Relation between varying fine structure constant and cosmological components

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(Received 28 October 2020; accepted 18 February 2021; published 12 March 2021)

We propose a simple model based on the assumption that the varying fine structure constant α is an effect of the cosmological expansion to investigate the relation between the varying α and the cosmological components. For a spatially flat, homogeneous, and isotropic universe, the current proportion of cosmological components and age of the universe predicted by the model are consistent with the cosmological observations. Furthermore, the predicted current variation rate of α is also close to the atomic clock measurements. For the early universe, we predict a very strict constraint, which is compatible with the upper limit given by the investigations of cosmic microwave background and big bang nucleosynthesis.

DOI: 10.1103/PhysRevD.103.063504

I. INTRODUCTION

Fundamental constants in physics reflect the essential properties of nature and some are used to define the units of measurement. However, the constancy of some fundamental constants has been controversial ever since Dirac proposed the "large numbers hypothesis" [1]. There is no point in discussing the constancy of dimensional constants because only their ratios to some units can be measured. The change of dimensional constants can be eliminated by redefining the units so that it has no substantial effect on the laws of physics. Therefore, only changes of dimensionless constants are meaningful and can be measured directly in experiments [2].

The fine structure constant α, which determines the strength of the electromagnetic interaction, is one of the most important dimensionless number in physics. In the last decades, the varying α has been investigated via various observations including the atomic clock measurements [3], the isotope ratio studies [4,5], the astronomical observations of white dwarfs [6,7] and galaxy clusters [8–11], and so on. In particular, Webb *et al.* found an evidence for a cosmological evolution of α by analyzing absorption lines in quasar spectra [12–15]. As the observational data accumulated, they further revealed a spatial variation of α [16–20]. In addition, the investigations of cosmic microwave background (CMB) at $z \approx 10^3$ and big bang nucleosynthesis (BBN) at $10^8 \leq z \leq 10^{10}$ both give an upper limit $|\Delta\alpha/\alpha| < 10^{-2}$ for the early universe [21–23]. These investigations show a preference for variations of α. A varying α is likely to involve underlying dynamics and associate new physics. Therefore, the research of varying α has a profound impact on the understanding of natural laws.

The fine structure constant can be expressed as a combination of several fundamental dimensional constants $\alpha \equiv e^2/\hbar c$ in cgs units with the elementary charge e, the reduced Planck's constant $\hbar$, and the speed of light c. Although the varying α could be interpreted by postulating that either the speed of light or the elementary charge varies with time, these nonstandard cosmological models are still highly controversial and facing many challenges [24–29]. More recently, some work investigated the varying α driven by the interaction of a scalar field and the electromagnetic field [30–33]. The modified gravity theories [34,35] and the $\Lambda(\alpha)$ cold dark matter models [36–38] are also employed to explain the varying α.

In this paper, we do not intend to discuss which fundamental dimensional constants are responsible for a varying α, but rather to take it as the quantity associated with the properties of space-time of that point. The observations of Type Ia supernovae (SNe Ia) in distant galaxies imply an accelerated expansion of the universe. It motivates us to consider that the varying α may be an effect of the varying expansion rate of space-time. Based on this assumption, we will propose a simple model to establish a relation between the varying α and the evolution of the various cosmological components. Then, we can obtain the proportions of matter and dark energy in the current universe by fitting the experimental data and further estimate the current age of the universe. In addition, the model will give the variation rate of α to compare with the experimental measurements and place a constraint on the variation of α in the early universe.

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II. MODEL

The expansion rate of the universe is determined by the total density of the universe, which is related to the specific proportion of the various cosmological components. Since the spatial curvature of the universe has been constrained to a very small value by the experiments of the SNe Ia, CMB, baryon acoustic oscillations (BAOs), and the measurement of the Hubble constant (H_0), the discussion of time varying α is in a flat universe [39–42]. In addition, the relativistic component decreases rapidly with the cosmological expansion and is negligible compared with the nonrelativistic component in the current universe. Therefore, the Friedmann equation, which describes the evolution of a spatially flat, homogeneous, and isotropic universe, can be simplified to

$$\tilde{H}^2 = \Omega_M a^{-3} + \Omega_\Lambda, \tag{1}$$

where the dimensionless Hubble parameter $\tilde{H}$ is defined as the expansion rate of the universe scaled by its current value, $\tilde{H} \equiv H/H_0$. The Hubble parameter is defined as $H \equiv \dot{a}/a$, a is scale factor, and the overdot denotes a derivative with respect to proper time. The relation between scale factor a and redshift z is $a = 1/(1+z)$. The $\Omega_M = 1 - \Omega_\Lambda$ and Ω_Λ are the matter and dark energy densities of the universe with respect to the current critical density, respectively.

According to Eq. (1), the first and second derivatives of $\tilde{H}$ with respect to proper time are, respectively,

$$\dot{\tilde{H}} = -\frac{3}{2} H_0 (\tilde{H}^2 - \Omega_\Lambda), \tag{2}$$

$$\ddot{\tilde{H}} = -3 H_0 \tilde{H} \dot{\tilde{H}}. \tag{3}$$

It is easy to see that the derivative of the expansion rate above first order with respect to proper time can be reexpressed as a function of $\tilde{H}$ and $\dot{\tilde{H}}$. So only the four quantities, Ω_Λ, $\tilde{H}$, H_0, and $\dot{\tilde{H}}$, appear in the equations and only two of them are independent.

If the varying α is indeed an effect of the cosmological expansion, the general form of the variation rate of α should be able to be expressed as any combination of the four quantities,

$$\frac{\dot{\alpha}}{\alpha_0} = \sum_{r,p,q,s} C_{r,p,q,s} \Omega_\Lambda^r \tilde{H}^p H_0^q (\dot{\tilde{H}})^s, \tag{4}$$

where the powers r, p, q, s are arbitrary integers and $C_{r,p,q,s}$ is the coefficient of the corresponding term. Although the general form is extremely complex, dimensional analysis allows for strict constraints on the possible forms. First of all, Ω_Λ is a dimensionless constant which can be absorbed into the coefficient $C_{r,p,q,s}$. In addition, $\tilde{H}$ is a dimensionless function and the dimensions of H_0 and $\dot{\tilde{H}}$ are both $[\mathrm{T}^{-1}]$. So, the dimension of $\dot{\alpha}/\alpha_0$ is $[\mathrm{T}^{-1}]$ which requires $q + s = 1$ and Eq. (4) can be expressed as

$$\frac{\dot{\alpha}}{\alpha_0} = \sum_{p,q} C_{p,q} \tilde{H}^p H_0^q (\dot{\tilde{H}})^{1-q} = \sum_{p,q} C_{p,q} \tilde{H}^p \dot{\tilde{H}} \left(\frac{\dot{\tilde{H}}}{H_0}\right)^{-q}. \tag{5}$$

The $\dot{\tilde{H}}/H_0$ can be reexpressed as polynomials of Ω_Λ and $\tilde{H}$ by using Eq. (2). After expanding and combining the polynomials, Eq. (5) will be simplified to

$$\frac{\dot{\alpha}}{\alpha_0} = \sum_p C_p \tilde{H}^p \dot{\tilde{H}}, \tag{6}$$

where the power p is an arbitrary integer and C_p is the coefficient of the corresponding term. Only $\tilde{H}$ and $\dot{\tilde{H}}$ survive in Eq. (6) in the end because only two quantities are independent in the cosmological model.

In principle, each term may have a contribute to the variation of α. But following the principle of simplicity of physical laws, we consider that only one term plays the most important role and the contribution of other terms can be absorbed by adjusting the corresponding coefficient. Therefore, Eq. (6) can be further simplified to

$$\frac{\dot{\alpha}}{\alpha_0} \propto \tilde{H}^p \dot{\tilde{H}} \propto (\dot{\tilde{H}^I}). \tag{7}$$

The second proportional relation is representing the former as a total derivative with respect to proper time, where $I = p + 1$ is an undetermined integer parameter.

After integrating Eq. (7) and using the current values as boundary condition, the relative variation of α is expressed as

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha_z - \alpha_0}{\alpha_0} = A(\tilde{H}^I - 1), \tag{8}$$

where α_z and α_0 are the values of the fine structure constant at redshift z and present, respectively, A is also an undetermined parameter. By using the Friedmann equation, Eq. (8) can be replaced by

$$\frac{\Delta\alpha}{\alpha} = A[(\Omega_M a^{-3} + \Omega_\Lambda)^{\frac{I}{2}} - 1]. \tag{9}$$

A relation is established between the varying α and the cosmological components in Eq. (9). Then, the proportion of matter and dark energy in the current universe could be obtained with the experimental measurements of varying α.

The age of the universe at the scale factor a can be obtained by solving the Friedmann equation

$$t = \frac{2}{3H_0}\frac{1}{\sqrt{\Omega_\Lambda}}\ln\left(\frac{\sqrt{\Omega_\Lambda a^3+\Omega_M}+\sqrt{\Omega_\Lambda a^3}}{\sqrt{\Omega_M}}\right). \tag{10}$$

The current age of the universe is corresponding to the scale factor $a = 1$ in Eq. (10),

$$t_0 = \frac{2}{3H_0}\frac{1}{\sqrt{\Omega_\Lambda}}\ln\left(\frac{1+\sqrt{\Omega_\Lambda}}{\sqrt{\Omega_M}}\right). \tag{11}$$

It can be estimated with the predicted proportion of cosmological components.

In order to get the variation rate of α with respect to proper time, $\Delta\alpha/\alpha$ should be represented as a function of proper time. The inverse solution of Eq. (10) gives the scale factor a varying with proper time,

$$a(t) = \left(\frac{\Omega_M}{\Omega_\Lambda}\right)^{\frac{1}{3}}\left[\sinh\left(\frac{3}{2}H_0\sqrt{\Omega_\Lambda}t\right)\right]^{\frac{2}{3}}. \tag{12}$$

Equation (9) can be expressed as a function of proper time with Eq. (12),

$$\frac{\Delta\alpha}{\alpha} = A\left\{\left[\sqrt{\Omega_\Lambda}\coth\left(\frac{3}{2}H_0\sqrt{\Omega_\Lambda}t\right)\right]^I - 1\right\}. \tag{13}$$

Then, the variation rate of α with proper time is given by

$$\frac{\dot{\alpha}}{\alpha_0} = -\frac{3}{2}AIH_0\Omega_\Lambda^{\frac{I+1}{2}}\frac{\cosh^{(I-1)}\left(\frac{3}{2}H_0\sqrt{\Omega_\Lambda}t\right)}{\sinh^{(I+1)}\left(\frac{3}{2}H_0\sqrt{\Omega_\Lambda}t\right)}, \tag{14}$$

$$\left.\frac{\dot{\alpha}}{\alpha_0}\right|_{t=t_0} = -\frac{3}{2}AIH_0\Omega_M. \tag{15}$$

The $\dot{\alpha}/\alpha$ in the condition $t = t_0$ corresponds to the current variation rate of α. It can be estimated after determining the parameters A, I and the current matter density Ω_M.

III. RESULTS

In this section, we will use the model and the experimental data of varying α to get the proportion of cosmological components and the current age of the universe, and estimate the variation rate of α and the constraint in the early universe.

We take the experimental data of the absorption lines in quasar spectra to fit the parameters in the model. The data set contains the 293 samples published in Ref. [17] (the two outliers have been removed), the 20 samples published in Ref. [18] (14 of them were remeasurements of points already in Ref. [17] and were taken priority), the 21 samples published in the Ref. [43], and the 4 new high-redshift samples published in Ref. [19]. Most of the measurements are observed at the Keck telescope and VLT (Very Large Telescope). The Keck telescope is situated in the northern hemisphere (at ~20°N) and mainly observes the northern sky, while the VLT is situated in the southern hemisphere (at ~25°S) and mainly observes the southern sky. Since the statistical significance of the dipole of the Keck samples is much lower than that of the VLT samples, the Keck samples are less affected by the spatial anisotropy than the VLT samples [17]. Therefore, we intend to get a preliminary verification of the model with the Keck samples and then obtain the final results by combing all the samples.

A. The prediction of cosmological components

In order to avoid that results are overly sensitive to an extreme sample, we follow the weighted mean method to divide all the Keck samples into 12 bins and approximately 12 points contribute to each bin [17]. Meanwhile, to make the predictions as reliable as possible, we intend to fix the power I to further simplify the parameter space. By comparing the different values, the model has a simple form and a relatively optimal fitting result when the power I is equal to -2. It means that the term which provides the greatest contribution to the variation rate of α is $\tilde{H}^{-3}\dot{\tilde{H}}$ in Eq. (6). Equation (7) can be expressed as

$$\frac{\dot{\alpha}}{\alpha_0} \propto \tilde{H}^{-3}\dot{\tilde{H}} \propto (\dot{\tilde{H}^{-2}}) \propto (\dot{\rho^{-1}}), \tag{16}$$

where ρ is the total density of the universe. The variation rate of α is proportional to the variation rate of the inverse of ρ. After substituting Ω_M with $1-\Omega_\Lambda$, Eq. (9) is simplified to

$$\frac{\Delta\alpha}{\alpha} = A[((1-\Omega_\Lambda)(1+z)^3+\Omega_\Lambda)^{-1} - 1]. \tag{17}$$

There are only two undetermined parameters A and Ω_Λ in the model. The best fit of the parameters in Eq. (9) is determined by minimizing

$$\chi^2 = \sum_i \frac{[(\frac{\Delta\alpha}{\alpha})_{\mathrm{pre},i} - (\frac{\Delta\alpha}{\alpha})_{\mathrm{exp},i}]^2}{\sigma^2_{\mathrm{exp},i}}, \tag{18}$$

where $(\frac{\Delta\alpha}{\alpha})_{\mathrm{pre},i}$ and $(\frac{\Delta\alpha}{\alpha})_{\mathrm{exp},i}$ are the predicted value and the experimental value, respectively. $\sigma_{\mathrm{exp},i}$ is the associated 1σ error of the experimental data, where $\sigma^2_{\mathrm{exp},i} = \sigma^2_{\mathrm{stat},i} + \sigma^2_{\mathrm{rand},i}$. $\sigma_{\mathrm{stat},i}$ is the uncertainty estimate of each $\Delta\alpha/\alpha$ value. $\sigma_{\mathrm{rand},i}$ is an estimate of the aggregation of any errors which average to zero over a large number of systems and is in quadrature with $\sigma_{\mathrm{stat},i}$ [17].

Figure 1 shows that the scatter plot $\Delta\alpha/\alpha$ versus the quasar absorption redshift and our fitting results. The dark blue squares are the weighted mean values of the binned Keck samples and the black bars are the associated 1σ errors. We fit the experimental data by using Eq. (17). The minimal χ^2 of the fitting is 4.99, when the parameters Ω_Λ and A take the value 0.765 and 8.3×10^{-6}, respectively. The red curve is the optimal fitting. The light blue, yellow, and gray hatched region are the confidence intervals of

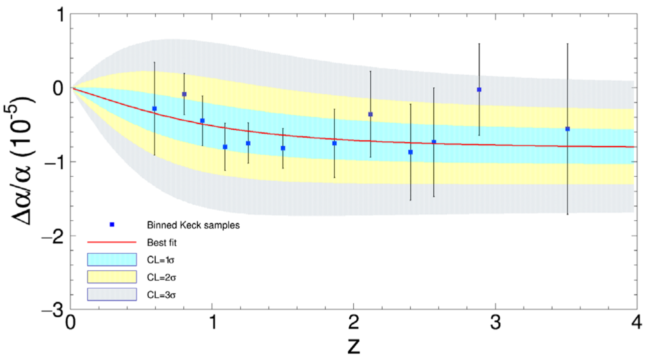

FIG. 1. Fitting the binned Keck samples with Eq. (17). The dark blue squares are the weighted mean values of the binned Keck samples and the black bars are the corresponding errors. The red curve is the optimal fitting. The minimal χ^2 of the fitting is 4.99, when the parameters Ω_Λ and A take the value 0.765 and 8.3×10^{-6}, respectively. The light blue, yellow, and gray hatched region are the confidence intervals for 1σ, 2σ, and 3σ, respectively.

$\Delta\alpha/\alpha$ for 1σ, 2σ, and 3σ, respectively. Figure 2 shows the χ^2 within 0.5 of its minimal value ($\chi^2 = 4.99$) in the (Ω_Λ, A) plane. There is a relatively obvious optimal region in the parameter space that points to Ω_Λ in the range [0.65, 0.85] and A in the range $[7.5, 9.0] \times 10^{-6}$. The optimal parameter point is unique in the whole space. We can see that the current proportion of dark energy obtained by the optimal fitting is bigger than the *Planck* 2018 results $\Omega_\Lambda = 0.6889 \pm 0.0056$ [44], but it is still roughly in the acceptable range. It suggests that the relation established in Eq. (17) is reasonable and the varying α could indeed be an effect of the cosmological expansion.

After getting the preliminary verification of the model with the Keck samples, we will further obtain the final results by combining all the samples. Intuitively, the trend of the VLT samples with redshift is different from that of the Keck samples at the redshift $z > 1.5$. But they will show the similar behavior after aligning the Keck and VLT samples. We analyze the seven samples measured simultaneously by the Keck and VLT telescopes and show the measurement deviations in the panel (a) in Fig. 3. It can be seen that the samples measured at $z < 2.0$ by the two telescopes are approximately consistent while the measurement deviations are large at $z > 2.0$. We consider that the Keck and VLT measurements are same at $z = 0$ and fit the deviations with a simple proportional function $\Delta\alpha/\alpha_{(\text{VLT-Keck})} = kz$. The slope given by the optimal fitting is $k = (0.52 \pm 0.30) \times 10^{-5}$. In order to further verify the conclusion with more data, we choose the Keck and VLT samples measured in the same direction $\theta = 0°$ and the angle of their directions $\theta < 1°$ with the difference of redshift $\Delta z < 0.1$, respectively. The deviations are shown in the panels (b) and (c) in Fig. 3. The slopes given by the optimal fittings are $k = (0.58 \pm 0.35) \times 10^{-5}$ and $k = (0.57 \pm 0.31) \times 10^{-5}$, respectively. All the analyses support that the deviations between Keck and VLT samples have the similar increasing trend with the redshift. After aligning the Keck and VLT samples with the proportional function $\Delta\alpha/\alpha_{(\text{VLT-Keck})} = (0.57 \pm 0.31) \times 10^{-5} z$, we can combine all the samples to obtain the final results.

We divide the four new high-redshift samples into one bin and then divide the remaining samples approximately evenly into nine bins. And we adjust the weights of the Keck and VLT samples in each bin to be equal except for

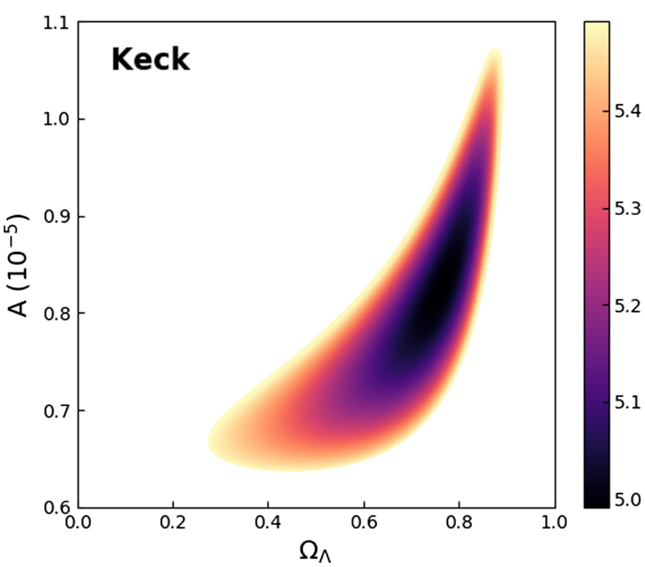

FIG. 2. The χ^2 of fitting the binned Keck samples within 0.5 of its minimal value ($\chi^2 = 4.99$) in the (Ω_Λ, A) plane.

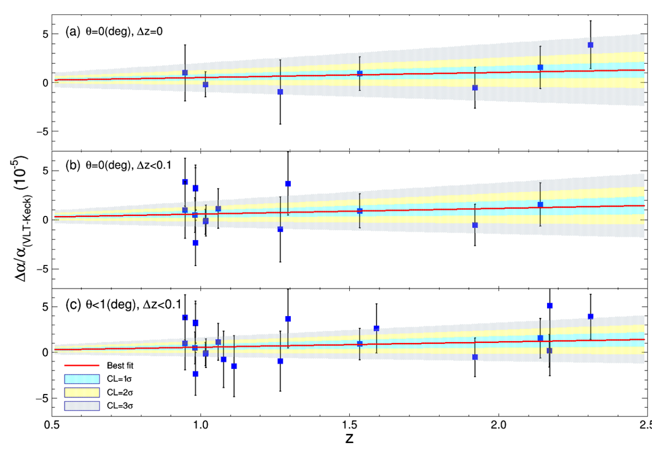

FIG. 3. The panel (a), (b), and (c) show the deviations between Keck and VLT samples measured in the same direction with same redshift, the same direction $\theta = 0°$ with the difference of redshift $\Delta z < 0.1$, and the angle of directions $\theta < 1°$ with the difference of redshift $\Delta z < 0.1$, respectively. The red lines are the optimal fittings with the proportional function $\Delta\alpha/\alpha_{(\text{VLT-Keck})} = kz$. The three slopes given by the optimal fittings are $k = (0.52 \pm 0.30) \times 10^{-5}$, $k = (0.58 \pm 0.35) \times 10^{-5}$, and $k = (0.57 \pm 0.31) \times 10^{-5}$. The light blue, yellow, and gray hatched region are the confidence intervals for 1σ, 2σ, and 3σ, respectively.

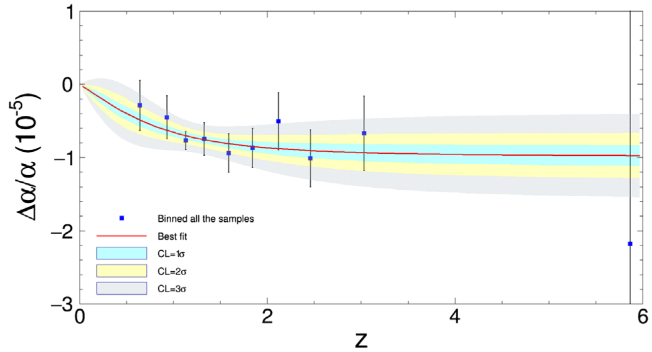

FIG. 4. Fitting the binned all the samples with Eq. (17). The dark blue squares are the weighted mean values of the binned samples and the black bars are the corresponding errors. The red curve is the optimal fitting. The minimal χ^2 of the fitting is 2.50, when the parameters Ω_Λ and A take the value 0.713 and 9.8×10^{-6}, respectively. The light blue, yellow, and gray hatched region are the confidence intervals for 1σ, 2σ, and 3σ, respectively..

the high-redshift bin where only VLT samples is available. By comparing the different values, the model has a simple form and a relatively optimal fitting result when the power I is still equal to -2. Similarly, Fig. 4 shows that the scatter plot $\Delta\alpha/\alpha$ versus the quasar absorption redshift and our optimal fitting results by using Eq. (17) for the binned all the samples. The minimal χ^2 of the fitting is 2.50, when the parameters Ω_Λ and A take the value 0.713 and 9.8×10^{-6}, respectively. The red curve is the optimal fitting and the light blue, yellow, and gray hatched region are the confidence intervals of $\Delta\alpha/\alpha$ for 1σ, 2σ, and 3σ, respectively. Figure 5 shows the χ^2 within 0.5 of its minimal value ($\chi^2 = 2.50$) in the (Ω_Λ, A) plane. There is a relatively obvious optimal region in the parameter space that points to Ω_Λ in the range [0.6, 0.8] and A in the range $[9.0, 10.5] \times 10^{-6}$. The optimal parameter point is also unique in the whole space. It can be seen that the current proportion of dark energy predicted by using all the samples much more strongly points to about 70% than that only using the Keck samples.

After determining the fitting parameter $\Omega_\Lambda = 0.713$, the current age of the universe estimated by Eq. (11) is about 13.61 billion years with the $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. These estimations are consistent with the Planck 2018 results ($\Omega_\Lambda = 0.6889 \pm 0.0056$ and $t_0 = 13.787 \pm 0.020$ Gyr) [44].

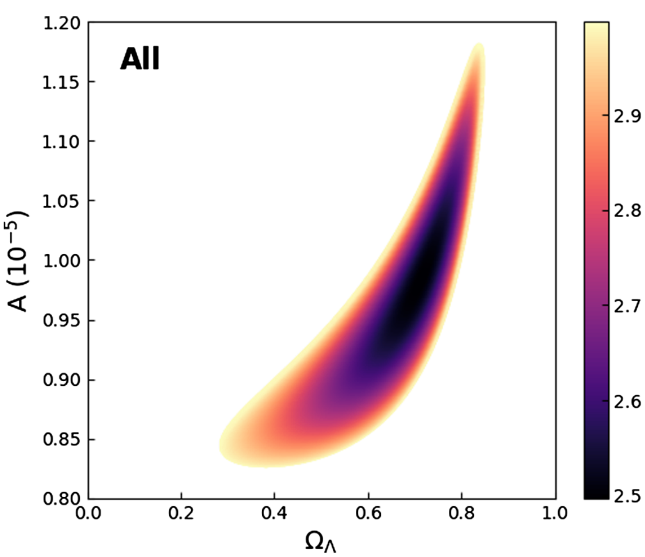

FIG. 5. The χ^2 of fitting the binned all the samples within 0.5 of its minimal value ($\chi^2 = 2.50$) in the (Ω_Λ, A) plane.

B. The variation rate of α

The variation rate of α can also be estimated with the model. Figure 6 shows that the $\Delta\alpha/\alpha$ varies as a function of the Hubble constant times proper time $H_0 t$, which is predicted in the present work with Eq. (13). For the early universe, Eq. (13) with $I = -2$ is approximately

$$\left.\frac{\Delta\alpha}{\alpha}\right|_{t\to 0} = -A = -9.8 \times 10^{-6}. \tag{19}$$

This place a very strict constraint on the variation of α in the early universe and is compatible with the upper limit $|\Delta\alpha/\alpha| < 10^{-2}$ given by the investigations of CMB, BAO, and BBN [21–23]. Then, α increases with the increasing of the proportion of dark energy and the variation rate accelerates first and then slows down. The blue point denotes the current universe. We estimate the current variation rate of α is about $6.0 \times 10^{-16}\ \mathrm{yr}^{-1}$ by using Eq. (15) and the growth rate of α is slowing down. The predicted current variation rate of α is close to the atomic clock measurements $(-1.6 \pm 2.3) \times 10^{-17}\ \mathrm{yr}^{-1}$ [3]. In the future, the expansion rate of the universe will tend to be stable and α will hardly change any more when the universe is almost entirely composed of dark energy. Equation (13) with $I = -2$ is approximately

$$\left.\frac{\Delta\alpha}{\alpha}\right|_{t\to +\infty} = A(\Omega_\Lambda^{-1} - 1) = 3.9 \times 10^{-6}. \tag{20}$$

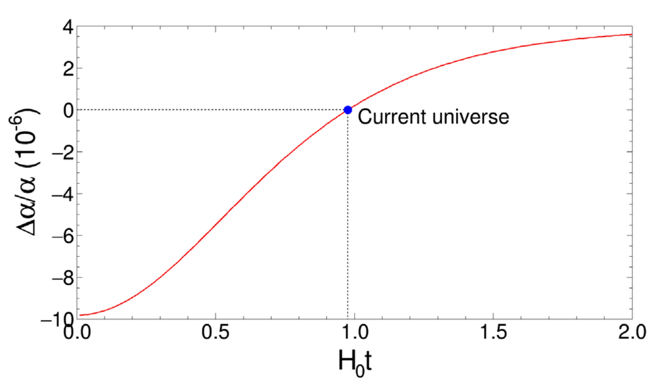

FIG. 6. The curve of $\Delta\alpha/\alpha$ varying with $H_0 t$ is predicted with Eq. (13). The blue point denotes the current universe.

The α, which eventually stabilizes, will be 3.9×10^{-6} bigger than the current value.

All the above discussion is about the evolution of α over time based on a spatially flat, homogeneous, and isotropic universe at large scale. However, the universe is inhomogeneous at smaller scale and its expansion rate is also a function of spatial coordinates [45]. Therefore, α will show the anisotropy of space at a smaller scale. The measurement of the anisotropy of Hubble parameter could serve as a test of the anisotropy of α in the future.

IV. CONCLUSIONS

In summary, the observations for the cosmological evolution of α are believed to have great consequences for fundamental physics, especially for grand unification and theories of the early universe. The implications for cosmology of a varying α are currently examined by physicists and astronomers. In the present work, we propose a model based on the assumption that the varying α is an effect of the cosmological expansion and establish a relation between the varying α and the cosmological components. For a spatially flat, homogeneous, and isotropic universe, we predict that the current proportion of matter and dark energy are 28.7% and 71.3%, respectively, and further estimate that the current age of the universe is about 13.61 billion years with the Hubble constant $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. These results are consistent with the current cosmological observations. Furthermore, we predict a strict constraint $\Delta\alpha/\alpha = -9.8 \times 10^{-6}$ for the early universe, which is compatible with the upper limit given by the investigations of CMB and BBN. These suggest that the established relation is reasonable and the varying α could indeed be an effect of the varying expansion rate of the universe. Then, we further estimate the current variation rate of α is about $6.0 \times 10^{-16}\ \mathrm{yr^{-1}}$ and predict the growth rate of α is slowing down. The finally stable α will be 3.9×10^{-6} bigger than the current value. The clearer correlations await the accumulation of experimental data in the future.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grant No. 11490563, the Continuous Basic Scientific Research Project under Grant No. WDJC-2019-13, and the National Key Research and Development Project under Grant No. 2016YFA0400502.

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