


Measurement of branching fraction and search for CP violation in $B \rightarrow \phi\phi K$

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We report the measurement of branching fractions and CP -violation asymmetries in $B \rightarrow \phi\phi K$ decays based on a 711 fb^{-1} data sample containing $772 \times 10^6 B\bar{B}$ events. The data were recorded at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. For $B^+ \rightarrow \phi\phi K^+$, the branching fraction and CP -violation asymmetry measured below the η_c threshold ($m_{\phi\phi} < 2.85 \text{ GeV}/c^2$) are $[3.43_{-0.46}^{+0.48}(\text{stat}) \pm 0.22(\text{syst})] \times 10^{-6}$ and $-0.02 \pm 0.11(\text{stat}) \pm 0.01(\text{syst})$, respectively. Similarly, the branching fraction obtained for $B^0 \rightarrow \phi\phi K^0$ below the η_c threshold is $[3.02_{-0.66}^{+0.75}(\text{stat}) \pm 0.20(\text{syst})] \times 10^{-6}$. We also measure the CP -violation asymmetry for $B^+ \rightarrow \phi\phi K^+$ within the η_c region ($m_{\phi\phi} \in [2.94, 3.02] \text{ GeV}/c^2$) to be $+0.12 \pm 0.12(\text{stat}) \pm 0.01(\text{syst})$.

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B -meson decays to three-body $\phi\phi K$ final states proceed via a $b \rightarrow s\bar{s}s$ loop (penguin) transition, which requires the creation of an additional $s\bar{s}$ pair. The same final state can also originate from the tree-level process $B \rightarrow \eta_c(\rightarrow \phi\phi)K$. Figure 1 shows the dominant Feynman diagrams that contribute to these decays. The interference between penguin and tree amplitudes is maximal when the $\phi\phi$ invariant mass lies close to the η_c mass ($m_{\phi\phi} \in [2.94, 3.02] \text{ GeV}/c^2$). No CP violation is expected from this interference, as the relative weak phase between the two amplitudes is $\arg(V_{tb}V_{ts}^*/V_{cb}V_{cs}^*) \approx 0$, where V_{ij} denote CKM matrix elements [1]. A potential new physics (NP) contribution to the loop, however, can introduce a nonzero CP -violating phase. In particular, the CP asymmetry can be as large as 40% in the presence of NP [2]. Thus, an observation of large CP violation in $B \rightarrow \phi\phi K$ would indicate the presence of physics beyond the Standard Model. In addition to being an NP probe, the decay is sensitive to the possible production of a glueball candidate near $2.3 \text{ GeV}/c^2$ that can subsequently decay to $\phi\phi$ [3]. We can also search for a structure at $2.35 \text{ GeV}/c^2$ observed in the $m_{\phi\phi}$ distribution in two-photon collisions [4] and dubbed the $X(2350)$.

Based on a 78 fb^{-1} data sample, Belle reported the first evidence for the decay with a branching fraction $\mathcal{B}(B^+ \rightarrow \phi\phi K^+) = [2.6_{-0.9}^{+1.1}(\text{stat}) \pm 0.3(\text{syst})] \times 10^{-6}$ [5] below the η_c threshold ($m_{\phi\phi} < 2.85 \text{ GeV}/c^2$) [6]. The result was consistent with the corresponding theory prediction, which lies in the range $(1.3 - 4.2) \times 10^{-6}$ [7,8]. The BABAR experiment performed a measurement of this decay using

their full dataset of $464 \times 10^6 B\bar{B}$ events [9]. The branching fraction obtained with the same $m_{\phi\phi}$ requirement was $\mathcal{B}(B^+ \rightarrow \phi\phi K^+) = (5.6 \pm 0.5 \pm 0.3) \times 10^{-6}$, about three standard deviations above Belle's result and larger than theoretical estimates. The $B^0 \rightarrow \phi\phi K^0$ channel was observed with a branching fraction of $(4.5 \pm 0.8 \pm 0.3) \times 10^{-6}$. BABAR also reported CP asymmetries for charged B decays as $-0.10 \pm 0.08 \pm 0.02$ below the η_c threshold and $+0.09 \pm 0.10 \pm 0.02$ within the η_c region.

In this paper, we update our earlier result [5] with a significantly larger data sample containing $772 \times 10^6 B\bar{B}$ events. The data were collected at the $\Upsilon(4S)$ resonance with the Belle detector [10] at the KEKB asymmetric-energy e^+e^- collider [11]. The subdetectors relevant for our study are a silicon vertex detector (SVD), a central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), and time-of-flight scintillation counters (TOF). All these are located inside a 1.5 T axial magnetic field.

To reconstruct $B^+ \rightarrow \phi\phi K^+$ and $B^0 \rightarrow \phi\phi K^0$ decay candidates, we combine a pair of ϕ mesons with a charged kaon and K_S^0 , respectively. All charged tracks except for those from the K_S^0 must have a distance of closest approach with respect to the interaction point (IP) of less than 0.2 cm in the transverse $r - \phi$ plane, and less than 5.0 cm along the z axis. The z axis is defined as the direction opposite that of the e^+ beam. We identify charged kaons based on a likelihood ratio $\mathcal{R}_{K/\pi} = \mathcal{L}_K/(\mathcal{L}_K + \mathcal{L}_\pi)$, where \mathcal{L}_K and \mathcal{L}_π denote the individual likelihood for kaons and pions, respectively. These are calculated using specific ionization in the CDC and information from the ACC and the TOF. A requirement $\mathcal{R}_{K/\pi} > 0.6$ is applied to select kaon candidates. The kaon identification efficiency, averaged over the momentum range, is 90%, with a pion misidentification rate of about 10%.

We reconstruct the ϕ candidates from pairs of oppositely charged kaons with an invariant mass in the range $1.00 - 1.04 \text{ GeV}/c^2$, corresponding to $\pm 5\sigma$ (σ is the width

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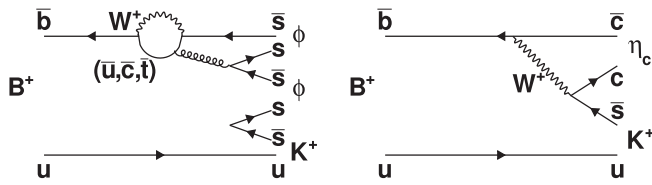


FIG. 1. Dominant Feynman diagrams that contribute to the decays (left) $B^+ \rightarrow \phi\phi K^+$ and (right) $B^+ \rightarrow \eta_c K^+$. Replacement of the spectator u quark with a d quark will lead to the corresponding diagrams for $B^0 \rightarrow \phi\phi K^0$ and $B^0 \rightarrow \eta_c K^0$.

of the mass distribution) around the nominal ϕ mass [12]. This is referred to as the M_{KK} signal region in the following discussion. The K_S^0 candidates are reconstructed from two oppositely charged tracks, assumed to be pions, and are further required to satisfy a criterion on the output of a neural network (NN) algorithm [13]. The algorithm uses the following input variables: the K_S^0 momentum in the lab frame; the distance of closest approach along the z axis between the two pion tracks; the flight length in the $r - \phi$ plane; the angle between the K_S^0 momentum and the vector joining the IP to the K_S^0 decay vertex; the angle between the K_S^0 momentum in the lab frame and the pion momentum in the K_S^0 rest frame; the distances of closest approach in the $r - \phi$ plane between the IP and the two pion tracks; the number of CDC hits for each pion track; and the presence or absence of SVD hits for each pion track. We require that the invariant mass lie between $491 \text{ MeV}/c^2$ and $504 \text{ MeV}/c^2$, which corresponds to a $\pm 3\sigma$ window in resolution around the nominal K_S^0 mass [12].

B -meson candidates are identified with two kinematic variables: the beam-energy-constrained mass $M_{bc} \equiv \sqrt{E_b^2/c^4 - |\sum_i \vec{p}_i/c|^2}$, and the energy difference $\Delta E \equiv \sum_i E_i - E_b$, where E_b is the beam energy, and \vec{p}_i and E_i are the momentum and energy, respectively, of the i -th decay product of the B candidate. All these quantities are evaluated in the e^+e^- center-of-mass (CM) frame. We perform a fit for each B candidate, constraining its decay products to originate from a common vertex. Candidate events with $M_{bc} \in [5.230, 5.289] \text{ GeV}/c^2$ and $|\Delta E| < 0.1 \text{ GeV}$ are retained for further study. The M_{bc} requirement corresponds to approximately $(-16\sigma, +3\sigma)$ in resolution around the nominal B mass [12], and the ΔE requirement denotes a $\pm 10\sigma$ window around zero. We apply such loose requirements on M_{bc} and ΔE as these are used in a maximum-likelihood fit to obtain the signal yield (described later). We define a signal region as $M_{bc} \in [5.272, 5.289] \text{ GeV}/c^2$ and $|\Delta E| < 0.05 \text{ GeV}$.

After application of the above selection criteria, the average number of B candidates found per event selected in data are 1.7 and 1.6 for $B^+ \rightarrow \phi\phi K^+$ and $B^0 \rightarrow \phi\phi K^0$, respectively. In the case of multiple B candidates, we choose the candidate with the lowest χ^2 value for the aforementioned B -vertex fit. From Monte Carlo (MC)

simulation the best candidate selection method is found to have an efficiency of 68% (65%) to correctly identify the B -meson candidate in $B^+ \rightarrow \phi\phi K^+$ ($B^0 \rightarrow \phi\phi K^0$) decays. In only about 6% of the total signal events, the B candidate is misreconstructed due to swapping of kaons between the two ϕ candidates, or of one daughter track with that from the rest of the event. Such misreconstructed events are treated as a part of the signal.

The dominant background is from the $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) continuum process. To suppress this background, observables based on event topology are used. The event shape in the CM frame is expected to be spherical for $B\bar{B}$ events and jetlike for continuum events. We use an NN [13] to combine the following six variables: a Fisher discriminant formed out of 16 modified Fox-Wolfram moments [14]; the cosine of the angle between the B momentum and the z axis; the cosine of the angle between the B thrust axis [15] and the z axis; the cosine of the angle between the thrust axis of the B candidate and that of the rest of the event; the ratio of the second- to the zeroth-order Fox-Wolfram moments (all quantities are calculated in the CM frame); and the vertex separation along the z axis between the B candidate and the remaining tracks. The NN training and validation are performed with signal and $q\bar{q}$ MC simulated events. The signal sample is generated with the EVTGEN program [16], assuming a uniform distribution over the three-body phase space of the final state.

The neural network output (O_{NN}) ranges between -1.0 and 1.0 , where events near -1.0 (1.0) are more continuum-(signal)-like. We apply a loose criterion $O_{NN} > -0.5$ to reduce the continuum background. The relative signal efficiency loss due to this requirement is about 6% (3%) for $B^+ \rightarrow \phi\phi K^+$ ($B^0 \rightarrow \phi\phi K^0$) decays, whereas the fraction of continuum events rejected is 76% (66%). As the remainder of the O_{NN} distribution strongly peaks near 1.0 for signal, it is difficult to model with an analytic function. However, the transformed variable

$$O'_{NN} = \log \left[\frac{O_{NN} - O_{NN,\min}}{O_{NN,\max} - O_{NN}} \right], \quad (1)$$

where $O_{NN,\min} = -0.5$ and $O_{NN,\max} \simeq 1.0$, has a Gaussian-like distribution that is easier to model. Thus, we use this transformed variable in our signal fit.

Backgrounds due to B decays, mediated by the dominant $b \rightarrow c$ transition, are studied with MC samples of such decays. For both $B^+ \rightarrow \phi\phi K^+$ and $B^0 \rightarrow \phi\phi K^0$ channels, the M_{bc} and ΔE distributions are found to peak in the signal region. To investigate the source of these contributions, we inspect the $m_{\phi\phi}$ distribution, which displays several peaks corresponding to the η_c and other charmonium resonances. To suppress these peaking backgrounds, we exclude candidates for which the $m_{\phi\phi}$ value is greater than $2.85 \text{ GeV}/c^2$. This requirement also allows us to compare our results with the earlier ones from Belle [5] and

BABAR [9]. We calculate the detection efficiencies for candidate events below the η_c threshold to be 12.4% and 12.0% for $B^+ \rightarrow \phi\phi K^+$ and $B^0 \rightarrow \phi\phi K^0$, respectively.

Charmless backgrounds that do not produce only kaons in the final state may still contribute to the $M_{bc} - \Delta E$ signal region when a final-state particle is misidentified. These are studied with a $B\bar{B}$ MC sample in which one of the B mesons decays via $b \rightarrow u, d, s$ transitions with known or estimated branching fractions [12]. Only 40 events survive from an MC sample equivalent to 50 times the size of the data sample. This small component is combined with the events surviving from $b \rightarrow c$ transitions to form an overall $B\bar{B}$ background component. In addition to this $B\bar{B}$ background that does not peak in M_{bc} or ΔE , we can have contributions from $B \rightarrow \phi KKK$ and $B \rightarrow KKKKK$ decays (described later), which have the same final-state particles as the signal.

The signal yield is obtained with an unbinned extended maximum-likelihood fit to the three variables M_{bc} , ΔE , and O'_{NN} . We define a probability density function (PDF) for each event category, i.e., signal, $q\bar{q}$, and $B\bar{B}$ backgrounds:

$$\mathcal{P}_j^i \equiv \frac{1}{2}(1 - q_i \mathcal{A}_{CP,j}) \mathcal{P}_j(M_{bc}^i) \mathcal{P}_j(\Delta E^i) \mathcal{P}_j(O'_{NN}^i), \quad (2)$$

where i denotes the event index, q_i is the charge of the B candidate ($q_i = \pm 1$ for B^\pm), and \mathcal{P}_j and $\mathcal{A}_{CP,j}$ are the PDF and CP asymmetry, respectively, for the event category j . The latter is defined as

$$\mathcal{A}_{CP} = \frac{N_{B^-} - N_{B^+}}{N_{B^-} + N_{B^+}}, \quad (3)$$

where N_{B^+} (N_{B^-}) is the number of B^+ (B^-) events. We find equal detection efficiencies for the B^+ ($12.3 \pm 0.1\%$) and B^- ($12.4 \pm 0.1\%$) decays. For neutral B decays, we replace the factor $\frac{1}{2}(1 - q_i \mathcal{A}_{CP,j})$ by 1 in Eq. (2). We also do not perform a CP -violation study in this case, since we would need to tag the recoiling B candidate for that, causing further loss in efficiency on top of the small signal yield. As the correlations among M_{bc} , ΔE , and O'_{NN} are found to be small ($\lesssim 5\%$), the product of three individual PDFs is a good approximation for the total PDF. The extended likelihood function is

$$\mathcal{L} = \frac{e^{-\sum_j n_j}}{N!} \prod_i \left[\sum_j n_j \mathcal{P}_j^i \right], \quad (4)$$

where n_j is the yield of event category j , and N is the total number of candidate events. From the fitted signal yield (n_{sig}), we calculate the branching fraction as

$$\mathcal{B}(B \rightarrow \phi\phi K) = \frac{n_{\text{sig}}}{\epsilon N_{B\bar{B}} [\mathcal{B}(\phi \rightarrow K^+ K^-)]^2}, \quad (5)$$

TABLE I. List of PDFs used to model the M_{bc} , ΔE , and, O'_{NN} distributions for various event categories for $B \rightarrow \phi\phi K$. The notation G, AG, 2G, ARG, and Poly1 denote Gaussian, asymmetric Gaussian, sum of two Gaussians, ARGUS [17] function, and first-order polynomial, respectively.

Event category	M_{bc}	ΔE	O'_{NN}
Signal	G + ARG	2G + Poly1	G + AG
$q\bar{q} + B\bar{B}$	ARG	Poly1	G

where ϵ and $N_{B\bar{B}}$ are the detection efficiency and the number of $B\bar{B}$ events, respectively. In case of $B^0 \rightarrow \phi\phi K^0$, we multiply the denominator by a factor of $\frac{1}{2}$ to account for $K^0 \rightarrow K_S^0$, as well as by the subdecay branching fraction $\mathcal{B}(K_S^0 \rightarrow \pi^+ \pi^-)$ [12].

As the expected yield of the nonpeaking $B\bar{B}$ background is small, and it is distributed similarly to $q\bar{q}$ in M_{bc} and ΔE , we merge $q\bar{q}$ and $B\bar{B}$ backgrounds into a single component. We find that the difference in the O'_{NN} distribution between the two backgrounds contributes a negligible systematic uncertainty. Table I lists the PDF shapes used to model M_{bc} , ΔE , and O'_{NN} distributions for various event categories of $B \rightarrow \phi\phi K$ candidates. The yield and PDF shape parameters of the combined background are floated in $B^+ \rightarrow \phi\phi K^+$. For the neutral channel, however, the background PDF shapes are fixed to their MC values after correcting for small differences between data and simulation, as obtained from the charged decay. Similarly, for the signal components, we fix the M_{bc} , ΔE , and O'_{NN} shapes to MC values and correct for small data-MC differences according to values obtained from a control sample of $B^+ \rightarrow D_s^+ \bar{D}^0$ decays, where $D_s^+ \rightarrow \phi(\rightarrow K^+ K^-) \pi^+$ and $\bar{D}^0 \rightarrow K^+ \pi^-$.

We apply the above 3D fit to $B^+ \rightarrow \phi\phi K^+$ and $B^0 \rightarrow \phi\phi K^0$ candidate events to determine the signal yield (and \mathcal{A}_{CP} in the first case). Figures 2 and 3 show M_{bc} , ΔE , and O'_{NN} projections of the fits. The fit results are listed in Table II. We find signal yields of $85.0_{-9.5}^{+10.2}$ for $B^+ \rightarrow \phi\phi K^+$ and $26.5_{-5.1}^{+5.8}$ for $B^0 \rightarrow \phi\phi K^0$, and an \mathcal{A}_{CP} value of -0.02 ± 0.11 for the first case. We also apply the 3D fit to $B^+ \rightarrow \phi\phi K^+$ candidate events with $m_{\phi\phi}$ within the η_c region to calculate the signal yield and \mathcal{A}_{CP} value. The corresponding M_{bc} and ΔE projections are shown in Fig. 4, with the fit results listed in Table II. We obtain a signal yield of $73.2_{-8.3}^{+9.0}$ and an \mathcal{A}_{CP} value of $+0.12 \pm 0.12$ in the η_c region. The signal significance is calculated as $\sqrt{-2 \log(\mathcal{L}_0/\mathcal{L}_{\text{max}})}$, where \mathcal{L}_0 and \mathcal{L}_{max} are the likelihood values with the signal yield fixed to zero and for the nominal fit, respectively. We include systematic uncertainties that impact only the signal yield into the likelihood curve via a Gaussian convolution before calculating the final significance.

To estimate the contribution of $B \rightarrow \phi KKK$ and $B \rightarrow KKKKK$ decays in the M_{KK} signal region (SR), we repeat the 3D fit in the following two sidebands: SB1 is denoted

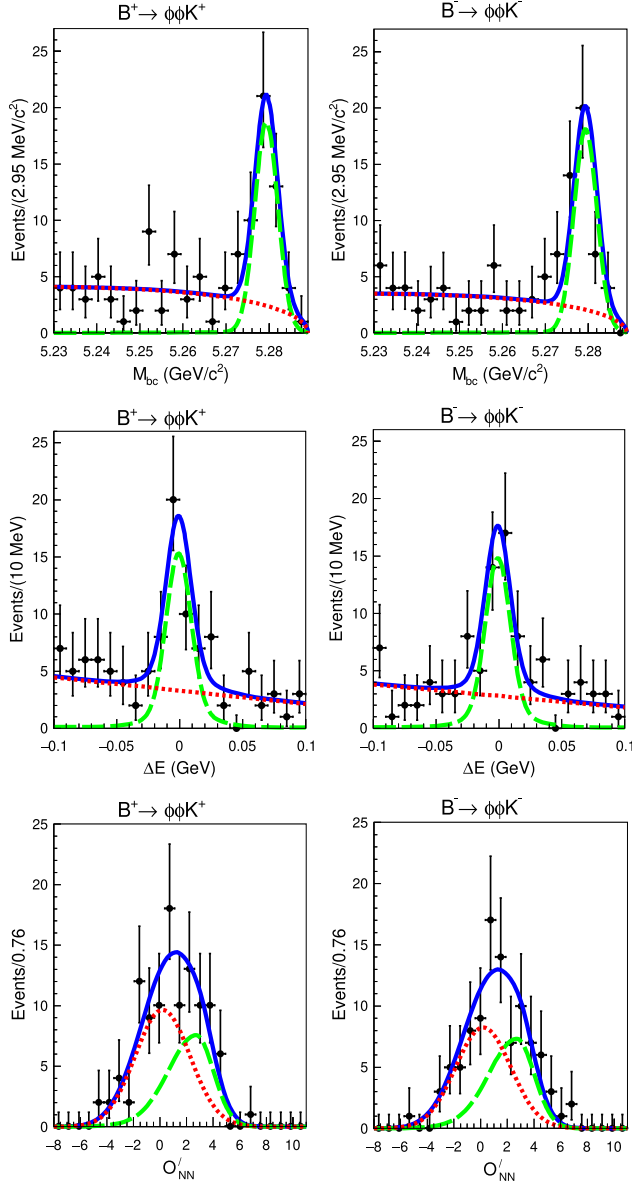


FIG. 2. Projections of $B^\pm \rightarrow \phi\phi K^\pm$ candidate events onto (top) M_{bc} , (middle) ΔE , and (bottom) O'_{NN} . Black points with error bars are the data, solid blue curves are the total PDF, dashed green curves are the signal component, and dotted red curves are the combined $q\bar{q}$ and $B\bar{B}$ background components.

by the sum of ($M_{K_1K_2} \in [1.04, 1.2]$ GeV/c^2 and $M_{K_3K_4} \in [1.0, 1.04]$ GeV/c^2) and ($M_{K_1K_2} \in [1.0, 1.04]$ GeV/c^2 and $M_{K_3K_4} \in [1.04, 1.2]$ GeV/c^2), and SB2 is denoted by $M_{K_1K_2} \in [1.04, 1.2]$ GeV/c^2 and $M_{K_3K_4} \in [1.04, 1.2]$ GeV/c^2 . In Fig. 5 we plot the distribution of data events in the $M_{K_1K_2}$ vs $M_{K_3K_4}$ plane showing SR, SB1, and SB2. The resonant $B \rightarrow \phi\phi K$ yield in SR is obtained by solving the following three linear equations:

$$N_0 = n_s + r_{a0} \times n_a + r_{b0} \times n_b, \quad (6)$$

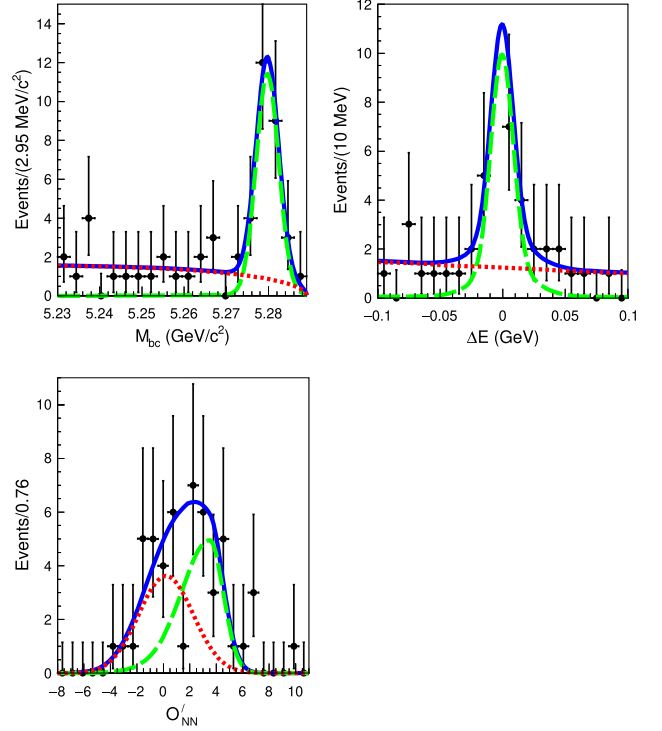


FIG. 3. Projections of $B^0 \rightarrow \phi\phi K^0$ candidate events onto (top left) M_{bc} , (top right) ΔE , and (bottom) O'_{NN} . The legends of the plots are defined in the same manner as in Fig. 2.

$$N_1 = r_{s1} \times n_s + n_a + r_{b1} \times n_b, \quad (7)$$

$$N_2 = r_{s2} \times n_s + r_{a2} \times n_a + n_b, \quad (8)$$

where N_0 , N_1 , and N_2 are the yields obtained in SR, SB1, and SB2, respectively; n_s , n_a , and n_b are the $B \rightarrow \phi\phi K$ yield in SR, $B \rightarrow \phi KKK$ yield in SB1, and $B \rightarrow KKKK$ yield in SB2, respectively. Lastly, r_{s1} and r_{s2} are the ratios of $B \rightarrow \phi\phi K$ yields in SB1 and SB2 to that in SR; r_{a0} and r_{a2} are the ratios of $B \rightarrow \phi KKK$ yields in SR and SB2 to

TABLE II. Number of candidate events (n_{cand}), detection efficiency (ϵ), total and resonant signal yield (n_{sig}), significance, branching fraction (\mathcal{B}) and CP asymmetry (\mathcal{A}_{CP}) obtained from a fit to data for $B \rightarrow \phi\phi K$ decays below and within the η_c region. Quoted uncertainties are statistical only, and significances defined in the text are given in terms of standard deviations.

	$B^+ \rightarrow \phi\phi K^+$	$B^0 \rightarrow \phi\phi K^0$	$B^+ \rightarrow \phi\phi(\eta_c)K^+$
n_{cand}	207	51	84
$\epsilon(\%)$	12.4	12.0	15.4
Total n_{sig}	$85.0^{+10.2}_{-9.5}$	$26.5^{+5.8}_{-5.1}$	$73.2^{+9.0}_{-8.3}$
Significance	14.9	7.2	16.7
Resonant n_{sig}	$81.8^{+10.1}_{-9.4}$	$23.7^{+5.7}_{-5.0}$...
$\mathcal{B}(10^{-6})$	$3.43^{+0.48}_{-0.46}$	$3.02^{+0.75}_{-0.66}$...
\mathcal{A}_{CP}	-0.02 ± 0.11	...	$+0.12 \pm 0.12$

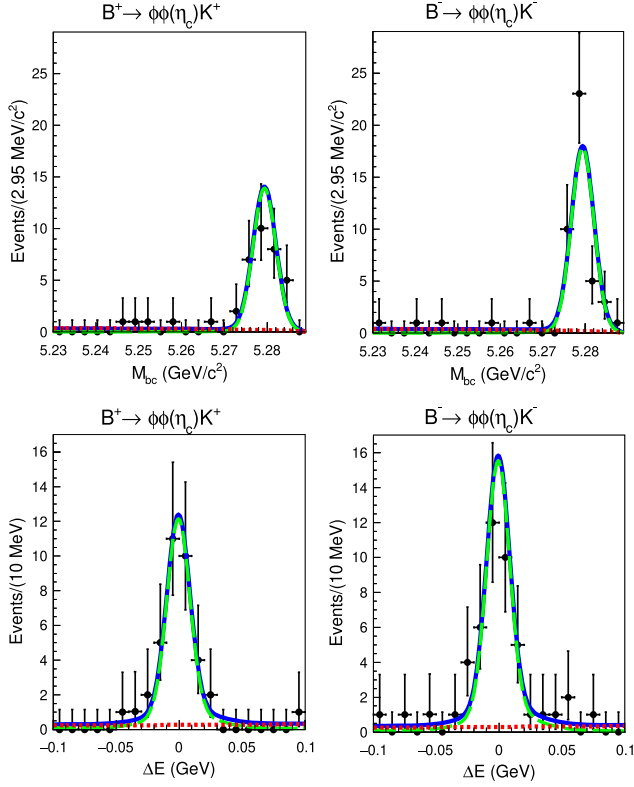


FIG. 4. Projections of $B^\pm \rightarrow \phi\phi K^\pm$ candidate events within the η_c region onto (top) M_{bc} and (bottom) ΔE . The legends of the plots are defined in the same manner as in Fig. 2.

that in SB1; and r_{b0} and r_{b1} are the ratios of $B \rightarrow KKKK$ yields in SR and SB1 to that in SB2. All these ratios are obtained from an MC study. We obtain the resonant $B \rightarrow \phi\phi K$ yield in SR (n_s) as $81.8^{+10.1}_{-9.4}$ and $23.7^{+5.7}_{-5.0}$ for the charged and neutral mode, respectively. These n_s values are used in the branching fraction calculation of Eq. (5).

The background-subtracted distributions [18] of $m_{\phi\phi}$ and $m_{\phi_1 K}$ obtained for $B^\pm \rightarrow \phi\phi K^\pm$ below the η_c threshold are shown in Fig. 6. These are broadly compatible with the

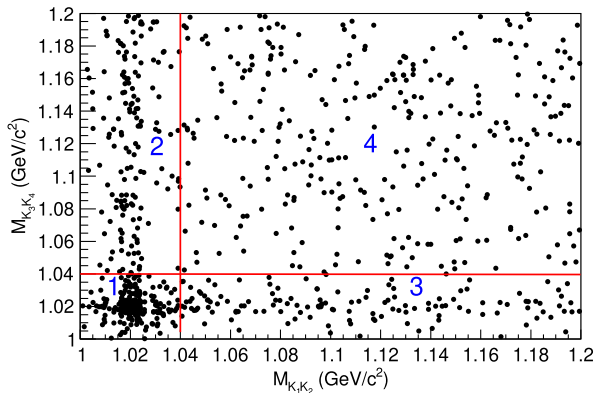


FIG. 5. Distribution of data events in the $M_{K_1 K_2}$ vs $M_{K_3 K_4}$ plane which shows the M_{KK} signal region (region 1) and two sidebands SB1 (region 2 and 3) and SB2 (region 4).

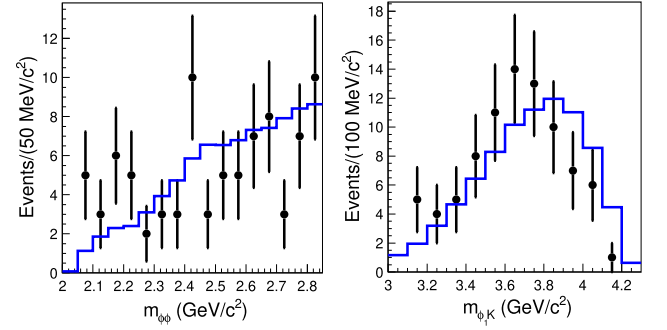


FIG. 6. Background-subtracted signal yield as a function of $m_{\phi\phi}$ (left) and $m_{\phi_1 K}$ (right) for $B^\pm \rightarrow \phi\phi K^\pm$. Black points with error bars are data and solid blue histograms denote the expectation from a phase-space MC sample.

predictions of a three-body phase space MC sample. In particular, we do not find any enhancement in the $m_{\phi\phi}$ spectrum, including the $2.3 \text{ GeV}/c^2$ region [3] where a glueball and $X(2350)$ candidates are predicted.

Systematic uncertainties in the branching fraction are listed in Table III. The uncertainties due to PDF shapes are estimated by varying all the fixed shape parameters by their errors. In particular, for fixed signal shape parameters, we vary the data-MC corrections by their uncertainties as determined using the control sample of $B^+ \rightarrow D_s^+ \bar{D}^0$ decays. Potential fit bias is checked by performing an ensemble test comprising 1000 pseudoexperiments, where signal is taken from the corresponding MC sample, and the PDF shapes are used to generate background events. We obtain a Gaussian normalized residual distribution of unit width, and add its mean and uncertainty in width in quadrature to calculate the systematic error. Uncertainty due to continuum suppression is obtained with the $B^+ \rightarrow D_s^+ \bar{D}^0$ control sample by comparing, between data and simulation, fit results obtained with and without the O_{NN} requirement. A $D^{*+} \rightarrow D^0(K^-\pi^+)\pi^+$ control sample is used to determine the systematic uncertainty due to the

TABLE III. Systematic uncertainties (in %) in the branching fractions. Values listed in the top three rows impact the signal yield and are included in the calculation of signal significance.

Source	$B^\pm \rightarrow \phi\phi K^\pm$	$B^0 \rightarrow \phi\phi K^0$
Signal PDF	+1.5	+1.3
Background PDF	-1.7	-1.9
	...	+3.0
		-1.9
Fit bias	± 1.7	± 2.0
Efficiency variation	± 2.1	± 2.1
$\mathcal{R}_{K/\pi}$ requirement	± 5.2	± 4.3
$q\bar{q}$ suppression	± 0.5	± 0.5
Track reconstruction	± 1.8	± 1.4
K_S^0 reconstruction	...	± 0.9
Number of $B\bar{B}$ events	± 1.4	± 1.4
Total	± 6.5	+6.5 -6.3

TABLE IV. Systematic uncertainties in \mathcal{A}_{CP} .

Source	$B^\pm \rightarrow \phi\phi K^\pm$	$B^\pm \rightarrow \phi\phi(\eta_c)K^\pm$
Detection asymmetry	± 0.008	± 0.008
Signal PDF shape	$^{+0.002}_{-0.003}$	± 0.002
Total	± 0.01	± 0.01

$\mathcal{R}_{K/\pi}$ requirement. We use partially reconstructed $D^{*+} \rightarrow D^0(K_S^0\pi^+\pi^-)\pi^+$ decays to assign the systematic uncertainty due to charged-track reconstruction (0.35% per track). The uncertainty due to K_S^0 reconstruction is estimated from $D^0 \rightarrow K_S^0 K_S^0$ decays [19]. We estimate the uncertainty due to efficiency variation across the Dalitz plot by comparing the yield obtained in bins of $m_{\phi\phi}$ and $m_{\phi_1 K}$ between data and phase-space signal MC samples. We fit the background-subtracted $m_{\phi\phi}$ and $m_{\phi_1 K}$ distributions in data (Fig. 6) using a first- and second-order Chebyshev polynomial, respectively. The bin-by-bin scale factors, given by the ratio of signal yields in data and MC events, are propagated to the phase space MC sample. We then calculate the difference in efficiency between reweighted and original MC events. The uncertainties in the polynomial coefficients are also considered in this calculation. The efficiency differences move in both positive and negative directions, with their magnitudes lying in the range 0.7–2.1%. We assign the maximum difference (2.1%) as the systematic uncertainty. The total systematic uncertainty is obtained by adding all the above contributions in quadrature.

We consider two possible sources of systematic uncertainties contributing to \mathcal{A}_{CP} , as listed in Table IV. The first is due to the intrinsic detector bias on charged kaon detection and is estimated using $D_s^+ \rightarrow \phi\pi^+$ and $D^0 \rightarrow K^-\pi^+$ decays [20]. The second arises due to the potential variation of the PDF shapes. We calculate its contribution by following a procedure similar to that used in estimating the PDF shape uncertainties in the branching fractions.

In summary, we have measured the branching fractions and CP -violation asymmetries in $B \rightarrow \phi\phi K$ decays based on the full $\Upsilon(4S)$ data sample of $772 \times 10^6 B\bar{B}$ events collected by the Belle detector at the KEKB asymmetric-energy e^+e^- collider. We obtain the branching fraction and

CP asymmetry for $B^\pm \rightarrow \phi\phi K^\pm$ below the η_c threshold ($m_{\phi\phi} < 2.85 \text{ GeV}/c^2$) as

$$(3.43_{-0.46}^{+0.48} \pm 0.22) \times 10^{-6} \quad (9)$$

and

$$-0.02 \pm 0.11 \pm 0.01, \quad (10)$$

respectively. We also report the CP -violation asymmetry for $B^\pm \rightarrow \phi\phi K^\pm$ in the η_c region ($m_{\phi\phi} \in [2.94, 3.02] \text{ GeV}/c^2$) to be

$$+0.12 \pm 0.12 \pm 0.01, \quad (11)$$

consistent with no CP violation. The obtained value of the branching fraction of $B^\pm \rightarrow \phi\phi K^\pm$ decay is consistent with and supersedes our previous result [5]. The measured branching fraction for $B^0 \rightarrow \phi\phi K^0$ below the η_c threshold is

$$(3.02_{-0.66}^{+0.75} \pm 0.20) \times 10^{-6}. \quad (12)$$

We find no evidence for glueball production in these decays.

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