Eddington gravity with matter: An emergent perspective

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We describe an action principle, within the framework of the Eddington gravity, which incorporates the matter fields in a simple manner. Interestingly, the gravitational field equations derived from this action is identical to Einstein's equations, in contrast with the earlier attempts in the literature. The cosmological constant arises as an integration constant in this approach. In fact, the derivation of the field equations *demands* the existence of a nonzero cosmological constant, thereby providing the *raison d'être* for a nonzero cosmological constant, implied by the current observations. Several features of our approach strongly support the paradigm that gravity is an emergent phenomenon and, in this perspective, our action principle could have a possible origin in the microstructure of the spacetime. We also discuss several extensions of the action principle, including the one which can incorporate torsion into the spacetime. We also show that an Eddington-like action can be constructed to obtain the field equations of the Lanczos-Lovelock gravity.

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I. INTRODUCTION AND MOTIVATION

The usual starting point for obtaining the gravitational field equations in general relativity is the Hilbert action, with the metric treated as the dynamical variable. Even though it is possible to derive Einstein's equations from the variation of this action (with respect to the metric), it is not completely straightforward because the variational problem is ill-posed. This is because of the presence of second derivatives of the metric in the Ricci scalar, which is the Lagrangian for the Hilbert action [1–5]. The well-posed version of the Hilbert action can only be obtained by adding suitable boundary terms to the Hilbert action, which crucially depend on the choice of the boundary surface [6–12]. Though these boundary terms do not affect Einstein's equations, they have important thermodynamical as well as geometrical implications [13–15].

Such complications, associated with the Hilbert action, provide one possible motivation for an alternative proposal, originally suggested by Eddington [16,17]. In this approach, one takes the gravitational Lagrangian to be $\sqrt{\det(R_{ab})}$, where R_{ab} is the Ricci tensor. Since the Ricci tensor R_{ab} [in the (0,2) form] can be constructed solely from the connection Γ^p_{qr}, we can consider the *connection* to be the dynamical variable and vary it in the Eddington action. Interestingly, the outcomes are the Einstein equations with a *nonzero* cosmological constant. Thus Eddington's action is a viable alternative to the Hilbert action and, as we will describe below, is also well-posed.

There is, however, a major issue with the Eddington proposal, viz. that it does not include the matter degrees of freedom. Surprisingly, the inclusion of matter in the Eddington framework has turned out to be not very straightforward. Most of the proposals in the literature are motivated by the Born-Infeld-like structure [18–26]. Broadly speaking, this requires the action to be dependent on the connection Γ^p_{qr}, the matter degrees of freedom Ψ and the metric g_{ab}, though the connection and the metric are considered independent. Setting the variation of the action with respect to the metric to zero, i.e., $(\delta\mathcal{A}[\Gamma, g, \Psi]/\delta g = 0)$, one obtains $g = g(\Gamma, \Psi)$. This result, when substituted back to the action, yields the on-shell action, $\mathcal{A}[\Gamma, \Psi]$, which is a function of the connection and the matter fields alone. The final step is the variation of this action with respect to the connection which yields the desired gravitational field equations. However, none of these attempts, as far as we know, yields the Einstein equations; rather, they lead to additional corrections [21,27–31].

In this paper, we will discuss a completely new approach to deriving the Einstein equations in the spirit of Eddington gravity. Most importantly, we will construct an action principle such that its variation, with respect to the connection, leads precisely to the Einstein equations *without* any additional corrections.

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In addition, as we will demonstrate, our variational principle will be well-posed.[1]

The paper is organized as follows: In Sec. II, the modified action for the gravity (incorporating the matter field) along the lines of Eddington gravity is presented. Its variation, leading to the gravitational field equation, is described in Sec. III. Finally, the relationship of this action with the microstructure of the spacetime is briefly discussed in Sec. IV.

Notations and Conventions: We will assume $c = 1 = \hbar$ and use the mostly positive signature convention. We will work in d-dimensional spacetime except when specified otherwise. Latin sub/superscripts run over all the spacetime indices.

II. ACTION FOR EDDINGTON GRAVITY WITH MATTER

In this section, we will introduce our action principle for gravity plus matter and describe some of its key features. We will vary this action and derive the field equations in the next section.

Motivated by the original form of the Lagrangian associated with the Eddington gravity, we propose the following action, in $d > 2$ spacetime dimensions, to describe gravity coupled to matter:

$$\mathcal{A} = \int d^d x \sqrt{|\det\,(R_{(ab)}(\Gamma) - \kappa \bar{T}_{ab})|};$$
$$\bar{T}_{ab} = T_{ab} - \frac{1}{(d-2)} T g_{ab}, \tag{1}$$

where $\kappa = 8\pi G$, with G being the Newton gravitational constant. There are several features of this action which are worth emphasizing.

(i) The action given above (with an integration measure $d^d x$ rather than $\sqrt{|\det g|} d^d x$) is indeed a scalar, since the determinant of any second rank tensor field transforms identically to $\sqrt{|\det g|}$, thereby making the action a generally covariant scalar. The action is also dimensionless. These features, of course, should be obvious to those familiar with the standard Eddington gravity, which is obtainable from Eq. (1) by setting $T_{ab} = 0$.

(ii) In the above action, $R_{(ab)}(\Gamma) \equiv (1/2)[R_{ab}(\Gamma) + R_{ba}(\Gamma)]$ is the symmetric part of the Ricci tensor, which is constructed solely from the connection Γ^a_{bc}. Thus the gravitational sector is independent of the metric and depends only on the connection. For simplicity in the main discussion, we will assume that the connection is symmetric, i.e., $\Gamma^a_{bc} = \Gamma^a_{cb}$. This assumption can be relaxed—and torsion can be included through the antisymmetric part of the connection—rather easily, as demonstrated in Appendix C of [33].

(iii) Since we are not assuming any *a priori* relation between Γ^a_{bc} with the metric g_{cd}, it follows that the Ricci tensor need not be symmetric. This is due to the term $\partial_a \Gamma^c_{bc}$ in the Ricci tensor, which is not symmetric in (a, b) unless Γ^a_{bc} is given by the Christoffel symbol corresponding to a metric. Therefore, we have constructed the action out of the symmetric part of the Ricci tensor.

(iv) The T_{ab} is the matter energy-momentum tensor, whose trace is denoted by T. This can—and indeed it does—depend on the metric. However, we will assume that T_{ab} is independent of the connection; this criterion is satisfied by almost all the matter stress tensors we will be interested in. As already stated, the connection and the metric are treated as independent variables at this stage, somewhat in the spirit of the Palatini formulation in Einstein's standard theory (the assumption that the matter sector does not explicitly depend on the connection is an assumption usually made in the standard Palatini approach, as well).

(v) The most significant departure of the action principle proposed above from those in the previous literature is in the treatment of the matter degrees of freedom. Instead of working with a separate matter Lagrangian, we will work with the matter energy-momentum tensor T_{ab} itself. Though it may appear somewhat surprising at first sight, everything will work out satisfactorily due to the following two facts. First, given a Lagrangian for the matter field, the energy-momentum tensor can be uniquely determined and hence there is a clear correspondence between the two. Second, as we shall show, the gravitational field equations will lead to $\nabla_a T^a_b = 0$ (as in Einstein's standard theory) from which one can derive the equations of motion for the matter field. To reiterate, variation of the above action with respect to the connection Γ^a_{bc} will yield the gravitational field equations, which, as we will demonstrate in the next section, will be identical to the Einstein equations sourced by T_{ab} (and a cosmological constant). Then Bianchi identity will yield the field equations for the matter field through $\nabla_a T^a_b = 0$ (we will comment again on this aspect later, after the derivation of the field equations have been presented).

We conclude this section with some brief comments on the relation between a second rank tensor and a matrix.

[1]Another key motivation for this approach is the following. The action we will be using here is closely related to an effective action in the emergent gravity paradigm, when we integrate out certain microscopic degrees of freedom of spacetime in a path integral. We will not pursue this idea in this paper—except for a brief comment in the last section—but will discuss it in a separate work [32].

Given any second rank tensor S_{ab} (equivalently S^a_b, or, S^{ab}), one can construct a matrix $\mathcal{M}^a_b$, such that the ath row and the bth column of the matrix coincides with the (a,b)th element of the tensor [given in (0,2), (1,1) or (2,0) form]. The determinant of the tensor is then *defined* as the determinant of the matrix to which its components are mapped.[2] This is precisely what we do while computing the determinant of the metric tensor g_{ab} in general relativity; the determinant in Eq. (1) is computed exactly as we compute the determinant of the metric tensor g_{ab} in general relativity [once again, this should be clear to those who are familiar with standard Eddington gravity, which is obtainable from Eq. (1) on setting $T_{ab}=0$].

III. VARIATION OF THE ACTION AND THE GRAVITATIONAL FIELD EQUATIONS

We will now vary the action in Eq. (1) for arbitrary variation of the connection and shall obtain the gravitational field equations. For this purpose, it will be convenient to define the tensor $M_{ab}\equiv R_{(ab)}-\kappa\bar{T}_{ab}$, which, by construction, is symmetric. As mentioned earlier, the definition of the determinant appearing in Eq. (1) requires us to map the (components of) tensor M_{ab} to (the elements of) a matrix $\mathcal{M}^a{}_b$. Given the matrix $\mathcal{M}^a{}_b$, one can define the inverse matrix $\mathcal{N}^a{}_b$, such that $\mathcal{M}^a{}_b\mathcal{N}^b{}_c=\delta^a_c=\mathcal{N}^a{}_b\mathcal{M}^b{}_c$. Again, one can map the inverse matrix $\mathcal{N}^a{}_b$ back to a tensor N^{ab}, such that $N^{ab}M_{bc}=\delta^a_c=M_{cb}N^{ba}$ (for some pedagogical details, see Appendix A of [33]). This tensor N^{ab} will be useful in the ensuing analysis.

The variation of our action in Eq. (1), under arbitrary variation of the symmetric connection Γ^a_{bc}, leads to:

$$\begin{aligned}\delta\mathcal{A}&=\int d^dx\frac{1}{2\sqrt{|\det(\mathcal{M})|}}|\det(\mathcal{M})|\times\mathcal{N}^a{}_b\delta\mathcal{M}^b{}_a\\&=\frac{1}{2}\int d^dx\sqrt{|\det(\mathcal{M})|}N^{ab}\delta M_{ba},\end{aligned}\tag{2}$$

where $\det(\mathcal{M})$ denotes the determinant of the matrix $\mathcal{M}^a_b$. Since the matter energy-momentum tensor $\bar{T}_{ab}$ is independent of the connection, the variation of the tensor M_{ab} (due to an arbitrary variation of the connection) will arise only from the Ricci term. The variation of the Ricci tensor, due to variation of the connection, is given by:

$$\delta R_{(ab)}=\nabla_c\delta\Gamma^c_{ab}-\nabla_{(a}\delta\Gamma^c_{b)c},\tag{3}$$

so the variation of the action $\mathcal{A}$ becomes

$$\begin{aligned}\delta\mathcal{A}&=\frac{1}{2}\int d^dx\sqrt{|\det(M)|}N^{ba}(\nabla_c\delta\Gamma^c_{ab}-\nabla_a\delta\Gamma^c_{bc})\\&=\frac{1}{2}\int d^dx\sqrt{|\det(M)|}\delta^{cd}_{ab}N^{pa}\nabla_d\delta\Gamma^b_{cp};\\\delta^{ab}_{cd}&\equiv\delta^a_c\delta^b_d-\delta^a_d\delta^b_c,\end{aligned}\tag{4}$$

where we have used the fact that N^{ab} is symmetric. In deriving the above variation of the action we have adopted the usual convention of writing the determinant of a matrix, $\det(\mathcal{M})$, as the determinant of the tensor, $\det(M)$, since no confusion is likely to arise in the subsequent discussion.

The above variation can be simplified further. As a first step, we will rewrite the above expression by separating out a total derivative term:

$$\begin{aligned}\delta\mathcal{A}=&\frac{1}{2}\int d^dx\,\nabla_d[\sqrt{|\det(M)|}\delta^{cd}_{ab}N^{pa}\delta\Gamma^b_{cp}]\\&-\frac{1}{2}\int d^dx\,\nabla_d[\sqrt{|\det(M)|}\,\delta^{cd}_{ab}N^{pa}]\delta\Gamma^b_{cp}.\end{aligned}\tag{5}$$

We next want to convert the total divergence term to a surface term, as is usually done, so that it will vanish with the usual boundary conditions, viz. $\delta\Gamma^b_{cp}=0$ at the boundary. This is, of course, trivial if the expression had a $\sqrt{|\det(g)|}$ in place of a $\sqrt{|\det(M)|}$ in the first integrand. Due to the presence of the $\sqrt{|\det(M)|}$ factor, it may appear that such a conversion of the total divergence term to a boundary term will not be possible in the present context. Fortunately, it turns out that one can indeed convert the first term in Eq. (5) into a surface term because the following identity holds (see Appendix B of [33] for a derivation):

$$\nabla_c[\sqrt{|\det(M)|}V^c]=\partial_c[\sqrt{|\det(M)|}V^c],\tag{6}$$

for any vector field V^c and second rank tensor field M_{ab}, whose determinant is $\det(M)$. Using this result, the variation of the action in Eq. (5) becomes

$$\begin{aligned}\delta\mathcal{A}=&\frac{1}{2}\int d^dx\,\partial_d[\sqrt{|\det(M)|}\delta^{cd}_{ab}N^{pa}\,\delta\Gamma^b_{cp}]\\&-\frac{1}{2}\int d^dx\nabla_d[\sqrt{|\det(M)|}\,\delta^{cd}_{ab}N^{pa}]\delta\Gamma^b_{cp}.\end{aligned}\tag{7}$$

The first term provides the boundary contribution arising out of the action when it is varied with respect to the connection, while the second term will provide the gravitational field equations. Even though the boundary term will not contribute to the field equations, it is worth emphasizing a few points about the boundary contribution, as it is intimately connected with the question of whether the action principle is well-posed. Let us take the usual boundary of a four-dimensional volume, made out of two

[2]Some pedagogical subtleties in defining the determinant of an arbitrary second rank tensor, not adequately emphasized in the literature and textbooks, are discussed in Appendix A of [33].

constant time hypersurfaces, $t = t_1$, $t = t_2$, along with a timelike surface at spatial infinity. Then we will be fixing Γ^a_{bc} at both the $t = t_1$, $t = t_2$ hypersurfaces (and assuming that $\delta\Gamma^a_{bc}$ vanishes at spatial infinity). Thus, in the present context, the field equations must be of second order in Γ^a_{bc} for the variational problem to be well-posed. As evident from the second term of the above variation, the field equations depend on at most the second derivatives of the connection and hence the variational problem is indeed well-posed. This is unlike the *metric* variation of the Hilbert action, in which case not only the metric, *but also* its normal derivatives need to be fixed at the boundaries. Thus the action for Eddington gravity will not require any additional boundary terms, in sharp contrast to the Hilbert action.

Neglecting the boundary contribution and setting $\delta\mathcal{A} = 0$ for arbitrary variation of the connection in the bulk, we obtain the field equations to be:

$$\nabla_d[\sqrt{|\det(M)|}\,\delta^{cd}_{ab}N^{pa}] + \nabla_d[\sqrt{|\det(M)|}\,\delta^{pd}_{ab}N^{ca}] = 2\nabla_b[\sqrt{|\det(M)|}\,N^{pc}] - \delta^c_b\nabla_d[\sqrt{|\det(M)|}\,N^{pd}] - \delta^p_b\nabla_d[\sqrt{|\det(M)|}\,N^{cd}] = 0. \tag{8}$$

In writing the first line, we have taken care of the fact that $\delta\Gamma^b_{cp}$ is symmetric in c, p. This equation can also be simplified further, by multiplying both sides using δ^b_c, from which we immediately obtain $\nabla_a[\sqrt{|\det(M)|}\,N^{ba}] = 0$. Substituting this expression back in Eq. (8), we finally obtain

$$\nabla_c[\sqrt{|\det(M)|}\,N^{ab}] = 0. \tag{9}$$

This provides the gravitational field equations arising out of the connection Γ^a_{bc} and is indeed a second order differential equation in Γ^a_{bc}, leading to a well-posed boundary value problem, as mentioned before.

If we set $T_{ab} = 0$, so that $M_{ab} = R_{(ab)}$, this equation reduces to the one we would have obtained in the context of standard Eddington gravity. In the current context, with the presence of matter, this equation has the same structure as that of Eddington gravity, with $R_{(ab)}$ replaced by M_{ab}. We can proceed exactly as in the case of in Eddington gravity, as should be obvious to those familiar with the standard Eddington gravity analysis. Nonetheless, we will spell out the relevant algebraic details below.

The field equation arising from the variation of the connection, given by Eq. (9), contains second derivatives of the connection. However, as in standard Eddington gravity, this equation can be immediately integrated to give the first integral (which will involve only the first derivatives of the connection). To do this we only have to note that Eq. (9) requires us to find a second rank symmetric tensor density which has a vanishing covariant derivative with respect to the connection Γ^a_{bc}, which is used to define the derivative operator ∇_c. Because N^{ab} and M_{ab} are inverses of each other, Eq. (9) requires $\nabla_c M_{ab} = 0$. If we expand out the covariant derivative in this equation, we can express Γ^a_{bc} in terms of N^{ab} and the derivatives of M_{ab}, exactly as we would relate the metric to the connection using the condition $\nabla_c g_{ab} = 0$. Therefore, when we set $M_{ab} \propto g_{ab}$ the connection Γ^a_{bc} and the metric will be related in the standard manner (that is, the connection used in ∇_c will be the one compatible with the metric). This will also, in turn, make $N^{ab} \propto g^{ab}$ and $\det(M) \propto \det(g)$. With this choice, Eq. (9) reduces to $\nabla_c[\sqrt{|\det(g)|}g^{ab}] = 0$, which is just the standard compatibility condition between the metric and Γ^a_{bc} used to construct the connection, thereby closing the logical loop (this is exactly the same as what is done in standard Eddington gravity, where $T_{ab} = 0$).

Before proceeding further, we will mention one subtlety in the above argument in the current context, where $T_{ab} \neq 0$. This is related to an interesting and hidden role played by the principle of equivalence in the presence of matter. Suppose we introduce *some* metric q_{ab} from which the connection Γ^a_{bc} can be obtained in the standard manner. Then the first integral to Eq. (9) is indeed given by:

$$\sqrt{|\det(M)|}N^{ab} \propto \sqrt{|\det(q)|}q^{ab}, \tag{10}$$

where q^{ab} is the inverse of the metric tensor q_{ab}. At this stage, *formally*, we actually have *two* metric tensors in play: g_{ab}, which could occur in the matter sector of the action (through $\bar{T}_{ab}$) and the tensor q_{ab}, which is introduced as the one compatible with the connection Γ^c_{ab}, and arises in the gravitational sector through Eq. (10). The principle of equivalence, however, requires us to identify these two metric tensors (i.e., set $q_{ab} = g_{ab}$) and make Γ^a_{bc} the Christoffel symbol associated with either of them. To see this, note that the principle of equivalence allows us to choose a coordinate system around any event $\mathcal{P}$ such that the local physics reduces to that of special relativity and all gravitational effects vanish to first order. This, in turn, is possible only if we can choose a coordinate system such that the metric reduces to the Minkowski form (η_{ab}) at $\mathcal{P}$ and the Christoffel symbols derived from the metric vanish at $\mathcal{P}$. Such a choice of coordinate system is clearly not possible if there are *two* nontrivially different metrics g_{ab} and q_{ab} (as well as their corresponding connections). A single coordinate's transformation will not be able to reduce two nontrivially different metrics into a locally flat form simultaneously. Since we want *both* the geometrical effects governed by q_{ab} and the behavior of matter governed by g_{ab} to reduce to special relativistic form in the *same* freely falling frame, it is necessary that we identify $g_{ab} = q_{ab}$ (a more general class of theories, called bimetric theories of gravity, is possible if we relax this condition but we will not be concerned with such generalizations in this work).

Returning to the main discussion, the first integral to Eq. (10) leads to the identification $M_{ab} \propto g_{ab} \equiv \lambda g_{ab}$ where λ is an integration constant. The introduction of the metric, compatible with the connection, also makes the Ricci tensor symmetric $R_{ab} = R_{ba}$ so that the equation $M_{ab} = \lambda g_{ab}$ leads to

$$R_{ab} - \kappa \bar{T}_{ab} = \lambda g_{ab}. \tag{11}$$

Taking the trace of this equation, we obtain, on the left hand side, $R + (2/(d-2))\kappa T$ and on the right hand side λd, thereby yielding $R = \lambda d - (2/(d-2))\kappa T$. So, with some simple algebra, Eq. (11) can be rewritten in terms of the Einstein tensor $G_{ab} \equiv R_{ab} - (1/2) R g_{ab}$ as:

$$G_{ab} + \left(\frac{d-2}{2}\right)\lambda g_{ab} \equiv G_{ab} + \Lambda g_{ab} = \kappa T_{ab}, \tag{12}$$

where $\Lambda \equiv [(d/2) - 1]\lambda$ is the d-dimensional cosmological constant. So we have obtained precisely the Einstein equations with a cosmological constant term. In contrast to other approaches to include matter in the Eddington gravity, where the gravitational field equations themselves are modified, while in the present approach we obtain the exact Einstein's equations.

A. Comments on the result

We will now make several key comments about our result.

(a) The role played by the cosmological constant in the above derivation is note-worthy. In sharp contrast to the standard derivation of the Einstein equations from the Hilbert action, our approach *demands* the existence of a nonzero cosmological constant. In order to arrive at the above field equation we *must* have $\Lambda \neq 0$, i.e., a nonzero cosmological constant is absolutely necessary if the field equation arising out of the Eddington gravity is to make any sense. This is, of course, gratifying because in the usual approaches, the cosmological constant is an "optional" parameter; you can set it to a zero value or to a nonzero value, as desired. Within such an approach, the observational facts, indicating the existence of a nonzero cosmological constant, have no fundamental explanation. That is, the standard derivation of the Einstein equation from the Hilbert action goes through without any hitch even if the cosmological constant is zero. But in our approach, the derivation of the field equations *demands a nonzero cosmological constant*.

(b) The cosmological constant arises in the first integral to the equations of motion in the form of an integration constant. It has been stressed in the previous literature that this is indeed the only way the cosmological constant problem can be addressed (see e.g., [15,34]). This is reminiscent of the manner in which the cosmological constant arises in the emergent gravity paradigm (see e.g., [14]). Just as in these approaches, its numerical value has to be fixed using some other general principle (see e.g., [35]) since it is an integration constant. We will say more about the connection between this approach and emergent gravity paradigm in the last section.

(c) We never varied the metric in the action principle and only needed to vary the connection. The resulting equations of motion, Eq. (9), involves second derivatives of the connection, but it was trivial to find the first integral to this equation of the form $M_{ab} = \lambda g_{ab}$ which involves only the first derivatives of the connection and hence the second derivatives of the metric. All of these are true even in the standard approach to Eddington gravity. The only modification is that, in the absence of matter, the action in Eq. (1) is independent of the metric, while our action has a possible dependence on the metric though the energy-momentum tensor T_{ab}. The variational principle treats the connection and the metric as independent variables and we have *only* varied the connection. Of course, just because some quantity appears in the action does *not* mean that we must vary it. More formally, the complete specification of *any* variational principle has three ingredients: (i) The form of the action functional, (ii) The entities which are varied, (iii) The nature of the variation and the boundary conditions. We have complete liberty to prescribe these; as long as we choose a consistent set of these three ingredients we do have a valid variational problem, as in our case.

(d) Closely related to the above comment is the *dual* role played by the field equations, as in Eq. (9). Its first integral, given by $M_{ab} = \lambda g_{ab}$, achieves *two* things: First, this choice transforms Eq. (9) to read as $\nabla_c(\sqrt{|\det g|} g^{ab}) = 0$, which is the standard compatibility condition between the metric and the connection. We obtain this without having to vary the metric, unlike, say, in the standard Palatini approach (this happens in the usual Eddington gravity without matter as well). Second, using $M_{ab} = R_{ab} - \kappa T_{ab}$, we obtain the Einstein equations in the form $R_{ab} - \kappa T_{ab} = \lambda g_{ab}$. The original equation, i.e., Eq. (9) (which is second order in the connection), would suggest that the connection is the dynamical variable of the theory but once we obtain the first integral, $R_{ab} - \kappa T_{ab} = \lambda g_{ab}$ (which is first order in the connection but second order in the metric) we see that the metric acquires a status similar to the dynamical variable, even though we never needed to vary the metric in the action principle. So the metric becomes, in the Wheelerian language, a "dynamical variable without (being) a dynamical variable." This is not only consistent with the emergent gravity paradigm but even strongly suggests thinking of the metric as an emergent variable.

(e) The Bianchi identity immediately gives $\nabla_a T^a_b = 0$, which is a consistency condition on T^a_b, which is used in the action. In addition, this will lead to the equations of motion for the matter field without having to vary the matter variables separately ("spacetime tells matter how to move"). This is perfectly adequate in a completely classical theory, in which the action is just a tool to get the equations of motion incorporating the relevant symmetries in the most economical way. However, there could be some contexts, like in the study of quantum fields in a fixed curved geometry, in which we would like to get the equations of motion for the matter sector from variation of matter variables in the total action. The same issue arises in the emergent gravity paradigm as well (see the discussion in the third paragraph after Eq. (40) in [36]), and can be taken care of by the following prescription. The total action is taken to be the sum of $L_{\rm matter}$ and the action in Eq. (1). The connection is varied at first and the solution to the gravitational field equations is substituted into the action to obtain the on-shell action as far as gravity is concerned. The matter degrees of freedom are then varied in this on-shell action to obtain matter equations of motion; one can also perform a path integral over matter variables in the on-shell action to do standard quantum field theory in curved spacetime. Note that the gravitational part of the on-shell action will be the one obtained by replacing $\det\,(R_{(ab)} - \kappa\bar{T}_{ab})$ by $\lambda^d \det\,(g)$, which is devoid of any matter degrees of freedom. Thus the new on-shell action will involve only the matter Lagrangian $L_{\rm matter}$ plus an additional metric dependent term, proportional to $\lambda^{d/2}$. Therefore, the variation of the matter degrees of freedom is identical to the variation of the matter Lagrangian $L_{\rm matter}$ in a given curved spacetime and will lead to the correct evolution equation for the matter fields.

(f) We next comment on the case of nonvanishing torsion within the context of the Eddington gravity. The first step, again, is to choose the appropriate Lagrangian and thus the action. Following the previous discussion, it seems legitimate to consider the Lagrangian to depend on the Ricci tensor constructed out of the symmetric Christoffel connection and the contorsion tensor, i.e., one may again consider the following Lagrangian $\sqrt{|\det\,(\bar{R}_{(ab)} - 8\pi G\bar{T}_{ab})|}$. Here, $\bar{R}_{ab}$ is the Ricci tensor which depends on the spacetime torsion as well and hence is not symmetric. But it turns out that the variation of the above action with respect to the symmetric Christoffel connection and the contorsion tensor does not yield appropriate expressions for the gravitational field equations. However, as shown in Appendix C of [33], a specific modification of the action does yield the correct gravitational field equations for the Einstein-Cartan theory. Of course, in the absence of any Fermionic or nonminimally coupled matter field, the torsion tensor will vanish identically on-shell and then the Einstein-Cartan gravitational field equations will reduce to the Einstein equations.

(g) Once we have broken free from the compulsion to vary the metric, it is possible to construct the Eddington-type action for Lanczos-Lovelock models (coupled to matter) as well (for a review of Lanczos-Lovelock models, see [37]). In this case, for the mth order Lanczos-Lovelock gravity, we take the action to be:

$$\mathcal{A} = \int d^dx\sqrt{|\det\,(\mathcal{R}^{(m)}_{(ab)} - \kappa\bar{T}^{(m)}_{ab})|};$$
$$\bar{T}^{(m)}_{ab} = T_{ab} - \frac{1}{(d-2m)}Tg_{ab}, \tag{13}$$

where

$$\mathcal{R}^{(m)}_{ab} = P_a^{pqr}R_{bpqr} = m\delta^{c_1d_1c_2d_2\cdots c_md_m}_{a_1b_1a_2b_2\cdots a_mb_m} \times (g^{b_2e_2}R^{a_2}{}_{e_2c_2d_2}\cdots g^{b_me_m}R^{a_m}{}_{e_mc_md_m}) \times \delta^{a_1}_a g^{b_1e_1}g_{bq}R^q{}_{e_1c_1d_1}. \tag{14}$$

Treating the metric and the connection as independent and varying the connection, somewhat lengthy algebra (see Appendix D of [33] for details) leads to the standard field equations for the Lanczos-Lovelock model:

$$\mathcal{R}^{(m)}_{(ab)} - \frac{1}{2}L_{(m)}g_{ab} + \Lambda g_{ab} = \kappa T_{ab}; \quad \Lambda = \left(\frac{d-2m}{2m}\right)\lambda. \tag{15}$$

Note that for $d = 2m$, i.e., in the critical dimension for the mth order pure Lovelock gravity, the effect from the cosmological constant term identically vanishes.

IV. DISCUSSION: EXTENSIONS AND THE BROADER PERSPECTIVE

We have explicitly demonstrated that there exists a first order formalism, in the same spirit as the Eddington gravity, which includes matter and reproduces the Einstein equations. The dynamical variable in the action is the connection, whose variation leads to the Einstein equations. This is in contrast with the other approaches in the literature to include matter in Eddington gravity, where the gravitational field equations are different from the Einstein equations. The variational principle proposed in this work is also well-posed, unlike in the case of the Hilbert action. This is because our action differs from the Hilbert action in two crucial respects: (a) The gravitational part of the action involves only the connection and has no reference to the metric, and (b) fixing the connection at the boundary turns out to be sufficient to render the variational principle well-posed.

Another remarkable feature of our analysis (which is common to the standard Eddington gravity to a certain extent) is the emergence of a cosmological constant naturally; more importantly, the cosmological constant *has to be* nonzero for the variational problem to make sense. This fact renders our action to be a *better* choice to derive the Einstein equations than the Hilbert action, since the latter does not demand a nonzero cosmological constant.

We will now mention several possible extensions of this action principle and their consequences.

To begin with, it is possible to construct a more general class of actions and still obtain the standard Einstein equations, along the lines of how we have proceeded. One such class of actions can be constructed as follows: Let $M_{ab} = L^2[R_{ab} - \kappa T_{ab}]$, where L is a constant length scale introduced for dimensional reasons (which does not affect the variation or the equations of motion) and let $X \equiv |\det M|/|\det g|$ be the ratio of the two determinants, which will transform as a scalar under coordinate transformations. We take the Lagrangian to be an *arbitrary* scalar function of X so that the (dimensionless) action for the gravity plus matter system is given by:

$$\mathcal{A} = \int \frac{d^d x}{L^d} \sqrt{|\det g|} f\left(\frac{|\det M|}{|\det g|}\right) = \int \frac{d^d x}{L^d} \sqrt{|\det g|} f(X). \tag{16}$$

The choice of $f(X) = X$ reduces this action to the one in Eq. (1). For other choices of $f(X)$, even the gravitational sector has a dependence in the metric. However, we treat the connection and the metric as independent and vary *only* the connection in the action. It is shown in Appendix E of [33] that the variation of the action in Eq. (16) also leads to Einstein's equations.

Second, let us consider the possible origin of the action in Eq. (1). The occurrence of a determinant in the Lagrangian is strongly suggestive of a path integral origin. To make this connection precise, consider the standard result of a Gaussian path integral in $d = 4$ Euclidean space, leading to an effective action:

$$\begin{aligned} &\int \mathcal{D}v^a \exp\left[-\int \frac{d^4 x}{L^4} \sqrt{|\det (g)|}\, v^a (L^2 M_{ab}) v^b\right] \\ &\quad \propto \exp\left[-\frac{1}{2}\int \frac{d^4 x}{L^4} \sqrt{|\det (g)|} \ln\left(|\det (L^2 M_{ab})|\right)\right] \\ &\quad \propto \exp\left(-\mathcal{A}_{\rm eff}[M_{ab}]\right), \end{aligned} \tag{17}$$

where v^a is a vector field that is integrated out and $M_{ab} = R_{(ab)}(\Gamma) - \kappa \bar{T}_{ab}$, is as defined earlier. Here $R_{(ab)}$ is the symmetric part of the Ricci tensor and is treated as a functional of the connection. The L is a constant length scale introduced purely for dimensional reasons (which will be of the order of Planck length in the emergent paradigm). The path integral thus gives rise to the following effective action:

$$\mathcal{A}_{\rm eff} = \frac{1}{2}\int \frac{d^4 x}{L^4} \sqrt{|\det (g)|} \ln\left(|\det [L^2 (R_{ab}(\Gamma) - \kappa \bar{T}_{ab})]|\right), \tag{18}$$

which is very similar to our action in Eq. (1) except for the logarithmic dependence. The variation of this action *also* leads to the Einstein equations, as shown in Appendix E of [33] [the action in Eq. (18) is equivalent to the one in Eq. (16) for the choice $f(X) = \ln X$, when we use the fact that the metric dependent terms are not varied in the action; the result follows from that for the class of actions in Eq. (16)]. The interpretation of this analysis and its connection with the microscopic degrees of freedom on null surfaces will be explored in a separate publication [32].

We conclude by pointing out a key broader implication of the results in this paper, including those of this section, for quantum gravity. As far as classical theories are concerned, it is only the equations of motion that are relevant. The action principle is more of an exercise in elegance and economy and, of course, is the simplest route to incorporate the expected symmetries of the theory. The situation, however, is quite different in quantum theory. In the path integral formalism, for example, it is important to know the form of the action principle as well as the status of dynamical variables. We have now shown that one can obtain Einstein's equations from different choices of action functionals and dynamical variables (recall that we only varied the connection and kept the metric frozen). All of them are equivalent at the classical level but their quantum versions will be quite different. It is conceivable that some of them will lead to a tractable model for quantum gravity, at least in the matter-free case.

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