Revised f_{NL} parameter in a curvaton scenario

Lei-Hua Liu,[1,*] Bin Liang,[2] Ya-Chen Zhou,[1] Xiao-Dan Liu,[1] Wu-Long Xu,[3] and Ai-Chen Li[3,4]

[1]*Department of Physics, College of Physics, Mechanical and Electrical Engineering, Jishou University, Jishou 416000, China*

[2]*College of Civil Engineering, Hunan University of Technology, Zhuzhou 412007, China*

[3]*Institute of Theoretical Physics, Beijing University of Technology, Beijing 100124, China*

[4]*Departamento de matematica da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal*

(Received 2 December 2020; accepted 2 February 2021; published 16 March 2021)

We revise the non-Gaussianity of canonical curvaton scenario with a generalized δN formalism, in which it can handle the generic potentials. In various curvaton models, the energy density is dominant in different periods including the secondary inflation of curvaton, matter domination, and radiation domination. Our method can unify to deal with these periods since the nonlinearity parameter f_{NL} associated with non-Gaussianity is a function of equation of state w. We first investigate the most simple curvaton scenario, namely, the chaotic curvaton with quadratic potential. Our study shows that most parameter spaces are satisfied with observational constraints. And our formula will nicely recover the well-known value of the f_{NL} parameter in the absence of nonlinear evolution. From the micro-origin of curvaton, we also investigate the pseudo-Nambu-Goldstone curvaton. Our result clearly indicates that the second short inflationary process for pseudo-Nambu-Goldstone curvaton is ruled out in light of these observations. Finally, our method sheds a new way of investigating the non-Gaussianity of curvaton mechanism, especially for exploring the non-Gaussianity in minimal supersymmetric Standard Model curvaton model.

DOI: 10.1103/PhysRevD.103.063515

I. INTRODUCTION

The traditional diagram of producing the curvature perturbation is sourced by the quantum fluctuations of inflationary field. In this broad class of single inflationary field theories, it experiences some initial condition problems associated with its corresponding potential. In order to relax the restrictions of single field inflation, one nice alternative called the curvaton mechanism was proposed [1–3], in which the energy density of the curvaton is subdominant compared with that of the inflaton during inflationary period. After inflation decay, the role of curvaton is more and more significant producing the isocurvature perturbation, which can be transferred into curvature perturbation seeding the temperature fluctuation on cosmological microwave background (CMB).

Due to the appearance of CMB, there are huge data waiting for the investigation. In particular, the most common method is calculating the power spectrum of the scalar field (driving the curvature perturbation) characterized by the two-point correlation function, its corresponding spectral index and tensor to scalar ratio. However, most data are still mysterious, thereby expecting a new theoretical method for exploring these treasures. Under this background, the calculation of non-Gaussianity (NG) identified with the three-point correlation function was proposed [4]. Combining with the curvaton scenario, NG, associated with its fraction of energy density among the total energy density, could also be produced as curvaton dominates over the energy density [5–7]. Upon relaxing this condition (the curvaton dominates over the energy density), it could yield large NG [8]. However, the current observation constrains these models [9], namely, characterizing by the local nonlinearity parameter f_{NL} that cannot be large. This local f_{NL} parameter is suppressed by the quadratic potential plus quartic potential [10] and also in the string axionic potential [11,12]. Furthermore, the observable f_{NL} parameter also puts an enhanced constraint on the decay epoch of the curvaton and its field value at the horizon exit [13]. The implications of NG features in curvaton scenario were also studied in Refs. [14–16]. On contrary, NG could be produced in various curvaton models [17–19].

In most curvaton scenarios, curvaton usually is considered as an independent field. If taking the thermal effects into account, the large NG is a necessary product due to the observed curvature perturbation [20], even the curvaton can be realized in low energy inflation compared with traditional curvaton mechanism [21]. Further, a similar curvaton mechanism can also be achieved due to the coupling between the inflaton and the curvaton [22]. From another

*liuleihua8899@hotmail.com

perspective of independent curvaton, it naturally embeds into two-field inflationary theory, in which it can produce a sizable NG within observations [23]. Very recently, the authors of Ref. [24] rigorously realized the curvaton mechanism under the covariant framework of field space. Taking the curvaton and the inflaton into account for perturbation, NG can be generated by inflaton curvaton mixed model [18] and even the curvaton can drive the second inflationary process [25]. However, current observational constraints are not capable of distinguishing between the inflaton curvaton mixed model and single field inflation [26]. As curvaton explicitly couples to the superheavy matter, it will lead to observational signal including NG [27,28]. From another aspect, curvaton is dubbed as some scalar fields, i.e., pseudo-Nambu-Goldstone boson or right-handed sneutrino curvaton, etc. [29–33]. Due to the unification of the string theory, the curvaton scenario can also be applied into the string cosmology framework [34,35], in which it yields considerable NG. Since the energy scale of the inflation is far from the Planck scale, the curvaton scenario can be embedded into the minimal supersymmetric Standard Model (MSSM) [36]. Another origin comes via the inflaton decay [37].

The NG is associated with the three-point correlation function, and in order to investigate the NG, δN formalism was proposed [6] depending on the surface of the energy density slicing. Its huge merit is only needed in the relation of the corresponding background field and e-folding number. Based on a previous work, δN formalism was systematically developed by [38]. δN formalism has become a standard procedure to evaluate the power spectrum and NG in the multifield inflationary framework including the curvaton scenario (the canonical kinetic term of the field space). Reference [39] modified the δN formalism at the slice of the curvaton energy density; the method could proceed the curvaton mechanism in various periods [matter domination (MD), radiation domination (RD), second inflationary period] explicitly associated with equation of state (EOS) w. However, this traditional δN formalism cannot analytically evaluate the various curvaton models (distinct potentials). In order to compensate this flaw, Refs. [40,41] also proposed a modified δN formalism, in which this method could deal with various curvaton potentials analytically in principle. However, they assumed that different periods have a simple attractor solution characterized by an ordinary parameter c, in which the kinetic term is neglected and its contribution is enrolled into this parameter. This estimation of their method is too coarse compared to traditional calculation. The best way is to include the contribution of EOS w since it is model independent. In light of these above theoretical motivations, we suggest a generalized δN formalism unified to evaluate the nonlinearity parameter $f_{\rm NL}$.

This paper is organized as follows. In Sec. II, we revise the δN formalism based on [38] and meanwhile we also give our central formula of the nonlinearity parameter $f_{\rm NL}$. In Sec. III, we study the most classical curvaton model whose potential is quadratic and pseudo-Nambu-Goldstone curvaton. Section IV gives our main conclusions.

All of the calculations are adopted in the natural units in which $G = M_P = c = 1$, where G is the Newton constant, M_P is the Planck mass, and c is the speed of light.

II. THE GENERALIZED δN FORMALISM OF CURVATON DECAY

In this section, we generalize the δN formalism. In light of Refs. [39] and [40,41], our extending framework contains their merits. The main advantage of [40,41] for the $f_{\rm NL}$ parameter is that they build the explicit relation of the onset of oscillation of curvaton and the curvaton value as inflation ends, which is not included in the traditional δN formalism. As for Ref. [39], the authors constructed the $f_{\rm NL}$ parameter associated with EOS w except the fraction of the curvaton energy density to the total energy density denoted by $r_{\rm decay}$. First, we will review the δN formalism.

A. Recap of δN formalism for curvaton decay

In a traditional curvaton scenario, it will generate the non-Gaussianity essentially characterizing by nonlocal non-Gaussianity parameter $f_{\rm NL}$. In order to obtain its explicit formula, the most common method for copying is the so-called δN formalism [38], since it only requires relation between the background field and the e-folding number. The curvature perturbation can be expanded as order by order,

$$\zeta(x) = \zeta_1(x) + \frac{1}{2!}\zeta_2(x) + \frac{1}{3!}\zeta_3(x), \tag{1}$$

with $\zeta_2 = \frac{6}{5} f_{\rm NL}\zeta_1^2$ and $\zeta_3 = \frac{54}{25} g_{\rm NL}\zeta_1^3$, where ζ_1 is explicitly proportional to Gaussian field, ζ_2 and ζ_3 are related to non-Gaussian field associated with Gaussian field for nonlocal non-Gaussianity parameters $f_{\rm NL}$ and $g_{\rm NL}$. Here, we are only concerned with the $f_{\rm NL}$ parameter since $g_{\rm NL} \propto f_{\rm NL}^2$ and it will be suppressed at higher order. The $f_{\rm NL}$ parameter originates from the three-point correlation functions,

$$\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = (2\pi)^3 B(k_1, k_2, k_3)\delta^3\left(\sum_{n=1}^{3} k_n\right), \tag{2}$$

where $B(k_1,k_2,k_3) = \frac{5}{6} f_{\rm NL}(P(k_1)P(k_2) + 2\,\mathbf{perm}.)$, where $P(k_i)$ is the power spectrum of ζ_{k_i} field.

In order to relate to the e-folding number N, once adopting uniform density hypersurfaces of curvaton, the curvature perturbation can then be denoted in terms of nonlinear curvature perturbation,

$$\zeta(x) = \delta N(x) + \frac{1}{3}\int_{\bar{\rho}(t_0)}^{\rho(x)} \frac{d\tilde{\rho}}{\tilde{\rho}+\tilde{P}}, \tag{3}$$

where δN is the perturbed expansion, $\tilde{\rho}$ is the local energy density, and $\tilde{P}$ is the local pressure. Given that the curvaton decay occurs in MD period, then one naturally neglects the contribution of pressure. Subsequently, by integrating both sides of Eq. (3) and choosing the flat slice, one obtains

$$\rho_\chi = \bar{\rho}_\chi \exp(3\zeta_\chi). \tag{4}$$

For curvaton field, its perturbation can be defined by

$$\chi_* = \bar{\chi} + \delta_1\chi_*, \tag{5}$$

where $\delta_1\chi_*$ denotes the vacuum fluctuations of the curvaton field. For depicting the curvature perturbation of curvaton, we need to relate the Hubble crossing value to the initial amplitude of the curvaton oscillation. In order to achieve this goal, one can use the Taylor expansion to build their relation,

$$g(\chi_*) = g(\bar{\chi} + \delta_1\chi_*) = \bar{g} + \sum_{n=1}^{\infty}\frac{g^{(n)}}{n!}\left(\frac{\delta_1\chi_{\rm osc}}{g'}\right)^n, \tag{6}$$

where $g' = \frac{dg}{d\chi_*}$ and $\chi_{\rm osc}$ denote the value of the curvaton field that begins to oscillate. Apparently, $g(\chi_*)$ depends on the model. Until present, the discussion of curvature perturbation of the curvaton is generic which means that the curvaton potential is general. In order to relate to some specific curvaton models, Ref. [38] assumes the simplest potential (quadratic potential) for curvaton. Apparently, it shows that $g(\chi_*) \propto \chi_*$. Subsequently, one can consider this potential as energy density and then expand it to the second order of perturbation of the curvaton field for comparison; finally, we find that

$$\zeta_{\chi 1} = \frac{2}{3}\frac{\delta_1\chi}{\bar{\chi}}, \tag{7}$$

$$\zeta_{\chi 2} = -\frac{3}{2}\left(1 - \frac{gg''}{g'^2}\right). \tag{8}$$

Next, we need to find the relation between ζ_χ and ζ. Following the sudden decay approximation, this relation can be analytically obtained, which is realized on a uniform total density hypersurface as $H = \Gamma_\chi$ (the decay rate of curvaton). On this curvaton decay hypersurface, one accordingly has

$$\rho_r(t_{\rm decay}) + \rho_\chi(t_{\rm decay}) = \bar{\rho}(t_{\rm decay}), \tag{9}$$

where $\bar{\rho}$ denotes the background field energy density. Meanwhile, we have $\delta N = \zeta$ on the curvaton decay hypersurface. Observing that the production of curvaton decay is relativistic and total pressure $P = \frac{1}{3}\rho$, consequently one easily obtains

$$\rho_r = \bar{\rho}_r \exp[4(\zeta_r - \zeta)], \tag{10}$$

$$\rho_\chi = \bar{\rho}_\chi \exp[3(\zeta_\chi - \zeta)]. \tag{11}$$

Using these two formulas into Eq. (9) and defining a dimensionless quantity $\Omega_\chi = \bar{\rho}_\chi/(\bar{\rho}_\chi + \bar{\rho}_r)$, after some algebra, one obtains

$$(1-\Omega_\chi)\exp[4(\zeta_r - \zeta)] + \Omega_\chi \exp[3(\zeta_\chi - \zeta)] = 1. \tag{12}$$

Once this central formula of δN formalism is derived, we can set the relations between the ζ_χ and ζ. Expanding up to the second order of Eq. (12), we collect these relations,

$$\zeta_1 = r_{\rm decay}\zeta_{\chi 1}, \tag{13}$$

$$\zeta_2 = \left[\frac{3}{2r_{\rm decay}}\left(1 + \frac{gg''}{g'^2}\right) - 2 - r_{\rm decay}\right]\zeta_{\chi 2}^2, \tag{14}$$

where we define

$$r_{\rm decay} = \frac{3\Omega_{\chi,\rm decay}}{4 - \Omega_{\chi,\rm decay}} = \frac{3\bar{\rho}_\chi}{3\bar{\rho}_\chi + 4\bar{\rho}_r}. \tag{15}$$

It naturally yields nonlinearity parameter using the sudden decay approximation [6,7],

$$f_{\rm NL} = \frac{5}{4r_{\rm decay}}\left(1 + \frac{gg''}{g'^2}\right) - \frac{5}{3} - \frac{5r_{\rm decay}}{6}. \tag{16}$$

We observe that this nonlinearity parameter highly depends on the $r_{\rm decay}$ and meanwhile mildly depends on the structure of model shown in g and g'. Although we adopted the simplest potential for curvaton, the final result is almost quadratic potential independent. Actually, one can roughly estimate this result when expanding the energy density of curvaton up to the second order. Subsequently, one can discover via Eqs. (4) and (6) that the background of curvaton will be canceled comparing them through their equations.

Furthermore, the generic potential of curvaton should be taken into account. The time of occurrence of curvaton mechanism (various decays of curvaton models will happen in RD or MD) is also different. In order to compensate these two missing places into curvaton mechanism, some distinct generalized δN formalisms are proposed.

B. Generalized δN formalism

In this section, we construct a generalized δN formalism with a generic potential and EOS w. Consequently, it is valid for broad kinds of curvaton models. In Ref. [39], the authors innovatively assumed that the curvaton decay occurs on a uniform curvaton density slice. Being different from the definition of the total energy density in Sec. II A, they found that

$$\zeta = \zeta_\chi + \frac{1}{4}\ln\left(\frac{4\bar{\rho}_r + 3(\bar{\rho}_\chi + \bar{P}_\chi)}{4\rho_r + 3(\bar{\rho}_\chi + \bar{P}_\chi)}\right). \tag{17}$$

By inserting Eq. (11) into Eq. (17),

$$\left(1 - \frac{1-3w}{4}\Omega_\chi\right)\exp[4(\chi - \chi_r)] = (1-\Omega_\chi)\exp[4(\zeta_r - \zeta_\chi)] + \frac{3(1+w)}{4}\Omega_\chi, \tag{18}$$

where $w = \frac{\bar{P}_\chi}{\bar{\rho}_\chi}$. Following the standard procedure, the relation between ζ and ζ_χ can be derived order by order,

$$\zeta_1 = \tilde{r}_{\rm decay}\zeta_{\chi 1}, \tag{19}$$

$$\frac{\zeta_2}{\zeta_{\chi 2}^2} = \frac{3(1+w)}{2\tilde{r}_{\rm decay}}\left(1 + \frac{gg''}{g'^2}\right) + \frac{1-3w}{\tilde{r}_{\rm decay}} - 4, \tag{20}$$

where $\tilde{r}_{\rm decay} = \frac{3(1+w)\Omega_\chi}{4+(3w-1)\Omega_\chi}$ is introduced. Apparently, the nonlinearity parameter associated with non-Gaussianity can be explicitly derived by

$$f_{\rm NL} = \frac{5}{4}\frac{1+w}{\tilde{r}_{\rm decay}}\left(1 + \frac{gg''}{g'^2}\right) + \frac{5}{6}\frac{1-3w}{\tilde{r}_{\rm decay}} - \frac{10}{3}. \tag{21}$$

Observe that the value of the $f_{\rm NL}$ parameter will get enhanced in the limit of $w \to 0$ which is equivalent to $\tilde{r}_{\rm decay} \to 0$. Reference [39] described that this case will appear in the secondary inflation. The similar process was also discussed in various curvaton models [42,43]. Consequently, one can conclude that w is a possible criterion for assessing the occurrence of secondary inflationary process.

This nonlinearity parameter is tiny and different compared to (16). This difference comes from the slice of energy density. In inflationary period, there are at least two components if the existence of curvaton field is required. In order to remove the influence of the other field from the non-Gaussianity, this method is necessary and more precise compared to traditional δN formalism. The huge merit of this generalized δN formalism is that it can deal with the second scalar field in different epochs. Especially for the curvaton mechanism, it is usually dubbed as pressless matter, namely, that it happens in MD before it decays. With the introduction of this method, the curvaton mechanism can be fulfilled in various eras. Consequently, it will extend the application of curvaton mechanism.

However, one cannot manage it analytically with generic potential besides the quadratic potential. References [40,41] accordingly proposed another generalized δN formalism for dealing with the generic potential analytically. In their method, the nonlinearity parameter is written as

$$f_{\rm NL} = -\frac{5}{6}r_{\rm decay} - \frac{5}{3} + \frac{5}{2r_{\rm decay}}(1+A), \tag{22}$$

where A is given as

$$A = \left[\frac{V'(\chi_{\rm osc})}{V(\chi_{\rm osc})} - \frac{3X(\chi_{\rm osc})}{\chi_{\rm osc}}\right]^{-1}\left[\frac{X'(\chi_{\rm osc})}{1-X(\chi_{\rm osc})} + \frac{V''(\chi_{\rm osc})}{V'(\chi_{\rm osc})} - (1-X(\chi_{\rm osc}))\frac{V''(\chi_*)}{V'(\chi_{\rm osc})}\right] + \left[\frac{V'(\chi_{\rm osc})}{V(\chi_{\rm osc})} - \frac{3X(\chi_{\rm osc})}{\chi_{\rm osc}}\right]^{-2}\left[\frac{V''(\chi_{\rm osc})}{V(\chi_{\rm osc})} - \left(\frac{V'(\chi_{\rm osc})}{V(\chi_{\rm osc})}\right)^2 - \frac{3X'(\chi_{\rm osc})}{\chi_{\rm osc}} + \frac{3X(\chi_{\rm osc})}{\chi_{\rm osc}^2}\right]. \tag{23}$$

Here A is characterized by a curvaton with a generic energy potential, in which it experiences a nonuniform onset of its oscillation. Its validity only requires starting a sinusoidal oscillation by satisfying

$$H_{\rm osc}^2 = \frac{V'(\chi_{\rm osc})}{c\chi_{\rm osc}}, \tag{24}$$

where c is given as $9/2$ and 5 when the curvaton begins to oscillate during MD and RD, respectively. The information of different periods is explicitly included in parameter c characterized by the attractor solution.

In order to relate the method of Ref. [39], we need to find the correspondence between Eqs. (22) and (21). Before finding the correspondence, the relation between Eqs. (22) and (16) is necessary since these two methods are adopted in the total energy density slice. Maybe this slice for [40,41] is not explicit. However, one can easily check that the whole calculation is depended on the total energy density in the curvaton dominant period after inflation. Furthermore, the total energy slice is approximately equal to the curvaton energy density slice after inflation, since the curvaton is dominant which is also an assumption for original curvaton scenario. In light of this logic, we should find the correspondence between Eqs. (22) and (16) and then explicitly adopt this correspondence for Eq. (22). Comparing with Eqs. (22) and (16), an explicit correspondence can be found by

$$1 + 2A = \frac{gg''}{g'^2}. \tag{25}$$

Using this correspondence into Eq. (21), we obtain

$$f_{\rm NL} = \frac{5}{2}\frac{1+w}{\tilde{r}_{\rm decay}}(1+A) + \frac{5}{6}\frac{1-3w}{\tilde{r}_{\rm decay}} - \frac{10}{3}. \qquad (26)$$

In this formula, we observe that $\tilde{r}_{\rm decay}$ is also the function of w. Following the traditional logic, we will work with the $f_{\rm NL}$ parameter in terms of $r_{\rm decay}$ and w. In order to achieve this goal, the relation between $r_{\rm decay}$ and $\tilde{r}_{\rm decay}$ is mandatory. In light of their relation, the nonlinearity parameter can be rewritten as

$$f_{\rm NL} = \frac{5(3Aw+3A+4)}{6r_{\rm decay}(w+1)} + \frac{5(3Aw^2+3Aw-4)}{6(w+1)}. \qquad (27)$$

Thus, we obtain the central result of this paper, in which it can tackle the generic potential analytically and can assess the existence of second inflationary process for curvaton field. In the next section, we will investigate the nonlinearity parameter $f_{\rm NL}$ in various curvaton models under the observational constraints.

III. CASE STUDY

The realization of curvaton mechanism depends on the models, particularly on the potential of curvaton. The shape of potential for curvaton will lead to the difference in various curvaton models, e.g., chaotic curvaton model, axionic curvaton, etc.

Before discussing the non-Gaussianity identified with the nonlinearity parameter $f_{\rm NL}$, the consideration of power spectrum of curvaton must be taken into account. Recalling that our derivation of the $f_{\rm NL}$ parameter is mainly according to the framework of [40,41], they found that the power spectrum of curvaton is nearly scale invariant for different values of k in various models of curvaton (exactly speaking for the various potentials of curvaton). Furthermore, Ref. [44] also studied that power spectrum only depends on $r_{\rm decay}$ and χ explicitly. Thus, the power spectrum of curvaton is the same for various models of curvaton. This issue can be easily checked in [38,40,41].

The second issue should also be clarified, which is related to the period of occurrence for curvaton mechanism. In a traditional curvaton mechanism, it happens in the MD whose corresponding value of $w = 0$ behaves like a presssureless matter [1–3]. However, this similar mechanism can be realized in different periods, e.g., Ref. [44] considered a curvaton mechanism that occurred in RD due to the decay of inflaton inspired by [45,46], in which the key ingredient is the explicit coupling between the curvaton field and the inflaton field. Once the curvaton is obtained, the curvaton field will also decay into the Standard Model's degrees of freedom as inflaton decay (the generation of curvaton comes via inflaton decay). Consequently, it will lead to the amount of isocurvature perturbation without thermalizing with the Standard Model degrees. From the current constraint [9], the power spectrum of isocurvature perturbation compared with curvature perturbation cannot be large. In order to transfer this isocurvature perturbation into curvature perturbation, Refs. [32,47] proposed the viable curvaton mechanism embedded into MSSM in light of [48] by considering the thermalization. Thus, the curvaton mechanism can be realized in various epochs under the framework of MSSM. Furthermore, the curvaton mechanism can also be achieved by the curvaton brane leading to the large NG [39] whose corresponding value $w = -1$. From the central formula of (21), it is also known that proceeding with the curvaton mechanism is adopted for various epochs. Meanwhile, the variants of δN formalism contain the method proceeding with the curvaton mechanism in distinct periods. In light of the above theoretical motivations, we could find that the curvaton mechanism will be realized in various periods corresponding to different values of w.

A. Chaotic curvaton

Chaotic curvaton indicates that the potential of curvaton is quadratic. These kinds of curvaton have been investigated broadly, in particular, for the non-Gaussianity characterized by nonlinearity parameter $f_{\rm NL}$ [6,7]. In light of quadratic potential, Ref. [38] proposed a generalized δN formalism to investigate the non-Gaussianity, in which curvature perturbation can be derived up to any order. We accordingly concern the second order of curvature perturbation associated with the $f_{\rm NL}$ parameter.

We will give an analysis of the $f_{\rm NL}$ parameter for chaotic curvaton based on our central result (26). In our previous work [44], we have clearly shown that $A = -\frac{1}{2}$ as the potential of curvaton proportional to χ^2 where χ denotes the value of curvaton field, in which it is explicitly consistent with simple analysis of Ref. [41] (only adopting the different notation for the fraction of curvaton energy density among the total energy density). Accordingly, the central result of the $f_{\rm NL}$ parameter becomes

$$f_{\rm NL} = -\frac{5(3w-5)}{12r_{\rm decay}(w+1)} - \frac{5(3w^2+3w+8)}{12(w+1)}. \qquad (28)$$

We will use this formula to investigate the non-Gaussianity compared to the previous relevant work. This $f_{\rm NL}$ parameter is a generic formula for curvaton associated with non-Gaussianity.

Case a.—$\mathbf{w \to -1}$

In various models, the EOS w can have different values. In Ref. [39], the authors constructed a curvaton scenario under the framework of brane world, in which the corresponding $w \to -1$. In this case, it clearly indicates that the $f_{\rm NL}$ parameter is divergent by exceeding the range of current observational constraints [49].

Case b.—$\mathbf{w \to 0}$

In this case, the curvaton behaves as the pressureless matter. The parameter $f_{\rm NL}$ simplifies into

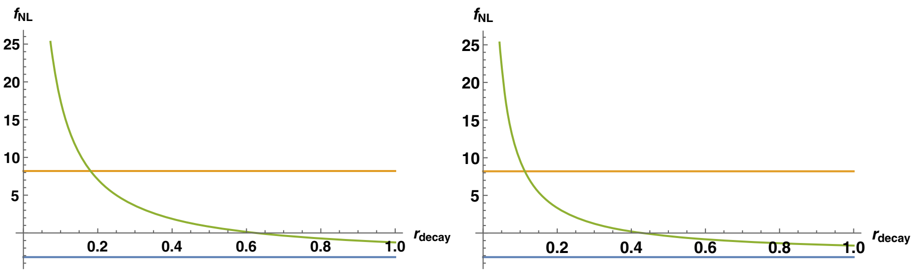

FIG. 1. The left panel shows the nonlinearity parameter $f_{\rm NL}$ for case b and the right panel for case c. The brown and blue lines denote the upper and lower bounds for the $f_{\rm NL}$ parameter. The corresponding value of $r_{\rm decay}$ is 0.18 and 0.11 with respect to case b and case c, respectively.

$$f_{\rm NL} = \frac{25}{12 r_{\rm decay}} - \frac{10}{3}. \tag{29}$$

In the limit of $r_{\rm decay} \to 1$, $f_{\rm NL} = -\frac{5}{4}$ nicely recovers with Eq. (26) in Ref. [38] in the absence of nonlinear evolution for the curvature perturbation of curvaton (also emphasized in Ref. [6]), in which the curvaton scenario is the simplest curvaton model whose potential is $\frac{1}{2} m_\chi^2 \chi^2$ (χ denotes the curvaton field) and behaves as pressureless matter according to our analysis. Meanwhile, curvaton dominates the energy density. For large non-Gaussianity, it requires that $r_{\rm decay} \to 0$. In order to better understand the possible range of $r_{\rm decay}$, we plotted Eq. (29).

Case c.—$\mathbf{w \to \frac{1}{3}}$

In this case, curvaton decay is a relativistic process. Then, the parameter $f_{\rm NL}$ becomes

$$f_{\rm NL} = \frac{5}{4 r_{\rm decay}} - \frac{35}{12}. \tag{30}$$

A similar analysis is given in case b. In the limit of $r_{\rm decay} \to 1$, $f_{\rm NL} \to -\frac{5}{3}$. The value is almost the same as in case b, in which one cannot distinguish the tiny difference between case b and case c. Frankly speaking, curvaton is an independent and extra field during inflationary process (even including the preheating process); however, curvaton can be induced by the inflaton decay whose realization occurs from the transfer of entropy perturbation to curvature perturbation [44]; in order to realize this transfer, the curvaton can be embedded into MSSM [47].

We have discussed the nonlinearity parameter with various cases of chaotic curvaton, whose potential is proportional to χ^2. Although we cannot distinguish the difference between case b and case c via observational constraints, it is expected to obtain the distinct values for the corresponding cases. Reference [9] tells that $f_{\rm NL} = 2.5 \pm 5.7$, afterward, combining with Eqs. (29) and (30), we can plot them for a comparison. In Fig. 1, the constraints of $r_{\rm decay}$ for case b and case c, respectively, are explicitly depicted. The corresponding values are 0.18 for the left panel (case b) and 0.11 for the right panel (case c). This trend is logical since case c illustrates that curvaton behaves as relativistic matter, meaning the curvaton will last for a longer time occurrence of its decay.

For the careful reader, they may find that there is still some lose information on the transition from $w \to 0$ to $w \to \frac{1}{3}$, since the curvaton will become the relativistic matter as the long time occurrence of curvaton decay (from MD to RD). If we consider this case, $r_{\rm decay}$ will be a small number, but what will be the precise value. We need a more detailed investigation on the parameter $f_{\rm NL}$ varying w. In order to achieve this goal, we show the density plot of the nonlinearity parameter $f_{\rm NL}$ depending on the parameters $r_{\rm decay}$ and w in Fig. 2. It clearly indicates that the non-Gaussianity will get dramatically enhanced as $w \to 0$ and $r_{\rm decay} \to 0$, which is consistent with our previous discussion. Interestingly, the parameter $f_{\rm NL}$ is still within the observational constraints [49], in which w approaches -1 before curvaton decays. This is one of our new findings for

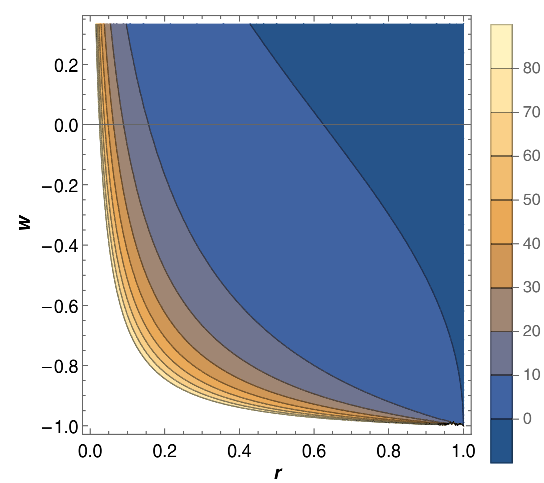

FIG. 2. Contour plot of nonlinearity parameter (28): the horizontal line corresponds to $r_{\rm decay}$ whose range is $0 \leqslant r_{\rm decay} \leqslant 1$ including the whole possible value. The vertical line denotes the value of equation of state w locating from -1 to $\frac{1}{3}$, in which it includes that dark energy epoch, radiation domination period, matter domination period, and it could indicate the transition from one era to another era. The right panel shows that the value of $f_{\rm NL}$ matches its corresponding color.

chaotic curvaton model. As the decay of curvaton continues, we find that there are lots of parameter spaces satisfied with observational constraints, showing the blue area of Fig. 2 as $r_{\rm decay} < 0.5$.

B. Pseudo-Nambu-Goldstone curvaton

In this case, we will further consider the origin of curvaton from microscopic physics, namely, pseudo-Nambu-Goldstone boson with a broken $U(1)$ symmetry. The curvaton mass will be suppressed by the approximating symmetry. Since curvaton has the periodicity of $U(1)$ leading to minima and maxima along the potential, it will generate the blue and red tiled curvature perturbation of curvaton. What we are concerned with is the potential of pseudo-Nambu-Goldstone curvaton. It reads as

$$V(\chi) = \Lambda^4\left[1-\cos\left(\frac{\chi}{f}\right)\right], \tag{31}$$

where f and Λ denote the energy scale. In order to obtain its corresponding $f_{\rm NL}$ parameter, the relation between the χ_* and $\chi_{\rm osc}$ is mandatory. To achieve this goal, we need the modified KG equation (24), and one can derive

$$\ln\left[\frac{\tan(\chi_{\rm osc}/2f)}{\tan(\chi_*/2f)}\right] = -\frac{N_*}{3H_{\rm inf}^2}\frac{\Lambda^4}{f^2} - \frac{1}{2(c-3)}\frac{\chi_{\rm osc}/f}{\sin(\chi_{\rm osc}/f)}, \tag{32}$$

where N_* denotes the e-folding number at the horizon exit and $H_{\rm inf}$ represents the Hubble parameter during inflation. After some algebras, we can represent χ_* in terms of $\chi_{\rm osc}$,

$$\chi_* = \frac{1}{f}\left[\operatorname{arccot}\left(\exp\left(-\frac{\frac{3f\chi_{\rm osc}\csc(\frac{\chi_{\rm osc}}{f})}{c-3} + \frac{2\Lambda^4 N_*}{H_{\rm inf}^2}}{6f^2}\right)\cot\left(\frac{f\chi_{\rm osc}}{2}\right)\right)\right] + \text{constant}. \tag{33}$$

In this calculation, the constant can be set to zero and the maxima of $\chi_{\rm osc}$ is around 0.08 based on the periodic condition. It is worthwhile to plot their relation after choosing suitable parameters in Planck units. Figure 3 clearly indicates that the maximal value of curvaton field is approximately equal to 0.7 whose value is lighter than Planck mass at the horizon exit. Compared with inflaton field, it is a light field that makes its energy density subdominant during inflation. Once their explicit relation is found, we can find the formula of A corresponding to pseudo-Nambu-Goldstone curvaton. Due to the complication of formula of A, all the formulas will be tackled by *Mathematica*. Being armed with these formulas, we will plot the nonlinearity parameter in various epochs including second inflationary process, RD, and MD. Being different by investigating chaotic curvaton, A is also a function of c whose various values correspond to different periods. Due

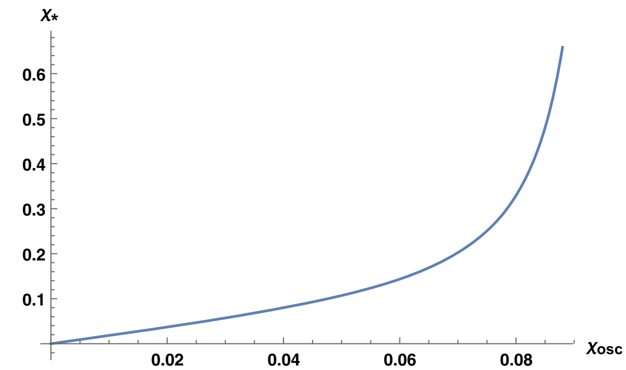

FIG. 3. The relation of $\chi_{\rm oscc}$ and χ_* according to their explicit relation (33). During the whole range of $\chi_{\rm osc}$, the corresponding maximal value of χ_* is 0.7. The parameters are set as $N_* = 50$, $f = 3.36\times10^{-2}$, $c = 9/2$ (MD as an instance), $\Lambda = 3.56\times10^{-4}$, and $H_{\rm inf} = 10^{-5}$ as adopted in Ref. [40].

to this parameter, we cannot vary with w to analyze the nonlinearity parameter $f_{\rm NL}$. Finally, we only study the individual case referring to specific w and c.

Case a.—$w = -1$ and $c = 3$

The explicit of $f_{\rm NL}$ parameter is too complicated to express due to the complication of A. Actually, most curvaton models with various potentials cannot express A explicitly since the relation between $\chi_{\rm osc}$ and χ_* is almost not possible, which takes place by numerical methods as shown in Refs. [40,41]. Once we have this knowledge and meanwhile observe that $w = -1$ and $c = 3$, it will lead to the divergence of the $f_{\rm NL}$ parameter from Eq. (27). For a better understanding of this case, the plot will be given. In Fig. 4, we can clearly see that the $f_{\rm NL}$ parameter varies with

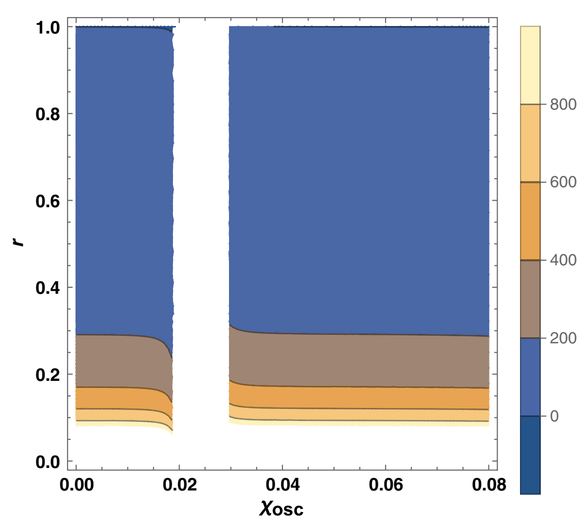

FIG. 4. The horizontal line corresponds to $r_{\rm decay}$ whose range is $0\leqslant r_{\rm decay}\leqslant 1$ including the whole possible value. The vertical line denotes the value of equation of $\chi_{\rm osc}$ locating from 0 to 0.08. The right panel shows that the value of the $f_{\rm NL}$ parameter matches its corresponding color. The parameters are set the same as in Fig. 3.

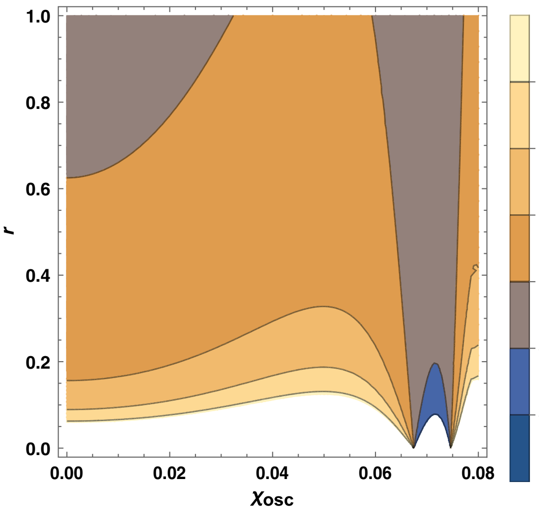

FIG. 5. The horizontal line corresponds to $r_{\rm decay}$ whose range is $0 \leqslant r_{\rm decay} \leqslant 1$ including the whole possible value. The vertical line denotes the value of equation of $\chi_{\rm osc}$ locating from 0 to 0.08. The right panel shows that the value of the $f_{\rm NL}$ parameter matches its corresponding color. The parameters are set the same as in Fig. 3.

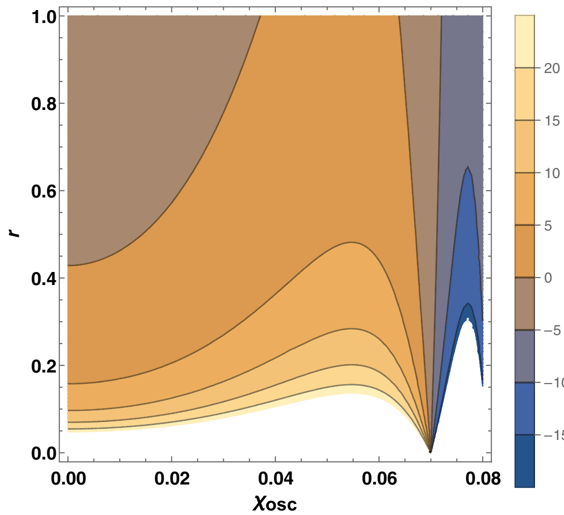

FIG. 6. The horizontal line corresponds to $r_{\rm decay}$ whose range is $0 \leqslant r_{\rm decay} \leqslant 1$ including the whole possible value. The vertical line denotes the value of equation of $\chi_{\rm osc}$ locating from 0 to 0.08. The right panel shows that the value of the $f_{\rm NL}$ parameter matches its corresponding color. The parameters are set the same as in Fig. 3.

$\chi_{\rm osc}$ and $r_{\rm decay}$. The observational constraint gives the upper limit whose value is less than 10. From Fig. 4, it is almost impossible to find this value, in particular, as $r < 0.3$, $f_{\rm NL}$ parameter already exceeds the upper limit of the observational constraint. Additionally, there is also divergence as $\chi_{\rm osc}$ is between 0.018 and 0.03. The varying trend of the $f_{\rm NL}$ parameter will flip by crossing these divergent areas. To sum up, the secondary inflation for curvaton will not happen in light of our discussion.

Case a.—$w = 0$ and $c = 9/2$

In this case, axionic curvaton behaves like pressureless matter. Its plot will also be gotten. Figure 5 clearly indicates that most parameter spaces are satisfied with observational constraints [9] especially for $r_{\rm decay} > 0.2$. The value of the $f_{\rm NL}$ parameter will become negative as $0.067 \leq \chi_{\rm osc} \leq 0.075$. If the observation can constrain the sign of the $f_{\rm NL}$ parameter, it will give a strong constraint of our mechanism for curvaton. Comparing with Ref. [40], our formula is not so highly depended on the field value of χ, in which we replace χ_* with $\chi_{\rm osc}$ to investigate. In this case, the upper limit $r_{\rm decay}$ is smaller as compared to chaotic curvaton, which means that the fraction of curvaton among the total energy could be less even in MD.

Case c.—$w = \frac{1}{3}$, $c = 5$

In this case, we will study the nonlinearity parameter in RD. Generically, the trend of Fig. 6 is similar to Fig. 5. It contains lots of parameter spaces satisfied with observational constraints. The difference comes for the upper limit of $r_{\rm decay}$, its value is even smaller whose range could reach 0.1, and the discussion is the same since one can consider that curvaton is the production in MD as shown in our previous work [44]. Another distinct place is that the sign of the $f_{\rm NL}$ parameter flips around $0.07 \leq \chi_{\rm osc} \leq 0.08$.

In this section, we apply our extending $f_{\rm NL}$ parameter to different curvaton models. First, in light of this framework [40], it is already known that the power spectrum does not vary dramatically with the energy scale. According to this point, we are only concerned with the nonlinearity parameter $f_{\rm NL}$. Our findings are the generic curvaton mechanism that will not experience the second inflationary process, although there is a tiny choice of parameter space for chaotic curvaton. As for a pseudo-Nambu-Goldstone curvaton, our findings show that no matter what curvaton behaves as pressure or pressureless matter, most of the parameter spaces satisfy with observational constraints [9]. The only differences are determined by their decay process, and this point is illustrated in Ref. [41] identified with comparison between $t_{\rm decay}$ and $t_{\rm reheating}$.

IV. CONCLUSION

In this paper, we constructed a generalized δN formalism consisting of merits of Refs. [39,40]. Our method deals with curvaton models with generic potentials only requiring sinusoidal oscillation; meanwhile, it can also handle curvaton mechanism in various periods explicitly shown by EOS w (secondary inflation, MD, RD) with corresponding parameter c in Sec. II B. For achieving a successful curvaton mechanism in different era, Refs. [32,47] proposed the curvaton mechanism embedded into MSSM with the consideration of thermalization, in which the effect of thermalization could transfer the isocurvature perturbation into curvature perturbation ensuring nearly scale invariant power spectrum. From another perspective, Ref. [40] analyzed the non-Gaussianity associated with the $f_{\rm NL}$ parameter. Although the method could work with different periods (MD, RD, etc.), the authors simply assumed that the different epochs correspond to various values of c by neglecting the contribution of the kinetic term. It is unavoidable to wrongly estimate the precise contribution of kinetic terms. In order to compensate this flaw, we adopted the advantage of Ref. [39], directly associated with EOS w, for investigation.

Once the key result for the nonlinearity parameter $f_{\rm NL}$ (27) was obtained, we implemented it into two curvaton models. One is the chaotic curvaton and the other one the Pseudo-Nambu-Goldstone curvaton. In light of the framework of [41], we were only concerned with non-Gaussianity identified with the $f_{\rm NL}$ parameter since the power spectrum is nearly scale invariant in various models. In a traditional curvaton scenario, it behaves as pressless matter with $w = 0$. However, we are not capable of distinguishing the period of occurrence of curvaton mechanism from the observations. Thus, the curvaton mechanism can be fulfilled in various periods, e.g., the curvaton mechanism is achieved by the decay of inflaton field [44] during the preheating period, in which it may be realized in RD as inflaton practically decays. Furthermore, the variants of δN formalism could proceed with various epochs of curvaton mechanism [39–41]. Accordingly, we discussed two specific curvaton models in distinct era.

For the chaotic curvaton, we investigate the $f_{\rm NL}$ parameter. In the limit of $r_{\rm decay} \to 1$, $f \to -\frac{5}{4}$ nicely recovers the analysis of Ref. [38] in case a of chaotic curvaton and the $f_{\rm NL}$ parameter will be divergent in the limit of $r_{\rm decay} \to 0$. For case a, it indicates that the secondary inflationary process is ruled out by observational constraints. However, there will be occurrence of second inflationary process if there is a transition from DE era to MD as shown in Fig. 2.

The original curvaton mechanism assumed that it was an extra and independent field compared to inflaton field. One possibility of accounting for its origin is pseudo-Nambu-Goldstone curvaton. In this model, the value of the $f_{\rm NL}$ parameter shows the similar varying trend with chaotic curvaton as shown in Figs. 4–6. Due to the complication of A as in Eq. (23), we could not transit w from one era to another taking place by parameter c. These figures explicitly show that most parameter spaces are satisfied with observational constraints which determine the upper limit of $r_{\rm decay} > 0.1$. And the case a will be ruled out by these observations.

Finally, we emphasized on the further validity of our new formula for the $f_{\rm NL}$ parameter. For the traditional curvaton mechanism, the curvaton corresponds to a pressureless matter with $w = 0$, and our formula nicely recovers the classical result $f_{\rm NL} = -5/4$ in the limit of $r_{\rm decay} \approx 1$. For its validity in RD, the curvaton mechanism can be realized as inflaton decay. As our previous discussions mentioned, isocurvature perturbation can be transferred into curvature perturbation by considering thermalization. This idea was proposed in [48,50]. The inflaton coupling is not a constant anymore; subsequently, it can translate into fluctuations in the reheating temperature. As a result, our formula of Eq. (27) can be naturally applied into the framework of MSSM for curvaton model construction. Furthermore, we can also implement our method to explore the non-Gaussianity in MSSM curvaton model.

ACKNOWLEDGMENTS

The work of L.-H. L. was funded by Hunan Provincial Natural Science Foundation No. 2020JJ5452 and Hunan Provincial Department of Education, No. 19B464. The work of W.-L. X. was funded by NSFC 1175012.

[1] K. Enqvist and M. S. Sloth, Nucl. Phys. **B626**, 395 (2002).
[2] D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002).
[3] T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001); **539**, 303(E) (2002).
[4] J. M. Maldacena, J. High Energy Phys. 05 (2003) 013.
[5] N. Bartolo, S. Matarrese, and A. Riotto, Phys. Rev. D **69**, 043503 (2004).
[6] D. H. Lyth and Y. Rodriguez, Phys. Rev. Lett. **95**, 121302 (2005).
[7] N. Bartolo, S. Matarrese, and A. Riotto, Phys. Rev. Lett. **93**, 231301 (2004).
[8] D. H. Lyth, C. Ungarelli, and D. Wands, Phys. Rev. D **67**, 023503 (2003).
[9] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **594**, A17 (2016).
[10] K. Mukaida, K. Nakayama, and M. Takimoto, Phys. Rev. D **89**, 123515 (2014).
[11] K. Dimopoulos, K. Kohri, D. H. Lyth, and T. Matsuda, J. Cosmol. Astropart. Phys. 03 (2012) 022.
[12] M. Kawasaki, T. Kobayashi, and F. Takahashi, J. Cosmol. Astropart. Phys. 03 (2013) 016.
[13] M. K. Sharma, K. Myrzakulov, and M. A. Ajmi, Mod. Phys. Lett. A **35**, 2050259 (2020).
[14] Q. G. Huang, Phys. Lett. B **669**, 260 (2008).
[15] Q. G. Huang and Y. Wang, J. Cosmol. Astropart. Phys. 09 (2008) 025.
[16] J. O. Gong, C. Lin, and Y. Wang, J. Cosmol. Astropart. Phys. 03 (2010) 004.
[17] K. Harigaya, M. Ibe, M. Kawasaki, and T. T. Yanagida, Phys. Rev. D **87**, 063514 (2013).
[18] J. Fonseca and D. Wands, J. Cosmol. Astropart. Phys. 06 (2012) 028.
[19] S. Enomoto, K. Kohri, and T. Matsuda, Phys. Rev. D **87**, 123520 (2013).

[20] K. Mukaida, K. Nakayama, and M. Takimoto, J. Cosmol. Astropart. Phys. 06 (2014) 013.
[21] K. Dimopoulos, D. H. Lyth, and Y. Rodriguez, J. High Energy Phys. 02 (2005) 055.
[22] Y. Rodriguez, Mod. Phys. Lett. A **20**, 2057 (2005).
[23] S. Enomoto and T. Matsuda, Phys. Rev. D **87**, 083513 (2013).
[24] L. H. Liu and T. Prokopec, arXiv:2005.11069.
[25] C. T. Byrnes, M. Corts, and A. R. Liddle, Phys. Rev. D **90**, 023523 (2014).
[26] W. H. Kinney, A. Moradinezhad Dizgah, B. A. Powell, and A. Riotto, Phys. Rev. D **86**, 023527 (2012).
[27] L. Li, S. Lu, Y. Wang, and S. Zhou, J. High Energy Phys. 07 (2020) 231.
[28] S. Kumar and R. Sundrum, J. High Energy Phys. 04 (2020) 077.
[29] N. Haba, T. Takahashi, and T. Yamada, J. Cosmol. Astropart. Phys. 06 (2018) 011.
[30] K. Dimopoulos, D. H. Lyth, A. Notari, and A. Riotto, J. High Energy Phys. 07 (2003) 053.
[31] M. Bastero-Gil, V. Di Clemente, and S. F. King, Phys. Rev. D **67**, 103516 (2003).
[32] K. Enqvist, S. Kasuya, and A. Mazumdar, Phys. Rev. Lett. **90**, 091302 (2003).
[33] L. Y. Lee and C. M. Lin, Gen. Relativ. Gravit. **44**, 2433 (2012).
[34] Y. F. Cai, S. Li, and Y. S. Piao, Phys. Lett. B **671**, 181 (2009).
[35] J. Zhang, Y. F. Cai, and Y. S. Piao, J. Cosmol. Astropart. Phys. 05 (2010) 001.
[36] A. Mazumdar and S. Nadathur, Phys. Rev. Lett. **108**, 111302 (2012).
[37] C. T. Byrnes, M. Corts, and A. R. Liddle, Phys. Rev. D **94**, 063525 (2016); Y. F. Cai and Y. Wang, Phys. Rev. D **82**, 123501 (2010).
[38] M. Sasaki, J. Valiviita, and D. Wands, Phys. Rev. D **74**, 103003 (2006).
[39] Y. F. Cai and Y. Wang, Phys. Rev. D **82**, 123501 (2010).
[40] M. Kawasaki, T. Kobayashi, and F. Takahashi, Phys. Rev. D **84**, 123506 (2011).
[41] T. Kobayashi and T. Takahashi, J. Cosmol. Astropart. Phys. 06 (2012) 004.
[42] Y. F. Cai and H. Y. Xia, Phys. Lett. B **677**, 226 (2009).
[43] Q. G. Huang, J. Cosmol. Astropart. Phys. 11 (2008) 005.
[44] L. H. Liu and W. L. Xu, Chin. Phys. C **44**, 085103 (2020).
[45] J. Jiang, Q. Liang, Y. F. Cai, D. A. Easson, and Y. Zhang, Astrophys. J. **876**, 136 (2019).
[46] Y. F. Cai, R. Brandenberger, and X. Zhang, J. Cosmol. Astropart. Phys. 03 (2011) 003.
[47] R. Allahverdi, K. Enqvist, A. Jokinen, and A. Mazumdar, J. Cosmol. Astropart. Phys. 10 (2006) 007.
[48] G. Dvali, A. Gruzinov, and M. Zaldarriaga, Phys. Rev. D **69**, 023505 (2004).
[49] Y. Akrami *et al.* (Planck Collaboration), Astron. Astrophys. **641**, A10 (2020).
[50] K. Enqvist, A. Mazumdar, and M. Postma, Phys. Rev. D **67**, 121303 (2003).