Reply to "Comment on 'Constraining the annihilating dark matter mass by the radio continuum spectral data of the NGC4214 galaxy"'

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We reply to the comment by Heesen and Brüggen. While we acknowledge the merit of their criticism, we argue that the limits obtained by our method are still valid, within our assumption. All the involved uncertainties should be addressed in depth in order to determine how they affect our limits.

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Here we reply to the comment by Heesen and Brüggen [\[1\]](#page-1-0) that using the radio continuum-star formation (radio-SFR) relation is a more robust way to determine the alleged signal of dark matter. They argue that a prior is needed on the expected radio continuum emission and they suggest that the radio-SFR relation is one of the most universal and tightest relations known in galaxies [\[1\].](#page-1-0) As the data of the NGC4214 galaxy are consistent with the best-fitting radio-SFR relation, it cannot provide any meaningful upper limits for the dark matter annihilation cross section [\[1\]](#page-1-0).

In fact, since the data of the NGC4214 galaxy are consistent with the best-fitting relation, this implies that the room for dark matter annihilation is small and thus we can generate some upper limits of annihilation cross section as there is no radio excess signal. Strictly speaking, the result shown in [\[1\]](#page-1-0) and the result in our analysis [\[2\]](#page-1-1) are consistent and compatible with each other. Both results show that no excess is found and no signal of dark matter could be reported. However, as pointed out in [\[1\],](#page-1-0) the scatter of the relation is not small (∼0.2 dex) so that the constraints obtained would be less stringent. Therefore, the drawback of using the radio-SFR relation is that the scatter is too large to get any stringent limits of the annihilation cross section. This point has also been discussed quantitatively in [\[1\]](#page-1-0). Nevertheless, the comment in [\[1\]](#page-1-0) emphasizes that "...what really determines the significance of the [dark matter] detection is the deviation from the radio-SFR relation.'' In fact, the significance of the constraints depends on which method of analysis is used and the uncertainty associated with that analysis. Different methods would have different associated uncertainties. The final limits would depend on these method-dependent uncertainties. In our analysis, the uncertainties of the observational radio spectral data are not very large so that a power-law line could be fitted with a relatively small uncertainty ($\approx 6\%$ [\[3\]](#page-1-2)). Therefore, a small additional contribution of dark matter annihilation may be able to cause a deviation from the power-law relation. The likelihood between the theoretical prediction and the spectral data is sensitive to the uncertainty associated with the spectral data. Therefore, it is relatively easier for the potential dark matter contribution to violate the observed power-law spectrum. The violation at any statistical significance (e.g., ruled out at 2σ) can be calculated by the χ^2 values [\[2\]](#page-1-1). Generally speaking, the limits obtained based on our analysis are more stringent because the associated uncertainties are smaller.

Another important point is that the radio-SFR relation is an empirical relation and it is not a benchmark relation. Some studies show that the radio-SFR relation is not a simple power-law relation and it may also depend on the stellar mass of the galaxies [\[4\].](#page-1-3) Therefore, assuming it to be the best established prior is somewhat overstated. Nevertheless, there is an advantage of using the radio-SFR relation as it originates from a sample of galaxies, which may reveal some general features of galaxies that are useful for dark matter analysis.

In our article [\[2\]](#page-1-1), we assume a constant spectral index for the cosmic-ray synchrotron spectrum and deviations from this assumed spectrum are attributed to a possible dark matter signal. One possible limitation is that it relies on the data of a single galaxy only. Such a limitation and the corresponding systematic uncertainties have been discussed in our article [\[2\].](#page-1-1) Nevertheless, many galaxies like M31 [\[5\]](#page-1-4) and NGC4449 galaxies [\[3\]](#page-1-2) manifest constant spectral index with small uncertainties, which show that it might be a good prior for our analysis. However, in their Comment, Heesen and Brüggen mention that any deviation from a power-law spectrum can easily be explained by processes such as cosmic-ray injection, cosmic-ray transport, and energy losses [\[1\]](#page-1-0). Therefore, they argue that ignoring these processes would make an invalid claim of a dark matter signal. In fact, in our original article, as there is no significant deviation from the constant spectral index, we did not claim any positive dark matter signal and we finally obtained the upper limits of an annihilation cross section. If there is any significant deviation from the constant spectral index (a positive signal) revealed from the spectrum, it is true that the deviation might possibly originate from other cosmic-ray injection or other processes like cosmic-ray transport or energy losses. However, because there is no positive signal, whether other processes or injection can explain the deviation is irrelevant. It does not affect the upper limits obtained in our analysis. Moreover, cosmic-ray transport or energy losses could steepen the spectral index in the high frequency regime, like the radio spectrum in the NGC1569 galaxy [\[6\]](#page-1-5). Nevertheless, in our analysis, a dark matter signal dominates in the low radio frequency regime (see Fig. 4 in [\[2\]\)](#page-1-1) which is different from the spectral steepening due to cosmic-ray transport and energy losses.

Finally, Heesen and Brüggen argue that based on the residence time of 10 Myr estimated in [\[7\]](#page-1-6), the diffusion coefficient should be about $D = 3 \times 10^{28}$ cm²/s [\[1\],](#page-1-0) which is ∼100 larger than what we used in our study [\[2\].](#page-1-1) If the diffusion coefficient is larger, the predicted dark matter radio emission would be suppressed and the limits of the annihilation cross section constrained should be larger. However, the estimated residence time 10 Myr used in [\[7\]](#page-1-6) is based on the Milky Way data. Also, the assumption of 1 kpc diffusion length in [\[1\]](#page-1-0) is also questionable. The diffusion length is about the same order of the turbulence scale, which can be as small as 90 pc, like in the Large Magellanic Cloud stated in [\[7\].](#page-1-6) If the diffusion length is 90 pc with the 10 Myr diffusion time, then the diffusion coefficient estimated is $\sim 10^{26}$ cm²/s, which is the same order of magnitude of our estimation. In fact, we can also estimate the diffusion coefficient theoretically. The diffusion coefficient depends on the injection scale L and the turbulent velocity V: $D = LV$ [\[8\].](#page-1-7) For $L = 100$ pc and $V = 50$ km/s for a typical galaxy, the diffusion coefficient is $D \sim 10^{26}$ cm²/s. Therefore, although there is no concrete observed value of the diffusion coefficient for the NGC4214 galaxy, our estimation in [\[2\]](#page-1-1) is still reasonable.

In summary, although the data of the NGC4214 galaxy are consistent with the best-fitting radio-SFR relation, this fact does not directly imply that any dark matter contribution could not be disentangled by the radio spectra, even if it is small. Moreover, all the involved uncertainties should be addressed in depth in order to determine how they affect our limits.

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