Tetraquarks with open charm flavor

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Inspired by the recent report of the exotic states $X_0(2900)$ and $X_1(2900)$ with four different quark flavors in the D^-K^+ invariant-mass distributions of the decay process $B^+ \to D^+D^-K^+$ by the LHCb Collaboration, we systematically investigate the tetraquarks composed of $ud\bar{s}\bar{c}$ with meson-meson and diquark-antidiquark structures in the quark delocalization color-screening model. We find that the $X_0(2900)$ can be interpreted as the molecular state $\bar{D}^*K^*$ with $IJ^P = 00^+$. Moreover, two bound states are obtained by the channel coupling calculation, with energies 2341.2 MeV for $IJ^P = 00^+$ and 2489.7 MeV for $IJ^P = 01^+$, respectively. We also extend our study to the $uc\bar{d}\bar{s}$ systems and find that there is no S-wave bound state, so the $D_{s0}(2317)$ cannot be identified as the DK molecular state in the present calculation. Besides, several resonance states with the diquark-antidiquark configuration are possible in both $ud\bar{s}\bar{c}$ and $uc\bar{d}\bar{s}$ systems. The states composed of $ud\bar{s}\bar{b}$ are also possible open bottom tetraquark candidates. All of these open charm and open bottom bound states and resonances are worth investigating in future experiments.

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I. INTRODUCTION

The search for exotic states beyond conventional hadron configurations is a long-standing challenge in hadron physics. So far, many tetraquark and pentaquark candidates have been proposed, and most of them are composed of hidden charm or bottom quarks. Very recently, the LHCb Collaboration reported the discovery of two new exotic structures, $X_0(2900)$ and $X_1(2900)$, in the D^-K^+ invariant-mass distributions of the decay process $B^+ \to D^+D^-K^+$ [1]. Since they are observed in the D^-K^+ channel, the lowest quark content of these two states should be $ud\bar{s}\bar{c}$, which implies that both $X_0(2900)$ and $X_1(2900)$ could be open charm tetraquarks. Their spin-parity quantum numbers are $J^P = 0^+$ and 1^-, respectively, and their masses and widths are

$$M_{X_0(2900)} = 2.866 \pm 0.007 \text{ GeV},$$
$$\Gamma_{X_0(2900)} = 57.2 \pm 12.9 \text{ MeV},$$
$$M_{X_1(2900)} = 2.904 \pm 0.005 \text{ GeV},$$
$$\Gamma_{X_0(2900)} = 110.3 \pm 11.5 \text{ MeV}.$$

Motivated by the LHCb observation, a lot of theoretical works have attempted to explain these two exotic states [2–12]. In the very recent work in Ref. [2], the $X_0(2900)$ was interpreted as a $cs\bar{u}\bar{d}$ isosinglet compact tetraquark with mass 2863 ± 12 MeV, and the analogous $bs\bar{u}\bar{d}$ tetraquark was predicted at 6213 ± 12 MeV. These two open charm tetraquarks are also observed in the framework of QCD sum rules [6,9,11], the extended relativized quark model [7], the one-boson exchange model [8,10], and so on. A nonresonance explanation—the triangle singularity mechanism—was also proposed [5]. Before the LHCb observation, several novel exotic charmed mesons were predicted using a coupled channel unitary approach in Ref. [13], where a $C = 1$, $S = -1$, $I = 0$, $J^P = 0^+$ state was obtained with mass 2848 MeV and width 23–59 MeV. Once the discovery of the $X_0(2900)$ and $X_1(2900)$ is confirmed, a new exotic state with four different quark flavors will be verified and it will help us to understand the low-energy behavior of the QCD and the nature of the strong interactions.

In fact, the $X(5568)$ has been proposed as an exotic state with open flavors $us\bar{d}\bar{b}$ or $ds\bar{u}\bar{b}$ by the D0 Collaboration [14]. Unfortunately, this state was not confirmed by other

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collaborations, including the LHCb Collaboration [15], CMS Collaboration [16], CDF Collaboration [17] and ATLAS Collaboration [18]. At the same time, another tetraquark with four different quark flavors $ud\bar{s}\bar{b}$ (or its charge conjugate) was proposed, which could definitely be observed via the weak decay mode $J/\psi K^- K^- \pi^+$ [19]. We have investigated tetraquarks composed of $us\overline{db}$ and $ud\bar{s}\bar{b}$ in the framework of the quark delocalization color-screening model (QDCSM) [20] and found that the $X(5568)$ cannot be explained as a molecular state or a diquark-antidiquark resonance of $us\bar{d}\bar{b}$. Nevertheless, two tetraquarks composed of $ud\bar{s}\bar{b}$ were obtained with a diquark-antidiquark structure. So, it is more likely that tetraquarks composed of $ud\bar{s}\bar{b}$ will form bound states than one composed of $us\bar{d}\bar{b}$.

Because of the heavy flavor symmetry, it is natural to extend the study to the tetraquarks composed of $ud\bar{s}\bar{c}$ and $uc\bar{d}\bar{s}$. The aims of this work are as follows. (1) We study two structures of the open charm tetraquarks to see if the newly reported $X_0(2900)$ and $X_1(2900)$ can be explained as open charm tetraquarks in the constituent quark model, and explore the structure of these two states. (2) We perform a systemic search of the open charm tetraquark systems to check if there are any other open charm tetraquarks. For example, the $D^*_{s0}(2317)$, first observed by the *BABAR* Collaboration [21], appears as a very narrow resonance below the DK threshold and decays to $D_s^+\pi^0$. One common interpretation is that it may be a DK molecular state. Very recently, lattice QCD studies observed the DK and $D\bar{K}$ scattering process and found a near-threshold $IJ^P = 00^+$ bound state DK, corresponding to the $D^*_{s0}(2317)$ [22]. Thus, it is also interesting to see whether there is any bound state below the threshold of DK, which may be used to explain the $D^*_{s0}(2317)$ in the quark approach.

The structure of the paper is as follows. A brief introduction of the quark model and wave functions are given in Sec. II. Section III is devoted to the numerical results and discussions. Our summary is given in the last section.

II. MODEL AND WAVE FUNCTIONS

A. The model (QDCSM)

The QDCSM has been widely described in the literature [23,24], and we refer the reader to those works for the details. Here we just present the salient features of the model. The Hamiltonian of the QDCSM includes three parts: the rest masses of quarks, the kinetic energy, and the interaction potentials. The potentials are composed of the color confinement (CON), the one-gluon exchange (OGE), and the one-Goldstone-boson exchange (OBE). The detailed form of the tetraquark system is shown below:

$$H = \sum_{i=1}^{4}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{CM} + \sum_{j>i=1}^{4} V_{ij}, \tag{1}$$

$$V_{ij} = V_{ij}^{\mathrm{CON}} + V_{ij}^{\mathrm{OGE}} + V_{ij}^{\mathrm{OBE}}, \tag{2}$$

$$V_{ij}^{\mathrm{CON}} = -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c (f(r_i j) + a_{ij}^0), \tag{3}$$

$$f(r_{ij}) = \begin{cases} r_{ij}^2 & \text{if } i, j \text{ are in the same cluster,} \\ \frac{1-e^{-\mu_{ij} r_{ij}^2}}{\mu_{ij}} & \text{otherwise,} \end{cases} \tag{4}$$

$$V_{ij}^{\mathrm{OGE}} = \frac{1}{4}\alpha_s \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left[\frac{1}{r_{ij}} - \frac{\pi}{2}\delta(\mathbf{r}_{ij})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{3m_i m_j}\right) - \frac{3}{4m_i m_j r_{ij}^3} S_{ij}\right], \tag{5}$$

$$V_{ij}^{\mathrm{OBE}} = V_\pi(\mathbf{r}_{ij}) \sum_{a=1}^{3} \lambda_i^a \cdot \lambda_j^a + V_K(\mathbf{r}_{ij}) \sum_{a=4}^{7} \lambda_i^a \cdot \lambda_j^a + V_\eta(\mathbf{r}_{ij})[(\lambda_i^8 \cdot \lambda_j^8)\cos\theta_P - (\lambda_i^0 \cdot \lambda_j^0)\sin\theta_P], \tag{6}$$

$$V_\chi(\boldsymbol{r}_{ij}) = \frac{g_{\mathrm{ch}}^2}{4\pi}\frac{m_\chi^2}{12 m_i m_j}\frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2} m_\chi \left\{\left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij})\right]\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \left[H(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} H(\Lambda_\chi r_{ij})\right] S_{ij}\right\}, \qquad \chi = \pi, K, \eta, \tag{7}$$

$$S_{ij} = \left\{3\frac{(\boldsymbol{\sigma}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{\sigma}_j \cdot \boldsymbol{r}_{ij})}{r_{ij}^2} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right\}, \tag{8}$$

$$H(x) = (1 + 3/x + 3/x^2)Y(x), \qquad Y(x) = e^{-x}/x, \tag{9}$$

where S_{ij} is the quark tensor operator, $Y(x)$ and $H(x)$ are standard Yukawa functions, T_c is the kinetic energy of the center of mass, α_s is the quark-gluon coupling constant, and g_{ch} is the coupling constant for the chiral field, which is determined from the $NN\pi$ coupling constant through

$$\frac{g_{\mathrm{ch}}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{\pi NN}^2}{4\pi}\frac{m_{u,d}^2}{m_N^2}. \tag{10}$$

The other symbols in the above expressions have their usual meanings. All model parameters are determined by fitting the meson spectrum and are shown in Table I. The calculated masses of the mesons in comparison with experimental values are shown in Table II.

The quark delocalization in the QDCSM is realized by specifying the single-particle orbital wave function of the QDCSM as a linear combination of left and right

TABLE I. Model parameters: $m_\pi = 0.7$ fm^{-1}, $m_K = 2.51$ fm^{-1}, $m_\eta = 2.77$ fm^{-1}, $\Lambda_\pi = 4.2$ fm^{-1}, $\Lambda_K = \Lambda_\eta = 5.2$ fm^{-1}, $g^2_{ch}/(4\pi) = 0.54$, $\theta_p = -15^0$.

b (fm)	m_u (MeV)	m_d (MeV)	m_s (MeV)	m_c (MeV)	m_b (MeV)
0.518	313	313	470	1270	4500

a_c (MeV fm^{-2})	a^0_{uu} (fm^2)	a^0_{us} (fm^2)	a^0_{uc} (fm^2)	a^0_{sc} (fm^2)	a^0_{ub} (fm^2)	a^0_{sb} (fm^2)
58.03	−0.733	−0.309	1.278	1.358	1.701	1.808
$\alpha_{s_{uu}}$	$\alpha_{s_{us}}$	$\alpha_{s_{uc}}$	$\alpha_{s_{sc}}$	$\alpha_{s_{ub}}$	$\alpha_{s_{sb}}$	
1.50	1.46	1.450	1.44	1.41	1.40	

TABLE II. Masses (in MeV) of the mesons obtained from the QDCSM. Experimental values are taken from the Particle Data Group [25].

Meson	M_{the}	M_{exp}
π	140	140
ρ	772	770
D	1865	1869
D^*	2008	2008
D_s	1968	1968
D_s^*	2062	2112
K	495	495
K^*	892	892
B	5280	5280
B^*	5319	5325
B_s	5367	5367
B_s^*	5393	5415

Gaussians, which are the single-particle orbital wave functions used in the ordinary quark cluster model,

$$\begin{aligned}
\psi_\alpha(\mathbf{s}_i,\epsilon) &= (\phi_\alpha(\mathbf{s}_i) + \epsilon\phi_\alpha(-\mathbf{s}_i))/N(\epsilon),\\
\psi_\beta(-\mathbf{s}_i,\epsilon) &= (\phi_\beta(-\mathbf{s}_i) + \epsilon\phi_\beta(\mathbf{s}_i))/N(\epsilon),\\
N(\epsilon) &= \sqrt{1+\epsilon^2+2\epsilon e^{-s_i^2/4b^2}},\\
\phi_\alpha(\mathbf{s}_i) &= \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\mathbf{r}_\alpha - \mathbf{s}_i/2)^2},\\
\phi_\beta(-\mathbf{s}_i) &= \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\mathbf{r}_\beta + \mathbf{s}_i/2)^2}.
\end{aligned} \tag{11}$$

Here $\mathbf{s}_i$, $i = 1, 2, \ldots, n$ are the generating coordinates, which are introduced to expand the relative motion wave function [24]. The delocalization parameter $\epsilon(\mathbf{s}_i)$ is not adjustable, but rather is determined variationally by the dynamics of the multiquark system itself. In this way, the multiquark system can choose its favorable configuration in a larger Hilbert space.

B. Wave functions

In this work we study the tetraquark systems in two structures: the meson-meson structure and the diquark-antidiquark structure. The resonating-group method (RGM) [26], a well-established method for studying a bound state or a scattering problem, is used to calculate the energy of all of these states. The wave function of the four-quark system is of the form

$$\Psi = \mathcal{A}[[\psi^L\psi^\sigma]_{JM}\psi^f\psi^c], \tag{12}$$

where ψ^L, ψ^σ, ψ^f, and ψ^c are the orbital, spin, flavor, and color wave functions, respectively, which are shown below. The symbol $\mathcal{A}$ is the antisymmetrization operator. For the meson-meson structure of $u\bar{s} - d\bar{c}$, $\mathcal{A}$ is defined as

$$\mathcal{A} = 1 - P_{13}, \tag{13}$$

for the $u\bar{d} - c\bar{s}$ it is

$$\mathcal{A} = 1 - P_{24}, \tag{14}$$

and for the diquark-antidiquark structure $ud - \bar{s}\bar{c}$ it is

$$\mathcal{A} = 1 - P_{12}. \tag{15}$$

The orbital wave function is the same in two configurations, and the spin wave functions are the same too. However, the flavor and color wave functions are constructed differently for different structures.

1. Orbital wave function

The orbital wave function is of the form

$$\psi^L = \psi_1(\boldsymbol{R}_1)\psi_2(\boldsymbol{R}_2)\chi_L(\boldsymbol{R}). \tag{16}$$

where $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are the internal coordinates for cluster 1 and cluster 2. $\boldsymbol{R} = \boldsymbol{R}_1 - \boldsymbol{R}_2$ is the relative coordinate between the two clusters 1 and 2. ψ_1 and ψ_2 are the internal cluster orbital wave functions of clusters 1 and 2, which are fixed in the calculation, and $\chi_L(\boldsymbol{R})$ is the relative motion wave function between the two clusters, which is expanded in the Gaussian basis as

$$\chi_L(\boldsymbol{R}) = \frac{1}{\sqrt{4\pi}}\left(\frac{3}{2\pi b^2}\right)\sum_{i=1}^{n} C_i \times \int \exp\left[-\frac{3}{4b^2}(\boldsymbol{R}-\boldsymbol{s}_i)^2\right] Y_{LM}(\hat{\boldsymbol{s}}_i) d\hat{\boldsymbol{s}}_i, \quad (17)$$

where $\boldsymbol{s}_i$ are the generating coordinates and n is the number of Gaussian bases, which is determined by the stability of the results. By doing this, the integro-differential equation of the RGM can be reduced to an algebraic equation, i.e., a generalized eigenequation. Then, we can obtain the energy of the system by solving this generalized eigenequation. The details of solving the RGM equation can be found in Ref. [26].

2. Flavor wave function

For the meson-meson configuration, as a first step we write the wave functions of the meson cluster as

$$\begin{aligned}
&\chi^{I1}_{00} = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}), \qquad \chi^{I2}_{10} = \frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u}),\\
&\chi^{I3}_{\frac{1}{2}\frac{1}{2}} = u\bar{c}, \qquad \chi^{I4}_{00} = c\bar{s}, \qquad \chi^{I5}_{\frac{1}{2}\frac{1}{2}} = u\bar{s},\\
&\chi^{I6}_{\frac{1}{2}\frac{1}{2}} = c\bar{d}, \qquad \chi^{I7}_{\frac{1}{2}-\frac{1}{2}} = d\bar{s}. \qquad \chi^{I8}_{\frac{1}{2}-\frac{1}{2}} = d\bar{c},\\
&\chi^{I9}_{\frac{1}{2}-\frac{1}{2}} = -c\bar{u},
\end{aligned} \quad (18)$$

where the superscript of χ is the index of the flavor wave function for a meson, and the subscript stands for the isospin I and the third component I_z. The flavor wave functions with the meson-meson structure are

$$\begin{aligned}
&\psi^{f_1}_{00} = \chi^{I4}_{00}\chi^{I1}_{00}, \qquad \psi^{f_2}_{11} = \chi^{I4}_{10}\chi^{I2}_{10},\\
&\psi^{f_3}_{00} = \sqrt{\frac{1}{2}}\left[\chi^{I6}_{\frac{1}{2}\frac{1}{2}}\chi^{I7}_{\frac{1}{2}-\frac{1}{2}} - \chi^{I9}_{\frac{1}{2}-\frac{1}{2}}\chi^{I5}_{\frac{1}{2}\frac{1}{2}}\right],\\
&\psi^{f_4}_{11} = \sqrt{\frac{1}{2}}\left[\chi^{I6}_{\frac{1}{2}\frac{1}{2}}\chi^{I7}_{\frac{1}{2}-\frac{1}{2}} + \chi^{I9}_{\frac{1}{2}-\frac{1}{2}}\chi^{I5}_{\frac{1}{2}\frac{1}{2}}\right],\\
&\psi^{f_5}_{00} = \sqrt{\frac{1}{2}}\left[\chi^{I3}_{\frac{1}{2}\frac{1}{2}}\chi^{I7}_{\frac{1}{2}-\frac{1}{2}} - \chi^{I8}_{\frac{1}{2}\frac{1}{2}}\chi^{I5}_{\frac{1}{2}\frac{1}{2}}\right],\\
&\psi^{f_6}_{11} = \sqrt{\frac{1}{2}}\left[\chi^{I3}_{\frac{1}{2}\frac{1}{2}}\chi^{I7}_{\frac{1}{2}-\frac{1}{2}} + \chi^{I8}_{\frac{1}{2}\frac{1}{2}}\chi^{I5}_{\frac{1}{2}\frac{1}{2}}\right].
\end{aligned} \quad (19)$$

For the diquark-antidiquark configuration, we first show the functions of the diquark and antidiquark, respectively:

$$\begin{aligned}
&\chi^{I1}_{10} = \frac{1}{\sqrt{2}}(ud+du), \qquad \chi^{I2}_{00} = \frac{1}{\sqrt{2}}(ud-du),\\
&\chi^{I3}_{\frac{1}{2}\frac{1}{2}} = cu, \qquad \chi^{I4}_{\frac{1}{2}-\frac{1}{2}} = cd,\\
&\chi^{I5}_{\frac{1}{2}-\frac{1}{2}} = -\bar{s}\bar{u}, \qquad \chi^{I6}_{\frac{1}{2}\frac{1}{2}} = \bar{s}\bar{d},\\
&\chi^{I7}_{00} = \bar{c}\bar{s}.
\end{aligned} \quad (20)$$

Then, the flavor wave functions for the diquark-antidiquark structure can be obtained by coupling the wave functions of two clusters,

$$\begin{aligned}
&\psi^{f_1}_{00} = \sqrt{\frac{1}{2}}\left[\chi^{I3}_{\frac{1}{2}\frac{1}{2}}\chi^{I5}_{\frac{1}{2}-\frac{1}{2}} - \chi^{I4}_{\frac{1}{2}-\frac{1}{2}}\chi^{I6}_{\frac{1}{2}\frac{1}{2}}\right],\\
&\psi^{f_2}_{10} = \sqrt{\frac{1}{2}}\left[\chi^{I3}_{\frac{1}{2}\frac{1}{2}}\chi^{I5}_{\frac{1}{2}-\frac{1}{2}} + \chi^{I4}_{\frac{1}{2}-\frac{1}{2}}\chi^{I6}_{\frac{1}{2}\frac{1}{2}}\right],\\
&\psi^{f_3}_{11} = \chi^{I2}_{00}\chi^{I7}_{00}, \qquad \psi^{f_4}_{00} = \chi^{I1}_{10}\chi^{I7}_{00}.
\end{aligned} \quad (21)$$

3. Spin wave function

The spin wave function of a meson cluster is

$$\begin{aligned}
&\chi^{\sigma1}_{11} = \alpha\alpha, \qquad \chi^{\sigma2}_{10} = \sqrt{\frac{1}{2}}(\alpha\beta+\beta\alpha),\\
&\chi^{\sigma3}_{1-1} = \beta\beta, \qquad \chi^{\sigma4}_{00} = \sqrt{\frac{1}{2}}(\alpha\beta-\beta\alpha).
\end{aligned} \quad (22)$$

Then, the spin wave functions of the four-quark system are

$$\begin{aligned}
&\psi^{\sigma_1}_{00} = \chi^{\sigma4}_{00}\chi^{\sigma4}_{00},\\
&\psi^{\sigma_2}_{00} = \sqrt{\frac{1}{3}}[\chi^{\sigma1}_{11}\chi^{\sigma3}_{1-1} - \chi^{\sigma2}_{10}\chi^{\sigma2}_{10} + \chi^{\sigma3}_{1-1}\chi^{\sigma1}_{11}],\\
&\psi^{\sigma_3}_{11} = \chi^{\sigma4}_{00}\chi^{\sigma1}_{11}, \qquad \psi^{\sigma_4}_{11} = \chi^{\sigma1}_{11}\chi^{\sigma4}_{00},\\
&\psi^{\sigma_5}_{11} = \sqrt{\frac{1}{2}}[\chi^{\sigma1}_{11}\chi^{\sigma2}_{10} - \chi^{\sigma2}_{10}\chi^{\sigma1}_{11}],\\
&\psi^{\sigma_6}_{22} = \chi^{\sigma1}_{11}\chi^{\sigma1}_{11}.
\end{aligned} \quad (23)$$

4. Color wave function

The color wave function of a meson cluster is

$$\chi^{1}_{[111]} = \sqrt{\frac{1}{3}}(r\bar{r}+g\bar{g}+b\bar{b}), \quad (24)$$

and the wave function of the four-quark system with the meson-meson structure is

$$\psi^{c_1} = \chi^{1}_{[111]}\chi^{1}_{[111]}. \quad (25)$$

For the diquark-antidiquark structure, the color wave functions of the diquark clusters are

$$\begin{aligned}
&\chi^{1}_{[2]} = rr, \qquad \chi^{2}_{[2]} = \frac{1}{\sqrt{2}}(rg+gr), \qquad \chi^{3}_{[2]} = gg,\\
&\chi^{4}_{[2]} = \frac{1}{\sqrt{2}}(rb+br), \qquad \chi^{5}_{[2]} = \frac{1}{\sqrt{2}}(gb+bg), \qquad \chi^{6}_{[2]} = bb,\\
&\chi^{7}_{[11]} = \frac{1}{\sqrt{2}}(rg-gr), \qquad \chi^{8}_{[11]} = \frac{1}{\sqrt{2}}(rb-br),\\
&\chi^{9}_{[11]} = \frac{1}{\sqrt{2}}(gb-bg),
\end{aligned} \quad (26)$$

and the color wave functions of the antidiquark clusters are

$$\chi^1_{[22]} = \bar{r}\bar{r}, \quad \chi^2_{[22]} = -\frac{1}{\sqrt{2}}(\bar{r}\bar{g}+\bar{g}\bar{r}), \quad \chi^3_{[22]} = \bar{g}\bar{g},$$
$$\chi^4_{[22]} = \frac{1}{\sqrt{2}}(\bar{r}\bar{b}+\bar{b}\bar{r}), \quad \chi^5_{[22]} = -\frac{1}{\sqrt{2}}(\bar{g}\bar{b}+\bar{b}\bar{g}), \quad \chi^6_{[22]} = \bar{b}\bar{b},$$
$$\chi^7_{[211]} = \frac{1}{\sqrt{2}}(\bar{r}\bar{g}-\bar{g}\bar{r}), \quad \chi^8_{[211]} = -\frac{1}{\sqrt{2}}(\bar{r}\bar{b}-\bar{b}\bar{r}),$$
$$\chi^9_{[211]} = \frac{1}{\sqrt{2}}(\bar{g}\bar{b}-\bar{b}\bar{g}). \tag{27}$$

Then, the wave functions for the four-quark system with the diquark-antidiquark structure can be obtained by coupling the wave functions of the diquark and antidiquark clusters, which are

$$\psi^{c_1} = \sqrt{\frac{1}{6}}\Big[\chi^1_{[2]}\chi^1_{[22]} - \chi^2_{[2]}\chi^2_{[22]} + \chi^3_{[2]}\chi^3_{[22]} + \chi^4_{[2]}\chi^4_{[22]} - \chi^5_{[2]}\chi^5_{[22]} + \chi^6_{[2]}\chi^6_{[22]}\Big],$$
$$\psi^{c_2} = \sqrt{\frac{1}{3}}\Big[\chi^7_{[11]}\chi^7_{[211]} - \chi^8_{[11]}\chi^8_{[211]} + \chi^9_{[11]}\chi^9_{[211]}\Big]. \tag{28}$$

Finally, by multiplying the wave functions ψ^L, ψ^σ, ψ^f, and ψ^c according to the definite quantum number of the system, we can acquire the total wave functions.

III. RESULTS AND DISCUSSIONS

In the present work we investigate tetraquarks with two kinds of quark components: $ud\bar{s}\bar{c}$ and $uc\bar{d}\bar{s}$. We consider two structures: meson-meson and diquark-antidiquark. The quantum numbers of the tetraquarks we study here are $I = 0, 1$, $J = 0, 1, 2$ and the parity is $P = +$. All of the orbital angular momenta are set to zero because we are interested in the ground states in this work. To check whether or not there is any bound state in such a tetraquark system, we do a dynamic bound-state calculation. Both the single-channel and channel-coupling calculations are carried out in this work. All of the general features of the calculated results are as follows.

A. Tetraquarks $ud\bar{s}\bar{c}$

For tetraquarks composed of $ud\bar{s}\bar{c}$, the possible quantum numbers are $IJ^P = 00^+$, 01^+, 02^+, 10^+, 11^+ and 12^+. The energies of the meson-meson structure are listed in Table III, where the second column gives the indices of the wave functions of every channel, and the wave functions for each degree of freedom are from Eqs. (17), (21), and (23). The third column is the corresponding channel. $E_{\rm th}$ denotes the theoretical threshold of every channel, and $E_{\rm sc}$ and $E_{\rm cc}$ represent the energies of the single-channel and channel-coupling calculation, respectively.

TABLE III. The energies (in MeV) of the meson-meson structure for tetraquarks $ud\bar{s}\bar{c}$.

	$[\psi^{f_i}\psi^{\sigma_j}\psi^{c_k}]$	Channel	$E_{\rm th}$	$E_{\rm sc}$	$E_{\rm cc}$
$IJ^P = 00^+$	$[\psi^{f_5}\psi^{\sigma_1}\psi^{c_1}]$	$\bar{D}K$	2360.0	2367.0	2341.2
	$[\psi^{f_5}\psi^{\sigma_2}\psi^{c_1}]$	$\bar{D}^*K^*$	2900.5	2820.7	
$IJ^P = 01^+$	$[\psi^{f_5}\psi^{\sigma_4}\psi^{c_1}]$	$\bar{D}^*K$	2503.0	2509.6	2489.7
	$[\psi^{f_5}\psi^{\sigma_3}\psi^{c_1}]$	$\bar{D}K^*$	2757.5	2761.3	
	$[\psi^{f_5}\psi^{\sigma_5}\psi^{c_1}]$	$\bar{D}^*K^*$	2900.5	2904.2	
$IJ^P = 02^+$	$[\psi^{f_5}\psi^{\sigma_5}\psi^{c_1}]$	$\bar{D}^*K^*$	2900.5	2908.4	
$IJ^P = 10^+$	$[\psi^{f_6}\psi^{\sigma_1}\psi^{c_1}]$	$\bar{D}K$	2360.0	2369.5	2369.3
	$[\psi^{f_6}\psi^{\sigma_2}\psi^{c_1}]$	$\bar{D}^*K^*$	2900.5	2907.6	
$IJ^P = 11^+$	$[\psi^{f_6}\psi^{\sigma_4}\psi^{c_1}]$	$\bar{D}^*K$	2503.0	2511.0	2511.0
	$[\psi^{f_6}\psi^{\sigma_3}\psi^{c_1}]$	$\bar{D}K^*$	2757.5	2766.5	
	$[\psi^{f_6}\psi^{\sigma_5}\psi^{c_1}]$	$\bar{D}^*K^*$	2900.5	2908.0	
$IJ^P = 12^+$	$[\psi^{f_6}\psi^{\sigma_5}\psi^{c_1}]$	$\bar{D}^*K^*$	2900.5	2905.3	

From Table III we can see that the energies of every single channel are above the corresponding theoretical threshold, except for the $\bar{D}^*K^*$ state with $IJ^P = 00^+$. The energy of this state is 2820.7 MeV, about 80 MeV lower than the threshold of $\bar{D}^*K^*$. It will become a resonance state by coupling to the open channel $\bar{D}K$. Since the energy is close to the newly reported $X_0(2900)$ and the spin-parity quantum numbers are $J^P = 0^+$, which is also consistent with $X_0(2900)$, it is reasonable to identify the $X_0(2900)$ as a molecular state $\bar{D}^*K^*$ with $IJ^P = 00^+$ in our quark model calculation.

We also investigate the effect of the multichannel coupling. It is obvious from Table III that two bound states are obtained after the channel-coupling calculation. One is the tetraquark state with $IJ^P = 00^+$, the energy of which is 2341.2 MeV, almost 20 MeV lower than the threshold of $\bar{D}K$. Although the energy of this state is close to the mass of $D^*_{s0}(2317)$, it cannot be used to explain the $D^*_{s0}(2317)$. Because the quark components here are $ud\bar{s}\bar{c}$, it cannot decay to the $D_s^+\pi^0$ channel. Another one is the tetraquark state with $IJ^P = 01^+$, the energy of which is 2489.7 MeV, 13.3 MeV lower than the threshold of $\bar{D}^*K$. In the same way, it cannot be used to identify $D_{s1}(2460)$, though the energy is close to $D_{s1}(2460)$. The states with other quantum numbers are unbound after the channel coupling, which indicates that the effect of the channel coupling for these systems is very small and cannot help much. Therefore, the channel coupling plays an important role in forming bound states for both the $IJ^P = 00^+$ and $IJ^P = 01^+$ tetraquark systems, while it can be neglected for the systems with other quantum numbers.

In Refs. [27–29] a new type of quark-quark interaction induced by a light quark–instanton interaction was applied to the study of the baryon spectrum and the short-range part of the baryon-baryon interaction. The authors found that the instanton-induced potential contained a color-magnetic term, which was essential in the hyperfine splittings of the baryon and the short-range exchange force between two baryons. It is also interesting to apply this interaction to the tetraquark systems. We add the instanton-induced potential $V_{ij}^{\rm ins}$ in Eq. (1), and the model Hamiltonian changes to

$$H = \sum_{i=1}^{4}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{CM} + \sum_{j>i=1}^{4}(V_{ij}^{\rm CON} + V_{ij}^{\rm OGE} + V_{ij}^{\rm OBE} + V_{ij}^{\rm ins}), \quad (29)$$

where

$$V_{ij}^{\rm ins} = -\frac{1}{2}W_{ij}\left[\frac{16}{15} + \frac{1}{10}(\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j) + \frac{3}{10}(\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j)(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\right]\delta(\mathbf{r}_{ij}). \quad (30)$$

The coefficient W_{ij} is proportional to the inverse of the effective quark masses ($W_{ij} \propto \frac{1}{m_i m_j}$) [27]. The authors of Ref. [27] reduced the contribution of the one-gluon exchange by decreasing the quark-gluon coupling constant α_s, and adjusted the coefficient W_{ij} to obtain the mass difference of $N - \Delta$. Here we use a similar scheme to fit the mass of mesons, and we can obtain the same meson masses listed in Table II. The adjusted parameters are listed in Table IV.

Then, we recalculate the tetraquarks composed of $ud\bar{s}\bar{c}$ by including the instanton-induced potential. At the same time, the quark delocalization and color screening, which introduce the medium-range attraction in the QDCSM, should be taken out here. We obtain similar results to those in the QDCSM. For example, for the $IJ^P = 00^+$ state the energy of $\bar{D}K$ is 2365.1 MeV, which is above the threshold of $\bar{D}K$. However, for the $\bar{D}^*K^*$ state the energy is 2828.9 MeV, about 72 MeV lower than the threshold of $\bar{D}^*K^*$, which indicates that the $\bar{D}^*K^*$ with $IJ^P = 00^+$ could be a resonance state here. Besides, a bound state with a mass of 2334.4 MeV is also obtained by the channel-coupling calculation. By comparing with the results listed in Table III, these results including the instanton-induced potential are consistent with those of the QDCSM. Besides, both of these models support identifying the $X_0(2900)$ as the molecular state $\bar{D}^*K^*$ with $IJ^P = 00^+$.

TABLE IV. Adjusted parameters.

$\alpha_{s_{uu}}$	$\alpha_{s_{us}}$	$\alpha_{s_{uc}}$	$\alpha_{s_{sc}}$
1.248	1.229	1.225	1.220
W_{uu} (MeV fm^3)	W_{us} (MeV fm^3)	W_{uc} (MeV fm^3)	$W_{\rm sc}$ (MeV fm^3)
66.195	41.471	15.173	9.487

With regard to the tetraquarks in a diquark-antidiquark structure, the energy of each single channel is higher than the theoretical threshold of the corresponding channel, which are shown in Table V. After the channel coupling calculation, the energy of the $IJ^P = 00^+$ system was pushed down to 2206.7 MeV, 153 MeV lower than the theoretical threshold, which indicates that the $IJ^P = 00^+$ state of the diquark-antidiquark structure could be a bound state. For the systems with other quantum numbers, although the effect of channel coupling is much stronger than that of the meson-meson structure, the energy is still above the theoretical threshold of the corresponding channel. So there are no bound states for the systems with $IJ^P = 01^+$, 02^+, 10^+, 11^+ or $IJ^P = 12^+$ in the diquark-antidiquark structure. However, it is possible for some states to be resonance states, because the colorful subclusters of a diquark (ud) and antidiquark ($\bar{s}\bar{c}$) cannot fall apart due to color confinement. To check this possibility, we carry out an adiabatic calculation of the effective potentials for the $ud\bar{s}\bar{c}$ system with a diquark-antidiquark structure.

The effective potential is obtained from the formula $V_E(S) = E(S) - E_{\rm th}$, where $E_{\rm th}$ is the threshold of the corresponding lowest channel and $E(S)$ is the energy at each S, which is the distance between two subclusters. Here $E(S)$ is obtained as

$$E(S) = \frac{\langle\Psi(S)|H|\Psi(S)\rangle}{\langle\Psi(S)|\Psi(S)\rangle},$$

where $\langle\Psi(S)|H|\Psi(S)\rangle$ and $\langle\Psi(S)|\Psi(S)\rangle$ are the Hamiltonian matrix and the overlap of the state. The effective potentials as functions of the distance between the diquark and antidiquark for the $ud\bar{s}\bar{c}$ system are shown in Fig. 1, where sc1, sc2, and sc3 stand for the potential of the first, second, and third single channel, respectively, as shown in Table V.

TABLE V. Energies (in MeV) of the diquark-antidiquark structure for tetraquarks $ud\bar{s}\bar{c}$.

	$[\psi^{f_i}\psi^{\sigma_j}\psi^{c_k}]$	$E_{\rm th}$	$E_{\rm sc}$	$E_{\rm cc}$
$IJ^P = 00^+$	$[\psi^{f_4}\psi^{\sigma_1}\psi^{c_2}]$	2360.0	2512.4	2206.7
	$[\psi^{f_4}\psi^{\sigma_2}\psi^{c_1}]$		2575.8	
$IJ^P = 01^+$	$[\psi^{f_4}\psi^{\sigma_3}\psi^{c_2}]$	2503.0	2572.2	2534.7
	$[\psi^{f_4}\psi^{\sigma_4}\psi^{c_1}]$		3023.4	
	$[\psi^{f_4}\psi^{\sigma_5}\psi^{c_1}]$		2825.4	
$IJ^P = 02^+$	$[\psi^{f_4}\psi^{\sigma_1}\psi^{c_1}]$	2900.5	3130.7	
$IJ^P = 10^+$	$[\psi^{f_3}\psi^{\sigma_2}\psi^{c_2}]$	2360.0	2851.5	2519.7
	$[\psi^{f_3}\psi^{\sigma_1}\psi^{c_1}]$		3147.1	
$IJ^P = 11^+$	$[\psi^{f_3}\psi^{\sigma_3}\psi^{c_2}]$	2503.0	2928.6	2807.5
	$[\psi^{f_3}\psi^{\sigma_4}\psi^{c_2}]$		2912.7	
	$[\psi^{f_3}\psi^{\sigma_5}\psi^{c_1}]$		3130.6	
$IJ^P = 12^+$	$[\psi^{f_3}\psi^{\sigma_1}\psi^{c_2}]$	2900.5	3013.7	

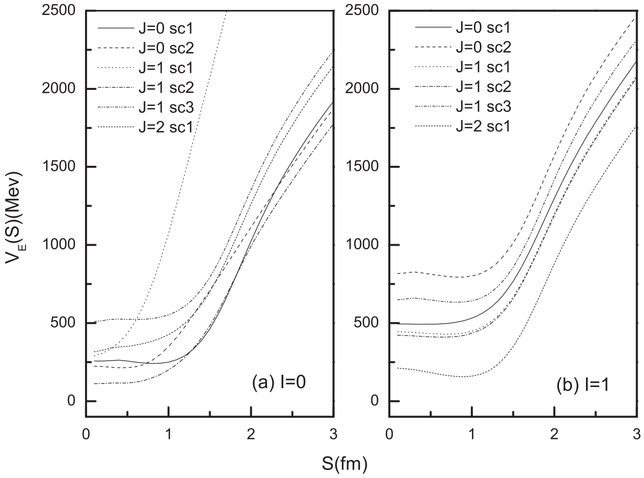

FIG. 1. Effective potentials as functions of the distance between the diquark (ud) and antidiquark ($\bar{s}\bar{c}$) for the $ud\bar{s}\bar{c}$ system.

From the left panel of Fig. 1 we can see that the effective potential of each channel with $IJ^P = 01^+$ increases when the two subclusters fall apart, which means that the diquark and antidiquark tend to clump together without hinderance. This behavior indicates that the odds of the states being a diquark-antidiquark structure, meson-meson structure, or other structures are the same. Besides, from Tables III and V we can see that the energy of each channel with $IJ^P = 01^+$ in the diquark-antidiquark structure is higher than that in the meson-meson structure. So the state prefers to be two free mesons. Therefore, none of these states is an observable resonance state in the present calculation. The results are different for the channels with other quantum numbers. Take the $IJ^P = 11^+$ state for example, from Fig. 1(b) we can see that the energy of the state rises when the two subclusters get very close to each other, so there is a hinderance for the state of the diquark-antidiquark structure changing to the meson-meson structure even if the energy of the state is lower in meson-meson structure. Therefore, it is possible to form a wide resonance because of the small repulsive core. The resonance energies are about 2500–3100 MeV. However, all of these states will couple to the open channels. To confirm whether any of the states can survive as a resonance state after coupling to the open channels, further study of the scattering process of the open channels is needed in future work. In addition, among all of these resonance states, we notice that the energy of the $IJ^P = 11^+$ resonance is 2912.7 MeV, which is close to the newly reported $X_1(2900)$. However, the spin-parity quantum numbers are $J^P = 1^+$, which is different to the experimental data 1^-. Therefore, it may not be used to explain the $X_1(2900)$ state here. The tetraquark systems with P wave should be considered to study the exotic state $X_1(2900)$.

B. Tetraquarks $uc\bar{d}\bar{s}$

For tetraquarks composed of $uc\bar{d}\bar{s}$, the energies of the meson-meson structure and the diquark-antidiquark structure are listed in Tables VI and VII, respectively. For the meson-meson configuration, we can see from Table VI that the energies of every single channel approach the corresponding theoretical threshold, which means that there are no bound states for every single channel.

TABLE VI. Energies (in MeV) of the meson-meson structure for tetraquarks $uc\bar{d}\bar{s}$.

	$[\psi^{f_i}\psi^{\sigma_j}\psi^{c_k}]$	Channel	$E_{\rm th}$	$E_{\rm sc}$	$E_{\rm cc}$
$IJ^P = 00^+$	$[\psi^{f_1}\psi^{\sigma_1}\psi^{c_1}]$	$D_s\eta$	2252.3	2259.9	2256.6
	$[\psi^{f_1}\psi^{\sigma_2}\psi^{c_1}]$	$D_s\omega$	2787.0	2791.1	
	$[\psi^{f_3}\psi^{\sigma_1}\psi^{c_1}]$	DK	2360.0	2368.6	
	$[\psi^{f_3}\psi^{\sigma_2}\psi^{c_1}]$	D^*K^*	2900.5	2907.5	
$IJ^P = 01^+$	$[\psi^{f_1}\psi^{\sigma_4}\psi^{c_1}]$	$D_s^*\eta$	2346.9	2352.5	2347.0
	$[\psi^{f_1}\psi^{\sigma_3}\psi^{c_1}]$	$D_s\omega$	2692.4	2698.1	
	$[\psi^{f_1}\psi^{\sigma_5}\psi^{c_1}]$	$D^*\omega$	2787.0	2791.2	
	$[\psi^{f_3}\psi^{\sigma_4}\psi^{c_1}]$	D^*K	2503.0	2510.2	
	$[\psi^{f_3}\psi^{\sigma_3}\psi^{c_1}]$	DK^*	2757.5	2765.0	
	$[\psi^{f_3}\psi^{\sigma_5}\psi^{c_1}]$	D^*K^*	2900.5	2907.2	
$IJ^P = 02^+$	$[\psi^{f_1}\psi^{\sigma_5}\psi^{c_1}]$	$D_s^*\omega$	2787.0	2791.4	2790.9
	$[\psi^{f_3}\psi^{\sigma_5}\psi^{c_1}]$	D^*K^*	2900.5	2905.5	
$IJ^P = 10^+$	$[\psi^{f_2}\psi^{\sigma_1}\psi^{c_1}]$	$D_s\pi$	2108.1	2116.0	2114.6
	$[\psi^{f_2}\psi^{\sigma_2}\psi^{c_1}]$	$D_s^*\rho$	2835.1	2838.6	
	$[\psi^{f_4}\psi^{\sigma_1}\psi^{c_1}]$	DK	2360.0	2368.6	
	$[\psi^{f_4}\psi^{\sigma_2}\psi^{c_1}]$	D^*K^*	2900.5	2906.0	
$IJ^P = 11^+$	$[\psi^{f_2}\psi^{\sigma_4}\psi^{c_1}]$	$D_s^*\pi$	2202.7	2209.6	2208.2
	$[\psi^{f_2}\psi^{\sigma_3}\psi^{c_1}]$	$D_s\rho$	2740.5	2746.0	
	$[\psi^{f_2}\psi^{\sigma_5}\psi^{c_1}]$	$D_s^*\rho$	2835.1	2838.8	
	$[\psi^{f_4}\psi^{\sigma_4}\psi^{c_1}]$	D^*K	2503.0	2510.2	
	$[\psi^{f_4}\psi^{\sigma_3}\psi^{c_1}]$	DK^*	2757.5	2765.0	
	$[\psi^{f_4}\psi^{\sigma_5}\psi^{c_1}]$	D^*K^*	2900.5	2906.4	
$IJ^P = 12^+$	$[\psi^{f_2}\psi^{\sigma_5}\psi^{c_1}]$	$D_s^*\rho$	2835.1	2839.0	2836.5
	$[\psi^{f_4}\psi^{\sigma_5}\psi^{c_1}]$	D^*K^*	2900.5	2906.8	

TABLE VII. Energies (in MeV) of the diquark-antidiquark structure for tetraquarks $uc\bar{d}\bar{s}$.

	$[\psi^{f_i}\psi^{\sigma_j}\psi^{c_k}]$	$E_{\rm th}$	$E_{\rm sc}$	$E_{\rm cc}$
$IJ^P = 00^+$	$[\psi^{f_1}\psi^{\sigma_2}\psi^{c_2}]$	2360.0	2939.9	2712.9
	$[\psi^{f_1}\psi^{\sigma_1}\psi^{c_1}]$		2965.7	
$IJ^P = 01^+$	$[\psi^{f_1}\psi^{\sigma_3}\psi^{c_2}]$	2503.0	2938.3	2735.3
	$[\psi^{f_1}\psi^{\sigma_5}\psi^{c_2}]$		2970.6	
	$[\psi^{f_1}\psi^{\sigma_4}\psi^{c_1}]$		2938.1	
$IJ^P = 02^+$	$[\psi^{f_1}\psi^{\sigma_5}\psi^{c_2}]$	2900.5	3024.3	
$IJ^P = 10^+$	$[\psi^{f_2}\psi^{\sigma_1}\psi^{c_1}]$	2360.0	2850.0	2584.3
	$[\psi^{f_2}\psi^{\sigma_2}\psi^{c_2}]$		2965.7	
$IJ^P = 11^+$	$[\psi^{f_2}\psi^{\sigma_3}\psi^{c_2}]$	2503.0	2938.3	2675.2
	$[\psi^{f_2}\psi^{\sigma_5}\psi^{c_2}]$		2930.2	
	$[\psi^{f_2}\psi^{\sigma_4}\psi^{c_1}]$		2938.1	
$IJ^P = 12^+$	$[\psi^{f_2}\psi^{\sigma_5}\psi^{c_2}]$	2900.5	3057.7	

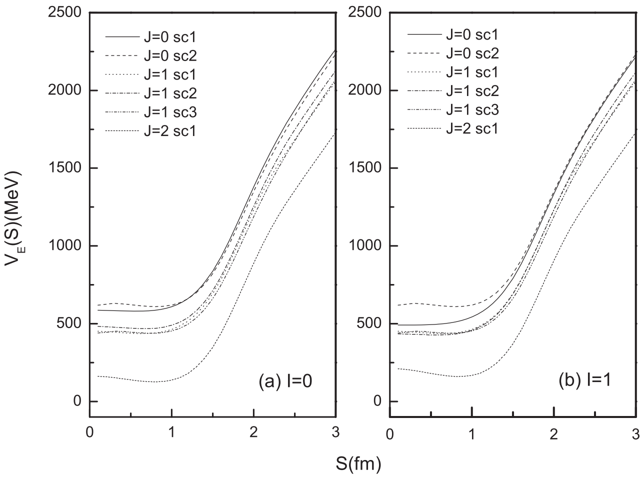

FIG. 2. Effective potentials as functions of the distance between the diquark (uc) and antidiquark ($\bar{d}\bar{s}$) for the $uc\bar{d}\bar{s}$ system.

The channel-coupling effect is very small and cannot help much, and the energies are still higher than the theoretical thresholds, which indicates that no S-wave bound states of the $uc\bar{d}\bar{s}$ system in the meson-meson structure can be formed in our quark model calculation. In particular, the DK state is unbound here, which shows that the $D^*_{s0}(2317)$ cannot be identified as the DK molecular state in the present calculation.

For the diquark-antidiquark structure, it is obvious that the energy of every system is much higher than that of the meson-meson structure. Thus, there is no bound state with a diquark-antidiquark structure. To check if there are any resonance states, we also perform an adiabatic calculation of the effective potentials for this $uc\bar{d}\bar{s}$ system with a diquark-antidiquark structure, which are shown in Fig. 2. It is clear that the variation tendency of the potentials of the $uc\bar{d}\bar{s}$ system with $I = 0$ or $I = 1$ is similar to that of the $ud\bar{s}\bar{c}$ system with $I = 1$. The energy of the state increases a little when the two subclusters get too close, which causes a hinderance for the state decaying to two mesons even if the energy of the diquark-antidiquark structure is higher than that of the meson-meson structure. As a result, it is possible to form wide resonance states here, with resonance energies from about 2580 to 3100 MeV. All of these resonances should be checked further by coupling to the open channels.

IV. SUMMARY

In this work we systematically observed the S-wave tetraquarks composed of $ud\bar{s}\bar{c}$ and $uc\bar{d}\bar{s}$ in the framework of the QDCSM. We considered two structures: meson-meson and diquark-antidiquark. A dynamic bound-state calculation was carried out to look for bound states in such open charm tetraquark systems. In the calculation, both the single channel and channel coupling were implemented. We also performed an adiabatic calculation of the effective potentials to check for any possible resonance states.

We obtained the following results for the $ud\bar{s}\bar{c}$ systems. (1) The single-channel dynamical calculation showed that there is a bound state $\bar{D}^*K^*$ with energy 2820.7 MeV and quantum numbers $IJ^P = 00^+$, which could explain the newly reported $X_0(2900)$. Similar results can be obtained by including the instanton-induced potential in the quark model. However, the $\bar{D}^*K^*$ can decay to the $\bar{D}K$ channel. To confirm whether the states of $\bar{D}^*K^*$ can survive as a resonance state after coupling to the scattering state, further study of the scattering process of $\bar{D}K$ is needed. The energy of the $\bar{D}^*K^*$ state would be pushed up by coupling to the open channel, much closer to the mass of the $X_0(2900)$. Besides, the two bound states $\bar{D}K$ and $\bar{D}^*K$ were obtained in the channel-coupling calculation. Their energies are 2341.2 and 2489.7 MeV, with quantum numbers $IJ^P = 00^+$ and $IJ^P = 01^+$, respectively. (2) The effective potentials of the diquark-antidiquark structure indicate that several wide resonances are possible in the present calculation, with resonance energies of about 2500–3100 MeV. Among all of these resonance states, we found that the energy of a resonance with $IJ^P = 11^+$ is 2912.7 MeV, which is close to the newly reported $X_1(2900)$, but the parity is opposite to the reported one. Thus, it may not be used to explain the $X_1(2900)$ state here.

We obtained the following results for the $uc\bar{d}\bar{s}$ systems: (1) The dynamical calculation showed that there are no S-wave bound states in the meson-meson structure or the diquark-antidiquark structure. Thus, the $D_{s0}(2317)$ cannot be identified as the DK molecular state in the present calculation. Actually, more and more theoretical work tended to explain the $D_{s0}(2317)$ as a mixture of $c\bar{s}$ and the DK state [30–34]. We will investigate the $D_{s0}(2317)$ in an unquenched quark model in future work. (2) The effective potentials of the diquark-antidiquark structure also show the possibility of several wide resonances in the quark model calculation, with resonance energies of about 2580–3100 MeV.

By comparing two systems, we found that the tetraquarks composed of $ud\bar{s}\bar{c}$ are more likely to form bound states than those composed of $uc\bar{d}\bar{s}$, which is similar to the $ud\bar{s}\bar{b}$ and $ub\bar{d}\bar{s}$ (or $ds\bar{u}\bar{b}$) systems. If the $X_0(2900)$ is confirmed as an exotic state with four different quark flavors $ud\bar{s}\bar{c}$, the state composed of $ud\bar{s}\bar{b}$ is also worthy of attention. We have obtained two tetraquark candidates consisting of $ud\bar{s}\bar{b}$ [20]: an $IJ^P = 00^+$ state with mass 5701 MeV, and an $IJ^P = 01^+$ state with mass 5756 MeV. Exploring these open bottom tetraquark states would also be interesting future work.

Besides, we studied the open charm tetraquarks in two structures in this work. There are also other structures, e.g., K-type structures. Structure mixing may lower the energies of the systems. However, this is not an easy job. The overcompleteness problem has to be solved.

In addition, to confirm the existence of the resonances with open charm, a study of the scattering process of the corresponding open channels is needed. All of these open charm bound states and resonances are worth investigating in future experiments.

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