Spin-parities of the $P_c(4440)$ and $P_c(4457)$ in the one-boson-exchange model

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The LHCb collaboration has recently observed three pentaquark peaks, the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$. They are very close to a pair of heavy baryon-meson thresholds, with the $P_c(4312)$ located 8.9 MeV below the $\bar{D}\Sigma_c$ threshold, and the $P_c(4440)$ and $P_c(4457)$ located 21.8 and 4.8 MeV below the $\bar{D}^*\Sigma_c$ one. The spin-parities of these three states have not been measured yet. In this work we assume that the $P_c(4312)$ is a $J^P = \frac{1}{2}^-$ $\bar{D}\Sigma_c$ bound state, while the $P_c(4440)$ and $P_c(4457)$ are $\bar{D}^*\Sigma_c$ bound states of unknown spin-parity, where we notice that the consistent description of the three pentaquarks in the one-boson-exchange model can indeed determine the spin and parities of the later, i.e., of the two $\bar{D}^*\Sigma_c$ molecular candidates. For this determination we revisit first the one-boson-exchange model, which in its original formulation contains a short-range deltalike contribution in the spin-spin component of the potential. We argue that it is better to remove these deltalike contributions because, in this way, the one-boson-exchange potential will comply with the naïve expectation that the form factors should not have a significant impact in the long-range part of the potential (in particular the one-pion-exchange part). Once this is done, we find that it is possible to consistently describe the three pentaquarks, to the point that the $P_c(4440)$ and $P_c(4457)$ can be predicted from the $P_c(4312)$ within a couple of MeV with respect to their experimental location. In addition the so-constructed one-boson-exchange model predicts the preferred quantum numbers of the $P_c(4440)$ and $P_c(4457)$ molecular pentaquarks to be $\frac{3}{2}^-$ and $\frac{1}{2}^-$, respectively.

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I. INTRODUCTION

The observation of three pentaquarklike resonances by the LHCb collaboration [1]—the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$—provides three of the most robust candidates so far for a hadronic molecule, a type of exotic hadron conjectured four decades ago [2,3]. As a matter of fact molecular pentaquarks, i.e., bound states of a charmed antimeson and a charmed baryon, were predicted in a series of theoretical works [4–9]. Subsequent theoretical analyses after the experimental observation of the LHCb pentaquarks [1] have further explored the molecular hypothesis [10–19] or considered nonmolecular explanations [20–24], have indicated the importance of their decays to confirm (or falsify) their nature [25–28] and have discussed the existence of new unobserved pentaquark states [29,30]. However a crucial piece of information required to determine the nature of the LHCb pentaquarks is their quantum numbers, which have not been experimentally determined yet, for which molecular and nonmolecular interpretations usually yield different predictions (with molecular interpretations overwhelmingly preferring negative parity). It is interesting to notice that the $P_c(4440)$ and $P_c(4457)$ were previously identified as a single peak, $P_c(4450)$ [31], where the later collection of data by the LHCb has finally uncovered the double peak structure. In this regard, the previous investigations about the old $P_c(4450)$ peak are still expected to be largely relevant for the new peaks, from

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its nature (be it either molecular [32–37] or nonmolecular [38–44]) to its possible partner states [45], the role of the $\bar{D}\Lambda_c(2595)$ threshold [46,47], etc.

Hadronic molecules are bound states among two or more hadrons. Their existence is contingent on the hadron-hadron potential. In this regard the one-boson-exchange (OBE) model provides a physically compelling and intuitive picture of hadron interactions [48,49], which can help us to predict prospective molecular states. According to this model, the potential between two hadrons is a consequence of the exchange of a series of light mesons, of which the most prominent ones are the π, σ, ρ and ω mesons. The OBE model was originally developed to describe the nucleon-nucleon potential, providing the first quantitative successful description of nuclear scattering observables and the deuteron [48,49]. Besides, it also provided the original theoretical motivation for the existence of hadronic molecules [2], with subsequent explorations often relying on this model to make predictions or to explain already known states [35,50–52].

The OBE model is endowed with a certain degree of ambiguity though. The most important limitation of the OBE model is that it requires the introduction of form factors and cutoffs to mimic the finite size of the hadrons involved. The cutoff cannot be determined *a priori* and is in principle dependent on external information (e.g., experimental measurements). Even if the cutoff is required to be of natural size—we expect it to lie within the 1–2 GeV range—this still leaves room for wildly different predictions. Yet, when applied to hadronic molecules, such a limitation is easy to overcome provided that there is a clear molecular candidate: the cutoff can be effectively determined from the condition of reproducing the aforementioned molecular candidate [53,54].

But phenomenological models, even the most successful ones such as the OBE model, usually end up requiring a certain amount of tweaking (see for instance Ref. [55] for a lucid exposition of a few of the quirks of the OBE model). For the OBE model as applied to nuclear physics, it was quickly realized that the correct description of the deuteron properties requires the cutoff to be $\Lambda_\pi > 1.3$ GeV for the pion contribution. The theoretical reason is the distortion of the long-range properties of the one-pion-exchange (OPE) potential by the form factors, which can be prevented if the cutoff is hard enough. The present manuscript indicates that this type of long-range distortion also happens for hadronic molecules, but proposes a different solution adapted to the particular circumstances of the application of the OBE model to the molecular pentaquarks.

The problem is as follows: if the $P_c(4312)$ is indeed a $J^P = \frac{1}{2}^-$ $\bar{D}\Sigma_c$ molecule with a binding energy of 8.9 MeV, it can be described within the OBE model with a monopolar form factor and a cutoff $\Lambda = 1119$ MeV. If we use the simplest OBE model possible, i.e., we use the same form factor and cutoff for all the exchanged mesons, then we can predict the $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ binding energies from the cutoff that we already determined from the $P_c(4312)$. In particular we arrive at

$$B_E\left(\frac{1^-}{2}\right) \simeq 74\,\text{MeV} \quad \text{and} \quad B_E\left(\frac{3^-}{2}\right) \simeq 3\,\text{MeV}, \qquad (1)$$

which are incompatible with the binding energies of the $P_c(4440)$ and $P_c(4457)$ as $\bar{D}^*\Sigma_c$ bound states, $B_E = 21.8$ and 4.8 MeV, respectively. This happens regardless of which state we identify with the $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ quantum numbers. The failure of the naïve OBE model to naturally explain the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ with the same cutoff can be traced back to a particular artifact generated by the form factors. The unregularized spin-spin piece of the OBE potential contains a contact-range and a finite-range piece, which we write schematically as

$$V_S \propto \left[-\delta^{(3)}(m\vec{r}) + \frac{e^{-mr}}{4\pi m r}\right], \qquad (2)$$

with m the mass of the exchanged meson. It happens that the inclusion of a form factor regularizes the contact-range Dirac-delta piece of the OBE potential, making it finite range. The expectation is that the finite range of the regularized delta will be considerably shorter than the range of the Yukawa-like piece. However this condition is not actually fulfilled for a monopolar form factor and a cutoff $\Lambda \sim 1$ GeV. This is obvious in the OPE contribution of the OBE potential, which is in fact distorted at distances comparable with the Compton wavelength of the pion. In particular the excessive binding of the $\frac{1}{2}^-$ $\bar{D}^*\Sigma_c$ in Eq. (1) can be traced back to the regularized delta contribution stemming from the OPE potential: while the Yukawa-like piece of the OPE potential is repulsive in this system, the deltalike piece provides the system with a strong, probably unphysical, short-range attraction. If we remove the delta-like contribution to the OPE potential by hand, we end up with the set of predictions

$$B_E\left(\frac{1^-}{2}\right) = 13.2\text{ MeV} \quad \text{and} \quad B_E\left(\frac{3^-}{2}\right) = 11.6\text{ MeV}, \qquad (3)$$

which are much closer to the expected binding energies of the molecular pentaquarks. There are similar deltalike contributions in the spin-spin piece of the vector-meson-exchange potential. This piece of the OBE potential is of shorter range than the OPE piece. The removal of their delta-like contributions is not as crucial as in the OPE piece, yet it should better be done if we want the OBE model to be internally consistent, in which case we arrive at the predictions:

$$B_E\left(\frac{1-}{2}\right) = 4.2\ \text{MeV} \quad \text{and} \quad B_E\left(\frac{3-}{2}\right) = 18.3\ \text{MeV}, \quad (4)$$

where the binding energies are in fact very close (within 1–3 MeV) to what we would expect from a molecular $P_c(4440)$ and $P_c(4457)$, namely 21.8 and 4.8 MeV. Owing to the compatibility of this set of predictions with the current experimental determination of the $P_c(4440)$ and $P_c(4457)$ masses, the removal of the Dirac-delta contributions could indeed be considered as the preferred solution to the form-factor problem. In this case the OBE model as applied to hadronic molecules ends up having the phenomenological success of its original nuclear physics version, modulo the larger experimental uncertainties associated with hadronic molecules. The seemingly *ad hoc* removal of the Dirac-delta contributions, which has also been considered for instance in Ref. [56], finds a natural explanation within the renormalized OBE model of Ref. [55].

The manuscript is structured as follows: in Sec. II we briefly explain how the heavy hadron-hadron interaction is constrained by heavy-quark spin symmetry (HQSS), where we also advocate a notation based on the quark model for heavy-hadron interactions [57]. In Sec. III we explain the OBE model, while in Sec. IV we explain the regulator artifact within the OBE model. Then in Sec. V we show the predictions we arrive at after removing this artifact. We discuss the relation with renormalization and effective field theory ideas in Sec. VI. Finally in Sec. VII we summarize our results.

II. HEAVY-QUARK SPIN SYMMETRY

In this section we review a few basic consequences of HQSS for heavy antimeson-baryon molecules. As applied to molecular states, HQSS refers to the fact that interactions among heavy hadrons do not depend on the spin of the heavy quarks within the hadrons. This can automatically be taken into account by writing the heavy hadron interactions in a suitable notation. The standard notation for this purpose is to group heavy hadrons with the same light-quark structure in a single superfield, which we review in Sec. II A. Here we advocate instead for a simpler notation in terms of the light-quark degrees of freedom, which has been recently presented in Ref. [57] (though we note that it has been intermittently used in the literature for a long time [58]), which we explain in Sec. II B.

A. Heavy-superfield notation

The P and P^* heavy mesons are $|Q\bar{q}\rangle$ states with total spin $J = 0$ and 1, respectively. They can be grouped into the single nonrelativistic superfield:

$$H_Q = \frac{1}{\sqrt{2}}[P + \vec{P}^* \cdot \vec{\sigma}], \quad (5)$$

which has been adapted from its relativistic version [59] and has good transformation properties with respect to heavy-quark spin rotations. In the formula above H_Q is a 2×2 matrix and $\vec{\sigma}$ are the Pauli matrices. The Σ_Q and Σ_Q^* heavy baryons are $|Qqq\rangle$ states with total spin $J = \frac{1}{2}$ and $\frac{3}{2}$. They can be written together as the following nonrelativistic superfield [60]

$$\vec{S}_Q = \frac{1}{\sqrt{3}}\vec{\sigma}\Sigma_Q + \vec{\Sigma}_Q^*, \quad (6)$$

which corresponds to the relativistic heavy-baryon superfield written in Ref. [61]. From these superfields, the simplest contact-range, no-derivative Lagrangian involving the heavy (anti)meson and heavy baryon fields is [45]

$$\mathcal{L} = C_a \vec{S}_Q^\dagger \cdot \vec{S}_Q \mathrm{Tr}[\bar{H}_Q^\dagger \bar{H}_Q] + C_b \sum_{i=1}^{3} \vec{S}_Q^\dagger \cdot (J_i \vec{S}_Q)\mathrm{Tr}[\bar{H}_Q^\dagger \sigma_i \bar{H}_Q], \quad (7)$$

where J_i with $i = 1, 2, 3$ refers to the spin-1 angular momentum matrices and with C_a and C_b coupling constants. If we particularize for the $\bar{D}\Sigma_c$ family of molecules, we obtain the following potential:

$$V\left(\bar{D}\Sigma_c, J = \frac{1}{2}\right) = C_a, \quad (8)$$

$$V\left(\bar{D}\Sigma_c^*, J = \frac{3}{2}\right) = C_a, \quad (9)$$

$$V\left(\bar{D}^*\Sigma_c, J = \frac{1}{2}\right) = C_a - \frac{4}{3}C_b, \quad (10)$$

$$V\left(\bar{D}^*\Sigma_c, J = \frac{3}{2}\right) = C_a + \frac{2}{3}C_b, \quad (11)$$

$$V\left(\bar{D}^*\Sigma_c^*, J = \frac{1}{2}\right) = C_a - \frac{5}{3}C_b, \quad (12)$$

$$V\left(\bar{D}^*\Sigma_c^*, J = \frac{3}{2}\right) = C_a - \frac{2}{3}C_b, \quad (13)$$

$$V\left(\bar{D}^*\Sigma_c^*, J = \frac{5}{2}\right) = C_a + C_b. \quad (14)$$

We notice that, for simplicity, we have ignored isospin when writing the Lagrangian of Eq. (7) and the potentials of Eqs. (8)–(14). Isospin can be trivially taken into account by adding a subindex indicating the isospin of the two-body state in the couplings: C_{Ia}, C_{Ib} with $I = \frac{1}{2}, \frac{3}{2}$.

B. Light-quark notation

Actually the heavy-quark symmetric interactions can be derived in an easier and more direct way if we consider that the heavy-quark acts as a spectator [57]. Instead of building superfields for the P and P^* heavy mesons, we can simply express the interactions in terms of the light-quark subfield within the heavy mesons, q_L. Equivalently, for the Σ_Q and Σ_Q^* heavy baryons we can use the light-diquark subfield within them: d_L. After introducing these fields, the lowest-order contact-range Lagrangian can be simply written as

$$\mathcal{L} = C_a (q_L^\dagger q_L)(d_L^\dagger d_L) + C_b (q_L^\dagger \vec{\sigma}_L q_L) \cdot (d_L^\dagger \vec{S}_L d_L), \tag{15}$$

where $\vec{\sigma}_L$ and $\vec{S}_L$ refer to the spins of the q_L and d_L subfields, respectively. This leads to the contact-range potential

$$V_C(q_L d_L) = C_a + C_b \vec{\sigma}_{L1} \cdot \vec{S}_{L2}, \tag{16}$$

where we have labeled the heavy meson and baryon with the light quark and light diquark inside them with the subscript 1 and 2. The contact-range potential is now written for the light-quark fields within the heavy hadrons. The translation into the heavy-hadron degrees of freedom can be encapsulated in a series of rules. In particular for the heavy mesons the light-quark spin operators are translated into

$$\langle P | \vec{\sigma}_L | P \rangle = 0, \tag{17}$$

$$\langle P^* | \vec{\sigma}_L | P^* \rangle = \vec{S}_1, \tag{18}$$

where $\vec{S}_1$ refers to the spin-1 matrices as applied to the heavy vector meson. For the heavy baryons the correspondence is

$$\langle \Sigma_Q | \vec{J}_L | \Sigma_Q \rangle = \frac{2}{3} \vec{\sigma}_2, \tag{19}$$

$$\langle \Sigma_Q^* | \vec{J}_L | \Sigma_Q^* \rangle = \frac{2}{3} \vec{S}_2, \tag{20}$$

where $\vec{\sigma}_2$ refers to the Pauli matrices (applied to the heavy spin-$\frac{1}{2}$ baryon) and $\vec{S}_2$ are the spin-$\frac{3}{2}$ angular momentum matrices. With these substitutions it is easy to check that the contact-range potential of Eq. (16) written in the light-quark field basis is indeed equivalent to the contact-range potential of Eqs. (9)–(14) written in the particle basis. The notation in terms of the light-quark subfields is however much more compact and we will adopt it for the rest of this work.

III. THE ONE-BOSON-EXCHANGE POTENTIAL

In this section we present the OBE model that we use in this work. In the OBE model the potential between two hadrons is generated by the exchange of a series of light mesons, which includes the π, the σ, the ρ and the ω (plus a few extra light mesons in its more sophisticated versions, e.g., the η, the $a_0(980)$ or even the a_1 [62]). We will propose here a minimalistic OBE model containing only the four aforementioned light mesons (π, σ, ρ, ω), which owing to their combination of range and coupling strength are usually assumed to provide the bulk of the hadron-hadron interaction (or at least this is the case when we are dealing with nucleons). Nonetheless, as a cross-check of the previous choice, we will consider the effects of additionally including the η meson, which has a similar range as the other light mesons we are considering (but a smaller coupling).

The OBE model results in a description of the forces among hadrons that is both simple and physically compelling, representing a natural generalization of the original idea by Yukawa for explaining nuclear forces. Yet there are disadvantages in the OBE model, which usually include a large number of coupling constants and the requirement of form factors and a cutoff to remove the unphysical short-range components of the potential. Here the choice of coupling constants will be done in terms of experimentally known information or by recourse to phenomenological models. For the form factor we will choose a standard multipolar form, while the cutoff will be determined by the condition of reproducing a known molecular candidate, the $P_c(4312)$ in this case. By determining the cutoff in this way we are partially *renormalizing* the OBE model, i.e., removing cutoff ambiguities in terms of observable information. This concept is based on the fully renormalized OBE model of Ref. [55], which in turn helps to understand a few of the tweaks required in the original OBE model (e.g., the excessively large coupling to the ω vector meson that is usually required in nuclear physics). We stress however that we have not implemented a renormalized OBE model in this work but merely adapted a few of the ideas of Ref. [55].

A. The Lagrangian

First we write down the Lagrangians that encode the couplings between the heavy hadrons and the light mesons (π, σ, ρ, ω). We use the light-quark notation introduced in Sec. II B. For the effective light-quark field degree of freedom within the heavy mesons the Lagrangian reads as follows

$$\mathcal{L}_{q_L q_L \pi} = \frac{g_1}{\sqrt{2} f_\pi} q_L^\dagger \vec{\sigma}_L \cdot \nabla (\vec{\tau} \cdot \vec{\pi}) q_L, \tag{21}$$

$$\mathcal{L}_{q_L q_L \sigma} = g_{\sigma 1} q_L^\dagger \sigma q_L, \tag{22}$$

$$\mathcal{L}_{q_L q_L \rho} = g_{\rho 1} q_L^\dagger \vec{\tau} \cdot \vec{\rho}_0 q_L - \frac{f_{\rho 1}}{4M_1} \epsilon_{ijk} q_L^\dagger \sigma_{L,k} \vec{\tau} \cdot (\partial_i \vec{\rho}_j - \partial_j \vec{\rho}_i) q_L, \quad (23)$$

$$\mathcal{L}_{q_L q_L \omega} = g_{\omega 1} q_L^\dagger \omega_0 q_L - \frac{f_{\omega 1}}{4M_1} \epsilon_{ijk} q_L^\dagger \sigma_{L,k} (\partial_i \omega_j - \partial_j \omega_i) q_L, \quad (24)$$

where g_1 is the pion axial coupling, $f_\pi = 132$ MeV the pion decay constant, $g_{\sigma 1}$ the coupling to the sigma meson, while g_{V1} and f_{V1} with $V = \rho, \omega$ are the electric- and magnetic-type couplings to the vector mesons; M_1 is a mass scale that we introduce for f_{V1} to be dimensionless. Finally $\vec{\tau}$ refers to the isospin matrices as applied to the charmed meson, which coincide with the Pauli matrices. For the effective light-diquark field within the heavy baryons we have

$$\mathcal{L}_{d_L d_L \pi} = \frac{g_2}{\sqrt{2} f_\pi} d_L^\dagger \vec{S}_L \cdot \nabla (\vec{T} \cdot \vec{\pi}) d_L, \quad (25)$$

$$\mathcal{L}_{d_L d_L \sigma} = g_{\sigma 2} d_L^\dagger \sigma d_L, \quad (26)$$

$$\mathcal{L}_{d_L d_L \rho} = g_{\rho 2} d_L^\dagger \vec{T} \cdot \vec{\rho}_0 d_L - \frac{f_{\rho 2}}{4M_2} \epsilon_{ijk} d_L^\dagger S_{L,k} \vec{T} \cdot (\partial_i \vec{\rho}_j - \partial_j \vec{\rho}_i) d_L + \frac{h_{\rho 2}}{2M_2^2} d_L^\dagger Q_{L,ij} \vec{T} \cdot \partial_i \partial_j \vec{\rho}_0 d_L, \quad (27)$$

$$\mathcal{L}_{d_L d_L \omega} = g_{\omega 2} d_L^\dagger \omega_0 d_L - \frac{f_{\omega 2}}{4M_2} \epsilon_{ijk} d_L^\dagger S_{L,k} (\partial_i \omega_j - \partial_j \omega_i) d_L + \frac{h_{\rho 2}}{2M_2^2} d_L^\dagger Q_{L,ij} \partial_i \partial_j \omega_0 d_L. \quad (28)$$

The spin of the light-diquark field is $S_L = 1$, which means that there are three possible type of interactions with a vector field: electric, magnetic and quadrupole type. They correspond to the g_{V2}, f_{V2} and h_{V2} couplings. The mass M_2 is introduced to make the f_{V2} and h_{V2} couplings dimensionless. $\vec{T}$ represents the isospin matrices for the charmed baryons, which are identical to the spin-1 matrices but applied to isospin space instead. For the quadrupole-type term we have introduced the spin-2 tensor

$$Q_{L,ij} = \frac{1}{2}[S_{L,i} S_{L,j} + S_{L,j} S_{L,i}] - \frac{\vec{S}_L^2}{3} \delta_{ij}, \quad (29)$$

which can be translated into its charmed baryon version with

$$\langle \Sigma_c | Q_{L,ij} | \Sigma_c \rangle = 0, \quad (30)$$

$$\langle \Sigma_c^* | Q_{L,ij} | \Sigma_c^* \rangle = \frac{1}{3} Q_{2,ij}, \quad (31)$$

with $Q_{2,ij}$ analogous to Eq. (29), but written in terms of the spin-$\frac{3}{2}$ angular momentum matrices. We expect the quadrupole-type term to be small though.

The previous Lagrangians have been derived in the heavy-quark limit, $m_Q \to \infty$. But the mass of the charm quark is finite and HQSS-breaking terms can in principle be included. Yet the relative size of the $1/m_Q$ terms is expected to be of the order of $\Lambda_{\rm QCD}/m_Q \sim 15\%$ in the charm sector. In Sec. V A we will consider the effect of these uncertainties in more detail.

Finally a few comments are in order at this point: first, the electric-type and quadrupole interactions of the vector mesons (ρ, ω) with the charmed mesons and baryons depend only on the zeroth component of the vector meson fields, which at first sight is anti-intuitive. This is actually a result of the heavy quark limit and our choice of parametrization for the heavy hadron fields, which we explain in Appendix A. Second, the relation with the multipole expansion can be better understood from a comparison with the electromagnetic Lagrangian of the heavy hadrons, which we explain in Appendix B. Third, the Lagrangian for the interaction of the heavy hadrons with the η, which we do not include in the basic version of the OBE model, can be found in Appendix C.

B. The OBE potential

The OBE potential can be easily derived from the previous Lagrangians for the light-quark and light-diquark fields. We write the potential in the following form

$$V_{\rm OBE} = \zeta V_\pi + V_\sigma + V_\rho + \zeta V_\omega, \quad (32)$$

where $\zeta = \pm 1$ is a sign, for which the convention is

$$\zeta = +1 \quad \text{for } q_L d_L (\text{e.g. } \bar{D}\Sigma_c), \quad (33)$$

$$\zeta = -1 \quad \text{for } \bar{q}_L d_L (\text{e.g. } D\Sigma_c), \quad (34)$$

that is, we take $\zeta = +1$ for the most representative type of molecule, the (hidden-charm) $\bar{D}\Sigma_c$ in this case. In momentum space the different components of the OBE potential read

$$V_\pi(\vec{q}) = -\frac{g_1 g_2}{2 f_\pi^2} \vec{\tau}_1 \cdot \vec{T}_2 \frac{\vec{\sigma}_{L1} \cdot \vec{q} \vec{S}_{L2} \cdot \vec{q}}{\vec{q}^2 + m_\pi^2}, \quad (35)$$

$$V_\sigma(\vec{q}) = -\frac{g_{\sigma 1} g_{\sigma 2}}{\vec{q}^2 + m_\sigma^2}, \quad (36)$$

$$V_\rho(\vec{q}) = \vec{\tau}_1 \cdot \vec{T}_2 \left[\frac{g_{\rho 1}}{\vec{q}^2 + m_\rho^2} \left(g_{\rho 2} - \frac{h_{\rho 2}}{2M_2^2} \vec{q} \cdot (Q_{L2} q) \right) - \frac{f_{\rho 1}}{2M_1} \frac{f_{\rho 2}}{2M_2} \frac{(\vec{\sigma}_{L1} \times \vec{q}) \cdot (\vec{S}_{L2} \times \vec{q})}{\vec{q}^2 + m_\rho^2} \right], \quad (37)$$

$$V_\omega(\vec{q}) = \frac{g_{\omega 1}}{\vec{q}^2 + m_\omega^2}\left(g_{\omega 2} - \frac{h_{\omega 2}}{2M_2^2}\vec{q}\cdot(Q_{L2}q)\right) - \frac{f_{\omega 1}}{2M_1}\frac{f_{\omega 2}}{2M_2}\frac{(\vec{\sigma}_{L1}\times\vec{q})\cdot(\vec{S}_{L2}\times\vec{q})}{\vec{q}^2+m_\omega^2}. \quad (38)$$

If we Fourier transform the previous expressions to coordinate space then we have

$$V_\pi(\vec{r}) = +\vec{\tau}_1\cdot\vec{T}_2\frac{g_1 g_2}{6f_\pi^2}[-\vec{\sigma}_{L1}\cdot\vec{S}_{L2}\delta(\vec{r}) + \vec{\sigma}_{L1}\cdot\vec{S}_{L2}m_\pi^3 W_Y(m_\pi r) + S_{L12}(\vec{r})m_\pi^3 W_T(m_\pi r)], \quad (39)$$

$$V_\sigma(\vec{r}) = -g_{\sigma 1}g_{\sigma 2}m_\sigma W_Y(m_\sigma r), \quad (40)$$

$$V_\rho(\vec{r}) = \vec{\tau}_1\cdot\vec{T}_2\Big[g_{\rho 1}g_{\rho_2}m_\rho W_Y(m_\rho r) + g_{\rho 1}\frac{h_{\rho 2}}{2M_2^2}Q_{L2}(\hat{r})m_\rho^3 W_T(m_\rho r) + \frac{f_{\rho 1}}{2M_1}\frac{f_{\rho 2}}{2M_2}\Big(-\frac{2}{3}\vec{\sigma}_{L1}\cdot\vec{S}_{L2}\delta(\vec{r}) + \frac{2}{3}\vec{\sigma}_{L1}\cdot\vec{S}_{L2}m_\rho^3 W_Y(m_\rho r) - \frac{1}{3}S_{L12}(\hat{r})m_\rho^3 W_T(m_\rho r)\Big)\Big], \quad (41)$$

$$V_\omega(\vec{r}) = g_{\omega 1}g_{\omega_2}m_\omega W_Y(m_\omega r) + g_{\omega 1}\frac{h_{\omega 2}}{2M_2^2}Q_{L2}(\hat{r})m_\omega^3 W_T(m_\omega r) + \frac{f_{\omega 1}}{2M_1}\frac{f_{\omega 2}}{2M_2}\Big(-\frac{2}{3}\vec{\sigma}_{L1}\cdot\vec{S}_{L2}\delta(\vec{r}) + \frac{2}{3}\vec{\sigma}_{L1}\cdot\vec{S}_{L2}m_\omega^3 W_Y(m_\omega r) - \frac{1}{3}S_{L12}(\hat{r})m_\omega^3 W_T(m_\omega r)\Big), \quad (42)$$

where we have introduced the dimensionless functions

$$W_Y(x) = \frac{e^{-x}}{4\pi x}, \quad (43)$$

$$W_T(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)\frac{e^{-x}}{4\pi x}, \quad (44)$$

while S_{L12} represents the standard tensor operator

$$S_{L12}(\hat{r}) = 3\vec{\sigma}_{L1}\cdot\hat{r}\vec{S}_{L2}\cdot\hat{r} - \vec{\sigma}_{L1}\cdot\vec{S}_{L2}, \quad (45)$$

and $Q_{L2}(\hat{r})$ is a second type of tensor operator

$$Q_{L2}(\hat{r}) = \hat{r}\cdot(Q_{L2}\hat{r}) = Q_{L2,ij}\hat{r}_i\hat{r}_j, \quad (46)$$

with $Q_{L2,ij}$ defined in Eq. (29). This second type of tensor operator is theoretically interesting, but probably not particularly relevant as the $h_{\omega 2}$ coupling is expected to be small, see Sec. III D for a more detailed discussion.

Finally it is important to notice that we have computed the OBE potentials under the assumption that the exchanged light mesons (π, σ, ρ, ω) have zero width. This approximation is evident for the π and ω mesons, which are narrow, but it is also known to work well for the ρ meson. The situation is more subtle for the σ meson, yet even in this case a broad σ meson can be substituted with a zero-width one (sometimes called σ') by a suitable redefinition of its mass and coupling. Yet here we will treat the σ meson more as an effective degree of freedom of the OBE model (i.e., as a "$\sigma_{\rm OBE}$") than as a physical particle. We refer to Ref. [48] and references therein for a complete discussion of this topic.

C. Form factors

We have derived the previous OBE potential under the assumption that the interactions between heavy hadrons and light mesons are pointlike. Hadrons have however a finite size, which can be taken into account by the introduction of a form factor for each vertex. In momentum space we will simply have

$$V_M(\vec{q};\Lambda_1,\Lambda_2) = V_M(\vec{q})F_{M1}(\vec{q},\Lambda_1)F_{M2}(\vec{q},\Lambda_2). \quad (47)$$

We will assume a monopolar form factor for vertices 1 and 2:

$$F_{Mi}(\vec{q},\Lambda_i) = \frac{\Lambda_i^2 - m_M^2}{\Lambda_i^2 + \vec{q}^2}. \quad (48)$$

In principle we can use different cutoffs for different vertices to take into account the different internal structure of the heavy mesons and heavy baryons. Yet this is only necessary if we want to describe heavy meson-meson, heavy meson-baryon and heavy baryon-baryon molecules consistently. If we are only interested in the heavy meson-baryon system then we can simply assume a unique cutoff for both vertices 1 and 2.

If we now Fourier transform the momentum space potential with a monopolar form factor into coordinate space, then the outcome is that we simply have to make the following substitutions:

$$\delta(\vec{x}) \to m^3 d(x,\lambda), \quad (49)$$

$$W_Y(x) \to W_Y(x,\lambda), \quad (50)$$

$$W_T(x) \to W_T(x,\lambda), \quad (51)$$

where

$$d(x,\lambda) = \frac{(\lambda^2-1)^2}{2\lambda}\frac{e^{-\lambda x}}{4\pi}, \quad (52)$$

$$W_Y(x,\lambda) = W_Y(x) - \lambda W_Y(\lambda x) - \frac{(\lambda^2-1)}{2\lambda}\frac{e^{-\lambda x}}{4\pi}, \quad (53)$$

$$W_T(x,\lambda) = W_T(x) - \lambda^3 W_T(\lambda x) - \frac{(\lambda^2-1)}{2\lambda}\lambda^2\left(1+\frac{1}{\lambda x}\right)\frac{e^{-\lambda x}}{4\pi}. \quad (54)$$

The corresponding expressions for form factors of higher polarity (e.g., dipolar) can be consulted in the Appendix of Ref. [54].

D. Couplings

For the axial coupling between the D and D^* heavy mesons and the pion we take

$$g_1 = 0.60, \quad (55)$$

which is compatible with $g_1 = 0.59 \pm 0.01 \pm 0.07$ as extracted from the $D^* \to D\pi$ decay [63,64]. For the Σ_c and Σ_c^* heavy baryons the axial coupling is not experimentally available, but there is a lattice QCD calculation [65]

$$g_2 = 0.84 \pm 0.2, \quad (56)$$

which is the value we adopt here. We notice in passing that there are several conventions for the axial coupling to the heavy baryons in the literature and here we are effectively using the one in Ref. [65]. Other two popular conventions are the ones by Cho [61] and Yan [66], which are related to our convention by the relations $g_2 = -g_{2,\mathrm{Cho}}$ and $g_2 = \frac{3}{2} g_{1,\mathrm{Yan}}$ (in Ref. [66] the axial coupling to the heavy baryons is denoted as g_1).

For the couplings to the σ meson, in the case of the nucleon-nucleon interaction it can be determined from the linear sigma model [67] yielding

$$g_{\sigma NN} = \sqrt{2}\frac{M_N}{f_\pi} \simeq 10.2. \quad (57)$$

For the case of the D, D^* mesons and Σ_c, Σ_c^* baryons we can estimate the coupling to the σ from the quark model. By assuming that the σ only couples to the u and d quarks, we expect

$$g_{\sigma 1} = \frac{g_{\sigma 2}}{2} = \frac{g_{\sigma NN}}{3} \simeq 3.4. \quad (58)$$

The choice of couplings for the ρ and ω mesons is more laborious. First, from SU(3)-flavor symmetry and the Okubo-Zweig-Iizuka (OZI) rule we expect that

$$g_{\rho 1} = g_{\omega 1}, \qquad g_{\rho 2} = g_{\omega 2}, \quad (59)$$

$$f_{\rho 1} = f_{\omega 1}, \qquad f_{\rho 2} = f_{\omega 2}, \quad (60)$$

$$h_{\rho 2} = h_{\omega 2}. \quad (61)$$

For the determination of the electric, magnetic and quadrupole couplings we will use the vector-meson dominance assumption. The original formulation of this idea states that hadrons do not couple directly to the electromagnetic field but by means of the neutral vector meson fields, ρ_3^μ and ω^μ, where μ refers to the Lorentz indices of these fields, and the subindex $_3$ indicates that we are dealing with the neutral rho meson. A practical way to apply this idea is to derive the electromagnetic Lagrangian from the substitutions

$$\rho_3^\mu \to e\lambda_\rho A^\mu, \quad (62)$$

$$\omega^\mu \to e\lambda_\omega A^\mu. \quad (63)$$

We can fix λ_ρ and λ_ω from the nucleon case, in which case we obtain

$$\lambda_\rho = \frac{1}{2g_{\rho NN}} = \frac{1}{2g_\rho}, \quad (64)$$

$$\lambda_\omega = \frac{1}{2g_{\omega NN}} = \frac{1}{6g_\rho}, \quad (65)$$

where in the right-hand side we have written $g_{\rho NN}$ and $g_{\omega NN}$ in terms of the universal ρ coupling (Sakurai's universality [68])

$$g_\rho = \frac{m_\rho}{2f_\pi} \sim 2.9, \quad (66)$$

where we have also made use of the relation $g_{\omega NN} = 3g_{\rho NN}$, which is derived from SU(3)-flavor symmetry and the OZI rule. In can be trivially checked that this choice correctly reproduces that the proton and neutron charges are $e_p = +e$ and $e_n = 0$, respectively.

The application to the heavy hadrons requires a few modifications. For instance, vector-meson dominance is expected to reproduce the total charge of the light quarks only. It does not apply to the heavy quark, which we consider to couple directly to the electromagnetic field. Thus the application to the $\bar{D}^0$ ($\bar{c}u$) charmed meson yields

$$g_{\rho 1}\left(\frac{1}{2g_\rho} + \frac{1}{6g_\rho}\right) = \frac{2}{3}e, \quad (67)$$

from which we deduce

$$g_{\rho 1} = g_\rho \simeq 2.9. \quad (68)$$

For the magnetic moments we define the following quantity for the sake of convenience

$$f_{\rho 1} = \kappa_{\rho 1} g_{\rho 1}, \tag{69}$$

which is related to the $\bar{D}^{*0}$ magnetic moment, $\mu(\bar{D}^{*0})$, by the relation

$$\frac{2}{3}\kappa_\rho = \frac{2M_1}{e}\mu(\bar{D}^{*0}). \tag{70}$$

If we set the scaling mass to be the nucleon mass $M_1 = M_N$, $\kappa_{\rho 1}$ simply coincides with $\mu(\bar{D}^{*0})$ in units of the nuclear magneton. If we use the quark model $\mu(\bar{D}^0) = \mu_u$, with $\mu_u = 1.85\mu_N$, we find

$$\kappa_{\rho 1}(M_1 = M_N) \simeq 2.8. \tag{71}$$

Notice that the definition of $\kappa_{\rho 1}$ is dependent on the mass scale M_1 in the Lagrangian. For $M_1 = m_D$ it happens that $\kappa_{\rho 1} \simeq 5.5$. It should be noticed that the vector-meson dominance model we have presented here can be further refined to obtain improved determinations of $g_{\rho 1}$ and $\kappa_{\rho 1}$. For instance, Ref. [69] applies a more sophisticated vector-meson dominance model to the weak decays of the charmed mesons, which translates into the couplings [53]

$$g_{\rho 1} \simeq 2.6 \quad \text{and} \quad \kappa_{\rho 1}(M_1 = M_N) \simeq 2.3 \pm 0.4. \tag{72}$$

As can be appreciated this determination is compatible with the one in Eqs. (68) and (71) within errors. We will use the set derived from Ref. [69], i.e., the values in Eq. (72), to follow the same convention as in our previous works.

Now we apply the previous ideas to the Σ_c and Σ_c^* baryons. First we define the reduced couplings

$$f_{\rho 2} = \kappa_{\rho 2} g_{\rho 2}, \qquad h_{\rho 2} = \eta_{\rho 2} g_{\rho 2}. \tag{73}$$

We now apply vector-meson dominance to arrive at the relations

$$g_{\rho 2} = 2 g_\rho, \tag{74}$$

$$\kappa_{\rho 2} = \frac{3}{4}\left(\frac{2M_2}{e}\right)\mu(\Sigma_c^{*++}), \tag{75}$$

$$\eta_{\rho 2} = \frac{9}{2}\left(\frac{M_2^2}{e}\right)Q(\Sigma_c^{*++}), \tag{76}$$

where $\mu(\Sigma_c^{*++})$ and $Q(\Sigma_c^{*++})$ are the magnetic and quadrupole moment of the Σ_c^{++} baryon. From the quark model (and the assumption that the charm quark provides a minor contribution to the magnetic and quadrupole moments) we expect $\mu(\Sigma_c^{*++}) = 2\mu_u$ and $Q(\Sigma_c^{*++}) = 0$. We note that a nonvanishing quadrupole moment will require the light-diquark wave function to have a D-wave component, which is not the case in the naïve quark model. Thus for $M_2 = M_N$ we arrive at

$$\kappa_{\rho 2} \simeq 2.8, \qquad \eta_{\rho 2} \simeq 0. \tag{77}$$

The fact that the quadrupole vector-meson coupling vanishes in the naïve quark model probably indicates a relatively small contribution from this piece of the potential. This is actually good news in the sense that it simplifies the OBE potential. However the estimations from the quark model have been superseded by recent lattice QCD calculations, at least for the magnetic moment of the Σ_c^{++} baryon [70]. If we use the magnetic moment of the Σ_c^{++} to determine $\kappa_{\rho 2}$, we first note that the vector-meson dominance relation reads

$$\kappa_{\rho 2} = \frac{9}{8}\left(\frac{2M_2}{e}\right)\mu(\Sigma_c^{++}). \tag{78}$$

Reference [70] obtains $\mu(\Sigma_c^{++}) = 1.499(202)$, which leads to

$$\kappa_{\rho 2} \simeq 1.7 \pm 0.2. \tag{79}$$

This is the value we will adopt here. The charmed-antimeson and charmed-baryon masses that we use in this work, together with the couplings, can be consulted in Tables I and II.

TABLE I. Masses and quantum numbers of the light mesons of the OBE model (π, σ, ρ, ω) and the heavy hadrons (D, D^*, Σ_c, Σ_c^*). Notice that we work in the isospin-symmetric limit and take the isospin-averaged masses of the π, D, D^*, Σ_c and Σ_c^* as listed in the PDG [71]. For the ρ and ω we approximate their masses to the closest tens of MeV, while for the σ we settle for a conventional value of its mass within the OBE model (which does not necessarily coincide with its physical mass).

Light Meson	$I^G(J^{PC})$	M (MeV)
π	$1^-\ (0^{-+})$	138
σ	$0^+\ (0^{++})$	600
ρ	$1^+\ (1^{--})$	770
ω	$0^-\ (1^{--})$	780
Heavy Hadron	$I(J^P)$	**M (MeV)**
D	$\frac{1}{2}(0^-)$	1867
D^*	$\frac{1}{2}(1^-)$	2009
Σ_c	$1(\frac{1}{2}^+)$	2454
Σ_c^*	$1(\frac{3}{2}^+)$	2518

TABLE II. Couplings of the light mesons of the OBE model (π, σ, ρ, ω) to the heavy-meson and heavy-baryon fields. For the magnetic-type coupling of the ρ and ω vector mesons we have used the decomposition $f_V = \kappa_V g_V$, with $V = \rho, \omega$. M refers to the mass scale (in MeV) involved in the magnetic-type couplings.

Coupling	Value for P/P^*
g_1	0.60
$g_{\sigma 1}$	3.4
$g_{\rho 1}$	2.6
$g_{\omega 1}$	2.6
$\kappa_{\rho 1}$	2.3
$\kappa_{\omega 1}$	2.3
M_1	940

Coupling	Value for Σ_Q/Σ_Q^*
g_2	0.84
$g_{\sigma 2}$	6.8
$g_{\rho 2}$	5.8
$g_{\omega 2}$	5.8
$\kappa_{\rho 2}$	1.7
$\kappa_{\omega 2}$	1.7
$\eta_{\rho 2}$	0
$\eta_{\omega 2}$	0
M_1	940

E. Wave functions and partial wave projection

The wave function for a heavy meson-baryon system is

$$|\Psi\rangle = \Psi_{JM}(\vec{r})|IM_I\rangle, \tag{80}$$

where $|IM_I\rangle$ is the isospin wave function and Ψ_{JM} the spin and spatial wave function, which can be written as a sum over partial waves

$$\Psi_{JM}(\vec{r}) = \sum_{LS} \psi_{LSJ}(r)|^{2S+1}L_J\rangle. \tag{81}$$

We use the spectroscopic notation $^{2S+1}L_J$, which denotes a partial wave with total spin S, orbital angular momentum L and total angular momentum J:

$$|^{2S+1}L_J\rangle = \sum_{M_S, M_L} \langle LM_L SM_S|JM\rangle |SM_S\rangle Y_{LM_L}(\hat{r}), \tag{82}$$

where $\langle LM_L SM_S|JM\rangle$ are the Clebsch-Gordan coefficients, $|SM_S\rangle$ the spin wave function and $Y_{LM_L}(\hat{r})$ the spherical harmonics. For the $P\Sigma_Q$ and $P\Sigma_Q^*$ systems the spin wave functions are trivial

$$|SM_S(P\Sigma_Q)\rangle = \left|\frac{1}{2} M_S\right\rangle, \tag{83}$$

$$|SM_S(P\Sigma_Q^*)\rangle = \left|\frac{3}{2} M_S\right\rangle, \tag{84}$$

as they correspond to the spin wave functions of the heavy baryon (the heavy meson P is a pseudoscalar). For the $P^*\Sigma_Q$ and $P^*\Sigma_Q^*$ systems,

$$|SM_S(P^*\Sigma_Q)\rangle = \sum_{M_{S1}, M_{S2}} \left\langle 1M_{S1} \frac{1}{2} M_{S2}|SM_S\right\rangle \times |1M_{S1}\rangle \left|\frac{1}{2} M_{S2}\right\rangle, \tag{85}$$

$$|SM_S(P^*\Sigma_Q^*)\rangle = \sum_{M_{S1}, M_{S2}} \left\langle 1M_{S1} \frac{3}{2} M_{S2}|SM_S\right\rangle \times |1M_{S1}\rangle \left|\frac{3}{2} M_{S2}\right\rangle, \tag{86}$$

with $|1M_{S1}\rangle$, $|J_2 M_{S2}\rangle$ the spin wave function of particles 1 and 2.

The partial-wave projection of the potential depends on the matrix elements of the spin-spin, tensor and quadrupole tensor operators, which are independent of J and M,

$$\langle S'L'J'M'|\mathbf{O}_{12}|SLJM\rangle = \delta_{JJ'}\delta_{MM'}\mathbf{O}^J_{SL,S'L'}, \tag{87}$$

with $\mathbf{O}_{12} = C_{12}, S_{12}, Q_2$, which are in turn defined as

$$C_{12} = \vec{a}_1 \cdot \vec{a}_2, \tag{88}$$

$$S_{12} = 3\vec{a}_1 \cdot \hat{r}\vec{a}_2 \cdot \hat{r} - \vec{a}_1 \cdot \vec{a}_2, \tag{89}$$

$$Q_{2,ij} = \frac{1}{2}[a_{2i}a_{2j} + a_{2j}a_{2i}] - \frac{\vec{a}_2^2}{3}\delta_{ij}, \tag{90}$$

with $\vec{a}_1$ ($\vec{a}_2$) the corresponding spin operator for the $\bar{D}$, $\bar{D}^*$ mesons (Σ_c, Σ_c^* baryons). In this work we are using the light-quark notation, which means that we have written the potentials in terms of the light-quark spin. The correspondence between the light-quark spin operators and C_{12}, S_{12} is given by

$$\vec{\sigma}_{L1} \cdot \vec{S}_{L2} = f_{12}C_{12}, \tag{91}$$

$$S_{L12} = f_{12}S_{12}, \tag{92}$$

$$Q_{L2} = f_2 Q_2, \tag{93}$$

where f_{12} and f_2 are factors related to the conversion from the light-quark to the hadron spin degrees of freedom (for all nonvanishing cases $f_{12} = \frac{2}{3}$ and $f_2 = \frac{1}{3}$). The specific matrix elements of the spin-spin, tensor and quadrupole-tensor operators can be consulted in Tables III–V for all the molecular configurations that contain an S-wave (i.e., the ones that are more likely to bind).

TABLE III. Matrix elements of the spin-spin operator for the partial waves we are considering in this work.

Molecule	Partial Waves	J^P	$\vec{\sigma}_{L1}\cdot\vec{S}_{L2} = f_{12}\times\vec{a}_1\cdot\vec{a}_2$
$\bar{D}\Sigma_c$	${}^2S_{1/2}$	$\frac{1}{2}^-$	0×0
$\bar{D}\Sigma_c^*$	${}^4S_{3/2}$-${}^4D_{3/2}$	$\frac{3}{2}^-$	$0\times\begin{pmatrix}0&0\\0&0\end{pmatrix}$
$\bar{D}^*\Sigma_c$	${}^2S_{1/2}$-${}^4D_{1/2}$	$\frac{1}{2}^-$	$\frac{2}{3}\times\begin{pmatrix}-2&0\\0&1\end{pmatrix}$
$\bar{D}^*\Sigma_c$	${}^2D_{3/2}$-${}^4S_{1/2}$-${}^4D_{1/2}$	$\frac{3}{2}^-$	$\frac{2}{3}\times\begin{pmatrix}-2&0&0\\0&1&0\\0&0&1\end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	${}^2S_{1/2}$-${}^4D_{1/2}$-${}^6D_{1/2}$	$\frac{1}{2}^-$	$\frac{2}{3}\times\begin{pmatrix}-\frac{5}{2}&0&0\\0&-1&0\\0&0&\frac{3}{2}\end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	${}^2D_{3/2}$-${}^4S_{3/2}$-${}^4D_{3/2}$-${}^6D_{3/2}$-${}^6G_{3/2}$	$\frac{3}{2}^-$	$\frac{2}{3}\times\begin{pmatrix}-\frac{5}{2}&0&0&0&0\\0&-1&0&0&0\\0&0&-1&0&0\\0&0&0&\frac{3}{2}&0\\0&0&0&0&\frac{3}{2}\end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	${}^2D_{5/2}$-${}^4D_{5/2}$-${}^4G_{5/2}$-${}^6S_{5/2}$-${}^6D_{5/2}$-${}^6G_{5/2}$	$\frac{5}{2}^-$	$\frac{2}{3}\times\begin{pmatrix}-\frac{5}{2}&0&0&0&0&0\\0&-1&0&0&0&0\\0&0&-1&0&0&0\\0&0&0&\frac{3}{2}&0&0\\0&0&0&0&\frac{3}{2}&0\\0&0&0&0&0&\frac{3}{2}\end{pmatrix}$

TABLE IV. Matrix elements of the tensor operator for the partial waves we are considering in this work.

Molecule	Partial Waves	J^P	$S_{L12}(\hat{r}) = f_{12}\times S_{12}(\hat{r})$
$\bar{D}\Sigma_c$	${}^2S_{1/2}$	$\frac{1}{2}^-$	0×0
$\bar{D}\Sigma_c^*$	${}^4S_{3/2}$-${}^4D_{3/2}$	$\frac{3}{2}^-$	$0\times\begin{pmatrix}0&0\\0&0\end{pmatrix}$
$\bar{D}^*\Sigma_c$	${}^2S_{1/2}$-${}^4D_{1/2}$	$\frac{1}{2}^-$	$\frac{2}{3}\times\begin{pmatrix}0&\sqrt{2}\\\sqrt{2}&-2\end{pmatrix}$
$\bar{D}^*\Sigma_c$	${}^2D_{3/2}$-${}^4S_{1/2}$-${}^4D_{1/2}$	$\frac{3}{2}^-$	$\frac{2}{3}\times\begin{pmatrix}0&-1&1\\-1&0&2\\1&2&0\end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	${}^2S_{1/2}$-${}^4D_{1/2}$-${}^6D_{1/2}$	$\frac{1}{2}^-$	$\frac{2}{3}\times\begin{pmatrix}0&-\frac{7}{2\sqrt{5}}&-\frac{3}{\sqrt{5}}\\-\frac{7}{2\sqrt{5}}&-\frac{8}{5}&-\frac{3}{10}\\-\frac{3}{\sqrt{5}}&-\frac{3}{10}&-\frac{12}{5}\end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	${}^2D_{3/2}$-${}^4S_{3/2}$-${}^4D_{3/2}$-${}^6D_{3/2}$-${}^6G_{3/2}$	$\frac{3}{2}^-$	$\frac{2}{3}\times\begin{pmatrix}0&\frac{7}{2\sqrt{10}}&-\frac{7}{2\sqrt{10}}&\frac{3}{\sqrt{35}}&-3\sqrt{\frac{6}{35}}\\\frac{7}{2\sqrt{10}}&0&\frac{8}{5}&-\frac{3}{10}\sqrt{\frac{7}{2}}&0\\-\frac{7}{2\sqrt{10}}&\frac{8}{5}&0&-\frac{3}{2\sqrt{14}}&-\frac{3}{5}\sqrt{\frac{3}{7}}\\\frac{3}{\sqrt{35}}&-\frac{3}{10}\sqrt{\frac{7}{2}}&-\frac{3}{2\sqrt{14}}&-\frac{6}{7}&\frac{9\sqrt{6}}{35}\\-3\sqrt{\frac{6}{35}}&0&-\frac{3}{5}\sqrt{\frac{3}{7}}&\frac{9\sqrt{6}}{35}&-\frac{15}{7}\end{pmatrix}$

(Table continued)

TABLE IV. *(Continued)*

Molecule	Partial Waves	J^P	$S_{L12}(\hat{r}) = f_{12} \times S_{12}(\hat{r})$
$\bar{D}^*\Sigma_c^*$	${}^2D_{5/2}$-${}^4D_{5/2}$-${}^4G_{5/2}$-${}^6S_{5/2}$-${}^6D_{5/2}$-${}^6G_{5/2}$	$\frac{5}{2}^-$	$\frac{2}{3}\times\begin{pmatrix} 0 & \frac{1}{2}\sqrt{\frac{7}{5}} & -\sqrt{\frac{21}{10}} & -\sqrt{\frac{3}{5}} & 2\sqrt{\frac{6}{35}} & -3\sqrt{\frac{2}{35}} \\ \frac{1}{2}\sqrt{\frac{7}{5}} & \frac{8}{7} & \frac{16\sqrt{6}}{35} & \frac{\sqrt{21}}{10} & -\frac{1}{7}\sqrt{\frac{3}{2}} & -\frac{12\sqrt{2}}{35} \\ -\sqrt{\frac{21}{10}} & \frac{16\sqrt{6}}{35} & -\frac{8}{7} & 0 & \frac{9}{70} & -\frac{3\sqrt{3}}{14} \\ -\sqrt{\frac{3}{5}} & \frac{\sqrt{21}}{10} & 0 & 0 & \frac{2\sqrt{14}}{5} & 0 \\ 2\sqrt{\frac{6}{35}} & -\frac{1}{7}\sqrt{\frac{3}{2}} & \frac{9}{70} & \frac{3\sqrt{14}}{5} & \frac{6}{7} & \frac{27\sqrt{3}}{35} \\ -3\sqrt{\frac{2}{35}} & -\frac{12\sqrt{2}}{35} & -\frac{3\sqrt{3}}{14} & 0 & \frac{27\sqrt{3}}{35} & -\frac{6}{7} \end{pmatrix}$

TABLE V. Matrix elements of the quadrupolelike tensor operator for the partial waves we are considering in this work.

Molecule	Partial Waves	J^P	$Q_{L2}(\hat{r}) = f_2 \times Q_2(\hat{r})$
$\bar{D}\Sigma_c$	${}^2S_{1/2}$	$\frac{1}{2}^-$	0×0
$\bar{D}\Sigma_c^*$	${}^4S_{3/2}$-${}^4D_{3/2}$	$\frac{3}{2}^-$	$\frac{1}{3}\times\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\bar{D}^*\Sigma_c$	${}^2S_{1/2}$-${}^4D_{1/2}$	$\frac{1}{2}^-$	$\frac{1}{3}\times\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
$\bar{D}^*\Sigma_c$	${}^2D_{3/2}$-${}^4S_{1/2}$-${}^4D_{1/2}$	$\frac{3}{2}^-$	$\frac{1}{3}\times\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	${}^2S_{1/2}$-${}^4D_{1/2}$-${}^6D_{1/2}$	$\frac{1}{2}^-$	$\frac{1}{3}\times\begin{pmatrix} 0 & \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{1}{5} & \frac{2}{5} \\ \frac{1}{\sqrt{5}} & \frac{2}{5} & -\frac{4}{5} \end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	${}^2D_{3/2}$-${}^4S_{3/2}$-${}^4D_{3/2}$-${}^6D_{3/2}$-${}^6G_{3/2}$	$\frac{3}{2}^-$	$\frac{1}{3}\times\begin{pmatrix} 0 & -\sqrt{\frac{2}{5}} & \sqrt{\frac{2}{5}} & -\frac{1}{\sqrt{35}} & \sqrt{\frac{6}{35}} \\ -\sqrt{\frac{2}{5}} & 0 & \frac{1}{5} & \frac{\sqrt{14}}{5} & 0 \\ \sqrt{\frac{2}{5}} & \frac{1}{5} & 0 & \sqrt{\frac{2}{7}} & \frac{4}{5}\sqrt{\frac{3}{7}} \\ -\frac{1}{\sqrt{35}} & \frac{\sqrt{14}}{5} & \sqrt{\frac{2}{7}} & -\frac{2}{7} & \frac{3\sqrt{6}}{35} \\ \sqrt{\frac{6}{35}} & 0 & \frac{4}{5}\sqrt{\frac{3}{7}} & \frac{3\sqrt{6}}{35} & -\frac{5}{7} \end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	${}^2D_{5/2}$-${}^4D_{5/2}$-${}^4G_{5/2}$-${}^6S_{5/2}$-${}^6D_{5/2}$-${}^6G_{5/2}$	$\frac{5}{2}^-$	$\frac{1}{3}\times\begin{pmatrix} 0 & -\frac{2}{\sqrt{35}} & 2\sqrt{\frac{6}{35}} & \frac{1}{\sqrt{15}} & -2\sqrt{\frac{2}{105}} & \sqrt{\frac{2}{35}} \\ -\frac{2}{\sqrt{35}} & \frac{1}{7} & \frac{2\sqrt{6}}{35} & -\frac{2}{5}\sqrt{\frac{7}{3}} & \frac{2}{7}\sqrt{\frac{2}{3}} & \frac{16\sqrt{2}}{35} \\ 2\sqrt{\frac{6}{35}} & \frac{2\sqrt{6}}{35} & -\frac{1}{7} & 0 & -\frac{6}{35} & \frac{2\sqrt{3}}{7} \\ \frac{1}{\sqrt{15}} & -\frac{2}{5}\sqrt{\frac{7}{3}} & 0 & 0 & \frac{\sqrt{14}}{5} & 0 \\ -2\sqrt{\frac{2}{105}} & \frac{2}{7}\sqrt{\frac{2}{3}} & \frac{6}{35} & \frac{\sqrt{14}}{5} & \frac{2}{7} & \frac{9\sqrt{3}}{35} \\ \sqrt{\frac{2}{35}} & \frac{16\sqrt{2}}{35} & \frac{2\sqrt{3}}{7} & 0 & \frac{9\sqrt{3}}{35} & -\frac{2}{7} \end{pmatrix}$

IV. THE CONSISTENT DESCRIPTION OF THE PENTAQUARK TRIO

In this section we investigate whether the OBE model can describe the LHCb pentaquark trio consistently. We find that the removal of the short-range Dirac-delta contributions to the OBE potential is a necessary step for achieving this goal. We discuss the possible interpretations and justifications of this modification to the OBE model.

A. Predictions of the $P_c(4440)$ and $P_c(4457)$

In this manuscript, following the ideas of Refs. [53,54], we propose the determination of the cutoff from the condition of reproducing the mass of a known molecular candidate. As there are three hidden-charm pentaquarks, we are left with three possibilities: $P_c(4312)$ (as a $\bar{D}\Sigma_c$ bound state), $P_c(4440)$ and $P_c(4457)$ (as $\bar{D}^*\Sigma_c$ bound states). Owing to the aforementioned regulator artifact in the spin-spin piece of the OBE potential, the most suitable choice is the $P_c(4312)$, which for the parameters of Table II is reproduced for

$$\Lambda_1 = 1119 \text{ MeV}. \tag{94}$$

In the naïve OBE model, this cutoff leads to the predictions

$$M\left(\frac{1}{2}\right) = 4388 \text{ MeV} \quad \text{and} \quad M\left(\frac{3}{2}\right) = 4459 \text{ MeV}, \tag{95}$$

which are not compatible with the experimental masses of the $P_c(4440)$ and the $P_c(4457)$, i.e.,

$$M_{P_{c2}} = 4440.3 \pm 1.3^{+4.1}_{-4.6} \text{ MeV} \quad \text{and}$$
$$M_{P_{c3}} = 4457.3 \pm 0.6^{+4.1}_{-1.7} \text{ MeV}. \tag{96}$$

As already explained, the reason for this mismatch is the distortion of the OBE potential at relatively long distances owing to the deltalike contribution to the spin-spin interaction, which we will explain in what follows.

B. The one-pion-exchange potential with a monopolar form factor

Now if we inspect the OPE contribution to the OBE potential, it contains a spin-spin and a tensor piece

$$V_\pi = \vec{\sigma}_{L1} \cdot \vec{S}_{L2} V_{\pi(S)} + S_{L12}(\hat{r}) V_{\pi(T)}, \tag{97}$$

The spin-spin piece reads

$$V_{\pi(S)} = \frac{g_1 g_2}{6 f_\pi^2} \vec{\tau}_1 \cdot \vec{T}_2 m_\pi^3 \times \left[-d\left(m_\pi r, \frac{\Lambda}{m_\pi}\right) + W_Y\left(m_\pi r, \frac{\Lambda}{m_\pi}\right) \right], \tag{98}$$

where d and W_Y are the regularized deltalike and Yukawa-like contributions defined in Eqs. (49) and (50).

As can be seen from Eq. (98) and Fig. 1, these two contributions have opposite sign: the deltalike contribution will generate a strong short-range attraction/repulsion that is unphysical. If the range of this unphysical contribution is short enough, it will have no observable effect in the predictions of the OBE model. However the problem is that this is not the case. If we compute the OPE potential contribution with a monopolar cutoff $\Lambda_1 = 1119$ MeV, the OPE potential changes sign at $r = 1.1$ fm, which is comparable with the range of the OPE potential $R_\pi = 1/m_\pi = 1.4$ fm. This is unsettling to say the least: the

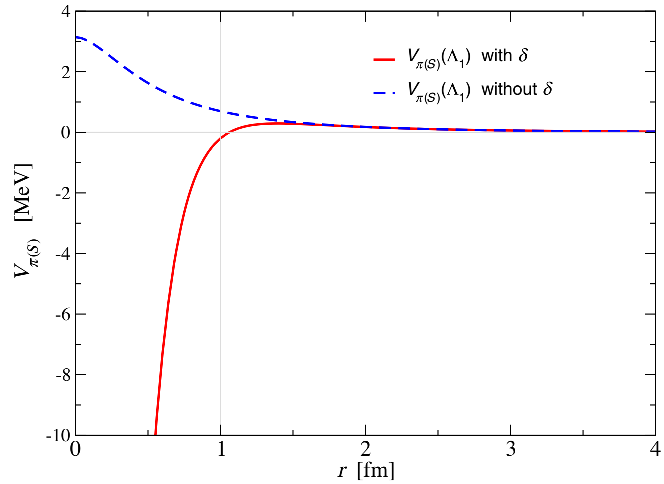

FIG. 1. The spin-spin piece of the OPE potential as a function of the radius r with and without the deltalike contributions. For simplicity we show this piece for $\vec{\tau}_1 \cdot \vec{T}_2 = +1$, which corresponds to $I = \frac{3}{2}$.

modifications of the form factors to the OBE potential are expected to be short-ranged but certainly not of the order of the pion range. This indicates that it is better to remove this contribution. If we remove the deltalike contributions of the pion and the vector mesons, we end up with the predictions

$$M\left(\frac{1}{2}\right) = 4458.0 \text{ MeV} \quad \text{and} \quad M\left(\frac{3}{2}\right) = 4443.9 \text{ MeV}, \tag{99}$$

which are basically compatible with the experimental determination of the masses of the $P_c(4440)$ and $P_c(4457)$.

V. THE PENTAQUARK MULTIPLET

In this section we compute the predictions of the OBE model for the hidden-charm molecular pentaquarks. We determine the cutoff in the calculation from the condition of reproducing the $P_c(4312)$, as explained in Sec. IV. From this condition and the OBE potential we can simply determine the full hidden-charm molecular spectrum. We also explain how we estimate the uncertainties of the OBE model.

A. Error estimations

The OBE model has a series of uncertainties, mostly stemming from the choice of the coupling constants. This error source can be dealt with by assigning a relative uncertainty to the OBE potential:

$$V(P_c) = V_{\text{OBE}}(1 \pm \delta_{\text{OBE}}), \tag{100}$$

where $V(P_c)$ is the molecular pentaquark potential in a given channel and V_{OBE} is the central value of the OBE potential with the central value of the couplings, see Table II for details. We will assume the relative uncertainty to be $\delta_{\rm OBE} = 30\%$, which is equivalent to assume that the average relative uncertainty of the coupling constants in Table II is $\delta_{\rm coupling} = \delta_{\rm OBE}/2 \sim 15\%$ (assuming a Gaussian uncertainty distribution for the couplings).[1] With the average uncertainty $\delta_{\rm OBE}$ we can recalculate the cutoff Λ_1 by determining the location of the $P_c(4312)$, leading to

$$\Lambda_1 = 1.119^{+0.190}_{-0.094}\ \text{GeV}. \tag{101}$$

The error in the binding energies is simply obtained by propagating the $(1 \pm \delta_{\rm OBE})$ uncertainty in the OBE potential, with the condition of recalculating the cutoff as to reproduce the $P_c(4312)$. This condition implies the partial renormalization of the OBE model, which manifests in the fact that the errors derived from the overall uncertainty in the potential are rather small. For the particular case of the $\bar{D}^*\Sigma_c$ bound states we arrive at

$$B_E\left(\frac{1^-}{2}\right) = 4.2^{+0.6}_{-0.7}\ \text{MeV} \quad \text{and} \quad B_E\left(\frac{3^-}{2}\right) = 18.3^{+0.6}_{-0.0}\ \text{MeV}, \tag{102}$$

where the errors, besides being small, are also asymmetric. There is a second error source: HQSS is not exact for finite heavy quark masses. The relative size of HQSS violations is expected to be of the order of $\delta_{\rm HQSS} \sim \Lambda_{\rm QCD}/m_Q$, with $\Lambda_{\rm QCD} \sim (200\text{–}300)$ MeV and m_Q the mass of the heavy quark. This error manifests in random variations of the OBE potential around its expected HQSS limit

$$V(P_c) = V_{\rm OBE}^{(m_Q=\infty)}(1 \pm \delta_{\rm HQSS}), \tag{103}$$

where in the charm sector we expect $\delta_{\rm HQSS} \sim 15\%$. It is worth stressing the difference between the OBE error of Eq. (100) and the HQSS error of Eq. (103): the OBE error takes into account the error in the coupling constants but assumes that these couplings are identical for all the possible molecules, while the HQSS error considers that these couplings might be different for each of the molecular states. For the $\bar{D}^*\Sigma_c$ bound states the HQSS uncertainty is

$$B_E\left(\frac{1^-}{2}\right) = 4.2^{+5.3}_{-3.3}\ \text{MeV} \quad \text{and} \quad B_E\left(\frac{3^-}{2}\right) = 18.3^{+11.6}_{-9.2}\ \text{MeV}, \tag{104}$$

which is considerably larger than the OBE uncertainty. The reason why this happens is that the OBE uncertainty is *renormalized away*: changes in the couplings of the light mesons to the heavy hadrons are compensated by a change in the cutoff. On the contrary HQSS violations imply that the couplings are different for the ground and excited spin states of a heavy hadron, i.e., the couplings for the D and D^* (or Σ_c and Σ_c^*) are a bit different. This uncertainty is not absorbed by the cutoff variation and results in a larger error. Finally for the full error we will sum in quadrature the OBE and HQSS errors.

In addition to the binding energies of the molecular pentaquarks, we also compute the S-wave scattering lengths of the charmed antimeson-baryon systems. The reason is to identify molecular configurations in which the attraction is strong, but not strong enough to bind. The basis of this idea is a well-known relation between the two-body scattering length and binding energy, a_2 and B_2, that works in the limit in which the bound state is weakly bound

$$a_2 = \frac{1}{\sqrt{2\mu B_2}} + \mathcal{O}\left(\frac{\sqrt{2\mu B_2}}{m_\pi}\right), \tag{105}$$

with μ the reduced mass of the system and m_π the pion mass. For a shallow bound state, i.e., $m_\pi > \sqrt{2\mu B_2} > 0$, the scattering length is positive and large ($m_\pi a_2 \gg 1$). For $B_2 \to 0$ the scattering length diverges and for a system that almost binds, the scattering length is negative and large. We notice that we compute the scattering lengths under the assumption that the charmed antimeson and the charmed baryon are stable hadrons with respect to the strong interaction, which is not true in general. This is not important as we are actually using the scattering length as a tool to identify configurations that are close to binding.

B. Predictions

With the OBE model regularized without the deltalike contributions, we can predict the seven possible S-wave $\bar{D}^{(*)}\Sigma_c^{(*)}$ molecules. The results are summarized in Table VI. For the isodoublet ($I = \frac{1}{2}$) molecular pentaquarks, the states predicted in the OBE model are indeed very similar to the ones obtained in *scenario B* of the contact-range EFT of Ref. [13] and the pionful one of Ref. [19]. Here we note that within the pionless and pionful descriptions of Refs. [13,19] there are two coupling constants whose values have to be determined from experimental information. Thus two scenarios were considered: scenario A, in which $P_c(4440)$ and $P_c(4457)$ are the $J = \frac{1}{2}$ and $\frac{3}{2}$ $\bar{D}^*\Sigma$ molecules, and scenario B for the opposite identification. Our OBE model naturally selects scenario B.

[1]The previous 15% figure is an educated guess, as it is markedly difficult to assess the specific errors in the couplings. The biggest uncertainty lies probably in the g_σ coupling, which comes from the nonlinear sigma model and the quark model: though difficult to determine, it could very well be about 20%–30%. For g_ρ and g_ω the uncertainty is expected to be smaller, maybe at the 10% level, as can be deduced from comparing (at least in the heavy meson case) the value deduced from Sakurai's universality ($g_\rho \simeq 2.9$) with the one calculated in the lattice ($g_\rho = 2.6 \pm 0.1 \pm 0.4$) [72], or the value deduced from semileptonic decays and vector meson dominance ($g_\rho = 2.6$) [69].

TABLE VI. Scattering lengths (a_2 in fm) and binding energies (B_2 in MeV) of prospective isodoublet and isoquartet hidden-charm antimeson-baryon molecules. The column "Molecule" refers to the two-body system under consideration, while I and J^P denote the isospin and total angular momentum and parity of the system. The error comes from an estimated relative uncertainty for the OBE potential of the order of 30% and from HQSS violations of the order of 15%, where the second error source dominates. M refers to the predicted mass (the central value) of a particular heavy antimeson-baryon molecule (if it binds). The calculation of the scattering length assumes that the hadrons are stable.

Molecule	I	J^P	a_2 (fm)	B_2 (MeV)	M (MeV)
$\bar{D}\Sigma_c$	$\frac{1}{2}$	$\frac{1}{2}^-$	$1.9^{+1.0}_{-0.4}$	Input	Input
$\bar{D}\Sigma_c^*$	$\frac{1}{2}$	$\frac{3}{2}^-$	$1.9^{+0.9}_{-0.4}$	$9.3^{+7.7}_{-5.7}$	4376.0
$\bar{D}^*\Sigma_c$	$\frac{1}{2}$	$\frac{1}{2}^-$	$2.5^{+2.3}_{-0.6}$	$4.2^{+5.3}_{-3.4}$	4458.0
$\bar{D}^*\Sigma_c$	$\frac{1}{2}$	$\frac{3}{2}^-$	$1.4^{+0.5}_{-0.3}$	$18.3^{+11.6}_{-9.2}$	4443.9
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{1}{2}^-$	$2.6^{+2.5}_{-0.7}$	$2.9^{+4.5}_{-2.6}$	4523.8
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{3}{2}^-$	$1.9^{+1.0}_{-0.4}$	$9.2^{+7.9}_{-5.8}$	4517.5
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{5}{2}^-$	$1.3^{+0.4}_{-0.3}$	$22.4^{+13.1}_{-10.6}$	4504.3

Molecule	I	J^P	a_2 (fm)	B_2 (MeV)	M (MeV)
$\bar{D}\Sigma_c$	$\frac{3}{2}$	$\frac{1}{2}^-$	$-1.8^{+1.2}_{-2.9}$	...	...
$\bar{D}\Sigma_c^*$	$\frac{3}{2}$	$\frac{3}{2}^-$	$-1.8^{+1.2}_{-3.2}$	...	...
$\bar{D}^*\Sigma_c$	$\frac{3}{2}$	$\frac{1}{2}^-$	$7.1^{+\infty(-19.5)}_{-4.8}$	$0.4^{+2.2}_{\dagger}$	4461.8
$\bar{D}^*\Sigma_c$	$\frac{3}{2}$	$\frac{3}{2}^-$	$-0.8^{+0.5}_{-1.4}$	...	...
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}$	$\frac{1}{2}^-$	$3.9^{+9.8}_{-1.7}$	$1.4^{+3.2}_{-1.8}$	4325.3
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}$	$\frac{3}{2}^-$	$-10.6^{+11.2}_{-\infty(10.0)}$	...	...
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}$	$\frac{5}{2}^-$	$-0.6^{+0.4}_{-0.8}$	...	...

For the isoquartet ($I = \frac{3}{2}$) molecular pentaquarks, we find that the $J = \frac{1}{2}$ $\bar{D}^*\Sigma_c$ and the $J = \frac{1}{2}$ $\bar{D}^*\Sigma_c^*$ bind. Yet this conclusion is not particularly strong: these two molecular pentaquarks are weakly bound and once we consider the error in the binding energies the outcome is that there is a fair likelihood that they will not bind. The isoquartet $J = \frac{3}{2}$ $\bar{D}^*\Sigma_c^*$ molecule is close to binding, as can be inferred from the large negative scattering length. Conversely, the uncertainties in the OBE model mean that this molecular pentaquark might be able to bind. The other isoquartet molecules display mild attraction, a conclusion which can be deduced from the negative (but natural) values of the scattering length shown in Table VI.

C. Inclusion of the η meson as a theoretical cross-check

Finally we will consider the effect of including the η meson in the OBE model. With this we want to check whether our theoretical error estimations are reliable or not. As a matter of fact we have a partial check in the prediction of the $P_c(4440)$ and $P_c(4457)$ pentaquark masses in Table VI, which are compatible with the experimental ones. We find it worth mentioning that the recent theoretical analysis of Ref. [73] indicates possible evidence of a narrow $P_c(4380)$ $\bar{D}\Sigma^*$ molecule within the current experimental data, a result which is also compatible with Table VI. Yet a robust experimental confirmation would still require the detection of the full pentaquark multiplet and their quantum numbers. Be it as it may, besides experiment, another method to cross-check our results is a theory-with-theory comparison. By including the η we are indeed comparing the OBE model with itself.

The OBE model is a phenomenological model and its formulation is dependent on a set of arbitrary choices that might affect its predictions. The choice of the form-factor cutoff, which we addressed in Sec. IV, is the most obvious example. Yet the selection of which light mesons to include in the description is equally important. Here we advocate for a minimalistic OBE in which only the π, σ, ρ and ω are taken into account. Interestingly whether this choice is good enough can be explicitly tested by including additional light mesons. The obvious candidate is the η meson, the mass of which ($m_\eta = 548$ MeV) is comparable to the one of the σ meson and thus they will both have a comparable range. The derivation of the η-exchange potential is presented in Appendix C, while here we simply notice that it contains a spin-spin and tensor pieces but not a central one. As a consequence the inclusion of the η meson has no effect for the $\bar{D}\Sigma_c$ and $\bar{D}\Sigma_c^*$ molecules, in which the OBE potential is purely central. In particular the $P_c(4312)$ pentaquark, which we use for determining the form-factor cutoff, is unaffected. Thus we end up with the same cutoff as before, i.e., $\Lambda_1 = 1119$ MeV. However η exchange will affect the location of the other pentaquarks, where in Table VII we show a detailed comparison of the η-less and η-full OBE models.

In Table VII we observe that in general the effects of including the η are relatively modest, of the order of 0.5–1.0 MeV in the binding energies in most cases. That is, the effects of η exchange are within the error bands we had already computed, providing further support for our error estimations. This also indicates that the inclusion of η exchange is not necessary for the pentaquarks as molecular states. Yet we warn that this conclusion might be specific to pentaquarks, as there might be other molecular states in which η exchange could play an important role [74]. However for the particular case of the pentaquarks it has been argued that pion exchanges might be perturbative [19]. If this were to be the case, then it is not surprising that η exchange plays a minor role in the OBE model, as it is considerably weaker than pion-exchange owing to SU(3)-flavor geometric factors and also to $f_\eta > f_\pi$, which further weakens η exchange.

TABLE VII. Comparison of the scattering lengths and binding energies in our original OBE model without η exchange ($a_2^{\not\eta}$ and $B_2^{\not\eta}$, in fm and MeV units, respectively) and after including η exchange (a_2^{η} and B_2^{η}). The columns "Molecule," I and J^P read as in Table VI, while $M^{\not\eta}$ and M^{η} refer to the masses of the pentaquarks in the η-less and η-full OBE model. The error estimations are estimated in the same way as in Table VI (i.e., assume a 30% uncertainty in the strength of the OBE potential and HQSS violations at the 15% level). The addition of the η meson in the OBE model leaves the prediction for the $\bar{D}\Sigma$ and $\bar{D}\Sigma^*$ molecules unchanged, as these molecules depend only on the central components of the potential while η exchange only generates spin-spin and tensor components. In general the predictions of the η-less and η-full OBE model overlap within the estimated error bands and are thus indistinguishable at the theoretical level.

Molecule	I	J^P	$a_2^{\not\eta}$ (fm)	a_2^{η} (fm)	$B_2^{\not\eta}$ (MeV)	B_2^{η} (MeV)	$M^{\not\eta}$ (MeV)	M^{η}
$\bar{D}\Sigma_c$	$\frac{1}{2}$	$\frac{1}{2}^-$	$1.9^{+1.0}_{-0.4}$	$1.9^{+1.0}_{-0.4}$	Input	Input	Input	Input
$\bar{D}\Sigma_c^*$	$\frac{1}{2}$	$\frac{3}{2}^-$	$1.9^{+0.9}_{-0.4}$	$1.9^{+0.9}_{-0.4}$	$9.3^{+7.7}_{-5.7}$	$9.3^{+7.7}_{-5.7}$	4376.0	4376.0
$\bar{D}^*\Sigma_c$	$\frac{1}{2}$	$\frac{1}{2}^-$	$2.5^{+2.3}_{-0.6}$	$2.4^{+2.0}_{-0.6}$	$4.2^{+5.3}_{-3.4}$	$4.8^{+5.7}_{-3.6}$	4458.0	4457.4
$\bar{D}^*\Sigma_c$	$\frac{1}{2}$	$\frac{3}{2}^-$	$1.4^{+0.5}_{-0.3}$	$1.3^{+0.5}_{-0.2}$	$18.3^{+11.6}_{-9.2}$	$17.3^{+11.1}_{-8.8}$	4443.9	4442.9
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{1}{2}^-$	$2.6^{+2.5}_{-0.7}$	$2.4^{+2.1}_{-0.6}$	$2.9^{+4.5}_{-2.6}$	$3.8^{+5.4}_{-3.2}$	4523.8	4522.9
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{3}{2}^-$	$1.9^{+1.0}_{-0.4}$	$1.9^{+1.0}_{-0.4}$	$9.2^{+7.9}_{-5.8}$	$9.4^{+8.0}_{-5.8}$	4517.5	4517.3
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{5}{2}^-$	$1.3^{+0.4}_{-0.3}$	$1.3^{+0.6}_{-0.3}$	$22.4^{+13.1}_{-10.6}$	$21.1^{+12.3}_{-10.1}$	4504.3	4503.0
Molecule	I	J^P	$a_2^{\not\eta}$ (fm)	a_2^{η} (fm)	$B_2^{\not\eta}$ (MeV)	B_2^{η} (MeV)	$M^{\not\eta}$ (MeV)	M^{η}
$\bar{D}\Sigma_c$	$\frac{3}{2}$	$\frac{1}{2}^-$	$-1.8^{+1.2}_{-2.9}$	$-1.8^{+1.2}_{-2.9}$	…	…	…	…
$\bar{D}\Sigma_c^*$	$\frac{3}{2}$	$\frac{3}{2}^-$	$-1.8^{+1.2}_{-3.2}$	$-1.8^{+1.2}_{-3.2}$	…	…	…	…
$\bar{D}^*\Sigma_c$	$\frac{3}{2}$	$\frac{1}{2}^-$	$7.1^{+\infty(-19.5)}_{-4.8}$	$5.7^{+\infty(-97.3)}_{-3.4}$	$0.4^{+2.2}_{\dagger}$	$0.7^{+2.0}_{\dagger}$	4461.8	4461.5
$\bar{D}^*\Sigma_c$	$\frac{3}{2}$	$\frac{3}{2}^-$	$-0.8^{+0.5}_{-1.4}$	$-0.8^{+0.4}_{-1.1}$	…	…	…	…
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}$	$\frac{1}{2}^-$	$3.9^{+9.8}_{-1.7}$	$3.6^{+5.2}_{-1.6}$	$1.4^{+3.2}_{-1.8}$	$2.0^{+3.2}_{-1.7}$	4525.3	4524.7
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}$	$\frac{3}{2}^-$	$-10.6^{+11.2}_{-\infty(10.0)}$	$-15.6^{+17.8}_{-\infty(7.9)}$	…	…	…	…
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}$	$\frac{5}{2}^-$	$-0.6^{+0.4}_{-0.8}$	$-0.5^{+0.5}_{-0.8}$	…	…	…	…

VI. THE OBE MODEL FROM A MODERN PERSPECTIVE

In this section we will consider the OBE model—in particular the removal of the Dirac-delta contributions—from a modern understanding grounded on renormalization and EFT ideas. From a phenomenological perspective this removal is motivated because the range of the regularized Dirac-delta contributions is unexpectedly long, resulting in the distortion of the OPE potential at distances comparable to the pion Compton wavelength. From a modern perspective this distortion will be considered a *regulator artifact*, which should be taken care of by means of the renormalization procedure. In the following lines we will explain these points of view in order to put the OBE model in context.

From a traditional perspective the existence of short-range ambiguities in the OBE model is apparent from the fact that the unregularized OBE potential is singular, with the tensor contributions diverging as $1/r^3$ for distances $mr \ll 1$, with m the mass of the exchanged boson. This type of potentials require regularization if we want to have a unique solution of the Schrödinger equation [75] (for more modern treatments of singular interactions see Refs. [76–79]). However we do not expect the behavior of the OBE potential at distances shorter than the size of the hadrons to be physical. Thus, we remove these unphysical short-range contributions by regularizing the OBE potential. This is the reason that justified the inclusion of form factors in the original OBE model and this is also the reason why we removed the Dirac deltas in Sec. IV.

Nowadays we know that the removal of short-range ambiguities requires not only regularization but also renormalization. By this we mean the following: we expect to trade-off the short-range ambiguities by observable information. In the original OBE model we simply regularize the potential by choosing a sensible form factor and cutoff. The renormalization process is more systematic: we explicitly include a contact-range potential to model the unknown short-range physics. By fitting the couplings in this contact-range potential to experimental information we are effectively absorbing the dependence on the form factor and the cutoff in these couplings. The price to pay is a reduction in the predictive power, as we have to include new parameters in the theory which have to be determined from experimental data.

Yet renormalization helps to understand in hindsight the success of phenomenological models. In the particular case of the OBE potential, choosing the form factor and the cutoff as to reproduce experimental information basically amounts to an implicit (but usually incomplete) renormalization process (for an explicit and complete renormalization of the OBE potential we recommend Ref. [55]). In particular we will consider in detail the following two perspectives

(i) the removal of the Dirac-delta contributions as a renormalization choice, and
(ii) the relation of the OBE model with EFT descriptions of the pentaquarks.

A. Renormalization and the OBE model

As we have already mentioned, the most important difference of the OBE model as presented in this work with its standard implementation is the removal of the contact-range pieces that appear in the spin-spin component of the potential. That is, we have identified that the OBE potential can be divided into two pieces

$$V_{\mathrm{OBE}} = V_{\mathrm{OBE(F)}} + V_{\mathrm{OBE(C)}}, \tag{106}$$

where F and C refer to the finite- and contact-range piece contributions. The contact-range piece conforms to the general structure of Eq. (16), i.e.,

$$V_{\mathrm{OBE(C)}} = C_a^{\mathrm{OBE}} + C_b^{\mathrm{OBE}} \vec{\sigma}_{L1} \cdot \vec{S}_{L2} \tag{107}$$

where the couplings are given by

$$C_a^{\mathrm{OBE}} = 0, \tag{108}$$

$$C_b^{\mathrm{OBE}} = +[C_b^{\delta\pi} + C_b^{\delta\rho} + C_b^{\delta\omega}], \tag{109}$$

where $C_b^{\delta\pi}$, $C_b^{\delta\rho}$ and $C_b^{\delta\omega}$ are determined by the Dirac-delta contributions to the spin-spin component of the OBE potential (hence the notation $\delta\pi$, $\delta\rho$, $\delta\omega$). These components come from π exchange and the magneticlike piece of ρ and ω exchange[2]:

$$C_b^{\delta\pi} = -\frac{g_1 g_2}{6 f_\pi^2} \vec{\tau}_1 \cdot \vec{T}_2, \tag{110}$$

$$C_b^{\delta\rho} = -\frac{2}{3} \frac{f_{\rho 1}}{2M_1} \frac{f_{\rho 2}}{2M_2} \vec{\tau}_1 \cdot \vec{T}_2, \tag{111}$$

$$C_b^{\delta\omega} = -\frac{2}{3} \frac{f_{\omega 1}}{2M_1} \frac{f_{\omega 2}}{2M_2}. \tag{112}$$

The removal of these contact-range pieces is equivalent to the following substitution:

$$V_{\mathrm{OBE}} = V_{\mathrm{OBE(F)}} + V_{\mathrm{OBE(C)}} \to V_{\mathrm{OBE}} = V_{\mathrm{OBE(F)}},$$

that is, instead of using the complete OBE potential we restrict ourselves to the finite-range piece of the OBE potential, where the justification we have provided so far is the unexpected long-range distortion of the finite-range pieces when the $V_{\mathrm{OBE(C)}}$ piece is kept.

Although there is indeed a physical justification for this removal, it would be interesting whether this can be justified from a renormalization perspective. A straightforward justification is the explicit inclusion of a contact-range component in the potential

$$V = \delta V_C + V_{\mathrm{OBE(F)}} + V_{\mathrm{OBE(C)}}, \tag{113}$$

where δV_C merely represents the unknown short-range physics not explicitly included in the OBE model, where the structure of δV_C still follows Eq. (16):

$$\delta V_C = \delta C_a + \delta C_b \vec{\sigma}_{L1} \cdot \vec{S}_{L2}, \tag{114}$$

Obviously the choice,

$$\delta V_c = -V_{\mathrm{OBE(C)}}, \tag{115}$$

will do the trick, where this choice corresponds to

$$\delta C_a = 0 \quad \text{and} \quad \delta C_b = -C_b^{\mathrm{OBE}}. \tag{116}$$

From this point of view the removal of the Dirac-delta contributions merely amounts to playing a shell game between unknown short-range interactions and the Dirac-delta contributions already present in the OBE potential. Complementarily the observation that with this choice we end up with a correct prediction of the masses of the $P_c(4440)$ and $P_c(4457)$ pentaquarks motivates and provides physical content to Eq. (116).

Here it is worth noticing that the rationale of renormalization is very different from that in the original OBE model, where this strong distortion of the pion contribution to the potential at long distances was avoided by the use of a large enough cutoff, usually $\Lambda_\pi > 1.3$ GeV. Besides, the finite-range piece of the spin-spin piece of the OPE potential is attractive in the S-wave singlet and triplet partial waves, which in turn leads to a repulsive Dirac-delta contribution. But for the heavy antimeson-baryon system it is difficult to have a large enough cutoff that still reproduces the three pentaquark poles.[3] In any case it would be interesting to check whether the present identification of the quantum numbers of $P_c(4440)$ and $P_c(4457)$ will still be correct in a fully renormalized OBE model as the one presented in Ref. [55], i.e., with a cutoff that is not fixed but floats within a given range.

[2]Notice that there is no Dirac-delta contribution stemming from the central pieces of the potential, i.e., from the exchange of the scalar meson σ or from the electric-type couplings of the ρ and ω.

[3]This will require making the ω-meson contribution considerably more repulsive by breaking the SU(3) relation $g_\rho = g_\omega$, which is what happens in the OBE model as applied to the nucleon-nucleon system.

B. The EFT description and the OBE model

The previous interpretation is rather formal, merely showing that it is possible to renormalize away the Dirac deltas appearing in the OBE potential. It is more interesting though to compare the OBE model with the EFT approach, as this would provide us with some interesting insights.

Within EFT we divide physics into long- and short-range contributions, where the long-range contributions are assumed to be known while the short-range ones are not. If we are dealing with a nonrelativistic problem, what we can do is to build an effective potential which can be decomposed into two pieces

$$V_{\rm EFT} = V_C + V_F. \qquad (117)$$

Here V_C and V_F refers to the contact- and a finite-range piece, which represent the long- and short-range physics, respectively. Actual EFTs are arranged in terms of a power counting, which orders the different contributions to V_C and V_F from more to less relevant. However the issue of power counting is not particularly relevant to the current discussion: thus we will implicitly assume that we are working at leading order (LO); i.e., we only take into account the most important contributions to the effective potential. Notice that the decomposition into long- and short-range physics is not unique and neither is the choice of power counting. Thus the meaning and interpretation of the contact-range potential V_C will depend on these choices.

Here we will compare the OBE model with the EFTs of Refs. [13,19], which differ on their choices for the finite-range part of the effective potential. In Ref. [13] pion exchanges are considered to be perturbative and thus subleading, which means that the LO potential reads

$$V_F^{\not\pi} = 0. \qquad (118)$$

In Ref. [19] pion exchanges are considered to be nonperturbative and included at LO

$$V_F^{\pi} = V_\pi, \qquad (119)$$

where V^π coincides with the OPE potential calculated here. For these two EFTs [13,19] the LO contact-range potential takes the form

$$V_C = C_a + C_b \vec{\sigma}_{L1} \cdot \vec{S}_{L2}, \qquad (120)$$

where C_a and C_b are couplings that have to be determined from experimental information, in particular from the masses of the pentaquarks. It happens that there are three pentaquarks to which to fit C_a and C_b. Besides, the coupling C_b represents a spin-dependent interaction and its determination depends on which is the spin of the $P_c(4440)$ and $P_c(4457)$ pentaquarks. Thus these EFTs consider two possible scenarios (A and B) for which the spin of these two pentaquarks is

(A) the $P_c(4440)$ ($P_c(4457)$) is $J = 1/2$ ($J = 3/2$),
(B) the $P_c(4457)$ ($P_c(4440)$) is $J = 1/2$ ($J = 3/2$).

Scenario A corresponds to the standard quark model expectation that hadrons with higher spin should have higher mass, while scenario B describes the opposite situation. These EFTs cannot discriminate *a priori* between these two possibilities, but only *a posteriori* if the predictions of each scenario are different enough.

The OBE model operates differently: the OBE potential is assumed to be a complete description of the hadron-hadron interaction. Thus we do not include a contact-range potential. Indeed we could interpret the OBE model as a EFT for which at LO

$$V_C = 0 \quad \text{and} \quad V_F = V_{\rm OBE(F)}. \qquad (121)$$

Yet this interpretation is admittedly liberal and we have included it for illustrative purposes only: it would be difficult to construct an EFT when the mass of the vector mesons is so close to the expected breakdown scale. Besides, the fact that predictions depend on the cutoff shows that it is indeed not an EFT. Be it as it may, from an operational standpoint we have a free parameter in the OBE model, the form-factor cutoff Λ, which we have determined from the $P_c(4312)$. From this cutoff we can predict the masses and spins of the other two pentaquarks, while within EFT we could not predict their spins. We present a summary of the EFT and OBE model predictions in Table VIII, where it turns out that the predictions of the OBE model coincide with those of scenario B in Refs. [13,19].

Further support towards scenario B from the OBE model can be found from analyzing the values of the C_a and C_b couplings. Though these are running couplings, they still might carry information about the short-range physics not explicitly included within the EFT description, i.e., about σ, ρ and ω exchange. The mechanism for this is the saturation hypothesis, i.e., that the value of the EFT couplings are saturated by light meson exchange. This idea works in pion-nucleon [80] and nucleon-nucleon [81] scattering, though its application in a purely nonperturbative context such as nucleon-nucleon scattering (or hadronic molecules) is considerably less clean and depends on the EFT cutoff being of the same order of magnitude as the exchanged mesons from which saturation comes.

Following the formalism of Ref. [82], we expect the EFT couplings in Refs. [13,19] to be saturated primarily by scalar and meson vector exchange

$$C_a^{\rm sat}(\Lambda \sim m_\sigma, m_V) \propto C_a^S + C_a^V, \qquad (122)$$

$$C_b^{\rm sat}(\Lambda \sim m_\sigma, m_V) \propto C_b^V, \qquad (123)$$

TABLE VIII. Comparison of the predicted masses of the pentaquarks in the OBE model and the EFT frameworks of Refs. [13,19]. Following the notation of Table VI, the columns "Molecule," I and J^P refer to the type of two-body system, its isospin and total spin and parity. For the masses, $M_{\rm OBE}$ is the predicted mass in the OBE model, $M_{({\rm EFT}\not{\pi})}$ to the mass within the EFT of Ref. [13] (in which pions are subleading) while $M_{({\rm EFT}\pi)}$ to the EFT of Ref. [19] (in which pions are leading). Both EFTs are unable to determine the quantum numbers of the $P_c(4440)$ and $P_c(4457)$ from first principles and thus a choice has to be made: in scenario A the $P_c(4440)$ is the $J^P = \frac{1}{2}^-$ $\bar{D}^*\Sigma_c$ molecule, while in scenario B it is the $P_c(4457)$ which is the $J^P = \frac{1}{2}^-$ $\bar{D}^*\Sigma_c$ molecule. The scenarios are indicated by a superscript in the mass ($M^A_{\rm EFT}$ and $M^B_{\rm EFT}$). The mass ranges are derived from varying the cutoff Λ within the EFT descriptions, where for the EFT of Ref. [13] (Ref. [19]) the cutoff window is $\Lambda = 0.5$–1.0 GeV ($\Lambda = 0.75$–1.5 GeV). In general the predictions of the OBE model agree well with scenario B, but not with scenario A.

Molecule	I	J^P	$M_{\rm OBE}$ (MeV)	$M^A_{({\rm EFT}\not{\pi})}$ (MeV) [13]	$M^B_{({\rm EFT}\not{\pi})}$ (MeV) [13]	$M^A_{({\rm EFT}\pi)}$ (MeV) [19]	$M^B_{({\rm EFT}\pi)}$ (MeV) [19]
$\bar{D}\Sigma_c$	$\frac{1}{2}$	$\frac{1}{2}^-$	Input	4312–4313	4306–4308	4314–4320	4313–4320
$\bar{D}\Sigma_c^*$	$\frac{1}{2}$	$\frac{3}{2}^-$	4376.0	4371–4372	4376–4377	4378–4383	4373–4385
$\bar{D}^*\Sigma_c$	$\frac{1}{2}$	$\frac{1}{2}^-$	4458.0	Input	Input	Input	Input
$\bar{D}^*\Sigma_c$	$\frac{1}{2}$	$\frac{3}{2}^-$	4443.9	Input	Input	Input	Input
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{1}{2}^-$	4523.8	4500–4501	4523–4524	4483–4500	4513–4523
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{3}{2}^-$	4517.5	4511	4517	4507–4512	4511–4516
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{5}{2}^-$	4504.3	4523–4524	4500–4501	4520–4523	4497–4501

where the superscript $^{\rm sat}$ indicates that we are considering the part of the couplings given by saturation and with $\Lambda \sim m_\sigma$, m_V indicating that saturation is expected to work only at EFT cutoffs close to the mass of the exchanged mesons, in this case σ, ρ and ω, suggesting $\Lambda \sim 0.6$–0.8 GeV. The value of the saturated couplings is expected to be proportional to the potential contribution $V_M(\vec{q})$ of the light mesons M evaluated at $|\vec{q}| = 0$ once we have removed the spurious Dirac-delta contributions [82]. This gives us

$$C_a^{{\rm sat}(\sigma)}(\Lambda \sim m_\sigma) \propto -\frac{g_{\sigma 1} g_{\sigma 2}}{m_\sigma^2}, \tag{124}$$

$$C_a^{{\rm sat}(V)}(\Lambda \sim m_\rho) \propto +\frac{g_{V1} g_{V2}}{m_V^2}(1 + \vec{\tau}_1 \cdot \vec{T}_2), \tag{125}$$

$$C_b^{{\rm sat}(V)}(\Lambda \sim m_\rho) \propto +\frac{f_{V1} f_{V2}}{6M^2}(1 + \vec{\tau}_1 \cdot \vec{T}_2), \tag{126}$$

where $V = \rho$, ω and we have made the simplification that $m_\rho = m_\omega = m_V$, and we do not include the pion as it is not expected to contribute for saturation at $\Lambda \gg m_\pi$. The proportionality constant is unknown and will depend on the details of the renormalization process. However, assuming this proportionality constant is the same for $C_a^{\rm sat}$ and $C_b^{\rm sat}$, we can compute their ratio

$$\frac{C_b^{{\rm sat}(V)}}{C_a^{{\rm sat}(V+\sigma)}} \simeq 0.123. \tag{127}$$

We will compare this reference value with the ratio we obtain in EFT at a momentum space cutoff $\Lambda = 750$ MeV, which is close to the vector meson masses. For the EFT of Ref. [13] in which pions are subleading, we obtain the ratios

$$\left.\frac{C_b^{({\rm EFT}\not{\pi})}}{C_a^{({\rm EFT}\not{\pi})}}\right|^{A}_{\Lambda = 750\ {\rm MeV}} = -0.176, \tag{128}$$

$$\left.\frac{C_b^{({\rm EFT}\not{\pi})}}{C_a^{({\rm EFT}\not{\pi})}}\right|^{B}_{\Lambda = 750\ {\rm MeV}} = +0.158, \tag{129}$$

which for scenario B is qualitatively compatible with the OBE estimate. For the EFT of Ref. [19], in which pions are included at leading order, the comparison is not direct. The reason is that the momentum space version of the one pion exchange potential used in Ref. [19] contains the spurious Dirac-delta contributions that we have removed in the OBE model here. Of course within the EFT framework these spurious contributions are not a problem because they can be reshuffled into the contact-range couplings. However, if we want to make a comparison with the saturated couplings then we have to explicitly remove this spurious pion contribution given by Eq. (110) and which evaluation yields $C_b^{\delta\pi} = 0.375\ {\rm fm}^2$. Now, if we remove $C_b^{\delta\pi}$, we will obtain the ratios

$$\left.\frac{C_b^{({\rm EFT}\pi)} - C_b^{\delta\pi}}{C_a^{({\rm EFT}\pi)}}\right|^{A}_{\Lambda = 750\ {\rm MeV}} = -0.229, \tag{130}$$

$$\left.\frac{C_b^{({\rm EFT}\pi)} - C_b^{\delta\pi}}{C_a^{({\rm EFT}\pi)}}\right|^{B}_{\Lambda = 750\ {\rm MeV}} = +0.136, \tag{131}$$

where scenario B is again closer to the ratio expected within the OBE model (in fact it basically reproduces the expected ratio).

It is interesting to notice that the EFT couplings can also be used to check the necessity of including certain light mesons within the OBE model. For instance, if we remove the σ meson, the $C_a^{\rm sat}$ coupling will not be saturated by this meson, and we will end up with the ratio

$$\frac{C_b^{{\rm sat}(V)}}{C_a^{{\rm sat}(V)}} \simeq 0.426, \tag{132}$$

which is not compatible with scenario A or B in the two EFTs of Refs. [13,19] by a factor of three, give or take. This favors the inclusion of the σ in our OBE model. Coversely, if we remove the ω or the ρ the ratios will change to

$$\frac{C_b^{{\rm sat}(\rho)}}{C_a^{{\rm sat}(\rho+\sigma)}} \simeq 0.193, \qquad \frac{C_b^{{\rm sat}(\omega)}}{C_a^{{\rm sat}(\omega+\sigma)}} \simeq -0.282, \tag{133}$$

which, surprisingly, are somewhat similar to the EFT ratios for scenarios A and B, respectively, see Eqs. (130) and (131). Removing the ω (ρ) leads to a version of scenario B (A) with larger hyperfine splittings than in Refs. [13,19]. However from flavor symmetry we expect the ρ and ω to have similar couplings and thus we do not consider removing one without the other. Removing the vector mesons completely would lead to $C_b/C_a \simeq 0$, which results in a spectrum were the isospin splitting comes from OPE only: it would be similar to scenario B but with smaller hyperfine splittings. If we now turn our attention to the η meson, whose exchange potential is computed in Appendix C, this meson will be able to saturate the C_b coupling:

$$C_a^{{\rm sat}(\eta)}(\Lambda \sim m_\eta) \propto 0, \tag{134}$$

$$C_b^{{\rm sat}(\eta)}(\Lambda \sim m_\eta) \propto \frac{g_1 g_2}{18 f_\eta^2}, \tag{135}$$

with g_1 and g_2 the axial couplings for the charmed antimeson and baryon and $f_\eta \simeq 150$ MeV the weak decay constant of the η meson. In this case we obtain the ratio

$$\frac{C_b^{{\rm sat}(V+\eta)}}{C_a^{{\rm sat}(V+\sigma)}} \simeq 0.109, \tag{136}$$

which is compatible with the η-less ratio within the 10% level and with the EFT ratios in scenario B within the (25–45)% level. Owing to the qualitative nature of saturation, it is probably difficult to argue strongly for or against the inclusion of the η in the OBE as applied to the molecular pentaquarks, as its effect seems to be small in this case. This is in contrast with the σ meson, for which the evidence is more conclusive. As already mentioned, the previous conclusions about η exchange are *molecule specific*: while in the pentaquarks it probably is a minor effect, in other two-hadron systems (particularly if they involve strange hadrons [74]) it might represent an important contribution to binding.

VII. SUMMARY

In this paper we have investigated the spectroscopy of the hidden-charm pentaquarks from the point of view of the OBE model. In particular we considered the impact of the short-range delta-like contributions in the OBE potentials. The removal of these contributions, in combination with the condition of reproducing the mass of the $P_c(4312)$ pentaquark as a $\bar{D}\Sigma_c$ bound state, leads to the following predictions for the $\bar{D}^*\Sigma_c$ molecules:

$$M\left(\frac{1^-}{2}\right) = 4458.0^{+3.4}_{-5.3}\ \text{MeV} \quad \text{and}$$
$$M\left(\frac{3^-}{2}\right) = 4443.9^{+9.2}_{-11.6}\ \text{MeV}, \tag{137}$$

which are close to the experimental masses of the $P_c(4440)$ and $P_c(4457)$ pentaquarks. This suggests the identification of the $P_c(4440)$ with the $J = \frac{3}{2}$ $\bar{D}^*\Sigma_c$ bound state and the $P_c(4457)$ with the $J = \frac{1}{2}$ one. In fact the expectation from OPE alone is that the $J = \frac{3}{2}$ molecule should be more bound than the $J = \frac{1}{2}$ one [7], as a consequence of (the spin-spin component of) OPE being attractive (repulsive) in the $J = \frac{3}{2}$ ($\frac{1}{2}$) channel. The combination of OPE with short-range physics, as in Ref. [83] (which uses the hidden-gauge approach to which it adds pion-exchange diagrams), leads to the same conclusion. The recent work of Ref. [84] also explains the molecular pentaquark spectrum on the basis of OPE and proposes the same spin-parity identification as here, but suggest that the reason why the $J = \frac{3}{2}$ molecule is more bound is the tensor component of the OPE potential (instead of the spin-spin component, as in Ref. [7]). Be it as it may, we warn that theoretical predictions in the OBE model have significant uncertainties and that these uncertainties cannot be systematically estimated, as we are dealing with a model (instead of an effective field theory). Another aspect to consider is the decays of the pentaquarks: within the molecular picture the natural expectation is that the pentaquarks would predominantly decay into $\bar{D}\Lambda_c$ and $\bar{D}^*\Lambda_c$, with the decay mediated by pion and vector meson exchange. In principle it could be possible to extend the OBE model to the $\Sigma_c \to \Lambda_c$ transition to calculate this decay width, but this is beyond the scope of this work. It is also worth mentioning that the decays of molecular pentaquarks into $J/\Psi N$ and $\eta_c N$ are expected to be suppressed, which raises the question of whether this is compatible with the original detection of the P_c's in the $J/\Psi p$ channel by the LHCb [1]. However the nonobservation of the P_c's by the GlueX collaboration [85] sets higher limits on the branching ratios of the pentaquarks into

$J/\Psi p$, which are in the singlet digit percent level and thus might be compatible with the molecular picture. Yet this remains to be confirmed by concrete calculations of both charmed antimeson-baryon and charmonium decays.

Besides proposing a possible identification for the quantum numbers of the three hidden-charm pentaquarks, we predict the existence of other four molecular pentaquarks with $I = \frac{1}{2}$. This prediction indeed confirms the conclusion of Ref. [13], which used a contact-range effective field theory to describe the molecular pentaquarks, and of Ref. [14], which used the hidden-gauge formalism (constrained by HQSS) instead. Among the predicted states there is the $J = \frac{5}{2}$ $\bar{D}^*\Sigma_c^*$ molecule, which was conjectured in Refs. [6,45] and reproduced in a few recent theoretical works [13–15,86]. Finally, in the isoquartet sector ($I = \frac{3}{2}$) there might be two or three molecular pentaquarks that bind. We note that this will impact the size of the proposed isospin-breaking decay $\Gamma(P_c \to J/\Psi\Delta^+)/\Gamma(P_c \to J/\Psi p)$ calculated in Ref. [25] (in a similar way, for instance, as the presence of a bound or virtual state in the $D\bar{D}$ system will affect the decay of the $X(3872)$ to $D^0\bar{D}^0\pi^0$ [87]). Conversely, the experimental measurement of the isospin-breaking decay ratio proposed in Ref. [25] might provide important clues regarding the existence of isoquartet molecular pentaquarks.

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APPENDIX A: LAGRANGIAN FOR VECTOR MESON EXCHANGE FOR HEAVY HADRONS AND THE CHOICE OF A VELOCITY PARAMETER

The Lagrangians describing the interaction of the vector meson with the charmed mesons and baryons contains electric-, magnetic- and quadrupolelike components, i.e., it is a multipole expansion. It is worth noticing that the electric- and quadrupole-like terms depend only on the zeroth component of the vector meson field, which is expected to be suppressed for nonrelativistic processes. If we take the electric-type term as an example, the Lagrangian can be generically written as

$$\mathcal{L}_{hhV} = g_V h^\dagger V^0 h, \tag{A1}$$

with g_V a coupling, h a nonrelativistic field representing the heavy hadron (e.g., $h = H_Q, S_Q$ for superfield notation to $h_Q = q_L, d_L$ for subfield notation) and V^0 the zeroth component of the vector meson field. This prompts the question of how it is possible to derive a potential from a Lagrangian that generates vanishing amplitudes in the heavy quark limit (i.e., when the heavy hadron masses go to infinity). Indeed from the previous Lagrangian we can derive the nonrelativistic amplitude

$$\mathcal{A}(h \to hV^\mu) = g_V \epsilon^\mu \delta_{\mu 0}, \tag{A2}$$

where ϵ^μ refers to the polarization of the vector meson and $\delta_{\mu\nu}$ is the Kronecker delta. If we evaluate this amplitude for a physical vector meson, for which the polarization vector obeys the relation $q^\mu \epsilon_\mu = 0$ with q^μ the four-momentum of the vector meson, and assume that the initial and final hadrons are on-shell, then we have $q^\mu = p'^\mu - p^\mu$, i.e., the difference of the final and initial heavy hadron momenta. it is apparent that in the heavy quark limit q^0 scales as $1/m_Q$ with m_Q the mass of the heavy quark inside the heavy hadron. For $m_Q \to \infty$ the relation $q^\mu \epsilon_\mu = 0$ simplifies to $\vec{q} \cdot \vec{\epsilon} = 0$, which implies that $\epsilon^0 = 0$ for a physical vector meson in this limit, thus resulting in the aforementioned vanishing amplitude.

There are however two factors to consider here: (i) the potential is a nonobservable quantity which actually results from the exchange of a virtual vector meson instead of a physical one (i.e., we end up with a nonvanishing potential in the $m_Q \to \infty$ limit because $q^\mu \epsilon_\mu = 0$ only applies to physical states), (ii) the fact that the Lagrangian vanishes for on shell heavy hadrons in this limit does not mean that the g_V coupling is detached from physical processes, with this detachment being an artifact of the notation instead. To specifically address this second point, we notice that the full Lagrangians for a relativistic hadron field and a vector meson are

$$\mathcal{L}_{MMV} = g_V (i M^\dagger \overleftrightarrow{\partial}_\mu M) V^\mu, \tag{A3}$$

$$\mathcal{L}_{BBV} = g_V \bar{B} \gamma_\mu V^\mu B, \tag{A4}$$

where M and B are meson and baryon fields, respectively, V^μ the vector meson field and γ_μ the Dirac matrices. As we are dealing with heavy hadrons, we consider them to move with constant speed $v = (v^0, \vec{v})$, with $v^\mu v_\mu = 1$, prompting the (schematic) field redefinitions

$$M_v(x) \propto e^{i m_Q v \cdot x} M(x), \tag{A5}$$

$$B_v(x) \propto e^{i m_Q v \cdot x} B(x), \tag{A6}$$

with m_Q the mass of the heavy quark inside the heavy hadron, which in the heavy quark limit can be basically identified with the heavy hadron mass, and v the velocity parameter. After a few manipulations we end up with the Lagrangians

$$\mathcal{L}_{MMV} = g_V M_v^\dagger v \cdot V M_v, \tag{A7}$$

$$\mathcal{L}_{BBV} = g_V B_v^\dagger v \cdot V B_v, \tag{A8}$$

which are formally identical, leading to the more simple notation:

$$\mathcal{L}_{hhV} = g_V h_v^\dagger v \cdot V h_v, \tag{A9}$$

where h_v stands for a heavy hadron field with velocity v, for which we obtain the nonrelativistic amplitude

$$\mathcal{A}(h_v \to h_v V^\mu) = g_V \epsilon \cdot v. \tag{A10}$$

If we choose $v = (1, \vec{0})$, we end up with Eq. (A1).

Finally, for deriving the potential for the exchange of a vector meson we first define

$$\mathcal{A} = \epsilon^\mu \mathcal{A}_\mu \to \mathcal{A}_\mu = g_V v_\mu. \tag{A11}$$

With this new amplitude plus the convenient normalization we have used in Eqs. (A1) and (A11), we can write the potential as

$$\begin{aligned} V(\vec{q}) &= \frac{\mathcal{A}^\mu(\vec{q})\mathcal{A}_\mu(-\vec{q})}{\vec{q}^2 + m_V^2}, \\ &= \frac{g_V^2}{\vec{q}^2 + m_V^2}, \end{aligned} \tag{A12}$$

where $\vec{q}$ is the exchanged momentum, m_V the mass of the vector meson and with the second line being a consequence of $v^2 = 1$, meaning that the potential is independent of the velocity parameter. The addition of the isospin degrees of freedom in the case of the ρ is trivial. Further details about the mechanics of the calculation of the potential for heavy hadrons can be consulted in Ref. [88].

APPENDIX B: LAGRANGIANS FOR THE MAGNETIC AND QUADRUPOLE MOMENTS

In this Appendix we discuss the magnetic and quadrupole couplings of a heavy hadron to the electromagnetic field. This is useful for the derivation of the magnetic- and quadrupolelike couplings to the vector mesons in the vector-meson dominance model. In particular we write

$$\mathcal{L}_\mu = \mu(h) h^\dagger \left[\frac{1}{|S_3|}\epsilon_{ijk} S_i \partial_j A_k\right] h, \tag{B1}$$

$$\mathcal{L}_Q = Q(h) h^\dagger \left[\frac{1}{|Q_{33}|} Q_{ij}\partial_i \partial_j A_0\right] h, \tag{B2}$$

for the magnetic-dipole and electric-quadrupole coupling of a heavy hadron field h to the photon field $A_\mu = (A_0, \vec{A})$. In the magnetic term, $\mu(h)$ is the magnetic-dipole moment of the heavy hadron h and $\vec{S}$ represents the spin operator of this heavy hadron, which we assume to be spin-S (with $S \geq \frac{1}{2}$ if we want the magnetic moment to be nonvanishing). In the quadrupole term, $Q(h)$ is the electric-quadrupole moment of the heavy hadron and Q_{ij} is a spin-2 tensor that can be constructed from the spin operator $\vec{S}$:

$$Q_{ij} = \frac{1}{2}[S_i S_j + S_j S_i] - \frac{1}{3} S(S+1)\delta_{ij}, \tag{B3}$$

which requires $S \geq 1$ to be nonvanishing. Finally $|S_3| = S$ and $|Q_{33}| = \frac{1}{3} S(2S-1)$ refer to the evaluation of the S_3 and Q_{33} operators for a state with maximum third component of the spin. These definitions ensure that

$$\langle SS|\hat{\mu}_3|SS\rangle = \mu(h), \tag{B4}$$

$$\langle SS|\hat{Q}_{33}|SS\rangle = Q(h), \tag{B5}$$

where $|SS\rangle$ represents a spin state of the heavy hadron h where the third component is $S_3 = +S$, while $\hat{\mu}_3$ and $\hat{Q}_{33}$ are the $i = 3$ and $ij = 33$ components of the magnetic and tensor operators, which can be identified with

$$\hat{\mu}_i = \mu(h) \frac{1}{|S_3|} S_i, \tag{B6}$$

$$\hat{Q}_{ij} = Q(h) \frac{1}{|Q_{33}|} Q_{ij}. \tag{B7}$$

Conversely, the moments of order n can be defined analogously as

$$M^{(n)}_{i_1 \cdots i_n} = M^{(n)}(h) \frac{1}{|T^{(n)}_{3\cdots 3}|} T^{(n)}_{i_1 \cdots i_n}, \tag{B8}$$

with $M^{(n)}(h)$ the n-polar moment of hadron h, $T^{(n)}$ a spin nth order tensor constructed from the hadron spin operator $\vec{S}$ and $|T^{(n)}_{3\ldots 3}|$ the evaluation of its $i_2 = i_2 = \cdots = i_3 = 3$ component for the $|SS\rangle$ spin state.

For the heavy-baryon sextet, assuming that the multipole moments are dominated by the light-quarks, only the magnetic-dipole and electric-quadrupole moments will be relevant; as discussed, the quadrupole moment is expected to be small (it requires either HQSS breaking or a sizable D-wave component for the light quark pair).

APPENDIX C: LAGRANGIAN FOR THE η MESON

In this Appendix we discuss the inclusion of the η meson in the OBE model. Even though it is well known within the standard OBE model as applied to nucleon-nucleon interactions that the contribution of the η meson is not particularly important, it is nonetheless interesting to include it explicitly not only to test this hypothesis but also to assess the systematic uncertainties of the OBE model.

To include the η we first notice that the pion, kaon and η mesons are pseudo Nambu-Goldstone bosons associated with the breakdown of chiral symmetry, which can be grouped together into the field

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \tag{C1}$$

Analogously the charmed meson and baryon fields can also be arranged into SU(3)-flavor matrices

$$H = \begin{pmatrix} \bar{D}^0 \\ D^- \\ \bar{D}_s \end{pmatrix}, \tag{C2}$$

$$S = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c^{+\prime} \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c^{0\prime} \\ \frac{1}{\sqrt{2}}\Xi_c^{+\prime} & \frac{1}{\sqrt{2}}\Xi_c^{0\prime} & \Omega_c^0 \end{pmatrix}. \tag{C3}$$

The flavor structure of the interaction between the pseudo Nambu-Goldstone and charmed meson and baryon fields will take the form

$$H_a M_{ab} H_b \quad \text{and} \quad S_{ab} M_{bc} S_{ca}, \tag{C4}$$

from which we can deduce that the Lagrangians will take the form

$$\mathcal{L}_{q_L q_L \eta} = \frac{1}{\sqrt{3}} \frac{g_1}{\sqrt{2} f_\eta} q_L^\dagger \vec{\sigma}_L \cdot \vec{\nabla}\eta q_L, \tag{C5}$$

$$\mathcal{L}_{d_L d_L \eta} = \frac{1}{\sqrt{3}} \frac{g_2}{\sqrt{2} f_\eta} d_L^\dagger \vec{S}_L \cdot \vec{\nabla}\eta d_L, \tag{C6}$$

where g_1 and g_2 are the axial coupling to the charmed mesons and baryons, respectively, which are expected to be identical to those for the pion, while for the weak decay constant we use $f_\eta \simeq 150$ MeV instead of f_π. It is interesting to notice that the previous Lagrangians can also be obtained from the substitution rules

$$\tau^a \pi^a \quad \text{and} \quad T^a \pi^a \to \frac{\eta}{\sqrt{3}}, \tag{C7}$$

which can be traced back to the form of the pseudo Nambu-Goldstone meson octet matrix, Eq. (C1). From the previous Lagrangians we deduce the momentum space potential

$$V_\eta(\vec{q}) = -\frac{g_1 g_2}{2 f_\eta^2} \frac{1}{3} \frac{\vec{\sigma}_{L1} \cdot \vec{q} \vec{S}_{L2} \cdot \vec{q}}{\vec{q}^2 + m_\eta^2} \tag{C8}$$

or, if we Fourier-transform into coordinate space

$$\begin{aligned} V_\eta(\vec{r}) = +\frac{1}{3} \frac{g_1 g_2}{6 f_\eta^2} [&-\vec{\sigma}_{L1} \cdot \vec{S}_{L2} \delta(\vec{r}) \\ &+ \vec{\sigma}_{L1} \cdot \vec{S}_{L2} m_\eta^3 W_Y(\mu_\eta r) \\ &+ S_{L12}(\vec{r}) m_\eta^3 W_T(m_\eta r)], \end{aligned} \tag{C9}$$

which is to be regularized with a suitable form factor and cutoff. To include it into the full OBE potential, we notice

$$V_{\text{OBE}} = \zeta V_\pi + V_\eta + V_\sigma + V_\rho + \zeta V_\omega, \tag{C10}$$

with $\zeta = \pm 1$ the sign for distinguishing the $q_L d_L$ and $\bar{q}_L d_L$ cases, and where we notice that the G parity of the η meson is $G = +1$, and thus this piece of the potential does not depend on whether we have particles or antiparticles.

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