Entanglement renormalization for gauge invariant quantum fields

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The continuous multiscale entaglement renormalization ansatz (cMERA) [Haegeman *et al.*, Phys. Rev. Lett. **110**, 100402 (2013)] is a variational wave functional for ground states of quantum field theories. So far, only scalar bosons and fermions have been considered. In this paper we explain how to generalize the cMERA framework to *gauge invariant* quantum fields. The fundamental difficulty to be addressed is how to make the gauge constraints (*local* linear constraints in the Hilbert space) compatible with the UV structure of the cMERA wave functional (which is generated by a *quasilocal* entangler). For simplicity, we consider U(1) gauge theory in $d + 1$ spacetime dimensions, a noninteracting theory with *massless* Hamiltonian $H_{U(1)}$ and Gaussian scale invariant ground state $|\Psi_{U(1)}\rangle$. We propose a gauge invariant cMERA wave functional $|\Psi_{U(1)}^{\Lambda}\rangle$ that, by construction, accurately reproduces the long distance properties of $|\Psi_{U(1)}\rangle$ while remaining somewhat unentangled at short distances. Moreover, $|\Psi_{U(1)}^{\Lambda}\rangle$ is the exact ground state of a gauge invariant, local Hamiltonian $H_{U(1)}^{\Lambda}$ whose low energy properties coincide with those of $H_{U(1)}$. Our construction also extends the cMERA formalism to *massive* (nongauge invariant) vector boson quantum fields.

I. INTRODUCTION

Gauge theories stand among the most successful theories of physical reality, describing a wide range of phenomena—from the standard model of particle physics [1,2] and general relativity [3] to topological phases of quantum matter [4]. They are characterized by an explicit redundancy in the choice of degrees of freedom used to represent the physical system. This redundancy is the price to be paid in order to retain a more tractable and intuitive description, for instance one in terms of a local Hamiltonian. Gauge theories fit into the more general framework of *constrained theories*, whose quantization is itself a rich and interesting subject [5]. A quantum gauge theory can be formulated so that physical states are confined to a particular, *gauge invariant* subspace of the total Hilbert space of the theory.

To go beyond perturbative treatments of gauge theory, one must often resort to numerical simulations. In lattice gauge theory [6,7], spacetime is discretized into a lattice in such a way that gauge invariance is preserved. Then stochastic methods, such as Monte Carlo sampling, are used to study certain aspects of the discretized theory.

For instance, and most prominently, such techniques have been used to successfully extract the mass spectrum of quantum chromodynamics (QCD) [8,9]. In spite of their remarkable success, simulation strategies based on stochastic sampling suffer from the fermionic sign and complex action problems at finite fermionic density [10,11] and, more generally, are not capable of simulating dynamics. For such important problems, alternative formulations are still much needed.

In the past two decades, tensor networks have arisen as a useful new framework to treat quantum many-body problems on the lattice. By exploiting the entanglement structure of certain many-body wave functions, such as ground states and low energy states of local Hamiltonians, tensor networks offer efficient parametrizations and solutions to problems of otherwise unmanageable computational complexity. Much work has been devoted to applying tensor network algorithms to lattice gauge theories [12–67], with the expectation of advancing our numerical capabilities past the breaking points of standard techniques, such as the sign problem mentioned above in the case of Monte Carlo simulation (see [66,67] for recent reviews). Successful simulations in one spatial dimension [12–50] and partial success in two spatial dimensions [51–62] are certainly

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encouraging. However, very significant improvements will be required before, e.g., QCD in three spatial dimensions can be meaningfully tackled. Finally, *continuous* tensor networks have been introduced more recently to simulate quantum field theories (QFTs) directly in the continuum, that is, without introducing a lattice [68,69]. Compared to lattice tensor networks, continuous tensor network techniques are still in their infancy, but one might hope that once they are better understood in one spatial dimension they will be more easily extended to higher dimensions.

The particular tensor network we will be interested in here is the multiscale entanglement renormalization ansatz (MERA) [70,71]. On the lattice, the MERA can be interpreted as a quantum circuit that builds an entangled many-body wave function starting from an unentangled or product state by introducing entanglement via (nearest-neighbor) local unitary gates that effectively act at progressively smaller length scales. Running this procedure backwards, we obtain a dual interpretation of the MERA as encoding a (discrete version of a) renormalization group flow, where entanglement at short length scales is progressively removed from the wave function. For lattice gauge theories in the Hamiltonian formalism, MERA has been seen to offer a proper framework to represent gauge invariant ground states [51,52]. In this case the renormalization group transformations exactly preserve the gauge constraints along the flow.

More specifically, in this work we will be concerned with the *continuous* MERA (cMERA), which was proposed by Haegeman, Osborne, Verschelde and Verstraete in Ref. [68]. While the original formulation of Ref. [68] applies to both interacting and noninteracting fields, in practice cMERA is only well-understood for noninteracting QFTs, in which case it this variational ansatz is a Gaussian wave functional (see however [72–74] for proposals that go beyond a purely Gaussian wave functional, e.g., through the use perturbation theory). It is of course not obvious why one would need a variational ansatz for a free QFT, which can be solved exactly without much effort. Nonetheless, the Gaussian cMERA offers a useful demonstration that MERA can be brought into the continuum, while also providing insights into the entanglement structure of ground state wave functionals. Moreover, similarly to MERA [75,76], Gaussian cMERA has found application as a conjectured realization of the holographic principle of quantum gravity, namely as a toy model for the AdS/CFT correspondence [77–85]. Quite promisingly, Ref. [86] recently proposed a particular realization of Gaussian cMERA, dubbed magic cMERA, which has a UV structure analogous to that of the continuous matrix product state (cMPS), another continuous tensor network. This is important because cMPS techniques work equally well for both noninteracting and interacting QFTs. Thanks to this connection it is now finally possible to produce and efficiently manipulate strongly correlated (that is, highly

non-Gaussian) cMERA wave functionals using cMPS techniques [87].

Reference [68] formulated cMERA for scalar bosons and for fermions. In this paper we take a step further and extend the cMERA formalism to gauge invariant quantum fields. Our main motivation is simple. If, as we expect, the cMERA program is to eventually give rise to a useful numerical simulation framework for interacting QFTs, then understanding how to handle gauge invariant quantum fields is a priority, given the central role gauge theories play in modern physics. A second motivation for our work comes from current applications of cMERA as a toy models for the AdS/CFT correspondence. There the CFT theory is often taken to be a gauge theory with a large gauge group. Therefore a gauge invariant cMERA could also be useful to build improved toy model of the AdS/CFT correspondence.

In this work we will illustrate how the cMERA formalism can be extended to gauge theories by considering the simple case of noninteracting U(1) gauge theory, or electromagnetism without matter fields, in $d + 1$ spacetime dimensions as a proof-of-principle example. U(1) gauge theory is ideal for our purposes, because the Hamiltonian is quadratic and this allows us to show, explicitly and exactly, how the *local* linear constraints in Hilbert space implementing gauge invariance can coexist with the *quasilocal* character of the *entangler* that generates the cMERA wave functional. Our eventual goal is to address interacting gauge theories, where the interaction may be due to either coupling to matter fields or to considering non-Abelian gauge groups (or to both at the same time). While a cMERA framework for interacting gauge theories may need to build on the proposals of Ref. [87], we expect that the compatibility between the quasilocal character of the cMERA and the local character of the gauge constraints, as demonstrated here for a noninteracting theory, will work in a similar way in the interacting case.

We devote Sec. II to reviewing, and setting our notation for, both cMERA and U(1) gauge theory. In Sec. III we introduce a Gaussian cMERA that approximates the ground state of U(1) gauge theory, and elaborate on its properties. In particular, we will see that our proposal is a natural extension to gauge fields of the *magic* entanglement renormalization scheme of Ref. [86], thus paving the way to subsequently building strongly correlated cMERA wave functionals for interacting gauge theories. Finally, we include a series of appendices elaborating on particular aspects of our construction.

II. REVIEW OF BACKGROUND MATERIAL

In this section we briefly review some required background material. First we introduce the cMERA formalism of Ref. [68] and a particular realization thereof, the magic cMERA [86]. Then we review U(1) gauge theory, which is *massless*, and the closely related *massive* vector boson QFT.

A. cMERA

The cMERA [68] (denoted $|\Psi^\Lambda(s)\rangle$ or $|\Psi^\Lambda\rangle$ depending on the context) is an ansatz that aims to approximate some target wave functional of interest, typically the ground state of a QFT Hamiltonian in d spatial dimensions. It is produced through a unitary *entangling evolution in scale*, which yields a one-parameter family of cMERA states

$$|\Psi^\Lambda(s)\rangle \equiv \mathcal{P}\exp\left(-i\int_0^s ds'\,[L + K(s')]\right)|\Lambda\rangle. \quad (1)$$

Here $\mathcal{P}\exp$ denotes a path-ordered exponential, s is the scale parameter, $L + K(s)$ is the Hermitian operator that generates the entangling evolution and $|\Lambda\rangle$ is the initial state. This initial state is taken to be unentangled, with correlation functions that vanish when evaluated at different points:

$$\langle\Lambda|\mathcal{O}(x)\mathcal{O}(y)|\Lambda\rangle = 0, \qquad x \neq y. \quad (2)$$

(We can think of $|\Lambda\rangle$ as the continuum limit of an unentangled/product state of a lattice system.)

The generator of the evolution in scale is split into two contributions, L and $K(s)$, which play two distinct roles. L is the generator of *nonrelativistic* scale transformations that rescale both space and the field degrees of freedom. As such, L only depends on the field content of the theory under study, and not on the specific form of the Hamiltonian whose ground state we aim to approximate with the cMERA. For a generic field φ with nonrelativistic scaling dimension Δ_φ, we have

$$e^{isL}\varphi(\vec{x})e^{-isL} = e^{s\Delta_\varphi}\varphi(e^s\vec{x}). \quad (3)$$

On the other hand $K(s)$ is a quasilocal operator called *entangler* [see Eqs. (6) and (10) below for an example in one spatial dimension]. By *quasilocal* we mean that the operator, an integral of a quasilocal density, acts at a specific length scale. It is standard to denote this length scale as Λ^{-1}, where Λ is the corresponding momentum scale. Intuitively, the entangling evolution in scale builds the cMERA wave functional from the initial uncorrelated state $|\Lambda\rangle$ by progressively introducing entanglement at length scale Λ^{-1} as we keep enlarging both the rescaling space and the fields. The resulting state will contain correlations at a range of length scales above Λ^{-1}, but will (partially) preserve the unentangled character of the initial state $|\Lambda\rangle$ at shorter distances. This idea can be made more precise, and allows us to say that a cMERA state presents an entanglement UV cutoff at length scale Λ^{-1} [88].

Let $|\Psi\rangle$ be the target state that the ansatz wave functional $|\Psi^\Lambda(s)\rangle$ aims to approximate at distances x larger than Λ^{-1}, in the sense that, e.g., the correlators agree to high accuracy

for $\Lambda x \gg 1$. When the ansatz succeeds, we say that $|\Psi^\Lambda(s)\rangle$ is a good long distance (LD) approximation to $|\Psi\rangle$ and denote it by

$$|\Psi^\Lambda(s)\rangle \overset{\text{LD}}{\sim} |\Psi\rangle. \quad (4)$$

Of particular interest in this paper is the case where the entangler is independent of s, $K(s) \equiv K$. Then, in the limit $\lim_{s\to\infty}|\Psi^\Lambda(s)\rangle$ (assuming this limit exists) we obtain a fixed point wave functional known as a *scale invariant* cMERA $|\Psi^\Lambda\rangle$,

$$|\Psi^\Lambda\rangle \equiv \lim_{s\to\infty} e^{-is(L+K)}|\Lambda\rangle, \quad (5)$$

see Fig. 1. By construction, the fixed point wave functional $|\Psi^\Lambda\rangle$ is invariant under further evolution under $L + K$. We can think of $D^\Lambda \equiv L + K$ as a generator of an alternative notion of scale transformations, one that is adapted to the specific theory under study. For instance, in the context of a free boson CFT [89], D^Λ is the generator of (a quasilocal version of) relativistic scale transformations, whereas L generates nonrelativistic scale transformations. Then we say that $|\Psi^\Lambda\rangle$ is invariant under this alternative notion of scale transformations, which is why we call $|\Psi^\Lambda\rangle$ a scale invariant cMERA wave functional.

Ideally, given a target QFT Hamiltonian, the specific form of the entangler $K(s)$ should be determined variationally from a procedure such as energy minimization. While no general algorithms have been developed so far to determine the entangler variationally, in the particular case of free fields one can find examples of entanglers that give rise to interesting cMERA wave functionals. For example, for a free scalar $\phi(x)$ in one spatial dimension, with conjugate momentum $\pi(x)$, a possible family of entanglers is given by the translationally invariant, quasilocal quadratic operator [68]:
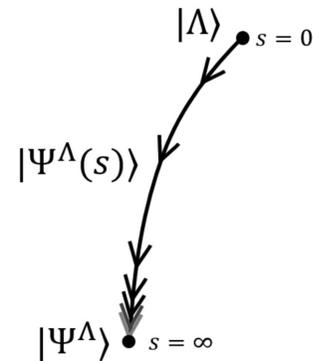
FIG. 1. The entangling evolution in scale generates a cMERA state $|\Psi^\Lambda(s)\rangle$ for each value $s \in [0, \infty)$. When the entangler is scale independent, $K(s) = K$, then in the large s limit we obtain a fixed point, scale invariant cMERA $|\Psi^\Lambda\rangle$.

$$K(s) = \frac{-i}{2} \int dx\, dy\, g(x-y,s)\psi(x)\psi(y) + \text{H.c.,} \quad (6)$$

where $\psi(x)$ is the annihilation operator

$$\psi(x) \equiv \sqrt{\frac{\Lambda}{2}}\phi(x) + i\sqrt{\frac{1}{2\Lambda}}\pi(x), \quad (7)$$

and $g(x,s)$ is some profile function at scale s. A simple example, independent of s, is the profile $g(x,s) = g(x) \sim e^{-(\Lambda x)^2}$ used in Ref. [68]. In Eq. (10) below we introduce an alternative choice of profile.

B. Magic cMERA

The variational parameters in cMERA correspond to different choices of the entangler K. There is a lot of freedom in choosing the entangler K, compatible with obtaining a good long distance approximation to a fixed target state, and one may be able to use this freedom to identify particularly useful subclasses of entanglers.

In one spatial dimension, consider a relativistic free *massless* boson conformal field theory (CFT), with Hamiltonian

$$H^{\text{CFT}} \equiv \frac{1}{2} \int dx\, [\pi(x)^2 + (\partial_x \phi(x))^2], \quad (8)$$

and ground state $|\Psi^{\text{CFT}}\rangle$, or more generally a relativistic free *massive* boson QFT, with Hamiltonian

$$H_m \equiv H^{\text{CFT}} + \frac{m^2}{2} \int dx\, \phi(x)^2, \quad (9)$$

and ground state $|\Psi_m\rangle$. It was recently shown [86] that the choice of entangler profile

$$g(x,s) \equiv g(x) = \frac{\Lambda}{4} e^{-\Lambda|x|} \quad (10)$$

leads to cMERA wave functionals $|\Psi^{\Lambda}\rangle$ and $|\Psi^{\Lambda}(s)\rangle$ with

$$|\Psi^{\Lambda}\rangle \overset{\text{LD}}{\sim} |\Psi^{\text{CFT}}\rangle, \quad (11)$$

for the massless case and

$$|\Psi^{\Lambda}(s)\rangle \overset{\text{LD}}{\sim} |\Psi_{m(s)}\rangle, \quad (12)$$

where $|\Psi_{m(s)}\rangle$ is the relativistic massive ground state $|\Psi_m\rangle$ for mass

$$m(s) \equiv \Lambda e^{-s}. \quad (13)$$

Moreover, the magic cMERA wave functional has two remarkable properties, that we summarize next.

(i) *Compatibility with cMPS:* $|\Psi^{\Lambda}(s)\rangle$ has the same UV structure as a cMPS [86]. As a result, cMPS techniques [69,90–102] can be used to numerically manipulate the cMERA wave functional efficiently. Most importantly, these cMPS techniques work equally well for both Gaussian and non-Gaussian wave functionals. Therefore they provide a much needed numerical venue for producing strongly correlated (i.e., highly non-Gaussian) cMERA wave functionals for interacting QFTs, as demonstrated in Ref. [87].

(ii) *Exact ground state of local Hamiltonian:* the magic cMERA $|\Psi^{\Lambda}(s)\rangle$ is the exact ground state of a strictly local QFT Hamiltonian $H^{\Lambda}(s)$, see Eq. (16) below. This is unexpected. Indeed, it can be seen that a generic choice of quasi-local entangler produces a wave functional that is the ground state of a Hamiltonian which is, at best, quasilocal [86].

Let us elaborate a bit more on this last property, since it will play an important role in our discussion of the U(1) gauge invariant cMERA. We introduce the local Hamiltonian

$$H^{\Lambda} \equiv H^{\text{CFT}} + A^{\Lambda}_{\text{UV}} \quad (14)$$

for the massless case, where

$$A^{\Lambda}_{\text{UV}} \equiv \frac{1}{2\Lambda^2} \int dx\, (\partial_x \pi(x))^2, \quad (15)$$

and the local Hamiltonian

$$H^{\Lambda}(s) \equiv H_{m(s)} + A^{\Lambda}_{\text{UV}}, \quad (16)$$

in the massive case, where $H_{m(s)}$ is the relativistic massive Hamiltonian H_m of Eq. (9) for mass $m(s)$ given by (13). Then Ref. [86] showed that $|\Psi^{\Lambda}\rangle$ is the exact ground state of H^{Λ} and $|\Psi^{\Lambda}(s)\rangle$ is the exact ground state of $H^{\Lambda}(s)$.

The parent Hamiltonian $H^{\Lambda}(s)$ is thus obtained from the relativistic massive $H_{m(s)}$ by adding the nonrelativistic UV regulator A^{Λ}_{UV}, which breaks Lorentz invariance and primarily affects the UV physics by modifying the dispersion relation for momenta above the cutoff scale Λ. On the other hand, the mass term in $H^{\Lambda}(s)$ introduces a mass gap in the low energy spectrum. Thus, for $s > 0$, that is $m < \Lambda$, we can think of m and Λ in $H^{\Lambda}(s)$ as providing IR and UV regulators to the relativistic, massless Hamiltonian H^{CFT}, respectively.

C. U(1) gauge theory

Next we summarize the quantization of a *gauge invariant, massless* vector boson field, which is nontrivial due to the presence of constraints (then below we will also discuss the *massive* case, which is no longer gauge invariant).

Our goal is to remind the reader of the characterization of the ground state in terms of annihilation operators, which will be useful in the forthcoming analysis. For a more detailed review of the quantization procedure see Appendix A.

Consider the free Maxwell Lagrangian for a bosonic vector field A_μ in $d + 1$ spacetime dimensions,

$$\mathcal{L} \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{17}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor. We will quantize the theory in the temporal gauge $A^0 = 0$. The spatial components A_i of the vector field and their corresponding conjugate momenta Π_i are promoted to operators satisfying the canonical commutation relations

$$[A_i(\vec{x}), \Pi_j(\vec{y})] = i\delta(\vec{x} - \vec{y}). \tag{18}$$

Due to gauge invariance, physical states are constrained to satisfy Gauss's law

$$\partial_i \Pi^i(\vec{x})|\text{phys}\rangle = 0, \quad \forall \vec{x} \in \mathbb{R}^d. \tag{19}$$

The Hamiltonian of the theory is given by

$$H_{\text{U}(1)} \equiv \frac{1}{2}\int d^dx \, [\Pi_i(\vec{x})\Pi^i(\vec{x})$$
$$- A^i(\vec{x})(\delta_{ij}\Delta - \partial_i\partial_j)A^j(\vec{x})], \tag{20}$$

$$= \frac{1}{2}\int d^dk \, [\Pi_i(-\vec{k})\Pi^i(\vec{k})$$
$$+ A^i(-\vec{k})(\delta_{ij}k^2 - k_ik_j)A^j(\vec{k})], \tag{21}$$

where $k = |\vec{k}|$. H can be diagonalized by changing to a basis consistent of the *longitudinal* polarization

$$A_\|(\vec{k}) = \frac{ik^i A_i(\vec{k})}{k}, \qquad \Pi_\|(\vec{k}) = \frac{ik^i \Pi_i(\vec{k})}{k}, \tag{22}$$

and the $d - 1$ orthogonal *transversal* polarizations [103]

$$A_{\perp,n}(\vec{k}), \qquad \Pi_{\perp,n}(\vec{k}), \quad n = 1, ..., d-1. \tag{23}$$

(Summation over any repeated index n for this basis will be implied throughout this paper.) This basis is additionally helpful because the longitudinal polarization is precisely the gauge degree of freedom, while the transversal polarizations are the physical (gauge invariant) degrees of freedom, as can be seen by performing a gauge transformation:

$$A_j(\vec{x}) \rightarrow A_j(\vec{x}) + \partial_j\omega(\vec{x})$$
$$\Rightarrow \begin{cases} A_\|(\vec{k}) & \rightarrow A_\|(\vec{k}) - k\omega(\vec{k}), \\ A_{\perp,n}(\vec{k}) & \rightarrow A_{\perp,n}(\vec{k}). \end{cases} \tag{24}$$

Consequently, the gauge constraint (19) becomes

$$\Pi_\|(\vec{k})|\text{phys}\rangle = 0, \quad \forall \vec{k} \in \mathbb{R}^d. \tag{25}$$

The Hamiltonian $H_{\text{U}(1)}$ restricted to the gauge invariant subspace reads

$$H_{\text{U}(1)} = \int d^dk \, (\Pi_{\perp,n}(-\vec{k})\Pi_{\perp,n}(\vec{k})$$
$$+ k^2 A_{\perp,n}(-\vec{k})A_{\perp,n}(\vec{k})), \tag{26}$$

$$= \int d^dk \, k \, a_{\perp,n}^\dagger(\vec{k})a_{\perp,n}(\vec{k}), \tag{27}$$

where

$$a_{\perp,n}(\vec{k}) \equiv \sqrt{\frac{k}{2}}A_{\perp,n}(\vec{k}) + i\sqrt{\frac{1}{2k}}\Pi_{\perp,n}(\vec{k}), \tag{28}$$

and we have removed an infinite constant term from the Hamiltonian in the usual way. What remains is nothing but the Hamiltonian for $d - 1$ free bosons, whose ground state $|\Psi_{\text{U}(1)}\rangle$ is defined via the annihilation operators:

$$a_{\perp,n}(\vec{k})|\Psi_{\text{U}(1)}\rangle = 0 \quad \forall \vec{k}, n = 1, ..., d-1. \tag{29}$$

Notice that both Eq. (29) and the gauge constraint (25) are constraints that are expressed in terms of operators that are linear in the field operators $A_i(\vec{x})$ and $\Pi_i(\vec{x})$. These constraints completely determine the ground state $|\Psi_{\text{U}(1)}\rangle$, which is therefore a Gaussian state.

For later reference, we parametrize the annihilation operators $a_\|(\vec{k})$ and $a_\perp(\vec{k})$ in terms of two functions $\alpha_\|(k)$, $\alpha_\perp(k)$ and write:

$$a_\|(\vec{k})|\Psi_{\text{U}(1)}\rangle = 0, \qquad a_{\perp,n}(\vec{k})|\Psi_{\text{U}(1)}\rangle = 0, \tag{30}$$

$$a_\|(\vec{k}) \equiv \sqrt{\frac{\alpha_\|(k)}{2}}A_\|(\vec{k}) + i\sqrt{\frac{1}{2\alpha_\|(k)}}\Pi_\|(\vec{k}), \tag{31}$$

$$a_{\perp,n}(\vec{k}) \equiv \sqrt{\frac{\alpha_\perp(k)}{2}}A_{\perp,n}(\vec{k}) + i\sqrt{\frac{1}{2\alpha_\perp(k)}}\Pi_{\perp,n}(\vec{k}). \tag{32}$$

We see that in order to recover (25) and (29) from the more general formulation (30)–(32), we just need to make the particular choice of functions $\alpha_\|(k)$, $\alpha_\perp(k)$ given by

$$\alpha_\|(k) = 0, \qquad \alpha_\perp(k) = k, \tag{33}$$

where in the particular case of the longitudinal polarization, $\alpha_\parallel(\vec{k}) = 0$ implies that the annihilation operator $a_\parallel(\vec{k})$ is proportional to $\Pi_\parallel(\vec{k})$, as required by the gauge constraint.

D. Massive vector boson quantum field theory

For what follows it is also useful to recall the *massive* vector boson theory, obtained by adding a (Proca) mass term to the Lagrangian

$$\mathcal{L}_m \equiv \mathcal{L} + \frac{1}{2}m^2 A_\mu A^\mu. \tag{34}$$

The relevant operator algebra is again generated by (18), but the mass term breaks gauge invariance so physical states are no longer restricted to satisfy (25). The massive Hamiltonian

$$H_m \equiv H_{U(1)} + \frac{1}{2}\int d^d x \left[\frac{(\partial_i \Pi^i(\vec{x}))^2}{m^2} + m^2 A_i(\vec{x}) A^i(\vec{x}) \right] \tag{35}$$

can be diagonalized in the polarization basis (22)–(23) as in the massless case, but the lack of gauge invariance implies that the longitudinal component $A_\parallel(\vec{k})$, $\Pi_\parallel(\vec{k})$ is now a legitimate propagating degree of freedom, instead of a gauge degree of freedom. The ground state of the theory is again of the form (30)–(32), this time with functions $\alpha_\parallel(k)$, $\alpha_\perp(k)$ given by

$$\alpha_\parallel(k) = \frac{m^2}{\sqrt{k^2 + m^2}}, \qquad \alpha_\perp(k) = \sqrt{k^2 + m^2} \tag{36}$$

Note that in the limit $m \to 0$, (36) reduces to (33) as expected (see also Fig. 2).
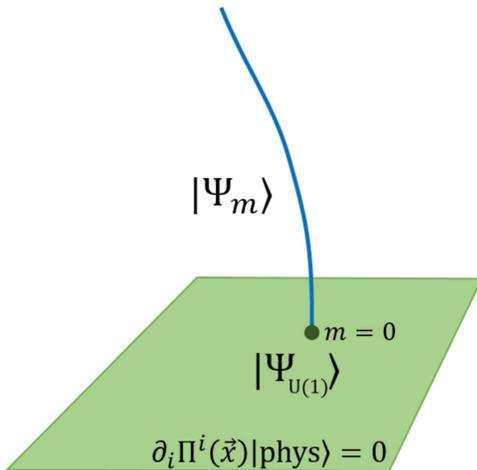
FIG. 2. The ground state $|\Psi_m\rangle$ of the massive vector boson Hamiltonian H_m depends on the mass m and is not gauge invariant. However, in the limit $m \to 0$ we recover massless vector boson Hamiltonian $H_{U(1)}$, whose ground state $|\Psi_{U(1)}\rangle$ is gauge invariant. The green surface represents the gauge invariant subspace, or physical subspace, of the Hilbert space.

III. cMERA

We are now ready to present our main result: a cMERA wave functional $|\Psi_{U(1)}^\Lambda\rangle$ that approximates the ground state $|\Psi_{U(1)}\rangle$ of Hamiltonian $H_{U(1)}$ in Eq. (20) for the U(1) *gauge invariant*, *massless* free vector boson.

Our construction actually corresponds to a scale invariant cMERA, that is, it is the fixed point of an entangling evolution in scale generated by a constant entangler K starting from an unentangled state $|\Lambda\rangle$ [as introduced earlier in Eq. (5)] and is an extension to gauge fields of the *magic* MERA of Ref. [86]. It has the following three key properties:

(i) *Gauge invariance*: the wave functional $|\Psi_{U(1)}^\Lambda\rangle$ is explicitly U(1) gauge invariant, that is, it fulfills the constraint (25).

(ii) *Correct large distance physics (I)*: the wave functional $|\Psi_{U(1)}^\Lambda\rangle$ accurately approximates the behavior (e.g., correlators, see Fig. 3) of the ground state $|\Psi_{U(1)}\rangle$ of $H_{U(1)}$ at distances $x \gg \Lambda^{-1}$, or

$$|\Psi_{U(1)}^\Lambda\rangle \overset{\text{LD}}{\sim} |\Psi_{U(1)}\rangle. \tag{37}$$

(iii) *Ground state of a local Hamiltonian (I)*: the cMERA $|\Psi^\Lambda\rangle$ is the exact ground state of a Hamiltonian $H_{U(1)}^\Lambda$, see Eq. (71) below, that is *local* and can be understood as a UV regulated version of $H_{U(1)}$.

Moreover, the intermediate cMERA wave functional $|\Psi^\Lambda(s)\rangle$ for any finite $s \in [0, \infty)$ is related to the massive vector boson described in the previous section in the following ways:

(iv) *Correct large distance physics (II)*: the wave functional $|\Psi^\Lambda(s)\rangle$ accurately approximates the behavior (e.g., correlators, see Fig. 3) of the ground state $|\Psi_{m(s)}\rangle$ of the relativistic massive vector boson Hamiltonian $H_{m(s)}$, or Hamiltonian H_m in Eq. (35) for mass $m(s) = \Lambda e^{-s}$, that is

$$|\Psi^\Lambda(s)\rangle \overset{\text{LD}}{\sim} |\Psi_{m(s)}\rangle. \tag{38}$$

(v) *Ground state of a local Hamiltonian (II)*: the cMERA $|\Psi^\Lambda(s)\rangle$ is the exact ground state of a Hamiltonian $H^\Lambda(s)$, see Eq. (68) below, that is *local* and can be understood as a UV regulated version of $H_{m(s)}$ for mass $m(s) = \Lambda e^{-s}$.

We emphasize that in our construction, for any finite s [finite mass $m(s)$] the cMERA wave functional $|\Psi^\Lambda(s)\rangle$ is not gauge invariant, and gauge invariance is only attained in the large s limit. That is, the entangling evolution in scale takes place outside the gauge invariant subspace of the Hilbert space. However, as we will show (see Fig. 5 below), an approximation to $|\Psi^\Lambda\rangle$ (e.g., in terms of correlators) can already be obtained from $|\Psi^\Lambda(s)\rangle$ at finite $s \gg 1$. Notice that this situation closely mimics the relativistic gauge invariant vector boson we are targeting: at finite mass m,

FIG. 4. The massive vector boson ground state $|\Psi_{m(s)}\rangle$ for $m(s) = \Lambda e^{-s}$ is approximated by the cMERA state $|\Psi^\Lambda(s)\rangle$. None of these wave functionals are gauge invariant. The massless vector boson ground state $|\Psi_{U(1)}\rangle$ is approximated by the scale-invariant cMERA state $|\Psi^\Lambda_{U(1)}\rangle$. These two wave functionals are gauge invariant.

We then consider the unentangled state $|\Lambda\rangle$ given by

$$\psi_i(\vec{x})|\Lambda\rangle = 0, \quad i = 1, ..., d. \tag{40}$$

This will be the starting point of our entangling evolution. For later convenience, we write (40) in the basis of polarizations in momentum space:

$$\psi_\parallel(\vec{k})|\Lambda\rangle = 0, \tag{41}$$

$$\psi_{\perp,n}(\vec{k})|\Lambda\rangle = 0, \quad n = 1, ..., d - 1, \tag{42}$$

Here we have defined

$$\psi_\parallel(\vec{k}) \equiv \sqrt{\frac{\Lambda}{2}} A_\parallel(\vec{k}) + i\sqrt{\frac{1}{2\Lambda}} \Pi_\parallel(\vec{k}), \tag{43}$$

$$\psi_{\perp,n}(\vec{k}) \equiv \sqrt{\frac{\Lambda}{2}} A_{\perp,n}(\vec{k}) + i\sqrt{\frac{1}{2\Lambda}} \Pi_{\perp,n}(\vec{k}), \tag{44}$$

for $n = 1, ..., d - 1$. The initial state $|\Lambda\rangle$ is clearly also of the Gaussian form (30)–(32), with both functions $\alpha_\parallel(k)$, $\alpha_\perp(k)$ set to a constant:

$$\alpha_\parallel(k) = \Lambda, \qquad \alpha_\perp(k) = \Lambda. \tag{45}$$

B. Entangling evolution in scale

Our next step is to define the generator of scale transformations as

FIG. 3. Correlator $\langle B(0)B(\vec{x})\rangle$ as a function of $x \equiv |\vec{x}|$ in $2 + 1$ dimensions, where $B \equiv \partial_1 A_2 - \partial_2 A_1$, for both a target state and the corresponding cMERA approximation, which matches the correlator of its target state for $x\Lambda \gg 1$. (top) Gauge invariant target state $|\Psi_{U(1)}\rangle$ and the corresponding cMERA $|\Psi^\Lambda_{U(1)}\rangle$. The cMERA correlator has a distributional contribution $\frac{1}{2\Lambda}(\Delta + \frac{\Lambda^2}{2})\delta(\vec{x})$ localized at the origin, not visible in the figure. (bottom) Massive target state $|\Psi_{m(s)}\rangle$ for $m(s) = \Lambda e^{-s}$ and the corresponding cMERA $|\Psi^\Lambda(s)\rangle$. The cMERA correlator has a distributional contribution $\frac{1}{2\Lambda}(\Delta + \frac{\Lambda^2 - m(s)^2}{2})\delta(\vec{x})$ localized at the origin, not visible in the figure.

the theory is not gauge invariant, and gauge invariance is only attained in the massless limit $m \to 0$. Figure 4 summarizes the relations between cMERA states and the ground states they target.

A. Unentangled state $|\Lambda\rangle$

We begin by defining annihilation operators

$$\psi_i(\vec{x}) \equiv \sqrt{\frac{\Lambda}{2}} A_i(\vec{x}) + i\sqrt{\frac{1}{2\Lambda}} \Pi_i(\vec{x}). \tag{39}$$

$$L \equiv \frac{1}{2} \int d^d\vec{x}\, \Pi_i(\vec{x}) \left(-\vec{x} \cdot \vec{\nabla} - \frac{d}{2} \right) A^i(\vec{x}) + \text{H.c.}, \quad (46)$$

$$= \int d^d\vec{k}\, \Pi_i(-\vec{k}) \left(\vec{k} \cdot \vec{\nabla}_{\vec{k}} + \frac{d}{2} \right) A^i(\vec{k}) + \text{H.c.}, \quad (47)$$

which assigns nonrelativistic scaling dimensions $\Delta_{A_i} = d/2$ and $\Delta_{\Pi_i} = d/2$ to the fields, and consider an entangler of the form

$$K = \frac{-i}{2} \int d^d\vec{x}\, d^d\vec{y}\, g^{ij}(\vec{x} - \vec{y}) \psi_i(\vec{x}) \psi_j(\vec{y}) + \text{H.c.}, \quad (48)$$

$$= \frac{-i}{2} \int d^d\vec{k}\, g^{ij}(\vec{k}) \psi_i(-\vec{k}) \psi_j(\vec{k}) + \text{H.c.}, \quad (49)$$

which is the natural generalization to vector bosons of the scalar boson entangler (6). We choose a rotation covariant form for the profile $g^{ij}(\vec{k}) = g(k)\delta^{ij} + f(k)k^i k^j$, and rewrite

$$K = \frac{-i}{2} \int d^d\vec{k}\, [g_\parallel(k)\psi_\parallel(-\vec{k})\psi_\parallel(\vec{k})$$
$$+ g_\perp(k)\psi_{\perp,n}(-\vec{k})\psi_{\perp,n}(\vec{k})] + \text{H.c.}, \quad (50)$$

where we have defined

$$g_\perp(k) \equiv g(k), \qquad g_\parallel(k) \equiv g(k) + k^2 f(k). \quad (51)$$

In our case, we choose

$$g_\parallel(k) = 1 - \frac{1}{2}\frac{\Lambda^2}{\Lambda^2 + k^2}, \qquad g_\perp(k) = \frac{1}{2}\frac{\Lambda^2}{\Lambda^2 + k^2}. \quad (52)$$

To have a picture of what these profiles look like in position space, notice that by inverse Fourier transforming we obtain

$$\mathcal{F}^{-1}\left[\frac{1}{\Lambda^2 + k^2} \right](x) \propto \frac{K_{\frac{d-2}{2}}(\Lambda x)}{(\Lambda x)^{\frac{d-2}{2}}}, \quad (53)$$

where $x = |\vec{x}|$ and K_n is the nth modified Bessel function of the second kind. This implies that the position space profile of the entangler decays exponentially at large distances, and for $d > 1$ it diverges at the origin. In particular, in $1 + 1$ spacetime dimensions, $g_\perp(x)$ is the same profile as that in Eq. (10).

Having made our choices for $|\Lambda\rangle$, L and K, the family of ansatz states $|\Psi^\Lambda(s)\rangle$ is defined via Eq. (1). Since K is a quadratic operator in the fields, and we start from a Gaussian state $|\Lambda\rangle$, the whole evolution takes place in the manifold of Gaussian states, and each $|\Psi^\Lambda(s)\rangle$ is of the form (30)–(32), i.e., it is given by a set of scale-dependent annihilation operators

$$a_\parallel^\Lambda(\vec{k}, s)|\Psi^\Lambda(s)\rangle = 0, \quad \forall\, \vec{k} \in \mathbb{R}^d, \quad (54)$$

$$a_{\perp,n}^\Lambda(\vec{k}, s)|\Psi^\Lambda(s)\rangle = 0, \quad \forall\, \vec{k} \in \mathbb{R}^d, \quad n = 1, \dots, d-1, \quad (55)$$

which are characterized by a pair of scale-dependent functions $\alpha_\parallel(k, s)$, $\alpha_\perp(k, s)$,

$$a_\parallel^\Lambda(\vec{k}, s) = \sqrt{\frac{\alpha_\parallel(k, s)}{2}} A_\parallel(\vec{k}) + i\sqrt{\frac{1}{2\alpha_\parallel(k, s)}} \Pi_\parallel(\vec{k}), \quad (56)$$

$$a_{\perp,n}^\Lambda(\vec{k}, s) = \sqrt{\frac{\alpha_\perp(k, s)}{2}} A_{\perp,n}(\vec{k}) + i\sqrt{\frac{1}{2\alpha_\perp(k, s)}} \Pi_{\perp,n}(\vec{k}). \quad (57)$$

Using Eq. (1) we can solve for $\alpha_\parallel(k, s)$ and $\alpha_\perp(k, s)$ in terms of $g_\parallel(k, s)$ and $g_\perp(k, s)$:

$$\alpha_\parallel(k, s) = \Lambda \exp\left(-2\int_0^s du\, g_\parallel(ke^{s-u}) \right), \quad (58)$$

$$\alpha_\perp(k, s) = \Lambda \exp\left(-2\int_0^s du\, g_\perp(ke^{s-u}) \right). \quad (59)$$

For our particular choice of entangler given by Eq (52), we have

$$\alpha_\parallel(k, s) = \frac{m(s)^2}{\Lambda} \sqrt{\frac{k^2 + \Lambda^2}{k^2 + m(s)^2}}, \quad (60)$$

$$\alpha_\perp(k, s) = \Lambda \sqrt{\frac{k^2 + m(s)^2}{k^2 + \Lambda^2}}, \quad (61)$$

where $m(s) = \Lambda e^{-s}$. Since for the transversal modes of the vector boson we used the same entangler as the one for a scalar boson in Ref. [86], $\alpha_\perp(k, s)$ is the same function as $\alpha(k, s)$ in Ref. [86].

C. Fixed-point wave functional and gauge invariance

In the limit $s \to \infty$, the constraint from Eq. (54) becomes the gauge invariance condition (25), so that the fixed-point state $|\Psi_{U(1)}^\Lambda\rangle$ belongs to the gauge invariant subspace. It is fully characterized (up to a global phase) by the gauge constraint and the $s \to \infty$ limit of the annihilation operators of the transversal modes:

$$\Pi_\parallel(\vec{k})|\Psi^\Lambda\rangle = 0, \quad (62)$$

$$a_{\perp,n}^\Lambda(\vec{k}, \infty)|\Psi^\Lambda\rangle = 0. \quad (63)$$

The state defined by the conditions (62)–(63) is a fixed point of the evolution generated by $L + K$. This can be shown in the same way as it was shown in Ref. [89] for the scalar boson. There is a subtlety regarding the $s \to \infty$ limit

since the theories for $s < \infty$ and $s = \infty$ are fundamentally distinct. We elaborate on this last point in Appendix B.

D. Comparison of Gaussian wave functionals

The fact that all states involved in this discussion are Gaussian, of the form (30)–(32), facilitates comparison among them, since it can be conducted at the level of α functions. It was argued in [89] that annihilation operators for noninteracting cMERA states interpolate between those of the target state at small momenta $k \ll \Lambda$ and those of the unentangled initial state at large momenta $k \gg \Lambda$. As numerically checked in [88], this leads to correlation functions with the corresponding interpolating behaviors. Indeed, the two-point functions of $|\Psi^\Lambda(s)\rangle$, which for Gaussian states encode all the other correlators, are intimately related to $\alpha_\parallel(k, s)$ and $\alpha_\perp(k, s)$:

$$\langle A_\parallel(\vec{k}) A_\parallel(\vec{q}) \rangle = \frac{1}{2} \frac{\delta(\vec{k} + \vec{q})}{\alpha_\parallel(\vec{k}, s)}, \tag{64}$$

$$\langle A_{\perp,n}(\vec{k}) A_{\perp,n}(\vec{q}) \rangle = \frac{1}{2} \frac{\delta(\vec{k} + \vec{q})}{\alpha_\perp(\vec{k}, s)}. \tag{65}$$

In the particular case of our current proposal, it follows from Eqs. (60)–(61) that

$$\alpha_\parallel(k, s) \sim \begin{cases} \frac{m(s)^2}{\sqrt{k^2 + m(s)^2}} & k \ll \Lambda, \\ \frac{m(s)^2}{\Lambda} & k \gg \Lambda, \end{cases} \tag{66}$$

$$\alpha_\perp(k, s) \sim \begin{cases} \sqrt{k^2 + m(s)^2} & k \ll \Lambda, \\ \Lambda & k \gg \Lambda. \end{cases} \tag{67}$$

We see that for $k \ll \Lambda$ these functions reproduce the target state's behavior [see Eq. (36)], while for $k \gg \Lambda$ they become constant, which is the behavior seen for the unentangled initial state $|\Lambda\rangle$ [see Eq. (45); the longitudinal case is special since the constant is rescaled along the evolution from Λ at $s = 0$ to 0 at $s = \infty$).

Figure 5 shows the correlator $\langle B(0) B(\vec{x}) \rangle$ for $|\Psi^\Lambda(s)\rangle$ as a function of s. We see that, aside from a delta at the origin (see also Appendix B), the correlators for large s converge to those of the fixed point wave functional $|\Psi^\Lambda_{U(1)}\rangle$. Thus, we can learn about the proporties of the gauge invariant $|\Psi^\Lambda_{U(1)}\rangle$ by studying the nongauge invariant $|\Psi^\Lambda(s)\rangle$ at finite but large s.

E. Local Hamiltonians with relativistic IR physics

In order to prove statements (iii) and (v) from the beginning of this section, consider the following family of Hamiltonians:

$$H^\Lambda(s) \equiv H_{m(s)} + B^\Lambda_{\mathrm{UV}}(s), \tag{68}$$
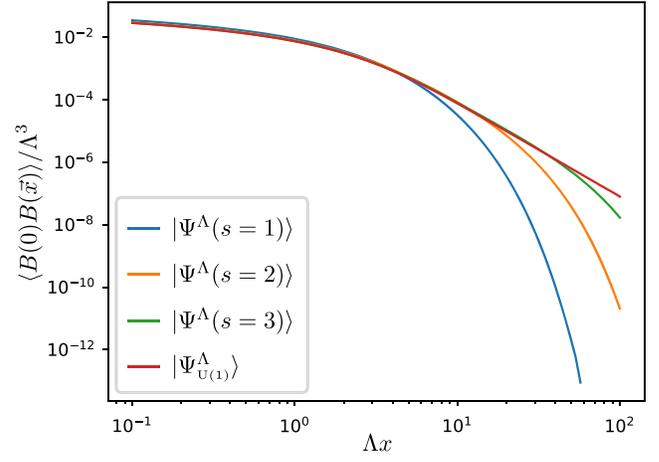
FIG. 5. Two-point correlator $\langle B(0) B(\vec{x}) \rangle$ for state $|\Psi^\Lambda(s)\rangle$ for $s = 1, 2, 3$ and for the fixed point wave functional $|\Psi^\Lambda_{U(1)}\rangle$ in $2 + 1$ dimensions (see also Fig. 3).

where $H_{m(s)}$ is the massive Hamiltonian from (35) with $m(s) = \Lambda e^{-s}$ and

$$B^\Lambda_{\mathrm{UV}}(s) \equiv \frac{1}{\Lambda^2} \int d^d x \, \Pi^i(\vec{x}) (\delta_{ij} \Delta - \partial_i \partial_j) \Pi^j(\vec{x}) + \frac{m(s)^2}{\Lambda^2} \int d^d x \, (\partial_i A^i(\vec{x}))^2. \tag{69}$$

For every $s \in [0, \infty]$, $H^\Lambda(s)$ is quadratic and can hence be easily diagonalized. We then find that $|\Psi^\Lambda(s)\rangle$ is the ground state of $H^\Lambda(s)$. The term $B^\Lambda_{\mathrm{UV}}(s)$ in (68) can be seen as a UV regulator for $H_{m(s)}$. Notice that the first line in (69) involves the transversal degrees of freedom, while the second line involves the longitudinal one:

$$B^\Lambda_{\mathrm{UV}}(s) = \frac{1}{\Lambda^2} \int d^d k \, k^2 \Pi_{\perp,n}(-\vec{k}) \Pi_{\perp,n}(\vec{k}) + \frac{m(s)^2}{\Lambda^2} \int d^d k \, k^2 A_\parallel(-\vec{k}) A_\parallel(\vec{k}). \tag{70}$$

The UV regulator term for the transversal modes is once again equivalent to the one found in [86]. In the limit $s \to \infty$, the longitudinal degree of freedom is restricted by the gauge constraint, and $|\Psi^\Lambda_{U(1)}\rangle$ can be given a rather compact parent Hamiltonian:

$$H^\Lambda_{U(1)} \equiv H_{U(1)} + \frac{1}{\Lambda^2} \int d^d x \, \Pi_i(\vec{x}) \Delta \Pi^i(\vec{x}). \tag{71}$$

with $H_{U(1)}$ the Maxwell Hamiltonian from (20).

IV. DISCUSSION

In this paper we have explained, through the concrete example of U(1) gauge theory (or electromagnetism without matter fields) in $d + 1$ spacetime dimensions, how to extend the cMERA formalism to gauge invariant quantum

fields. In particular, we have seen that the gauge constraint can be made compatible with the UV structure of the cMERA wave functional. As with previous cMERA constructions, the ansatz state can be understood as a the result of modifying the short-distance structure of the target state. Additionally, the resulting cMERA wave fuctional $|\Psi^\Lambda(s)\rangle$ has been seen to be the exact ground state of a local Hamiltonian, obtained from the original relativistic Hamiltonian by adding a non-relativistic term that modifies its UV behavior.

As in the case of the free scalar boson of Ref. [86], for finite s the entangling evolution in scale produces a cMERA ansatz for the massive theory with mass $m(s) = \Lambda e^{-s}$. However, for the vector boson analyzed here this came with an interesting twist: since a mass term breaks gauge invariance, the intermediate cMERA state $|\Psi^\Lambda(s)\rangle$ is not in the physical subspace of the gauge theory, and the whole cMERA evolution happens outside of it, with only the asymptotic fixed point cMERA state $|\Psi^\Lambda_{U(1)}\rangle$ being gauge invariant.

Our ultimate goal is to use cMERA to simulate interacting gauge theories in $d + 1$ spacetime dimensions. While for $d = 1$ non-Gaussian cMERAs for interacting QFTs can be numerically manipulated using cMPS techniques [87], no analogous strategy has yet been developed for $d > 1$.

ACKNOWLEDGMENTS

APPENDIX A: REVIEW OF THE QUANTIZATION OF THEORIES WITH CONSTRAINTS

The field theories we deal with in this paper are special in the sense that they involve constraints due to the Lagrangian not being regular. There are several nuances that should be taken into account when quantizing a constrained system, and in general there might not be a unique way of doing so. For instance, the Maxwell Lagrangian presents gauge invariance, and can thus be quantized by fixing the gauge in a variety of ways. Here we choose a quantization scheme that makes the massive and massless theories "compatible," in a sense that we will specify below. The motivation for this choice comes from the magic cMERA for a scalar field [86], where the evolution in scale that asymptotically generates the cMERA can be interpreted as the removal of an IR cutoff given by a mass (see main text). In this Appendix we review the canonical quantization procedure we make use of in the main text. For a more thorough explanation we refer the reader to a specialized textbook such as [5]. From now on, $\{,\}$ refers to the canonical Poisson bracket, and we usually omit the space dependence of the fields.

These are the general steps we will follow for the vector boson theories:

1. Given an irregular Lagrangian, find all the constraints to be imposed in the Hamiltonian formalism. This includes primary constraints (dependency relations between coordinates and momenta) and secondary constraints (constraints derived from demanding that other constraints are preserved by the time evolution generated by the Hamiltonian).

2. Once all constraints have been found, classify them as first class (if they Poisson commute with all other constraints) or second class (if they do not).

3. Impose second-class constraints as operator equations in the algebra of operators (technically speaking, what we do is replacing Poisson brackets by Dirac brackets when defining the algebra of operators, but for all practical purposes we can think of this step as firstly stated).

4. For first-class constraints we use two different quantization strategies: either we impose them as gauge invariance constraints on the Hilbert space (claiming only gauge invariant states are physical), or we add an additional constraint to make them second class, then apply the previous step (gauge fixing).

We begin with the massless vector boson (Maxwell) Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$
$$= -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu), \quad \text{(A1)}$$

and compute the conjugate momenta to the field components in the usual way

$$\Pi^0 \equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 A_0)} = 0, \tag{A2}$$

$$\Pi^i \equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 A_i)} = -F^{0i} = \partial_0 A_i - \partial_i A_0. \tag{A3}$$

We find a single primary constraint: $\Pi^0 = 0$. This constraint is now included in the Hamiltonian by means of a Lagrange multiplier u:

$$\mathcal{H} = \partial_0 A_\mu \Pi^\mu - \mathcal{L} + u\Pi^0, \tag{A4}$$

$$= \frac{1}{2}\Pi_i \Pi^i + \frac{1}{4}F_{ij}F^{ij} - A_0 \partial_i \Pi^i + u\Pi^0. \tag{A5}$$

We look for secondary constraints by imposing the preservation of the primary constraint under time evolution:

$$\partial_0 \Pi^0 = \{\Pi^0, \mathcal{H}\} = 0 \Rightarrow \partial_i \Pi^i = 0. \tag{A6}$$

Thus we obtain a secondary constraint, that we identify as Gauss's law. We look for additional secondary constraints and find that

$$\partial_0(\partial_i \Pi^i) = \{\partial_i \Pi^i, \mathcal{H}\} = 0 \tag{A7}$$

holds without any additional assumptions. Thus we have found all constraints.

The two constraints we obtained are first class since their Poisson bracket vanishes:

$$\{\Pi^0(\vec{x}), \partial_i \Pi^i(\vec{y})\} = 0. \tag{A8}$$

First-class constraints are a consequence of gauge invariance, which it is well known that the Maxwell field exhibits.

We now make the choice to quantize in the temporal gauge. We add the (partially) gauge-fixing constraint $A_0 = 0$ (temporal gauge), and check its consistency:

$$\dot{A}_0 = \{A_0, \mathcal{H}\} = \frac{\partial \mathcal{H}}{\partial \Pi^0} = u. \tag{A9}$$

Hence we can impose the preservation of the gauge fixing just by a condition on the Lagrange multiplier. The new set of constraints includes a first-class constraint

$$\partial_i \Pi^i = 0 \tag{A10}$$

and a pair of second-class constraints

$$A_0 = \Pi^0 = 0. \tag{A11}$$

Upon quantization, the second pair of constraints can be imposed (à la Dirac) as operator equations: the operator representation of A_0 and Π^0 vanishes identically.

The remaining first-class constraint generates ("residual") gauge transformations:

$$A_i \mapsto A_i + \partial_i \epsilon(x). \tag{A12}$$

This transformations do not affect the physical degrees of freedom, and any two states that differ by one of them should be identified. Consequently, we define the physical Hilbert space as the subspace of the total Hilbert space whose elements are invariant under $\partial_i \Pi^i$:

$$\partial_i \hat{\Pi}^i |\text{phys}\rangle = 0. \tag{A13}$$

And the relevant operator algebra is just the one generated by the *spatial* components of fields and momenta A_i, Π^i, with the usual canonical commutation relations:

$$[A_i(\vec{x}), \Pi^j(\vec{y})] = i\delta_{ij}\delta(\vec{x} - \vec{y}). \tag{A14}$$

In this setting we can now write down and diagonalize the Hamiltonian operator, as is done in the main text.

We now turn to the massive vector boson, given by the Proca Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu. \tag{A15}$$

The nonvanishing mass term spoils gauge invariance, but this does not mean that the Lagrangian is regular, since we obtain the same primary constraint

$$\Pi^0 \equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 A_0)} = 0, \tag{A16}$$

which we include it with a Lagrange multiplier u:

$$\mathcal{H} = \dot{A}_\mu \Pi^\mu - \mathcal{L} + u\Pi^0, \tag{A17}$$

$$= \frac{1}{2}\Pi_i \Pi^i + \frac{1}{4}F_{ij}F^{ij} + \frac{1}{2}m^2 A_\mu A^\mu - A_0 \partial_i \Pi^i + u\Pi^0. \tag{A18}$$

Looking for secondary constraints, we find

$$\partial_0 \Pi^0 = 0 \Rightarrow m^2 A_0 + \partial_i \Pi^i = 0, \tag{A19}$$

and

$$\partial_0(m^2 A_0 + \partial_i \Pi^i) = 0 \Rightarrow u = \partial_i A_i, \tag{A20}$$

thus the theory presents two constraints. These are second-class, since their Poisson bracket does not vanish:

$$\{\Pi^0(\vec{x}), m^2 A_0(\vec{y}) + \partial_i \Pi^i(\vec{y})\} = -m^2\delta(\vec{x} - \vec{y}). \tag{A21}$$

We proceed to quantize the theory "à la Dirac": we impose the constraints as operator equations, which effectively

removes A_0, Π^0 as independent operators, and impose canonical commutation relations on the rest of operators:

$$\Pi^0 = 0, \qquad A_0 = -\frac{\partial_i \Pi^i}{m^2}, \tag{A22}$$

$$[A_i(\vec{x}), A_j(\vec{y})] = [\Pi^i(\vec{x}), \Pi^j(\vec{y})] = 0, \tag{A23}$$

$$[A_i(\vec{x}), \Pi^j(\vec{y})] = \delta_{ij}\delta(\vec{x} - \vec{y}). \tag{A24}$$

As expected, the Hilbert space presents no gauge constraints, unlike the massless case. From here on the Hamiltonian can be reexpressed as in Eq. (35), and diagonalized by writing it in terms of the corresponding creation-annihilation operators.

It can be seen now why we have chosen this particular schemes for quantization of the massive and massless vector fields. The relevant operator algebras are identical: we have in both cases A_i, Π_i, the spatial components of the field and their momenta, and they satisfy canonical commutation relations. (This would not be as trivial if we had chosen, for example, the Coulomb gauge quantization for the gauge theory, where the commutation relations of the operators are modified from the canonical case.) The Hilbert spaces where these observables are represented are also taken to be the same, with the caveat that for the massless case only a subspace of the total Hilbert space is physical, since the number of physical degrees of freedom is reduced by the gauge invariance. This allows for a cMERA evolution to be defined consistently as in the main text, in a way that $\Psi^\Lambda(s)$ is a massive vector boson state for finite s and a massless vector boson state for $s = \infty$.

APPENDIX B: ON THE CONTINUITY OF THE $s \to \infty$ LIMIT

In this Appendix we point out a subtlety with the $s \to \infty$ limit of the longitudinal degrees of freedom in the cMERA from the main text. A reader familiar with the $m \to 0$ limit of the massive vector boson theory to the massless vector boson theory will find it analogous to what we present here. To be concrete, we study the particular case of $1 + 1$ dimensions, where there are no transversal degrees of freedom. We thus denote $A(\vec{k}) \equiv A_1(\vec{k}) = A_\parallel(\vec{k})$ and $\Pi(\vec{k}) \equiv \Pi_1(\vec{k}) = \Pi_\parallel(\vec{k})$. The massless case theory is pure gauge, having no physical degrees of freedom except for the zero mode $A(\vec{k} = 0)$, $\Pi(\vec{k} = 0)$ (which gives the quantization of the constant value of the electric field, the only physical degree of freedom of the classical theory). The massless Hamiltonian is given by

$$H = \frac{1}{2}\int dx\, \Pi(x)^2 = \frac{1}{2}\int dk\, \Pi(-k)\Pi(k), \tag{B1}$$

and its ground state is characterized by

$$\Pi(k)|\Psi\rangle = 0, \tag{B2}$$

which includes the gauge invariance constraint ($k \neq 0$) and the energy minimization for the single degree of freedom ($k = 0$). It is hard to make statements about the entanglement properties of this state, since, even though Fourier transformation of (B2) yields local annihilation operators

$$\Pi(x)|\Psi\rangle = 0, \tag{B3}$$

as befits an unentangled state, there are no local physical degrees of freedom to speak about their correlations or lack thereof.

Applying the formalism from the main text, we start from the unentangled state

$$\psi(x)|\Lambda\rangle = 0 \tag{B4}$$

and evolve with an entangler

$$K = \frac{-i}{2}\int dk\, g(k)\psi(-k)\psi(k) + \text{H.c.} \tag{B5}$$

with

$$g(k) = 1 - \frac{1}{2}\frac{\Lambda^2}{\Lambda^2 + k^2}. \tag{B6}$$

In position space, this entangler looks like this

$$K = \int dx\, A(x)\Pi(x) - \int dx\, dy\, e^{-\Lambda|x-y|}A(x)\Pi(y), \tag{B7}$$

which has an on-site part, and a bilocal part, which acts at a particular lengthscale Λ^{-1}. The cMERA states are then characterized by

$$\left(\sqrt{\frac{\alpha(k,s)}{2}}A(k) + i\sqrt{\frac{1}{2\alpha(k,s)}}\Pi(k)\right)|\psi^\Lambda(s)\rangle = 0, \tag{B8}$$

with

$$\alpha(k,s) = \frac{m(s)^2}{\Lambda}\sqrt{\frac{k^2 + \Lambda^2}{k^2 + m(s)^2}}. \tag{B9}$$

Figure 6 shows a qualitative plot of $\alpha(k,s)$. States at finite values of s are ground states of the following regularized version of the massive vector boson Hamiltonian:

$$H(s) = \int dx\left[\frac{\Pi(x)^2}{2} + \frac{m^2}{\Lambda^2}(\partial A(x))^2\right], \tag{B10}$$

where $m(s) = \Lambda e^{-s}$. This states are entangled, due to the action of K. At any finite time in the evolution, the state has
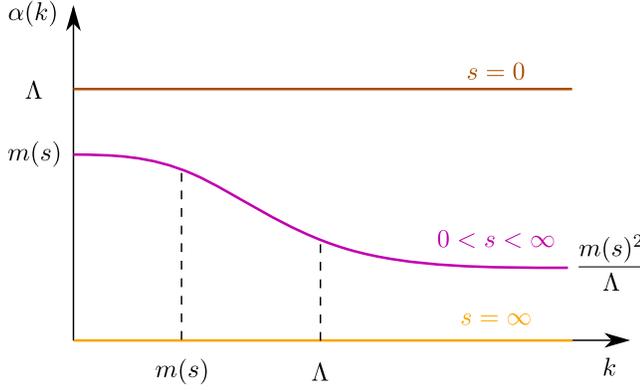
FIG. 6.   Evolution of $\alpha(k, s)$ (qualitative plot).

all the entanglement that has been introduced from the UV cutoff scale Λ^{-1} to the IR cutoff scale $m(s)^{-1}$. This entanglement does not disappear smoothly in the $s \to \infty$ limit because the convergence to the fixed point is not smooth. Take a look at the two-point function for $\Pi(x)$:

$$\langle \Pi(x)\Pi(y)\rangle \propto \mathcal{F}^{-1}[\alpha(k)](x-y). \qquad (B11)$$

Because of the behavior of $\alpha(k)$ we can decompose this correlator into two parts: an on-site delta and an integrable function of the distance:

$$\langle \Pi(x)\Pi(y)\rangle \sim \frac{m(s)^2}{\Lambda}\delta(x-y) + f_s(x-y). \qquad (B12)$$

As $s \to 0$, we have $\|f_s\|_2 \to 0$ so both terms in the correlator go to zero. However the value at $x = y$, corresponding to the squared norm of $\Pi(x)|\Psi^{\Lambda}(s)\rangle$ preserves the delta divergence for the whole evolution, while it should be zero in the gauge invariant subspace.

APPENDIX C: UV REGULARIZATION OF CORRELATION FUNCTIONS

In the main text we have provided a cMERA for the ground state of a vector boson theory. The cMERA states can be seen to approximate the long distance properties of their target states, while keeping their short distance properties closer to those of the original unentangled state. In this Appendix we study the UV structure of the proposed gauge invariant cMERA in more detail.

In [88], the existence of the short distance limit of two-point functions of cMERA states was used as a witness for UV regularization. These correlation functions usually take the following form in cMERA states:

$$\langle \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\rangle = C\delta(\vec{x}-\vec{y}) + f(|\vec{x}-\vec{y}|), \qquad (C1)$$

with C a constant and f some function such that

$$\lim_{\vec{x}\to\vec{y}}\langle \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\rangle < \infty, \qquad (C2)$$

that is, the short-distance limit of the correlator of two fields is finite, barring the on-site delta divergence. In our particular example, and focusing on the A fields, the two-point functions are given in terms of the α functions by (65). Removing the constant factor responsible for the δ functions, the corresponding limits are given by

$$\lim_{\vec{x}\to\vec{y}}\langle A_{\|}(\vec{x})A_{\|}(\vec{y})\rangle = \int d^d k \left(\frac{1}{\alpha_{\|}(k, s)} - \frac{\Lambda}{m(s)^2}\right), \qquad (C3)$$

$$\lim_{\vec{x}\to\vec{y}}\langle A_{\perp,n}(\vec{x})A_{\perp,n}(\vec{y})\rangle = \int d^d k \left(\frac{1}{\alpha_{\perp}(k, s)} - \frac{1}{\Lambda}\right), \qquad (C4)$$

where the expectation values are taken with respect to $|\Psi(k, s)\rangle$. For the α functions from (60)–(61) we have

$$\frac{1}{\alpha_{\|}(k, s)} \sim \frac{\Lambda}{m(s)^2}\left(1 - \frac{\Lambda^2 - m(s)^2}{2k^2} + \cdots\right), \qquad (C5)$$

$$\frac{1}{\alpha_{\perp}(k, s)} \sim \frac{1}{\Lambda}\left(1 + \frac{\Lambda^2 - m(s)^2}{2k^2} + \cdots\right), \qquad (C6)$$

and the UV divergence is removed in $d = 1$, but it remains in higher dimensions. Notice that since

$$\alpha_{\|}(k, s) = \frac{m(s)^2}{\alpha_{\perp}(k, s)}, \qquad (C7)$$

and both functions asymptote to constants, their asymptotic behavior is very much related, as can be seen in the expansions above.

If needed, we can build cMERAs where these states are more strongly UV regulated, at the cost of adding extra derivatives to the entangler and the parent Hamiltonians. Consider an entangler of the form (50) whose momentum space profile is given by

$$g_{\perp}(k) = \frac{1 + n\kappa^{2n-2}}{2(1 + \kappa^{2n-2})(1 + \kappa^2 + \kappa^{2n})}, \qquad \kappa \equiv \frac{k}{\Lambda}, \qquad (C8)$$

$$g_{\|}(k) = 1 - g_{\perp}(k), \qquad (C9)$$

for $n > 1$. $g_{\perp}(k)$ is a rational function of k^2 that goes to $\frac{1}{2}$ at $k = 0$ and decays as k^{-2n} at long distances. Its Fourier transform, namely the real space profile of the entangler, is therefore integrable for dimensions $d < 2n$. The resulting α functions corresponding to this entangler, if applied on the same initial state, are

$$\alpha_\perp(k, s) = \Lambda \sqrt{\frac{k^{2n} + k^2 \Lambda^{2n-2}}{k^{2n} + k^2 \Lambda^{2n-2} + \Lambda^{2n}}}$$
$$\times \sqrt{\frac{k^{2n} + k^2 m(s)^{2n-2} + m(s)^{2n}}{k^{2n} + k^2 m(s)^{2n-2}}}, \qquad (C10)$$

$$\alpha_\parallel(k, s) = \frac{m^2(s)}{\alpha_\perp(k, s)}, \qquad (C11)$$

with asymptotic fixed points:

$$\alpha_\perp(k) = \Lambda \sqrt{\frac{k^{2n} + k^2 \Lambda^{2n-2}}{k^{2n} + k^2 \Lambda^{2n-2} + \Lambda^{2n}}}, \qquad (C12)$$

$$\alpha_\parallel(k) = 0. \qquad (C13)$$

These states are more strongly UV regularized, as can be checked by expanding

$$\frac{1}{\alpha_\parallel(k, s)} \sim \frac{\Lambda}{m(s)^2} + \frac{\Lambda(m(s)^{2n} - \Lambda^{2n})}{2m(s)^2 k^{2n}} + ..., \qquad (C14)$$

$$\frac{1}{\alpha_\perp(k, s)} \sim \frac{1}{\Lambda} + \frac{\Lambda^{2n} - m(s)^{2n}}{2\Lambda k^{2n}} + ..., \qquad (C15)$$

Indeed, the short distance limit of the corresponding two-point functions will be finite for $d < 2n$, while their long distance behavior can still be seen to approximate that of the target state correlation functions.

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