Dispersive and absorptive CP violation in $D^0-\bar{D}^0$ mixing

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CP violation (CPV) in $D^0-\bar{D}^0$ mixing is described in terms of the dispersive and absorptive "weak phases" ϕ_f^M and ϕ_f^Γ. They parametrize CPV originating from the interference of D^0 decays with and without dispersive mixing, and with and without absorptive mixing, respectively, for CP conjugate hadronic final states f, $\bar{f}$. These are distinct and separately measurable effects. For CP eigenstate final states, indirect CPV only depends on ϕ_f^M (dispersive CPV), whereas ϕ_f^Γ (absorptive CPV) can only be probed with non-CP eigenstate final states. Measurements of the final state dependent phases ϕ_f^M, ϕ_f^Γ determine the intrinsic dispersive and absorptive mixing phases ϕ_2^M and ϕ_2^Γ. The latter are the arguments of the dispersive and absorptive mixing amplitudes M_{12} and Γ_{12}, relative to their dominant ($\Delta U = 2$) U-spin components. The intrinsic phases are experimentally accessible due to *approximate universality*: in the SM, and in extensions with negligible new CPV phases in Cabibbo favored/doubly Cabibbo suppressed (CF/DCS) decays, the deviation of $\phi_f^{M,\Gamma}$ from $\phi_2^{M,\Gamma}$ is negligible in CF/DCS decays $D^0 \to K^\pm X$, and below 10% in CF/DCS decays $D^0 \to K_{S,L} X$ (up to precisely known $O(\epsilon_K)$ corrections). In singly Cabibbo suppressed (SCS) decays, QCD pollution enters at $O(\epsilon)$ in U-spin breaking and can be significant, but is $O(\epsilon^2)$ in the average over $f = K^+K^-$, $\pi^+\pi^-$. SM estimates yield $\phi_2^M, \phi_2^\Gamma = O(0.2\%)$. A fit to current data allows $O(10)$ larger phases at 2σ, from new physics. A fit based on naively extrapolated experimental precision suggests that sensitivity to ϕ_2^M and ϕ_2^Γ in the SM may be achieved at the LHCb Phase II upgrade.

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I. INTRODUCTION

In the Standard Model (SM), CP violation (CPV) enters $D^0-\bar{D}^0$ mixing and D decays at $O(V_{cb}V_{ub}/V_{cs}V_{us}) \sim 10^{-3}$, due to the weak phase γ. Consequently, all three types of CPV [1] are realized: (i) direct CPV, (ii) CPV in pure mixing (CPVMIX), which is due to interference of the dispersive and absorptive mixing amplitudes, and (iii) CPV due to the interference of decay amplitudes with and without mixing (CPVINT). In this work, we are particularly interested in the latter two, which result from $D^0-\bar{D}^0$ mixing, and which we collectively refer to as "indirect CPV". We would like to answer the following questions: How large are the indirect CPV asymmetries in the SM? What is the minimal parametrization appropriate for the LHCb/Belle-II precision era? How large is the current window for new physics (NP)? Can this window be closed by LHCb and Belle-II?

In order to address these questions we first develop the description of indirect CPV in terms of the CP violating (CP-odd) and final state dependent dispersive and absorptive "weak phases." These phases, which we denote as ϕ_f^M and ϕ_f^Γ, respectively, for CP conjugate final states f and $\bar{f}$, parametrize CPVINT contributions originating from the interference of D^0 decays with and without dispersive (absorptive) mixing, respectively. These are distinct measurable effects, as we will see below. Their difference equals the CPVMIX weak phase.

An immediate consequence of our approach is that it yields simplified expressions for the indirect CP asymmetries, which have a transparent physical interpretation (unlike the more familiar description in terms of the mixing parameter $|q/p|$, and the weak phase ϕ_{λ_f}). In particular, the requirement that the underlying interfering amplitudes possess nontrivial CP-even "strong-phase" differences is manifest, and accounts for the differences between the ϕ_f^M and ϕ_f^Γ dependence of the CP asymmetries. For example, we will see that the time-dependent CPVINT asymmetries in decays to CP eigenstate final states are purely dispersive,

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i.e., they only depend on ϕ_f^M (apart from subleading direct CPV effects).

In the SM, the dispersive and absorptive $D^0 - \bar{D}^0$ mixing amplitudes are due to the long distance exchanges of all off-shell and on-shell intermediate states, respectively (short distance dispersive mixing is negligible). The CPVINT asymmetries are due to the CP-odd contributions of the subleading $\Delta C = 1$ transitions to the mixing amplitudes (via intermediate states) and the decay amplitudes (via final states). The combined effects of these two CPV contributions can be expressed in terms of the underlying final state dependent phases $\phi_f^{M,\Gamma}$, as noted above. Unfortunately, due to their nonperturbative nature, these phases cannot currently be calculated from first principles QCD. However, we will be able to make meaningful statements using $SU(3)_F$ flavor symmetry arguments.

In order to estimate the magnitudes and final state dependence of $\phi_f^{M,\Gamma}$ in the different classes of decays, we compare them to a theoretical pair of dispersive and absorptive phases. The latter are intrinsic to the mixing amplitudes, and follow from their U-spin decomposition. In general, they are defined as the arguments of the total dispersive and absorptive amplitudes, respectively, relative to a basis choice for the real axis in the complex mixing plane, given by the common direction of the dominant $\Delta U = 2$ mixing amplitudes. Hence, we denote them as ϕ_2^M and ϕ_2^Γ, respectively. Note that these phases are quark (or meson) phase convention independent and physical, like the phases $\phi_f^{M,\Gamma}$ directly measured in the decays. U-spin based estimates yield $\phi_2^M, \phi_2^\Gamma = O(0.2\%)$ in the SM. In principle, they could be measured on the lattice in the future. Their difference yields the CPVMIX phase, like the final state dependent phases.

In the SM, and for the Cabibbo favored and doubly Cabibbo suppressed decays (CF/DCS), the differences between ϕ_f^M and ϕ_2^M, or ϕ_f^Γ and ϕ_2^Γ are essentially known. This allows for precise experimental determinations of the theoretical phases, and their comparison with U-spin based estimates and future lattice measurements. A single pair of intrinsic dispersive and absorptive mixing phases suffices to parametrize all indirect CPV effects in CF/DCS decays, whereas for SCS decays this may cease to be the case as SM sensitivity is approached. We refer to this fortunate state of affairs as *approximate universality*. In particular, the approximate universality phases are identified with the intrinsic mixing phases, ϕ_2^M and ϕ_2^Γ. Once nonuniversality is hinted at in the SCS phases, the SCS observables could be dropped from the global fits. Instead, one could compare the CF/DCS based fit results for $\phi_2^{M,\Gamma}$ with measurements of $\phi_f^{M,\Gamma}$ and direct CPV in the SCS decays, to learn about the anatomy of the (subleading) SCS QCD penguin amplitudes. For example, in the SM one could separately determine their relative magnitudes, and strong phases.

Approximate universality generalizes beyond the SM under the following conservative assumptions regarding subleading decay amplitudes containing new weak phases: (i) they can be neglected in Cabibbo favored and doubly Cabibbo suppressed (CF/DCS) decays, given that an exotic NP flavor structure would otherwise be required in order to evade the ϵ_K constraint [2]; (ii) in singly Cabibbo suppressed (SCS) decays, their magnitudes are similar to, or smaller than the SM QCD penguin amplitudes, as already hinted at by current bounds on direct CPV in $D^0 \to K^+K^-, \pi^+\pi^-$ decays. These assumptions can ultimately be tested by future direct CPV measurements at LHCb and Belle-II.

The most stringent experimental bounds on indirect CPV phases have been obtained in the superweak limit [3–5], in which the SM weak phase γ and potential NP weak phases in the decay amplitudes are set to zero in the indirect CPV observables. In this limit, the dispersive and absorptive mixing phases satisfy $\phi_f^M = \phi_2^M$ and $\phi_f^\Gamma = \phi_2^\Gamma = 0$. Thus, indirect CPV is entirely due to short-distance NP. The superweak fits are highly constrained, given that only one CPV phase controls all indirect CPV. Comparison of superweak fit results with our estimate, $\phi_2^M, \phi_2^\Gamma = O(0.2\%)$ suggests that there is currently an O(10) window for NP in indirect CPV.

Moving forward, the increased precision at LHCb and Belle-II will require fits to the indirect CPV data to be carried out for both ϕ_2^M and ϕ_2^Γ, in the approximate universality framework. The addition of ϕ_2^Γ yields a less constrained fit. However, this should ultimately be overcome by a large increase in statistics.

Throughout this work we develop, in parallel, the description of indirect CPV for the three relevant classes of decays: (i) SCS (both CP eigenstate and non-CP eigenstate final states), (ii) CF/DCS decays to $K^\pm X$, and (iii) CF/DCS decays to K^0X, $\bar{K}^0X$. The last one requires special care due to the intervention of CPV in $K^0 - \bar{K}^0$ mixing. In Sec. II, the formalism for mixing and indirect CPV is presented, based on the final state dependent dispersive and absorptive CPVINT observables. A translation between the dispersive and absorptive CPV phases, ϕ_f^M, ϕ_f^Γ, and more widely used CPV parameters is also provided. In Sec. III, we apply this formalism to the derivation of general expressions for the time dependent decay widths and indirect CP asymmetries in terms of ϕ_f^M, ϕ_f^Γ. In CF/DCS decays to K^0X, $\bar{K}^0X$, the widths depend on two elapsed time intervals: the time at which the D decays, and the time at which the K decays, following their respective production. Approximate universality is discussed in Sec. IV. We begin with the U-spin decomposition of the mixing amplitudes in the SM, introduce the intrinsic mixing phases ϕ_2^M, ϕ_2^Γ, estimate their magnitudes, and derive their deviations from the final state dependent phases. In Sec. V we explain how to convert the expressions for the time dependent decay widths and indirect CP asymmetries, collected in Sec. III, to the approximate universality framework. In the case of CF/DCS decays to K^0X, $\bar{K}^0X$, the effects of ϵ_K on the K decay timescales of relevance for LHCb and Belle-II are compared. Superweak and approximate universality fits to the current data are presented in Sec. VI, together with future projections. We conclude with a summary of our results in Sec. VII. The Appendix contains expressions for a selection of time-integrated CP asymmetries, demonstrating that they can also be used to separately measure ϕ_2^M and ϕ_2^Γ.

II. FORMALISM

A. Mixing and time evolution

The time evolution of an arbitrary linear combination of the neutral D^0 and $\bar{D}^0$ mesons,

$$a|D^0\rangle + b|\bar{D}^0\rangle \tag{1}$$

follows from the time-dependent Schrödinger equation (see, e.g., [1]),

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H\begin{pmatrix} a \\ b \end{pmatrix} \equiv \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} a \\ b \end{pmatrix}. \tag{2}$$

The 2×2 matrices M and Γ are Hermitian. The former is referred to as the mass matrix, and the latter yields exponential decays of the neutral mesons. CPT invariance implies $H_{11} = H_{22}$. The transition amplitudes for $D^0 - \bar{D}^0$ mixing are given by the off-diagonal entries

$$\begin{aligned} \langle D^0|H|\bar{D}^0\rangle &= M_{12} - \frac{i}{2}\Gamma_{12}, \\ \langle \bar{D}^0|H|D^0\rangle &= M_{12}^* - \frac{i}{2}\Gamma_{12}^*. \end{aligned} \tag{3}$$

M_{12} is the dispersive mixing amplitude. In the SM it is dominated by the long-distance contributions of off-shell intermediate states. A significant short distance effect would be due to NP. Γ_{12} is the absorptive mixing amplitude, and is due to the long distance contributions of on-shell intermediate states, i.e., decays.

The D meson mass eigenstates are

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle, \tag{4}$$

where

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \tag{5}$$

The differences between the masses and widths of the mass eigenstates, $\Delta M_D = m_2 - m_1$ and $\Delta\Gamma_D = \Gamma_2 - \Gamma_1$, are expressed in terms of the observables

$$x = \frac{\Delta M_D}{\Gamma_D}, \qquad y = \frac{\Delta\Gamma_D}{2\Gamma_D}, \tag{6}$$

where the averaged D^0 lifetime and mass are denoted by Γ_D and M_D. We can define three "theoretical" physical mixing parameters: two CP conserving ones,

$$x_{12} \equiv 2|M_{12}|/\Gamma_D, \qquad y_{12} \equiv |\Gamma_{12}|/\Gamma_D, \tag{7}$$

and a CP violating pure mixing (CPVMIX) phase

$$\phi_{12} \equiv \arg\left(\frac{M_{12}}{\Gamma_{12}}\right) = \phi^M - \phi^\Gamma. \tag{8}$$

The CP-odd phases

$$\phi^M = \arg(M_{12}), \qquad \phi^\Gamma = \arg(\Gamma_{12}), \tag{9}$$

are separately meson and quark phase convention dependent and unphysical. The CP conserving parameters in (6) and (7) are related as

$$(x - iy)^2 = x_{12}^2 - y_{12}^2 - 2ix_{12}y_{12}\cos\phi_{12}, \tag{10}$$

yielding

$$|x| = x_{12}, \qquad |y| = y_{12}, \tag{11}$$

up to negligible corrections quadratic in $\sin\phi_{12}$. Two other useful relations are

$$\left(\left|\frac{q}{p}\right|^2, \left|\frac{p}{q}\right|^2\right) \times (x^2 + y^2) = x_{12}^2 + y_{12}^2 \pm 2x_{12}y_{12}\sin\phi_{12}.$$

Measurements of the D^0 meson mass and lifetime differences and CPV asymmetries imply that $x_{12}, y_{12} \sim 0.5\%$, while $\sin\phi_{12} \lesssim 0.1$, cf. Sec. VI. One is free to identify D_2 or D_1 with either the short-lived meson, or the heavier meson, by redefining $q \to -q$. This is equivalent to choosing a sign-convention for y, which in turn fixes the sign of x, or vice-versa, via the imaginary part of (10). In the HFLAV [6] convention, D_2 is identified with the would be CP-even state in the limit of no CPV. Given that the short-lived meson is approximately CP-even, this is equivalent to the choice $y > 0$.

The time-evolved mesons $D^0(t)$ and $\bar{D}^0(t)$ denote the mesons which start out as a D^0 and $\bar{D}^0$ at $t = 0$, respectively. Solving (2) for their time-dependent components yields,

$$\langle \bar{D}^0|D^0(t)\rangle = -e^{-i(M_D - i\frac{\Gamma_D}{2})t}\left(e^{i\pi/2}M_{12}^* + \frac{1}{2}\Gamma_{12}^*\right)t \times \frac{\sin[\frac{1}{2}(\Delta M_D - i\frac{1}{2}\Delta\Gamma_D)t]}{\frac{1}{2}(\Delta M_D - i\frac{1}{2}\Delta\Gamma_D)t},$$
$$\langle D^0|D^0(t)\rangle = \langle \bar{D}^0|\bar{D}^0(t)\rangle = e^{-i(M_D - i\frac{\Gamma_D}{2})t}\cos\left[\frac{1}{2}\left(\Delta M_D - i\frac{1}{2}\Delta\Gamma_D\right)t\right], \tag{12}$$

with $\langle D^0|\bar{D}^0(t)\rangle$ obtained from $\langle \bar{D}^0|D^0(t)\rangle$ by substituting $M_{12}^* \to M_{12}$ and $\Gamma_{12}^* \to \Gamma_{12}$. The phase $\pi/2$ in the first relation of (12) originates from the time derivative in (2), and is a dispersive CP-even "strong phase." We will keep track of its role in the derivation of the indirect CP asymmetries in Sec. III. For the time intervals relevant to experiment, i.e., $t \lesssim 1/\Gamma_D$, (12) reduces to

$$\langle \bar{D}^0|D^0(t)\rangle = e^{-i(M_D - i\frac{\Gamma_D}{2})t}\left(e^{-i\pi/2}M_{12}^* - \frac{1}{2}\Gamma_{12}^*\right)t$$
$$\langle D^0|\bar{D}^0(t)\rangle = e^{-i(M_D - i\frac{\Gamma_D}{2})t}\left(e^{-i\pi/2}M_{12} - \frac{1}{2}\Gamma_{12}\right)t$$
$$\langle D^0|D^0(t)\rangle = \langle \bar{D}^0|\bar{D}^0(t)\rangle = e^{-i(M_D - i\frac{\Gamma_D}{2})t} \times \left(1 - \frac{1}{8}[x_{12}^2 - y_{12}^2 - 2ix_{12}y_{12}\cos\phi_{12}]\Gamma_D^2 t^2\right), \tag{13}$$

up to negligible corrections entering at $O(t^3)$ and beyond, where use has been made of (10) in the last relation.

B. The decay amplitudes

The amplitudes for D^0 and $\bar{D}^0$ decays to CP conjugate final states f and $\bar{f}$ are denoted as

$$A_f = \langle f|\mathcal{H}|D^0\rangle, \qquad \bar{A}_f = \langle f|\mathcal{H}|\bar{D}^0\rangle,$$
$$A_{\bar{f}} = \langle \bar{f}|\mathcal{H}|D^0\rangle, \qquad \bar{A}_{\bar{f}} = \langle \bar{f}|\mathcal{H}|\bar{D}^0\rangle, \tag{14}$$

where $\mathcal{H}$ is the $|\Delta C| = 1$ weak interaction effective Hamiltonian. The tree-level dominated decay amplitudes can, in general, be written as

$$A_f = A_f^0 e^{+i\phi_f^0}[1 + r_f e^{i(\delta_f + \phi_f)}],$$
$$A_{\bar{f}} = A_{\bar{f}}^0 e^{i(\Delta_f^0 + \phi_{\bar{f}}^0)}[1 + r_{\bar{f}} e^{i(\delta_{\bar{f}} + \phi_{\bar{f}})}],$$
$$\bar{A}_{\bar{f}} = A_f^0 e^{-i\phi_f^0}[1 + r_f e^{i(\delta_f - \phi_f)}],$$
$$\bar{A}_f = A_{\bar{f}}^0 e^{i(\Delta_f^0 - \phi_{\bar{f}}^0)}[1 + r_{\bar{f}} e^{i(\delta_{\bar{f}} - \phi_{\bar{f}})}], \tag{15}$$

where A_f^0 and $A_{\bar{f}}^0$ are the magnitudes of the dominant SM contributions, the ratios r_f and $r_{\bar{f}}$ are the relative magnitudes of the subleading amplitudes (which are CKM suppressed in the SM, and potentially contain NP contributions), ϕ_f^0, $\phi_{\bar{f}}^0$, ϕ_f, and $\phi_{\bar{f}}$ are CP-odd weak phases and Δ_f^0, δ_f, and $\delta_{\bar{f}}$ are CP-even strong phases. With the exception of the weak phases, the quantities entering (15) are understood to be phase space dependent for three-body and higher multiplicity decays. Note that ϕ_f^0 and $\phi_{\bar{f}}^0$ are quark and meson phase convention dependent. However, this dependence cancels in physical observables.

In the case of decays to CP eigenstates, $\Delta_f^0 = 0(\pi)$ for CP-even (odd) final states. Equation (15) therefore reduces to

$$A_f = A_f^0 e^{+i\phi_f^0}[1 + r_f e^{i(\delta_f + \phi_f)}],$$
$$\bar{A}_f = \eta_f^{CP} A_f^0 e^{-i\phi_f^0}[1 + r_f e^{i(\delta_f - \phi_f)}], \tag{16}$$

where $\eta_f^{CP} = +(-)$ for CP-even (odd) final states.

For SCS decays, the choice of the dominant and subleading SM amplitudes in (15) and (16) is convention dependent. For example, using CKM unitarity, the leading SCS D^0 decay amplitudes could be chosen to be proportional to $V_{cs}^* V_{us}$, $V_{cd}^* V_{ud}$, or their difference $V_{cs}^* V_{us} - V_{cd}^* V_{ud}$. The last choice is a particularly convenient one that is motivated by U-spin flavor symmetry, cf. Sec. IV A. In all cases, the subleading SM amplitudes are $\propto V_{cb}^* V_{ub}$, and are included in the second term on the right-hand side (rhs) of each relation in (15), (16). However, the physical observables must be convention independent.

We divide the CF/DCS decays into two categories: (i) decays to $K^\pm X$, where indirect CPV requires interference between a CF and a DCS decay chain, e.g., $D^0 \to K^-\pi^+$ and $D^0 \to \bar{D}^0 \to K^-\pi^+$, respectively; (ii) decays to $K^0 X$, $\bar{K}^0 X$, where indirect CPV is dominated by interference between two CF decay chains, e.g., $D^0 \to \bar{K}^0\pi^+\pi^-$ and $D^0 \to \bar{D}^0 \to K^0\pi^+\pi^-$, with subsequent decays $K^0/\bar{K}^0 \to \pi^+\pi^-$. In the SM, the CF and DCS D^0 decay amplitudes are proportional to $V_{cs}^* V_{ud}$ and $V_{cd}^* V_{us}$, respectively. Thus, only the first terms in (15) are present. We choose the CF and DCS amplitudes to be $A_f, \bar{A}_{\bar{f}}$ and $A_{\bar{f}}, \bar{A}_f$, respectively. For the computation of the indirect CP asymmetries in case (i), all four amplitudes in (15) must be included, whereas in case (ii) we will see that the contributions of the two DCS amplitudes can be neglected to good approximation.

C. The CPVINT observables

The time dependent hadronic decay amplitudes sum over contributions with and without mixing, e.g., for CP conjugate decay modes,

$$A(\bar{D}^0(t) \to f) = A_f \langle D^0|\bar{D}^0(t)\rangle + \bar{A}_f \langle \bar{D}^0|\bar{D}^0(t)\rangle,$$
$$A(D^0(t) \to \bar{f}) = \bar{A}_{\bar{f}} \langle \bar{D}^0|D^0(t)\rangle + A_{\bar{f}} \langle D^0|D^0(t)\rangle. \quad (17)$$

Factoring out the unmixed contributions, the time dependent *CP* asymmetries are seen to depend on the ratios $A_f\langle D^0|\bar{D}^0(t)\rangle/\bar{A}_f\langle \bar{D}^0|\bar{D}^0(t)\rangle$, and their *CP* conjugates. In turn, (13) implies that the *CP* asymmetries are determined by the quantities $M_{12}A_f/\bar{A}_f$ and $\Gamma_{12}A_f/\bar{A}_f$, as well as their *CP* conjugates. Keeping this in mind, we are now ready to define the CPV phases ϕ_f^M and ϕ_f^Γ, responsible for dispersive and absorptive CPVINT, respectively.[1]

1. SCS decays to CP eigenstates

For SCS decays to *CP* eigenstate final states, ϕ_f^M and ϕ_f^Γ are the arguments of the CPVINT observables

$$\lambda_f^M \equiv \frac{M_{12}}{|M_{12}|}\frac{A_f}{\bar{A}_f} = \eta_f^{CP}\left|\frac{A_f}{\bar{A}_f}\right| e^{i\phi_f^M},$$
$$\lambda_f^\Gamma \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_f}{\bar{A}_f} = \eta_f^{CP}\left|\frac{A_f}{\bar{A}_f}\right| e^{i\phi_f^\Gamma}. \quad (18)$$

They are given by

$$\phi_f^{M(\Gamma)} = \phi^{M(\Gamma)} + 2\phi_f^0 + 2r_f \cos\delta_f \sin\phi_f, \quad (19)$$

to first order in r_f, cf. (9), (16). We will see that $\phi_f^M, \phi_f^\Gamma \approx 0$ (rather than π), given the sign of the *CP* conserving observable y_{CP}^f, $f = \pi^+\pi^-$, K^+K^-, cf. (60), (62).

2. SCS decays to non-CP eigenstates

For SCS decays to non-*CP* eigenstate final states, e.g., $D^0 \to K^{*+}K^-$, two pairs of observables are introduced,

$$\lambda_f^M \equiv \frac{M_{12}}{|M_{12}|}\frac{A_f}{\bar{A}_f} = \left|\frac{A_f}{\bar{A}_f}\right| e^{i(\phi_f^M - \Delta_f)},$$
$$\lambda_f^\Gamma \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_f}{\bar{A}_f} = \left|\frac{A_f}{\bar{A}_f}\right| e^{i(\phi_f^\Gamma - \Delta_f)}, \quad (20)$$

and

$$\lambda_{\bar{f}}^M \equiv \frac{M_{12}}{|M_{12}|}\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}} = \left|\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}}\right| e^{i(\phi_f^M + \Delta_f)},$$
$$\lambda_{\bar{f}}^\Gamma \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}} = \left|\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}}\right| e^{i(\phi_f^\Gamma + \Delta_f)}. \quad (21)$$

[1] In [7] it was noted that a nonzero value for $\arg[M_{12}^2 A_f \bar{A}_{\bar{f}}^* A_{\bar{f}} \bar{A}_{\bar{f}}^*]$ or $\arg[\Gamma_{12}^2 A_f \bar{A}_{\bar{f}}^* A_{\bar{f}} \bar{A}_{\bar{f}}^*]$, equivalent to $2\phi_f^M$ and $2\phi_f^\Gamma$, respectively, cf. (18), (20), (21), implies *CP* violation. However, the phenomenology of these phases was not discussed.

The dispersive and absorptive CPV phases now satisfy, cf. (9), (15),

$$\phi_f^{M(\Gamma)} = \phi^{M(\Gamma)} + \phi_f^0 + \phi_{\bar{f}}^0 + r_f \cos\delta_f \sin\phi_f + r_{\bar{f}} \cos\delta_{\bar{f}} \sin\phi_{\bar{f}}, \quad (22)$$

while the overall strong phase difference in the decay amplitude ratios is given by

$$\Delta_f = \Delta_f^0 - r_f \sin\delta_f \cos\phi_f + r_{\bar{f}} \sin\delta_{\bar{f}} \cos\phi_{\bar{f}}, \quad (23)$$

to first order in r_f and $r_{\bar{f}}$.

3. CF/DCS decays to $K^\pm X$

For CF/DCS decays to $K^\pm X$, e.g., $D^0 \to K^\pm\pi^\mp$, the definitions in (20), (21) apply (recall that A_f is the CF amplitude), however we introduce overall minus signs in the equalities, i.e.,

$$\lambda_f^M = -\left|\frac{A_f}{\bar{A}_f}\right| e^{i(\phi_f^M - \Delta_f)}, \qquad \lambda_f^\Gamma = -\left|\frac{A_f}{\bar{A}_f}\right| e^{i(\phi_f^\Gamma - \Delta_f)}$$
$$\lambda_{\bar{f}}^M = -\left|\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}}\right| e^{i(\phi_f^M + \Delta_f)}, \qquad \lambda_{\bar{f}}^\Gamma = -\left|\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}}\right| e^{i(\phi_f^\Gamma + \Delta_f)}. \quad (24)$$

Thus, the dispersive and absorptive CPV phases satisfy

$$\phi_f^{M(\Gamma)} = \phi^{M(\Gamma)} + \phi_f^0 + \phi_{\bar{f}}^0 + \pi + r_f \cos\delta_f \sin\phi_f + r_{\bar{f}} \cos\delta_{\bar{f}} \sin\phi_{\bar{f}}, \quad (25)$$

and the expression for the strong phase in (23) is not modified. The sign convention in (24) yields ϕ_f^M, $\phi_f^\Gamma \approx 0$ (rather than π), as in SCS decays. In the SM and, more generally, in models with negligible new weak phases in CF/DCS decays, the second line in (25) is absent, and the dispersive and absorptive phases are separately equal for all decays in this class. Moreover, the absence of direct CPV yields the relation $|A_{\bar{f}}/\bar{A}_{\bar{f}}| = |A_f/\bar{A}_f|^{-1}$.

4. CF/DCS decays to $K^0X, \bar{K}^0X$

Next, we define the CPVINT observables for $D^0/\bar{D}^0$ decays to final states $f = [\pi^+\pi^-]X$, where the square brackets indicate that the pion pair originates from decays of a K_S or K_L, i.e., two step transitions of the form $D^0 \to [K_{S,L} \to \pi^+\pi^-] + X$. In order to achieve SM sensitivity to CPVINT, the contributions of CPV in the K system must be taken into account. The neutral K mass eigenkets are written as,

$$|K_S\rangle = p_K|K^0\rangle + q_K|\bar{K}^0\rangle,$$
$$|K_L\rangle = p_K|K^0\rangle - q_K|\bar{K}^0\rangle. \quad (26)$$

The corresponding eigenbras are given in the "reciprocal basis" [7,8],

$$\langle\tilde{K}_S| = \frac{1}{2}(p_K^{-1}\langle K^0| + q_K^{-1}\langle\bar{K}^0|),$$
$$\langle\tilde{K}_L| = \frac{1}{2}(p_K^{-1}\langle K^0| - q_K^{-1}\langle\bar{K}^0|), \tag{27}$$

where CPT invariance has been assumed. To excellent approximation (see, e.g., [1]),

$$\left|\frac{p_K}{q_K}\right| = 1 + 2\mathrm{Re}[\epsilon_K]. \tag{28}$$

The experimental values of the real and imaginary parts of the kaon CPV parameter ϵ_K are [9],

$$\epsilon_R \equiv \mathrm{Re}[\epsilon_K] = (1.62 \pm 0.01) \times 10^{-3},$$
$$\epsilon_I \equiv \mathrm{Im}[\epsilon_K] = (1.53 \pm 0.01) \times 10^{-3}. \tag{29}$$

We have obtained them from the quoted measurements of η_{00} and η_{+-}, ignoring correlations in their errors.

In general, due to the presence of the two intermediate states $K_S X$ and $K_L X$, there are four pairs of CPVINT observables,

$$\lambda^M_{K_a X} \equiv \frac{M_{12}}{|M_{12}|}\frac{A_{K_a X}}{\bar{A}_{K_a X}}, \qquad \lambda^\Gamma_{K_a X} \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_{K_a X}}{\bar{A}_{K_a X}},$$
$$\lambda^M_{\overline{K_a X}} \equiv \frac{M_{12}}{|M_{12}|}\frac{A_{\overline{K_a X}}}{\bar{A}_{\overline{K_a X}}}, \qquad \lambda^\Gamma_{\overline{K_a X}} \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_{\overline{K_a X}}}{\bar{A}_{\overline{K_a X}}}, \qquad a = S, L, \tag{30}$$

where the first and second lines correspond to the CP conjugate final states $f = [\pi^+\pi^-]X$ and $\bar{f} = \overline{[\pi^+\pi^-]X}$, respectively. Note that for the important case of $X = \pi^+\pi^-$, $\bar{f}$ corresponds to interchange of the Dalitz plot variables $(p_K + p_{\pi^+})^2 \leftrightarrow (p_K + p_{\pi^-})^2$ in f. We can express the CPVINT observables (30) in the form

$$\lambda^{M,\Gamma}_{K_{S/L}X} = \pm\left|\frac{A_{K_{S/L}X}}{\bar{A}_{K_{S/L}X}}\right| e^{i(\phi^{M,\Gamma}[K_{S/L}X] - \Delta[K_{S/L}X])},$$
$$\lambda^{M,\Gamma}_{\overline{K_{S/L}X}} = \pm\left|\frac{\bar{A}_{\overline{K_{S/L}X}}}{A_{\overline{K_{S/L}X}}}\right| e^{i(\phi^{M,\Gamma}[K_{S/L}X] + \Delta[K_{S/L}X])}, \tag{31}$$

where the overall plus and minus signs refer to the K_S and K_L, respectively. The four CPVINT phases and two strong phases in (31) are $\phi^{M,\Gamma}[K_{S/L}X]$ and $\Delta[K_{S/L}X]$, respectively.

The D decay amplitudes in (30) satisfy,

$$A_{K_{S/L}X} = \frac{1}{2}(\pm q_K^{-1}A_{\bar{K}^0X} + p_K^{-1}A_{K^0X}),$$
$$\bar{A}_{K_{S/L}X} = \frac{1}{2}(p_K^{-1}\bar{A}_{K^0X} \pm q_K^{-1}\bar{A}_{\bar{K}^0X}),$$
$$A_{\overline{K_{S/L}X}} = \frac{1}{2}(q_K^{-1}A_{\overline{K^0X}} \pm p_K^{-1}A_{\overline{\bar{K}^0X}}),$$
$$\bar{A}_{\overline{K_{S/L}X}} = \frac{1}{2}(\pm p_K^{-1}\bar{A}_{\overline{\bar{K}^0X}} + q_K^{-1}\bar{A}_{\overline{K^0X}}), \tag{32}$$

where we have used the reciprocal basis (27), and the first and second terms on the rhs in each relation are the dominant CF and subleading DCS contributions, respectively.

In the SM and, more generally, in models with negligible new CPV phases in CF/DCS decays, the DCS decay amplitudes introduce relative corrections of $O(\theta_C^2)$ to the weak phases, strong phases, and magnitudes of $\lambda^{M,\Gamma}_{K_{S/L}X}$, $\lambda^{M,\Gamma}_{\overline{K_{S/L}X}}$, making it a good approximation to neglect them. (We assess the impact of the DCS amplitudes on approximate universality in Sec. IV C 3.) In this limit, (30) reduces to

$$\lambda^M_f \equiv \lambda^M_{K_S X} = -\lambda^M_{K_L X} = \frac{M_{12}}{|M_{12}|}\frac{p_K}{q_K}\frac{A_{\bar{K}^0X}}{\bar{A}_{K^0X}},$$
$$\lambda^\Gamma_f \equiv \lambda^\Gamma_{K_S X} = -\lambda^\Gamma_{K_L X} = \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{p_K}{q_K}\frac{A_{\bar{K}^0X}}{\bar{A}_{K^0X}},$$
$$\lambda^M_{\bar{f}} \equiv \lambda^M_{\overline{K_S X}} = -\lambda^M_{\overline{K_L X}} = \frac{M_{12}}{|M_{12}|}\frac{p_K}{q_K}\frac{A_{\overline{K^0X}}}{\bar{A}_{\overline{\bar{K}^0X}}},$$
$$\lambda^\Gamma_{\bar{f}} \equiv \lambda^\Gamma_{\overline{K_S X}} = -\lambda^\Gamma_{\overline{K_L X}} = \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{p_K}{q_K}\frac{A_{\overline{K^0X}}}{\bar{A}_{\overline{\bar{K}^0X}}}. \tag{33}$$

Thus, in the limit of negligible new CPV phases in CF/DCS decays, it is a good approximation to consider a single pair of CPVINT observables for final state $f = [\pi^+\pi^-]X$, and a single pair for $\bar{f} = \overline{[\pi^+\pi^-]X}$, which we have denoted in (33) as λ^M_f, λ^Γ_f and $\lambda^M_{\bar{f}}$, $\lambda^\Gamma_{\bar{f}}$, respectively. They can be expressed in terms of dispersive and absorptive CPVINT phases as

$$\lambda^{M(\Gamma)}_f = \left|\frac{p_K A_{\bar{K}^0X}}{q_K\bar{A}_{K^0X}}\right| e^{i(\phi_f^{M(\Gamma)} - \Delta_f)},$$
$$\lambda^{M(\Gamma)}_{\bar{f}} = \left|\frac{p_K \bar{A}_{K^0X}}{q_K A_{\bar{K}^0X}}\right| e^{i(\phi_f^{M(\Gamma)} + \Delta_f)}, \tag{34}$$

where the amplitude relations,

$$|\bar{A}_{\overline{\bar{K}^0X}}/A_{\bar{K}^0X}| = |A_{\overline{K^0X}}/\bar{A}_{K^0X}| = 1, \tag{35}$$

valid in the limit of vanishing direct CPV, have been employed in the second relation. Note that the weak phases

$\phi^{M,\Gamma}[K_{S/L}X]$ and strong phases $\Delta[K_{S/L}X]$, defined in general in (31), reduce to $\phi_f^{M,\Gamma}$ and Δ_f, respectively.

The strong phase difference Δ_f (between $\bar{A}_{K^0X}$ and $A_{\bar{K}^0X}$) is generally nonvanishing and phase space dependent for multi-body intermediate states, e.g., $X = \pi^+\pi^-$. The weak phases satisfy

$$\phi_f^{M(\Gamma)} = \phi^{M(\Gamma)} + 2\phi^0_{\bar{K}^0X} + \arg(p_K/q_K), \qquad (36)$$

where $\phi^0_{\bar{K}^0X}$ is the weak phase of the CF amplitudes $A_{\bar{K}^0X}$, $A_{\overline{K^0X}}$, cf. (15), while $\arg(p_K/q_K)$ introduces a dependence on CPV in the K system, cf. Sec. IV C 3. Note that ϕ_f^M and ϕ_f^Γ are separately equal for all final states in this class.

In the case of two-body (and quasi two-body) intermediate states, the CPVINT observables in (34) reduce to

$$\lambda_f^{M(\Gamma)} = \eta_f^{CP}\left|\frac{p_K}{q_K}\right| e^{i\phi_f^{M(\Gamma)}}, \qquad (37)$$

where

$$\eta_f^{CP} \equiv (-)^L \times CP[X], \qquad (38)$$

L is the orbital angular momentum of the intermediate states $K_{S/L}X$, and $CP[X] = +(-)$ for CP-even (odd) X. For example, $\eta_f^{CP} = -1$ for $f = K_S\omega$, $K_S\pi^0$, and $\eta_f^{CP} = +1$ for $f = K_S f_0$. (Equivalently, $\eta^f_{CP} = +1(-1)$ for CP-even (odd) intermediate state K_SX.)

Finally, we point out that in all three classes of D^0 decays discussed in this section, the quark (CKM) phase convention dependence cancels in ϕ_f^M and ϕ_f^Γ, i.e., between the first two terms on the rhs of (19), the first three terms on the rhs of (22), and between all three terms in (36), cf. Sec. IV C. Moreover, they are always related to the pure mixing phase ϕ_{12} as

$$\phi_{12} = \phi_f^M - \phi_f^\Gamma, \qquad (39)$$

i.e., the final state dependent effects are common to the dispersive and absorptive phases.

5. Relation to other parametrizations of CPVINT

It is instructive to relate the parametrization of indirect CPV effects in terms of absorptive and dispersive phases to the more familiar one currently in use. The latter consists of the CPVMIX parameter,

$$|q/p| - 1, \qquad (40)$$

and the final state dependent phenomenological CPVINT phases ϕ_{λ_f}, which appear in the arguments of the observables λ_f, see, e.g., [1]. We begin with the definitions of the λ_f, corresponding to the absorptive and dispersive observables $\lambda_f^{M,\Gamma}$, in the different classes of decays. For SCS decays to CP eigenstate final states, they correspond to the observables in (18), and are given by[2]

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f} = -\eta_f^{CP}|\lambda_f| e^{i\phi_{\lambda_f}}. \qquad (41)$$

For SCS decays to non-CP eigenstate final states, and CF/DCS decays to $K^\pm X$, the λ_f corresponding to the observables in (20), (21), and (24) are given by,

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f} = \mp|\lambda_f| e^{i(\phi_{\lambda_f} + \Delta_f)},$$
$$\lambda_{\bar{f}} \equiv \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} = \mp|\lambda_{\bar{f}}| e^{i(\phi_{\lambda_f} - \Delta_f)}, \qquad (42)$$

where the $\mp$ sign conventions in the right-most relations apply to the SCS and CF/DCS cases, respectively.

Finally, for CF/DCS decays to K^0X, $\bar{K}^0X$ (given negligible new CPV phases in the decay amplitudes, and neglecting the DCS contributions) the λ_f correspond to the absorptive and dispersive observables in (33), (34), and are given by

$$\lambda_f \equiv \frac{q}{p}\frac{q_K}{p_K}\frac{\bar{A}_{K^0X}}{A_{\bar{K}^0X}} = -|\lambda_f| e^{i(\phi_{\lambda_f} + \Delta_f)},$$
$$\lambda_{\bar{f}} \equiv \frac{q}{p}\frac{q_K}{p_K}\frac{\bar{A}_{\overline{\bar{K}^0X}}}{A_{\overline{K^0X}}} = -|\lambda_f| e^{i(\phi_{\lambda_f} - \Delta_f)}, \qquad (43)$$

for final states $f = [\pi^+\pi^-]X$ and $\bar{f} = \overline{[\pi^+\pi^-]X}$. In the case of two-body or quasi two-body intermediate states, corresponding to the observables in (37), these expressions reduce to,

$$\lambda_f^{M(\Gamma)} = \eta_f^{CP}\left|\frac{q}{p}\frac{q_K}{p_K}\right| e^{i\phi_{\lambda_f}}. \qquad (44)$$

The sign conventions in the right-most relations of (41)–(44) yield all $\phi_{\lambda_f} \approx 0$ (HFLAV convention for D_2), or all $\approx\pi$, for the three classes of decays.

The CPV parameters $|q/p| - 1$ and ϕ_{λ_f} are expressed in terms of the absorptive and dispersive CPV phases as

$$\left|\frac{q}{p}\right| - 1 = \frac{x_{12}y_{12}\sin\phi_{12}}{x_{12}^2 + y_{12}^2}[1 + O(\sin\phi_{12})], \qquad (45)$$

[2] In our convention for λ_f^M, λ_f^Γ, the numerators correspond to the transitions $\bar{D}^0 \to D^0 \to f$, whereas in λ_f they correspond to $D^0 \to \bar{D}^0 \to f$.

where $\phi_{12} = \phi_f^M - \phi_f^\Gamma$, cf. (39), and

$$\tan 2\phi_{\lambda_f} = -\left(\frac{x_{12}^2 \sin 2\phi_f^M + y_{12}^2 \sin 2\phi_f^\Gamma}{x_{12}^2 \cos 2\phi_f^M + y_{12}^2 \cos 2\phi_f^\Gamma}\right). \qquad (46)$$

Equation (46) is obtained by multiplying both sides of (5) by $(\bar{A}_f/A_f)^2$ and $(\bar{A}_f\bar{A}_{\bar{f}}/A_f A_{\bar{f}})$ for CP eigenstate and non-CP eigenstate final states, respectively, and holds for all classes of decays. To lowest order in the CPV phases, it equates the phenomenological CPVINT phase ϕ_{λ_f} to a sum over the dispersive and absorptive CPVINT phases, ϕ_f^M and ϕ_f^Γ, weighted by the ratios $x_{12}^2/(x_{12}^2 + y_{12}^2)$ and $y_{12}^2/(x_{12}^2 + y_{12}^2)$, respectively. These weights are, respectively, the leading dispersive and absorptive contributions to the CP averaged mixing probability, $|\langle \bar{D}^0|D^0(t)\rangle|^2 + |\langle D^0|\bar{D}^0(t)\rangle|^2$, cf. (13).

Indirect CPV can be equivalently described in terms of the parameters emphasized in this work, i.e., ϕ_f^M, ϕ_f^Γ, x_{12}, y_{12}, or the more familiar ones $|q/p|$, ϕ_{λ_f}, x, y, cf. (11), (39), (45), (46). Indeed, (39) implies that the same number of independent parameters is employed in each case.

Finally, we remark on the CPV observables Δx_f [10] and Δy_f, which have been measured in tandem by the LHCb collaboration [11] in $D^0 \to K_S\pi^+\pi^-$ decays. They are defined in terms of ϕ_{λ_f} and $|q/p|$ as[3]

$$2\Delta x_f = x \cos\phi_{\lambda_f}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) + y \sin\phi_{\lambda_f}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right),$$
$$2\Delta y_f = y \cos\phi_{\lambda_f}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) - x \sin\phi_{\lambda_f}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right).$$

The observable $-\Delta y_f$ is equivalent to the familiar CPVINT asymmetry ΔY_f for SCS decays to CP eigenstate final states, cf. (59). Translating to the dispersive/absorptive parametrization via (45), (46), we obtain[4]

$$\Delta x_f = -y_{12} \sin\phi_f^\Gamma, \qquad \Delta y_f = x_{12} \sin\phi_f^M, \qquad (47)$$

to leading order in $\sin\phi_f^{M,\Gamma}$. Thus, the use of the parameters Δx_f and Δy_f is equivalent to the CPVINT parametrization in terms of ϕ_f^M and ϕ_f^Γ, respectively, modulo the corresponding dispersive and absorptive mixing factors. (It is amusing that interchange of the Δx and Δy labels turns out to be appropriate). Interestingly, we will see that experimental sensitivity to ϕ_f^Γ (or Δx_f) requires a nontrivial strong phase difference between decay amplitudes, i.e., non-CP eigenstate final states, e.g., $f = K_S\pi^+\pi^-, K^+\pi^-$.

[3]To be fully general, we have replaced ϕ with ϕ_{λ_f}, and added a subscript f to Δx and Δy in the definitions of [10].

[4]We have used the relations $y\cos\phi_{\lambda_f} = y_{12}\cos\phi_f^\Gamma$, and $x\cos\phi_{\lambda_f} = x_{12}\cos\phi_f^M$, which hold up to negligible relative corrections quadratic in the CPV phases.

III. THE INDIRECT *CP* ASYMMETRIES

We can now derive expressions for the time-dependent decay widths and CP asymmetries in terms of the absorptive and dispersive CPV phases. (A discussion of CPV in certain time-integrated decays is deferred to the Appendix.)

A. Semileptonic decays

We begin with the CPVMIX "wrong sign" semileptonic CP asymmetry,

$$a_{\rm SL} \equiv \frac{\Gamma(D^0(t) \to \ell^- X) - \Gamma(\overline{D^0}(t) \to \ell^+ X)}{\Gamma(D^0(t) \to \ell^- X) + \Gamma(\overline{D^0}(t) \to \ell^+ X)},$$
$$= \frac{|\langle \bar{D}^0|D^0(t)\rangle|^2 - |\langle D^0|\overline{D^0}(t)\rangle|^2}{|\langle \bar{D}^0|D^0(t)\rangle|^2 + |\langle D^0|\overline{D^0}(t)\rangle|^2}. \qquad (48)$$

In the second line the semileptonic decay amplitude factors have been cancelled, given negligible direct CPV in these decays, i.e., $|\bar{A}_{\ell^- X}| = |A_{\ell^+ X}|$. In turn, the expressions for the mixed amplitudes in (12) or (13) yield the semileptonic asymmetry,

$$a_{\rm SL} = \frac{2x_{12}y_{12}}{x_{12}^2 + y_{12}^2}\sin\phi_{12}. \qquad (49)$$

Note that the CP-even phase difference between the interfering dispersive and absorptive mixing amplitudes, required to obtain CPVMIX, is provided by the dispersive mixing phase $\pi/2$ in the first line of (12).

B. Hadronic decays

The hadronic decay amplitudes sum over contributions with and without mixing, cf. (17) (substitute $f \leftrightarrow \bar{f}$ for the CP conjugate final states). The corresponding time-dependent decay rates are identified with their magnitudes squared. They are expressed in terms of the CPVINT observables $\lambda_{f,\bar{f}}^M$, $\lambda_{f,\bar{f}}^\Gamma$, cf. (18), (20), (21), as ($\tau \equiv \Gamma_D t$),

$$\Gamma(\bar{D}^0(t) \to f) = e^{-\tau}|\bar{A}_f|^2\Big\{1 - \tau{\rm Re}[i\lambda_f^M x_{12} + \lambda_f^\Gamma y_{12}] + \frac{\tau^2}{4}((|\lambda_f^M|^2 - 1)x_{12}^2 + (|\lambda_f^\Gamma|^2 + 1)y_{12}^2 + 2x_{12}y_{12}{\rm Im}[\lambda_f^{M*}\lambda_f^\Gamma])\Big\},$$
$$\Gamma(D^0(t) \to f) = e^{-\tau}|A_f|^2\Big\{1 - \tau{\rm Re}[ix_{12}/\lambda_f^M + y_{12}/\lambda_f^\Gamma] + \frac{\tau^2}{4}((1/|\lambda_f^M|^2 - 1)x_{12}^2 + (1/|\lambda_f^\Gamma|^2 + 1)y_{12}^2 + 2x_{12}y_{12}{\rm Im}[1/(\lambda_f^{M*}\lambda_f^\Gamma)])\Big\}, \qquad (50)$$

with the expressions for $\Gamma(\bar{D}^0(t) \to \bar{f})$ and $\Gamma(D^0(t) \to \bar{f})$ obtained via the substitutions $f \to \bar{f}$ in (50). Note that throughout this work appropriate normalization factors are implicit in all decay width formulas, including (50). The expressions in (50) are applied to the following cases: SCS decays to *CP* eigenstates, SCS decays to non-*CP* eigenstates, and CF/DCS decays to $K^\pm X$. The description of CF/DCS decays to $K^0 X$, $\bar{K}^0 X$ requires a separate treatment, cf. Sec. III C.

1. SCS decays to CP eigenstates

This category includes, for example, the decays $D^0 \to K^+K^-/\pi^+\pi^-$. (We comment on the decay $D^0 \to K^0\bar{K}^0$ at the end of Sec. IV C 1). The time-dependent decay widths $D^0(t) \to f$ and $\bar{D}^0(t) \to f$, expressed in terms of ϕ_f^M, ϕ_f^Γ, cf. (19), and the direct *CP* asymmetry,

$$a_f^d \equiv 1 - |\bar{A}_f/A_f| = -2r_f \sin\delta_f \sin\phi_f, \tag{51}$$

cf. (16), are given by

$$\begin{aligned}\Gamma(D^0(t) \to f) &= e^{-\tau}|A_f|^2(1 + c_f^+\tau + c_f'^+\tau^2),\\ \Gamma(\bar{D}^0(t) \to f) &= e^{-\tau}|\bar{A}_f|^2(1 + c_f^-\tau + c_f'^-\tau^2),\end{aligned} \tag{52}$$

where the coefficients $c_f^\pm$, $c_f'^\pm$ satisfy

$$\begin{aligned}c_f^\pm &= \eta_{CP}^f[\mp x_{12}\sin\phi_f^M - y_{12}\cos\phi_f^\Gamma(1 \mp a_f^d)],\\ c_f'^\pm &= \frac{1}{2}y_{12}^2 \pm \frac{1}{4}(x_{12}^2 + y_{12}^2)(a_{\rm SL} - 2a_f^d).\end{aligned} \tag{53}$$

Terms involving a_f^d have been expanded to first order in CPV quantities, and the semileptonic *CP* asymmetry, expressed in terms of ϕ_{12}, is given in (49).

The $O(\tau^2)$ terms in the SCS widths are usually neglected, due to an $O(x_{12}, y_{12})$ suppression relative to the $O(\tau)$ term. Thus, it has been traditional to express the SCS widths in the approximate exponential forms,

$$\begin{aligned}\Gamma(D^0(t) \to f) &= |A_f|^2 \exp[-\hat{\Gamma}_{D^0\to f}\tau],\\ \Gamma(\overline{D^0}(t) \to f) &= |\bar{A}_f|^2 \exp[-\hat{\Gamma}_{\overline{D^0}\to f}\tau],\end{aligned} \tag{54}$$

where the decay rate parameters satisfy

$$\hat{\Gamma}_{D^0/\bar{D}^0\to f} = 1 - c^\pm, \tag{55}$$

cf. (53). As the goal of SM sensitivity comes into view, i.e., $\phi_f^M, \phi_f^\Gamma = O(\text{few}) \times 10^{-2}$, this will not necessarily be a good approximation, as can be seen by comparing the *CP*-odd terms in $c_f^\pm$, and the *CP*-even term in $c_f'^\pm$. However, the *CP*-odd terms in $c_f'^\pm$ are further suppressed by CPV parameters, and can be neglected. Thus, to good approximation,

$$c_f'^\pm = \frac{1}{2}y_{12}^2. \tag{56}$$

Measurements of the time-dependent decay rates at linear order in τ yield the known *CP* conserving observables,

$$y_{CP}^f \equiv -\frac{(c_f^+ + c_f^-)}{2}, \tag{57}$$

and the CPVINT asymmetries,

$$\Delta Y_f \equiv \frac{(c_f^+ - c_f^-)}{2}. \tag{58}$$

The average of ΔY_f over $f = K^+K^-, \pi^+\pi^-$ is denoted by A_Γ. In the exponential approximation, the corresponding definitions are,

$$\begin{aligned}y_{CP}^f &\equiv \frac{\hat{\Gamma}_{D^0\to f_{CP}} + \hat{\Gamma}_{\overline{D^0}\to f_{CP}}}{2} - 1,\\ \Delta Y_f &\equiv \frac{\hat{\Gamma}_{\bar{D}^0\to f} - \hat{\Gamma}_{D^0\to f}}{2}.\end{aligned} \tag{59}$$

Applying (53), and neglecting contributions quadratic in CPV, we obtain

$$y_{CP}^f = \eta_f^{CP} y_{12}\cos\phi_f^\Gamma. \tag{60}$$

The experimental average over $f = K^+K^-, \pi^+\pi^-$ [6] yields $y_{CP}^f/\eta_f^{CP} > 0$, or

$$y_{CP}^f = \eta_f^{CP} y_{12} = \eta_f^{CP}|y|, \tag{61}$$

to excellent approximation. Furthermore, fits to the data [6,12] yield $xy > 0$ at 3σ, or $\phi_{12} \approx 0$ (rather than π), cf. (10). Thus, we learn that both

$$\phi_f^M \approx 0, \qquad \phi_f^\Gamma \approx 0. \tag{62}$$

At first order in CPV, (53) yields the relation (already noted in (47) for the CPVINT part),

$$\Delta Y_f = \eta_{CP}^f(-x_{12}\sin\phi_f^M + a_f^d y_{12}). \tag{63}$$

The direct CPV contribution in (63) is formally subleading, cf. Sec. IV C 1. In general, it can be disentangled experimentally from the dispersive CPV contribution with the help of time integrated CPV measurements, in which a_f^d enters without mixing suppression, cf. the Appendix.

It is noteworthy that ΔY_f depends on ϕ_f^M, but not on ϕ_f^Γ. This is because *CP* asymmetries require a nontrivial *CP*-even phase difference δ between the interfering amplitudes, i.e., they are proportional to $\sin\delta$. In general, for *CP* eigenstate final states there is a CP-even phase difference between decays with and without dispersive mixing, namely the $\pi/2$ dispersive phase in (12). However, there is none between decays with and without absorptive mixing (the strong phase between A_f and $\bar{A}_f$ is trivial). Therefore, in general, ϕ_f^Γ can only be measured in decays to non-CP eigenstate final states, where the requisite CP-even phase is provided by the strong phase difference Δ_f between A_f and $\bar{A}_f$, as we will see explicitly below. Finally, in the case of CP averaged decay rates, interference terms are in general proportional to $\cos\delta$, rather than $\sin\delta$. Therefore, in the CP averaged time dependent decay rates for CP eigenstate final states, the interference between decays with and without dispersive mixing will vanish at leading order in the mixing, i.e., $O(\tau)$, only leaving a dependence on y_{12}. This is borne out by the expression for y_{CP}^f in (60).

2. SCS decays to non-CP eigenstates

This category includes, for example, the decays $D^0 \to \rho\pi$, $K^{*+}K^-$. The time dependent decay widths are of the form

$$\Gamma(D^0(t)\to f) = e^{-\tau}|A_f|^2(1+\sqrt{R_f}c_f^+\tau + R_f c_f'^+\tau^2),$$
$$\Gamma(\bar{D}^0(t)\to f) = e^{-\tau}|\bar{A}_f|^2\left(1+\frac{1}{\sqrt{R_f}}c_f^-\tau + \frac{1}{R_f}c_f'^-\tau^2\right), \tag{64}$$

for final state f, and

$$\Gamma(D^0(t)\to \bar{f}) = e^{-\tau}|A_{\bar{f}}|^2(1+\sqrt{R_{\bar{f}}}c_{\bar{f}}^+\tau + R_{\bar{f}} c_{\bar{f}}'^+\tau^2),$$
$$\Gamma(\bar{D}^0(t)\to \bar{f}) = e^{-\tau}|\bar{A}_{\bar{f}}|^2\left(1+\frac{1}{\sqrt{R_{\bar{f}}}}c_{\bar{f}}^-\tau + \frac{1}{R_{\bar{f}}}c_{\bar{f}}'^-\tau^2\right), \tag{65}$$

for final state $\bar{f}$, where

$$R_f \equiv |\bar{A}_f/A_f|^2, \qquad R_{\bar{f}} \equiv |\bar{A}_{\bar{f}}/A_{\bar{f}}|^2. \tag{66}$$

In general, the ratios satisfy $R_f, R_{\bar{f}} = O(1)$ for SCS decays. The coefficients $c_f^\pm$ and $c_{\bar{f}}^\pm$ in (64), (65), expressed in terms of ϕ_f^M, ϕ_f^Γ, and Δ_f, cf. (20)–(23), are given by

$$c_f^\pm = \mp x_{12}\sin(\phi_f^M - \Delta_f) - y_{12}\cos(\phi_f^\Gamma - \Delta_f),$$
$$c_{\bar{f}}^\pm = \mp x_{12}\sin(\phi_f^M + \Delta_f) - y_{12}\cos(\phi_f^\Gamma + \Delta_f). \tag{67}$$

The coefficients in the $O(\tau^2)$ terms satisfy

$$c_f'^\pm = \frac{1}{4}[R_f^{\mp1}(y_{12}^2 - x_{12}^2) + (x_{12}^2 + y_{12}^2)(1\pm a_{\rm SL})],$$
$$c_{\bar{f}}'^\pm = \frac{1}{4}[R_{\bar{f}}^{\mp1}(y_{12}^2 - x_{12}^2) + (x_{12}^2 + y_{12}^2)(1\pm a_{\rm SL})]. \tag{68}$$

As in the prior case of decays to CP eigenstates, the CP-even terms in $c_{f,\bar{f}}'^\pm$ should be kept, with future sensitivity at the level of SM indirect CPV in mind. However, the CP-odd terms ($\propto a_{\rm SL}$) can be neglected.

The time dependent measurements yield pairs of CPVINT asymmetries (normalized rate differences for $D^0(t)\to f$ vs $\bar{D}^0(t)\to\bar{f}$, and $D^0(t)\to\bar{f}$ vs $\bar{D}^0(t)\to f$) at linear order in τ,

$$\Delta Y_f \equiv \frac{\sqrt{R_f}c_f^+ - c_{\bar{f}}^-/\sqrt{R_{\bar{f}}}}{2},$$
$$\Delta Y_{\bar{f}} \equiv \frac{\sqrt{R_{\bar{f}}}c_{\bar{f}}^+ - c_f^-/\sqrt{R_f}}{2}. \tag{69}$$

To first order in CPV parameters, (67) yields the expressions,

$$\Delta Y_f = \sqrt{R_f}\Big[-x_{12}\sin\phi_f^M\cos\Delta_f - y_{12}\sin\phi_f^\Gamma\sin\Delta_f - \frac{1}{2}(a_f^d + a_{\bar{f}}^d)(x_{12}\sin\Delta_f - y_{12}\cos\Delta_f)\Big],$$
$$\Delta Y_{\bar{f}} = \frac{1}{\sqrt{R_f}}\Big[-x_{12}\sin\phi_f^M\cos\Delta_f + y_{12}\sin\phi_f^\Gamma\sin\Delta_f + \frac{1}{2}(a_f^d + a_{\bar{f}}^d)(x_{12}\sin\Delta_f + y_{12}\cos\Delta_f)\Big], \tag{70}$$

where the direct CP asymmetries,

$$a_f^d = 1 - |\bar{A}_{\bar{f}}/A_f| = -2r_f\sin\phi_f\sin\delta_f,$$
$$a_{\bar{f}}^d = 1 - |\bar{A}_f/A_{\bar{f}}| = -2r_{\bar{f}}\sin\phi_{\bar{f}}\sin\delta_{\bar{f}}, \tag{71}$$

cf. (15), enter via the deviation of $\sqrt{R_f R_{\bar{f}}}$ from unity. In (70), replacing the numerator and denominator in the ratio R_f, cf. (66), with their CP averaged counterparts would introduce a negligible higher order correction in the CPV parameters.

Note that the CP-even phase differences for dispersive and absorptive CPVINT are given by $\Delta_f - \pi/2$ and Δ_f, respectively, where $\pi/2$ is the "dispersive" phase in the first line of (12), thus accounting for the factors $\cos\Delta_f$ and $\sin\Delta_f$ in the first two terms of ΔY_f and $\Delta Y_{\bar{f}}$ in (70). In particular, Eq. (70) confirms that sensitivity to the absorptive phase ϕ_f^Γ requires a strong phase difference between decay amplitudes, i.e., non-CP eigenstate final states, as argued at the end of Sec. III B 1.

3. CF/DCS decays to $K^{\pm}X$

This category consists of the CF/DCS decays $D^0 \to K^{\pm}X$, with a single K in the final state. As noted previously, we choose the DCS decay amplitudes in (15), (20), (21), and (24), to be $A_{\bar{f}}$ and $\bar{A}_f$, e.g., $\bar{f} = K^+\pi^-$. Thus, we denote the time dependent CF/DCS decays to "wrong-sign" (WS) final states as $D^0(t) \to \bar{f}$ and $\bar{D}^0(t) \to f$, while the "right-sign" (RS) decays are $D^0(t) \to f$ and $\bar{D}^0(t) \to \bar{f}$. The $O(\tau^2)$ terms in (50) and its CP conjugate cannot be neglected, given that the decay amplitude ratios entering $\lambda^{M,\Gamma}_{f,\bar{f}}$ are now of $O(1/\theta_C^2)$. The RS and WS decay widths following from (50) and (62) can be expressed as

$$\Gamma(D^0(t) \to f) = e^{-\tau}|A_f|^2(1 + \sqrt{R_f}c^+_{\mathrm{RS},f}\tau + R_f c'^+_{\mathrm{RS},f}\tau^2),$$
$$\Gamma(\bar{D}^0(t) \to \bar{f}) = e^{-\tau}|\bar{A}_{\bar{f}}|^2\left(1 + \frac{1}{\sqrt{R_{\bar{f}}}}c^-_{\mathrm{RS},f}\tau + \frac{1}{R_{\bar{f}}}c'^-_{\mathrm{RS},f}\tau^2\right) \tag{72}$$

and

$$\Gamma(D^0(t) \to \bar{f}) = e^{-\tau}|A_f|^2(R_f^+ + \sqrt{R_f^+}c^+_{\mathrm{WS},f}\tau + c'^+_{\mathrm{WS},f}\tau^2),$$
$$\Gamma(\bar{D}^0(t) \to f) = e^{-\tau}|\bar{A}_{\bar{f}}|^2(R_f^- + \sqrt{R_f^-}c^-_{\mathrm{WS},f}\tau + c'^-_{\mathrm{WS},f}\tau^2) \tag{73}$$

where $R_f^{\pm}$ are the DCS to CF ratios

$$R_f^+ = |A_{\bar{f}}/A_f|^2, \qquad R_f^- = |\bar{A}_f/\bar{A}_{\bar{f}}|^2, \tag{74}$$

the ratios R_f, $R_{\bar{f}}$ are defined in (66), and the coefficients $c^{\pm}_{\mathrm{RS(WS)},f}$, $c'^{\pm}_{\mathrm{RS(WS)},f}$, to first order in CPV parameters, are given by

$$\begin{aligned}
c^{\pm}_{\mathrm{RS},f} &= -x_{12}\sin\Delta_f + y_{12}\cos\Delta_f \\
&\quad \pm (x_{12}\sin\phi_f^M\cos\Delta_f + y_{12}\sin\phi_f^\Gamma\sin\Delta_f), \\
c^{\pm}_{\mathrm{WS},f} &= (1 \mp a_f^d)[x_{12}\sin\Delta_f + y_{12}\cos\Delta_f] \\
&\quad \pm x_{12}\sin\phi_f^M\cos\Delta_f \mp y_{12}\sin\phi_f^\Gamma\sin\Delta_f, \\
c'^{\pm}_{\mathrm{RS},f} &= \frac{1}{4}[(x_{12}^2 + y_{12}^2)(1 \pm a_{\mathrm{SL}}) + \xi^{\pm}(y_{12}^2 - x_{12}^2)], \\
c'^{\pm}_{\mathrm{WS},f} &= \frac{1}{4}(x_{12}^2 + y_{12}^2)[1 \pm a_{\mathrm{SL}} \mp 2a_f^d] + \frac{1}{4}R_f^{\pm}(y_{12}^2 - x_{12}^2),
\end{aligned} \tag{75}$$

with $\xi^+ = R_f^{-1}$, $\xi^- = R_{\bar{f}}$. The (CF) direct CP asymmetry, a_f^d, appearing in (75) is given by

$$a_f^d = 1 - |\bar{A}_{\bar{f}}/A_f| = -2r_f\sin\phi_f\sin\delta_f, \tag{76}$$

and vanishes in the SM. In the SM, the $O(\tau^2)$ coefficients are well approximated as

$$c'^{\pm}_{\mathrm{RS(WS)},f} = \frac{1}{4}(x_{12}^2 + y_{12}^2). \tag{77}$$

The prefactors in (73) are, to excellent approximation, equal to the RS time dependent decay widths,

$$\Gamma(D^0(t) \to f) \sim e^{-\tau}|A_f|^2,$$
$$\Gamma(\bar{D}^0(t) \to \bar{f}) \sim e^{-\tau}|\bar{A}_{\bar{f}}|^2, \tag{78}$$

where the subleading DCS contributions in (72) have been neglected.

A fit to the time-dependence in (73), (78) yields measurements of $R_f^{\pm}$, $c^{\pm}_{\mathrm{WS},f}$, $c'^{\pm}_{\mathrm{WS},f}$, and the indirect CP asymmetries,

$$\begin{aligned}
\delta c_{\mathrm{WS},f} &\equiv \frac{1}{2}(c^+_{\mathrm{WS},f} - c^-_{\mathrm{WS},f}) = x_{12}\sin\phi_f^M\cos\Delta_f \\
&\quad - y_{12}\sin\phi_f^\Gamma\sin\Delta_f - a_f^d(x_{12}\sin\Delta_f + y_{12}\cos\Delta_f), \\
\delta c'_{\mathrm{WS},f} &\equiv \frac{c'^+_{\mathrm{WS},f} - c'^-_{\mathrm{WS},f}}{c'^+_{\mathrm{WS},f} + c'^-_{\mathrm{WS},f}} = a_{SL} - 2a_f^d.
\end{aligned} \tag{79}$$

Note that the last terms in (79) for $\delta c_{\mathrm{WS},f}$ and $\delta c'_{\mathrm{WS},f}$ are absent in the SM and, more generally, in models with negligible CP violating NP in CF/DCS decays. As in (70), the $\cos\Delta_f$ and $\sin\Delta_f$ dependence in the first two terms of $\delta c_{\mathrm{WS},f}$ originates from the total CP-even phase differences $\Delta_f - \pi/2$ and Δ_f, between decays with and without dispersive mixing and decays with and without absorptive mixing, respectively. This again confirms that strong phase differences are required in order to measure the absorptive CPV phases, ϕ_f^Γ.

C. CF/DCS decays to $K^0X, \bar{K}^0X$

We derive expressions for the time-dependent D^0 and $\bar{D}^0$ decay rates for two step CF/DCS decays of the form

$$D^0(t) \to [K_{S,L}(t') \to \pi^+\pi^-] + X, \tag{80}$$

to final states $f = [\pi^+\pi^-]X$. These decays depend on two elapsed time intervals, t and t', at which the D and K decay following their respective production.

The $D^0(t)$ and $\bar{D}^0(t)$ decay amplitudes now sum over contributions with and without $D^0 - \bar{D}^0$ mixing, and with and without $K^0 - \bar{K}^0$ mixing. The kaon time evolution is conveniently described in the mass basis,

$$|K_S(t)\rangle = e^{-iM_S t}e^{-\Gamma_S t/2}|K_S\rangle,$$
$$|K_L(t)\rangle = e^{-iM_L t}e^{-\Gamma_L t/2}|K_L\rangle, \tag{81}$$

where $M_{S,L}$, $\Gamma_{S,L}$, and $\tau_{S,L}$ are the corresponding masses, widths, and lifetimes. The time-dependent amplitudes for the decay of an initial D^0 to final state $f = [\pi^+\pi^-]X$, and for the CP conjugate decay of an initial $\bar{D}^0$ to final state $\bar{f} = \overline{[\pi^+\pi^-]X}$, are given by

$$\begin{aligned} A_f(t,t') = &\sum_{a=S,L} A(K_a \to \pi^+\pi^-) \\ &\times e^{-(iM_a+\frac{1}{2}\Gamma_a)t'}(A_{K_aX}\langle D^0|D^0(t)\rangle \\ &+ \bar{A}_{K_aX}\langle \bar{D}^0|D^0(t)\rangle), \\ \bar{A}_{\bar{f}}(t,t') = &\sum_{a=S,L} A(\bar{K}_a \to \pi^+\pi^-) \\ &\times e^{-(iM_a+\frac{1}{2}\Gamma_a)t'}(A_{\overline{K_aX}}\langle D^0|\bar{D}^0(t)\rangle \\ &+ \bar{A}_{\overline{K_aX}}\langle \bar{D}^0|\bar{D}^0(t)\rangle), \end{aligned} \tag{82}$$

where expressions for the D decay amplitudes A_{K_aX}, etc. appear in (32). The $K_{S,L} \to \pi\pi$ decay amplitudes satisfy,

$$\begin{aligned} A(K_S \to \pi^+\pi^-) &= p_K A_{+-} + q_K \bar{A}_{+-}, \\ A(K_L \to \pi^+\pi^-) &= p_K A_{+-} - q_K \bar{A}_{+-}, \end{aligned} \tag{83}$$

with

$$A_{+-} \equiv \langle \pi^+\pi^-|H|K^0\rangle, \bar{A}_{+-} \equiv \langle \pi^+\pi^-|H|\bar{K}^0\rangle. \tag{84}$$

The amplitudes $\bar{A}_f(t,t')$ and $A_{\bar{f}}(t,t')$ are obtained by substituting $|D^0(t)\rangle \to |\bar{D}^0(t)\rangle$ and vice versa in the first and second relations of (82), respectively. Expressing the amplitudes in terms of the CPVINT observables in (30) yields the general expressions, valid to linear order in τ:

$$\begin{aligned} A_f(t,t') &= e^{-(iM_D+\frac{1}{2}\Gamma_D)t}\sum_{a=S,L} A(K_a \to \pi^+\pi^-)A_{K_aX}e^{-(iM_a+\frac{1}{2}\Gamma_a)t'}\left(1-\frac{1}{2}\tau\left[i\frac{x_{12}}{\lambda^M_{K_aX}}+\frac{y_{12}}{\lambda^\Gamma_{K_aX}}\right]\right), \\ \bar{A}_{\bar{f}}(t,t') &= e^{-(iM_D+\frac{1}{2}\Gamma_D)t}\sum_{a=S,L} A(\bar{K}_a \to \pi^+\pi^-)\bar{A}_{\overline{K_aX}}e^{-(iM_a+\frac{1}{2}\Gamma_a)t'}\left(1-\frac{1}{2}\tau\left[ix_{12}\lambda^M_{\overline{K_aX}}+y_{12}\lambda^\Gamma_{\overline{K_aX}}\right]\right), \end{aligned} \tag{85}$$

where $\bar{A}_f(t,t')$ is obtained by substituting $A_{K_aX} \to \bar{A}_{K_aX}$ and $\lambda^{M(\Gamma)}_{K_aX} \to 1/\lambda^{M(\Gamma)}_{K_aX}$ in the first relation, and $A_{\bar{f}}(t,t')$ is obtained by substituting $\bar{A}_{\overline{K_aX}} \to A_{\overline{K_aX}}$ and $\lambda^{M(\Gamma)}_{\overline{K_aX}} \to 1/\lambda^{M(\Gamma)}_{\overline{K_aX}}$ in the second relation.

The time-dependent decay rates are obtained by squaring the magnitudes of the amplitudes in (85), e.g., $\Gamma_f(t,t') = |A_f(t,t')|^2$ etc., and assuming that CP violating NP is negligible in CF/DCS decays. Therefore, as in the SM, we assume vanishing direct CPV in the CF decays, neglect the DCS amplitudes (their impact is discussed in Sec. IV C 3), and employ the expressions for the CPVINT observables given in (34). We work to first order in CPV quantities, and also employ the relations (see, e.g., [1])

$$\begin{aligned} |A(K_S \to \pi^+\pi^-)|^2 &= 4|p_K A_{+-}|^2(1-2\epsilon_R) \\ &= 4|q_K \bar{A}_{+-}|^2(1+2\epsilon_R), \\ A(K_S \to \pi^+\pi^-)A(K_L \to \pi^+\pi^-)^* &= 4|p_K A_{+-}|^2\epsilon_K^* \\ &= 4|q_K \bar{A}_{+-}|^2\epsilon_K^*, \\ |A(K_L \to \pi^+\pi^-)|^2 &= O(\epsilon_K^2). \end{aligned} \tag{86}$$

In particular, the last relation in (86) implies that we can neglect the purely K_L contributions to the widths. The expressions for the time-dependent decay rates are then of the form,

$$\begin{aligned} \Gamma_f(t,t') = &e^{-\tau}|\bar{A}_{+-}|^2|A_{\bar{K}^0X}|^2\{e^{-\Gamma_S t'}[c^+ + \sqrt{R_f}c_f^+\tau + R_f c_f'^+\tau^2] + e^{-\Gamma_K t'}[(b^+ + \sqrt{R_f}b_f^+\tau)\cos(\Delta M_K t') \\ &+ (d^+ + \sqrt{R_f}d_f^+\tau)\sin(\Delta M_K t')]\}, \\ \bar{\Gamma}_f(t,t') = &e^{-\tau}|\bar{A}_{+-}|^2|\bar{A}_{K^0X}|^2\left\{e^{-\Gamma_S t'}\left[c^- + \frac{1}{\sqrt{R_f}}c_f^-\tau + \frac{1}{R_f}c_f'^-\tau^2\right] + e^{-\Gamma_K t'}\left[\left(b^- + \frac{1}{\sqrt{R_f}}b_f^-\tau\right)\cos(\Delta M_K t')\right.\right. \\ &\left.\left. + \left(d^- + \frac{1}{\sqrt{R_f}}d_f^-\tau\right)\sin(\Delta M_K t')\right]\right\}, \end{aligned} \tag{87}$$

for final state f, and

$$\begin{aligned} \Gamma_{\bar{f}}(t,t') = &e^{-\tau}|\bar{A}_{+-}|^2|\bar{A}_{K^0X}|^2\left\{e^{-\Gamma_S t'}\left[c^+ + \frac{1}{\sqrt{R_f}}c_{\bar{f}}^+\tau + \frac{1}{R_f}c_{\bar{f}}'^+\tau^2\right] + e^{-\Gamma_K t'}\left[\left(b^+ + \frac{1}{\sqrt{R_f}}b_{\bar{f}}^+\tau\right)\cos(\Delta M_K t')\right.\right. \\ &\left.\left. + \left(d^+ + \frac{1}{\sqrt{R_f}}d_{\bar{f}}^+\tau\right)\sin(\Delta M_K t')\right]\right\}, \end{aligned}$$

$$\bar{\Gamma}_{\bar{f}}(t,t') = e^{-\tau}|\bar{A}_{+-}|^2|A_{\bar{K}^0X}|^2\{e^{-\Gamma_S t'}[c^- + \sqrt{R_f}c^-_{\bar{f}}\tau + R_f c'^-_{\bar{f}}\tau^2] + e^{-\Gamma_K t'}[(b^- + \sqrt{R_f}b^-_{\bar{f}}\tau)\cos(\Delta M_K t') + (d^- + \sqrt{R_f}d^-_{\bar{f}}\tau)\sin(\Delta M_K t')]\}, \tag{88}$$

for final state $\bar{f}$, where

$$R_f \equiv |\bar{A}_{K^0X}/A_{\bar{K}^0X}|^2, \tag{89}$$

$\Delta M_K \equiv M_L - M_S$, and $\Gamma_K \equiv (\Gamma_L + \Gamma_S)/2$. We have taken $|A_{+-}| = |\bar{A}_{+-}|$, given that the two magnitudes differ by negligible corrections of $O(\epsilon_K^2, \epsilon'_K)$. The coefficients in (87), (88) depend on the quantities ϕ_f^M, ϕ_f^Γ, Δ_f, cf. (34)–(36), and ϵ_K. For the purely $K_S X$ contributions ($e^{-\Gamma_S t'}$ dependence), they are given by

$$\begin{aligned}
c^\pm &= 1 \pm 2\epsilon_R,\\
c_f^\pm &= (\pm x_{12} - y_{12}\sin\phi_f^\Gamma)\sin\Delta_f - (y_{12} \pm x_{12}\sin\phi_f^M)\cos\Delta_f,\\
c_{\bar{f}}^\pm &= (\mp x_{12} + y_{12}\sin\phi_f^\Gamma)\sin\Delta_f - (y_{12} \pm x_{12}\sin\phi_f^M)\cos\Delta_f,\\
c'^\pm_f &= \frac{1}{4}(x_{12}^2 + y_{12}^2 + [y_{12}^2 - x_{12}^2]R_f^{\mp 1}),\\
c'^\pm_{\bar{f}} &= \frac{1}{4}(x_{12}^2 + y_{12}^2 + [y_{12}^2 - x_{12}^2]R_f^{\pm 1}).
\end{aligned} \tag{90}$$

CP-odd contributions to the coefficients $c'^\pm_f$, $c'^\pm_{\bar{f}}$ are of $O[(x_{12}^2, y_{12}^2)\times(\epsilon_K, \phi_{12})]$ and have been neglected, i.e., they are $O(x_{12}, y_{12})$ suppressed relative to the CP-odd terms arising at $O(\tau)$. Interference between the amplitudes containing intermediate $K_S X$ and $K_L X$ ($e^{-\Gamma_K t'}$ dependence) yields,

$$\begin{aligned}
b^\pm &= \mp 2\epsilon_R, \qquad d^\pm = \mp 2\epsilon_I,\\
b_f^\pm &= 2(\pm x_{12}\cos\Delta_f + y_{12}\sin\Delta_f)\epsilon_I,\\
b_{\bar{f}}^\pm &= 2(\pm x_{12}\cos\Delta_f - y_{12}\sin\Delta_f)\epsilon_I,\\
d_f^\pm &= 2(\mp x_{12}\cos\Delta_f - y_{12}\sin\Delta_f)\epsilon_R,\\
d_{\bar{f}}^\pm &= 2(\mp x_{12}\cos\Delta_f + y_{12}\sin\Delta_f)\epsilon_R.
\end{aligned} \tag{91}$$

We have neglected interference contributions of $O(x_{12}^2\epsilon_K, y_{12}^2\epsilon_K)$ arising at $O(\tau^2)$ in (87), (88). Again, they are $O(x_{12}, y_{12})$ suppressed relative to the CP-odd terms arising at $O(\tau)$.

The indirect CP asymmetries are obtained by taking normalized rate differences between Γ_f and $\bar{\Gamma}_{\bar{f}}$, and between $\Gamma_{\bar{f}}$ and $\bar{\Gamma}_f$. To first order in CPV quantities, the phases ϕ_f^M, ϕ_f^Γ only enter the CP asymmetries of the purely K_S contributions, while the CP asymmetries induced by $K_S - K_L$ interference only probe ϵ_K. The first set of CP asymmetries, between the coefficients in (90), are given by ($\delta c'$ is negligible),

$$\begin{aligned}
\delta c &\equiv \frac{1}{2}(c^+ - c^-) = 2\epsilon_R,\\
\delta c_f &\equiv \frac{1}{2}(c_f^+ - c_{\bar{f}}^-)\\
&= -(y_{12}\sin\phi_f^\Gamma\sin\Delta_f + x_{12}\sin\phi_f^M\cos\Delta_f),\\
\delta c_{\bar{f}} &\equiv \frac{1}{2}(c_{\bar{f}}^+ - c_f^-)\\
&= (y_{12}\sin\phi_f^\Gamma\sin\Delta_f - x_{12}\sin\phi_f^M\cos\Delta_f).
\end{aligned} \tag{92}$$

Again, $\Delta_f \neq 0, \pi$ is required in order to measure ϕ_f^Γ, due to the lack of a nontrivial CP-even phase in the absorptive mixing amplitude. The six CP asymmetries in the second set of coefficients, cf. (91), are

$$\begin{aligned}
\delta b &\equiv \frac{1}{2}(b^+ - b^-) = -2\epsilon_R,\\
\delta d &\equiv \frac{1}{2}(d^+ - d^-) = -2\epsilon_I,\\
\delta b_f &\equiv \frac{1}{2}(b_f^+ - b_{\bar{f}}^-)\\
&= 2(x_{12}\cos\Delta_f + y_{12}\sin\Delta_f)\epsilon_I,\\
\delta b_{\bar{f}} &\equiv \frac{1}{2}(b_{\bar{f}}^+ - b_f^-)\\
&= 2(x_{12}\cos\Delta_f - y_{12}\sin\Delta_f)\epsilon_I,\\
\delta d_f &\equiv \frac{1}{2}(d_f^+ - d_{\bar{f}}^-)\\
&= -2(x_{12}\cos\Delta_f + y_{12}\sin\Delta_f)\epsilon_R,\\
\delta d_{\bar{f}} &\equiv \frac{1}{2}(d_{\bar{f}}^+ - d_f^-)\\
&= 2(-x_{12}\cos\Delta_f + y_{12}\sin\Delta_f)\epsilon_R.
\end{aligned} \tag{93}$$

In principle, each of the CP asymmetries in (92), (93) can be measured by fitting to the dependence of the decay rates on t and t'.

In Sec. IV B we will see that in the SM, ϕ_f^M and ϕ_f^Γ are expected to be of same order as ϵ_K, implying that the CPVINT asymmetries in (92) and (93) are also of same order. Thus, the impact of ϵ_K, particularly at linear order in τ, on the asymmetry measurements needs to be considered. We will address this point in Sec. V, taking into account the typical decay times t' for the intermediate K^0's detected at LHCb and Belle-II.

In the case of two body (and quasi two body) intermediate states, e.g., $X = \pi^0, \omega, f_0$, expressions for the time dependent decay rates and CP asymmetries are obtained by setting $R_f = 1$ [and $|\bar{A}_{K^0X}| = |A_{\bar{K}^0X}|$ in (87)], and $\sin\Delta_f = 0$, $\cos\Delta_f = \eta_f^{CP}$ in (90)–(93), where η_f^{CP} is defined in (37). The resulting decay widths are

$$\Gamma_f(t,t') = e^{-\tau}|\bar{A}_{+-}|^2|A_{\bar{K}^0X}|^2\{e^{-\Gamma_S t'}[c^+ + c_f^+\tau + c'\tau^2] + e^{-\Gamma_K t'}[(b^+ + b_f^+\tau)\cos(\Delta M_K t') + (d^+ + d_f^+\tau)\sin(\Delta M_K t')]\}, \quad (94)$$

$$\bar{\Gamma}_f(t,t') = e^{-\tau}|\bar{A}_{+-}|^2|A_{\bar{K}^0X}|^2\{e^{-\Gamma_S t'}[c^- + c_f^-\tau + c'\tau^2] + e^{-\Gamma_K t'}[(b^- + b_f^-\tau)\cos(\Delta M_K t') + (d^- + d_f^-\tau)\sin(\Delta M_K t')]\}, \quad (95)$$

with coefficients,

$$c^\pm = 1 \pm 2\epsilon_R, \qquad c_f^\pm = -\eta_f^{CP}(y_{12} \pm x_{12}\sin\phi_f^M),$$
$$c' = \frac{1}{2}y_{12}^2, \qquad b^\pm = \mp 2\epsilon_R, \qquad b_f^\pm = \pm 2\eta_f^{CP}x_{12}\epsilon_I,$$
$$d^\pm = \mp 2\epsilon_I, \qquad d_f^\pm = \mp 2\eta_f^{CP}x_{12}\epsilon_R. \quad (96)$$

The corresponding CP asymmetries, as defined in (92), (93), are given by

$$\delta c = 2\epsilon_R, \qquad \delta c_f = -\eta_f^{CP}x_{12}\sin\phi_f^M,$$
$$\delta b = -2\epsilon_R, \qquad \delta b_f = 2\eta_f^{CP}x_{12}\epsilon_I,$$
$$\delta d = -2\epsilon_I, \qquad \delta d_f = -2\eta_f^{CP}x_{12}\epsilon_R. \quad (97)$$

Note that δc_f is purely dispersive, similarly to ΔY_f for SCS decays to CP eigenstates, cf. (63) (again, the only CP-even phase available for charm CPVINT is the dispersive mixing phase $\pi/2$).

Finally, the CP conserving observable, y_{CP}^f, for SCS decays to CP eigenstates, cf. (57), (59), can be carried over to the case of two body and quasi two body intermediate states discussed above. It is analogously defined as

$$y_{CP}^f \equiv -\frac{c_f^+ + c_f^-}{2}. \quad (98)$$

However, the K_S decay time dependence, $e^{-\Gamma_S t'}$, in (94), (95), must be accounted for in order to avoid additional systematic errors in its extraction. Employing (96) yields

$$y_{CP}^f = \eta_f^{CP}y_{12} = \eta_f^{CP}|y|, \quad (99)$$

up to negligible corrections quadratic in CPV parameters. For example, we expect $y_{CP}^f = -y_{12}$ for $X = \omega, \pi^0$ (opposite in sign to y_{CP}^f for K^+K^-, $\pi^+\pi^-$), and $y_{CP}^f = +y_{12}$ for $X = f_0$.

IV. APPROXIMATE UNIVERSALITY

In the previous section, all indirect CPV effects were parametrized in full generality, in terms of final state dependent pairs of dispersive and absorptive weak phases $(\phi_f^M, \phi_f^\Gamma)$. In order to understand how best to parametrize indirect CPV effects in the upcoming precision era, we need to estimate the final state dependence. We accomplish this via a U-spin flavor symmetry decomposition of the SM $D^0-\bar{D}^0$ mixing amplitudes. Crucially, this also yields estimates of indirect CPV effects in the SM.

A. U-spin decomposition

The SM $D^0-\bar{D}^0$ mixing amplitudes Γ_{12} and M_{12} have flavor transitions $\Delta C = -\Delta U = 2$ and $\Delta S = \Delta D = 0$. We can write them as

$$\Gamma_{12}^{\rm SM} = -\sum_{i,j=d,s}\lambda_i\lambda_j\Gamma_{ij}, \qquad M_{12}^{\rm SM} = -\sum_{i,j=d,s,b}\lambda_i\lambda_j M_{ij}, \quad (100)$$

where $\lambda_i \equiv V_{ci}V_{ui}^*$. At the quark level, the transition amplitudes Γ_{ij} and M_{ij} are identified with box diagrams containing, respectively, on-shell and off-shell internal i and j quarks. Thus, they possess the flavor structures (Dirac structure is unimportant for our discussion) $\Gamma_{ij}, M_{ij} \sim (\bar{u}c)^2(\bar{i}i)(\bar{j}j) \sim (\bar{u}c)^2(\bar{i}j)(\bar{j}i)$, or

$$\Gamma_{ss} \sim (\bar{s}s)^2, \qquad \Gamma_{dd} \sim (\bar{d}d)^2, \qquad \Gamma_{sd} \sim (\bar{s}s)(\bar{d}d), \quad (101)$$

and similarly for the M_{ij}. Employing CKM unitarity $(\lambda_d + \lambda_s + \lambda_b = 0)$, the U-spin decomposition of $\Gamma_{12}^{\rm SM}$ is given by

$$\Gamma_{12}^{\rm SM} = \frac{(\lambda_s - \lambda_d)^2}{4}\Gamma_2 + \frac{(\lambda_s - \lambda_d)\lambda_b}{2}\Gamma_1 + \frac{\lambda_b^2}{4}\Gamma_0, \quad (102)$$

where the U-spin amplitudes $\Gamma_{2,1,0}$ are the $\Delta U_3 = 0$ elements of the $\Delta U = 2,\ 1,\ 0$ multiplets, respectively. This can be seen from their quark flavor structures,

$$\Gamma_2 = \Gamma_{ss} + \Gamma_{dd} - 2\Gamma_{sd} \sim (\bar{s}s - \bar{d}d)^2 = O(\epsilon^2),$$
$$\Gamma_1 = \Gamma_{ss} - \Gamma_{dd} \sim (\bar{s}s - \bar{d}d)(\bar{s}s + \bar{d}d) = O(\epsilon),$$
$$\Gamma_0 = \Gamma_{ss} + \Gamma_{dd} + 2\Gamma_{sd} \sim (\bar{s}s + \bar{d}d)^2 = O(1). \quad (103)$$

The orders in the U-spin breaking parameter ϵ at which they enter are also included, corresponding to the power of the U-spin breaking spurion $M_\epsilon \sim \epsilon(\bar{s}s - \bar{d}d)$ required to construct each Γ_i. The U-spin decomposition of M_{12} is analogous to (102), with the exception of additional contributions to M_1 and M_0, given by $(M_{sb} - M_{db})$ and $(M_{sb} + M_{db} + M_{bb})$, respectively, and corresponding to box diagrams with internal b quarks at the quark level. The small value of λ_b implies that we can neglect the $\Delta U = 1, 0$ contributions to the mass and width differences, even though the $\Delta U = 2$ piece is of higher order in ϵ. Thus, x_{12} and y_{12} are due to Γ_2 and M_2, respectively, and arise at $O(\epsilon^2)$ [13–15]. Similarly, CPV in mixing arises at $O(\epsilon)$ due to Γ_1 and M_1, while the contributions of Γ_0 and M_0 are negligible.

The U-spin amplitudes Γ_i, M_i are of the form,

$$M_i = \eta_i^M |M_i| e^{2i\xi}, \qquad \Gamma_i = \eta_i^\Gamma |\Gamma_i| e^{2i\xi}, \qquad \eta_i^M, \eta_i^\Gamma = \pm. \tag{104}$$

The exponential factors originate from the choice of meson phase convention, and trivially cancel in physical observables. However, the η_i in (104) are physical, can *a priori* be of either sign, and can be determined from experiment. For example, since $\phi_{12} \approx 0$, we already know that

$$\arg[M_2/\Gamma_2] = 0, \tag{105}$$

or that $\eta_2^M = \eta_2^\Gamma$. Moreover, as we shall see shortly, cf. (125), existing measurements also imply that

$$\eta_2^M = \eta_2^\Gamma = +. \tag{106}$$

The inclusive [16–23] and exclusive [13–15,24,25] approaches to estimating $\Delta\Gamma_D$ yield several observations of relevance to our discussion of CPV below. In the inclusive OPE based approach, the flavor amplitudes satisfy $\Gamma_{ij} \sim \Gamma_D$. This is reflected in the ability of this approach to accommodate the charm meson lifetimes [23,26]. The individual Γ_{ij} contributions to y_{12} are, therefore, about five times larger than the experimental value [27], suggesting that U-spin violation is large, e.g., $\mathcal{O}(\epsilon^2) \sim 20\%$ for Γ_2, cf. (103), (120).[5] The exclusive approach estimates sums over exclusive decay modes. Unfortunately, the charm quark mass is not sufficiently light for D^0 meson decays to be dominated by a few final states. Moreover, the strong phase differences entering y_{12}, and the off-shell decay amplitudes in x_{12} are not calculable from first principles. However, there is consensus in the literature that accounting for y_{12} near 1% requires significant contributions from high multiplicity final states ($n \geq 4$), due to the large $SU(3)_F$ breaking near threshold. This observation is consistent with the large U-spin breaking required (potentially from duality violations) in the OPE/HQE approach.

[5]Inclusive OPE based GIM-cancelations between the Γ_{ij} yield y four orders of magnitude below experiment. Evidently, m_c and $(m_s - m_d)/\Lambda_{\rm QCD}$ are not sufficiently large and small, respectively, for this approach to properly account for U-spin breaking in y_{12}.

B. CPV phases intrinsic to mixing

We introduce three intrinsic CPV mixing phases, defined with respect to the direction of the dominant $\Delta U = 2$ dispersive and absorptive mixing amplitudes in the complex plane,

$$\begin{aligned} \phi_2^\Gamma &\equiv \arg\left[\frac{\Gamma_{12}}{\frac{1}{4}(\lambda_s - \lambda_d)^2 \Gamma_2}\right], \\ \phi_2^M &\equiv \arg\left[\frac{M_{12}}{\frac{1}{4}(\lambda_s - \lambda_d)^2 M_2}\right], \\ \phi_2 &\equiv \arg\left[\frac{q}{p}\frac{(\lambda_s - \lambda_d)^2 \Gamma_2}{4}\right], \end{aligned} \tag{107}$$

where Γ_{12}, M_{12}, and q/p can contain NP contributions. These phases can be viewed as the pure mixing analogs of the final state dependent phases ϕ_f^M, ϕ_f^Γ, and ϕ_{λ_f}, respectively. Note that they are quark and meson phase convention independent, like the final state dependent ones, as required for physical phases. For later use we give the expressions for the (phase convention dependent) arguments of M_{12} and Γ_{12} in terms of ϕ_2^M and ϕ_2^Γ, respectively, cf. (104),

$$\begin{aligned} \phi^M &= 2\arg[\lambda_s - \lambda_d] + 2i\xi + \pi(1 - \eta_2^M)/2 + \phi_2^M, \\ \phi^\Gamma &= 2\arg[\lambda_s - \lambda_d] + 2i\xi + \pi(1 - \eta_2^\Gamma)/2 + \phi_2^\Gamma. \end{aligned} \tag{108}$$

Employing (105), the theoretical or intrinsic mixing phases are seen to satisfy the relations

$$\phi_{12} = \phi_2^M - \phi_2^\Gamma, \tag{109}$$

and the analog of (46),

$$\tan 2\phi_2 = -\left(\frac{x_{12}^2 \sin 2\phi_2^M + y_{12}^2 \sin 2\phi_2^\Gamma}{x_{12}^2 \cos 2\phi_2^M + y_{12}^2 \cos 2\phi_2^\Gamma}\right). \tag{110}$$

Combining the two relations, ϕ_2 can be related to ϕ_{12}, and ϕ_2^Γ or ϕ_2^M, to first order in CPV quantities, as

$$\begin{aligned} \tan 2(\phi_2 + \phi_2^\Gamma) &\approx -\frac{x_{12}^2}{x_{12}^2 + y_{12}^2} \sin 2\phi_{12} \\ \tan 2(\phi_2 + \phi_2^M) &\approx \frac{y_{12}^2}{x_{12}^2 + y_{12}^2} \sin 2\phi_{12}. \end{aligned} \tag{111}$$

Together with (45), the above relations allow translation between ϕ_2 and $|q/p|$, and any two out of the three phases ϕ_2^M, ϕ_2^Γ, and ϕ_{12}.

We estimate the magnitudes of the theoretical phases in the SM ($\Gamma_{12} = \Gamma_{12}^{\rm SM}$, $M_{12} = M_{12}^{\rm SM}$), as well as their deviations from the corresponding final state dependent phases ϕ_f^Γ, ϕ_f^M, and ϕ_{λ_f}, using U-spin based arguments and experimental input. To very good approximation, the CKM hierarchy $|\lambda_b/(\lambda_s - \lambda_d)| \ll 1$ yields,

$$\phi_2^\Gamma = \mathrm{Im}\left(\frac{2\lambda_b}{\lambda_s - \lambda_d}\frac{\Gamma_1}{\Gamma_2}\right) = \left|\frac{\lambda_b}{\theta_C}\right| \sin\gamma \times \frac{\Gamma_1}{\Gamma_2}. \qquad (112)$$

Taking into account the U-spin breaking hierarchy $\Gamma_1/\Gamma_2 = \mathcal{O}(1/\epsilon)$, cf. (103), yields the rough SM estimates[6]

$$\phi_2^\Gamma \sim \left|\frac{\lambda_b}{\theta_C}\right| \sin\gamma \times \frac{1}{\epsilon}, \qquad (113)$$

and similarly for ϕ_2^M. In terms of the most recent CKM fits [28,29], we obtain

$$\phi_{12} \sim \phi_2^\Gamma \sim \phi_2^M \sim (2.2\times 10^{-3}) \times \left[\frac{0.3}{\epsilon}\right]. \qquad (114)$$

The third phase, ϕ_2, is seen to be of same order, barring large cancelations, cf. (110).

An alternative expression for ϕ_2^Γ in the SM follows from (112), via the relation $|\Gamma_2| \cong |y|\Gamma_D/\lambda_s^2$,

$$|\phi_2^\Gamma| = \left|\frac{\lambda_b \lambda_s \sin\gamma}{y}\right| \frac{|\Gamma_1|}{\Gamma_D} = 0.005\left(\frac{0.66\%}{|y|}\right)\frac{|\Gamma_1|}{\Gamma_D} \sim 0.005\epsilon, \qquad (115)$$

where in the second relation we have incorporated the current central value of $|y|$ [6], and in the last relation we have taken $\Gamma_1 \sim \epsilon\Gamma_D$ (recall that the inclusive approach yields $\Gamma_{ij} \sim \Gamma_D$). The estimates for ϕ_2^Γ in (114) and (115) are consistent with each other (for illustrative purposes, if we identify their respective ϵ factors, the two estimates would coincide for $\epsilon \approx 0.36$).

The ϵ dependence in (115) has been shifted to the numerator, compared to (114) [note that $y = O(\epsilon^2)$]. This allows us to obtain an approximate upper bound on ϕ_2^Γ, which we briefly describe here. A detailed discussion will be given elsewhere [30]. We rewrite the ratio of widths in (115) as

$$\frac{|\Gamma_1|}{\Gamma_D} = \frac{|\Gamma_{sd}|}{\Gamma_D}\epsilon_1, \qquad (116)$$

where, cf. (103),

$$\epsilon_1 \equiv \frac{|\Gamma_{dd} - \Gamma_{ss}|}{|\Gamma_{sd}|} = O(\epsilon). \qquad (117)$$

Moreover, $SU(3)_F$ flavor symmetry arguments yield the bound

$$\frac{|\Gamma_{sd}|}{\Gamma_D} < 1 + O(\epsilon). \qquad (118)$$

The $O(\epsilon)$ correction in (118) originates from differences between the D^0 decay matrix elements for U-spin related DCS and CF final states, modulo the CKM factors. It is expected to be small since it does not depend on U-spin breaking from phase space differences.[7] (It is interesting to note that $|\Gamma_{sd}|/\Gamma_D \approx 0.6$–$0.75$ has been obtained in the OPE based approach [19].) Thus, we obtain the absorptive CPV upper bound,

$$|\phi_2^\Gamma| < 0.005\left(\frac{0.66\%}{|y|}\right)\epsilon_1[1 + O(\epsilon)], \qquad (119)$$

where, conservatively, $\epsilon_1 < 1$.

Combining (118) with the measured value of y also yields the lower bound, cf. (103),

$$(\epsilon_2)^2 \equiv \frac{|\Gamma_{ss} + \Gamma_{dd} - 2\Gamma_{sd}|}{|\Gamma_{sd}|} > 0.14\left(\frac{|y|}{0.66\%}\right)[1 + O(\epsilon)]. \qquad (120)$$

Given that $(\epsilon_2)^2 = O(\epsilon^2)$, (120) confirms the existence of large U-spin breaking in $D^0 - \bar{D}^0$ mixing.

In principle, Γ_1 can be estimated via the exclusive approach, as more data on SCS D^0 decay branching ratios and direct CP asymmetries become available. It relies on the U-spin decomposition of exclusive contributions to Γ_1. Details can be found in [31]. Unfortunately, the potentially large contributions from high multiplicity final states would complicate this program, as in the case of $\Delta\Gamma_D$.

C. Final state dependence

The misalignments between the final state dependent phases ϕ_f^M, ϕ_f^Γ, ϕ_{λ_f}, and their theoretical counterparts are equal in magnitude, satisfying

$$\delta\phi_f \equiv \phi_f^\Gamma - \phi_2^\Gamma = \phi_f^M - \phi_2^M = \phi_2 - \phi_{\lambda_f}. \qquad (121)$$

Below, we discuss the size of $\delta\phi_f$ in the SM for (i) SCS decays, (ii) CF/DCS decays to $K^\pm X$, and (iii) CF/DCS decays to $K^0 X$, $\bar{K}^0 X$.

1. SCS decays

The amplitudes for the SCS decay modes $D^0 \to f$ and $\bar{D}^0 \to f$ in the SM can be written as, see e.g., [32],

$$A_f = \frac{1}{2}(\lambda_s^* - \lambda_d^*)\mathcal{A}_{f,1} + \lambda_b^*\mathcal{A}_{f,0},$$
$$\bar{A}_f = \frac{1}{2}(\lambda_s - \lambda_d)\bar{\mathcal{A}}_{f,1} + \lambda_b\bar{\mathcal{A}}_{f,0}, \qquad (122)$$

with substitutions $f \to \bar{f}$ for the CP conjugate modes. The first and second terms in each relation are the $\Delta U = 1$ and

[6] We thank Yuval Grossman for this estimate.

[7] Phase space differences enter the rhs of (118) at $O(\epsilon^2)$ [30].

$\Delta U = 0$ transition amplitudes, respectively, where the former is due to the current-current operators Q_1, Q_2, and the latter is dominated by their QCD penguin contractions. Generically, both amplitudes are $O(1)$ in $SU(3)_F$ breaking, and the $\Delta U = 0$ amplitude is parametrically suppressed by $O(\lambda_b/\theta_C)$. (Two exceptions are mentioned below).

The amplitudes for decays to CP eigenstates are generally of the form given in (16). In the case of SCS decays, comparison with (122) yields the weak phase,

$$\arg\left[\eta_f^{CP}\frac{A_f}{\bar{A}_f}\right] = -2\arg[\lambda_s - \lambda_d] - 2i\xi + 2r_f\cos\delta_f\sin\phi_f, \tag{123}$$

where the sum of the first two terms on the rhs is identified with $2\phi_f^0$ (the second term originates from the choice of meson phase convention), and in the SM,

$$\delta_f = \arg[\mathcal{A}_{f,0}/\mathcal{A}_{f,1}], \qquad \phi_f = -\gamma, \qquad r_f = \left|\frac{\lambda_b}{\theta_C}\frac{\mathcal{A}_{f,0}}{\mathcal{A}_{f,1}}\right|. \tag{124}$$

Combining (108) and (123) yields the following expressions for the CPVINT phases ϕ_f^M, ϕ_f^Γ, cf. (18), (19),

$$\begin{aligned}\phi_f^M &= \pi(1-\eta_2^M)/2 + \phi_2^M - 2r_f\cos\delta_f\sin\gamma,\\ \phi_f^\Gamma &= \pi(1-\eta_2^\Gamma)/2 + \phi_2^\Gamma - 2r_f\cos\delta_f\sin\gamma.\end{aligned} \tag{125}$$

Given that ϕ_f^M, $\phi_f^\Gamma \approx 0$ (rather than π) for $f = \pi^+\pi^-$, K^+K^-, cf. (62), we learn that the first term on the rhs of each relation in (125) must vanish, i.e., $\eta_2^M = \eta_2^\Gamma = +$, as claimed in (106). In turn, the misalignment in (121) for a CP eigenstate final state, is given by

$$\delta\phi_f = -2r_f\cos\delta_f\sin\gamma = -a_f^d\cot\delta_f, \tag{126}$$

where the direct CP asymmetry, a_f^d, has been defined in (51).

It is instructive to rewrite the CPVINT asymmetry ΔY_f, cf. (63), in terms of ϕ_2^M, and the subleading decay amplitude parameters r_f, ϕ_f, and δ_f, cf. (124),

$$\frac{\Delta Y_f}{\eta_{CP}^f} = -x_{12}\sin\phi_2^M - 2r_f\sin\phi_f(x_{12}\cos\delta_f + y_{12}\sin\delta_f). \tag{127}$$

Previously, we saw that the leading amplitude contribution is purely dispersive for CP eigenstate final states, because the requisite CP-even phase difference is only present in the dispersive mixing amplitude ($\delta = \pi/2$). Similarly, it is now clear that the strong phase dependence of the dispersive and absorptive contributions entering at first order in the subleading amplitudes, cf. (127), can be attributed to the strong phase differences $\pi/2 + \delta_f$ and δ_f, between their respective interfering decay chains.

In the case of SCS decays to non-CP eigenstates, the misalignments of the CPVINT phases, cf. (20)–(22), generalize as

$$\begin{aligned}\delta\phi_f &= -(r_f\cos\delta_f + r_{\bar{f}}\cos\delta_{\bar{f}})\sin\gamma\\ &= -(a_f^d\cot\delta_f + a_{\bar{f}}^d\cot\delta_{\bar{f}})/2,\end{aligned} \tag{128}$$

where r_f, δ_f are defined as in (124); $r_{\bar{f}}$, $\delta_{\bar{f}}$ correspond to the substitutions $f \to \bar{f}$ therein; and $\phi_f = \phi_{\bar{f}} = -\gamma$. The direct CP asymmetries have been defined in (71).

The misalignments (126), (128) for SCS decays are nonperturbative, and incalculable at present, like the direct CP asymmetries. However, the strong phases are expected to satisfy $\delta_{f,\bar{f}} = O(1)$, due to large rescattering at the charm mass scale, yielding the order of magnitude estimates $\delta\phi_f = \mathcal{O}(\lambda_b\sin\gamma/\theta_C)$. In particular, the misalignments, like the direct CP asymmetries a_f^d are $O(1)$ in $SU(3)_F$ breaking. Thus, they are parametrically suppressed relative to the theoretical phases in the SM, cf. (112),

$$\frac{\delta\phi_f}{\phi_2^M}, \frac{\delta\phi_f}{\phi_2^\Gamma} = \mathcal{O}(\epsilon). \tag{129}$$

For example, the recent LHCb discovery [33] of a nonvanishing difference between the $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ direct CP asymmetries yields the world average [6],

$$\Delta a_{CP}^{\rm dir} \equiv a_{K^+K^-}^d - a_{\pi^+\pi^-}^d = -0.00164 \pm 0.00028. \tag{130}$$

In the U-spin symmetric limit, $a_{\pi^+\pi^-}^d = -a_{K^+K^-}^d$ [34], implying the rough estimate $\delta\phi_f \sim 0.08\%$ for these decays. Dividing by the SM estimates for ϕ_2^M and ϕ_2^Γ in (114) or (115) yields significant misalignments, consistent with the parametric suppression in (129) for sizable $\epsilon \sim 0.4$.

Fortunately, the K^+K^- and $\pi^+\pi^-$ misalignments, like the direct CP asymmetries [34], are equal and opposite in the U-spin limit, i.e.,

$$\begin{aligned}(\delta\phi_{K^+K^-} + \delta\phi_{\pi^+\pi^-}) &= O(\epsilon\delta\phi_{K^+K^-,\pi^+\pi^-}),\\ (a_{K^+K^-}^d + a_{\pi^+\pi^-}^d) &= O(\epsilon a_{K^+K^-,\pi^+\pi^-}^d).\end{aligned} \tag{131}$$

Thus, the average of $\phi_f^{M,\Gamma}$ over $f = K^+K^-, \pi^+\pi^-$ satisfies,

$$\frac{1}{2}(\phi_{K^+K^-}^{M,\Gamma} + \phi_{\pi^+\pi^-}^{M,\Gamma}) = \phi_2^{M,\Gamma}[1 + O(\epsilon^2)], \tag{132}$$

and the average of the time dependent CP asymmetries in (63) satisfies,

$$A_\Gamma = -x_{12}\phi_2^M[1 + O(\epsilon^2)], \qquad (133)$$

where we have used the relations $x_{12} \sim y_{12}$ and $\delta\phi_f \sim a_f^d$.

As has already been noted, large U-spin violation is likely to play an important role in mixing. Moreover, the $\delta\phi_f$ for SCS decays are inherently nonperturbative. Therefore, while (129) implies that the order of magnitude estimates (114), (115) for $\phi_2^{M,\Gamma}$ apply equally well to the measured phases $\phi_f^{M,\Gamma}$ in the SM, $O(1)$ variations cannot be ruled out. The latter possibility would correspond to the weakest form of approximate universality. Ultimately, precision measurements of the indirect and direct CP asymmetries in a host of SCS decays will clarify the situation.

We point out that in the presence of NP in SCS decays, the expressions for the misalignments, $\delta\phi_f$, in the second relations of (126), (128) remain valid. In particular, the direct CP asymmetries $a_{f,\bar{f}}^d$ and the strong phases $\delta_{f,\bar{f}}$ now depend on the total subleading amplitudes, i.e., the sums of the QCD penguin and NP amplitudes. The $\delta\phi_f$ would be of same order as in the SM, provided that the CP-odd NP amplitudes are similar in size, or smaller than the SM QCD penguin amplitudes, as already hinted at by the current bounds on direct CPV in $D^0 \to K^+K^-, \pi^+\pi^-$ decays.

Finally, we mention two SCS decay modes, $D^0 \to K^0\bar{K}^0$ and $D^0 \to K^{*0}\bar{K}^0$, which violate the $O(\epsilon)$ counting in (129). For $D^0 \to K^0\bar{K}^0$, the first term in (122) is suppressed by $O(\epsilon)$ (as reflected in the rate), yielding $O(1/\epsilon)$ enhancements of $\delta\phi_f$, the direct CP asymmetry [35,36], and the misalignment, i.e., $\delta\phi_f/\phi_2^{M,\Gamma} = O(1)$ in the SM. For $D^0 \to K^{*0}\bar{K}^0$, the first term in (122) is not formally suppressed by $O(\epsilon)$. However, a large accidental cancelation between contributions related by $K^{*0} \leftrightarrow \bar{K}^0$ interchange (again reflected in the measured decay rate), again enhances $\delta\phi_f$, and the direct CP asymmetry [37]. Thus, in effect, the misalignment could be $O(1)$, as for $K^0\bar{K}^0$.

2. CF/DCS decays to $K^\pm X$

The CPVINT observables in this class are given in (20), (21), with the modified sign convention of (24). The CKM factors enter the CF/DCS amplitudes as $A_f \propto V_{cs}^*V_{ud}$ (CF) and $\bar{A}_f \propto V_{cd}V_{us}^*$ (DCS). Thus, in the SM and, more generally, in models with negligible new weak phases in CF/DCS decays, Eqs. (25) and (108) yield the absorptive and dispersive phases,

$$\phi_f^{M(\Gamma)} = \phi_2^{M(\Gamma)} + \arg\left[-\frac{V_{cs}^*V_{ud}}{V_{cd}V_{us}^*}(\lambda_s - \lambda_d)^2\right]. \qquad (134)$$

Employing CKM unitarity, the misalignments, given by the second term on the rhs, are seen to satisfy

$$\delta\phi_f = O\left(\frac{\lambda_b^2}{\lambda_d^2}\right). \qquad (135)$$

Thus, for CF/DCS decays to $K^\pm X$, the misalignments vanish up to a negligible (and precisely known) final-state independent correction of $O(10^{-6})$. This represents the strongest form of approximate universality, i.e., the universal limit $\phi_f^{M(\Gamma)} = \phi_2^{M(\Gamma)}$. In particular, CPVINT measurements in these decays directly determine the theoretical phases.

3. CF/DCS decays to K^0X, $\bar{K}^0X$

We begin with a discussion of the misalignments in this class of decays in the limit that the DCS decays are neglected. Expressions for the CPVINT observables and time-dependent decay widths in this approximation are given in (33)–(36) and Sec. III C, respectively. The misalignments follow from (36). One ingredient is the phase of q_K/p_K. To excellent approximation [1], this ratio satisfies the relation

$$\frac{q_K}{p_K} = \frac{A_0}{\bar{A}_0}(1 - 2\epsilon_K), \qquad (136)$$

where $A_{0,2}$ denote the $K^0 \to (\pi\pi)_{I=0,2}$ amplitudes, respectively, i.e., they are $\Delta I = 1/2, 3/2$ transitions. Keeping track of the CKM factors, these amplitudes can be written as

$$\begin{aligned} A_{0(2)} &= V_{ud}V_{us}^*\mathcal{A}_{0(2)} + V_{td}V_{ts}^*\mathcal{B}_{0(2)} \\ &= V_{ud}V_{us}^*\mathcal{A}_{0(2)}[1 + r_{0(2)}], \end{aligned} \qquad (137)$$

yielding

$$\arg\left[\frac{q_K}{p_K}\right] = 2\arg[V_{ud}V_{us}^*] - 2\epsilon_I + 2\mathrm{Im}[r_0]. \qquad (138)$$

A second ingredient is the CP-odd phase in the ratio of CF amplitudes, $A_{\bar{K}^0X}/\bar{A}_{K^0X}$,

$$\begin{aligned} 2\phi_{\bar{K}^0X}^0 &= 2\arg[V_{cs}^*V_{ud}] - 2i\xi \\ &= 2\arg[V_{us}^*V_{ud}] + 2\arg[\lambda_s^*] - 2i\xi. \end{aligned} \qquad (139)$$

Finally, combining (108), (138), and (139) yields the final state independent absorptive and dispersive phases,

$$\phi_f^{M(\Gamma)} = \phi_2^{M(\Gamma)} + 2\epsilon_I + \left|\frac{\lambda_b}{\lambda_s}\right|\sin\gamma - 2\mathrm{Im}[r_0]. \qquad (140)$$

The last term in (140) is nonperturbative in origin. However, it enters the kaon CPV observable, ϵ_K'/ϵ_K, as[8]

[8]In a phase convention commonly employed for discussions of ϵ_K'/ϵ_K, $\mathrm{Im}[r_{0(2)}] = \mathrm{Im}[A_{0(2)}]/\mathrm{Re}[A_{0(2)}]$.

$$\mathrm{Re}\left[\frac{\epsilon'_K}{\epsilon_K}\right] = (1.66 \pm 0.23) \times 10^{-3} \quad [10]$$
$$= -\frac{\omega}{\sqrt{2}|\epsilon|}(\mathrm{Im}[r_0] - \mathrm{Im}[r_2]), \tag{141}$$

where $\omega \equiv (\mathcal{A}_2/\mathcal{A}_0) \approx 1/22$. Equating the measured value of $\mathrm{Re}[\epsilon'_K/\epsilon_K]$ with the first term on the rhs of the second relation in (141), i.e., assuming modest cancelation with A_2 [38], yields the estimate

$$\mathrm{Im}[r_0] \approx 1.2 \times 10^{-4}. \tag{142}$$

Similarly, the dominant chirally enhanced penguin operator (Q_6) contribution to A_0 yields [38],

$$\mathrm{Im}[r_0] \approx 1.5 \times 10^{-4} B_6^{(1/2)}, \tag{143}$$

where the matrix element parameter $B_6^{(1/2)} = 1$ in the large N_C limit. (A recent study [39] claiming that the SM prediction for ϵ'/ϵ could be significantly smaller than the measured value obtains Im $[r_0] < 10^{-4}$).

Thus, in the limit that the DCS amplitudes are neglected, the misalignments satisfy

$$\delta\phi_f = 2\epsilon_I + \left|\frac{\lambda_b}{\lambda_s}\right| \sin\gamma = 3.7 \times 10^{-3}, \tag{144}$$

up to a small CP-odd ratio of $K \to \pi\pi$ amplitudes, given by $-2\mathrm{Im}[r_0] = O(10^{-4})$. The latter lies an order of magnitude below our SM estimates for the theoretical phases ϕ_2^M, ϕ_2^Γ in (114), (115) and can be neglected.

Finally, we address the impact of the DCS amplitudes. Expanding the CPVINT observables in (30) to first order in the DCS amplitudes, the weak and strong phases in $\lambda^{M,\Gamma}_{K_{S/L}X}$ are seen to be related to those in $\lambda_f^{M,\Gamma}$ (cf. (31) and (34), respectively), as

$$\begin{aligned}
\phi^M[K_{S/L}X] &= \phi_f^M \pm (r_f \cos\delta_f + r_{\bar{f}} \cos\delta_{\bar{f}})\delta\phi_f,\\
\phi^\Gamma[K_{S/L}X] &= \phi_f^\Gamma \pm (r_f \cos\delta_f + r_{\bar{f}} \cos\delta_{\bar{f}})\delta\phi_f,\\
\Delta[K_{S/L}X] &= \Delta_f \pm (r_f \sin\delta_f - r_{\bar{f}} \sin\delta_{\bar{f}}),
\end{aligned} \tag{145}$$

where $\delta\phi_f$ is given in (144). We recall that $\phi_f^{M,\Gamma}$ are the CPV phases in the absence of the DCS amplitudes, r_f and $r_{\bar{f}}$ are the magnitudes of DCS to CF amplitude ratios,

$$r_f = \left|\frac{A_{K^0X}}{A_{\bar{K}^0X}}\right|, \qquad r_{\bar{f}} = \left|\frac{\bar{A}_{\bar{K}^0X}}{\bar{A}_{K^0X}}\right|, \tag{146}$$

and δ_f, $\delta_{\bar{f}}$ are the strong phase differences of the corresponding amplitude ratios. Finally, their magnitudes are related as

$$\begin{aligned}
|\lambda^M_{K_{S/L}X}| &= |\lambda_f^M|(1 - [r_f \cos\delta_f - r_{\bar{f}} \cos\delta_{\bar{f}}]),\\
|\lambda^M_{\overline{K_{S/L}X}}| &= |\lambda_{\bar{f}}^M|(1 + [r_f \cos\delta_f - r_{\bar{f}} \cos\delta_{\bar{f}}]),
\end{aligned} \tag{147}$$

and similarly for $M \to \Gamma$.

Expressions for the time dependent decay widths, including the DCS amplitudes, are obtained via insertion of the CPVINT observables (31) and the full expressions for the decay amplitudes (32) into the general formulas (85) for the time-dependent amplitudes. The result can be brought into the same general form as (87), (88). Effectively, the prefactors in Eqs. (87) and (88), the ratios $\sqrt{R_f}$, and the expressions (90), (91) for the coefficients are modified at $O(r_f, r_{\bar{f}})$, i.e., $O(\theta_C^2)$. For example, the coefficients contain new CP-even terms of $O(r_{f,\bar{f}})$, and new CP-odd terms of $O(\epsilon_K r_{f,\bar{f}})$. These corrections produce relative shifts in the CP averaged decay rates, as well as the indirect CP asymmetries listed in (92), (93), (97), of $O(\theta_C^2)$.

Our primary focus here is on the absorptive and dispersive CPVINT phases. As previously noted, they only reside in the pure K_S contributions to the time dependent widths (to first order in CPV). In particular, $\phi_f^{M,\Gamma}$ are replaced by $\phi^{M,\Gamma}[K_SX]$ in the coefficients $c_f^\pm$, $c_{\bar{f}}^\pm$, cf. (145), (90). Consequently, the misalignments (144) are modified as

$$\begin{aligned}
\delta\phi_f &\equiv \phi^{M(\Gamma)}[K_SX] - \phi_2^{M(\Gamma)}\\
&= \left(2\epsilon_I + \left|\frac{\lambda_b}{\lambda_s}\right| \sin\gamma\right)(1 + r_f \cos\delta_f + r_{\bar{f}} \cos\delta_{\bar{f}})\\
&= \left(2\epsilon_I + \left|\frac{\lambda_b}{\lambda_s}\right| \sin\gamma\right)(1 + O[\theta_C^2]).
\end{aligned} \tag{148}$$

Thus, while the DCS corrections to the CPVINT phases are final state dependent, they are of $O(2\theta_C^2\epsilon_I)$, or $O(0.1\phi_2^{M,\Gamma})$ in the SM. This represents a more generic form of approximate universality than what we found in the previous two classes of decays, i.e., an $O(10\%)$ variation among the ϕ_f^M and ϕ_f^Γ, corresponding to a similar variation in the CPVINT asymmetries. The shifts in the asymmetries remain at this order when taking all of the DCS corrections to the widths into account. We therefore conclude that their inclusion in (87), (88) is not warranted for the interpretation of CPVINT data at SM sensitivity.

V. IMPLEMENTATION OF APPROXIMATE UNIVERSALITY

In this section, we discuss how to convert the general expressions for the time dependent decay widths and indirect CP asymmetries obtained in Sec. III B to the approximate universality parametrization, in the three classes of decays. For CF/DCS decays to K^0X, $\bar{K}^0X$, we pay special attention to ϵ_K induced effects at LHCb and Belle-II.

A. SCS decays

For SCS decays, the theoretical absorptive and dispersive CPV phases replace the final state dependent ones via the substitutions,

$$\phi_f^M \to \phi_2^M, \qquad \phi_f^\Gamma \to \phi_2^\Gamma, \tag{149}$$

in the expressions for the time dependent decay widths and CP asymmetries. For decays to CP eigenstates, they enter the expressions for the decay widths (52) (via Eq. (53) for $c_f^\pm$) and the CP asymmetry ΔY_f (63). For decays to non-CP eigenstates, they enter the expressions for the decay widths (64), (65) (via Eq. (67) for $c_{\bar f}^\pm$) and the indirect CP asymmetries ΔY_f, $\Delta Y_{\bar f}$ (70). Note that the misalignments $\delta\phi_f$ are dropped on the RHS of (149), as they are not calculable from first principles QCD. Moreover, while formally of $O(\epsilon)$ in U-spin breaking relative to $\phi_2^{M,\Gamma}$, they could, in principle, yield $O(1)$ variations in ϕ_f^M and ϕ_f^Γ in the SM. In Sec. VI B we discuss a strategy for fits carried out once SM sensitivity is achieved, and final state dependent effects in ϕ_f^M, ϕ_f^Γ become accessible to experiment.

The direct CPV (a_f^d) and misalignment ($\delta\phi_f$) contributions to the CPVINT asymmetries in (63), (70) are of same order, cf. (126). Therefore, consistency requires us to drop the a_f^d, $a_{\bar f}^d$ terms in the CPVINT asymmetries, if we neglect $\delta\phi_f$ in (149). For example, for CP eigenstate final states, and in the approximate universality parametrization, (63) reduces to,

$$\Delta Y_f = -\eta_f x_{12} \sin\phi_2^M, \tag{150}$$

and similarly for the non-CP eigenstates (the first line of each asymmetry in (70) is kept, with $\phi_f^{M,\Gamma} \to \phi_2^{M,\Gamma}$). However, we recall that in the average of ΔY_f over $f = K^+K^-, \pi^+\pi^-$, i.e., A_Γ, the error incurred by dropping $\delta\phi_f$ and a_f^d is of $O(\epsilon^2)$, cf. (132), (133).

B. CF/DCS decays to $K^\pm X$

For CF/DCS decays to $K^\pm X$, substitute

$$\phi_f^M \to \phi_2^M, \qquad \phi_f^\Gamma \to \phi_2^\Gamma, \tag{151}$$

in the expressions for the decay widths (73) (via Eq. (75) for the coefficients $c^\pm$), and the indirect CP asymmetries δc_f (79). However, in contrast to the SCS decays, the misalignments are entirely negligible, cf. (135).

C. CF/DCS decays to K^0X, $\bar K^0X$

In CF/DCS decays to K^0X, $\bar K^0X$, the final state dependent phases for $f = \pi^+\pi^-X$ are replaced by the theoretical phases via the substitutions,

$$\phi_f^{M,\Gamma} \to \phi_2^{M,\Gamma} + 2\epsilon_I + \left|\frac{\lambda_b}{\lambda_s}\right| \sin\gamma, \tag{152}$$

in the widths (87), (88) (via Eq. (90) for the coefficients $c_f^\pm$, $c_{\bar f}^\pm$), and in the indirect CP asymmetries δc_f, $\delta c_{\bar f}$ (92). The sum of the last two terms in (152) equals the misalignment $\delta\phi_f$ (144), up to negligible corrections lying an order of magnitude below our SM estimates of $\phi_2^{M,\Gamma}$, cf. (142), (143), (148).

At LHCb, the bulk of observed $K^0/\bar K^0 \to \pi^+\pi^-$ decays take place within a time interval[9] $t' \lesssim \tau_S/3$, while at Belle-II they can be detected over far longer time intervals,[10] e.g., $t' \lesssim O(10\tau_S)$. This has important consequences for the impact of ϵ_K on the CP asymmetries, e.g., in $D^0 \to K_S\pi^+\pi^-$ decays, which we discuss below.

The total time dependent CP asymmetries, following from (87), (88), (92), (93), can be expressed (up to an overall normalization factor) as

$$\begin{aligned}\Gamma_f - \bar\Gamma_{\bar f} = {} & -2e^{-\tau}|\bar A_{+-}|^2|A_{\bar K^0X}|^2\{2\epsilon_R F_0(t') \\ & + \sqrt{R_f}\tau[2\epsilon_I(x_{12}\cos\Delta_f + y_{12}\sin\Delta_f)F_1(t') \\ & + (x_{12}\cos\Delta_f \sin\tilde\phi_2^M + y_{12}\sin\Delta_f\sin\tilde\phi_2^\Gamma)e^{-\Gamma_S t'}]\},\end{aligned} \tag{153}$$

and

$$\begin{aligned}\Gamma_{\bar f} - \bar\Gamma_f = {} & -2e^{-\tau}|\bar A_{+-}|^2|A_{\bar K^0X}|^2\{2\epsilon_R F_0(t') \\ & + \sqrt{R_f}\tau[2\epsilon_I(x_{12}\cos\Delta_f - y_{12}\sin\Delta_f)F_1(t') \\ & + (x_{12}\cos\Delta_f \sin\tilde\phi_2^M - y_{12}\sin\Delta_f\sin\tilde\phi_2^\Gamma)e^{-\Gamma_S t'}]\},\end{aligned} \tag{154}$$

where, for convenience, we have introduced the phase

$$\tilde\phi_2^{M,\Gamma} \equiv \phi_2^{M,\Gamma} + |\lambda_b/\lambda_s|\sin\gamma. \tag{155}$$

The CKM term in (155) is $\approx 6.6 \times 10^{-4}$. The functions F_0, F_1 satisfy,

$$F_0(t) = -e^{-\Gamma_S t} + e^{-\Gamma_K t}\left(\cos\Delta m_K t + \frac{\epsilon_I}{\epsilon_R}\sin\Delta m_K t\right),$$

$$F_1(t) = e^{-\Gamma_S t} - e^{-\Gamma_K t}\left(\cos\Delta m_K t - \frac{\epsilon_R}{\epsilon_I}\sin\Delta m_K t\right). \tag{156}$$

[9]We thank Marco Gersabek for correspondence on this point.

[10]We thank David Cinabro for correspondence on this point.

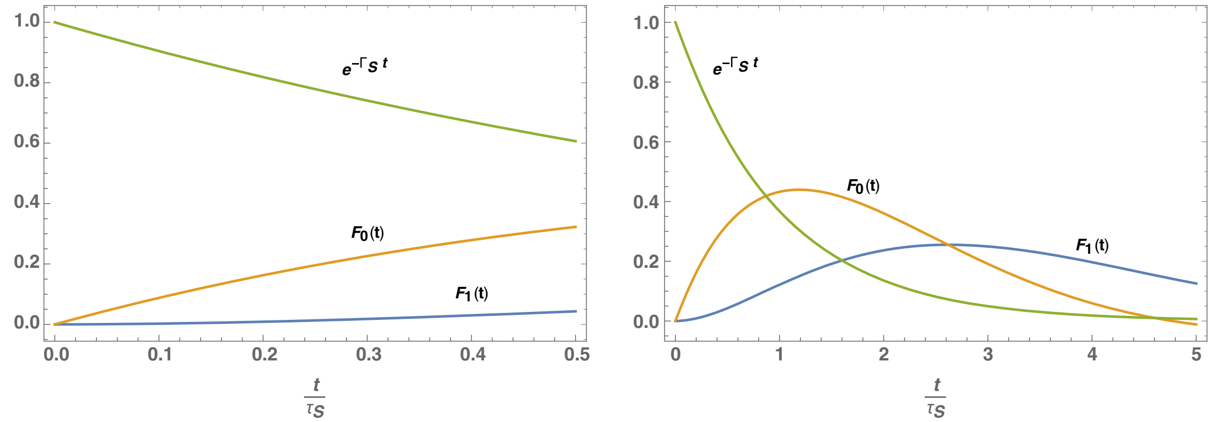

FIG. 1. The functions $F_0(t)$, $F_1(t)$, and $\exp[-\Gamma_S t]$, plotted over a short time interval of relevance to LHCb (left), and a longer time interval of relevance to Belle-II (right), cf. (153)–(156).

Note that the ratio $\epsilon_I/\epsilon_R = 1$, up to a small $\approx 5\%$ correction, cf. (29). Negligible CP asymmetries entering at $O(\tau^2)$ have not been included in (153), (154). Dividing by the sums over the CP conjugate decay widths yields the normalized time dependent CP asymmetries,

$$\frac{\Gamma_f - \bar{\Gamma}_{\bar{f}}}{\Gamma_f + \bar{\Gamma}_{\bar{f}}} = -\{2\epsilon_R e^{\Gamma_S t'} F_0(t') + \sqrt{R_f}\tau[2\epsilon_I(x_{12}\cos\Delta_f + y_{12}\sin\Delta_f)e^{\Gamma_S t'}F_1(t') + (x_{12}\cos\Delta_f \sin\tilde{\phi}_2^M + y_{12}\sin\Delta_f \sin\tilde{\phi}_2^\Gamma)]\}, \tag{157}$$

and

$$\frac{\Gamma_{\bar{f}} - \bar{\Gamma}_f}{\Gamma_{\bar{f}} + \bar{\Gamma}_f} = -\{2\epsilon_R e^{\Gamma_S t'} F_0(t') + \sqrt{R_f}\tau[2\epsilon_I(x_{12}\cos\Delta_f - y_{12}\sin\Delta_f)e^{\Gamma_S t'}F_1(t') + (x_{12}\cos\Delta_f \sin\tilde{\phi}_2^M - y_{12}\sin\Delta_f \sin\tilde{\phi}_2^\Gamma)]\}. \tag{158}$$

The function F_0 is associated with direct CPV via integration over τ, and agrees with the expression obtained in [40]. The functions F_1 and $e^{-\Gamma_S t'}$ are associated with the contributions of ϵ_K and $\phi_2^{M,\Gamma}$ to the CPVINT asymmetries, respectively. In Fig. 1, we plot the three functions over a short time interval of relevance to LHCb, and a longer time interval of relevance to Belle-II. Over the entire timescale for observed K^0's at LHCb, e.g., $t' \lesssim 0.5\tau_S$, the function F_1 undergoes a remarkable cancelation down to the few percent level, while $e^{-\Gamma_S t'} = O(1)$. Thus, at LHCb, the contributions of ϵ_K to the CPVINT asymmetries are highly suppressed compared to those of $\phi_2^{M,\Gamma}$ (recall that $\phi_2^{M,\Gamma} \sim \epsilon_{I,R}$ in the SM).

The cancellation in F_1 at short times takes place between the contributions to CPVINT from $K_L - K_S$ interference [$\delta b_{f,\bar{f}}$, $\delta d_{f,\bar{f}}$ in (93)], and from the ϵ_I term in $\phi_f^{M,\Gamma}$ (144) [via $\delta c_{f,\bar{f}}$ in (92)]. Thus, for simplicity, analyses of CPVINT in $D^0 \to K_{S,L}\pi^+\pi^-$ decays at LHCb could omit a fit to the interference terms [$\propto e^{-\Gamma_K t'}\tau$ in (87), (88)], if they substitute

$$\phi_f^{M,\Gamma} \to \phi_2^{M,\Gamma} + |\lambda_b/\lambda_s| \sin\gamma, \tag{159}$$

rather than (152). In contrast, over the longer K^0 decay timescales that can be explored at Belle-II, the cancelation in F_1 subsides, and ϵ_K ultimately dominates the CPVINT asymmetries in the SM, cf. Fig. 1 (right). Thus, Belle-II CPVINT analyses must fit for $K_L - K_S$ interference and employ the substitutions in (152), in order to extract $\phi_2^{M,\Gamma}$. Finally, the function F_0 undergoes some cancelation at small time intervals, e.g., $t' \lesssim \tau_S/3$, leading to moderate suppression of direct CPV at LHCb.

VI. CURRENT STATUS AND PROJECTIONS

We perform two global analyses of the current experimental data, collected in Table I, in order to assess the current sensitivity to the phases ϕ_2^M and ϕ_2^Γ. (The $x_{\rm CP}$, $y_{\rm CP}$, Δx, Δy entries in Tables I and III correspond to $K_S\pi^+\pi^-$). We also report on future projections.

A. Superweak limit

Until recently, fits to measurements of indirect CPV were sensitive to values of ϕ_{12} down to the 100 mrad level. This level of precision probed for large short-distance NP effects. In particular, the effects of weak phases in the subleading decay amplitudes could be safely neglected in the indirect CPV observables. In this limit, referred to as the superweak limit, a nonvanishing ϕ_{12} would be entirely due to short-distance NP in M_{12}, with the CPVINT phases satisfying

TABLE I. Experimental data used in the analysis, mostly from ref. [6]. Asymmetric errors have been symmetrized.

Observable	Value	Correlation Coeff.					Reference
y_{CP}	$(0.72 \pm 0.11)\%$						[41–48]
A_Γ	$(-0.031 \pm 0.020)\%$						[45,47,49–52]
x	$(0.53 \pm 0.19 \pm 0.06 \pm 0.07)\%$	1	0.054	−0.074	−0.031		[53]
y	$(0.28 \pm 0.15 \pm 0.05 \pm 0.05)\%$		1	0.034	−0.019		[53]
$\lvert q/p \rvert$	$(0.91 \pm 0.16 \pm 0.05 \pm 0.06)$			1	0.044		[53]
ϕ	$(-6 \pm 11 \pm 3 \pm 4)°$				1		[53]
x_{CP}	$(0.27 \pm 0.16 \pm 0.04)\%$	1	−0.17	0.04	−0.02		[6]
y_{CP}	$(0.74 \pm 0.36 \pm 0.11)\%$		1	−0.03	0.01		[6]
Δx	$(-0.053 \pm 0.07 \pm 0.022)\%$			1	−0.13		[6]
Δy	$(0.06 \pm 0.16 \pm 0.03)\%$				1		[6]
x	$(0.16 \pm 0.23 \pm 0.12 \pm 0.08)\%$	1	0.0615				[54]
y	$(0.57 \pm 0.20 \pm 0.13 \pm 0.07)\%$	0.0615	1				[54]
R_M	$(0.0130 \pm 0.0269)\%$						[55–59]
$(x^2 + y^2)/4$	$(0.0048 \pm 0.0018)\%$						[60]
$(x'_+)_{K\pi\pi}$	$(2.48 \pm 0.59 \pm 0.39)\%$	1	−0.69				[61]
$(y'_+)_{K\pi\pi}$	$(-0.07 \pm 0.65 \pm 0.50)\%$	−0.69	1				[61]
$(x'_-)_{K\pi\pi}$	$(3.50 \pm 0.78 \pm 0.65)\%$	1	−0.66				[61]
$(y'_-)_{K\pi\pi}$	$(-0.82 \pm 0.68 \pm 0.41)\%$	−0.66	1				[61]
R_D	$(0.533 \pm 0.107 \pm 0.045)\%$	1	0	0	−0.42	0.01	[62]
x^2	$(0.06 \pm 0.23 \pm 0.11)\%$	0	1	−0.73	0.39	0.02	[62]
y	$(4.2 \pm 2 \pm 1)\%$	0.	−0.73	1	−0.53	−0.03	[62]
$\cos\delta_{K\pi}$	$(0.84 \pm 0.2 \pm 0.06)$	−0.42	0.39	−0.53	1	0.04	[62]
$\sin\delta_{K\pi}$	$(-0.01 \pm 0.41 \pm 0.04)$	0.01	0.02	−0.03	0.04	1	[62]
R_D	$(0.3030 \pm 0.0189)\%$	1	0.77	−0.87			[63]
$(x'_+)^2_{K\pi}$	$(-0.024 \pm 0.052)\%$	0.77	1	−0.94			[63]
$(y'_+)_{K\pi}$	$(0.98 \pm 0.78)\%$	−0.87	−0.94	1			[63]
A_D	$(-2.1 \pm 5.4)\%$	1	0.77	−0.87			[63]
$(x'_-)^2_{K\pi}$	$(-0.020 \pm 0.050)\%$	0.77	1	−0.94			[63]
$(y'_-)_{K\pi}$	$(0.96 \pm 0.75)\%$	−0.87	−0.94	1			[63]
R_D	$(0.364 \pm 0.018)\%$	1	0.655	−0.834			[64]
$(x'_+)^2_{K\pi}$	$(0.032 \pm 0.037)\%$	0.655	1	−0.909			[64]
$(y'_+)_{K\pi}$	$(-0.12 \pm 0.58)\%$	−0.834	−0.909	1			[64]
A_D	$(2.3 \pm 4.7)\%$	1	0.655	−0.834			[64]
$(x'_-)^2_{K\pi}$	$(0.006 \pm 0.034)\%$	0.655	1	−0.909			[64]
$(y'_-)_{K\pi}$	$(0.20 \pm 0.54)\%$	−0.834	−0.909	1			[64]
R_D	$(0.351 \pm 0.035)\%$	1	−0.967	0.900			[65]
$(y'_{\mathrm{CPA}})_{K\pi}$	$(0.43 \pm 0.43)\%$	−0.967	1	−0.975			[65]
$(x'_{\mathrm{CPA}})^2_{K\pi}$	$(0.008 \pm 0.018)\%$	0.900	−0.975	1			[65]
R_D	$(0.3454 \pm 0.0028 \pm 0.0014)\%$	1	−0.883	0.745	−0.883	0.749	[66]
$(y'_+)_{K\pi}$	$(0.501 \pm 0.048 \pm 0.029)\%$		1	−0.944	0.758	−0.644	[66]
$(x'_+)^2_{K\pi}$	$(6.1 \pm 2.6 \pm 1.6)10^{-5}$			1	−0.642	0.545	[66]
$(y'_-)_{K\pi}$	$(0.554 \pm 0.048 \pm 0.029)\%$				1	−0.946	[66]
$(x'_-)^2_{K\pi}$	$(1.6 \pm 2.6 \pm 1.6)10^{-5}$					1	[66]

TABLE II. Results of fits to the current and future D mixing data within the superweak and approximate universality frameworks, where the phases are defined in Eq. (107).

	Superweak—current		Approx. univ.—current		Approx. univ.—future
Parameter	68% prob.	95% prob.	68% prob.	95% prob.	Estimated 68% prob.
$10^3 x_{12}$	3.6 ± 1.1	[1.3, 5.7]	3.7 ± 1.2	[1.3, 5.9]	± 0.017
$10^4 y_{12}$	60.3 ± 5.7	[49, 73]	59.6 ± 5.6	[49, 71]	± 0.19
$10^2 \phi_2^M$ [rad]	-0.5 ± 2.2	$[-6.1, 4.7]$	-1.0 ± 2.9	$[-10.0, 5.7]$	± 0.12
$10^2 \phi_2^\Gamma$ [rad]	0	0	-3.2 ± 9.9	$[-23, 16]$	± 0.17
$10^2 \phi_{12}$ [rad]	-0.5 ± 2.2	$[-6.1, 4.7]$	2.6 ± 9.7	$[-20, 22]$	± 0.21
$10^3 x$	3.6 ± 1.1	[1.3, 5.8]	3.7 ± 1.2	[1.3, 6.0]	± 0.017
$10^4 y$	60.3 ± 5.7	[49, 73]	59.5 ± 5.6	[48, 71]	± 0.19
$10^3(\lvert q/p\rvert - 1)$	-2.3 ± 9.0	$[-21, 16]$	8 ± 41	$[-73, 99]$	± 0.92
$10^2 \phi_2$ [rad]	0.12 ± 0.51	$[-0.96, 1.26]$	2.5 ± 7.2	$[-13, 17]$	± 0.13

$$\phi_f^M = \phi_2^M = \phi_{12}, \qquad \phi_f^\Gamma = 0, \qquad \phi_{\lambda_f} = \phi_2. \tag{160}$$

For example, the expression for the SCS time dependent CP asymmetry in (63) would reduce to[11]

$$\Delta Y_f = -\eta_{CP}^f x_{12} \sin \phi_2^M. \tag{161}$$

Thus, the phase ϕ_2^M (or ϕ_{12}) would be the only source of indirect CPV. Consequently, CPVMIX and CPVINT would be related as [3–5],

$$\tan 2\phi_2 \approx -\frac{x_{12}^2}{x_{12}^2 + y_{12}^2} \sin 2\phi_2^M, \tag{162}$$

or, equivalently, as

$$\tan \phi_2 \approx \left(1 - \left|\frac{q}{p}\right|\right) \frac{x}{y}, \tag{163}$$

where (162) is the superweak limit of (46).

Superweak fits to the data are highly constrained, given that there is only one CPV parameter controlling all of indirect CPV. The second column in Table II contains the results of our fit to the mixing parameters with current data in the superweak framework. We see that sensitivity to ϕ_2^M is ≈ 22 mrad at 1σ, and ≈ 54 mrad at 95% probability, while sensitivity to ϕ_2 is ≈ 5 mrad at 1σ, and ≈ 11 mrad at 95% probability.[12] Some superweak correlation plots are also shown in the first row of Fig. 2. The Heavy Flavor Averaging Group (HFLAV) [6] has obtained similar results,

$$\phi_2^M = -0.004 \pm 0.016(1\sigma), \qquad \phi_2 = 0.001 \pm 0.005(1\sigma). \tag{164}$$

Comparison with the SM ranges (114) implies that an order of magnitude window for NP remains, at 95% probability, in the CPVINT phases.

B. Approximate universality fits

It is encouraging that the 1σ error on ϕ_2 in the superweak fit (5 mrad), and the U-spin based SM estimates for $\phi_2^{M,\Gamma}$, ϕ_{12} in (114), (115) are only about a factor of two apart. However, this means that the approximate universality parametrization is advisable moving forward. Inspection of the relations between ϕ_2 and $\phi_2^{M,\Gamma}$ in (110), (111), reinforces this conclusion. Approximate universality fits are less constrained, given that they employ two CPV parameters rather than a single one to describe indirect CPV. Hopefully, this will be overcome in the high statistics LHCb and Belle-II precision era, and SM sensitivity in $\phi_2^{M,\Gamma}$ will be achieved. This possibility is assessed below.

We remark that an approximate universality fit for any two of the phases ϕ_2^M, ϕ_2^Γ, and ϕ_{12} is equivalent to a (traditional) two-parameter fit for ϕ_2 and $|q/p|$, with translations provided by (45), (109)–(111). General formulas for the decay widths, given in terms of ϕ_{λ_f} and $|q/p|$, can be converted to approximate universality formulas which depend on ϕ_2 and $|q/p|$, via the substitutions $\phi_{\lambda_f} \to \phi_2$ (SCS), $\phi_{\lambda_f} \to \phi_2$ (CF/DCS $K^\pm X$), $\phi_{\lambda_f} \to \phi_2 - 2\epsilon_I - |\lambda_b/\lambda_s| \sin\gamma$ (CF/DCS $K^0 X$, general), and $\phi_{\lambda_f} \to \phi_2 - |\lambda_b/\lambda_s| \sin\gamma$ (CF/DCS $K^0 X$, LHCb). These are analogous to the substitutions for $\phi_f^{M,\Gamma}$ in (149), (151), (152), and (159), respectively.

We begin with a fit to the current data, cf. Table I, for the phases ϕ_2^M and ϕ_2^Γ. We implement the substitutions for $\phi_f^{M,\Gamma}$ given in (149), (151), (159), and employ the expression for ΔY_f in (150). The $K_L - K_S$ interference terms in the

[11]In the superweak limit, the effects of weak phases in the SCS decay amplitudes are neglected in time dependent CP asymmetries, but they are kept in time integrated ones, where they are not suppressed by x_{12}, y_{12}.

[12]Smaller errors for ϕ_2 than ϕ_2^M in the superweak fit can be traced to the small central value of the prefactor in (162), $x_{12}^2/(x_{12}^2 + y_{12}^2) \approx 0.26$.

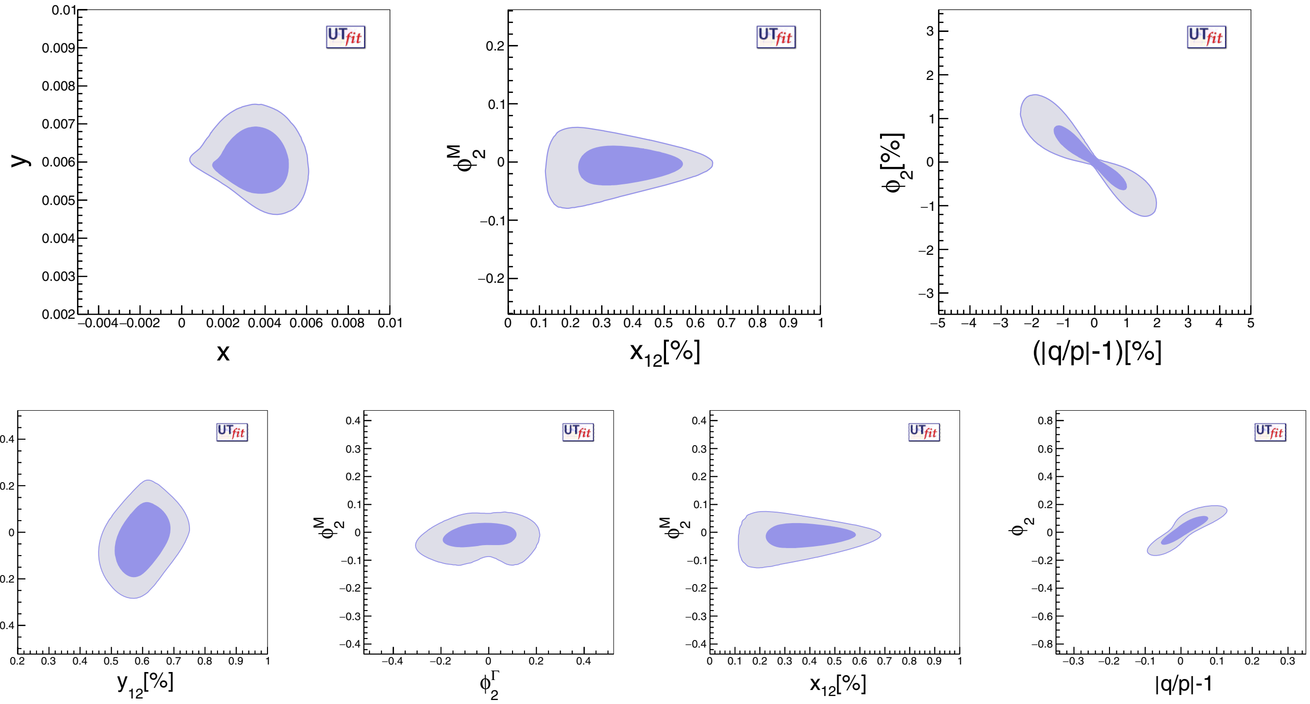

FIG. 2. P.d.f.'s for mixing parameters in the superweak (first row) and approximate universality scenarios, see text. Darker (lighter) regions correspond to 68% (95%) probability. Notice the order-of-magnitude difference in the scale of the rightmost plots.

$D \to K_{S,L}\pi^+\pi^-$ decay widths (87), (88) are ignored, as in the experimental analyses. As explained in Sec. V C, this does not affect the determination of $\phi_2^{M,\Gamma}$ at LHCb, provided that the substitution in (159) is employed. For the Belle $D^0 \to K_{S,L}\pi^+\pi^-$ analysis [53], omission of $K_L - K_S$ interference is not an issue, given its experimental precision.

The results of the approximate universality fit appear in the third column of Table II, and in the second row of correlation plots in Fig. 2. It is interesting to notice that the error on ϕ_2^M is about a factor of three smaller than the error on ϕ_2^Γ, and is similar to the corresponding superweak error. This can be traced, in part, to the observable $A_\Gamma = -\Delta Y_f$, for $f = \pi^+\pi^-, K^+K^-$. It has a relatively small experimental error, and it only depends on the product $x_{12}\sin\phi_2^M$ in the fit [compare (150), (161)]. However, both ϕ_2 and $|q/p| - 1$ are determined with order of magnitude larger uncertainties in the approximate universality framework, due to their dependence on both ϕ_2^M and ϕ_2^Γ.

In the future, as SM sensitivity in CPVINT is approached, a modified strategy will be appropriate. As discussed in Sec. IV C 1, significant and nonuniversal misalignment ratios $\delta\phi_f/\phi_2^{M,\Gamma}$ could manifest themselves in the SCS measurements, even though they are formally $O(\epsilon)$ in U-spin breaking. In contrast, the misalignments in CF/DCS decays are either negligible ($K^\pm X$), or known to very good approximation ($K^0 X, \bar{K}^0 X$), cf. Secs. IV C 2, IV C 3. Thus, at that this point one could simply drop the SCS observables from the global fits to ϕ_2^M, ϕ_2^Γ. Alternatively, one could only include the SCS final states $\pi^+\pi^-$ and K^+K^- in the global fits, via their averaged time dependent CP asymmetry A_Γ, thus taking advantage of the $O(\epsilon^2)$ suppression of the averaged QCD penguin pollution, cf. (133).

It is interesting to point out that simultaneous knowledge of $\phi_2^{M,\Gamma}$ from CF/DCS decays, and of the direct CP asymmetries in the SCS decays could be used to determine the relative magnitudes and strong phases of the corresponding subleading SCS decay amplitudes in the SM, i.e., r_f and δ_f. This can be seen for CP eigenstate final states via (51) with $\phi_f = \gamma$, (63) with $\phi_f^M = \phi_2^M + \delta\phi_f$, and (126), and similarly for non-CP eigenstate final states. Thus,

TABLE III. Estimated uncertainties on mixing parameters from CF/DCS decays in the LHCb Phase II Upgrade. Correlations from current results have been used where available.

$\delta(x_{\rm CP})$ 3.8×10^{-5}	$\delta(y_{\rm CP})$ 8.6×10^{-5}	$\delta(\Delta x)$ 1.7×10^{-5}	$\delta(\Delta y)$ 3.8×10^{-5}	[11] scaled by luminosity
$\delta(y'_+)_{K\pi}$ 3.2×10^{-5}	$\delta(y'_-)_{K\pi}$ 3.2×10^{-5}	$\delta(x'_+)^2_{K\pi}$ 1.7×10^{-6}	$\delta(x'_-)^2_{K\pi}$ 1.7×10^{-6}	[66] scaled by luminosity
$\delta(x_{K\pi\pi\pi})$ 2×10^{-5}	$\delta(y_{K\pi\pi\pi})$ 2×10^{-5}	$\delta(\lvert q/p\rvert_{K\pi\pi\pi})$ 2×10^{-3}	$\delta(\phi_{K\pi\pi\pi})$ $0.1°$	[67]

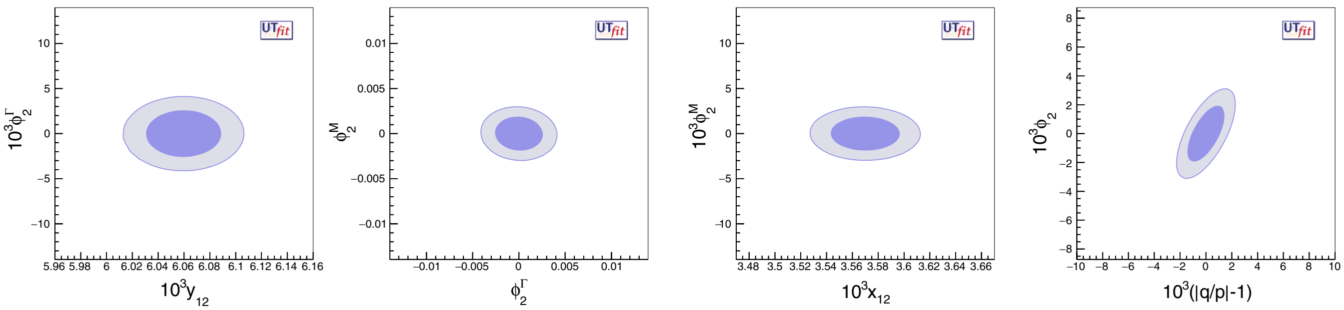

FIG. 3. P.d.f.'s for mixing parameters in the approximate universality future scenario, see text. Darker (lighter) regions correspond to 68% (95%) probability.

important information on the QCD anatomy of these decays could be obtained.

To illustrate the potential for probing the SM in the precision era, we use the (naïvely) estimated experimental sensitivities reported in Table III for the LHCb Phase II Upgrade era, for three decay modes: $D^0 \to K_{S,L}\pi^+\pi^-$, $K^+\pi^-$, and $K^+\pi^-\pi^+\pi^-$. We caution that scaling the errors on the individual measurements purely based on the expected statistics may be optimistic. The results of the fit are presented in the rightmost columns in Table II and in Fig. 3 (including the SCS observable A_Γ leads to marginal improvement in the sensitivity to ϕ_2^M in Phase II). They suggest that SM sensitivity to $\phi_2^{M,\Gamma}$ may be achievable, particularly if these phases lie on the high end of our U-spin based estimates. Moreover, additional input from Belle-II indirect CPV measurements at 50 ab^{-1} [68], e.g., for the decays $D^0 \to K_{S,L}\pi^+\pi^-$, $K^+\pi^-$, $K^+\pi^-\pi^0$, and A_Γ, may improve the sensitivity.

VII. DISCUSSION

In this paper we have developed the description of *CP* violation in $D^0 - \bar{D}^0$ mixing in terms of the final state dependent dispersive and absorptive weak phases ϕ_f^M and ϕ_f^Γ. They govern *CP* violation in the interference between decays with and without dispersive mixing, and with and without absorptive mixing, respectively. The expressions for the time dependent decay widths and *CP* asymmetries undergo extensive simplifications compared to the familiar parametrization in terms of $|q/p|$ and ϕ_{λ_f} (translations are provided), and become physically transparent. For instance, their dependence on the strong phases in the decay amplitudes, and the *CP*-even dispersive mixing phase $\pi/2$, are easily understood. This understanding extends to the strong phases of the subleading decay amplitudes, e.g., those responsible for direct *CP* violation in $D^0 \to K^+K^-, \pi^+\pi^-$. An important consequence is that the time dependent *CP* asymmetries for decays to *CP* eigenstate final states, e.g., $f = K^+K^-, \pi^+\pi^-$, depend on ϕ_f^M (dispersive *CP* violation), but not on ϕ_f^Γ (absorptive *CP* violation). Conversely, the ϕ_f^Γ can only be probed in decays to non-*CP* eigenstate final states, e.g., the CF/DCS final states $f = K^+\pi^-, K_{S,L}\pi^+\pi^-$.

We have applied the dispersive/absorptive formalism to the three classes of decays which contribute to $D^0 - \bar{D}^0$ mixing, (i) CF/DCS decays to $K^\pm X$, (ii) CF/DCS decays to K^0X, $\bar{K}^0X$, and (iii) SCS decays (both *CP* eigenstate and non-*CP* eigenstate final states). Derivations and expressions have been provided for the time dependent decay widths and asymmetries in all three cases. The CF/DCS decays to K^0X, $\bar{K}^0X$ require special care due to the effects of CPV in $K^0 - \bar{K}^0$ mixing. Moreover, their widths depend on two elapsed time intervals, the D and K decay times, following their respective production. The Appendix contains expressions for a selection of time-integrated *CP* asymmetries, demonstrating that they can also be used to separately measure ϕ_f^M and ϕ_f^Γ.

Measurements of the final state dependent phases ϕ_f^M and ϕ_f^Γ ultimately determine a pair of intrinsic mixing phases ϕ_2^M and ϕ_2^Γ, respectively, cf. (107). The latter are the arguments, in the complex mixing plane, of the total dispersive and absorptive mixing amplitudes M_{12} and Γ_{12}, relative to their dominant $\Delta U = 2$ (U-spin) components. The latter are responsible for the neutral D meson mass and width differences. The intrinsic mixing analog (ϕ_2) of the final state dependent phenomenological phases ϕ_{λ_f}, is similarly defined as the argument of q/p relative to the $\Delta U = 2$ mixing amplitude. The U-spin decomposition of the dispersive and absorptive mixing amplitudes yields the SM estimates $\phi_2^M, \phi_2^\Gamma = O(0.2\%)$, cf. (112)–(115), (119), with ϕ_2 of same order. We also obtain an upper bound on the absorptive phase in the SM, $|\phi_2^\Gamma| < 0.005$ [30], when taking $\Delta\Gamma_D$ equal to its measured central value, and conservatively assuming that a certain U-spin breaking parameter satisfies $\epsilon_1 < 1$, cf. (117), (119).

The intrinsic mixing phases are experimentally accessible due to *approximate universality*. In particular, we have shown that there is minimal uncontrolled final-state dependent pollution from the decay amplitudes in the measured phases ϕ_f^M, ϕ_f^Γ:

(i) For the CF/DCS $K^\pm X$ final states, e.g., $K^+\pi^-$, in the SM and in extensions with negligible new weak phases in these decays, the difference $\delta\phi_f$ between $\phi_2^{M,\Gamma}$ and $\phi_f^{M,\Gamma}$ is known, final state independent, and entirely negligible, i.e., it is $O(\lambda_b^2/\lambda_d^2) \sim 10^{-6}$, cf. (134), (135).

(ii) For the CF/DCS K^0X final states, e.g., $K_{S,L}\pi^+\pi^-$, in the SM and under the same NP assumptions, there are two contributions to the misalignments, $\delta\phi_f$: a small incalculable final state dependent one of $O(2\theta_C^2 \mathrm{Im}[\epsilon_K]) \sim 0.1\phi_2^{M,\Gamma}$, due to the subleading DCS amplitudes, and a precisely known one of $O(2\mathrm{Im}[\epsilon_K]) \sim \phi_2^{M,\Gamma}$ which can be subtracted from the measured values of $\phi_f^{M,\Gamma}$, cf. (148).

(iii) For the SCS decays, e.g., $f = K^+K^-, \pi^+\pi^-$, there is uncontrolled final state dependent QCD penguin pollution. In the SM, and for extensions with CP-odd QCD penguins of same order, the misalignments satisfy $\delta\phi_f/\phi_2^{M,\Gamma} = O(\epsilon)$ in U-spin breaking. This could be sizable for certain decays. A U-spin based estimate, taking into account ΔA_{CP}, yields the representative value $\epsilon \sim 0.4$, or $\delta\phi_{K^+K^-}, \delta\phi_{\pi^+\pi^-} = O(0.4)\phi_2^{M,\Gamma}$, cf. (129)–(132). Fortunately, the average over $\phi_{K^+K^-}^{M,\Gamma}$ and $\phi_{\pi^+\pi^-}^{M,\Gamma}$ differs from $\phi_2^{M,\Gamma}$ by $O(\epsilon^2)$.

Expressions for the time dependent decay widths in the approximate universality parametrization, i.e., in terms of ϕ_2^M, ϕ_2^Γ, have been discussed in detail for the three classes of decays, cf. Sec. V. Our results for the K^0X final states are particularly noteworthy. On the timescale of sequential K^0 decays at LHCb ($t \lesssim 0.5\tau_S$), the effect of kaon CP violation on the time dependent CP asymmetries (due to $K_LX - K_SX$ interference, and an $\mathrm{Im}[\epsilon_K]$ component in $\phi_f^{M,\Gamma}$) undergoes a cancelation at the few percent level. Thus, to very good approximation, LHCb analyses of these modes can neglect the effects of kaon CP violation in measurements of $\phi_2^{M,\Gamma}$ from the time dependent CP asymmetries. In contrast, over the longer K^0 decay timescales that can be explored at Belle-II, the cancelation subsides, and ϵ_K ultimately dominates the time dependent CP asymmetries. Thus, Belle-II analyses must fit for $K_L - K_S$ interference effects, and account for $\mathrm{Im}[\epsilon_K]$ in the extraction of $\phi_2^{M,\Gamma}$.

In the future, the values of $\phi_2^{M,\Gamma}$ obtained from the CF/DCS decays will allow a determination of the misalignments, $\delta\phi_f$, in the SCS decays. In combination with measurements of the SCS direct CP asymmetries, a_f^d, it will be possible to determine the anatomy of the QCD penguins in the SM, e.g., for $f = K^+K^-,\ \pi^+\pi^-$. In particular, taking the SM value γ for the weak phases of the penguin amplitudes relative to the dominant "tree" amplitudes, it will be possible to measure their relative magnitudes and strong phases. This would provide an important test of QCD dynamics, if lattice measurements of these quantities become available.

Past fits to the mixing data were sensitive to values of $\phi_{12} = \arg[M_{12}/\Gamma_{12}] = \phi_2^M - \phi_2^\Gamma$ down to the 100 mrad level. This level of precision probed for large short-distance new physics contributions. Thus, the effects of weak phases in the subleading decay amplitudes could be safely neglected in the indirect CPV observables. In this limit, referred to as superweak, the mixing phases satisfy $\phi_{12} = \phi_2^M$, and $\phi_2^\Gamma = 0$. We have carried out a fit to the current data set in this limit, yielding $\phi_2^M = (-0.5 \pm 2.2)\%$ at 1σ, consistent with the HFLAV fit result, and corresponding to an $O(10)$ window for New Physics at 2σ.

The approximate universality fit is less constrained, given the description of indirect CP violation in terms of two phases, ϕ_2^M and ϕ_2^Γ, rather than just one. Interestingly, in this case, our errors for ϕ_2^M ($\approx$29 mrad) are similar to the superweak fit result, and about a factor of three smaller than the errors for ϕ_2^Γ ($\approx$99 mrad). This is due, in part, to the observable $A_\Gamma = -\Delta Y_f$ ($f = \pi^+\pi^-,\ K^+K^-$), which depends on ϕ_2^M but not on ϕ_2^Γ, and has a relatively small experimental error. The phenomenologically motivated phase ϕ_2 is a weighted sum over ϕ_2^M and ϕ_2^Γ, where the weights are equal to the leading CP averaged dispersive and absorptive mixing probabilities, respectively, cf. (110). This explains why the error on ϕ_2 ($\approx$72 mrad) is similar to the error on ϕ_2^Γ.

The U-spin based estimates of ϕ_2^M and ϕ_2^Γ imply that probing the SM will require a precision of a few mrad or better for both phases. Given the large theoretical uncertainties, a null result as this sensitivity is approached would effectively close the window for new physics in charm indirect CP violation. Alternatively, the most likely origin for a significantly enhanced signal would be CP violating short distance new physics, yielding $\phi_2^M \gg \phi_2^\Gamma$, with the latter given by its SM value. A second possibility, light CP violating new physics, would enter both the dispersive and absorptive mixing amplitudes via new D^0 decay modes, likely enhancing both ϕ_2^M and ϕ_2^Γ. This appears unlikely, given the upper bounds on exotic D^0 decay rates. For instance, for invisible D^0 decays, the upper bound on the branching ratio, $\mathrm{Br_{inv}} < 9.4 \times 10^{-5}$ (90% CL) [9], constrains the invisible contribution to ϕ_2^Γ as $\delta\phi_2^\Gamma \lesssim \mathrm{Br_{inv}}/\theta_C^2 \sim 0.2\%$, i.e., the upper bound lies at the SM level (before taking into account additional suppression due to the relative magnitudes of the interfering invisible decay amplitudes, and their weak and strong phase differences). Moreover, the upper bound on contributions from $D^0 \to K^0+$ invisibles is about a factor of 30 smaller.[13]

Finally, based on available LHCb Phase II projections for the decays $D^0 \to K_{S,L}\pi^+\pi^-$, $K^+\pi^-$, $K^+\pi^-\pi^+\pi^-$, and A_Γ,

[13]An upper bound on $D^+ \to K^+ +$ invisibles, $\mathrm{Br}_{K^+ + \mathrm{inv}} < 8 \times 10^{-6}$ [69], yields $\delta\phi_2^\Gamma \lesssim (\mathrm{Br}_{K^+ + \mathrm{inv}}/\theta_C^2)(\Gamma_{D^+}/\Gamma_{D^0}) \sim 6.5 \times 10^{-5}$, well below the SM estimates, where we have assumed similar widths for the semi-invisible D^+ and D^0 decays.

we have estimated the precision that could be reached for $\phi_2^{M,\Gamma}$ in the upcoming high statistics charm era, using an approximate universality fit. Note that our results are intended to be illustrative, given that the LHCb phase II projections do not include systematic errors. The resulting 1σ errors for ϕ_2^M ($\approx$1.2 mrad) and ϕ_2^Γ ($\approx$1.7 mrad) suggest that sensitivity to $\phi_2^{M,\Gamma}$ in the SM may be achievable, particularly if these phases lie on the high end of the U-spin based estimates. Measurements of $\phi_2^{M,\Gamma}$ could one day become available on the lattice. Comparison with their measured values would provide the ultimate precision test for the SM origin of CP violation in charm mixing.

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APPENDIX: CPVINT PHASES ϕ_2^M, ϕ_2^Γ FROM TIME-INTEGRATED CP ASYMMETRIES

We give expressions for a few time integrated CP asymmetries, illustrating the possibility of determining the theoretical CPVINT phases purely from time-integrated decays. We begin with the tagged and untagged CP asymmetries for the CF/DCS final states $f = K^+\pi^-$, $\bar{f} = K^-\pi^+$ ($A_{\bar{f}}$, $\bar{A}_f$ are the DCS amplitudes):

$$A_{\rm CP}^{\rm tag,DCS(CF)} \equiv \frac{\int dt(\Gamma_{D^0(t)\to\bar{f}(f)} - \Gamma_{\bar{D}^0(t)\to f(\bar{f})})}{\int dt(\Gamma_{D^0(t)\to\bar{f}(f)} + \Gamma_{\bar{D}^0(t)\to f(\bar{f})})},$$
$$A_{\rm CP}^{\rm untag} \equiv \frac{\int dt(\Gamma_{D^0(t)\to\bar{f}} + \Gamma_{\bar{D}^0(t)\to\bar{f}} - \Gamma_{D^0(t)\to f} - \Gamma_{\bar{D}^0(t)\to f})}{\int dt(\Gamma_{D^0(t)\to\bar{f}} + \Gamma_{\bar{D}^0(t)\to\bar{f}} + \Gamma_{D^0(t)\to f} + \Gamma_{\bar{D}^0(t)\to f})}.$$

To obtain their dependence on the CPVINT phases, we must keep the subleading DCS amplitudes in (72), in analogy to the CF contributions in (73). Assuming no new weak phases in the CF/DCS decays as in the SM, hence no direct CPV, the amplitude ratios simplify as $R_f = 1/R_{\bar{f}} = R_f^{\pm}$, cf. (74). Thus, Eqs. (72) and (73) yield

$$\sqrt{R_f}A_{\rm CP}^{\rm tag,DCS} = x_{12}\sin\phi_f^M\cos\Delta_f - y_{12}\sin\phi_f^\Gamma\sin\Delta_f,$$
$$\frac{A_{\rm CP}^{\rm tag,CF}}{\sqrt{R_f}} = x_{12}\sin\phi_f^M\cos\Delta_f + y_{12}\sin\phi_f^\Gamma\sin\Delta_f. \tag{A1}$$

The absorptive and dispersive CPV phases are then readily separated as

$$\frac{A_{\rm CP}^{\rm tag,CF}}{\sqrt{R_f}} - \sqrt{R_f}A_{\rm CP}^{\rm tag,DCS} = -\frac{(1+R_f)A_{\rm CP}^{\rm untag}}{\sqrt{R_f}} = 2y_{12}\sin\phi_2^\Gamma\sin\Delta_f$$
$$\frac{A_{\rm CP}^{\rm tag,CF}}{\sqrt{R_f}} + \sqrt{R_f}A_{\rm CP}^{\rm tag,DCS} = 2x_{12}\sin\phi_2^M\cos\Delta_f, \tag{A2}$$

where Δ_f is the $K^+\pi^-$ strong phase, cf. (24). We have taken $\phi_f^{M,\Gamma} = \phi_2^{M,\Gamma}$, cf. (134), (135). Note that the untagged CP asymmetry is purely absorptive.

We end with the time integrated CP asymmetries for the SCS final states $f = \pi^+\pi^-, K^+K^-$:

$$A_{\rm CP,f}^{\rm SCS} \equiv \frac{\int dt(\Gamma_{D^0(t)\to f} - \Gamma_{\bar{D}^0(t)\to f})}{\int dt(\Gamma_{D^0(t)\to f} + \Gamma_{\bar{D}^0(t)\to f})}. \tag{A3}$$

We obtain the expression

$$A_{\rm CP,f}^{\rm SCS} = a_f^d + \frac{\langle t\rangle}{\tau_D}\Delta Y_f = a_f^d + \frac{\langle t\rangle}{\tau_D}(-x_{12}\sin\phi_f^M + y_{12}a_f^d), \tag{A4}$$

where $\langle t\rangle$ is the average (acceptance dependent) decay time of the D^0 mesons in the experimental sample. The ratio $\langle t\rangle/\tau_D$ is very close to 1 at the B factories; at LHCb, it exceeds 1 by about 5%–10% for the muon-tagged sample [33], while it is in the 1.7–1.8 range for the D^{*+}-tagged sample [70].[14] Recall that in the SM, for SCS decays,

$$\phi_f^M = \phi_2^M - a_f^d\cot\delta_f = \phi_2^M[1 + O(\epsilon)], \tag{A5}$$

where δ_f is the strong phase difference between the leading and subleading $D^0 \to f$ decay amplitudes, and a_f^d is the direct CP asymmetry, cf. (126). However, the average of ϕ_f^M over $f = K^+K^-, \pi^+\pi^-$ differs from ϕ_2^M by $O(\epsilon^2)$ in U-spin breaking, cf. (126), (129), (132).

The time integrated CP asymmetry difference $\Delta A_{CP} = A_{CP,K^+K^-} - A_{CP,\pi^+\pi^-}$ [33] can be expressed in terms of ϕ_2^M and the direct CP asymmetries as

[14] We thank T. Pajero for pointing this out to us.

$$\Delta A_{\rm CP} = a_K^d - a_\pi^d + \frac{\langle t_K\rangle + \langle t_\pi\rangle}{2\tau_D}(x_{12}[a_K^d \cot\delta_K - a_\pi^d \cot\delta_\pi] + y_{12}[a_K^d - a_\pi^d]) - \frac{\langle t_K\rangle - \langle t_\pi\rangle}{2\tau_D}(x_{12}[2\sin\phi_2^M - a_K^d\cot\delta_K - a_\pi^d\cot\delta_\pi] - y_{12}[a_K^d + a_\pi^d]), \tag{A6}$$

where the subscripts K and π refer to the K^+K^- and $\pi^+\pi^-$ final states, respectively. At LHCb the difference of the two average decay times satisfies $\langle t_K\rangle - \langle t_\pi\rangle \approx 0.12\tau_D$. The corrections to the first line in (A6) are negligible, as is well known. In particular, we find that the contribution proportional to the sum of the average decay times is of $O(x_{12}a_f^d, y_{12}a_f^d)$. The contribution proportional to the difference of decay times is of $O(0.1x_{12}\phi_2^M)$, given that $(a_K^d + a_\pi^d)$ and $(a_K^d \cot\delta_K + a_\pi^d \cot\delta_\pi)$ are formally of $O(\epsilon^2 \cdot \phi_2^M)$.

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