Probing compactified extra dimensions with gravitational waves

Yuchen Du, Shammi Tahura, Diana Vaman, and Kent Yagi
Department of Physics, University of Virginia, Charlottesville, Virginia 22904, USA

(Received 17 April 2020; accepted 9 December 2020; published 15 February 2021)

We study the effect of compact extra dimensions on the gravitational wave luminosity and waveform. We consider a toy model, with a compactified fifth dimension, and matter confined on a brane. We work in the context of five-dimensional (5D) general relativity, though we do make connections with the corresponding Kaluza-Klein effective 4D theory. We show that the luminosity of gravitational waves emitted in 5D gravity by a binary with the same characteristics (same masses and separation distance) as a 4D binary is 20.8% less relative to the 4D case, to leading post-Newtonian order. The phase of the gravitational waveform differs by 26% relative to the 4D case, to leading post-Newtonian order. Such a correction arises mainly due to the coupling between matter and dilaton field in the effective 4D picture and agrees with previous calculations when we set black holes' scalar charges to be those computed from the Kaluza-Klein reduction. The above corrections to the waveform and the luminosity are inconsistent with the gravitational-wave and binary pulsar observations, and they thus effectively rule out the possibility of such a simple compactified higher dimensions scenario. We also comment on how our results change if there are several compactified extra dimensions and show that the discrepancy with 4D general relativity only increases.

DOI: 10.1103/PhysRevD.103.044031

I. INTRODUCTION

Recent discoveries of gravitational waves have opened a new window for testing general relativity (GR), especially in the strong/dynamical field regime [1–7], in a way that is complementary to other tests including Solar System experiments [8,9], binary pulsars [10,11], and cosmological observations [12,13]. Gravitational wave events have been used to probe the fundamental pillars of GR, such as the equivalence principle, Lorentz/parity invariance, and massless gravitons [2].

One fundamental aspect of gravity that is important to probe is the presence of extra dimensions motivated by, e.g., string theory. For example, in flat D-dimensional (noncompact) spacetime, gravitational waves decay with distance traveled as $1/R^{(D-2)/2}$ [14]. This fact has been used to measure D with gravitational wave observations [4,15]. One can also use tidal deformabilities of compact objects to probe extra dimensions. In four-dimensional (4D) GR tidal deformabilities vanish for nonrotating black holes, but they do not for higher-dimensional black holes [16,17].

In this paper we consider gravitational waves in a 5D spacetime, with the fifth dimension being compactified on a circle and with the matter constrained on a brane. We work in 5D GR rather than in the context of the effective Einstein-Maxwell-dilaton 4D theory (considered in [18–21]), which arises from performing a Kaluza-Klein (KKreduction of the 5D theory. Our motivation is twofold. First, motivation is simplicity: it is easier to work with a single field (the metric) than a collection of fields. Also, by not performing the Kaluza-Klein reduction (which assumes that fields are independent of the compactified dimension) it allows us to account for the effect of the massive fluctuations that arise when integrating out the compact dimension. By working directly in higher-dimensional GR theory we are also able to generalize the 5D results to an arbitrary number of compactified extra dimensions. Second, we hope that our 5D analysis will be useful to assess the effect of the extra dimensions on gravitational wave detection when considering other paradigms for the geometry of the extra dimensions, e.g., Randall-Sundrum (RS) models.

Gravitational waves in higher-dimensional spacetimes with compactified or warped extra dimensions have been studied in the literature. Kaluza-Klein compactification leads to an extra scalar polarization mode(s) (the breathing mode) plus massive Kaluza-Klein modes, whose frequencies are typically much higher than what can be probed with ground-based detectors [22,23]. Such Kaluza-Klein modes also create a stochastic gravitational wave background [24] and modify the quasinormal modes after mergers [25,26]. Black hole and neutron star tidal deformabilities have been computed within braneworld models and have been applied to GW170817 to constrain the brane tension [27,28]. In the RS-II braneworld model [29], black holes may evaporate classically [30,31], which changes the orbital evolution of binary black holes and further modifies the waveform from that in 4D GR. This fact has been applied to the GW events, such as GW150914, to place bounds on the size of the extra dimension [2], though such bounds are much weaker than those coming from tabletop experiments [32,33]. Last, given that gravitational waves can propagate in the higher-dimensional bulk while electromagnetic waves are constrained on a brane, one can compare the propagation of two such waves to probe the extra dimensions [34–38], which has been applied to GW170817 [39–42]. Again, these bounds on the extra dimension size are much weaker than those from tabletop experiments.

Our paper is organized as follows. In Sec. II we present our conventions, notations, and general framework. In Sec. III, we extract the compactified 5D Newtonian potential and the modified Kepler's law for a binary at a fixed position in the extra dimension. In Sec. IV we review the Kaluza-Klein reduction of the 5D theory and point out that 5D matter, when seen from a 4D perspective, is nonminimally coupled. Section V contains our main result, namely the form of the gravitational waves sourced by the 5D binary. In Sec. VI we extract the luminosity of the gravitational waves, and in Sec. VII we compute the phase of the gravitational waveform as predicted by the compactified 5D model and compare it with observations. Throughout the paper we give generalizations of our formulas in the case of a compactified D-dimensional spacetime, with four noncompact dimensions. We conclude in Sec. VIII. We relegate some of the more technical details to Appendixes A–FA–F.

II. SETUP

We will consider pointlike mass sources in some higher-dimensional spacetime, and we will investigate their effect on the spacetime geometry and on the emission of gravitational waves from binaries, in perturbation theory.

To this end we will compute the metric perturbation $\tilde{h}_{\mu\nu}$ by direct integration of Einstein's equations and not from the quadrupole formula as it is customary, because the quadruple formula actually fails when there is a compactified dimension. The reason for this is that the validity of the quadrupole formula relies on integration by parts. When the spatial coordinates are noncompact, the boundary terms that accompany the integration by parts are zero. However, when there are compactified extra dimensions, there are nonzero boundary terms, which are not straightforward to evaluate and which will contribute, in addition, to the usual quadrupole integral. Please see Appendix A for the modified expression.

Thus, we need to solve the metric perturbation directly from the Einstein equations. We will use the relaxed Einstein equations in the harmonic gauge [43].

For simplicity in most of this paper we will consider a 5D spacetime with coordinates x^M, with four noncompact dimensions x^μ and a fifth dimension, $x^5 = w$, compactified on a circle of radius $\mathcal{R}$, though we will occasionally point out how our results change in the case of additional compactified extra dimensions:

$$\begin{aligned} x^M &= (x^\mu, x^5, \ldots) = (t, \vec{x}, w, \ldots) \\ &\sim (t, \vec{x}, w + 2\pi\mathcal{R}, \ldots), \\ M &= 0, 1, 2, 3, 5, \ldots, D. \end{aligned} \tag{2.1}$$

We further denote the spatial coordinates by

$$x^I = (\vec{x}, w, \ldots) = (x, y, z, w, \ldots), \quad I = 1,2,3,5,\ldots,D, \tag{2.2}$$

and the spatial noncompact coordinates by

$$x^i = (x, y, z), \quad i = 1, 2, 3. \tag{2.3}$$

We set the speed of light to $c = 1$.

Let us consider a perturbation of the flat spacetime

$$h_{MN} \equiv g_{MN} - \eta_{MN}, \quad \eta_{MN} = \mathrm{diag}(-1, 1, 1, 1, 1, \ldots), \tag{2.4}$$

and let us also define

$$\tilde{h}^{MN} \equiv \eta^{MN} - \tilde{g}^{MN}, \qquad \tilde{g}^{MN} \equiv \sqrt{-g}g^{MN}, \tag{2.5}$$

where (g) is the determinant of the metric g_{MN}. As advertised, we take $\tilde{h}^{MN}$ to satisfy the Lorenz, or de Donder, or harmonic gauge condition:

$$\partial_M \tilde{h}^{MN} = 0. \tag{2.6}$$

To linear order in h_{MN}, $\tilde{h}_{MN}$ reduces to the usual trace-reversed metric perturbation:

$$\tilde{h}_{MN} \simeq h_{MN} - \frac{1}{2} h \eta_{MN}. \tag{2.7}$$

Then, the relaxed Einstein equations state [43]

$$\Box \tilde{h}^{MN} = -16\pi G^{(D)} \tau^{MN}, \tag{2.8}$$

where $G^{(D)}$ is the gravitational constant in the D-dimensional spacetime, $\Box = \partial_M \partial_N \eta^{MN}$, and where τ^{MN} is given by

$$\begin{aligned} \tau^{MN} &= (-g)(T^{MN} + t^{MN}_{\mathrm{LL}}) \\ &\quad + \frac{1}{16\pi G^{(5)}} (\tilde{h}^{MP}{}_{,Q} \tilde{h}^{NQ}{}_{,P} - \tilde{h}^{PQ} \tilde{h}^{MN}{}_{,PQ}). \end{aligned} \tag{2.9}$$

Last, T^{MN} is the matter energy-momentum tensor while t^{MN}_{LL} is the Landau-Lifshitz [44] gravitational energy-momentum pseudotensor. In a D-dimensional spacetime we have [45]

$$16\pi G^{(D)}(-g)t_{\rm LL}^{MN} = \tilde{g}^{MN}{}_{,P}\tilde{g}^{PQ}{}_{,Q} - \tilde{g}^{MP}{}_{,P}\tilde{g}^{NQ}{}_{,Q} + \frac{1}{2}g^{MN}g_{PQ}\tilde{g}^{PR}{}_{,S}\tilde{g}^{QS}{}_{,R} - (g^{MP}g_{QR}\tilde{g}^{NR}{}_{,S}\tilde{g}^{QS}{}_{,P} + g^{NP}g_{QR}\tilde{g}^{MR}{}_{,S}\tilde{g}^{QS}{}_{,P}) + g_{PQ}g^{RS}\tilde{g}^{MP}{}_{,R}\tilde{g}^{NQ}{}_{,S} + \frac{1}{4(D-2)}(2g^{MP}g^{NQ} - g^{MN}g^{PQ})[(D-2)g_{RS}g_{R'S'} - g_{RR'}g_{SS'}]\tilde{g}^{RR'}{}_{,P}\tilde{g}^{SS'}{}_{,Q}. \quad (2.10)$$

From the relaxed Einstein equations (2.8) and the harmonic gauge condition (2.6) it is easy to see that τ^{MN} obeys the conservation law

$$\partial_M \tau^{MN} = \partial_M((-g)(T^{MN} + t_{\rm LL}^{MN})) = 0. \quad (2.11)$$

III. MODIFIED KEPLER'S LAW

Let us first consider the scenario where there is one extra noncompact spatial dimension. Thus $D = 5$, the background is flat, and we assume that there is one matter source which is pointlike, of mass m, at rest. Then, the energy-momentum tensor is

$$T^{MN}(x^\mu, w) = m\delta^{M0}\delta^{N0}\delta^3(\vec{x})\delta(w). \quad (3.1)$$

The only nontrivial linearized metric fluctuation is $\tilde{h}_{\rm T}^{00}$, and it satisfies

$$\Box \tilde{h}_{\rm T}^{00} = -16\pi G^{(5)}T^{00}, \quad (3.2)$$

where $G^{(5)}$ is the gravitational constant in 5D. The solution is

$$\tilde{h}_{\rm T}^{00}(\vec{x}, w) = \frac{4G^{(5)}m}{\pi(R^2 + w^2)}, \qquad R^2 \equiv \vec{x}^2 = x^2 + y^2 + z^2. \quad (3.3)$$

The 5D linearized metric fluctuation $h_{\rm T}^{00} = (2/3)\tilde{h}_{\rm T}^{00}$[1] corresponds to a Newtonian potential[2]

$$V^{(5)}(R, w) = -\frac{4}{3}\frac{G^{(5)}m}{\pi(R^2 + w^2)}. \quad (3.4)$$

If the extra dimension is not flat, but compact, with an identification $w \sim w + 2\pi\mathcal{R}$, an observer sees a mass m at every $w = 2n\pi\mathcal{R}$, where $\mathcal{R}$ is the radius of compactification and $n \in \mathbb{Z}$ [46]. Summing over all such sources, the resulting linearized metric fluctuation $\tilde{h}_{\rm T}^{00}$ is periodic $\tilde{h}_{\rm T}^{00}(\vec{x}, w) \sim \tilde{h}_{\rm T}^{00}(\vec{x}, w + 2\pi n\mathcal{R})$,

$$\tilde{h}_{\rm T}^{00}(t, \vec{x}, w) = \frac{4G^{(5)}m}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{\vec{x}^2 + (w - 2n\pi\mathcal{R})^2}, \quad (3.5)$$

and, correspondingly, the Newtonian gravitational potential is given by

$$V^{(5,c)}(\vec{x}, w) = -\frac{4}{3}\frac{G^{(5)}m}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{R^2 + (w - 2n\pi\mathcal{R})^2}. \quad (3.6)$$

If the observers are located at the same w coordinate as the source (think of the matter source and observer living on the same brane at $w = 0$, and ignore for simplicity the backreaction of the brane on the geometry), then we are interested in $V^{(5,c)}(\vec{x}, w = 0)$.[3] Setting $w = 0$ and evaluating the sum over n in (3.6) yields

$$V^{(5,c)}(\vec{x}, w = 0) = -\frac{4}{3}\frac{G^{(5)}m}{lR}\coth\left(\frac{\pi R}{l}\right) = -\frac{1}{3}\tilde{h}_{\rm T}^{00}(t, \vec{x}, w = 0), \quad (3.7)$$

where

$$l = 2\pi\mathcal{R} \quad (3.8)$$

denotes the length of the compactified extra dimension. For a generalization of (3.7) to the case when the observer and the source are located at different positions in the compact dimension, please see Appendix B.

There are two useful limits of (3.7): one is the decompactification limit, when $l \gg R$, and the other is the

[1] In D spacetime dimensions this relation gets modified to $h_{\rm T}^{00} = (D-3)/(D-2)\tilde{h}_{\rm T}^{00}$, and the linearized metric is given by $ds^2 = (-1 + (D-3)/(D-2)\tilde{h}_{\rm T}^{00})dt^2 + (1 + 1/(D-2)\tilde{h}_{\rm T}^{00})d\vec{x}\cdot d\vec{x}$.

[2] Working with a D-dimensional spacetime, Eq. (3.4) generalizes to

$$V^{(D)}(R, \rho) = -\frac{1}{2}\frac{D-3}{D-2}\tilde{h}_{\rm T}^{00} = -\frac{D-3}{(D-2)}\frac{4}{\pi^{(D-3)/2}}\Gamma\left(\frac{D-1}{2}\right)\frac{G^{(D)}m}{(R^2 + \rho^2)^{D-3}},$$
$$\rho^2 = x^I x^I - R^2.$$

[3] For an investigation whether localized matter can arise in the context of the effective field theory see [47].

opposite, with $l \ll R$. In the first case, the Newtonian potential assumes the form of a 4D noncompact *space*

$$V^{(5,c)}(R, w=0) = -\frac{4}{3}\frac{G^{(5)}m}{\pi R^2} + \mathcal{O}(R/l),$$
$$R = \vec{x}^2 = x^i x^i, \qquad l \gg R, \tag{3.9}$$

whereas in the second case it is equal to the Newtonian potential in a 3D noncompact space plus exponential corrections[4]

$$V^{(5,c)}(R, w=0) = -\frac{4}{3}\frac{G^{(5)}m}{lR}(1 + 2e^{-2\pi R/l} + \mathcal{O}(e^{-4\pi R/l})),$$
$$l \ll R. \tag{3.10}$$

The exponential corrections look like a Yukawa potential and can be interpreted as being due to massive gravitons. From a 4D perspective, these massive gravitons correspond to nonuniform Fourier modes of the massless 5D gravitons on the circle $w \sim w + l$. We will have more to say about this in the following sections.

Next, let us consider a quasicircular binary with component masses m_1 and m_2 and binary separation r_{12}, with $r_{12} \gg l$. The matter energy-momentum tensor is given by

$$T^{MN}(x) = \sum_{a=1,2} m_a \int d\tau \frac{\dot{x}_a^M \dot{x}_a^N}{\sqrt{-g_{PQ}(x)\dot{x}_a^P \dot{x}_a^Q}} \delta^5(x - x_a(\tau)), \tag{3.11}$$

where $x_a^M(\tau)$ is the trajectory of the pointlike mass m_a with τ representing an affine parameter on the worldline. We can use reparametrization invariance to identify $x^0(\tau) = \tau$ and, assuming that the matter sources are located at $w_1(t) = w_2(t) = 0$ (i.e., confined to the same brane), to leading order we have

$$\vec{x}_{12}(t) = \vec{x}_1(t) - \vec{x}_2(t)$$
$$= (r_{12}\cos(\Omega t), r_{12}\sin(\Omega t), 0). \tag{3.12}$$

Further using (3.10) yields the effective potential of such a binary

$$V_{\rm eff} \simeq \frac{1}{2}\mu r_{12}^2 \Omega^2 - \frac{G_{\rm N}\mu M}{r_{12}}(1 + 2e^{-2\pi r_{12}/l}), \tag{3.13}$$

where

$$M = m_1 + m_2 \tag{3.14}$$

is the total mass of the binary and

$$\mu = (m_1 m_2)/M \tag{3.15}$$

is the reduced mass, while Ω is the orbital angular frequency. $G_{\rm N}$ is the 4D Newton constant, with[5]

$$G_{\rm N} \equiv \frac{4}{3}\frac{G^{(5)}}{l}. \tag{3.16}$$

The distance between the two sources is solved from the condition of the local extremum of $V_{\rm eff}$ with respect to r_{12}. This leads to the following modification to Kepler's law:

$$r_{12} \simeq \left(\frac{G_{\rm N}M}{\Omega^2}\right)^{1/3} \times \left\{1 + \frac{2}{3}\left(\frac{G_{\rm N}M}{\Omega^2}\right)^{1/3}\frac{2\pi}{l}\exp\left[-\frac{2\pi}{l}\left(\frac{G_{\rm N}M}{\Omega^2}\right)^{1/3}\right] + \frac{2}{3}\exp\left[-\frac{2\pi}{l}\left(\frac{G_{\rm N}M}{\Omega^2}\right)^{1/3}\right]\right\}, \tag{3.17}$$

where we have retained the first order correction to 4D Kepler's law.

IV. PERFORMING THE KALUZA-KLEIN REDUCTION WITH 5D POINTLIKE MATTER SOURCES

One might be tempted to think that the physics of the binary system in a 5D spacetime is that of a binary (two pointlike masses) coupled to the fields obtained via the Kaluza-Klein reduction of the 5D metric, namely gravity, dilaton, and Maxwell fields, and with the latter two being set to zero. Then, to leading order, neglecting all corrections coming from massive modes on the fifth-dimensional circle, one recovers the 4D matter (the binary) plus gravity setup. However, this is not the case. To better understand this issue, we take a quick detour and review the Kaluza-Klein reduction of the 5D system composed of gravity plus pointlike sources. This is a self-contained section of the paper, and for the purpose of performing the Kaluza-Klein

[4]For a D-dimensional spacetime $R^{3,1} \times T^{D-4}$, with three noncompact spatial dimensions and the compact space being torus, the generalization of (3.10) is

$$V^{(D)}(x^I) = -\frac{2(D-3)}{D-2}\frac{G^{(D)}m}{\mathrm{Vol(Compact\ Space)}}\frac{1}{R}(1 + \mathcal{O}(e^{-2\pi R/l})),$$

where l is the length of the largest of the cycles of the torus.

[5]For a D-dimensional spacetime $R^{3,1} \times T^{D-4}$, with three noncompact spatial dimensions and the compact space being torus, the 4D Newton constant is given by

$$G_{\rm N} = \frac{2(D-3)}{D-2}\frac{G^{(D)}}{\mathrm{Vol(Compact\ Space)}},$$

with $G^{(D)}$ the D-dimensional gravitational constant.

reduction we introduce the notation G_{MN} for the 5D metric and $g_{\mu\nu}$ for the 4D metric.

Consider five-dimensional gravity

$$\mathcal{S}_{5\mathrm{D}} = \frac{1}{l\kappa}\int d^5x\sqrt{-\det G_{MN}}R[G_{MN}], \tag{4.1}$$

where $\kappa = 16\pi G$ (we work in units where $c = 1$) and $G = G^{(5)}/l$. The Kaluza-Klein reduction ansatz to 4D is (see, for example, [48])

$$G_{MN} = e^{-\varphi/3}\begin{pmatrix} g_{\mu\nu} + \kappa e^{\varphi}A_\mu A_\nu & \sqrt{\kappa}e^{\varphi}A_\mu \\ \sqrt{\kappa}e^{\varphi}A_\nu & e^{\varphi}\end{pmatrix}, \tag{4.2}$$

where all the fields in (4.2) are functions of the 4D coordinates, x^μ, only.

Substituting (4.2) into the 5D Einstein-Hilbert action yields the 4D Einstein-Maxwell-dilaton action

$$\mathcal{S}_{\mathrm{KK}} = \frac{1}{\kappa}\int d^4x\sqrt{-g}\left(R[g_{\mu\nu}] - \frac{\kappa}{4}e^{\varphi}F_{\mu\nu}F^{\mu\nu} - \frac{1}{6}\partial_\mu\varphi\partial^\mu\varphi\right). \tag{4.3}$$

Note that we can find solutions with a vanishing dilaton as long as the Maxwell field is pure gauge. (The dilaton equation is sourced by the Maxwell field, so setting the dilaton to zero in general would lead to an inconsistent Kaluza-Klein truncation.)

By introducing the rescaled dilaton and Maxwell field as

$$\varphi = -2\sqrt{3}\phi, \qquad A_\mu = \frac{2}{\sqrt{\kappa}}\bar{A}_\mu, \tag{4.4}$$

we can rewrite the Einstein-Maxwell-dilaton action as

$$\mathcal{S}_{\mathrm{KK}} = \frac{1}{\kappa}\int d^4x\sqrt{-g}(R[g_{\mu\nu}] - e^{-2\sqrt{3}\phi}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} - 2\partial_\mu\phi\partial^\mu\phi). \tag{4.5}$$

This agrees with (II.1) of [19]. We will use the rescaled dilaton (4.4) to make contact with [19] in Sec. VI.

Consider now adding a pointlike source of mass m to the 5D action:

$$\mathcal{S}_{\mathrm{matter,5D}} = -m\int d\tau\sqrt{-\dot{x}^M(\tau)\dot{x}^N(\tau)G_{MN}(x(\tau))}, \tag{4.6}$$

where τ is an affine parameter on the source's worldline and $\dot{x}^M = \frac{d}{d\tau}x^M$. This will source the 5D metric in the usual way, leading to the 5D perturbative analysis performed in the previous section and continued in the next.

Here we would like to point out that the 4D dilaton is also being sourced by the 5D matter (4.6). Specifically, for a source that is not moving in the fifth dimension (note that this is a solution to its equation of motion in the context of a 5D metric which is independent of the fifth coordinate), the reduction of the 5D action (4.6) yields

$$\mathcal{S}_{\mathrm{matter,KK}} = -m\int d\tau e^{-\varphi/6}\sqrt{-\dot{x}^\mu(\tau)\dot{x}^\nu(\tau)(g_{\mu\nu} + \kappa e^{\varphi}A_\mu A_\nu)}, \tag{4.7}$$

where the 4D fields are evaluated on the worldline.

In contrast, adding a 4D neutral source of mass m to the Einstein-Maxwell-dilaton action is done by considering

$$\mathcal{S}_{\mathrm{matter},4D} = -m\int d\tau\sqrt{-\dot{x}^\mu(\tau)\dot{x}^\nu(\tau)g_{\mu\nu}}. \tag{4.8}$$

The main message here is that 5D matter couples not only to the 4D graviton but also to the dilaton and Maxwell field. Most importantly, while we can find solutions with a vanishing dilaton to the 4D action with a 4D matter source, we cannot find solutions with a vanishing dilaton to the 4D Kaluza-Klein reduced action when the matter source is a 5D pointlike source; this can be understood by noticing that in (4.7) there is a linear coupling between the dilaton and the matter source, when expanding in small fluctuations about a vanishing dilaton. In effect, the 4D mass in the Kaluza-Klein reduced action is modulated by the dilaton,

$$m_{\mathrm{effective}} = m\exp(-\varphi/6), \tag{4.9}$$

as evidenced by (4.7).

Last, in the context of Kaluza-Klein reduction of a higher-dimensional gravitational theory, the 4D gravitational coupling constant is $1/(16\pi G)$, as we can see from (4.3), with

$$G = G^{(5)}/l. \tag{4.10}$$

However, G and G_{N} [which shows up in the Newtonian potential and is given in (3.16) for $D = 5$] are not equal: they differ by a factor. This is different from 4D GR, where G and G_{N} are equal to one another. The explanation for this mismatch stems from the fact that in an effective 4D theory, the interaction between two masses is not only gravitational, but there are additional contributions mediated by 4D scalars (e.g., dilaton) as well.

V. METRIC PERTURBATIONS

We now return to our main problem, namely finding the 5D metric fluctuations sourced by a binary in a spacetime with one compact dimension of radius $\mathcal{R}$.

We separately find the contribution of the matter energy-momentum tensor and of the Landau-Lifshitz pseudotensor to the metric fluctuations: we denote these by $\tilde{h}_{\mathrm{T}}^{MN}$ and $\tilde{h}_{\mathrm{t}}^{MN}$. There is one more contribution to the metric fluctuations from the remainder of the relaxed Einstein equation source τ^{MN} [see (2.9)]. However, to leading order, the extra terms in τ^{MN} contribute only to the 00 component of the metric fluctuations, $\tilde{h}^{00}$. We will compute the 0 components of the metric fluctuations not by direct integration, but by using the harmonic gauge (2.6). So, for the remaining fluctuations $\tilde{h}^{IJ} = (\tilde{h}^{ij}, \tilde{h}^{i5}, \tilde{h}^{55})$ we will evaluate first the contribution from the matter source, and then use this in the Landau-Lifshitz pseudotensor to evaluate the nonlinear metric fluctuations. Despite $\tilde{h}^{IJ}_{\mathrm{t}}$ being nonlinear, it is actually of the same order as $\tilde{h}^{IJ}_{\mathrm{T}}$ in a velocity expansion. Last, we add the two contributions to find the metric perturbation to second order in velocities, i.e., leading order in post-Newtonian expansion.

A. Metric perturbations: Contribution from the matter sources

We begin by computing the perturbations sourced by the matter stress-energy tensor. The energy-momentum tensor T^{MN} of a system of point masses at $w = 0$ is given by

$$T^{MN}(t, \vec{x}, w) = \sum_a \frac{P_a^M P_a^N}{P_a^0} \delta^3(\vec{x} - \vec{x}_a(t))\delta(w), \tag{5.1}$$

where $P_a^M = m\dot{x}_a^M / \sqrt{-g_{PQ}\dot{x}_a^P \dot{x}_a^Q}$ is the M component of the momentum of particle a. We parametrize the particles' trajectories with $\vec{x}_a = \vec{x}_a(t)$ and take $x_a^0 = t$. For a binary source, we specifically use (3.12).

From (2.8), the linearized fluctuations are given by

$$\tilde{h}^{MN}_{\mathrm{T}}(t,\vec{x},w) = -16\pi G^{(5)} \int dt' d^3\vec{x}' dw' \mathcal{G}^{(5,c)}(t,\vec{x},w;t',\vec{x}',w') \times T^{MN}(t',\vec{x}',w'), \tag{5.2}$$

where $\mathcal{G}^{(5,c)}(t,\vec{x},w;t',\vec{x}',w')$ is the (scalar) retarded compactified Green's function in 5D. The retarded Green's function in flat 5D can be represented as [49]

$$\mathcal{G}^{(5)}(t,\vec{x},w;t',\vec{x}',w') = -\frac{\theta(t-t')}{4\pi^2 r}\frac{\partial}{\partial r}\frac{\theta(t-t'-r)}{\sqrt{(t-t')^2 - r^2}}, \tag{5.3}$$

with

$$r^2 = (\vec{x} - \vec{x}')^2 + (w - w')^2, \tag{5.4}$$

and where θ denotes the Heaviside step-function.[6]

Then, starting from (5.3), we can write the compactified 5D retarded Green's function, $\mathcal{G}^{(5,c)}(x, y)$,

$$\mathcal{G}^{(5,c)}(t,\vec{x},w;t',\vec{x}',w') = -\sum_{n=-\infty}^{\infty} \frac{\theta(t-t')}{4\pi^2 r_n}\frac{\partial}{\partial r_n}\frac{\theta(t-t'-r_n)}{\sqrt{(t-t')^2 - r_n^2}}, \tag{5.5}$$

where $r_n^2 = (\vec{x} - \vec{x}')^2 + (w - w' - nl)^2$. For practical purposes, the compactified Green's function expression given in (5.5) is not very useful.[7] Instead, we will use the equivalent representation of the compactified retarded Green's function in terms of a sum/integral over Fourier modes[8] (see also Appendix D):

$$\mathcal{G}^{(5,c)}(x^\mu, w; x'^\mu, w') = -\frac{1}{l}\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip\cdot(x-x')} e^{i2\pi n(w-w')/l}}{-(p_0 + i\epsilon)^2 + \vec{p}^2 + (2\pi n/l)^2}, \tag{5.6}$$

where ϵ is an infinitesimally small positive number. In (5.6) each term in the sum can be interpreted as the 4D retarded Green's function of a massive particle of mass $m_s = (2\pi n/l)$. These massive particles are nothing else but the massive Kaluza-Klein graviton states. Thus we expect that in the limit when $r \gg l$, and for slow moving sources, Eq. (5.6) will reduce to the 4D retarded Green's function of a massless particle, corresponding to $n = 0$, plus exponentially suppressed corrections, with the leading order correction coming from the least massive mode, corresponding to $n = 1$. Indeed, the $n = 0$ term in the sum above corresponds to massless 4D excitations, and the retarded 4D Green's function $\theta(t-t')\delta(t-t'-|\vec{x}-\vec{x}'|)/(4\pi|\vec{x}-\vec{x}'|)$. The nonzero n terms are associated with massive 4D excitations. The retarded Green's function for a massive 4D scalar of mass m_s is

$$-\int \frac{d^4p}{(2\pi)^4} \frac{e^{ip\cdot(x-x')}}{-(p_0+i\epsilon)^2 + \vec{p}^2 + m_s^2} = -\frac{\theta(t-t')}{4\pi}\left[\frac{\delta(t-t'-|\vec{x}-\vec{x}'|)}{|\vec{x}-\vec{x}'|} - \theta(t-t'-|\vec{x}-\vec{x}'|)\frac{m_s J_1(m_s\sqrt{(t-t')^2 - |\vec{x}-\vec{x}'|^2})}{\sqrt{(t-t')^2 - |\vec{x}-\vec{x}'|^2}}\right]. \tag{5.7}$$

[6]For the reader accustomed to 4D expressions, we want to point out that even though the 5D retarded Green's function does not have support only on the light cone (as opposed to the massless 4D retarded Green's function which has support on the light cone only), it does have support inside the light cone, and it is therefore causal. This is one of the peculiar features of odd dimension spacetimes.

[7]However, for the purpose of demonstrating how one could use (5.5) in an explicit calculation, please see Appendix C for another derivation of the Newtonian potential in 5D GR.

[8]The Dirac-delta function, written as a distribution on the space of periodic functions with period l, is $\delta(w-w') = (1/l)\sum_{n=-\infty}^{\infty}\exp(i2\pi n(w-w')/l)$.

Consider next the propagation of a periodic signal $e^{i\omega t'} f(\vec{x}')$, with $f(\vec{x}')$ localized near the origin (similar to the case encountered with the binary sources). In the leading multipole expansion, for $|\vec{x}-\vec{x}'| \simeq |\vec{x}| = R$ we are left with evaluating

$$\begin{aligned}\int_{-\infty}^{t} dt' \Bigg\{ &\frac{\delta(t-t'-R)}{R} \\ &- \theta(t-t'-R) \frac{m_s J_1(m_s\sqrt{(t-t')^2-R^2})}{\sqrt{(t-t')^2-R^2}} \Bigg\} e^{i\omega t'} \\ &= \frac{e^{i\omega(t-R)}}{R} - m_s e^{i\omega t} I_{\frac{1}{2}}\left[\frac{R}{2}\left(\sqrt{m_s^2-\omega^2}-i\omega\right)\right] \\ &\quad \times K_{\frac{1}{2}}\left[\frac{R}{2}\left(\sqrt{m_s^2-\omega^2}+i\omega\right)\right] \\ &= \frac{e^{i\omega t}}{R} e^{-R\sqrt{m_s^2-\omega^2}}. \end{aligned} \tag{5.8}$$

If $m_s \gg \omega$ (which is the case for slow moving binary sources since $m_s = \frac{2\pi n}{l}$, $l \ll r_{12}$, $\Omega r_{12} \ll 1$), the approximate result from (5.8) would be simply $(1/R) e^{i\omega t} e^{-2\pi n\frac{R}{l}}$, which is the anticipated exponentially suppressed contribution.

So, putting everything together, the signal propagating from a source that is localized near the origin $f(\vec{x}', w') e^{i\omega t'}$ to a spacetime coordinate $(t, \vec{x}, w)$ is

$$\begin{aligned}\int dt' \int d^3\vec{x}' \int_0^l dw' \mathcal{G}^{(5,c)}(t,\vec{x},w;t',\vec{x}',w') f(\vec{x}',w') e^{i\omega t'} \\ \simeq -\frac{e^{i\omega(t-R)}}{4\pi l R} \int d^3\vec{x}' \int_0^l dw' f(\vec{x}',w') \\ - \sum_{n, n\neq 0} \frac{e^{i\omega t}}{4\pi l R} e^{-R\sqrt{(2\pi n/l)^2-\omega^2}} \int d^3\vec{x}' \\ \times \int_0^l dw' f(\vec{x}',w') e^{2\pi i n(w-w')/l}. \end{aligned} \tag{5.9}$$

In the limit of a small extra dimension and a slow moving source (i.e., $2\pi/l \gg \omega$), we find that the leading contribution is

$$-\frac{1}{4\pi l R} \int d^3\vec{x}' \int_0^l dw' f(\vec{x}',w') e^{i\omega(t-R)}, \tag{5.10}$$

which corresponds to a signal that propagates uniformly in w and radially in the noncompact space. The massive Kaluza-Klein gravitons give an exponentially suppressed contribution of the form

$$\begin{aligned} -2\sum_{n=1}^{\infty} \frac{e^{i\omega t}}{4\pi l R} e^{-2\pi n R/l} \int d^3\vec{x}' \\ \times \int_0^l dw' f(\vec{x}',w') \cos(2\pi n(w-w')/l). \end{aligned} \tag{5.11}$$

At large distances $R \gg l$ these massive state contributions can be safely ignored.

In particular, from (5.2), sourced by the binary equations (3.11) and (3.12) energy-momentum, and using the approximations in (5.10) and (5.11) in the far-field slow motion limit, we find, for example, the (x, y) component as

$$\begin{aligned} \tilde{h}_{\mathrm{T}}^{xy}(t,\vec{x},w=0) \\ \simeq -\frac{3}{2}\frac{G_{\mathrm{N}}\mu}{R} r_{12}^2 \Omega^2 \left(\sin[2\Omega(t-R)] + \sum_{n=1}^{\infty} e^{-\frac{2\pi R}{l}n} \sin(2\Omega t) \right), \end{aligned} \tag{5.12}$$

where $R = |\vec{x}|$ is the 3D distance between the sources and the observer and μ is the reduced mass defined in (3.15). The leading correction due to the extra compact dimension to the part of the metric fluctuation that is sourced by the matter energy-momentum tensor is given by the $n = 1$ term in the sum in (5.12), and it is an exponentially suppressed correction. Since $l \ll R$, the correction $\exp(-\frac{2\pi R}{l})$ is extremely small and can safely be ignored (after all we have already ignored corrections of order r_{12}/R, and we expect $l < r_{12}$).

B. Metric perturbations: The nonlinear contribution from the Landau-Lifshitz pseudotensor

We denote the nonlinear metric perturbations sourced by t_{LL}^{MN} in (2.8) as $\tilde{h}_{\mathrm{t}}^{MN}$. In the slow motion limit ($v \ll 1$), since $\tilde{h}_{\mathrm{T}}^{00} \sim \mathcal{O}(1)$, $\tilde{h}_{\mathrm{T}}^{0i} \sim \mathcal{O}(v)$, $\tilde{h}_{\mathrm{T}}^{ij} \simeq \mathcal{O}(v^2)$, and $\tilde{h}_{\mathrm{T}}^{M5} \simeq 0$, the leading order contribution for $I, J = 1, 2, 3, 5, \ldots, D$ comes from

$$\begin{aligned} \tilde{h}_{\mathrm{t}}^{IJ}(x) &= -16\pi G^{(D)} \int d^5y \mathcal{G}^{(D,c)}(x,y) t_{\mathrm{LL}}^{IJ}(y) \\ &\simeq -\frac{D-3}{4(D-2)} \int d^5y \mathcal{G}^{(D,c)}(x,y) \partial_M \tilde{h}_{\mathrm{T}}^{00}(y) \\ &\quad \times \partial_N \tilde{h}_{\mathrm{T}}^{00}(y)(2\eta^{IM}\eta^{JN} - \eta^{IJ}\eta^{MN}), \end{aligned} \tag{5.13}$$

where $\mathcal{G}^{(D,c)}(x,y)$ is the compactified retarded Green's function in D dimensions. Specializing to the case $D = 5$, we get

$$\begin{aligned} \tilde{h}_{\mathrm{t}}^{IJ}(x) \simeq -\frac{1}{6}(2\eta^{IM}\eta^{JN} - \eta^{IJ}\eta^{MN}) \\ \times \int d^5y \mathcal{G}^{(5,c)}(x,y) \partial_M \tilde{h}_{\mathrm{T}}^{00}(y) \partial_N \tilde{h}_{\mathrm{T}}^{00}(y), \end{aligned} \tag{5.14}$$

where $\mathcal{G}^{(5,c)}(x,y)$ was previously defined in (5.5) and (5.6). As discussed in the previous subsection, in the far field limit (when the distance to the source is much larger than the distances between sources) with the observer and the sources located at $w = 0$, and in the slow motion approximation, the compactified retarded 5D Green's function reduces effectively to a 4D retarded Green's function. The first order correction, which is proportional to $\exp(-\frac{2\pi R}{l})$ is negligible, and Eq. (5.14) becomes

$$\tilde{h}_{\rm t}^{IJ}(t,\vec{x},0) \simeq \frac{1}{4\pi l R}\frac{1}{6}(2\eta^{IN}\eta^{JM} - \eta^{IJ}\eta^{MN})\int d^3y \times \int_0^l dw \partial_M \tilde{h}_{\rm T}^{00}(t-R,\vec{y},w)\partial_N \tilde{h}_{\rm T}^{00}(t-R,\vec{y},w). \tag{5.15}$$

The most striking difference in the nonlinear contribution to the metric fluctuations in 5D with respect to the 4D case is the coefficient $1/6$ on the right-hand side of (5.15) relative to the more familiar coefficient of $1/8$ in 4D.

We now return to the specific case of a binary system at $w=0$, with masses m_1 and m_2 moving in the (x^1,x^2) plane. The first observation is that in (5.15), to leading order in velocities we only need to consider the action of the spatial derivatives on $\tilde{h}_{\rm T}^{00}$, which does not contain an explicit t dependence. Restricting now the summation over M, N indices to spatial indices K, L, consider the term $\partial_K \tilde{h}_{\rm T}^{00}\partial_L \tilde{h}^{00}$ in (5.15). Since $\tilde{h}_{\rm T}^{00} = \tilde{h}_{\rm T,1}^{00} + \tilde{h}_{\rm T,2}^{00}$, there will be four terms. However, we are only interested in the two crossing terms because noncrossing terms will be simply regularized and effectively be dropped out. In addition, we will replace the spatial derivative on y to the derivative with respect to the position of the sources (with a minus sign). We can do so because of translation invariance of the flat background which implies that the linearized fluctuation $\tilde{h}_{\rm T,a}^{00}$ only depends on $\vec{y}-\vec{y}_a$ and $w-w_a$. We will use $\partial_K^{(a)}$ to represent partial derivatives with respect to the coordinates of the source a, $\partial/\partial y_a^K$. With the help of this little trick, we can simplify (5.15):

$$\tilde{h}_{\rm t}^{IJ}(t,\vec{x},w) \simeq \frac{(2\eta^{IK}\eta^{JL} - \eta^{IJ}\eta^{KL})(\partial_K^{(1)}\partial_L^{(2)} + \partial_K^{(2)}\partial_L^{(1)})}{24\pi l R} \times \int_{\rm NZ} d^3y \int_0^l dw \tilde{h}_{\rm T,1}^{00}(t-R,\vec{y},w)\tilde{h}_{\rm T,2}^{00}(t-R,\vec{y},w), \tag{5.16}$$

where $\int_{\rm NZ} d^3y$ denotes integration in the near zone (NZ) region (i.e., in the vicinity of the sources) which is the region that contributes the most to the volume integral $\int d^3y$ [43]. Because of the near-zone approximation, the wave propagation is almost instantaneous and we can use for $\tilde{h}_{\rm T,a}^{00}$ in the NZ region the result from (3.5),

$$\tilde{h}_{\rm NZ,T,a}^{00}(t,\vec{y},w) = -\frac{4}{\pi}G^{(5)}m_a \sum_n \frac{1}{(\vec{y}-\vec{x}_a(t))^2 + (w-w_a+nl)^2}. \tag{5.17}$$

Substituting (5.17) into the integrand (5.16), we can use the infinite sums to extend the integration region over w first, and then we can perform the remaining sum exactly:

$$\begin{aligned}\int_0^l dw' \tilde{h}_{\rm NZ,T,1}^{00}(t,\vec{y},w')\tilde{h}_{\rm NZ,T,2}^{00}(t,\vec{y},w') &= \sum_{n_1,n_2}\int_0^l dw' \frac{16 m_1 m_2 (G^{(5)})^2}{\pi^2(R_1^2 + (w'-w_1+n_1 l)^2)(R_2^2 + (w'-w_2+n_2 l)^2)} \\ &= \int_{-\infty}^{+\infty} dw' \sum_{n_2}\frac{16 m_1 m_2 (G^{(5)})^2}{\pi^2(R_1^2 + w'^2)(R_2^2 + (w'+w_1-w_2+n_2 l)^2} \\ &= \frac{16 m_1 m_2 (G^{(5)})^2 (R_1+R_2)}{\pi R_1 R_2}\sum_{n_2}\frac{1}{(R_1+R_2)^2 + (w_1-w_2+n_2 l)^2} \\ &= l\frac{9 m_1 m_2 G_{\rm N}^2}{R_1 R_2}\frac{\sinh\frac{2\pi(R_1+R_2)}{l}}{\cosh\frac{2\pi(R_1+R_2)}{l} - \cos\frac{2\pi(w_1-w_2)}{l}} \\ &\simeq l\frac{9 m_1 m_2 G_{\rm N}^2}{R_1 R_2}\left(1 + 2e^{-\frac{2\pi(R_1+R_2)}{l}}\cos\frac{2\pi(w_1-w_2)}{l}\right),\end{aligned} \tag{5.18}$$

where $R_1 = |\vec{y}-\vec{x}_1(t)|$, where $R_2 = |\vec{y}-\vec{x}_2(t)|$, and where $G_{\rm N}$ was previously defined in (3.16). In the last step in (5.18) we used $l \ll r_{12} \le R_1 + R_2$, with r_{12} the binary separation distance. It is important to keep the explicit w_i dependence because we will still have to take derivatives with respect to the position of the sources in the 5D spacetime. Substituting (5.18) into (5.16) we obtain

$$\tilde{h}_{\rm t}^{IJ}(x) \simeq \frac{3}{2}\frac{m_1 m_2 G_{\rm N}^2}{2\pi R}(2\eta^{IK}\eta^{JL} - \eta^{IJ}\eta^{KL})(\partial_K^{(1)}\partial_L^{(2)} + \partial_K^{(2)}\partial_L^{(1)}) \times \left(-\pi r_{12} + l e^{-\frac{2\pi r_{12}}{l}}\cos\frac{2\pi(w_1-w_2)}{l}\right). \tag{5.19}$$

For more details on how the integration was performed in (5.19), please see Appendix E.

In particular, from (5.19), for a binary at $w_1 = w_2 = 0$ as in (3.11) and (3.12), we obtain, e.g., the (x, y) component of the metric perturbation as

$$\begin{aligned}\tilde{h}_{\mathrm{t}}^{xy}(t,\vec{x},0)&\\ \simeq -\frac{3}{2}\frac{m_1 m_2 G_{\mathrm{N}}^2}{R}&(\partial_1^{(1)}\partial_2^{(2)}+\partial_1^{(2)}\partial_2^{(1)})\left(r_{12}-\frac{l}{\pi}e^{-\frac{2\pi r_{12}}{l}}\right)\\ \simeq -\frac{3}{2}\frac{m_1 m_2 G_{\mathrm{N}}^2}{R r_{12}}&\left(1+2e^{-\frac{2\pi r_{12}}{l}}+\frac{4\pi r_{12}}{l}e^{-\frac{2\pi r_{12}}{l}}\right)\sin[2\Omega(t-R)].\end{aligned} \tag{5.20}$$

We can further use the modified Kepler's law $\Omega^2 = \frac{G_{\mathrm{N}}M}{r_{12}^3}(1 + 2e^{-\frac{2\pi r_{12}}{l}} + \frac{4\pi r_{12}}{l}e^{-\frac{2\pi r_{12}}{l}})$ to cast it into a more familiar form:

$$\tilde{h}_{\mathrm{t}}^{xy}(t,\vec{x},0) \simeq -\frac{3}{2}\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\sin[2\Omega(t-R)]. \tag{5.21}$$

C. Gravitational waves from a binary source in a 5D spacetime

Similar calculations to the ones we presented in explicit detail in the previous sections yield the following expressions for the other nonzero linearized fluctuations $\tilde{h}_{\mathrm{T}}^{IJ}$ (sourced by the matter energy-momentum tensor), as well as the leading order nonlinear fluctuations, $\tilde{h}_{\mathrm{t}}^{IJ}$ (sourced by the Landau-Lifshitz pseudotensor):

$$\begin{aligned}&\tilde{h}_{\mathrm{T}}^{xx}(t,\vec{x},0) \simeq 3\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\sin^2[\Omega(t-R)],\\ &\tilde{h}_{\mathrm{t}}^{xx}(t,\vec{x},0) \simeq -3\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\cos^2[\Omega(t-R)],\\ &\tilde{h}_{\mathrm{T}}^{yy}(t,\vec{x},0) \simeq 3\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\cos^2[\Omega(t-R)],\\ &\tilde{h}_{\mathrm{t}}^{yy}(t,\vec{x},0) \simeq -3\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\sin^2[\Omega(t-R)],\\ &\tilde{h}_{\mathrm{T}}^{zz}(t,\vec{x},0) \simeq 0, \qquad \tilde{h}_{\mathrm{t}}^{zz}(t,\vec{x},0) \simeq 0,\\ &\tilde{h}_{\mathrm{T}}^{ww}(t,\vec{x},0) \simeq 0,\\ &\tilde{h}_{\mathrm{t}}^{ww}(t,\vec{x},0) \simeq -3\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\left(1-4\frac{2\pi r_{12}}{l}e^{-\frac{2\pi r_{12}}{l}}\right),\end{aligned} \tag{5.22}$$

where we recall that μ is the reduced mass of the binary (3.15). [As a caveat, we would like to point out that we cannot set $z_1 = z_2 = w_1 = w_2 = 0$ until the derivatives in (5.19) have been taken, and the vanishing of $\tilde{h}_{\mathrm{t}}^{zz}$ is not trivial.] As advertised, both $\tilde{h}_{\mathrm{T}}^{IJ}$ and $\tilde{h}_{\mathrm{t}}^{IJ}$ are of the same order in velocities.

To second order in velocities, the nonzero metric fluctuations $\tilde{h}^{IJ}$ are obtained by adding the linearized $\tilde{h}_{\mathrm{T}}$ and nonlinear $\tilde{h}_{\mathrm{t}}$:

$$\begin{aligned}&\tilde{h}^{xy}(t,\vec{x},0) = \tilde{h}_{\mathrm{T}}^{xy}+\tilde{h}_{\mathrm{t}}^{xy} \simeq -3\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\sin[2\Omega(t-R)],\\ &\tilde{h}^{xx}(t,\vec{x},0) = \tilde{h}_{\mathrm{T}}^{xx}+\tilde{h}_{t}^{xx} \simeq -3\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\cos[2\Omega(t-R)],\\ &\tilde{h}^{yy}(t,\vec{x},0) = \tilde{h}_{\mathrm{T}}^{yy}+\tilde{h}_{\mathrm{t}}^{yy} \simeq 3\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\cos[2\Omega(t-R)],\\ &\tilde{h}^{ww}(t,\vec{x},0) = \tilde{h}_{\mathrm{T}}^{ww}+\tilde{h}_{\mathrm{t}}^{ww}\\ &\qquad \simeq -3\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\left(1-4\frac{2\pi r_{12}}{l}e^{-\frac{2\pi r_{12}}{l}}\right).\end{aligned} \tag{5.23}$$

According to our discussion in Secs. V A [see (5.11)] and V B [see (5.19)], the metric fluctuations $\tilde{h}^{IJ}(x^\mu, w)$ are equal to $\tilde{h}^{IJ}(x^\mu, w = 0)$, up to exponentially suppressed corrections. However, the biggest change to the luminosity of the gravitational waves and the phase of the gravitational waveform comes from the leading order terms retained in (5.23).

The extension of the results given in (5.23) to a compactified D-dimensional spacetime is straightforward:

$$\tilde{h}^{(D)IJ} = \frac{D-2}{2(D-3)}\begin{pmatrix}\tilde{h}^{(4)ij} & 0\\ 0 & (-4\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2)\delta^{pq}\end{pmatrix}, \quad i,j = 1,2,3 \quad \text{and} \quad p,q = 5,6,\ldots,D, \tag{5.24}$$

where $\tilde{h}^{(4)ij}$ denotes the 4D gravitational waves sourced by a binary with the same characteristics as ours: reduced mass μ, separation distance r_{12}, angular frequency Ω, and location $z = 0$:

$$\tilde{h}^{(4)ij} = \begin{pmatrix}-4\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\cos[2\Omega(t-R)] & -4\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\sin[2\Omega(t-R)] & 0\\ -4\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\sin[2\Omega(t-R)] & 4\frac{G_{\mathrm{N}}\mu}{R}r_{12}^2\Omega^2\cos[2\Omega(t-R)] & 0\\ 0 & 0 & 0\end{pmatrix}, \qquad i,j = 1,2,3.$$

Last, the remaining metric fluctuations $\tilde{h}^{0M}$ can be obtained either by direct integration or, more easily, by using the harmonic gauge (2.6) condition. In the next sections we will use the latter.

VI. THE LUMINOSITY OF GRAVITATIONAL WAVES

In this section we compute the luminosity of gravitational waves. We work in the harmonic gauge (2.6), without specializing to the more commonly used transverse-traceless gauge (for a comparison, see Appendix F).

There is one more subtlety we would like to comment on before we begin. In the D-dimensional gravitational theory, the only coupling constant is the gravitational constant $G^{(D)}$ of the Einstein-Hilbert action. After performing the Kaluza-Klein reduction, the effective 4D theory has the gravitational constant $G = G^{(D)}/\mathrm{Vol(Compact\ Space)}$ and Newton's constant is $G_{\mathrm{N}} = (2(D-3)/(D-2))G$. In contrast, in a strictly 4D theory of gravity coupled to matter, we would have $G = G_{\mathrm{N}}$.

In our subsequent comparisons between the predictions of the compactified higher-dimensional gravity theory and 4D GR we will identify Newton's constants in the two theories.

We use the gravitational energy-momentum pseudotensor t_{LL}^{MN} given in (2.10). Since t_{LL}^{MN} is already second order in the metric fluctuations, we can use the linearized approximation for $\tilde{h}^{MN}$, Eq. (2.7), to obtain

$$\begin{aligned}
16\pi G^{(D)} t_{\mathrm{LL}}^{MN} & \\
\simeq \tilde{h}^{MN}{}_{,P}\tilde{h}^{PQ}{}_{,Q} - \tilde{h}^{MP}{}_{,P}\tilde{h}^{NQ}{}_{,Q} + \frac{1}{2}\eta^{MN}\tilde{h}^{PR}{}_{,Q}\tilde{h}^{Q}{}_{P,R} & \\
- (\tilde{h}^{MP}{}_{,Q}\tilde{h}_{P}{}^{Q,N} + \tilde{h}^{NP}{}_{,Q}\tilde{h}_{P}{}^{Q,M}) & \\
+ \tilde{h}^{MP,Q}\tilde{h}^{N}{}_{P,Q} + \frac{1}{2}\tilde{h}^{PQ,M}\tilde{h}_{PQ}{}^{,N} - \frac{1}{4}\eta^{MN}\tilde{h}^{PQ,R}\tilde{h}_{PQ,R} & \\
- \frac{1}{4(D-2)}(2\tilde{h}^{,M}\tilde{h}^{,N} - \eta^{MN}\tilde{h}^{,P}\tilde{h}_{,P}), & \qquad (6.1)
\end{aligned}$$

where the indices are raised and lowered with the Minkowski metric. Further imposing the Lorenz gauge (2.6) and performing short-wavelength averaging,[9] Eq. (6.1) becomes

$$\langle (t_{\mathrm{LL}})_{MN}\rangle \simeq \frac{1}{32\pi G^{(D)}}\left\langle \partial_M\tilde{h}_{PQ}\partial_N\tilde{h}^{PQ} - \frac{1}{D-2}\partial_M\tilde{h}\partial_N\tilde{h}\right\rangle. \qquad (6.2)$$

The total energy carried away by gravitational waves is given by the following volume integral:

$$E_{\mathrm{GW}} = \int d^3\vec{x}\int_0^l dw t_{\mathrm{LL}}^{00}(t, x^I) \simeq l\int d^3\vec{x} t_{\mathrm{LL}}^{00}(t, \vec{x}), \qquad (6.3)$$

where in the last step we used that, to leading order, the metric fluctuations propagate uniformly in w. Then the rate of change of the radiated energy is

$$\begin{aligned}
\dot{E}_{\mathrm{GW}} = \frac{dE_{\mathrm{GW}}}{dt} &= \int d^3\vec{x}\int_0^l dw\partial_0 t_{\mathrm{LL}}^{00} \\
&= -\int d^3\vec{x}\int_0^l dw\partial_I t_{\mathrm{LL}}^{I0} \\
&= \oint dA\int_0^l dw(t_{\mathrm{LL}})_{0I}n^I, \qquad (6.4)
\end{aligned}$$

where we recall that our index conventions defined in (2.2) and (2.3) are $I, J = 1, 2, 3, 5$ and $i, j = 1, 2, 3$. In (6.4), dA is the differential area element on the 2-sphere at spatial infinity and n^I is the unit vector along the direction of propagation of the gravitational waves. From (5.10) and (5.23) we saw that the gravitational waves propagate radially in the noncompact directions and uniformly in w to leading order. The nonuniform propagation along the direction of compactification is due to the massive Kaluza-Klein modes which yield exponentially suppressed corrections. So, to leading order, the only nonzero components of n^I are n^i. Then, the rate of change of energy in a 2-sphere at a distance R from the source becomes

$$\dot{E}_{\mathrm{GW}} = l\oint d\Omega(t_{\mathrm{LL}})_{0k}n^kR^2. \qquad (6.5)$$

Using repeatedly the harmonic gauge and the fact that the perturbations in the far zone depend on the retarded time, we obtain the following identities to leading order in $1/R$:

$$\begin{aligned}
\partial_k\tilde{h}_{IJ} &\simeq -\dot{\tilde{h}}_{IJ}n_k, \\
\partial_k\tilde{h}_{00} &\simeq -\dot{\tilde{h}}_{ij}n^in^jn_k, \\
\partial_k\tilde{h}_{0I} &\simeq \dot{\tilde{h}}_{Ij}n^jn_k, \\
\dot{\tilde{h}}_{00} &\simeq \dot{\tilde{h}}_{ij}n^in^j, \\
\dot{\tilde{h}}_{0I} &\simeq -\dot{\tilde{h}}_{Ij}n^j, \qquad (6.6)
\end{aligned}$$

where a dot denotes a time derivative. In more detail, in writing $\partial_k\tilde{h}_{IJ} \simeq -\dot{\tilde{h}}_{IJ}n_k$, we used the fact that the metric fluctuations are spherical waves [see (5.23)], and to leading order in $1/R$, we can ignore the action of the ∂_k derivative on the $1/R$ factor. Then, when acting on the periodic function of $t - R$, we can trade off ∂_k for $n_k\partial_R$ and the latter for $-n_k\partial_t$.

Substituting (6.6) into (6.2) we derive the following result:

$$\begin{aligned}
\langle t_{0k}n^k\rangle \simeq -\frac{1}{32\pi G^{(D)}}\Big\langle \dot{\tilde{h}}_{IJ}\dot{\tilde{h}}^{IJ} + \frac{D-3}{D-2}\dot{\tilde{h}}_{ij}\dot{\tilde{h}}_{kl}n^in^jn^kn^l & \\
- 2\dot{\tilde{h}}_{Ij}\dot{\tilde{h}}^{Ik}n^jn_k - \frac{1}{D-2}\dot{\tilde{h}}^I{}_I\dot{\tilde{h}}^J{}J & \\
+ \frac{2}{D-2}\dot{\tilde{h}}^I{}_I\dot{\tilde{h}}_{ij}n^in^j\Big\rangle. & \qquad (6.7)
\end{aligned}$$

We are now ready to compute the luminosity of the gravitational waves from a binary source. For $D = 5$,

[9] When performing short-wavelength averaging, integration by parts is permitted [50].

substituting the perturbations derived in (5.23) into (6.5), and noting that to leading order we have $\partial_0\tilde{h}^I{}_I = \partial_0\tilde{h}^i{}_i = 0$, we find[10]

$$\dot{E}^{(5,c)}_{\rm GW} \simeq -\frac{19}{360}\frac{R^2}{G}\langle \dot{\tilde{h}}_{ij}\dot{\tilde{h}}^{ij}\rangle, \quad (6.8)$$

where we recall that G is the effective 4D theory gravitational constant (4.10), and we used the isotropy of the gravitational waves together with the following identities:

$$\int d^2\Omega n^i n^j = \frac{4\pi}{3}\delta^{ij},$$
$$\int d^2\Omega n^i n^j n^k n^l = \frac{4\pi}{15}(\delta^{ij}\delta^{kl} + \delta^{il}\delta^{jk} + \delta^{ik}\delta^{jl}). \quad (6.9)$$

Substituting the metric perturbations derived earlier in (5.23) into (6.8), and keeping terms only to leading order in velocity, the compactified 5D GR luminosity is

$$\dot{E}^{(5,c)}_{\rm GW} \simeq -\frac{304}{45}G\mu^2 r_{12}^4\Omega^6 = -\frac{76}{15}G_{\rm N}^{7/3}\mu^2 M^{4/3}\Omega^{10/3}. \quad (6.10)$$

In contrast, the luminosity of gravitational waves in a purely 4D gravitational theory, with the gravitational waves sourced by a binary with the same characteristics as ours, is equal to

$$\dot{E}^{(4)}_{\rm GW} \simeq -\frac{32}{5}G_{\rm N}\mu^2 r_{12}^4\Omega^6 = -\frac{32}{5}{G_{\rm N}}^{7/3}\mu^2 M^{4/3}\Omega^{10/3}. \quad (6.11)$$

We conclude that the luminosity of gravitational waves in a 5D spacetime with a compact fifth dimension differs by 20.8% from the corresponding 4D GR luminosity.

Let us now compare the luminosity derived earlier in (6.10) with the predictions of Einstein-Maxwell-dilaton theory studied in Refs. [19–21]. For neutral matter (i.e., the electric charges are zero), the energy of a binary is dissipated via gravitational and scalar (dilaton) radiation. We refer to them as $\dot{E}_g$ and $\dot{E}_\phi$, respectively. The luminosity depends on the scalar charge through the quantity

$$\alpha_a^0 = \frac{d\ln m_a(\phi)}{d\phi}, \quad (6.12)$$

where the superscript "0" refers to the quantity being evaluated at ϕ_∞ (a constant corresponding to the scalar field at spatial infinity), and where $m_a(\phi)$ is the effective 4D mass of a source a, which may depend on the dilaton. In general, for a circular binary, the leading order term in $\dot{E}_\phi$ is dipolar and depends on the difference in scalar charges of the binary constituents [51].

In our compactified (Kaluza-Klein) higher-dimensional gravity picture, the effective 4D mass of source a is given by

$$m_a(\phi) = m_a e^{-\phi/\sqrt{3}}, \quad (6.13)$$

as in (4.9), and where we used the dilaton rescaling as in (4.4). Since in our theory masses are coupled to the dilaton universally,

$$\alpha_1^0 = \alpha_2^0 = -1/\sqrt{3}, \quad (6.14)$$

the dipole radiation is zero (because $\alpha_1^0 - \alpha_2^0 = 0$), and so the leading contribution in $\dot{E}_\phi$ is quadrupolar. Therefore, both $\dot{E}_g$ and $\dot{E}_\phi$ are quadrupolar, and so is their sum, which is in agreement with our earlier findings (6.10). More precisely, given the Kaluza-Klein scalar charges (6.14), the leading order contribution to the luminosity in Einstein-Maxwell-dilaton theories [20,21] becomes

$$\dot{E}_g \simeq (1+\alpha_1^0\alpha_2^0)^{-1}\dot{E}^{(4)}_{\rm GW},$$
$$\dot{E}_\phi \simeq \frac{1}{6}(1+\alpha_1^0\alpha_2^0)^{-1}\alpha_1^0\alpha_2^0\dot{E}^{(4)}_{\rm GW}. \quad (6.15)$$

Thus, in total, we have

$$\dot{E}_g + \dot{E}_\phi \simeq (1+\alpha_1^0\alpha_2^0)^{-1}\left(1+\frac{1}{6}\alpha_1^0\alpha_2^0\right)\dot{E}^{(4)}_{\rm GW} = \frac{19}{24}\dot{E}^{(4)}_{\rm GW}, \quad (6.16)$$

which matches with our result in (6.10).

VII. CONSTRAINTS FROM GRAVITATIONAL WAVE OBSERVATIONS

In this section we compute the phase of the gravitational waveform in the frequency domain and compare it with observations. We restrict ourselves to the leading post-Newtonian contribution. We begin by deriving the frequency evolution of the gravitational waves from the energy-balance law

$$\frac{dE}{dt} = \dot{E}_{\rm GW}, \quad (7.1)$$

which simply states that the rate of change of the binding energy of the binary E is the same as the luminosity $\dot{E}_{\rm GW}$ of

[10] For general D dimensions,

$$\dot{E}^{(D,c)}_{\rm GW} \simeq -\frac{7D-16}{15(D-2)}\frac{R^2}{8G_N}\frac{2(D-3)}{D-2}\langle \dot{\tilde{h}}_{ij}\dot{\tilde{h}}^{ij}\rangle = -\frac{7D-16}{15(D-2)}\frac{D-2}{2(D-3)}16G_N^{7/3}\mu^2M^{4/3}\Omega^{10/3} = \frac{7D-16}{12(D-3)}\dot{E}^{(4)}_{\rm GW},$$

where $\dot{E}^{(4)}_{\rm GW}$ is defined in (6.11).

the energy radiated by gravitational waves. For a circular binary, the binding energy is the same as the effective potential, which is given in (3.13). However, since we are interested in calculating the leading post-Newtonian effect, we can ignore the exponentially suppressed correction. We can further use Kepler's law to rewrite the binding energy as

$$E = -\frac{1}{2}\mu(G_N M\Omega)^{2/3}. \tag{7.2}$$

Substituting (6.10) on the right-hand side of (7.1), and (7.2) into its left-hand side, we find

$$\dot{f}^{(5,c)} = \frac{76}{5}\pi^{8/3} f^{11/3} G_N^{5/3} \mathcal{M}^{5/3}, \tag{7.3}$$

where $f = \Omega/\pi$ is the gravitational waves frequency [this is manifest in (5.23)] and $\mathcal{M} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}$ denotes the chirp mass. On the other hand, the frequency evolution in 4D GR is given by

$$\dot{f}^{(4)} = \frac{96}{5}\pi^{8/3} f^{11/3} G_N^{5/3} \mathcal{M}^{2/3}, \tag{7.4}$$

which differs from the compactified 5D result in (7.3) by a numerical factor independent of the size of the extra dimension.

We now compute the gravitational wave phase in the frequency domain. The observed waveform is given by a linear combination of the + and × modes. In stationary phase approximation, the phase of gravitational waveform as a function of the frequency f is [52,53]

$$\Psi(f) = 2\pi f t(f) - \varphi(f) - \frac{\pi}{4}, \tag{7.5}$$

where

$$t(f) = t_0 + \int_\infty^f df \frac{dt}{df} = t_0 + \int_\infty^f df \frac{1}{\dot{f}} \tag{7.6}$$

and

$$\varphi(f) = \int dt 2\pi f = \varphi_0 + \int_\infty^f df \frac{2\pi f}{\dot{f}}. \tag{7.7}$$

Further using (7.3) we obtain

$$\Psi^{(5,c)}(f) = \frac{9}{304 G_N^{5/3} u^5} + 2\pi f t_0 - \varphi_0 - \frac{\pi}{4}, \tag{7.8}$$

where t_0 and φ_0 are the time and phase at the coalescence, respectively, and $u \equiv (\pi \mathcal{M} f)^{1/3}$ is the effective relative velocity of the binary components. On the other hand, the 4D GR result for the phase of the gravitational waves in the frequency domain is [52,53]

$$\Psi^{(4)}(f) = \frac{3}{128 G_N^{5/3} u^5} + 2\pi f t_0 - \varphi_0 - \frac{\pi}{4}. \tag{7.9}$$

Thus, we can rewrite (7.8) based on (7.9) as

$$\Psi^{(5,c)}(f) = \frac{3}{128 G_N^{5/3} u^5}(1 + \delta\hat{\varphi}) + 2\pi f t_0 - \varphi_0 - \frac{\pi}{4}, \tag{7.10}$$

with

$$\delta\hat{\varphi} \equiv \frac{5}{19} \sim 0.26. \tag{7.11}$$

We note that our results agree with those derived in the context of Einstein-Maxwell-dilaton theory discussed in Ref. [21] (with $\alpha_1 = \alpha_2 = -1/\sqrt{3}$ and the electric charges set to zero as discussed previously) to 0.3%. The difference arises due to a series expansion in luminosity in Ref. [21] which assumes that the scalar energy flux is small compared to the tensor energy flux. If one performs the calculation without making such an approximation, the result in Ref. [21] matches with ours exactly.

Let us now compare the predictions of our model, with a compactified fifth dimension with actual gravitational wave observations. From (7.11), one sees that the leading post-Newtonian term in (7.8) differs by 26% from that of (7.9), irrespective of the masses of the binary components. The LIGO/Virgo Collaborations used the events detected from the first and second observing runs and have placed upper bounds on $|\delta\hat{\varphi}|$ as ∼15% from single events and ∼10% from combined events [5]. Hence a discrepancy of 26% is inconsistent with the gravitational wave observations, and thus we can rule out the simple compactified 5D GR model considered in this paper.

We can easily generalize our previous results and compute the phase of the gravitational waves in an arbitrary number of dimensions D, with four noncompact dimensions and the rest compactified (periodic). Using the gravitational wave luminosity in a D-dimensional spacetime given in footnote 10, it is straightforward to derive $\delta\hat{\varphi}$ as

$$\delta\hat{\varphi}^{(D)} = \frac{5(D-4)}{7D-16}. \tag{7.12}$$

We plot $\delta\hat{\varphi}^{(D)}$ as a function of D in Fig. 1 and notice that $\delta\hat{\varphi}$ increases with D. This means that our model stays inconsistent with the LIGO/Virgo observations even if we increase the number of compact extra dimensions.

We now comment on some caveat in the above bounds. We used the bounds derived by the LIGO/Virgo Collaborations that assumed the correction to 4D GR in the phase enters only at 0Post-Newtonian (PN) order. Such a correction partially degenerates with the chirp mass that also enters first at 0PN order, though the mass also enters at

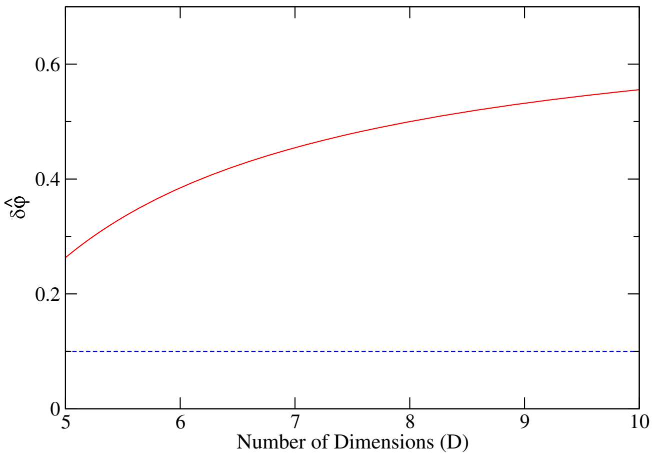

FIG. 1. The fractional difference ($\delta\hat{\varphi}$) of the GW phase with respect to that of the 4D GR as a function of the number of dimensions (D). The blue dashed line shows the upper bound placed on $\delta\hat{\varphi}$ from the combined events of the first and second observing runs of the LIGO/Virgo [5].

higher PN orders and thus the degeneracy can be partially broken. In reality, higher PN corrections to 4D GR also enter in the waveform phase. This may change the amount of correlation between the chirp mass and beyond-4D-GR effects and may weaken the bound on the 0PN correction. In [1], the LIGO/Virgo Collaborations carried out another analysis for GW150914 where they included phase corrections at various PN orders in the search parameter set. This enhances the correlation significantly and the bound on the 0PN correction now becomes $|\delta\hat{\varphi}| \lesssim 5$. If we quote this bound, we cannot rule out the compact extra dimension model considered here. Thus, to make a robust statement on whether one can rule out the model with gravitational-wave observations, one needs to compute corrections at higher PN orders and rederive bounds on the extra dimension effect.

Having said this, one can still rule out the model with binary pulsar observations for the following reason. A standard method for testing GR with binary pulsars is to determine the masses from at least three independent observables (such as post-Keplerian parameters including the periastron precession, Shapiro delay, and orbital decay rate) assuming GR and check the consistency. The orbital decay rate $\dot{P}$ is the only post-Keplerian parameter that depends on the gravitational-wave emission. Thus, even for the compact extra dimension model considered here, one can safely use the masses obtained from other post-Keplerian parameters under the 4D GR assumption since the conservative corrections are exponentially suppressed. One can then use the measurement of $\dot{P}$ to constrain the model without having to worry about the degeneracy between the extra dimension effect and masses. Such $\dot{P}$ measurements have been mapped to a bound on $\delta\hat{\varphi}$ as $|\delta\hat{\varphi}| \lesssim 10^{-3}$ [54,55], which is much stronger than the gravitational-wave bound. Thus, one can rule out the compact dimension model with the binary pulsar observations.[11]

VIII. CONCLUSIONS

In this paper we performed an analysis of gravitational waves sourced by a binary in a D-dimensional spacetime with four noncompact dimensions and a set of compactified extra dimensions. We worked under the assumptions that the two binary sources are pointlike and located on the same "brane" (i.e., at the same position in the compact coordinates). For the most part we took $D = 5$, but we have provided generalizations to arbitrary D throughout the paper. We worked within the framework of GR, and in the limit of small extra dimensions. We computed the gravitational waves sourced by the binary, the luminosity of the gravitational waves, and the phase of the gravitational waves, to leading order in the post-Newtonian expansion. We found that the luminosity of gravitational waves emitted in 5D gravity by a binary with the same characteristics (same masses and separation distance) as a 4D binary is 20.8% less relative to the 4D case, to leading post-Newtonian order. The phase of the gravitational waveform differs by 26% relative to the 4D case, to leading post-Newtonian order, while for general D, the fractional difference of the phase with that of 4D GR is $\frac{5(D-4)}{7D-16}$, which only increases with an increase in D. While there are exponential corrections that depend on the size of the extra dimensions, the leading order estimates for the gravitational wave phase we gave here are independent of size and depend only on the number of extra dimensions. Based on a comparison with gravitational-wave observations from the LIGO/Virgo Collaborations [5] and binary pulsar observations from radio astronomy [54,55] we can rule out this class of models for compact extra dimensions. The main source of discrepancy is the higher-dimensional gravity coupling with matter, which, when seen from a $4d$ perspective, means that matter will couple not only with the $4d$ metric, but with the dilaton as well. This dilaton coupling (or scalar charge) is responsible for fifth force effects that change the phase of the gravitational waves. Our results agree with those derived in the context of 4D Einstein-Maxwell-dilaton theory [19–21] provided that we set the binary's scalar charges equal to one another and equal to the Kaluza-Klein value.

The same fifth force effects are responsible for the difference between 4D Newton's constant $G_{\rm N}$ and the 4D gravitational coupling G: $G_N = \frac{2(D-3)}{D-2} G$, and for the Shapiro time-delay discrepancy with 4D GR. In a parametrized post-Newtonian expansion (PPN) the 4D

[11] A similar result was found in [56] though this reference effectively introduces matter after the KK reduction and thus is different from the setup we study here.

"physical" metric (which is obtained by performing a rescaling of the 4D metric with the dilaton in such a way to eliminate the matter-dilaton coupling [9] that is written as $g_{00}=-1+2U+\cdots, g_{ij}=\delta_{ij}(1+2\gamma U+\cdots)$, with $\gamma=1$ in 4D GR. A measurement of the frequency shift of radio photons to and from the Cassini spacecraft as they passed near the Sun gave $\gamma = 1+(2.1\pm 2.3)\times 10^{-5}$ [57]. On the other hand, the "physical metric" as read off from footnote 1 has $g_{00}=-1+\frac{2}{3}\tilde{h}_{\mathrm{T}00}$ and $g_{ij}=1+\frac{1}{3}\tilde{h}_{\mathrm{T}00}+\cdots$, which amounts to $\gamma\sim 1/2$. Therefore this class of compactified extra dimension models was ruled out based on Solar System measurements [58,59].

In string theory the massless dilaton is one of the many moduli (zero mass scalars) that arise in the compactification of the higher-dimensional spacetime. Stabilization of the moduli can be achieved, for example, by turning on fluxes for the Ramond-Ramond potentials [60]. This gives rise to a mass term for the moduli and eliminates the large contribution of the scalar fifth force, by turning a Coulomb potential into a Yukawa potential. It would be interesting to study gravitational waves in such a setup and place constraints on the various parameters. A somewhat simpler scenario is the Randall-Sundrum model, where the fifth dimension is large and warped. Our work here is intended as a first step in understanding how to set up the problem of solving for gravitational waves in a higher-dimensional space with compact dimensions, or warped, large extra dimensions. For example, we saw that quadrupole formulas need not apply, and we had to use a direct integration of the Einstein equations. Also, when it comes to the propagation of gravitational waves in spacetimes with warped, large extra dimensions, reducing the problem to 4D seems less appropriate, and working in the higher-dimensional space, as we did here, presents an advantage.

ACKNOWLEDGMENTS

We thank Eric Poisson and Leo Stein for useful comments. The work of D. V. and Y. D. was supported in part by the U.S. Department of Energy under Grant No. DE-SC0007984. S. T. and K. Y. acknowledge support from the Ed Owens Fund. K. Y. also acknowledges support from NSF Award No. PHY-1806776, a Sloan Foundation Research Fellowship, the COST Action GWverse CA16104, and JSPS KAKENHI Grant No. JP17H06358.

APPENDIX A: MODIFIED QUADRUPOLE FORMULA

Here we explicitly point out why the usual quadrupole formula

$$\int d^3\vec{x}\tau^{ij} = \frac{1}{2}\partial_0^2\int d^3\vec{x}\tau^{00}x^ix^j \tag{A1}$$

cannot be directly extended to higher dimensions if there are compact dimensions.

Consider the expression $\frac{1}{2}\partial_0^2\int d^3\vec{x}\int_0^l dw\tau^{00}x^Ix^J$, and then use repeatedly the conservation law for τ^{MN} and integrate by parts, while paying attention to the boundary terms:

$$\begin{aligned}\frac{1}{2}\partial_0^2\int d^3\vec{x}\int_0^l dw\tau^{00}x^Ix^J &= \int d^3\vec{x}\int_0^l dw\tau^{IJ}\\ &+\frac{1}{2}\int d^3\vec{x}(\partial_K\tau^{5K}x^Ix^J-\tau^{5I}x^J-\tau^{5J}x^I)|_{w=0}^{w=l}.\end{aligned} \tag{A2}$$

For $I,J=i,j=1,2,3$, the quadrupole formula applies because of the periodicity of the metric fluctuations in w. However, if either I or J are along the compact dimension, then there are extra terms relative to those expected based on the quadrupole formula. These terms are not straightforward to evaluate, and for this reason we rely on direct integration of the Einstein equations.

APPENDIX B: HIDDEN BRANE SCENARIO

We generalize the Newtonian potential by having the observer located at $w=0$, and the static mass source m on a hidden brane at $w=w_1$:

$$T_{00}(x^\mu,w)=m\delta^3(\vec{x})\delta(w-w_1). \tag{B1}$$

The corresponding Newtonian potential evaluated by the observer is

$$V^{(5,c)}(R,w_1)=-\frac{4}{3\pi}G^{(5)}m\sum_{n=-\infty}^{\infty}\frac{1}{R^2+(w_1+nl)^2}. \tag{B2}$$

Using Poisson summation, the above sum can be evaluated to give the following result:

$$\begin{aligned}V^{(5,c)}(R,w_1)&=-\frac{4}{3}\frac{G^{(5)}m}{lR}\tanh\frac{2\pi R}{l}\left(1+\operatorname{sech}\frac{2\pi R}{l}\cos\frac{2\pi w_1}{l}\right)\\ &\simeq-\frac{4}{3}\frac{G^{(5)}m}{lR}\left(1+2e^{-2\pi R/l}\cos\frac{2\pi w_1}{l}\right),\end{aligned} \tag{B3}$$

where in the last step we used $R\gg l$ and $R\gg w_1$. When $w_1=0$, Eq. (B3) reproduces the Newtonian potential in Sec. III in the limit of $2\pi R\gg l$.

APPENDIX C: NEWTONIAN POTENTIAL FROM INTEGRATING THE RETARDED 5D COMPACTIFIED GREEN'S FUNCTION

In this Appendix we offer an alternative derivation of the Newtonian potential obtained in Sec. III, by using the retarded 5D compactified Green's function. A static source of mass m located at the origin has

$$T_{00}(x^\mu, w) = m\delta^3(\vec{x})\delta(w). \tag{C1}$$

Substituting (5.5) and (C1) into (5.2) we obtain

$$\tilde{h}_{\mathrm{T},00}(x^\mu, w) = -\frac{4}{\pi}G^{(5)}m\sum_{n=-\infty}^{\infty}\frac{1}{r_n}\int_0^\infty dt'\frac{\partial}{\partial r_n}\frac{\theta(t-t'-r_n)}{\sqrt{(t-t')^2-r_n^2}}, \tag{C2}$$

where $r_n = \sqrt{x^2+y^2+z^2+(w-nl)^2}$. The integral in (C2) can be evaluated by reexpressing the ∂_{r_n} derivative in terms of a time derivative using

$$\frac{\partial}{\partial r_n}\left[\frac{\theta(\Delta t-r_n)}{(\Delta t^2-r_n^2)^{1/2}}\right] = -\frac{\partial}{\partial\Delta t}\left[\frac{\theta(\Delta t-r_n)}{(\Delta t^2-r_n^2)^{1/2}}\right] + \frac{\theta(\Delta t-r_n)}{[(\Delta t)^2-r_n^2]^{3/2}}(r_n-\Delta t), \tag{C3}$$

where $\Delta t = t-t'$. Substituting (C3) into (C2) we obtain

$$\tilde{h}_{\mathrm{T}}^{00}(x^\mu, w) = -\frac{4}{\pi}G^{(5)}m\sum_{n=-\infty}^{\infty}\frac{1}{r_n}\int_{r_n}^\infty d\Delta t\frac{r_n-\Delta t}{[(\Delta t)^2-r_n^2]^{3/2}} = \frac{4}{\pi}G^{(5)}m\sum_{n=-\infty}^{\infty}\frac{1}{r_n^2}, \tag{C4}$$

and the corresponding Newton's potential $V^{(5,c)} \equiv (-1/3)\tilde{h}_{\mathrm{T},00}$ matches with the one in Eq. (3.6).

APPENDIX D: RETARDED GREEN'S FUNCTION

The massless scalar D-dimensional flat space Euclidean Green's function $\mathcal{G}_\mathrm{E}(x,x') = \mathcal{G}_\mathrm{E}(x-x')$ is the inverse of the D-dimensional Laplacian

$$\Delta_{(D)}\mathcal{G}^{(D)}E(x-x') = \delta^D(x-x'),$$
$$\mathcal{G}_\mathrm{E}^{(D)}(x-x') = -\int\frac{d^Dp}{(2\pi)^D}\frac{e^{ip\cdot x}}{p\cdot p}. \tag{D1}$$

Starting from the unique Euclidean Green's function, in the Minkowski signature the retarded Green's function is obtained via the analytical continuation $p_\mathrm{E}^0 \to -i(p^0+i\epsilon)$. The Euclidean metric δ^{MN} gets replaced by the Minkowski metric η^{MN}, and the $i\epsilon$ prescription yields the *retarded* Green's function:

$$\mathcal{G}^{(D)}(x-x') = -\int\frac{d^Dp}{(2\pi)^D}\frac{e^{ip\cdot x}}{-(p^0+i\epsilon)^2+p^ip^i}. \tag{D2}$$

The location of the poles is in the lower half-plane, when viewed as a function of p^0 as a complex variable. To evaluate the integral one integrates over p^0, using Cauchy's theorem, and if $t-t'>0$, one picks up the contribution from the two poles, otherwise the retarded Green's function is zero. If $D=4$, we recover the familiar expression

$$\mathcal{G}^{(4)}(x-x') = -\theta(t-t')\int\frac{d^3p}{(2\pi)^3}\frac{\sin(p(t-t'))}{p}e^{i\vec{p}\cdot(\vec{x}-\vec{x}')} = -\theta(t-t')\frac{1}{4\pi r}\delta((t-t')-|\vec{x}-\vec{x}'|), \tag{D3}$$

where we used p to denote the magnitude of the spatial vector $\vec{p}$, i.e., $p=|\vec{p}|$. If $D=5$, then

$$\begin{aligned}\mathcal{G}^{(5)}(x-x') &= -\theta(t-t')\int\frac{d^4p}{(2\pi)^4}\frac{\sin(p(t-t'))}{p}e^{i\vec{p}\cdot(\vec{x}-\vec{x}')}\\ &= -\theta(t-t')\frac{4\pi}{(2\pi)^4}\int_0^\infty dp\,p^3\int_0^\pi d\theta\sin^2\theta\frac{\sin(p(t-t'))}{p}e^{ip|\vec{x}-\vec{x}'|\cos\theta}\\ &= -\theta(t-t')\frac{1}{4\pi^2|\vec{x}-\vec{x}'|}\int_0^\infty dp\,p\sin(p(t-t'))J_1(p|\vec{x}-\vec{x}'|)\\ &= -\theta(t-t')\frac{1}{4\pi^2|\vec{x}-\vec{x}'|}\frac{\partial}{\partial|\vec{x}-\vec{x}'|}\int_0^\infty dp\sin(p(t-t'))J_0(p|\vec{x}-\vec{x}'|)\\ &= -\theta(t-t')\frac{1}{4\pi^2|\vec{x}-\vec{x}'|}\frac{\partial}{\partial|\vec{x}-\vec{x}'|}\frac{\theta(t-t'-|\vec{x}-\vec{x}'|)}{\sqrt{(t-t')^2-(\vec{x}-\vec{x}')^2}}.\end{aligned} \tag{D4}$$

APPENDIX E: DIRECT INTEGRATION VS QUADRUPOLE FORMULA

Here we use post-Newtonian order counting to explain a somewhat subtle aspect of our calculations. For simplicity's sake, in this Appendix we restrict ourselves to 4D GR. Specifically, we will show that if in computing the spatial metric fluctuations $\tilde{h}^{ij}$, one uses the quadrupole formula, then one can safely neglect nonlinear source terms in the relaxed Einstein equations. However, if one directly integrates the relaxed Einstein equations, then the nonlinear terms cannot be neglected already at the leading post-Newtonian order. Since our goal is only to highlight the dependence on velocities, we will write our equations with squiggle lines, signaling that we are imprecise about numerical factors.

The relaxed Einstein equations in the harmonic gauge are given in (2.8), whose 4D solution is given by

$$\tilde{h}^{\mu\nu} \sim \frac{4}{R} \int d^3\vec{x}\tau^{\mu\nu}(\vec{x}, t-R), \tag{E1}$$

where we assumed that we are working in the far-field approximation, R is the distance from the source to the field point, and the right-hand side is evaluated at the retarded time.

1. Direct integration

We are interested in extracting the order of magnitude in post-Newtonian (PN) expansion estimates of the spatial components $\tilde{h}^{ij}$ for a compact binary. We will be somewhat careless about the indices and numerical factors since we only care about counting the PN order, namely powers of the relative velocity of the binary constituents, vs Eq. (E1) should receive contributions from both T^{ij} and $t^{ij}_{\rm LL}$. To leading PN order, the contribution from T^{ij} is roughly given by

$$\tilde{h}^{ij}_{\rm T} \sim \frac{\mu}{R} v^i v^j \sim \frac{\mu}{R} v^2, \tag{E2}$$

where μ is the reduced mass of the binary. To consider the contribution of $t^{ij}_{\rm LL}$, let us substitute $\tilde{g}^{ij} = \eta^{ij} - \tilde{h}^{ij}$ into (2.10) and look at one term (inside $t^{ij}_{\rm LL}$), for example,

$$(-g)t^{ij}_{\rm LL} \sim \tilde{h}^{00,i}\tilde{h}_{00}{}^{,j} \sim \tilde{h}_{00,i}\tilde{h}_{00,j}, \tag{E3}$$

so that

$$\tilde{h}^{ij}_{\rm t} \sim \frac{1}{R} \int d^3x \tilde{h}_{00,i}\tilde{h}_{00,j}. \tag{E4}$$

To compute (E4), let us consider partitioning the spacetime into a near zone and a far zone (FZ)[12] relative to the location of the sources. NZ is the region centered around the source with the size of the gravitational wavelength while FZ is the region exterior to it [43]. Within the NZ, the gravitational fields can be considered as almost instantaneous and retardation can be neglected.

[12]FZ is also called the radiation zone or the wave zone.

The integral in (E4) can be decomposed into the NZ and FZ integrals. It turns out that the former dominates the latter, so what we need is the NZ solution for $\tilde{h}_{00}$. For a compact binary, the leading NZ solution is given by [61]

$$h^{\rm NZ}_{00} \sim \frac{2m_1}{r_1} + \frac{2m_2}{r_2}, \tag{E5}$$

$$h^{\rm NZ}_{ij} \sim \left(\frac{2m_1}{r_1} + \frac{2m_2}{r_2}\right)\delta_{ij}, \tag{E6}$$

with $r_a \equiv |\vec{x} - \vec{x}_a|$ and where $a = 1, 2$ corresponds to one of the two sources. Thus, $h^{\rm NZ} \sim 4m_1/r_1 + 4m_2/r_2$ and

$$\tilde{h}^{\rm NZ}_{00} \sim h^{\rm NZ}_{00} - \frac{1}{2} h^{\rm NZ}\eta_{00} \sim \frac{4m_1}{r_1} + \frac{4m_2}{r_2}. \tag{E7}$$

We now substitute the above equation into the right-hand side of (E4). Those terms that only depend on r_1 or r_2 will diverge and be dropped upon regularization, so what matters is the cross term between sources 1 and 2. Ignoring numerical factors, we find

$$\begin{aligned} \tilde{h}^{ij}_{\rm t} &\sim \frac{m_1 m_2}{R} \int_{\rm NZ} d^3x \partial_i \left(\frac{1}{r_1}\right) \partial_j \left(\frac{1}{r_2}\right) + (1 \leftrightarrow 2) \\ &\sim \frac{m_1 m_2}{R} \int_{\rm NZ} d^3x \partial^{(1)}_i \left(\frac{1}{r_1}\right) \partial^{(2)}_j \left(\frac{1}{r_2}\right) + (1 \leftrightarrow 2) \\ &\sim \frac{m_1 m_2}{R} \partial^{(1)}_i \partial^{(2)}_j \int_{\rm NZ} d^3x \frac{1}{r_1 r_2} + (1 \leftrightarrow 2), \end{aligned} \tag{E8}$$

where we changed the partial derivatives to source derivatives $\partial^{(a)}_i \equiv \partial/\partial x^i_a$ so that derivatives can be taken outside of the integral. The remaining integral, is given by [62]

$$\int_{\rm NZ} d^3x \frac{1}{r_1 r_2} = -2\pi r_{12}, \tag{E9}$$

where $r_{12} = |\vec{x}_1 - \vec{x}_2|$ is the binary separation. Thus,

$$\begin{aligned} \tilde{h}^{ij}_{\rm t} &\sim \frac{m_1 m_2}{R} \partial^{(1)}_i \partial^{(2)}_j r_{12} + (1 \leftrightarrow 2) \\ &\sim \frac{m_1 m_2}{R} \frac{1}{r_{12}} \\ &\sim \frac{\mu}{R} \frac{M}{r_{12}} \\ &\sim \frac{\mu}{R} v^2, \end{aligned} \tag{E10}$$

where M is the total mass and we used the Kepler's law in the last equation. This scaling could, in fact, easily be obtained from the first line of (E8) by replacing all the length scales inside the integral with r_{12}. Notice that $\tilde{h}_{\rm t}^{ij}$ is of the same PN order as $\tilde{h}_{\rm T}^{ij}$. This means that the contribution from $t_{\rm LL}^{ij}$ cannot be neglected even at the leading order.

2. Quadrupole formula

So far we have seen that by using direct integration of the relaxed Einstein equations, where $\tilde{h}^{ij}$ is sourced by τ^{ij}, both the linearized $\tilde{h}_{\rm T}^{ij}$ and second order in fluctuation $\tilde{h}_{\rm t}^{ij}$ are of the same order in a velocity expansion. However, if we replace our starting point for the derivation of $\tilde{h}^{ij}$ with the quadrupole formula (see Appendix A)

$$\int \tau^{ij} d^3x = \frac{1}{2}\frac{d^2}{dt^2}\int d^3x \tau^{00} x^i x^j, \tag{E11}$$

then we would be using τ^{00} in extracting $\tilde{h}^{ij}$. In this case, the contribution from $t_{\rm LL}^{00}$ is subleading to that of T^{00}, as we will now show.

Using (E11) and (E1) one derives

$$\tilde{h}^{ij} \sim \frac{2}{R}\frac{d^2}{dt^2}\int d^3x \tau^{00} x^i x^j. \tag{E12}$$

The contribution from T^{00} has the same scaling as in (E2), i.e.,

$$\tilde{h}^{ij}_{(T^{00})} \sim \frac{\mu}{R} v^2. \tag{E13}$$

On the other hand, the contribution from t^{00} is given by

$$\begin{aligned}\tilde{h}^{ij}_{(t^{00})} &\sim \frac{1}{R}\frac{d^2}{dt^2}\int_{\rm NZ} d^3x \tilde{h}^{\rm NZ}_{00\ ,i}\tilde{h}^{\rm NZ}_{00\ ,j} x^i x^j \\ &\sim \frac{1}{R}\frac{d^2}{dt^2}\int_{\rm NZ} d^3x \partial_i\left(\frac{m_1}{r_1}\right)\partial_j\left(\frac{m_2}{r_2}\right) x^i x^j \\ &\sim \frac{1}{R}\Omega^2 \frac{m_1}{r_{12}^2}\frac{m_2}{r_{12}^2} r_{12}^2 r_{12}^3 \sim \frac{m_1 m_2}{R} r_{12}\Omega^2 \\ &\sim \frac{\mu}{R}\frac{M}{r_{12}}(r_{12}\Omega)^2 \sim \frac{\mu}{R} v^4,\end{aligned} \tag{E14}$$

where Ω is the binary angular frequency and we replaced all the length scale by r_{12} in the third line as noted in Sec. E 1. Notice that $\tilde{h}^{ij}_{(t^{00})}$ is of higher order in velocities than $\tilde{h}^{ij}_{(\rm T^{00})}$. Thus, once the expression for $\tilde{h}^{ij}$ is turned into the quadrupole formula, the contribution from t^{00} becomes subdominant and one only needs to consider T^{00} to leading order.

APPENDIX F: GW LUMINOSITY IN TRANSVERSE-TRACELESS (TT) GAUGE

The transverse traceless (TT) gauge for linearized gravitational fluctuations about a flat spacetime imposes the following conditions[13]:

$$\tilde{h}^{\rm TT}_{0M} = 0, \qquad (\tilde{h}^{\rm TT})^I{}_I = 0, \qquad \partial_J(\tilde{h}^{\rm TT})^{IJ} = 0. \tag{F1}$$

Starting from the trace-reversed metric fluctuations, $\tilde{h}_{IJ}$, one can show that the transverse traceless components are obtained by simply acting with a transverse-tracelss projector

$$\Lambda_{IJKL} = P_{IK}P_{JL} - \frac{1}{D-2}P_{IJ}P_{KL}, \tag{F2}$$

and where P_{IK} are projectors orthogonal to n^I, with n^I the direction of propagation of the waves and $n^I n^I = 1$:

$$P_{IJ} = \delta_{IJ} - n_I n_J, \qquad P_{IJ} n^J = 0. \tag{F3}$$

Therefore, the nonvanishing (same as the trace-reversed) metric fluctuations in the TT gauge are

$$\tilde{h}^{\rm TT}_{IJ} = \Lambda_{IJKL}\tilde{h}^{KL}, \quad (\tilde{h}^{\rm TT})^I{}_I = 0, \quad n^I\tilde{h}^{\rm TT}_{IJ} = \tilde{h}^{\rm TT}_{IJ} n^J = 0. \tag{F4}$$

It is easy to verify that

$$\begin{aligned}\tilde{h}^{\rm TT}_{IJ}(\tilde{h}^{\rm TT})^{IJ} &= \tilde{h}_{IJ}\tilde{h}^{IJ} - 2n^I n^J \delta^{KL}\tilde{h}_{IK}\tilde{h}_{JL} \\ &\quad + \left(\frac{D-3}{D-2}\right) n^I n^J n^K n^L \tilde{h}_{IJ}\tilde{h}_{KL} \\ &\quad - \left(\frac{1}{D-2}\right)(\tilde{h}^I{}_I)^2 + \left(\frac{2}{D-2}\right) n^K n^L \tilde{h}_{KL}\tilde{h}^I{}_I.\end{aligned} \tag{F5}$$

In particular, working in the TT gauge, Eq. (6.2) becomes simply

$$t_{0K} n^K = -\frac{1}{32\pi G^{(D)}}\langle \dot{\tilde{h}}^{\rm TT}_{IJ}(\dot{\tilde{h}}^{\rm TT})^{IJ}\rangle. \tag{F6}$$

We can nevertheless recover (6.7), starting from the TT gauge expression (F6) and substituting (F5), which is to be expected given that we are computing a gauge-invariant quantity.

[13]The TT gauge uses the residual gauge freedom of the Lorenz gauge to impose the additional conditions: $(\tilde{h}^{\rm TT})^M{}_M = 0$, $\tilde{h}^{\rm TT}_{0M} = 0$. Take n_I to be pointing in the direction of propagation of the waves, assuming they are plane waves: $\tilde{h}^{\rm TT}_{MN} = \tilde{h}^{\rm TT}_{MN}(t - n_I x^I)$. Then, from the harmonic gauge one finds that $n_I(\tilde{h}^{\rm TT})^{IJ} = \partial_I(\tilde{h}^{\rm TT})^{IJ} = 0$, $(\tilde{h}^{\rm TT})^I{}_I = 0$. For spherical waves these relations remain true to leading order in $1/R$.

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