonic decays between two pseudoscalar mesons [15–19]. In fact, there are two main kinds of LFQM, i.e., the standard LFQM [15,16] and the covariant LFQM [17–19]. In the standard LFQM, the constituent quark and antiquark in a bound state are required to be on-mass shells and the spin-orbit wave function is obtained by the interaction-independent Melosh transformation [20] from the ordinary equal-time static spin-orbit wave function assigned by the quantum number J^{PC}. The main characteristic of the standard LFQM is to use the sum of the LF energy of the constituent quark and antiquark for the meson mass in the spin-orbit wave function and any physical observable can be obtained directly in three-dimensional LF momentum space using the more phenomenologically accessible LF wave function such as Gaussian radial wave function $\phi(x, \mathbf{k}_\perp)$. However, as the standard LFQM itself is not amenable to analyze the zero-mode contribution, the covariant LFQM using the manifestly covariant Bethe-Salpeter (BS) model with the multipole type $q\bar{q}$ vertex was introduced [17], in which the constituents are off-mass shell. While the covariant BS model used in [17–19] allows one to analyze all the treacherous points such the zero modes and the off-mass shell instantaneous contributions in a systematic way, it is less realistic than the standard LFQM. Thus, in an effort to apply such treacherous points found in the covariant BS model to the standard LFQM, the effective replacement [17–19] of the LF vertex function $\chi(x, \mathbf{k}_\perp)$ obtained in the BS model with the more realistic Gaussian wave function $\phi(x, \mathbf{k}_\perp)$ in the standard LFQM has been made.

However, through the analysis of the vector meson decay constant together with the twist-2 and twist-3 distribution amplitudes (DAs) of the vector meson [21], we found the correspondence relation between χ and ϕ proposed in [17–19] encounters the self-consistency problem, e.g., the vector meson decay constants obtained in the standard LFQM were found to differ for different sets of the LF current components and polarization states of the vector meson [21]. We also resolved this self-consistency problem in the same work [21] by imposing the on-mass shell condition of the constituent quark and antiquark, i.e., replacement of the physical meson mass M with the invariant mass M_0 in the integrand of formulas for the physical quantities, in addition to the original correspondence relation between χ and ϕ. The remarkable finding from our new self-consistent correspondence relations (i.e., $\chi \to \phi$ and $M \to M_0$) between the two models [see, e.g., Eq. (49) in [21]] was that both zero mode and instantaneous contributions appeared in the covariant BS model became absent in the standard LFQM with the LF on-mass shell constituent quark and antiquark degrees of freedom. We then extended our self-consistent correspondence relations to analyze the decay amplitude related with twist-2 and twist-3 DAs of pseudoscalar mesons [22,23] and observed the same conclusion drawn from [21].

In the previous analysis [17–19] of the semileptonic decays between two pseudoscalar mesons using the covariant BS model, the LF covariant calculations was made in the Drell-Yan-West ($q^+ = q^0 + q^3 = 0$) frame (i.e., $q^2 = -\mathbf{q}_\perp^2 < 0$), which is advantageous in that only the valence contributions are needed unless the zero-mode contributions exist. The form factor $f_+(q^2)$ was obtained only from the plus component (J^+) of the weak current J^μ without encountering the zero-mode contribution. One needs, however, two different components of the current to obtain the form factor $f_0(q^2)$ [or $f_-(q^2)$], and J^+ and $\mathbf{J}_\perp = (J_x, J_y)$ were used to obtain it in [17–19].[1] However, $f_-(q^2)$ obtained from (J^+, $\mathbf{J}_\perp$) in the covariant BS model receives not only the instantaneous contribution but also the zero mode due to the $\mathbf{J}_\perp$ component. Employing the effective method presented in [17–19] to express the zero-mode contribution as a convolution of the zero-mode operator with the initial and final state LF vertex functions, the form factor $f_-(q^2)$ can also be expressed as the convolution form between the initial- and final-states LF vertex functions $\chi(x, \mathbf{k}_\perp)$ in the valence sector. To obtain $f_+(q^2)$ and $f_-(q^2)$ in the more realistic standard LFQM, the authors in [17–19] use the only correspondence relation between χ and ϕ without imposing the on-mass shell condition (i.e., $M \to M_0$).

In the recent work in [24], the authors investigated the self-consistency of the form factor $f_-(q^2)$ obtained from $(J^+, \mathbf{J}_\perp)$ by applying both the old correspondence ($\chi \to \phi$) and our new correspondence ($\chi \to \phi$ and $M \to M_0$) between the BS model and the standard LFQM. From their numerical calculations, the authors found from $f_-(q^2)$ in the standard LFQM that the zero-mode contribution to $f_-(q^2)$ is sizable for the case of using only ($\chi \to \phi$) relation but vanishes when using ($\chi \to \phi$ and $M \to M_0$) relations. This result is very supportive to assert that our new correspondence relations are universally applicable even to the weak transition form factors for a self-consistent description of the standard LFQM. In order to assert that the form factor $f_-(q^2)$ is truly self-consistent, however, it is essential to show that $f_-(q^2)$ obtained in the $q^+ = 0$ frame is independent of the components of the current, i.e., $f_-(q^2)$ obtained from (J^+, J^-) is the same as the one obtained from $(J^+, \mathbf{J}_\perp)$.

In this work, we shall show that our new correspondence relations ($\chi \to \phi$ and $M \to M_0$) guarantee the self-consistent description for the weak decay constant of a pseudoscalar meson and the semileptonic decays between two pseudoscalar mesons in the standard LFQM. To show this, we shall prove that (1) the decay constant $f_{\mathcal{P}}$ of a pseudoscalar meson ($\mathcal{P}$) is independent of the components of the current, and (2) $f_-(q^2)$ obtained from (J^+, J^-) is exactly the same as the one obtained from $(J^+, \mathbf{J}_\perp)$ in the $q^+ = 0$ frame. Those findings again entail that the zero-mode contribution as well as the instantaneous one appeared in the covariant BS model became absent in the standard LFQM.

Although we do not consider in this analysis, the $q^+ \neq 0$ frame may be used to compute the timelike process such as this semileptonic decay but then it is unavoidable to encounter the particle-number-nonconserving Fock state (or nonvalence) contribution [25]. The main source of difficulty in the LFQM phenomenology is the lack of

[1] While the method of Jaus [17] and ours [19] in obtaining the form factors are slightly different, the final results for $f_-(q^2)$ are the same with each other, i.e., $f_-(q^2)$ [see Eq. (4.3) in [17] and Eq. (42) in [19]] was obtained from using both J^+ and $\mathbf{J}_\perp$.

information on the non-wave-function vertex [26,27] in the nonvalence diagram arising from the quark-antiquark pair creation/annihilation. This should contrast with the usual LF valence wave function. In principle, there is a systematic program as was discussed in [28] to include the particle-number-nonconserving amplitude to take into account the nonvalence contributions. However, the program requires to find all the higher Fock-state wave functions while there has been relatively little progress in computing the basic wave functions of hadrons from first principles. In the very recent analysis [29] of the semileptonic $B_c \to \eta_c(J/\psi)$ decays in the framework of basis LF quantization, the frame dependence of the TFFs between $q^+ = 0$ and $q^+ \neq 0$ frames is discussed. The main reason for the frame dependence comes from the ignorance of the nonvalence contribution in the $q^+ \neq 0$ frame and it is not even possible to show that the form factors are independent of the components of the current in the $q^+ \neq 0$ frame unless the nonvalence contribution is correctly taken into account. However, our main findings in the $q^+ = 0$ frame may be incorporated in the same $q^+ = 0$ frame calculations of Ref. [29].

The paper is organized as follows: in Sec. II, we briefly review the decay constant $f_{\mathcal{P}}$ of a pseudoscalar meson in an exactly solvable model based on the covariant BS model of $(3+1)$-dimensional fermion field theory. We then present our LF calculation of $f_{\mathcal{P}}$ in the BS model using both plus and minus components of the current and discuss the treacherous points such as the zero-mode contribution and the instantaneous one when the minus component of the current is used. Linking the covariant BS model to the standard LFQM with our universal mapping between the two models [21–23], we obtain $f_{\mathcal{P}}$ from both plus and minus components of the current in the standard LFQM. Our main finding is that while $f_{\mathcal{P}}$ obtained from the minus component of the current in the covariant BS model receives both the zero mode and the instantaneous contributions, $f_{\mathcal{P}}$ obtained from the minus component of the current in the standard LFQM is free from such treacherous contributions and gives an identical result with the one obtained from the plus component of the current. In Sec. III, we obtain the transition form factors $f_\pm(q^2)$ in the standard LFQM using the same procedure discussed in Sec. II. Especially, we explicitly show that $f_-(q^2)$ obtained from (J^+, J^-) is exactly the same as the one obtained from $(J^+, \mathbf{J}_\perp)$ in the $q^+ = 0$ frame. This finding again supports the universality of our correspondence relations between the covariant BS model and the standard LFQM. In Sec. IV, we show our numerical results for the semileptonic $B \to D\ell\nu_\ell$ $(\ell = e, \mu, \tau)$ decays. In the Appendix, the explicit forms of the standard LFQM results for $f_\pm(q^2)$ are presented.

II. DECAY CONSTANT

A. $f_{\mathcal{P}}$ in the covariant BS model

In the solvable model, based on the covariant BS model of $(3+1)$-dimensional fermion field theory, the decay constant $f_{\mathcal{P}}$ of a pseudoscalar meson ($\mathcal{P}$) with the four-momentum P and mass M as a $q\bar{q}$ bound state is defined by the matrix element of the axial vector current

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 q|\mathcal{P}(P)\rangle = i f_{\mathcal{P}} P^\mu. \tag{1}$$

The matrix element $A^\mu \equiv \langle 0|\bar{q}\gamma^\mu\gamma_5 q|\mathcal{P}(P)\rangle$ is given in the one-loop approximation as a momentum integral

$$A^\mu = N_c \int \frac{d^4k}{(2\pi)^4} \frac{H_{\mathcal{P}} S^\mu}{(p^2 - m_1^2 + i\varepsilon)(k^2 - m_q^2 + i\varepsilon)}, \tag{2}$$

where N_c is the number of colors and $p = P - k$ and k are the internal momenta carried by the quark and antiquark propagators of mass m_1 and m_q, respectively. The $q\bar{q}$ bound-state vertex function $H_{\mathcal{P}}$ of a pseudoscalar meson is taken as multipole ansatz, i.e., $H_{\mathcal{P}}(p^2, k^2) = g/(p^2 - \Lambda^2 + i\epsilon)$ where g and Λ are constant parameters in this manifestly covariant model. The trace term is given by

$$S^\mu = \mathrm{Tr}[\gamma^\mu\gamma_5(\not{p} + m_1)\gamma_5(-\not{k} + m_q)]. \tag{3}$$

Performing the LF calculation, we take the reference frame where $P = (P^+, P^-, \mathbf{P}_\perp) = (P^+, M^2/P^+, \mathbf{0}_\perp)$ and use the metric convention $a\cdot b = \frac{1}{2}(a^+b^- + a^-b^+) - \mathbf{a}_\perp\cdot\mathbf{b}_\perp$. We then obtain the identity $\not{q} = \not{q}_{\mathrm{on}} + \frac{1}{2}\gamma^+\Delta_q^-$, where $\Delta_q^- = q^- - q_{\mathrm{on}}^-$ and the subscript (on) denotes the on-mass shell quark momentum, i.e., $p_{\mathrm{on}}^2 = m_1^2$ and $k_{\mathrm{on}}^2 = m_q^2$. Using this identity, one can separate the trace term into the on shell propagating part S_{on}^μ and the off-mass shell instantaneous one S_{inst}^μ as $S^\mu = S_{\mathrm{on}}^\mu + S_{\mathrm{inst}}^\mu$.

By the integration over k^- in Eq. (2) and closing the contour in the lower half of the complex k^- plane, one picks up the residue at $k^- = k_{\mathrm{on}}^-$ in the region of $0 < k^+ < P^+$ (or $0 < x < 1$) where $x = \frac{p^+}{P^+}$ and $1 - x = \frac{k^+}{P^+}$ are the LF longitudinal momentum fractions of the quark and antiquark. We denote the valence contribution to A^μ that is obtained by taking $k^- = k_{\mathrm{on}}^-$ in the region of $0 < x < 1$ region as $[A^\mu]_{\mathrm{val}}^{\mathrm{LF}}$. Then the Cauchy integration formula for the k^- integration in the valence region of Eq. (2) yields

$$[A^\mu]_{\mathrm{val}}^{\mathrm{LFBS}} = \frac{iN_c}{16\pi^3}\int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi(x, \mathbf{k}_\perp) S_{\mathrm{val}}^\mu, \tag{4}$$

where $\chi(x, \mathbf{k}_\perp) = \frac{g}{x^2(M^2 - M_0^2)(M^2 - M_\Lambda^2)}$ is the LF quark-meson vertex function and $M_{0(\Lambda)}^2 = \frac{\mathbf{k}_\perp^2 + m_1^2(\Lambda^2)}{x} + \frac{\mathbf{k}_\perp^2 + m_q^2}{1-x}$. The trace term in the valence contribution is given by $S_{\mathrm{val}}^\mu = S_{\mathrm{on}}^\mu + S_{\mathrm{inst}}^\mu$, where $S_{\mathrm{on}}^\mu = 4(m_1 k_{\mathrm{on}}^\mu + m_q p_{\mathrm{on}}^\mu)$ and $S_{\mathrm{inst}}^\mu = 2(m_1\Delta_k^- + m_q\Delta_p^-)g^{\mu+}$. We note from S_{inst}^μ that the off shell instantaneous contributions are nonzero for the minus component of the current while they are absent for the plus or perpendicular components of the current.

In our previous work [21], we check the LF covariance of $f_{\mathcal{P}}$ obtained from Eq. (4) using two different components (i.e., $\mu = +$ and $-$) of the current. We found that while $f^{(+)}_{\mathcal{P}}$ obtained from $\mu = +$ is free from the zero mode, $f^{(-)}_{\mathcal{P}}$ obtained from $\mu = -$ receives the zero mode. We also identified the zero-mode operator corresponding to the zero-mode contribution to $f^{(-)}_{\mathcal{P}}$ [see Eq. (B9) in [21]]. Since the LF calculations of $f_{\mathcal{P}}$ obtained from Eq. (4) were explicitly shown in [21,22], we recapitulate the essential features of obtaining the full LF result of $f^{(-)}_{\mathcal{P}}$. Then, we focus on the self-consistent standard LFQM analysis of $f_{\mathcal{P}}$ using our new correspondence relations (i.e., $\chi \to \phi$ and $M \to M_0$).

For $\mu = +$, the full result of $f_{\mathcal{P}}$ can be obtained only from the valence contribution with the on-mass shell quark propagating part, i.e., $S^+_{\rm full} = S^+_{\rm val} = S^+_{\rm on}$. The full solution of the decay constant obtained from $\mu = +$ is given by [21,22]

$$[f^{(+)}_{\mathcal{P}}]^{\rm LFBS}_{\rm full} = \frac{N_c}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\mathbf{k}_\perp \chi(x,\mathbf{k}_\perp)\frac{S^+_{\rm on}}{4P^+}, \tag{5}$$

where $S^+_{\rm full} = S^+_{\rm on} = 4P^+\mathcal{A}_1$ and $\mathcal{A}_1 = (1-x)m_1 + xm_q$.

For $\mu = -$, the valence contribution to the trace term comes not only from the on-shell propagating part but also from the off-shell instantaneous one, i.e., $S^-_{\rm val} = S^-_{\rm on} + S^-_{\rm inst}$. However, the valence contribution itself is not equal to the manifestly covariant result (or equivalently $[f^{(+)}_{\mathcal{P}}]^{\rm LFBS}_{\rm full}$) since the minus component of the current receives the zero-mode contribution as shown in [21]. In [21], we also found the zero-mode operator $S^-_{\rm Z.M.}$ corresponding to the zero-mode contribution at the trace level, i.e., $S^-_{\rm Z.M.} = \frac{4}{P^+}(m_q - m_1)(-Z_2)$ with $Z_2 = x(M^2 - M_0^2) + m_1^2 - m_q^2 + (1-2x)M^2$. Adding $S^-_{\rm Z.M.}$ to $S^-_{\rm val}$, we found that $S^-_{\rm full} = S^-_{\rm val} + S^-_{\rm Z.M.} = 4P^-\mathcal{A}_1$.

That is, in this manifestly covariant BS model, the full solution $[f^{(-)}_{\mathcal{P}}]^{\rm LFBS}_{\rm full}$ obtained from $\mu = -$ is completely equal to $[f^{(+)}_{\mathcal{P}}]^{\rm LFBS}_{\rm full}$ only if the zero-mode contribution is included in addition to the valence contribution. We should note that while $[f^{(+)}_{\mathcal{P}}]^{\rm LFBS}_{\rm full} = [f^{(+)}_{\mathcal{P}}]^{\rm LFBS}_{\rm on}$, $[f^{(-)}_{\mathcal{P}}]^{\rm LFBS}_{\rm full} = [f^{(-)}_{\mathcal{P}}]^{\rm LFBS}_{\rm on} + [f^{(-)}_{\mathcal{P}}]^{\rm LFBS}_{\rm inst} + [f^{(-)}_{\mathcal{P}}]^{\rm LFBS}_{\rm Z.M.}$.

For the sake of comparison with $[f^{(+)}_{\mathcal{P}}]^{\rm LFBS}_{\rm on}$ and also for later use in the standard LFQM analysis, we display the result of $[f^{(-)}_{\mathcal{P}}]^{\rm LFBS}_{\rm on}$ obtained from Eq. (4) with only the on-mass propagating part, $S^\mu_{\rm val} = S^-_{\rm on}$, as follows

$$[f^{(-)}_{\mathcal{P}}]^{\rm LFBS}_{\rm on} = \frac{N_c}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\mathbf{k}_\perp \chi(x,\mathbf{k}_\perp)\frac{P^+S^-_{\rm on}}{4M^2}, \tag{6}$$

where $S^-_{\rm on} = 4(m_1 k^-_{\rm on} + m_q p^-_{\rm on})$ with $k^-_{\rm on} = \frac{\mathbf{k}_\perp^2 + m_q^2}{(1-x)P^+}$ and $p^-_{\rm on} = \frac{\mathbf{k}_\perp^2 + m_1^2}{xP^+}$.

B. $f_{\mathcal{P}}$ in the standard LFQM

In the standard LFQM [15,16,30–36], the wave function of a ground state pseudoscalar meson as a $q\bar{q}$ bound state is given by

$$\Psi_{\lambda\bar{\lambda}}(x,\mathbf{k}_\perp) = \phi(x,\mathbf{k}_\perp)\mathcal{R}_{\lambda\bar{\lambda}}(x,\mathbf{k}_\perp), \tag{7}$$

where $\mathcal{R}_{\lambda\bar{\lambda}}$ is the spin-orbit wave function that is obtained by the interaction independent Melosh transformation from the ordinary spin-orbit wave function assigned by the quantum number J^{PC}. The covariant form of $\mathcal{R}_{\lambda\bar{\lambda}}$ with the definite spin $(S, S_z) = (0,0)$ constructed out of the LF helicity $\lambda(\bar{\lambda})$ of a quark (antiquark) is given by

$$\mathcal{R}_{\lambda\bar{\lambda}} = \frac{\bar{u}_\lambda(p_q)\gamma_5 v_{\bar{\lambda}}(p_{\bar{q}})}{\sqrt{2}[M_0^2 - (m_1 - m_q)^2]^{1/2}}, \tag{8}$$

which satisfies the unitarity condition, $\sum_{\lambda\bar{\lambda}}\mathcal{R}^\dagger_{\lambda\bar{\lambda}}\mathcal{R}_{\lambda\bar{\lambda}} = 1$. Its explicit matrix form is given by

$$\mathcal{R}_{\lambda\bar{\lambda}} = \frac{1}{\sqrt{2}\sqrt{\mathbf{k}_\perp^2 + \mathcal{A}_1^2}}\begin{pmatrix} -k^L & \mathcal{A}_1 \\ -\mathcal{A}_1 & -k^R \end{pmatrix}, \tag{9}$$

where $k^R = k_x + ik_y$ and $k^L = k_x - ik_y$.

For the radial wave function ϕ in Eq. (7), we use the Gaussian wave function

$$\phi(x,\mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}}\sqrt{\frac{\partial k_z}{\partial x}}\exp(-\vec{k}^2/2\beta^2), \tag{10}$$

where $\vec{k}^2 = \mathbf{k}_\perp^2 + k_z^2$ and β is the variational parameter fixed by the analysis of meson mass spectra [19,33–35]. The longitudinal component k_z is defined by $k_z = (x - \frac{1}{2})M_0 + \frac{(m_q^2 - m_1^2)}{2M_0}$, and the Jacobian of the variable transformation $\{x, \mathbf{k}_\perp\} \to \vec{k} = (\mathbf{k}_\perp, k_z)$ is given by $\frac{\partial k_z}{\partial x} = \frac{M_0}{4x(1-x)}[1 - (\frac{m_1^2 - m_q^2}{M_0^2})^2]$. The normalization of our Gaussian radial wave function is then given by

$$\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}|\phi(x,\mathbf{k}_\perp)|^2 = 1. \tag{11}$$

Using the plus component of the current, the standard LFQM calculation of Eq. (1) is obtained by

$$[f^{(+)}_{\mathcal{P}}]^{\rm SLF}_{\rm on} = \frac{\sqrt{2N_c}}{8\pi^3}\int_0^1 dx \int d^2\mathbf{k}_\perp \frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{\mathbf{k}_\perp^2 + \mathcal{A}_1^2}}\frac{S^+_{\rm on}}{4P^+}. \tag{12}$$

We should note that the main differences between the covariant BS model and the standard LFQM are attributed to the different spin structures of the $q\bar{q}$ system (i.e., off shellness vs on shellness) and the different meson-quark vertex functions (χ vs ϕ). In other words, while the results of the covariant BS model allow the nonzero binding energy $E_{\rm B.E.} = M^2 - M_0^2$, the SLF (referring to Standard LFQM) result is obtained from the condition of on-mass shell quark and antiquark (i.e., $M \to M_0$).

To find the exact correspondence between the covariant BS model and the standard LFQM, we first compare the physical quantities which are immune to the treacherous points such as the zero modes or the instantaneous contributions in the BS model. In the case of pseudoscalar meson decay constant, since $f^{(+)}_{\mathscr{P}}$ obtained from the plus component of the current satisfies this prerequisite condition, one can find the following correspondence relation, $\sqrt{2N_c}\frac{\chi(x,\mathbf{k}_\perp)}{1-x} \to \frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{\mathscr{A}_1^2+\mathbf{k}_\perp^2}}$, by comparing $[f^{(+)}_{\mathscr{P}}]^{\rm LFBS}_{\rm full} = [f^{(+)}_{\mathscr{P}}]^{\rm LFBS}_{\rm on}$ in Eq. (5) and $[f^{(+)}_{\mathscr{P}}]^{\rm SLF}_{\rm on}$ in Eq. (12). In most previous LFQM analyses, this correspondence (χ vs ϕ) has also been used for the mapping of other physical observables contaminated by the treacherous points.

In our previous analysis [21–23], we found that the correspondence relation including only LF vertex functions brings about the self-consistency problem, i.e., the same physical quantity obtained from different components of the current and/or the polarization vectors yields different results in the standard LFQM. Our new correspondence relations between the two models to iron out the self-consistency problem is given by [21–23]:

$$\sqrt{2N_c}\frac{\chi(x,\mathbf{k}_\perp)}{1-x} \to \frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{\mathscr{A}_1^2+\mathbf{k}_\perp^2}}, \qquad M \to M_0, \tag{13}$$

that is, the physical mass M included in the integrand of the BS amplitude should be replaced with the invariant mass M_0 since the results in the standard LFQM are obtained from the requirement of all constituents being on their respective mass shell. We should note that the correspondence in Eq. (13) between the covariant model and the LFQM has been verified through our previous analyses of pseudoscalar [22] and pseudotensor [23] twist-3 DAs of a pseudoscalar meson and the chirality-even twist-2 and twist-3 DAs of a vector meson [21].

The virtue of Eq. (13) to restore the self-consistency of the standard LFQM is that one can apply Eq. (13) only to the on-mass shell contribution in the BS model to get the full result in the standard LFQM. In other words, the treacherous points (i.e., zero mode and the instantaneous contribution) appeared in the covariant BS model are absorbed into the LF on-mass shell constituent quark and antiquark contributions and the full result in the standard LFQM is obtained only from the on-shell contribution regardless of the components of the currents being used. This remarkable feature also can be seen in this analysis of decay constant of pseudoscalar meson obtained from the "−" component of the currents. That is, applying Eq. (13) to $[f^{(-)}_{\mathscr{P}}]^{\rm LFBS}_{\rm on}$ given by Eq. (6), we obtain the SLF result for the minus component of the current as follows

$$[f^{(-)}_{\mathscr{P}}]^{\rm SLF}_{\rm on} = \frac{\sqrt{2N_c}}{8\pi^3}\int_0^1 dx \int d^2\mathbf{k}_\perp \frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{\mathbf{k}_\perp^2+\mathscr{A}_1^2}}\frac{P^+ S^-_{\rm on}}{4M_0^2}. \tag{14}$$

We confirm numerically that $[f^{(-)}_{\mathscr{P}}]^{\rm SLF}_{\rm on} = [f^{(+)}_{\mathscr{P}}]^{\rm SLF}_{\rm on}$, which contrasts with the covariant BS model calculation, in which $[f^{(-)}_{\mathscr{P}}]^{\rm LFBS}_{\rm on} \neq [f^{(+)}_{\mathscr{P}}]^{\rm LFBS}_{\rm on}$. We also should note that our confirmation for $[f^{(-)}_{\mathscr{P}}]^{\rm SLF}_{\rm on} = [f^{(+)}_{\mathscr{P}}]^{\rm SLF}_{\rm on}$ is independent of the form of the radial wave function, e.g., the power-law type wave function such as $\phi \propto \sqrt{\partial k_z/\partial x}(1+\vec{k}^2/\beta^2)^{-2}$ also shows $[f^{(-)}_{\mathscr{P}}]^{\rm SLF}_{\rm on} = [f^{(+)}_{\mathscr{P}}]^{\rm SLF}_{\rm on}$.

III. SEMILEPTONIC DECAYS BETWEEN TWO PSEUDOSCALAR MESONS

The transition form factors for the $\mathscr{P}(P_1) \to \mathscr{P}(P_2)\ell\nu_\ell$ semileptonic decays between two pseudoscalar mesons are given by

$$\langle P_2|V^\mu|P_1\rangle = f_+(q^2)(P_1+P_2)^\mu + f_-(q^2)q^\mu, \tag{15}$$

where $q^\mu = (P_1 - P_2)^\mu$ is the four-momentum transfer to the lepton pair($\ell\nu_\ell$) and $m_\ell^2 \le q^2 \le (M_1 - M_2)^2$. The two form factors $f_\pm(q^2)$ also satisfy

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_1^2 - M_2^2} f_-(q^2). \tag{16}$$

The matrix element $\mathscr{M}^\mu \equiv \langle P_2|V^\mu|P_1\rangle$ in the BS model is given by

$$\mathscr{M}^\mu = iN_c \int \frac{d^4k}{(2\pi)^4}\frac{H_{p_1}\mathscr{T}^\mu H_{p_2}}{N_{p_1}N_k N_{p_2}}, \tag{17}$$

where $N_k = k^2 - m_q^2 + i\epsilon$ and $N_{p_j} = p_j^2 - m_j^2 + i\epsilon$ with $p_j = P_j - k$ $(j = 1, 2)$. To be consistent with the analysis of the decay constant, we take the $q\bar{q}$ bound-state vertex functions $H_{p_j}(p_j^2, k^2) = g_j/(p_j^2 - \Lambda_j^2 + i\epsilon)$ of the initial $(j = 1)$ and final $(j = 2)$ state pseudoscalar mesons. The trace term is given by

$$\mathscr{T}^\mu = {\rm Tr}[\gamma_5(\not{p}_1 + m_1)\gamma^\mu(\not{p}_2 + m_2)\gamma_5(-\not{k} + m_q)]. \tag{18}$$

Performing the LF calculation of Eq. (17) in the valence region $(0 < k^+ < P_2^+)$ of the $q^+ = 0$ frame, where the pole $k^- = k^-_{\rm on} = (\mathbf{k}_\perp^2 + m_q^2 - i\epsilon)/k^+$ (i.e., the spectator quark) is located in the lower half of the complex k^- plane, the Cauchy integration formula for the k^- integral in Eq. (17) gives

$$[\mathscr{M}^\mu]^{\rm LFBS}_{\rm val} = N_c \int_0^1 \frac{dx}{(1-x)}\int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\chi_1(x,\mathbf{k}_\perp)\chi_2(x,\mathbf{k}'_\perp)\mathscr{T}^\mu_{\rm val}, \tag{19}$$

where

$$\chi_{1(2)} = \frac{g_{1(2)}}{x^2(M^2_{1(2)} - M_0^{(\prime)2})(M^2_{1(2)} - M^2_{\Lambda_{1(2)}})}, \quad (20)$$

with $M^2_{0(\Lambda_1)} = \frac{\mathbf{k}_\perp^2 + m_1^2(\Lambda_1^2)}{x} + \frac{\mathbf{k}_\perp^2 + m_q^2}{1-x}$ and $M'^2_{0(\Lambda_2)} = M^2_{0(\Lambda_1)} \times (m_1(\Lambda_1) \to m_2(\Lambda_2), \mathbf{k}_\perp \to \mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp)$. The explicit LF calculation of Eq. (19) in parallel with the manifestly covariant calculation of Eq. (17) can be found in [19]. As shown in Ref. [19], while $f_+(q^2)$ was obtained from J^+ and immune to the zero mode, the form factor $f_-(q^2)$ was obtained from $(J^+, \mathbf{J}_\perp)$ and received both the instantaneous and the zero-mode contributions. Of course, one cannot avoid such treacherous points in the BS model even if $f_-(q^2)$ is obtained from the two components (J^+, J^-) of the current.

In this work, we shall show that $f_-(q^2)$ in the standard LFQM is independent of the components of the current, i.e., regardless of using $(J^+, \mathbf{J}_\perp)$ or (J^+, J^-), as far as we apply Eq. (13) in the BS model to get the standard LFQM results. So, from now on, we discuss only for the on-mass shell contribution in the valence region of the $q^+ = 0$ frame. Of the trace terms $\mathcal{T}^\mu_{\rm val} = \mathcal{T}^\mu_{\rm on} + \mathcal{T}^\mu_{\rm inst}$, the on-shell contribution is given by

$$\mathcal{T}^\mu_{\rm on} = 4[p^\mu_{1\rm on}(p_{2\rm on}\cdot k_{\rm on}) - k^\mu_{\rm on}(p_{1\rm on}\cdot p_{2\rm on}) + p^\mu_{2\rm on}(p_{1\rm on}\cdot k_{\rm on}) + m_2 m_{\bar q} p^\mu_{1\rm on} + m_1 m_{\bar q} p^\mu_{2\rm on} + m_1 m_2 k^\mu_{\rm on}], \quad (21)$$

where

$$p_{1\rm on} = \left[xP_1^+, \frac{m_1^2 + \mathbf{k}_\perp^2}{xP_1^+}, -\mathbf{k}_\perp\right],$$
$$p_{2\rm on} = \left[xP_1^+, \frac{m_2^2 + (\mathbf{k}_\perp + \mathbf{q}_\perp)^2}{xP_1^+}, -\mathbf{k}_\perp - \mathbf{q}_\perp\right],$$
$$k_{\rm on} = \left[(1-x)P_1^+, \frac{m_q^2 + \mathbf{k}_\perp^2}{(1-x)P_1^+}, \mathbf{k}_\perp\right]. \quad (22)$$

The explicit form of the instantaneous contribution $\mathcal{T}^\mu_{\rm inst}$ can be found in [19]. On the one hand, the transition form factors $f_\pm(q^2)$ obtained from $(J^+, \mathbf{J}_\perp)$ are given by

$$f_+(q^2) = \frac{\mathcal{M}^+}{2P_1^+},$$
$$f_-^{(\perp)}(q^2) = \frac{\mathcal{M}^+}{2P_1^+} + \frac{\mathcal{M}^\perp\cdot\mathbf{q}_\perp}{\mathbf{q}_\perp^2}. \quad (23)$$

On the other hand, the form factor $f_-(q^2)$ obtained from (J^+, J^-) is given by

$$f_-^{(-)}(q^2) = -\frac{\mathcal{M}^+}{2P_1^+}\left(\frac{\Delta M_+^2 + \mathbf{q}_\perp^2}{\Delta M_-^2 - \mathbf{q}_\perp^2}\right) + \frac{P_1^+\mathcal{M}^-}{\Delta M_-^2 - \mathbf{q}_\perp^2}, \quad (24)$$

where $\Delta M^2_\pm = M_1^2 \pm M_2^2$. For convenience sake, the form factor $f_-(q^2)$ obtained from $(J^+, \mathbf{J}_\perp)$ and (J^+, J^-) is denoted by $f_-^{(\perp)}(q^2)$ and $f_-^{(-)}(q^2)$, respectively. In the manifestly covariant BS model given by Eq. (17), we note that while $[f_+^{(+)}]^{\rm LFBS}_{\rm full} = [f_+^{(+)}]^{\rm LFBS}_{\rm on}$, $[f_-^{(\perp)}]^{\rm LFBS}_{\rm full} = [f_-^{(\perp)}]^{\rm LFBS}_{\rm on} + [f_-^{(\perp)}]^{\rm LFBS}_{\rm inst} + [f_-^{(\perp)}]^{\rm LFBS}_{\rm Z.M.}$. The full result $f_-^{(-)}(q^2)$ has the same structure as $f_-^{(\perp)}(q^2)$, i.e., $[f_-^{(-)}]^{\rm LFBS}_{\rm full} = [f_-^{(-)}]^{\rm LFBS}_{\rm on} + [f_-^{(-)}]^{\rm LFBS}_{\rm inst} + [f_-^{(-)}]^{\rm LFBS}_{\rm Z.M.}$ although the explicit forms of the instantaneous and zero-mode contributions are different from those for $f_-^{(\perp)}(q^2)$.

For the calculation of the transition form factors $f_\pm(q^2)$, our new correspondence relations between the covariant BS model and the standard LFQM are given by

$$\sqrt{2N_c}\frac{\chi_1(x,\mathbf{k}_\perp)}{1-x} \to \frac{\phi_1(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2 + \mathbf{k}_\perp^2}}, \qquad M_1 \to M_0,$$
$$\sqrt{2N_c}\frac{\chi_2(x,\mathbf{k}'_\perp)}{1-x} \to \frac{\phi_2(x,\mathbf{k}'_\perp)}{\sqrt{\mathcal{A}_2^2 + \mathbf{k}'^2_\perp}}, \qquad M_2 \to M'_0. \quad (25)$$

In order to obtain the self-consistent description of our standard LFQM, we first compute $[f_+]^{\rm LFBS}_{\rm full} = [f_+]^{\rm LFBS}_{\rm on}$, $[f_-^{(\perp)}]^{\rm LFBS}_{\rm on}$, and $[f_-^{(-)}]^{\rm LFBS}_{\rm on}$ from the BS model and apply Eq. (25) to get the corresponding standard LFQM results, i.e., $[f_+]^{\rm SLF}_{\rm on}$, $[f_-^{(\perp)}]^{\rm SLF}_{\rm on}$ and $[f_-^{(-)}]^{\rm SLF}_{\rm on}$, respectively. The final standard LFQM results for $f_\pm(q^2)$ are given by

$$[f_+(q^2)]^{\rm SLF}_{\rm on} = \int_0^1 dx\int\frac{d^2\mathbf{k}_\perp}{16\pi^3}\frac{\phi_1(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2+\mathbf{k}_\perp^2}}\frac{\phi_2(x,\mathbf{k}'_\perp)}{\sqrt{\mathcal{A}_2^2+\mathbf{k}'^2_\perp}} \times\frac{(1-x)}{2}\left[\frac{\mathcal{T}^+_{\rm on}}{2P_1^+}\right], \quad (26)$$

$$[f_-^{(\perp)}(q^2)]^{\rm SLF}_{\rm on} = \int_0^1 dx\int\frac{d^2\mathbf{k}_\perp}{16\pi^3}\frac{\phi_1(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2+\mathbf{k}_\perp^2}}\frac{\phi_2(x,\mathbf{k}'_\perp)}{\sqrt{\mathcal{A}_2^2+\mathbf{k}'^2_\perp}} \times\frac{(1-x)}{2}\left[\frac{\mathcal{T}^+_{\rm on}}{2P_1^+} + \frac{\mathcal{T}_{\perp\rm on}\cdot\mathbf{q}_\perp}{\mathbf{q}_\perp^2}\right], \quad (27)$$

and

$$[f_-^{(-)}(q^2)]^{\rm SLF}_{\rm on} = \int_0^1 dx\int\frac{d^2\mathbf{k}_\perp}{16\pi^3}\frac{\phi_1(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2+\mathbf{k}_\perp^2}}\frac{\phi_2(x,\mathbf{k}'_\perp)}{\sqrt{\mathcal{A}_2^2+\mathbf{k}'^2_\perp}} \times\frac{(1-x)[P_1^+\mathcal{T}^-_{\rm on} - \frac{\mathcal{T}^+_{\rm on}}{2P_1^+}(\Delta M^2_{0+} + \mathbf{q}_\perp^2)]}{2(\Delta M^2_{0-} - \mathbf{q}_\perp^2)}, \quad (28)$$

respectively, where $\Delta M^2_{0\pm} = M_0^2 \pm M_0'^2$ obtained from the on-mass shell condition (i.e., $M^{(\prime)} \to M_0^{(\prime)}$) and $\mathcal{A}_i = (1-x)m_i + xm_q$ $(i = 1, 2)$. We numerically confirm that $[f_-^{(\perp)}(q^2)]^{\rm SLF}_{\rm on} = [f_-^{(-)}(q^2)]^{\rm SLF}_{\rm on}$, which supports the self-consistency of our standard LFQM. The explicit forms of the on shell trace terms and the form factors in

Eqs. (26)–(28) are given in the Appendix. We note that the form factors obtained in the spacelike region using the $q^+ = 0$ frame are analytically continued to the timelike region by changing $\mathbf{q}_\perp^2$ to $-q^2$ in the form factors.

Including the nonzero lepton mass (m_ℓ), the differential decay rate for the exclusive $\mathcal{P}(P_1) \to \mathcal{P}(P_2)\ell\nu_\ell$ process is given by [37,38]

$$\frac{d\Gamma}{dq^2} = \frac{8\mathcal{N}|\vec{p}^*|}{3}\left[\left(1 + \frac{m_\ell^2}{2q^2}\right)|H_+|^2 + \frac{3m_\ell^2}{2q^2}|H_0|^2\right], \quad (29)$$

where

$$|\vec{p}^*| = \frac{1}{2M_1}\sqrt{(M_1^2 + M_2^2 - q^2)^2 - 4M_1^2M_2^2} \quad (30)$$

is the modulus of the three momentum of the daughter meson in the parent meson rest frame, and the helicity amplitudes H_0 and H_t corresponding to the longitudinal parts of the spin-1 and spin-0 hadronic contributions, respectively, can be expressed in terms of f_+ and f_0 as follows:

$$H_+ = \frac{2M_1|\vec{p}^*|}{\sqrt{q^2}} f_+(q^2), \qquad H_0 = \frac{M_1^2 - M_2^2}{\sqrt{q^2}} f_0(q^2). \quad (31)$$

The normalization factor in Eq. (29) is

$$\mathcal{N} = \frac{G_F^2}{256\pi^3}\eta_{\rm EW}^2 |V_{Q_1\bar{Q}_2}|^2 \frac{q^2}{M_1^2}\left(1 - \frac{m_\ell^2}{q^2}\right)^2, \quad (32)$$

where $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi constant, $V_{Q_1\bar{Q}_2}$ is the relevant CKM mixing matrix element and the factor $\eta_{\rm EW} = 1.0066$ accounts for the leading order electroweak corrections [39].

The kinematics of the $\mathcal{P}(P_1) \to \mathcal{P}(P_2)\ell\nu_\ell$ decay can also be expressed in terms of the recoil variable w defined by

$$w = v_1 \cdot v_2 = \frac{M_1^2 + M_2^2 - q^2}{2M_1M_2}, \quad (33)$$

where $v_{1(2)} = \frac{P_{1(2)}}{M_{1(2)}}$ is the four velocity of the initial (final) meson and $q^2 = (P_1 - P_2)^2 = (P_\ell + P_\nu)^2$. While the minimum value of $w = 1$ (or $q^2 = q^2_{\rm max}$) corresponds to zero recoil of the final meson in the initial meson rest frame, the maximum value of w (or $q^2 = 0$) corresponds to the maximum recoil of the final meson recoiling with the maximum three momentum $|\vec{P}_2| = \frac{(M_1^2 - M_2^2)}{2M_1}$.

IV. NUMERICAL RESULTS

In our numerical calculations for the semileptonic $B \to D\ell\nu_\ell$ ($\ell = e,\ \mu,\ \tau$) decays, we use two sets of model parameters (m, β) for the linear and harmonic oscillator (HO) confining potentials given in Table I obtained from the calculation of the ground state meson mass spectra [19,35]. For the physical (B, D) meson masses, we use the central values quoted by the Particle Data Group (PDG) [40]. Our predictions for the decay constants of (D, B) mesons obtained from the model parameters in Table I are $f_D = 197(180)$ MeV and $f_B = 171(161)$ MeV for the linear (HO) parameters, respectively, while the current available experimental data are given by $f_D^{\rm exp} = 205.8(4.5)(0.4)(2.7)$ MeV [40] and $f_B^{\rm exp} = 229^{+39+34}_{-31-37}$ MeV [41].

TABLE I. The constituent quark mass m_q (in GeV) and the Gaussian parameters $\beta_{q\bar{q}}$ (in GeV) for the linear and HO confining potential obtained by the variational principle [19,35]. $q = u$ and d.

Model	m_q	m_c	m_b	β_{qc}	β_{qb}
Linear	0.22	1.8	5.2	0.4679	0.5266
HO	0.25	1.8	5.2	0.4216	0.4960

In Fig. 1, we show the q^2 dependences of $f_+(q^2)$ (solid line), $f_0(q^2)$ (dashed line), and $f_-(q^2)$ for $B \to D\ell\nu_\ell$ decay obtained from Eqs. (26)–(28) with the linear potential parameters. As one can see, our result for $f_-(q^2)$ (dot-dashed line) obtained from $(J^+, \mathbf{J}_\perp)$ [see Eq. (27)] shows a complete agreement with $f_-(q^2)$ (circle) obtained from (J^+, J^-) [see Eq. (28)] substantiating the self-consistency of our LFQM. We also should note that the form factors are displayed not only for the whole timelike kinematic region $[m_\ell^2 \le q^2 \le (M_B - M_D)^2]$ (in unit of GeV^2) but also for the

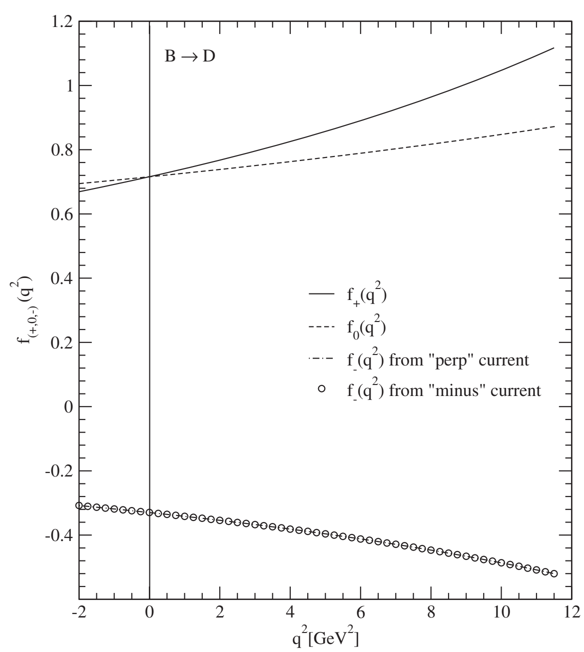

FIG. 1. The q^2 dependent form factors (f_+, f_0, f_-) of the $B \to D\ell\nu_\ell$ decay for both spacelike and the kinematic timelike regions, $-2 \le q^2 \le (M_B - M_D)^2$ GeV^2.

spacelike region ($-2 \leq q^2 \leq 0$) (in unit of GeV^2) to demonstrate the validity of our analytic continuation from spacelike region to the timelike by changing $\mathbf{q}_\perp^2$ to $-\mathbf{q}_\perp^2 (= q^2 > 0)$ in the form factors.

Our results of the form factors $(f_\pm, f_0)$ obtained from the linear (HO) potential parameters at the maximum recoil $(q^2 = 0)$ and minimum recoil $(q^2 = q^2_{\rm max})$ points are summarized in Table II. Our direct LFQM results for the form factors $f_i(q^2)$ $(i = \pm,\ 0)$ obtained from Eqs. (26)–(28) are well described by the following parametrization [9]

$$f_i(q^2) = \frac{f_i(0)}{1 - b_i(q^2/M_B^2) + c_i(q^2/M_B^2)^2}, \qquad (34)$$

where the parameters (b_i, c_i) can be obtained from our LFQM results in Eqs. (26)–(28) via $b_i = \frac{M_B^2}{f_i(0)} f'_i(0)$ and $c_i = b_i^2 - \frac{f''_i(0) M_B^4}{2 f_i(0)}$. The fitted parameters (b_i, c_i) for (f_+, f_0) are also summarized in Table III and those for f_- are obtained as $b_- = 0.970071(1.00817)$ and $c_- = 0.200821(0.2384)$ for the liner (HO) parameters, respectively. We should note that our direct LFQM results and the ones obtained from Eq. (34) are in excellent agreement with each other within 0.1% error.

In Fig. 2, we show the recoil variable w dependent form factor $f_+(w)$ (solid line) and $f_0(w)$ (dashed line) obtained from both linear (black lines) and HO (blue line) potential parameters and compare them with the data from the Belle experiment [42] and the lattice QCD (HPQCD collaboration) [5]. Our results are overall in good agreement with those from [5,42].

Of special interest, while our results for $f_+(w)$ and $f_0(w)$ obtained from the linear potential parameters (black line) are somewhat different from those obtained from the HO potential parameters (blue lines) at the maximum recoil point (i.e., $w \simeq 1.6$), both potential parameters give almost the same results at the zero recoil point (i.e., $w = 1$). This is related with the heavy-quark symmetry (HQS); i.e., in the

TABLE II. Form Factors of the $B \to D\ell\nu_\ell$ decay at $q^2 = 0$ and $q^2 = q^2_{\rm max}$ obtained from the linear (HO) potential parameters.

$f_+(0)$	$f_+(q^2_{\rm max})$	$f_0(q^2_{\rm max})$	$f_-(0)$	$f_-(q^2_{\rm max})$
0.7157	1.1235	0.8739	−0.3298	−0.5231
(0.6969)	(1.1209)	(0.8755)	(−0.3190)	(−0.5142)

TABLE III. The fitted parameters $b_{+(0)}$ and $c_{+(0)}$ for the parametric form factors in Eq. (34) obtained from the linear (HO) potential parameters.

$f_{+,0}(0)$	b_+	c_+	b_0	c_0
0.7157	0.955259	0.203408	0.428416	−0.014496
(0.6969)	(1.00776)	(0.245602)	(0.484403)	(−0.007704)

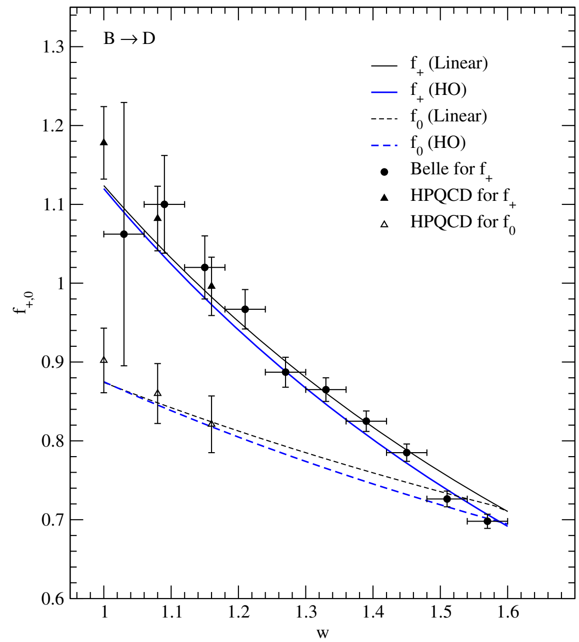

FIG. 2. The recoil variable w dependent form factors (f_+, f_0) of $B \to D\ell\nu_\ell$ obtained from the linear and HO potential parameters, and the result of the combined fit to experimental [42] and lattice QCD (HPQCD) [5] data.

infinite quark mass limit, the heavy-to-heavy transition form factors between two pseudoscalar mesons such as $B \to D\ell\nu_\ell$ decay are reduced to single universal Isgur-Wise function [43,44] $\mathcal{G}(w) = \frac{2\sqrt{M_B M_D}}{M_B + M_D} f_+(w)$, which should in principle satisfy the following normalization $\mathcal{G}(1) = 1$ in the exact HQS limit. Our LFQM results of $\mathcal{G}(1) = 0.988(0.984)$ obtained from the linear (HO) parameters are in good agreement with the exact HQS limit within 2% errors. Our results also should be compared with other theoretical predictions such as $\mathcal{G}(1) = 1.035(40)$ [5], $\mathcal{G}(1) = 1.0541(83)$ [6], and $\mathcal{G}(1) = 1.033(95)$ [45] from the lattice QCD and $\mathcal{G}(1) = 0.981^{+0.045}_{-0.048}$ from the QCD sum rules [46].

In Fig. 3, we show our results for the differential width of $B \to D\ell\nu_\ell$ $(\ell = e,\ \mu,\ \tau)$ decay obtained from both linear (black lines) and HO (blue lines) parameters. The solid lines represent our results for the light (e, μ) decay modes compared with the experimental data from Belle [42]. The dashed lines represent our results for the semitauonic $B \to D\tau\nu_\tau$ decay. We summarize our LFQM predictions on the branching ratios for $B \to D\ell\nu_\ell$ decays obtained from both linear and HO potential parameters in Table IV and compare ours with the results from PDG [40] and other theoretical predictions such as LCSR [9] and heavy quark effective theory [47]. For the numerical calculations of the branching ratios, we use the CKM matrix element $|V_{cb}| = (40.5 \pm 1.5) \times 10^{-3}$, the PDG values [40] of the lepton (e, μ, τ) and hadron (B, D) masses together with the lifetimes of $(B^0, B^\pm)$. As one can see from Table IV, our

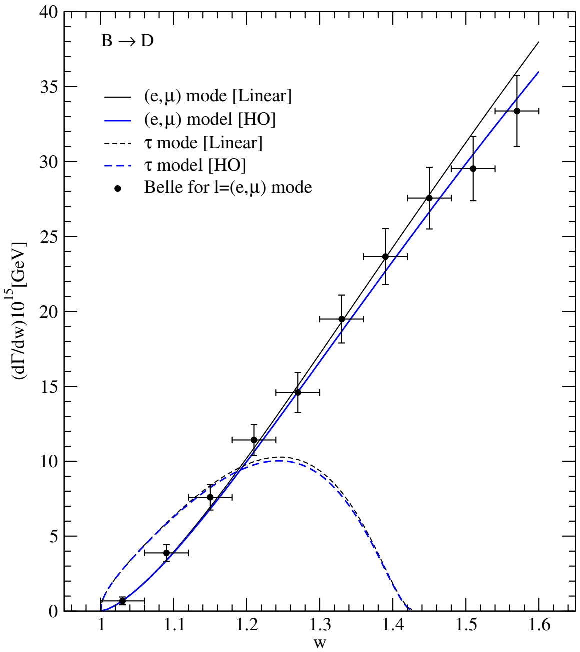

FIG. 3. Differential decay width of $B \to D\ell\nu_\ell$ ($\ell = e,\ \mu,\ \tau$) compared with the experimental data [42] measured from the light leptonic decay mode.

results obtained from the linear parameters are slightly larger than those obtained from the HO parameters. Our predictions for three decay modes such as $B^0 \to D^-\ell'\nu_{\ell'}$, $B^+ \to \bar{D}^0\ell'\nu_{\ell'}$, and $B^+ \to \bar{D}^0\tau\nu_\tau$ also agree with other theoretical results [9,47] as well as PDG values [40] within the errors. For the semitauonic $B^0 \to D^-\tau\nu_\tau$ decay, while three theoretical predictions agree with each other, those theoretical predictions are smaller than the data from PDG. From the results given in Table IV, our predictions for the ratio $\mathcal{R}(D) = \frac{\mathrm{Br}(B\to D\tau\nu_\tau)}{\mathrm{Br}(B\to D\ell'\nu_{\ell'})}$ ($\ell' = e, \mu$) are as follows

$$\mathcal{R}(D) = 0.284^{+0.046}_{-0.039}[0.286^{+0.046}_{-0.040}] \qquad (35)$$

for the linear [HO] potential parameters. Our predictions for the ratio $\mathcal{R}(D)$ are consistent with other theoretical predictions such as 0.300(8) [5] and 0.299(11) [6] from the lattice QCD and $0.320^{+0.018}_{-0.021}$ [9] within the errors. While our results are quite smaller than the experimental values, $\mathcal{R}^{\exp}(D) = 0.440(58)(42)$ from *BABAR* [1,2] and $\mathcal{R}^{\exp}(D) = 0.375(64)(26)$ from Belle [3], we also take note of a new preliminary result $\mathcal{R}^{\exp}(D) = 0.307(37)(16)$ [48] reported from the Belle collaboration, which is consistent with the SM at the 1.2σ level.

V. SUMMARY AND DISCUSSION

In this work, we discussed the self-consistent description on the decay constant $f_\mathcal{P}$ of a pseudoscalar ($\mathcal{P}$) meson and the weak form factors f_+ and f_- (or f_0) for the exclusive semileptonic $B \to D\ell\nu_\ell$ ($\ell = e,\ \mu,\ \tau$) decays in the standard LFQM. It has been a common perception in the LF formulation that while the plus component (J^+) of the LF current J^μ in the matrix element can be regarded as the "good" current, the perpendicular ($\mathbf{J}_\perp$) and the minus (J^-) components of the current were known as the "bad" currents since ($\mathbf{J}_\perp, J^-$) are easily contaminated by the treacherous points such as the LF zero mode and the off-mass shell instantaneous contributions.

To scrutinize such treacherous points when the usage of $\mathbf{J}_\perp$ or J^- is unavoidable, we employed the exactly solvable manifestly covariant BS model using the multipole type of $q\bar{q}$ bound state vertex function. Carrying out the LF calculations for f_P and $f_\pm(q^2)$ in the BS model, we found that $f_\mathcal{P}$ and $f_-(q^2)$ obtained from the so-called bad components of the current receive the zero-mode contributions as well as the instantaneous ones. We then linked the covariant BS model to the standard LFQM following the same universal correspondence Eq. (13) between the two models that we found in our previous analysis of the twist-2 and twist-3 DAs of pseudoscalar and vector mesons [21–23] and replaced the LF vertex function in the BS model with the more phenomenologically accessible Gaussian wave function provided by the LFQM analysis of meson mass spectra [33,34]. As in the previous analysis [21–23], it is striking to observe that the zero mode and the instantaneous contribution present in the BS model become absent in the LFQM. In other words, our LFQM results of the decay constant $f_\mathcal{P}$ and the TFFs $f_\pm(q^2)$ are shown to be independent of the components of the current without involving any of those treacherous contributions.

We then apply our current independent form factors $f_\pm(q^2)$ for the self-consistent analysis of $B \to D\ell\nu_\ell$ ($\ell = e,\ \mu,\ \tau$) decay using our LFQM constrained by the variational principle for the QCD-motivated effective Hamiltonian with the linear (or HO) plus Coulomb interaction [19,33–35]. The form factors $f_\pm(q^2)$ are obtained in

TABLE IV. Our LFQM predictions on the branching ratios (in %) for $B \to D\ell\nu_\ell$ ($\ell = e, \mu, \tau$) decays compared with the results from other theoretical predictions [9,47] and PDG [40]. $\ell' = e,\ \mu$.

Channel	Linear	HO	LCSR [9]	HQET [47]	PDG [40]
$B^0 \to D^-\ell'\nu_{\ell'}$	2.34 ± 0.18	2.25 ± 0.17	$2.086^{+0.230}_{-0.232}$	–	2.19 ± 0.12
$B^0 \to D^-\tau\nu_\tau$	0.66 ± 0.05	0.64 ± 0.05	$0.666^{+0.058}_{-0.057}$	0.64 ± 0.05	1.03 ± 0.22
$B^+ \to \bar{D}^0\ell'\nu_{\ell'}$	2.53 ± 0.19	2.44 ± 0.19	$2.260^{+0.249}_{-0.251}$	–	2.27 ± 0.11
$B^+ \to \bar{D}^0\tau\nu_\tau$	0.72 ± 0.05	0.70 ± 0.05	$0.724^{+0.063}_{-0.062}$	0.66 ± 0.05	0.77 ± 0.25

the $q^+ = 0$ frame ($q^2 = -\mathbf{q}_\perp^2 < 0$) and then analytically continued to the timelike region by changing $\mathbf{q}_\perp^2$ to $-q^2$ in the form factors. We obtain $\mathrm{Br}(B \to D\ell\nu_\ell)$ for both neutral and charged B mesons and compare with the experimental data as well as other theoretical model predictions. Our results for $\mathrm{Br}(B \to D\ell\nu_\ell)$ show reasonable agreement with the data except for the semitauonic $B^0 \to D^-\tau\nu_\tau$ decay. Our results for the ratio $\mathcal{R}(D)$ are consistent with other theoretical predictions as well as the new preliminary result from the Belle collaboration [48] although the previous data from *BABAR* [1,2] and Belle [3] show quite larger values than our predictions.

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APPENDIX: EXPLICIT FORMS FOR $f_+(q^2)$ AND $f_-(q^2)$

The on shell contributions of the trace terms in Eqs. (26)–(28) are given by

$$\begin{aligned}
\mathcal{T}^+_{\rm on} &= \frac{4P_1^+}{\bar{x}}(\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + \mathcal{A}_1\mathcal{A}_2),\\
\mathcal{T}^\perp_{\rm on} &= \frac{-2\mathbf{k}_\perp}{x\bar{x}}[2\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + \bar{x}(\mathbf{q}_\perp^2 + m_1^2 + m_2^2) + 2x^2 m_q^2\\
&\quad + 2x\bar{x}(m_1 m_q + m_2 m_q - m_1 m_2)] - \frac{2\mathbf{q}_\perp}{x\bar{x}}(\mathbf{k}_\perp^2 + \mathcal{A}_1^2),\\
\mathcal{T}^-_{\rm on} &= \frac{4}{x^2\bar{x}P^+}[\bar{x}(m_1\mathcal{A}_1 + \mathbf{k}_\perp^2)[m_2^2 + (\mathbf{k}_\perp + \mathbf{q}_\perp)^2]\\
&\quad + x^2\bar{x}M_0^2(\mathbf{k}_\perp^2 + \mathbf{k}_\perp\cdot\mathbf{q}_\perp) + x^2 m_1 m_2(m_q^2 + \mathbf{k}_\perp^2)\\
&\quad + x\bar{x}m_2 m_q(m_1^2 + \mathbf{k}_\perp^2)],
\end{aligned}\tag{A1}$$

where $\bar{x} = 1 - x$. The final standard LFQM results for $f_+(q^2)$ and $f_-(q^2)$ are given by

$$[f_+^{(+)}]^{\rm SLF}_{\rm on} = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\frac{\phi_1(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2+\mathbf{k}_\perp^2}}\frac{\phi_2(x,\mathbf{k}'_\perp)}{\sqrt{\mathcal{A}_2^2+\mathbf{k}'^2_\perp}} \times(\mathcal{A}_1\mathcal{A}_2 + \mathbf{k}_\perp\cdot\mathbf{k}'_\perp),\tag{A2}$$

$$\begin{aligned}
[f_-^{(\perp)}]^{\rm SLF}_{\rm on} &= \int_0^1 \bar{x}dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\frac{\phi_1(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2+\mathbf{k}_\perp^2}}\frac{\phi_2(x,\mathbf{k}'_\perp)}{\sqrt{\mathcal{A}_2^2+\mathbf{k}'^2_\perp}}\\
&\quad\times\Big[-\bar{x}M_0^2 + (m_2 - m_q)\mathcal{A}_1 - m_q(m_1 - m_q)\\
&\quad + \frac{\mathbf{k}_\perp\cdot\mathbf{q}_\perp}{q^2}[M_0^2 + M'^2_0 - 2(m_1 - m_q)(m_2 - m_q)]\Big],
\end{aligned}\tag{A3}$$

and

$$\begin{aligned}
[f_-^{(-)}]^{\rm SLF}_{\rm on} &= \int_0^1 \frac{dx}{x^2} \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\frac{\phi_1(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2+\mathbf{k}_\perp^2}}\frac{\phi_2(x,\mathbf{k}'_\perp)}{\sqrt{\mathcal{A}_2^2+\mathbf{k}'^2_\perp}}\\
&\quad\times\{a_0[x^2\bar{x}M_0^2(\mathbf{k}_\perp^2 + \mathbf{k}_\perp\cdot\mathbf{q}_\perp)\\
&\quad + \bar{x}(m_1\mathcal{A}_1 + \mathbf{k}_\perp^2)[m_2^2 + (\mathbf{k}_\perp + \mathbf{q}_\perp)^2]\\
&\quad + x^2 m_1 m_2(m_q^2 + \mathbf{k}_\perp^2) + x\bar{x}m_2 m_q(m_1^2 + \mathbf{k}_\perp^2)]\\
&\quad - x^2 b_0(\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + \mathcal{A}_1\mathcal{A}_2)\},
\end{aligned}\tag{A4}$$

where $a_0 = \frac{2}{M_0^2 - M'^2_0 - \mathbf{q}_\perp^2}$ and $b_0 = \frac{M_0^2 + M'^2_0 + \mathbf{q}_\perp^2}{M_0^2 - M'^2_0 - \mathbf{q}_\perp^2}$.

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