Heating up an environment around black holes and inside de Sitter space

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We study quantum fields on spacetimes having a bifurcate Killing horizon by allowing the possibility that left and right (ingoing and outgoing) modes have different temperatures. An example of such a state is the Unruh vacuum in a black hole background, in which ingoing modes are in the zero temperature state, while the outgoing ones are at the Hawking temperature. We construct the corresponding Wightman functions and study their properties with arbitrary and different temperatures for both types of movers. We consider, in particular, the Rindler for both massless and massive fields, the static de Sitter, and Schwarzschild black hole backgrounds for massive fields. We find that in all three cases, when any of the temperatures are different from the canonical one (Unruh, Hawking, and Gibbons-Hawking, correspondingly) the correlation functions have extra singularities at the horizon.

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I. INTRODUCTION

There is a well-known relation between the existence of a bifurcate Killing horizon on certain spacetime manifolds and the temperature of some specially privileged equilibrium thermal states. The Rindler, the de Sitter, and the Schwarzschild spacetimes are examples of such situations [1].

While there is vast literature devoted to the study of the above states, little attention has been paid to other possible thermal or pseudothermal states, let alone to what happens when interactions are switched on: does the fluctuation-dissipation theorem still work as in flat space? Are the privileged states attractor solutions of some sort of kinetic equations with exact modes instead of plane waves? Does thermalization of a given initial state happen in *strongly* curved space-times without substantial backreaction on the gravitational background? The backreaction during thermalization seems to be negligible practically for any reasonable (Hadamard) initial state in flat space-time [2,3].

When attempting to address some of the above questions one encounters many underwater stones; for example, describing thermal states other than the Gibbons-Hawking one [4] in the static de Sitter-Rindler wedge [5,6] is already nontrivial. Furthermore, if time translations are not a symmetry, as is the case in the global Lanczos' spherical coordinate system [7], secular divergences arise both in distributions and in anomalous averages [8–11]. The latter can be resummed for fields whose mass is bigger than a critical mass, via analogues of kinetic equations for both the distributions and anomalous averages. Such kinetic equations do not have Planckian or Boltzmannian solutions. Moreover, they have exploding solutions. Such a situation, when thermalization does not happen without taking into account backreaction on the background field, is similar to the one encountered in a constant electric field [12,13].

As regards to the Schwarzschild geometry and the black hole radiation, there are three distinguished states which are usually considered: the Boulware [14], Unruh [15], and Hartle-Hawking states. [16,17]. The Boulware state is the vacuum of both the ingoing and outgoing modes of the Schwarzschild background; the Unruh state is the vacuum of the ingoing modes and has the Planckian distribution at the Hawking temperature for the outgoing modes; finally, in the Hartle-Hawking state both the ingoing and the outgoing modes are thermally distributed at the Hawking temperature.

Can any of the aforementioned quantum states actually describe the fate of the quantum field at the end of the collapse [18]? Actually, in the formation of a black hole process one has to consider a different basis of modes [19] where the ingoing and outgoing modes are not treated as independent but rather a linear combination of them, which is regular at the center of the collapsing star. The corresponding state is inequivalent to either the Boulware, the

Unruh, or the Hartle-Hawking state. The question then arises whether the initial state before the collapse can thermalize to any of the above mentioned states.

A second set of questions regards the behavior of black holes surrounded by a gas with a temperature different from the Hawking one. Can one heat up a black hole by surrounding it with a gas of temperature different from the Hawking one? And how does the heating work in detail? In concrete astrophysical situations black holes indeed are surrounded by accretion disks which definitely have nothing to do with the Hawking radiation. It goes without saying about primordial black holes in early Universe.

In this paper, even without performing loop calculations (where the heating process is actually seen), we will argue that the answer to the first question in the last paragraph seems to be negative. Both in the static de Sitter space (see also [6]) and in the black hole background all correlation functions with temperatures different from the Hawking one have anomalous singularities at the horizon.

To simplify our discussion for the beginning we consider real scalar field theory in two dimensions, in either the Rindler, the static de Sitter or two-dimensional analog of the Schwarzschild background. We study the properties of the tree-level Wightman functions for a class of time translation invariant states, which include states having different temperatures for the ingoing and the outgoing modes. In all cases when the temperatures are different from the Unruh, Gibbons-Hawking, and Hawking ones in the corresponding situations there are anomalous singularities of the correlation functions at the horizon.

II. RINDLER SPACE-TIME

To put the results of the paper in perspective we start by discussing the Rindler spacetime. This section mainly contains a recapitulation of known facts. However, some of them are new.

A. Geometry, modes, and the Wightman function

The coordinate system for the Rindler right wedge of the Minkowski spacetime is obtained by applying the one-parameter subgroup of boosts, which leaves invariant the wedge to points of, say, the half-line $t = 0$, $x = e^{\alpha\xi}/\alpha > 0$:

$$X(\eta,\xi) = \begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} \cosh(\alpha\eta) & \sinh(\alpha\eta) \\ \sinh(\alpha\eta) & \cosh(\alpha\eta) \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{\alpha} e^{\alpha\xi} \end{pmatrix} = \begin{pmatrix} \frac{1}{\alpha} e^{\alpha\xi} \sinh(\alpha\eta) \\ \frac{1}{\alpha} e^{\alpha\xi} \cosh(\alpha\eta) \end{pmatrix}. \tag{2.1}$$

Here η is the parameter of the subgroup and is interpreted as the Rindler time, ξ is the space coordinate, and α is the proper acceleration. For real values of η and ξ the Rindler coordinates (2.1) cover only the right wedge; this is causally disconnected from the left wedge Fig. 1. The half-lines $x = \pm t$ with $x > 0$ are the past and the future horizons. When η and ξ are complex they cover the full Minkowski spacetime; in the following we will set the acceleration to 1 ($\alpha = 1$).

In the above coordinates the metric is static and conformal to Minkowski

$$ds^2 = e^{2\xi}(d\eta^2 - d\xi^2); \tag{2.2}$$

the invariant interval between two events in the wedge is given by

$$L_{12} = (X_1 - X_2)^2 = 2e^{(\xi_1+\xi_2)} \cosh(\eta_2 - \eta_1) - e^{2\xi_2} - e^{2\xi_1}. \tag{2.3}$$

For future reference please note the obvious symmetry of the interval in the exchange

$$\xi_1 \leftrightarrow \xi_2. \tag{2.4}$$

Lorentz transformations of the wedge corresponds to (time) translations in the η variable: $\eta \to \eta + \gamma$; dilatations in Minkowski space $X \to e^{\beta} X$ correspond to the shift $\xi \to \xi + \beta$. As regards to the light cone variables

$$u = t - x = -e^{(\xi-\eta)} = -e^{-U}, \quad v = t + x = e^{(\xi+\eta)} = e^{V}, \tag{2.5}$$

they are transformed as follows:

$$\begin{aligned} &u \to e^{-\gamma} u, \quad u \to e^{\beta} u, \quad U \to U + \gamma, \quad U \to U - \beta, \\ &v \to e^{\gamma} v, \quad v \to e^{\beta} v, \quad V \to V + \gamma, \quad V \to V + \beta. \end{aligned} \tag{2.6}$$

Though geodetically incomplete, the Rindler wedge is a globally hyperbolic manifold in itself; of course, a Cauchy surface, say $\eta = 0$, is not a Cauchy surface for the whole Minkowski spacetime. As a consequence, the modes constructed by canonical quantization in the Rindler wedge do not constitute a basis for the whole Minkowski spacetime. It is well known that to obtain general Hilbert space representations of the fields, including the ones carrying unitary representations of the Poincaré group, one needs to construct also the modes defined in the left wedge [20]. A less known but powerful alternative is to resort to the theory of generalized Bogoliubov transformations,[1] which makes use only of the modes of the right wedge [21,22]. The Klein-Gordon equation for a massive scalar field in two dimensions is as follows:

[1]Starting from pure states, generalized Bogoliubov transformations may produce mixed states while standard Bogoliubov transformations cannot.

$$(\partial_\eta^2 - \partial_\xi^2 + e^{2\xi}m^2)\varphi(\eta,\xi) = 0. \tag{2.7}$$

By separating the variables one gets a Schrodinger eigenvalue (textbook) problem in an exponential potential $V(\xi) = m^2 e^{2\xi}$. Normalizable modes are proportional to Macdonald functions $K_{i\omega}(me^{\xi})$ which are linear combinations of left-moving and right-moving waves.

The canonical field operator may be expanded as follows:

$$\varphi(X) = \frac{1}{\pi}\int_0^{+\infty}(e^{-i\omega\eta}b_\omega + e^{i\omega\eta}b_\omega^\dagger)K_{i\omega}(me^{\xi})\sqrt{\sinh\pi\omega}d\omega, \tag{2.8}$$

where the creation and annihilation operators obey the standard commutation relations:

$$[b_\omega, b_{\omega'}^\dagger] = \delta(\omega-\omega'), \qquad [b_\omega, b_{\omega'}] = 0.$$

The so-called Fulling vacuum [23,24] is identified by the condition

$$b_\omega|0_R\rangle = 0, \qquad \omega \geq 0. \tag{2.9}$$

It is a pure state and the corresponding two-point function is given by

$$\begin{aligned} W_\infty(X_1,X_2) &= \langle 0_R|\varphi(X_1)\varphi(X_2)|0_R\rangle \\ &= \frac{1}{\pi^2}\int_0^\infty e^{-i\omega(\eta_1-\eta_2)}K_{i\omega}(me^{\xi_1}) \\ &\quad\times K_{i\omega}(me^{\xi_2})\sinh\pi\omega\, d\omega. \end{aligned} \tag{2.10}$$

The thermal equilibrium average of an operator $\mathcal{O}$ at temperature $T = \beta^{-1}$ is defined in quantum mechanics as follows:

$$\langle\mathcal{O}\rangle = \frac{\mathrm{Tr}\, e^{-\beta H}\mathcal{O}}{\mathrm{Tr}\, e^{-\beta H}}, \tag{2.11}$$

where H is the Hamiltonian of the system. This definition does not directly work in quantum field theory, but there are states having the same general properties expressed in terms of their analyticity and periodicity properties in the complex time variable; they are known as the Kubo-Martin-Schwinger (KMS) states [25].

In the Rindler wedge, a Wightman function having the KMS property may be obtained by a generalized Bogoliubov transformation [21,22] of the Fulling vacuum. In the case under consideration it is given by:

$$\begin{aligned} &W_\beta(X_1(\eta_1,\xi_1),X_2(\eta_2,\xi_2)) \\ &\quad= \frac{1}{\pi^2}\int_0^\infty\left[\frac{e^{-i\omega(\eta_1-\eta_2)}}{1-e^{-\beta\omega}} + \frac{e^{i\omega(\eta_1-\eta_2)}}{e^{\beta\omega}-1}\right] \\ &\quad\quad\times K_{i\omega}(me^{\xi_1})K_{i\omega}(me^{\xi_2})\sinh\pi\omega\, d\omega. \end{aligned} \tag{2.12}$$

The two-point function (2.12) is time-translation invariant (and, therefore, it provides an equilibrium state) and *respects the exchange symmetry* (2.4). The formal proof of the KMS periodicity makes use of the exchange symmetry (2.4) but is otherwise straightforward.

When $\beta = 2\pi$ an explicit calculation of the integral shows that the lhs in Eq. (2.12) can be extended to the whole complex Minkowski spacetime (minus the causal cut), and it is actually Poincaré invariant [15,21,22]:

$$W_{2\pi}(X_1,X_2) = \frac{1}{2\pi}K_0\left(m\sqrt{-(X_1-X_2)^2}\right). \tag{2.13}$$

When $mL \to 0$ it has the standard ultraviolet (Hadamard behavior) divergence with the correct coefficient $1/4\pi$:

$$\frac{1}{2\pi}K_0(m\sqrt{-L}) \approx -\frac{1}{4\pi}\log(-m^2L). \tag{2.14}$$

Inside the Rindler wedge, the main contributions to the integral (2.12) for lightlike separations come from high energies $\omega \gg me^{\xi_{1,2}}$ ($\xi_{1,2}$ fixed), and the divergence does not depend on the temperature. This is true for any β. However, when $\beta \neq 2\pi$ there are extra (anomalous) singularities at the horizon—the boundary of the wedge, which of course is also lightlike. We will show this now.

When the temperature is an integer multiple of $(2\pi)^{-1}$ a simple formula is available [6,26]:

$$W_{\frac{2\pi}{N}}(X_1,X_2) = \sum_{k=0}^{N-1}W_{2\pi}\left(X_1\left(\eta_1 - \frac{2\pi ik}{n},\xi_1\right),X_2(\eta_2,\xi_2)\right). \tag{2.15}$$

Let us consider the simplest case $\beta = \pi$:

$$\begin{aligned} W_\pi &= \frac{1}{2\pi}K_0\left(m\sqrt{e^{2\xi_1}+e^{2\xi_2}-2e^{\xi_1+\xi_2}\cosh\Delta\eta}\right) \\ &\quad+\frac{1}{2\pi}K_0\left(m\sqrt{e^{2\xi_1}+e^{2\xi_2}+2e^{\xi_1+\xi_2}\cosh\Delta\eta}\right). \end{aligned} \tag{2.16}$$

Points of the horizons may be attained as follows:

$$\lim_{\lambda\to\pm\infty}X(\lambda,\chi\mp\lambda) = \lim_{\lambda\to\infty}\begin{pmatrix} e^{\chi\mp\lambda}\sinh\lambda \\ e^{\chi\mp\lambda}\cosh\lambda\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\pm e^{\chi} \\ e^{\chi}\end{pmatrix}. \tag{2.17}$$

The interval between two points having the same coordinate λ is spacelike; for instance,

$$L_{12} = (X_1(\lambda,\chi_1-\lambda) - X_2(\lambda,\chi_2-\lambda))^2 = -e^{-2\lambda}(e^{\chi_1}-e^{\chi_2})^2 < 0; \tag{2.18}$$

furthermore,

$$(X_1(\lambda-i\pi,\chi_1-\lambda) - X_2(\lambda,\chi_2-\lambda))^2 = -e^{-2\lambda}(e^{\chi_1}+e^{\chi_2})^2 = L_{12} + 4e^{-2\lambda}e^{\chi_1+\chi_2}. \tag{2.19}$$

The first term in (2.16) is singular for any two lightlike separated points in the Rindler wedge. When the two points are both approaching either the future or the past horizon also the second term diverges, and it does exactly as the first term; when $\lambda \to +\infty$,

$$W_\pi[X(\lambda,\chi_1-\lambda), X(\lambda,\chi_2-\lambda)] \approx -\frac{2}{4\pi}\log(-m^2 L_{12}), \quad \text{as } \lambda \to +\infty. \tag{2.20}$$

Similarly, for $\beta = \frac{2\pi}{N}$ and $\lambda \to +\infty$,

$$W_{\frac{2\pi}{N}}[X(\lambda,\chi_1-\lambda), X(\lambda,\chi_2-\lambda)] \approx -\frac{N}{4\pi}\log(-m^2 L_{12}) = -\frac{1}{2\beta}\log(-m^2 L_{12}). \tag{2.21}$$

In the horizon limit (2.17) the dominant contribution to the integral (2.12) comes from the infrared region $\omega \to 0$. Using Appendix A and the asymptotic form of the modes near the horizon, one can show that (2.21) remains true for general β. The calculation is similar to the one preformed in [6]. Such a dependence of the coefficient of the singularity at lightlike separation (at the horizon) implies that the thermal state cannot be continued to the entire Minkowski space-time.

It is possible to introduce more general time translation invariant (at tree-level) states by letting the temperature depend on the energy:

$$\mathcal{W}(X_1,X_2) = \frac{1}{\pi^2}\int_0^\infty \left[\frac{e^{-i\omega(\eta_1-\eta_2)}}{1-e^{-\beta(\omega)\omega}} + \frac{e^{i\omega(\eta_1-\eta_2)}}{e^{\beta(\omega)\omega}-1}\right] \times K_{i\omega}(me^{\xi_1})K_{i\omega}(me^{\xi_2})\sinh \pi\omega \, d\omega. \tag{2.22}$$

These sates also respect the exchange symmetry (2.4). However, when $m \neq 0$ it is not possible to disentangle two independent temperatures for the left- and right-movers. That is because the modes $e^{-i\omega t}K_{i\omega}$ are normalizable linear combinations of left- and right-movers. We will see below that for de Sitter and Schwarzschild fields the situation is different and such a possibility does exist.

B. General dimension

If there are d extra flat transverse spatial dimensions $\vec{x}$, then the Rindler metric is

$$ds_d^2 = e^{2\xi}(d\eta^2 - d\xi^2) - d\vec{x}^2. \tag{2.23}$$

The modes can be represented as $\varphi(\eta,\xi,\vec{x}) = e^{i\vec{k}\vec{x}}\varphi_{\vec{k}}(\eta,\xi)$, where $\varphi_{\vec{k}}(\eta,\xi)$ obeys Eq. (2.7) with the effective mass m^2+k^2. Therefore, the field operator can be expanded as

$$\varphi(\eta,\xi,\vec{x}) = \int_{-\infty}^{+\infty}\frac{d^d k}{(2\pi)^{\frac{d}{2}}}\int_0^{+\infty}\frac{d\omega}{\pi}\sqrt{\sinh \pi\omega} \times \left[e^{-i\omega\eta+i\vec{k}\vec{x}}b_{\omega,\vec{k}} + e^{i\omega\eta-i\vec{k}\vec{x}}b^\dagger_{\omega,\vec{k}}\right] \times K_{i\omega}\left(\sqrt{m^2+k^2}e^{\xi}\right). \tag{2.24}$$

The Wightman function at temperature β is as follows:

$$W_\beta(X_1,X_2) = \int_{-\infty}^{+\infty}\frac{d^d k}{(2\pi)^d}\int_{-\infty}^{+\infty}\frac{d\omega}{\pi^2}\frac{\sinh(\pi\omega)}{1-e^{-\beta\omega}} \times e^{-i\omega(\eta_1-\eta_2)}e^{i\vec{k}(\vec{x}_1-\vec{x}_2)}K_{i\omega}\left(\sqrt{m^2+k^2}e^{\xi_1}\right) \times K_{i\omega}\left(\sqrt{m^2+k^2}e^{\xi_2}\right). \tag{2.25}$$

Enforcing Poincaré invariance gives $\beta = 2\pi$ [21]; this is the well-known Bisognano-Wichmann theorem, valid also for interacting quantum fields [27]. The anomalous divergence on the horizons for generic $\beta \neq 2\pi$ goes precisely as in the previous section.

C. Stress energy tensor in 2D

Here we complete the discussion of the massive scalar field in 2D Rindler spacetime by examining the renormalized stress-energy tensor at various temperatures (some technical details can be found in Appendix B). To set up the notations let us summarize the standard expression resulting from point splitting regularization in the Poincaré invariant case $\beta = 2\pi$:

$$\langle T_{VV}\rangle_{2\pi} = -\frac{t_V t_V}{4\pi\epsilon^2}, \qquad \langle T_{UU}\rangle_{2\pi} = -\frac{t_U t_U}{4\pi\epsilon^2},$$
$$\langle T_{VU}\rangle_{2\pi} = \langle T_{\mathrm{UV}}\rangle_{2\pi} = -\frac{e^{V-U}}{8\pi}m^2\left[\gamma_e + \log(m) + \log\left(\epsilon\sqrt{t_\alpha t^\alpha}\right)\right], \tag{2.26}$$

where t_μ is the vector separating the two points of the Wightman function (2.12).

The above expressions lead to the covariantly conserved stress-energy tensor [28]:

$$\langle :T_{\mu\nu}:\rangle_{2\pi} = -\frac{1}{4\pi}m^2[\gamma + \log(m)]g_{\mu\nu}, \qquad (2.27)$$

where γ is the Euler-Mascheroni constant. This is obviously related to the expectation value in Minkowski space by the coordinate transformations (2.1).

Similarly, for $\beta = 2\pi/N$ point splitting regularization in (2.15) gives

$$\langle :T_{\mu\nu}:\rangle_{\frac{2\pi}{N}} = \sum_{n=1}^{N-1}\frac{m^2}{4}e^{V-U}K_2\left(2me^{\frac{V-U}{2}}\sin\left(\frac{n\pi}{N}\right)\right)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \left(-\frac{1}{4\pi}m^2[\gamma + \log(m)] + \frac{m^2}{2}\sum_{n=1}^{N-1}K_0\left(2me^{\frac{V-U}{2}}\sin\left(\frac{n\pi}{N}\right)\right)\right)g_{\mu\nu}, \qquad (2.28)$$

where $K_0(x)$ and $K_2(x)$ are MacDonald functions. Violation of Poincaré invariance is manifest.

Near the horizon this expression simplifies to

$$\langle :T_{\mu\nu}:\rangle_{\frac{2\pi}{N}} = \frac{1}{24}(N^2 - 1)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \mathcal{O}(e^{V-U}), \qquad (2.29)$$

while at the spatial infinity it gives

$$\langle :T_{\mu\nu}:\rangle_{\frac{2\pi}{N}} \approx -\frac{1}{4\pi}m^2[\gamma_e + \log(m)]g_{\mu\nu},$$

which coincides with the $\beta = 2\pi$ case. These two types of asymptotic behavior of the stress energy tensor are regular. Furthermore, the second one does not depend on β. On the other hand, the expectation value of the mixed components of stress-energy tensor T_μ^ν diverge at the horizon. For generic values of β, when both points in (2.25) are taken to the horizon, we get (see Appendix B)

$$W_\beta(X^+, X^-) \approx \int_{-\infty}^{\infty}\frac{d\omega}{\pi\omega}\frac{e^{-\frac{i\omega}{2}(V^+ + U^+ - V^- - U^-)}}{1 - e^{-\beta\omega}} \times \sin\left(\omega\log(me^{\frac{(V^+ - U^+)}{2}}/2) + \arg\Gamma(1 - i\omega)\right) \times \sin\left(\omega\log(me^{\frac{(V^- - U^-)}{2}}/2) + \arg\Gamma(1 - i\omega)\right). \qquad (2.30)$$

The expectation value may be obtained by taking into account

$$\partial_{V_+}\partial_{V_-}W_\beta(X^+, X^-) = \int_{-\infty}^{\infty}\frac{d\omega}{4\pi}\frac{\omega}{1 - e^{-\beta\omega}}e^{-i\omega(V^+ - V^-)} = -\frac{1}{4\pi(V^+ - V^-)^2} + \frac{\pi}{12\beta^2} = -\frac{t_V t_V}{4\pi\epsilon^2} + \frac{1}{24\pi} + \frac{\pi}{12\beta^2}. \qquad (2.31)$$

At the horizon for arbitrary temperatures we get

$$\langle :T_{\mu\nu}:\rangle_\beta = \frac{1}{24}\left(\left(\frac{2\pi}{\beta}\right)^2 - 1\right)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \mathcal{O}(e^{V-U}) \qquad (2.32)$$

to be compared with (2.29).

D. Massless case

We complete this section with a few remarks on the massless case, necessary to understand the novel features of the de Sitter case, which are presented in the next section. The discussion is kept short. Details will be fully discussed elsewhere.

The massless Klein-Gordon equation in the two-dimensional Rindler spacetime is

$$\Box\phi = \partial_\eta^2\phi - \partial_\xi^2\phi = 0. \qquad (2.33)$$

As regards the vacuum two-point function, as in the Minkowskian case [29,30], one would set in Fourier space

$$\tilde{W}_0(k) = \theta(k^0)\delta(k^2) = \tilde{W}_R(k) + \tilde{W}_L(k) = \frac{1}{k^+}\theta(k^+)\delta(k^-) + \frac{1}{k^-}\theta(k^-)\delta(k^+), \qquad (2.34)$$

where we introduced the light cone variables $k^\pm = k^0 \pm k^1$. Unfortunately, $\theta(k^0)$ is not a multiplier for $\delta(k^2)$. The standard regularization (see e.g., [29–32]) involves an arbitrary infrared regulator having the dimension of a mass:

$$W_R(\eta, \xi) = W_R(U) = -\frac{1}{4\pi}\log(i\mu(U - i\epsilon)), \qquad (2.35)$$

$$W_L(\eta, \xi) = W_L(V) = -\frac{1}{4\pi}\log(i\mu(V - i\epsilon)). \qquad (2.36)$$

We see here that W_R (resp. W_L) is analytic in the lower half-plane of the complex variable U (resp. V). We may, therefore, introduce the regularized thermal massless right two-point function as the following formal series:

$$W_{R,\beta}(x^-) = \sum_{n=0}^{\infty}W_R(U - in\beta) + \sum_{n=1}^{\infty}W'_R(U + in\beta) \qquad (2.37)$$

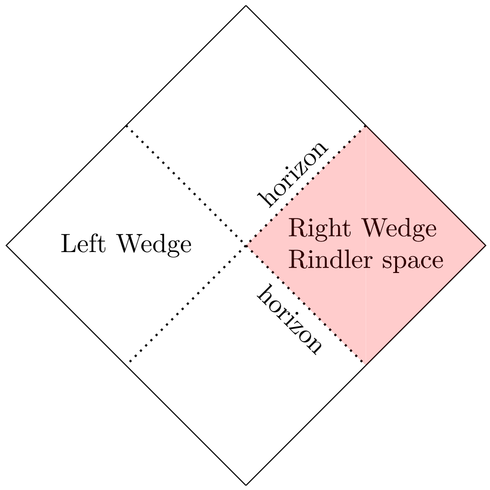

FIG. 1. Penrose diagram of the Rindler space. The Rindler space is bordered by a Killing horizon.

and similarly for $W_{L,\beta}(V)$. The total two-point function is the sum

$$W_{0,\beta}(\eta,\xi) = W_{L,\beta}(V) + W_{R,\beta}(U). \tag{2.38}$$

But the left and right movers are independent fields and may have independent temperatures. We may thus formally introduce the states characterized by functions of the form

$$W_{0,\beta_L\beta_R}(x) = W_{L,\beta_L}(x^+) + W_{R,\beta_R}(x^-). \tag{2.39}$$

This choice, of course, does not change the commutators that do not depend on the temperatures.

In conclusion we may introduce two independent temperatures for the left- and right-moving fields,

$$\phi = \phi_R + \phi_L, \tag{2.40}$$

by leaving, of course, the commutator untouched. But this possibility in Rindler space exists only for the massless field and not for the massive one.

III. STATIC PATCH OF de Sitter SPACE-TIME

A. Geometry, modes, and Wightman functions

The two-dimensional de Sitter space can be most easily visualized as the one-sheeted hyperboloid embedded in a three dimensional ambient Minkowski space:

$$dS_2 = \{X \in \mathbf{R}^3, X^\mu X_\mu = X_0^2 - X_1^2 - X_2^2 = -R^2\}. \tag{3.1}$$

X^μ denotes the coordinates of a given Lorentzian frame of the ambient spacetime; we set the radius R of the de Sitter space equal to 1. A suitable coordinate system for the static patch is

$$X(t,x) = \begin{cases} X^0 = \sinh t \,\mathrm{sech}\, x \\ X^1 = \tanh x = u \\ X^2 = \cosh t \,\mathrm{sech}\, x \end{cases}, \quad t \in (-\infty,\infty), x \in (-\infty,\infty). \tag{3.2}$$

The static coordinates cover a causal diamond of the entire two-dimensional de Sitter space (see Fig. 2; see also [6] for a full description); we refer to it as the static patch or the Rindler-de Sitter wedge. The metric and the massive scalar Klein-Gordon equation in these coordinates are written as follows:

$$ds^2 = \frac{dt^2 - dx^2}{\cosh x^2}, \qquad \partial_t^2\varphi - \partial_x^2\varphi + \frac{m^2\varphi}{\cosh^2 x} = 0. \tag{3.3}$$

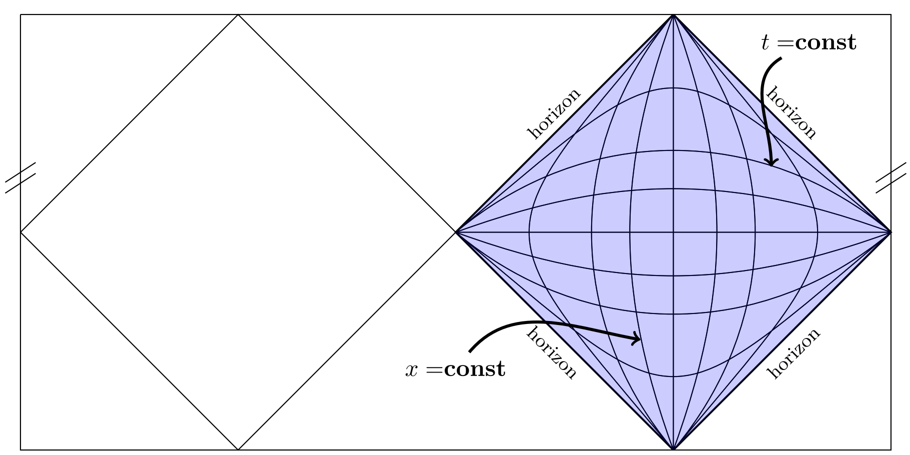

FIG. 2. Penrose diagram of the de Sitter manifold with Cauchy surfaces of different patches. The static patch is bordered by a bifurcate Killing horizon.

The scattering eigenfunctions of the Schrödinger operator with potential $m^2/\cosh^2 x$ [6]

$$\psi_\omega(x) = \sqrt{\sinh(\pi\omega)}\Gamma\left(\frac{1}{2} + i\mu - i\omega\right)\Gamma\left(\frac{1}{2} - i\mu - i\omega\right)\mathsf{P}^{i\omega}_{-\frac{1}{2}+i\mu}(\tanh x), \quad \mu^2 = m^2 - \frac{1}{4}$$

provide two modes for each energy level ω, namely $e^{-i\omega t}\psi_\omega(\pm x)$. The asymptotic behavior of the modes at $x \to \infty$ is governed by

$$\mathsf{P}^{i\omega}_{-\frac{1}{2}+i\mu}(\tanh x) \underset{x\to\infty}{\approx} \frac{e^{i\omega x}}{\Gamma(1 - i\omega)}, \tag{3.4}$$

$$\mathsf{P}^{i\omega}_{-\frac{1}{2}+i\mu}(-\tanh x) \underset{x\to\infty}{\approx} \left[\frac{\Gamma(-i\omega)e^{-i\omega x}}{\Gamma(\frac{1}{2} + i\mu - i\omega)\Gamma(\frac{1}{2} - i\mu - i\omega)} + \frac{\cosh(\mu\pi)\Gamma(i\omega)e^{i\omega x}}{\pi}\right]; \tag{3.5}$$

this shows that $e^{-i\omega t}\psi_\omega(x)$ is asymptotically right moving and $e^{-i\omega t}\psi_\omega(-x)$ left moving.

The expansion of the field operator written in terms of the above modes naturally splits into two commuting fields: a left mover ϕ_L and a right mover ϕ_R for all values of the mass m:

$$\begin{aligned}
&\phi(t,x) = \phi_R(t,x) + \phi_L(t,x),\\
&\phi_R(t,x) = \frac{1}{2\pi}\int_0^\infty [e^{-i\omega t}\psi_\omega(x)a_\omega + e^{i\omega t}\psi^*_\omega(x)a^\dagger_\omega]d\omega,\\
&\phi_L(t,x) = \frac{1}{2\pi}\int_0^\infty [e^{-i\omega t}\psi_\omega(-x)b_\omega + e^{i\omega t}\psi^*_\omega(-x)b^\dagger_\omega]d\omega;
\end{aligned} \tag{3.6}$$

the ladder operators obey the standard commutation relations

$$[a_{\omega_1}, a^\dagger_{\omega_2}] = \delta(\omega_1 - \omega_2), \qquad [b_{\omega_1}, b^\dagger_{\omega_2}] = \delta(\omega_1 - \omega_2),$$
$$[a_{\omega_1}, b_{\omega_2}] = [a_{\omega_1}, b^\dagger_{\omega_2}] = 0.$$

It maybe worthwhile to stress that the above separation into left and right movers is only possible in the static coordinate system because of the symmetry of the effective potential and, once more, it holds true for massive fields. Here the left- and right-moving modes only asymptotically depend on one of the two light cone variables $t \pm x$ near the corresponding side of the horizon.

In [6] we constructed general time translation invariant states

$$\begin{aligned}
\langle a^\dagger_\omega a_{\omega'}\rangle &= \delta(\omega - \omega')\frac{1}{e^{\beta_R(\omega)\omega} - 1} \quad \text{and}\\
\langle b^\dagger_\omega b_{\omega'}\rangle &= \delta(\omega - \omega')\frac{1}{e^{\beta_L(\omega)\omega} - 1}.
\end{aligned} \tag{3.7}$$

We gave, in particular, a full treatment for states of arbitrary global (inverse) temperature

$$\beta_L(\omega) = \beta_R(\omega) = \beta \tag{3.8}$$

and provided new integral representations for their correlation functions. Taking inspiration from the consideration of the Unruh state for black holes, we enlarge that study and consider different global temperatures for the left- and the right-moving modes:

$$\beta_L(\omega) = \beta_L, \qquad \beta_R(\omega) = \beta_R. \tag{3.9}$$

The Wightman function is the sum of two contributions

$$W_{\beta_L\beta_R}(X_1, X_2) = W_{L,\beta_L}(X_1, X_2) + W_{R,\beta_R}(X_1, X_2), \tag{3.10}$$

where

$$W_{L,\beta}(X_1, X_2) = \int_0^\infty \frac{d\omega}{4\pi^2}\left[e^{-i\omega(t_1-t_2)}\frac{\psi_\omega(-x_1)\psi^*_\omega(-x_2)}{1 - e^{-\beta\omega}} + e^{i\omega(t_1-t_2)}\frac{\psi^*_\omega(-x_1)\psi_\omega(-x_2)}{e^{\beta\omega} - 1}\right], \tag{3.11}$$

$$W_{R,\beta}(X_1, X_2) = \int_0^\infty \frac{d\omega}{4\pi^2}\left[e^{-i\omega(t_1-t_2)}\frac{\psi_\omega(x_1)\psi^*_\omega(x_2)}{1 - e^{-\beta\omega}} + e^{i\omega(t_1-t_2)}\frac{\psi^*_\omega(x_1)\psi_\omega(x_2)}{e^{\beta\omega} - 1}\right]. \tag{3.12}$$

The formal proof of the KMS periodicity property goes as follows:

$$W_{R,\beta}(X_2(t_2,x_2),X_1(t_1,x_1)) = \frac{1}{4\pi^2}\sum_{n=0}^{\infty}\int_0^{\infty} e^{-i\omega(t_2-t_1-in\beta)}\psi_\omega(x_2)\psi^*_\omega(x_1)d\omega + \frac{1}{4\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty} e^{i\omega(t_2-t_1+in\beta)}\psi^*_\omega(x_2)\psi_\omega(x_1)d\omega$$
$$= W_{R,\beta}(X_1(t_1-i\beta,x_1),X_2(t_2,x_2)). \tag{3.13}$$

There holds the exchange symmetry

$$W_{R,\beta_R}(X_1(t_1,x_1),X_2(t_2,x_2)) = W_{L,\beta_R}(X_1(t_1,-x_1),X_2(t_2,-x_2)). \tag{3.14}$$

When $\beta_L = \beta_R = 2\pi$ the Wightman function (3.10) respects the de Sitter isometry [4,6,33–37], i.e., it is a function of the complex de Sitter invariant variable

$$\zeta = -\frac{\cosh(t_2-t_1)+\sinh x_1 \sinh x_2}{\cosh x_1 \cosh x_2}$$

with the locality cut on the negative reals. The variable ζ and the geodesic distance L are related as follows: $\zeta = -\cosh L$ for timelike geodesics, $\zeta = \cos L$ for spacelike ones, and $\zeta = -1$ for lightlike separations.

Let us consider now the behavior at the horizon of Eq. (3.10). Points of the right (left) future horizon are obtained in the following limit:

$$\lim_{\lambda\to+\infty} X(\lambda,\pm(\lambda-\chi)) = \lim_{\lambda\to+\infty}\begin{pmatrix}\operatorname{sech}(\lambda-\chi)\sinh\lambda\\ \pm\tanh(\lambda-\chi)\\ \operatorname{sech}(\lambda-\chi)\cosh\lambda\end{pmatrix} = \begin{pmatrix}e^{\chi}\\ \pm1\\ e^{\chi}\end{pmatrix}. \tag{3.15}$$

Points of the left (right) past horizon are obtained in the limit $\lambda\to-\infty$ of the above expression. In all cases the interval between two points having the same finite coordinate λ is spacelike:

$$L_{12} = -\frac{2(\cosh(\chi_1-\chi_2)-1)}{\cosh(\lambda-\chi_1)\cosh(\lambda-\chi_2)} < 0, \tag{3.16}$$

becoming lightlike only in the limit $\lambda\to\pm\infty$.

Using the asymptotics of the modes in (3.4) and (3.5), and Eq. (A1), one can obtain the behavior of W_{R,β_R} and W_{L,β_L} separately at, e.g., the right side of the horizon. As we can see from Eq. (3.4), in this region W_{R,β_R} depends only on the difference $x_1 - x_2$, which does not grow when both points are taken to the same side of the horizon. It means that this contribution to the Wightman function is regular near the right side of the horizon. At the same time, in the same region W_{L,β_L} depends on both the x_1-x_2 and x_1+x_2, as we can see from Eq. (3.5). The latter sum is infinitely growing near the horizon. As a result, using (A1) one obtains that

$$W_{\beta_L\beta_R}(X(\lambda,\chi_1-\lambda),X(\lambda,\chi_2-\lambda)) \approx W_{L,\beta_L}(X(\lambda,\chi_1-\lambda),X(\lambda,\chi_2-\lambda)) \approx \frac{1}{\beta_L}\lambda, \quad \lambda\to+\infty. \tag{3.17}$$

Behavior near the left horizon can be found from the symmetry from Eq. (3.14), which implies that

$$W_{\beta_L\beta_R}(X(t_1,x_1),X(t_2,x_2)) = W_{\beta_R\beta_L}(X(t_1,-x_1),X(t_2,-x_2)).$$

Hence, parity $x\to-x$ plus rearrangement of temperatures $\beta_L\leftrightarrow\beta_R$ leaves the two-point function invariant. As a result, it follows that for $\lambda\to-\infty$

$$W_{\beta_L\beta_R}(X(\lambda,\chi_1-\lambda),X(\lambda,\chi_2-\lambda)) \approx W_{R,\beta_R}(X(\lambda,\chi_1-\lambda),X(\lambda,\chi_2-\lambda)) \approx \frac{1}{\beta_R}|\lambda|. \tag{3.18}$$

The lightlike singularity at the horizons depends on the state of the theory. In particular, at the right horizon it depends only on β_R, while at the left horizon it depends on β_L. This shows that such a peculiar behavior is present due to the interplay between the waves that are falling down and reflected from the $m^2/\cosh^2 x$ potential.

B. General dimension

The $(D+1)$-embedding coordinates and the invariant scalar product for the D-dimensional static patch are given by

$$X_0 = \sinh(t)\operatorname{sech}(x), \qquad X_i = \tanh(x)\vec{y}_i,$$
$$X_D = \cosh(t)\operatorname{sech}(x), \qquad \vec{y}_i\vec{y}_i = 1, \tag{3.19}$$

$$Z = \eta_{\mu\nu}X_1^\mu X_2^\nu = -\frac{\cosh(t_2-t_1)+\vec{y}_1\cdot\vec{y}_2\sinh x_1\,\sinh x_2}{\cosh x_1\,\cosh x_2}. \tag{3.20}$$

The Bunch-Davies Wightman function [4,35,36,38–42] corresponding to the inverse temperature $\beta_L = \beta_R = 2\pi$ [4,26,33,35,36] is given by

$$W_{2\pi}(Z) = \frac{\Gamma(\frac{D-1}{2}+i\mu)\Gamma(\frac{D-1}{2}-i\mu)}{2(2\pi)^{\frac{D}{2}}}(Z^2-1)^{-\frac{D-2}{4}}P^{-\frac{D-2}{2}}_{-\frac{1}{2}+i\mu}(Z), \tag{3.21}$$

where $\mu = \sqrt{m^2-(D-1)^2/4}$. It has the standard Hadamard singularity near $Z=-1$.

Points of the future and past horizons are attained in the following limits:

$$\lim_{\lambda\to\pm\infty} X(\lambda,(\lambda-\chi)) = \lim_{\lambda\to\infty}\begin{pmatrix}\mathrm{sech}(\lambda-\chi)\sinh\lambda\\ \tanh(\lambda-\chi)\vec{y}\\ \mathrm{sech}(\lambda-\chi)\cosh\lambda\end{pmatrix} = \begin{pmatrix}\pm e^{\chi}\\ \pm\vec{y}\\ e^{\chi}\end{pmatrix}. \tag{3.22}$$

Two events on the horizons are spacelike separated unless $\vec{y}_1=\vec{y}_2$. As in Eq. (2.15) for $\beta=\frac{2\pi}{N}$ in the horizon limit one gets:

$$W_{\frac{2\pi}{n}}(\lambda\to+\infty)\approx NW_{2\pi}(\lambda\to+\infty) \approx -N\frac{\Gamma(\frac{D-2}{2})}{2^{2+(D-2)\frac{3}{2}}\pi^{\frac{D}{2}}}e^{(D-2)\lambda}. \tag{3.23}$$

As in Rindler space the singularity of the propagator on the horizon depends on the temperature.

C. Stress-energy tensor in 2D

Let us introduce the light-cone coordinates of the static patch:

$$V=t+x,\qquad U=t-x,$$
$$ds^2=\frac{1}{\cosh^2(\frac{V-U}{2})}dUdV\equiv C(U,V)dUdV. \tag{3.24}$$

To set up notations let us discuss first the de Sitter invariant case $\beta=2\pi$. When the two arguments of the Wightman function are taken very close to each other, one has that

$$W_{2\pi}(X_+,X_-)\approx-\frac{1}{4\pi}\left(H_{-\frac{1}{2}+i\mu}+H_{-\frac{1}{2}-i\mu} + \log\left[\frac{(V_--V_+)(U_+-U_-)}{4\cosh^2(V-U)}\right]\right), \tag{3.25}$$

where $H_{-\frac{1}{2}+i\mu}=\psi(\frac{1}{2}+i\mu)+\gamma_e$ are the harmonic numbers; the definitions of $X_\pm$, $V_\pm$, and $U_\pm$ can be found in Appendix B. Since

$$\partial_{V_+}\partial_{V_-}W_{2\pi}(Z)\approx-\frac{1}{4\pi(V_+-V_-)^2}+\frac{1}{48\pi},\quad \text{and}$$
$$\partial_{U_+}\partial_{U_-}W_{2\pi}(Z)\approx-\frac{1}{4\pi(U_+-U_-)^2}+\frac{1}{48\pi}, \tag{3.26}$$

the covariant point splitting regularization gives

$$\langle T_{\mathrm{UV}}\rangle_{2\pi}=-\frac{m^2}{8\pi\cosh^2(\frac{V-U}{2})}\left(\psi\left(\frac{1}{2}+i\mu\right)+\psi\left(\frac{1}{2}-i\mu\right) + 2\gamma_e+\log\left[\epsilon^2 t_\alpha t^\alpha\right]\right), \tag{3.27}$$

$$\langle T_{UU}\rangle_{2\pi}=-\left(\frac{1}{4\pi\epsilon^2(t_\alpha t^\alpha)}+\frac{R}{24\pi}\right)\frac{t_U t_U}{t_\alpha t^\alpha}, \tag{3.28}$$

$$\langle T_{VV}\rangle_{2\pi}=-\left(\frac{1}{4\pi\epsilon^2(t_\alpha t^\alpha)}+\frac{R}{24\pi}\right)\frac{t_V t_V}{t_\alpha t^\alpha}. \tag{3.29}$$

After regularization we obtain the well-known answer [42]

$$\langle:T_{\mu\nu}:\rangle_{2\pi}=-\frac{1}{4\pi}m^2\left[\psi\left(\frac{1}{2}+i\mu\right)+\psi\left(\frac{1}{2}-i\mu\right)+2\gamma_e\right]g_{\mu\nu}. \tag{3.30}$$

The expectation value of the stress-energy tensor with two temperatures β_L and β_R can be obtained starting from Eq. (3.10). The most interesting case in this situation is the near horizon limit. For instance, close to the right horizon a lengthy but not difficult calculation gives

$$\partial_{V^+}\partial_{V^-}W_{\beta_L\beta_R}(X^+,X^-)\approx\int_{-\infty}^{\infty}\frac{d\omega}{4\pi}\frac{\omega}{e^{\beta_L\omega}-1}e^{i\omega(V^+-V^-)} = \frac{\pi}{12\beta_L^2}-\frac{1}{4\pi}\frac{1}{(V^+-V^-)^2} \tag{3.31}$$

and

$$\partial_{U^+}\partial_{U^-}W_{\beta_L\beta_R}(X^+,X^-) \approx\int_{-\infty}^{\infty}\frac{d\omega}{4\pi}\frac{\omega\sinh^2\pi\omega e^{i\omega(U^+-U^-)}}{\cosh\pi(\omega-\mu)\cosh\pi(\omega+\mu)} \times\left[\frac{1}{e^{\beta_R\omega}-1}+\frac{\cosh^2\mu\pi}{\sinh^2\pi\omega}\frac{1}{e^{\beta_L}-1}\right]. \tag{3.32}$$

The above expressions simplify when the temperatures of the left- and right-movers coincide: $\beta_R=\beta_L=\beta$. Then the regularized stress-energy tensor in the near horizon limit takes the form

$$\langle:T_{\mu\nu}:\rangle\approx\Theta_{\mu\nu}+\frac{R}{48\pi}g_{\mu\nu}, \tag{3.33}$$

where

$$\Theta_{UU} = -\frac{1}{12\pi} C^{1/2} \partial_U^2 C^{-1/2} + \frac{\pi}{12\beta^2} = \frac{\pi}{12}\left(\frac{1}{\beta^2} - \frac{1}{(2\pi)^2}\right),$$
$$\Theta_{VV} = -\frac{1}{12\pi} C^{1/2} \partial_V^2 C^{-1/2} + \frac{\pi}{12\beta^2} = \frac{\pi}{12}\left(\frac{1}{\beta^2} - \frac{1}{(2\pi)^2}\right),$$
$$\Theta_{UV} = \Theta_{VU} = 0.$$

Note that the de Sitter covariance is recovered only when $\beta = 2\pi$.

IV. SCHWARZSCHILD BLACK HOLE

Here we consider the radial part of the Schwarzschild metric (we call it the two-dimensional black hole):

$$ds^2 = \left(1 - \frac{r_g}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} = ds^2 = \left[1 - \frac{r_g}{r(r^*)}\right](dt^2 - dr^{*2}). \quad (4.1)$$

The tortoise coordinate,

$$r^* = r + r_g \log\left(\frac{r}{r_g} - 1\right), \quad (4.2)$$

is such that $r^* \approx r$ when $r \to +\infty$, while $r^* \to -\infty$ when $r \to r_g$; near the horizon the metric looks like the Rindler's one (2.2):

$$ds^2_{\mathrm{nh}} \approx e^{\frac{r^*}{r_g}}(dt^2 - dr^{*2}) \quad (4.3)$$

with the acceleration $\alpha = \frac{1}{2r_g}$. see Fig. 3 for the Penrose diagram'.

A. Modes and the Wightman function

In the tortoise coordinates the massive Klein-Gordon field equation,

$$\partial_t^2 \varphi - \partial_{r^*}^2 \varphi + m^2 g_{00} \varphi = 0, \quad (4.4)$$

is such that the mass term in (4.4) vanishes at the horizon where g_{00} does vanish. By separating the variables $\varphi(t, r^*) = e^{i\omega t}\varphi_\omega(r^*)$, we obtain

$$-\partial_{r^*}^2 \varphi_\omega(r^*) + m^2 g_{00}(r^*)\varphi_\omega(r^*) = \omega^2 \varphi_\omega(r^*). \quad (4.5)$$

When $\omega \leq m$ the modes decay exponentially at large r^* and are localized near the horizon [43]. There is no double degeneration as in the case of massive field in Rindler space.

The classically permitted region is at the left of the turning point solving the equation $\omega^2 - m^2 g_{00}(r^*_{\mathrm{turning}}) = 0$. Near the horizon, where the effective potential vanishes, the modes approximately behave as

$$\varphi_\omega(r^*) \approx \sqrt{\frac{2}{\pi}} \cos(\omega r^* + \delta_\omega), \quad \omega < m, \quad |\omega r^*| \gg 1. \quad (4.6)$$

We do not need their exact form for further considerations. They do not propagate at spatial infinity and do not contribute to the Hawking radiation [19,43]. On the other hand, they play an important role in the vicinity of the horizon.

When $\omega > m$ the situation is similar to the static de Sitter case: there are outgoing modes (right movers) $R_\omega(r^*)$ and ingoing modes (left movers) $L_\omega(r^*)$ which might be represented by resorting to special functions. We will instead apply semiclassical approximation methods, which in two dimensions[2] works well for all values of r^*; they lead to the following solutions:

$$R_\omega(r^*) = A_\omega 4\sqrt{\frac{\omega^2}{\omega^2 - m^2 g_{00}(r^*)}} \times \exp\left(i \,\mathrm{sgn}(\omega) \int_{r_0}^{r^*} \sqrt{\omega^2 - m^2 g_{00}(x)}\, dx\right), \quad (4.7)$$

$$L_\omega(r^*) = B_\omega 4\sqrt{\frac{\omega^2}{\omega^2 - m^2 g_{00}(r^*)}} \times \exp\left(-i \,\mathrm{sgn}(\omega) \int_{r_0}^{r^*} \sqrt{\omega^2 - m^2 g_{00}(x)}\, dx\right), \quad (4.8)$$

where r_0 is a reference point.

Here is the mode expansion of the field operator:

$$\varphi(t, r^*) = \int_0^m \frac{d\omega}{\sqrt{2\omega}} e^{-i\omega t} \varphi_\omega(r^*) c_\omega + \int_m^{+\infty} \frac{d\omega}{\sqrt{2\omega}} e^{-i\omega t}[R_\omega(r^*) a_\omega + L_\omega(r^*) b_\omega] + \mathrm{H.c.}, \quad (4.9)$$

---

[2] Indeed one has that

$$k(x) = \sqrt{(\omega r_g)^2 - (m r_g)^2 g_{00}(x)},$$
$$\left|\frac{d}{dx}\frac{1}{k(x)}\right| = \frac{1}{2 m r_g} \frac{1}{(\frac{\omega}{m} - g_{00}(x))^{3/2}} \frac{d g_{00}}{dx}.$$

Here $x = r_g/r^*$; $k(x)$ is the wave vector of the problem (4.5); the following inequality holds for all values of $\omega > m$:

$$\left|\frac{d}{dx}\frac{1}{k(x)}\right| < 1.$$

Thus, if $m r_g \gg 1$ the semiclassical approximation is applicable for all values of r^* and there is no reflection from the potential barrier in (4.5). Such a reflection is inevitable in four dimensions. We would like to thank Dmitriy Trunin for bringing to our attention this important simplification in 2D.

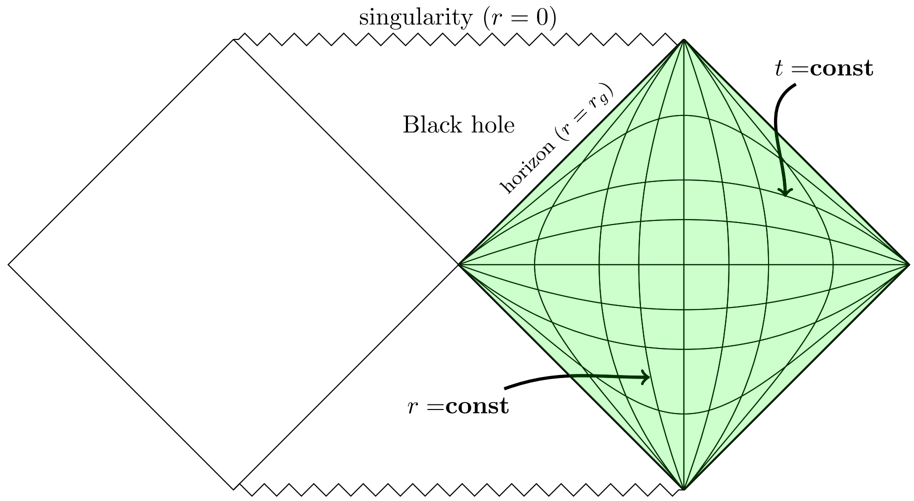

FIG. 3. Penrose diagram of the Schwarzschild black hole.

where

$$[a_\omega, a^\dagger_{\omega'}] = \delta(\omega - \omega'), \qquad [b_\omega, b^\dagger_{\omega'}] = \delta(\omega - \omega'),$$
$$[c_\omega, c^\dagger_{\omega'}] = \delta(\omega - \omega'), \tag{4.10}$$

all the other commutators being zero. One may show that in the approximation $mr_g \gg 1$ the canonical commutation relations give the normalization

$$|A_\omega|^2 = |B_\omega|^2 \approx \frac{1}{2\pi}. \tag{4.11}$$

Taking inspiration from the Rindler and de Sitter cases, we may now introduce a Wightman function depending on three (inverse) temperatures as follows:

$$\begin{aligned} W((t_1, r_1^*), (t_2, r_2^*))_{\beta_0\beta_L\beta_R} &= \int_{|\omega|<m} \frac{d\omega}{2\omega} \frac{e^{-i\omega(t_1-t_2)}}{1-e^{-\beta_0\omega}} \varphi_\omega(r_2^*)\varphi_\omega(r_1^*) \\ &+ \int_{|\omega|>m} \frac{d\omega}{2\omega} \left[\frac{e^{-i\omega(t_1-t_2)}}{1-e^{-\beta_L\omega}} L_\omega(r_1^*) L^*_\omega(r_2^*) \right. \\ &\left. + \frac{e^{-i\omega(t_1-t_2)}}{1-e^{\beta_R\omega}} R_\omega(r_1^*) R^*_\omega(r_2^*)\right]. \end{aligned} \tag{4.12}$$

The first term on the rhs of (4.12) shows again a double pole at $\omega = 0$. The way we treat it is explained Appendix A.

In Appendix C it is shown that when $m = 0$ and $\beta_R = \beta_L = 4\pi r_g$ the above expression coincides with the Hartle-Hawking Wightman function; when $\beta_L = \infty$ and $\beta_R = 4\pi r_g$ it corresponds to the Unruh state; finally, when $\beta_{R,L} = \infty$ it reproduces the Boulware state.

B. Singularity at the horizon

The additional singularity at the horizon is an infrared effect; the main contribution to it comes from the term

$$\varphi_\omega(r_1^*)\varphi_\omega(r_2^*) \approx \frac{1}{\pi} \cos(\omega(r_1^* + r_2^*) + 2\delta_\omega) \tag{4.13}$$

in (4.12) with $\omega \to 0$. To fix the phase we consider spacelike separated points near the horizon parametrized as follows:

$$r_1^* = \lambda, \qquad r_2^* = \lambda + \text{const}, \qquad t_1 = t_2 = -\lambda. \tag{4.14}$$

The (future) horizon limit corresponds to $\lambda \to -\infty$; in this limit the Wightman function has the following asymptotics:

$$W(t_1, r_1^* = \lambda | t_2, r_2^* = \lambda) = \frac{\lambda}{\beta_0} e^{2i\delta_0}, \ \text{as}\, \lambda \to -\infty. \tag{4.15}$$

As anticipated the limiting expression depends on the phase and goes to zero in the zero temperature limit. At low energies turning point

$$r^*_{\text{turning}} \approx r_g \log \frac{\omega^2}{m^2}, \quad \text{as } \omega \to 0 \tag{4.16}$$

is shifted to minus infinity. Since in this limit the Rindler space asymptotics should be reproduced, we have to set $\delta_0 = \frac{\pi}{2}$. At the end of the Appendix C we present another derivation of this.

C. Stress-energy tensor

In the light cone coordinates $V = t + r^*$, $U = t - r^*$ the metric (4.1) takes the form

$$ds^2 = C(U,V)dUdV, \qquad C(U,V) = \frac{\mathcal{W}(e^{\frac{V-U}{4M}-1})}{1+\mathcal{W}(e^{\frac{V-U}{4M}-1})}, \tag{4.17}$$

where $\mathcal{W}(r^*)$ is the Lambert function. Near the horizon

$$W((V^+,U^+),(V^-,U^-)) \approx \int_{-m}^{m} \frac{d\omega}{4\pi\omega} \frac{1}{e^{\beta_0\omega}-1} \left(e^{i\omega(V^+-U^-)+2i\delta_\omega} + e^{i\omega(V^+-V^-)} + e^{i\omega(U^+-U^-)} + e^{i\omega(U^+-V^-)-2i\delta_\omega}\right) + \int_{|\omega|>m} \frac{d\omega}{4\pi\omega}\left[\frac{e^{i\omega(V^+-V^-)}}{e^{\beta_R\omega}-1} + \frac{e^{i\omega(U^+-U^-)}}{e^{\beta_L\omega}-1}\right]. \tag{4.18}$$

By taking the limit of coinciding points, one gets

$$\partial_{U^+}\partial_{U^-}W \approx -\frac{1}{4\pi}\frac{1}{(U^+-U^-)^2} + \frac{\pi}{12\beta_0^2} + \frac{1}{2\pi}\left(\frac{Li_2(e^{-m\beta_L})}{\beta_L^2} - \frac{Li_2(e^{-m\beta_0})}{\beta_0^2}\right) + \frac{m}{2\pi}\left(\frac{\log(1-e^{-m\beta_0})}{\beta_0} - \frac{\log(1-e^{-m\beta_L})}{\beta_L}\right), \tag{4.19}$$

where $Li_2(x)$ is the polylogarithmic function.

Then for the components of the stress-energy tensor we obtain

$$T_{UU} \approx -\left[\frac{1}{4\pi\epsilon^2(t_\alpha t^\alpha)} + \frac{R}{24\pi}\right]\frac{t_U t_U}{t_\alpha t^\alpha} + \frac{\pi}{12}\left(\frac{1}{\beta_0^2} - \frac{1}{(8\pi M)^2}\right) + \frac{1}{2\pi}\left(\frac{Li_2(e^{-m\beta_L})}{\beta_L^2} - \frac{Li_2(e^{-m\beta_0})}{\beta_0^2}\right) + \frac{m}{2\pi}\left(\frac{\log(1-e^{-m\beta_0})}{\beta_0} - \frac{\log(1-e^{-m\beta_L})}{\beta_L}\right).$$
$$T_{VV} \approx -\left[\frac{1}{4\pi\epsilon^2(t_\alpha t^\alpha)} + \frac{R}{24\pi}\right]\frac{t_V t_V}{t_\alpha t^\alpha} + \frac{\pi}{12}\left(\frac{1}{\beta_0^2} - \frac{1}{(8\pi M)^2}\right) + \frac{1}{2\pi}\left(\frac{Li_2(e^{-m\beta_R})}{\beta_R^2} - \frac{Li_2(e^{-m\beta_0})}{\beta_0^2}\right) + \frac{m}{2\pi}\left(\frac{\log(1-e^{-m\beta_0})}{\beta_0} - \frac{\log(1-e^{-m\beta_R})}{\beta_R}\right). \tag{4.20}$$

The nondiagonal component near the horizon goes to zero:

$$T_{\mathrm{UV}} \approx \frac{m^2}{4} e^{\lambda/2r_g} \frac{|\lambda|}{\beta_0} \to 0. \tag{4.21}$$

Near the horizon the stress-energy tensor in similar to the original result of [44,45]

$$T_{\mu\nu} \approx \Theta_{\mu\nu} + \frac{R}{48\pi} g_{\mu\nu}, \tag{4.22}$$

where

$$\Theta_{UU} = -\frac{1}{12\pi} C^{1/2}\partial_U^2 C^{-1/2} + \frac{\pi}{12\beta_0^2} + L(\beta_L,\beta_0),$$
$$\Theta_{VV} = -\frac{1}{12\pi} C^{1/2}\partial_V^2 C^{-1/2} + \frac{\pi}{12\beta_0^2} + L(\beta_R,\beta_0),$$
$$\Theta_{UV} = \Theta_{VU} = 0,$$

and

$$L(\beta_1,\beta_2) = \left(\frac{Li_2(e^{-m\beta_1})}{\beta_1^2} - \frac{Li_2(e^{-m\beta_2})}{\beta_2^2}\right) + \frac{m}{2\pi}\left(\frac{\log(1-e^{-m\beta_2})}{\beta_2} - \frac{\log(1-e^{-m\beta_1})}{\beta_1}\right). \tag{4.23}$$

Some comments are in order here. When the three temperatures coincide the finite logarithmic and dilogarithmic contributions vanish. Furthermore, there are no finite contributions at all when they all are equal to the Hawking temperature $\beta = 4\pi r_g$. Second, while the additional singularity of the propagators is effective only on the nondiagonal components of the stress-energy tensor (in (u, v) coordinates), the exponential damping protects the covariant components (4.21). Additional singularity does arise in the mixed components of the stress energy tensor as follows:

$$T_V^V = T_U^U = \frac{m^2}{2}\langle\varphi\varphi\rangle \sim \frac{m^2}{2\beta_0}|\lambda|, \quad \text{as } \lambda \to -\infty. \tag{4.24}$$

V. OUTLOOK

Heating and thermalization are, however, nonstationary processes. To calculate the correlation functions in nonstationary situations one has to exploit the Schwinger-Keldysh diagrammatic technique [2,3]. The starting point is to choose an initial Cauchy surface[3] and an initial value of the correlation functions, i.e., an initial state. The Schwinger-Keldysh technique provides the time evolution towards the future of the correlation function in question.

Different types of Cauchy surfaces and initial values may, and in general do, lead to substantially different physical behaviors [9,11]. Even in highly symmetric curved space-time (such as de Sitter) the tree-level correlators of a generic state are not functions of geodesic distances. It goes without saying about generic space-times, which only partly resemble the de Sitter space [11]. In this sense the situation in strongly curved space-times is similar to the condensed matter phenomena rather than to high energy physics ones.

There is no *a priori* reason for the initial state in the early universe or in the vicinity of primordial black holes be necessarily the ground state or a thermal state at the canonical temperature. Here we considered a class of time translation invariant states in Rindler, static de Sitter, and two-dimensional black hole space-times. They can be thought of as initial states for thermalization or heating problems. They may also appear as attractor equilibrium states at the end of some processes. We have shown that when the various temperatures do not coincide with the canonical ones then the two-point Wightman functions have anomalous singularities at the horizons. That may affect the loop corrections. The latter are necessary to calculate and to resum to trace the fate of the initial state and of the correlation functions (see e.g., [2,3,9] for various related situations). Loop calculations will be considered elsewhere.

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APPENDIX A: LEADING INFRARED CONTRIBUTION

The behavior of various Wightman functions discussed above at the horizons is governed by the integral of the form:

$$\int_{-\infty}^{+\infty} \frac{d\omega}{\omega + i\epsilon} \frac{e^{i\omega\theta}}{e^{\beta(\omega+i\epsilon)} - 1}, \quad \text{where } |\theta| \gg 1.$$

The choice of the shifts of the poles here reproduces the results in the case $\beta = 2\pi/N$, but it can also be justified by general distributional methods [31]. The contour in closed in the upper half-plane for positive values of θ and in the lower half for negative ones. In the first case the double pole at $\omega = -i\epsilon$ does not contribute. Contributions from other poles are suppressed. For negative θ the leading contributions in the limit $\theta \to -\infty$ comes from the double pole at $\omega = -i\epsilon$:

$$\int_{-\infty}^{+\infty} \frac{d\omega}{\omega + i\epsilon} \frac{e^{i\omega\theta}}{e^{\beta(\omega+i\epsilon)} - 1} \approx \begin{cases} 0 & \text{if } \theta > 0 \\ \frac{2\pi}{\beta}\theta & \text{if } \theta < 0 \end{cases} \quad \text{as } |\theta| \gg 1, \tag{A1}$$

the answer depends on the sign of θ.

APPENDIX B: POINT-SPLITTING REGULARIZATION

To make the paper self-contained and set up the notations in this Appendix we summarize here the standard point splitting regularization procedure [44] of the expectation value of the stress-energy tensor in curved space-time:

$$\langle T_{\mu\nu}(x)\rangle = D_{\mu\nu}\langle\varphi(x^+)\varphi(x^-)\rangle|_{x^+=x^-=x}.$$

Here $D_{\mu\nu}$ is a differential operator; $x^\pm$ are points which are separated from x along a spacelike geodesic, and t^μ is tangent vector (see the Fig. 4). A point close enough to x^μ can be represented as follows:

$$x^\mu(\tau) = x^\mu + \tau t^\mu + \frac{1}{2}\tau^2 a^\mu + \frac{1}{6}\tau^3 b^\mu + \cdots, \tag{B1}$$

where τ is the proper length. And the coordinates of $x^\pm$ are $x^{\mu\pm} = x^\mu(\tau = \pm\epsilon)$.

General two-dimensional conformally flat metrics can be written $ds^2 = C(u,v)dudv$. The geodesic equations provide the relations between the parameters t^μ, a^μ, b^μ:

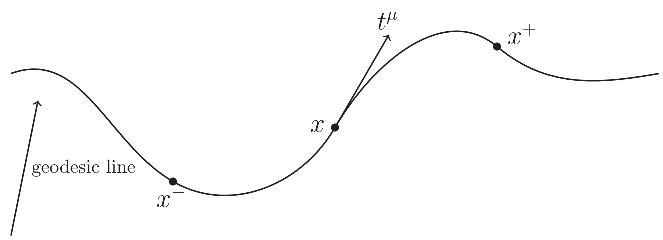

FIG. 4. Point-splitting.

[3]This method applies to globally hyperbolic spacetimes. In nonglobally hyperbolic space-times one should deal also with boundary conditions [46].

$$a^\mu = -\Gamma^\mu_{\nu\lambda} t^\nu t^\lambda, \qquad b^\mu = -\Gamma^\mu_{\nu\lambda}(a^\nu t^\lambda + t^\nu a^\lambda) - t^\sigma \partial_\sigma \Gamma^\mu_{\nu\lambda} t^\nu t^\lambda. \tag{B2}$$

It is enough to express a^μ and b^μ in terms of t^μ to find the finite part of the expectation value of the stress-energy tensor. Another building block is the parallel transport matrix $e^\mu_\nu(\tau)$, solving the following equation:

$$\frac{de^\mu_\nu}{d\tau} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\tau} e^\sigma_\nu = 0, \quad e^\mu_\nu(\tau = 0) = \delta^\mu_\nu. \tag{B3}$$

Again, one expands the parallel transport matrix in powers of τ:

$$e^\mu_\nu = \delta^\mu_\nu + \tau t^\mu_\nu + \frac{1}{2}\tau^2 a^\mu_\nu + \cdots,$$

where

$$t^\mu_\nu = -\Gamma^\mu_{\rho\nu} t^\rho, \quad a^\mu_\nu = \Gamma^\mu_{\rho\nu}\Gamma^\rho_{\alpha\beta} t^\alpha t^\beta + \Gamma^\mu_{\rho\sigma}\Gamma^\sigma_{\alpha\nu} t^\rho t^\alpha - t^\alpha t^\rho \partial_\alpha \Gamma^\mu_{\rho\nu}.$$

The expectation value of the stress-energy tensor in a state at inverse temperature β^{-1} is given by

$$\langle T_{\mu\nu}\rangle_\beta = \langle \partial_\alpha \varphi(x^+)\partial_\beta \varphi(x^-)\rangle_\beta \left(e^{+\alpha}_\mu e^{-\beta}_\nu - \frac{1}{2} g_{\mu\nu} g^{\sigma\rho} e^{+\alpha}_\sigma e^{-\beta}_\rho \right) + \frac{1}{2} m^2 g_{\mu\nu} \langle \varphi(x^+)\varphi(x^-)\rangle_\beta, \tag{B4}$$

where $e^{\pm\alpha}_\mu = e^\alpha_\mu(\tau = \pm\epsilon)$ and the limit $\epsilon \to 0$ is taken. The result will contain terms that depend on ϵ and direction-dependent terms. For example, for the massless field in the generic conformally flat background one has [45]

$$\langle T_{\mu\nu}\rangle = -\left[\frac{1}{4\pi\epsilon^2 (t_\alpha t^\alpha)} + \frac{R}{24\pi}\right]\left[\frac{t_\mu t_\nu}{t_\alpha t^\alpha} - \frac{1}{2} g_{\mu\nu}\right] + \Theta_{\mu\nu}, \tag{B5}$$

and the regularized stress-energy tensor reads:

$$\langle : T_{\mu\nu} : \rangle = \Theta_{\mu\nu} + \frac{R}{48\pi} g_{\mu\nu}, \tag{B6}$$

with

$$\Theta_{uu} = -\frac{1}{12\pi} C^{1/2} \partial_u^2 C^{-1/2} + \text{state dependent terms}, \tag{B7}$$

$$\Theta_{vv} = -\frac{1}{12\pi} C^{1/2} \partial_v^2 C^{-1/2} + \text{state dependent terms}, \tag{B8}$$

$$\Theta_{uv} = \Theta_{vu} = 0. \tag{B9}$$

The tensor is conserved for an invariant state only if one omits the direction-dependent terms, while averaging over directions leads to quantities, which are not covariantly conserved.

APPENDIX C: BOULWARE, UNRUH, AND HARTLE-HAWKING STATES

There are several different ways to define Boulware, Unruh, and Hartle–Hawking states for *massless scalar fields in four dimensions*. Not all of them can be straightforwardly generalized to the massive case. Here we repeat the standard constructions and consider their generalizations to the massive case.

1. Analytic continuation of the positive frequency modes

We look for a complete set of solutions of the massless Klein-Gordon equation in either the left or right (Schwarzschild) quadrant of the entire black hole spacetime in four dimensions (see Fig. 3). We require these functions to have a definite sign of frequency with respect to the timelike Killing vector $\frac{\partial}{\partial t}$ (in the left quadrant $-\frac{\partial}{\partial t}$):

$$\vec{u}_{\omega lm}(x) = (4\pi\omega)^{-1/2} e^{-i\omega t} \vec{R}_l(\omega|r) Y_{lm}(\theta,\varphi),$$
$$\overleftarrow{u}_{\omega lm}(x) = (4\pi\omega)^{-1/2} e^{-i\omega t} \overleftarrow{R}_l(\omega|r) Y_{lm}(\theta,\varphi), \tag{C1}$$

where $\vec{R}_l(\omega|r)$ and $\overleftarrow{R}_l(\omega|r)$ are solutions of the radial equation corresponding to outgoing and incoming waves, respectively [17], and $Y_{lm}(\theta,\varphi)$ are the standard spherical harmonics; $x = (t, r, \theta, \varphi)$. The Boulware two-point function is

$$W_B(x,x') = \sum_{lm} \int_0^\infty \frac{d\omega}{4\pi\omega} e^{-i\omega(t-t')} Y_{lm}(\theta,\phi) Y^*_{lm}(\theta',\phi') \times [\vec{R}_l(\omega|r)\vec{R}^*_l(\omega|r') + \overleftarrow{R}_l(\omega|r)\overleftarrow{R}^*_l(\omega|r')]. \tag{C2}$$

To define the Unruh state the Kruskal extension is needed:

$$U = t - r - 2M \log(r/2M - 1), \qquad \tilde{U} = -4Me^{-U/4M},$$
$$V = t + r + 2M \log(r/2M - 1), \qquad \tilde{V} = 4Me^{V/4M}.$$

The Unruh modes are positive-frequency with respect to U and near the paste horizon behave as follows:

$$y_{\omega lm} \sim e^{-i\omega\tilde{U}} Y_{lm}(\theta,\varphi). \tag{C3}$$

They are analytic functions in the lower half-plane of the complex variable $\tilde{U}$. On the other hand, the behavior of the modes (C1), on the past horizon of the right patch and, respectively, on the future horizon of left patch, is as follows:

$$\vec{u}^R_{\omega lm}(x) \approx (4\pi\omega)^{-1/2} \left|\frac{\tilde{U}}{4M}\right|^{i4M\omega} Y_{lm}(\theta,\varphi), \tag{C4}$$

$$\vec{u}^L_{\omega lm}(x) \approx (4\pi\omega)^{-1/2}\left|\frac{\tilde{U}}{4M}\right|^{-i4M\omega} Y_{lm}(\theta,\varphi). \tag{C5}$$

Then the normalized combinations,

$$\vec{y}_{\omega lm} = \frac{1}{\sqrt{|2\sinh(4\pi M\omega)|}}[e^{2\pi M\omega}\vec{u}^R_{\omega lm} + e^{-2\pi M\omega}(\vec{u}^L_{\omega lm})^*], \tag{C6}$$

have the same analyticity properties of the modes (C3) and are equivalent to them. We can then compute the Wightman function of the Unruh state when the two points are located in the right Schwarzschild patch:

$$W_U(x,x') = \sum_{lm}\int_{-\infty}^{+\infty} d\omega\left[\frac{\vec{u}_{\omega lm}(x)\vec{u}^*_{\omega lm}(x')}{1-e^{-\frac{2\pi\omega}{\kappa}}} + \overleftarrow{u}_{\omega lm}(x)\overleftarrow{u}^*_{\omega lm}(x')\theta(\omega)\right], \tag{C7}$$

where $\kappa = (4M)^{-1}$ is the surface gravity. In a similar manner, the modes that are positive frequency with regards to $\frac{\partial}{\partial \tilde{V}}$ are:

$$\overleftarrow{y}_{\omega lm} = \frac{1}{\sqrt{2\sinh(4\pi M\omega)}}[e^{-2\pi M\omega}(\overleftarrow{u}^R_{\omega lm})^* + e^{2\pi M\omega}\overleftarrow{u}^L_{\omega lm}]. \tag{C8}$$

They give rise to the Hartle-Hawking Wightman function:

$$W_H(x,x') = \sum_{lm}\int_{-\infty}^{+\infty} d\omega\left[\frac{\vec{u}_{\omega lm}(x)\vec{u}^*_{\omega lm}(x')}{1-e^{-\frac{2\pi\omega}{\kappa}}} + \frac{\overleftarrow{u}^*_{\omega lm}(x)\overleftarrow{u}_{\omega lm}(x')}{e^{\frac{2\pi\omega}{\kappa}}-1}\right]. \tag{C9}$$

The outgoing waves of the Unruh state are thermally distributed at temperature $T = \frac{\kappa}{2\pi} = \frac{1}{8\pi M}$; both the outgoing and incoming waves of the Hartle-Hawking state are thermally distributed at the same temperature. In the two-dimensional case these formulae reduce to

$$W_U(x,x') = \int_{-\infty}^{+\infty} d\omega\left[\frac{\vec{u}_\omega(x)\vec{u}^*_\omega(x')}{1-e^{-\frac{2\pi\omega}{\kappa}}} + \overleftarrow{u}_\omega(x)\overleftarrow{u}^*_\omega(x')\theta(\omega)\right],$$
$$W_H(x,x') = \int_{-\infty}^{+\infty} d\omega\left[\frac{\vec{u}_\omega(x)\vec{u}^*_\omega(x')}{1-e^{-\frac{2\pi\omega}{\kappa}}} + \frac{\overleftarrow{u}^*_\omega(x)\overleftarrow{u}_\omega(x')}{e^{\frac{2\pi\omega}{\kappa}}-1}\right]. \tag{C10}$$

2. Two dimensions again

The following construction is valid for any stationary background, provided there are left and right movers. Stationarity implies that a Wightman function with zero anomalous quantum averages depends only on the difference of times:

$$W(x,x') = \int_0^{+\infty} d\omega\int_0^{+\infty} d\omega'[\langle a_\omega a^\dagger_{\omega'}\rangle\vec{u}_\omega(x)\vec{u}^*_{\omega'}(x') + \langle a^\dagger_\omega a_{\omega'}\rangle\vec{u}^*_\omega(x)\vec{u}_{\omega'}(x') + \langle b_\omega b^\dagger_{\omega'}\rangle\overleftarrow{u}_\omega(x)\overleftarrow{u}^*_{\omega'}(x') + \langle b^\dagger_\omega b_{\omega'}\rangle\overleftarrow{u}^*_\omega(x)\overleftarrow{u}_{\omega'}(x')]. \tag{C11}$$

In this general setting the Unruh and the Hartle-Hawking states correspond, respectively, to the following choices:

$$\langle a^\dagger_\omega a_{\omega'}\rangle = \frac{1}{e^{\frac{2\pi\omega}{\kappa}}-1}\delta(\omega-\omega'), \qquad \langle b^\dagger_\omega b_{\omega'}\rangle = 0, \tag{C12}$$

$$\langle a^\dagger_\omega a_{\omega'}\rangle = \langle b^\dagger_\omega b_{\omega'}\rangle = \frac{1}{e^{\frac{2\pi\omega}{\kappa}}-1}\delta(\omega-\omega'), \tag{C13}$$

so that

$$W_U(x,x') = \int_0^{+\infty} d\omega\left[\frac{\vec{u}_\omega(x)\vec{u}^*_\omega(x')}{1-e^{-\frac{2\pi\omega}{\kappa}}} + \frac{\vec{u}^*_\omega(x)\vec{u}_\omega(x')}{e^{\frac{2\pi\omega}{\kappa}}-1} + \overleftarrow{u}_\omega(x)\overleftarrow{u}^*_\omega(x')\right], \tag{C14}$$

$$W_H(x,x') = \int_0^{+\infty} d\omega\left[\frac{\vec{u}_\omega(x)\vec{u}^*_\omega(x')}{1-e^{-\frac{2\pi\omega}{\kappa}}} + \frac{\vec{u}^*_\omega(x)\vec{u}_\omega(x')}{e^{\frac{2\pi\omega}{\kappa}}-1} + \frac{\overleftarrow{u}_\omega(x)\overleftarrow{u}^*_\omega(x')}{1-e^{-\frac{2\pi\omega}{\kappa}}} + \frac{\overleftarrow{u}^*_\omega(x)\overleftarrow{u}_\omega(x')}{e^{\frac{2\pi\omega}{\kappa}}-1}\right]. \tag{C15}$$

These expressions are equivalent to (C10) if the following condition holds:

$$u^*_{-\omega}(x)u_{-\omega}(x') = -u_\omega(x)u^*_\omega(x') \tag{C16}$$

for both outgoing and incoming waves; this condition is verified in the two-dimensional Schwarzschild spacetime.

In the massive case, gluing modes (4.6) in the classically permitted and forbidden regions gives the relation

$$e^{2i\delta_\omega} = \frac{i\omega - \sqrt{m^2-\omega^2}}{i\omega + \sqrt{m^2-\omega^2}}e^{-2i\omega r^*_{\text{turning}}}. \tag{C17}$$

From here one can show that

$$\varphi_\omega(r^*_1)\varphi_\omega(r^*_2) = \varphi_{-\omega}(r^*_1)\varphi_{-\omega}(r^*_2),$$

and this provides another justification of Eq. (4.12).

In the horizon limit modes, as with $|\omega| < m$, they behave as follows:

$$\phi_\omega(r^*) = C(\omega)K_{4i\omega M}(4Mm\xi) \sim \cos(\omega r^* + \delta_\omega),$$
$$\xi^2 = \frac{r}{2M} - 1 \tag{C18}$$

with

$$\delta_\omega = \frac{\pi}{2} + r_g\omega(2\log(mr_g) - 1) - \arg\Gamma(1 + i\omega r_g). \tag{C19}$$

It follows that $\delta_0 = \frac{\pi}{2}$. Noting that $\arg\Gamma(1 + i\omega r_g) = -\arg\Gamma(1 - i\omega r_g)$ again points towards (4.12).

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