Determining the Hubble constant without the sound horizon: Measurements from galaxy surveys

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Two sources of geometric information are encoded in the galaxy power spectrum: the sound horizon at recombination and the horizon at matter-radiation equality. Analyzing the BOSS 12th data release galaxy power spectra using perturbation theory with Ω_m priors from Pantheon supernovae but no priors on Ω_b, we obtain constraints on H_0 from the second scale, finding $H_0 = 65.1^{+3.0}_{-5.4}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$; this differs from the best fit of SH0ES at 95% confidence. Similar results are obtained if Ω_m is constrained from uncalibrated baryon acoustic oscillations: $H_0 = 65.6^{+3.4}_{-5.5}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. Adding the analogous lensing results from Baxter and Sherwin from 2020, the posterior shifts to $70.6^{+3.7}_{-5.0}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. Using mock data, Fisher analyses, and scale cuts, we demonstrate that our constraints do not receive significant information from the sound horizon scale. Since many models resolve the H_0 controversy by adding new physics to alter the sound horizon, our measurements are a consistency test for standard cosmology before recombination. A simple forecast indicates that such constraints could reach $\sigma_{H_0} \simeq 1.6\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ in the era of Euclid.

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I. INTRODUCTION

How do galaxy surveys measure the Hubble constant? Recent analyses have determined H_0 by comparing the angular scale of baryon acoustic oscillations (BAOs) with the theoretical size of the sound horizon scale at decoupling, r_d. A second "standard ruler" exists, however: the equality scale, i.e., the horizon wave number at matter radiation equality, whose angular scale can be measured from the power spectrum shape and physical scale predicted by theory. In this work, we explore the extent to which galaxy surveys can use this scale to place constraints on H_0 that are independent of the sound horizon.

Until recently, precise H_0 constraints have stemmed from two sources: the Cepheid-calibrated local distance ladder (e.g., Refs. [1,2]) and anisotropies in the cosmic microwave background (CMB) (e.g., Ref. [3]). Today, a host of additional constraints is available, arising from datasets such as galaxy and Lyman-alpha BAO (e.g., Refs. [4–10]), strong gravitational lensing (e.g., Refs. [11,12]), and gravitational wave observations [13,14]. Broadly, these fall into two camps: "indirect" measurements, which require a full cosmological model for interpretation; and "direct" probes, independent of early-universe physics.[1] Probes in the former category, including CMB and calibrated BAO, usually derive information from the sound horizon at recombination,[2] calculated assuming ΛCDM. Previously, a tension between direct and indirect measurements seemed apparent; however, this distinction has become less clear with the latest results from the Tip of the Red Giant Branch-calibrated distance ladder [15], strong lensing [12], and recalibrated megamaser results [16]. Nevertheless, there remains significant disagreement between indirect probes and the SH0ES distance ladder measurements [1], reaching a significance of approximately 5σ [17].

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[1]These are sometimes classified as "early" and "late" measurements, respectively, but the terminology can be confusing, since an early measurement might not involve high-redshift datasets.

[2]For the purposes of this work, there is little difference between "recombination" and "decoupling"; we thus use the terms interchangeably.

Two primary possibilities exist to resolve this: (a) unresolved systematics (e.g., Ref. [18]) or (b) incompleteness of the cosmological model. For the latter, a wide variety of new-physics models have been proposed; many of these resolve the tension by providing mechanisms to reduce the sound horizon at recombination. As of yet, there is no generally accepted solution.

Reference [19] proposed a new method to shed light on the discrepancy, using CMB lensing to measure H_0 without the sound horizon scale; constraints were instead derived from the angular equality scale, $L_{\rm eq}$. Being simply the projected wave number of modes entering the horizon at $z_{\rm eq} \sim 3400$, this produces a definitive feature in the convergence spectrum and can be used as a standard ruler. Importantly, the equality scale is sensitive to different redshifts than those of CMB and BAO analyses ($z \sim 1100$). This yields an important test: inconsistency of equality- and recombination-based H_0 constraints would give evidence for physics beyond ΛCDM operating at $z \gtrsim 10^3$. Combining *Planck* lensing with cosmological priors on Ω_m and A_s, Reference [19] obtained $H_0 = 73.8 \pm 5.1\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$; unfortunately, the projected improvements from future surveys were modest, owing to the intrinsically large cosmic variance.

Since the number of modes available to a three-dimensional galaxy survey is typically much greater than for CMB lensing, one might expect stronger constraints on H_0 from this avenue: indeed, this was the primary source of H_0 information from galaxy surveys two decades ago (see, e.g., Refs. [20,21] and [22] for a more recent attempt). In this work, we perform such a measurement with modern surveys.

This paper has the following structure. In Sec. II, we discuss the physics behind our approach, before considering the datasets and analysis pipeline in Sec. III. Results are presented in Sec. IV, both for the BOSS data and idealized mock catalogs, before we present a discussion in Sec. V.

II. EQUALITY AND THE SOUND HORIZON

A glance at the matter power spectrum reveals two features: the broadband peak at wave number $k_{\rm eq} \sim 10^{-2}h\,\mathrm{Mpc}^{-1}$ and the oscillatory behavior with period $\Delta k \sim 0.05h\,\mathrm{Mpc}^{-1}$. The behavior around $k_{\rm eq}$ is well known, arising from the transition between modes that enter the horizon in radiation-dominated and matter-dominated epochs. In galaxy surveys, resolving the peak is difficult (though possible with experiments such as SPHEREx [23]), due to relativistic effects and integral constraints (e.g., Ref. [24]), alongside cosmic variance and imaging systematics. More generally, $k_{\rm eq}$ information is encoded in the shape of the power spectrum and can be inferred from smaller (though still linear) scales, as seen by the approximate scaling solution to the linearly biased perturbation equations:

$$P_g(k) \approx \begin{cases} b_1^2 A_s \left(\frac{k}{k_{\rm eq}}\right)^{n_s} & k < k_{\rm eq} \\ b_1^2 A_s \left(1 + \log\left(\frac{k}{k_{\rm eq}}\right)\right)^2 \left(\frac{k}{k_{\rm eq}}\right)^{n_s - 4} & k > k_{\rm eq}. \end{cases} \tag{1}$$

This is facilitated in part by the addition of amplitude and bias information from redshift-space distortions or priors.[3]

In ΛCDM, the equality scale is simply related to cosmological parameters;

$$k_{\rm eq} = (2\Omega_{cb} H_0^2 z_{\rm eq})^{1/2}, \qquad z_{\rm eq} = 2.5 \times 10^4 \Omega_{cb} h^2 \Theta_{2.7}^{-4} \tag{2}$$

[25,26], where $\Theta_{2.7} \equiv T_{\rm CMB}/(2.7\ \mathrm{K})$ is the temperature of the CMB monopole, $\Omega_{cb} \equiv \Omega_{cdm} + \Omega_b$ (assuming neutrinos to be relativistic at $z_{\rm eq}$) is the CDM + baryon density fraction, and $h \equiv H_0/(100\ \mathrm{km\,s^{-1}\,Mpc^{-1}})$. Measuring $k_{\rm eq}$ in $h\,\mathrm{Mpc}^{-1}$ units probes the combination $\Omega_{cb} h \equiv (\omega_{cdm} + \omega_b)/h$ or, marginalizing over ω_b, ω_{cdm}/h. Given $k_{\rm eq}$ and a probe of Ω_{cb} (or, more commonly, Ω_m), we can thus solve for the Hubble constant.

Complicating this is the second scale: the sound horizon at z_d, the redshift of photon-baryon decoupling. This is given by

$$r_d \equiv r_s(z_d) = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz \approx \frac{55.154 h \exp\left[-72.3(\omega_\nu + 0.0006)^2\right]}{\omega_{cb}^{0.25351} \omega_b^{0.12807}} h^{-1}\,\mathrm{Mpc} \tag{3}$$

[27], where $H(z)$ and $c_s(z)$ are the Hubble parameter and sound speed. r_d sources two main features: the BAO wiggles with $\Delta k \approx 0.05h\,\mathrm{Mpc}^{-1}$, and a small-scale suppression of power on the baryonic Jeans scale [28]. Both have amplitudes scaling as ω_b/ω_{cb} and could be used to infer the physical scale of the sound horizon. In combination with the measured angular scale, this constrains H_0. From (3), such measurements carry the degeneracy $\omega_{cdm} \propto h^4$ (after ω_b marginalization), and a measurement using the Jeans suppression is degenerate with n_s.

An H_0 measurement from the full shape (FS) of the galaxy power spectrum will include information from both r_d and $k_{\rm eq}^{-1}$ standard rulers, while BAO analyses are sensitive only to r_d. To extract only information deriving from equality, one may wish to "marginalize over the sound horizon"; this is nontrivial since r_d is not a direct input to any Boltzmann code, emerging only following

[3]If the transfer function was a pure power law, we would expect full $A_s - k_{\rm eq}$ degeneracy; the logarithmic shape for $k > k_{\rm eq}$ reduces this, though we note that measuring $k_{\rm eq}$ in this manner is inherently model dependent.

simplifications such as tight coupling.[4] Here, we limit the H_0 information arising from the sound horizon simply by removing the usual informative prior on ω_b, and thus the external r_d calibration even if ω_{cb} is known precisely. For future data, this may be insufficient; the BAO features and Jeans suppression can, in principle, calibrate each other, sourcing an effective sound-horizon prior.

III. DATASETS AND ANALYSIS

A. Redshift-space power spectrum

Our main observational dataset is the 12th data release (DR12) [4] of the Baryon Oscillation Spectroscopic Survey (BOSS), part of SDSS-III [29,30]. Split across two redshift bins (at $z = 0.38$, 0.61) in each of the Northern and Southern galactic caps, the survey contains approximately 1.2×10^6 galaxy positions with a total volume of $5.8 h^{-3}\,\mathrm{Gpc}^3$. Here, we use the (unreconstructed) power spectrum monopole and quadrupole,[5] each in 48 k-bins for $k \in [0.01, 0.25] h\,\mathrm{Mpc}^{-1}$, with covariances generated from a suite of 2048 MultiDark-Patchy mocks [31,32], using the cosmology $\{\Omega_m = 0.307115, \Omega_b = 0.048, \sigma_8 = 0.8288, h = 0.6777, \sum m_\nu = 0\,\mathrm{eV}\}$.[6]

To extract maximal shape information, we model $P_\ell(k)$ with the effective field theory (EFT) of large-scale structure, following Ref. [33] (see also Ref. [34]). This includes one-loop perturbation theory, infrared resummation of long-wavelength modes, and counterterms parametrizing the impact of small-scale physics. The model is convolved with the survey window function and incorporates Alcock-Paczynski (AP) effects [35].[7] The procedure has been used in a number of works [26,36–39], including a rigorous test on huge volume simulations [40], showing any theory error to be strongly subdominant to the BOSS statistical error. Here, we utilize the CLASS-PT implementation [41], with Markov chain Monte Carlo performed using MONTEPYTHON3.3 alongside heavily optimized public likelihoods,[8] with convergence assumed once the Gelman-Rubin diagnostic is below 1.05.

We vary the parameter set

$$\left\{h, \omega_{cdm}, \omega_b, A_s/A_{s,\mathrm{Planck}}, n_s, \sum m_\nu\right\} \times \{b_1, b_2, b_{G_2}, b_4, c_{s,0}, c_{s,2}, P_{\mathrm{shot}}\}, \tag{4}$$

where $A_{s,\mathrm{Planck}} = 2.0989 \times 10^{-9}$ and $\sum m_\nu$ is the summed mass of three degenerate neutrinos. To aid convergence, we add a Gaussian prior on ω_b of width 50% centered at the *Planck* best fit and flat priors of [0, 0.18] eV and [0.87, 1.07] to $\sum m_\nu$ and n_s, respectively.[9] The second line gives nuisance parameters of the EFT model, which are allowed to vary independently in each of the four data patches necessitated by their differing redshifts and calibrations, subject to the weak Gaussian priors of Ref. [33]. Those entering the likelihood linearly (b_4, $c_{s,0}$, $c_{s,2}$, and P_{shot}) are marginalized analytically [39,42,43], reducing the total number of sampled parameters to $6 + 3 \times 4 = 18$.

Later, we will require mock data to explore the information content of our model. This is generated from the theory model using the baseline cosmology $\{\omega_{cdm} = 0.118, \omega_b = 0.022, A_s/A_{s,\mathrm{Planck}} = 1.025, h = 0.6777, n_s = 0.9649, \sum m_\nu = 0.06\,\mathrm{eV}\}$, similar to the MultiDark-Patchy parameters, but with massive neutrinos. Three samples are created: 1) fiducial, 2) with negligible BAO wiggles, and 3) with fewer baryons (to reduce both BAO and baryon damping effects). For each, we fit nuisance parameters to the observed $P_\ell(k)$ and do not include noise, such that all datasets may be simply compared. Set 2 is generated by increasing the BAO damping scale by 1000 times, while 3 reduces Ω_b by 10 times relative to the fiducial value, keeping ω_{cdm} and A_s fixed. When performing parameter inference, all mock datasets are analyzed using the MultiDark-Patchy covariance matrix described above.

B. Cosmological priors

Equality based measurements of H_0 are assisted by information on A_s (to constrain k_{eq}) and Ω_{cb} (to break the $H_0 - \Omega_{cb}$ degeneracy).[10] For the former, we employ a weak Gaussian prior of $A_s = (2.11 \pm 0.36) \times 10^{-9}$, centered on the *Planck* best fit [3]. The r_d dependence of this is minimal, since the CMB measurement is limited by the optical depth and hence derives from very large scales; however, to be maximally conservative, we choose the prior width to be 10 times that of the *Planck* constraint.

For the Ω_m prior, we principally use the marginalized result from Pantheon supernovae: $\Omega_m = 0.298 \pm 0.022$ [44]. This cannot constrain H_0 directly, since the supernova absolute magnitudes are unknown. An alternative source of Ω_m information is given by *uncalibrated* BAO measurements. A standard BAO analysis proceeds by comparing the radial and angular oscillatory scales to the ΛCDM sound horizon, providing information on Ω_m and $H_0 r_d$ through the evolution of the angular diameter distance and

[4]An *ad-hoc* rescaling of r_d would be dangerous, as it would not conserve the stress-energy $T_{\mu\nu}$; instead, one should self-consistently add any new physics model to the perturbation equations.

[5]fbeutler.github.io/hub/boss_papers.html.

[6]We caution that the spectra may have nontrivial imaging systematics at low k; investigation of these is beyond the scope of this work.

[7]We use a fiducial value of $\Omega_m = 0.31$ to apply the AP rescaling to the BOSS data.

[8]github.com/michalychforever/lss_montepython.

[9]Initial testing showed that these do not significantly affect the H_0 constraints; indeed, we can increase the ω_b prior width by a factor of 2 without changing σ_{H_0}, though this leads to slower convergence. This is further discussed in Sec. IV B.

[10]Knowledge of b_1 is also useful; this is provided by redshift-space distortions.

Hubble parameter (e.g., Ref. [5]). To remove the dependence on r_d, we rescale the sound horizon by a free parameter α_{r_d} when including BAO $H(z)r_d$ and $D_A(z)/r_d$ measurements. In this formalism, no knowledge of recombination physics is required, just the existence of a time-independent correlation function peak. Here, we use a range of galaxy BAO measurements from BOSS DR7 (6dFGS and Main Galaxy Samples) [45,46] and eBOSS DR14 Lyman-alpha measurements (including cross-correlations with quasars) [47,48]. We exclude the BOSS DR12 BAO measurements, since they are covariant with the FS dataset, which would cause additional complications. Alone, the uncalibrated BAOs are found to give the constraint $\Omega_m = 0.308^{+0.025}_{-0.030}$. For analyses using mock data, we center the priors on the true parameter values, keeping the same fractional width.

C. Additional datasets

H_0 constraints from CMB lensing were demonstrated in Ref. [19]. Because of the presence of projection integrals, the measurements are relatively free from r_d calibration, even with a restrictive prior on ω_b. The lensing power spectrum measures the combination $L_{\rm eq} \sim \Omega_m^{0.6} h$, with a different scaling than that of $k_{\rm eq}$; thus, we may expect some degeneracy breaking when this is combined with FS measurements. Here, we use the public *Planck* 2018 CMB-marginalized lensing likelihood [49], assuming zero covariance between this and the BOSS data.[11] For analyses including the lensing dataset, we impose a twice tighter prior on A_s of $(2.11 \pm 0.18) \times 10^{-9}$ (as in Ref. [19]), to break the significant $A_s - L_{\rm eq}$ degeneracy.

IV. RESULTS

A. Constraints from current datasets

Figure 1 shows the cosmological constraints obtained. Combining BOSS power spectra with Pantheon Ω_m priors, we obtain $H_0 = 65.1^{+3.0}_{-5.4}$ km s^{-1} Mpc^{-1}, below the best-fit SH0ES value at a 95% confidence level (including non-Gaussianity of the posterior), even though our analysis is not based on the sound horizon.[12] As expected, the ω_b posterior is broad since, unlike in previous analyses, we have not imposed a restrictive prior. There is a strong $h - \omega_{cdm} - \omega_b$ degeneracy, close to the expected linear relationship, rather than the $\omega_{cdm} \propto h^4$ scaling of r_d calibration. The $\omega_b - \omega_{cdm}$ degeneracy indicates that a small amount of ω_{cdm} information comes from the BAO wiggles, though, as argued below, we do not expect this to inform our H_0 constraints. We note little dependence on the A_s prior, with less than 10% degradation in σ_{H_0} if this is removed; this is expected since A_s can be measured from the power spectrum through the loop corrections.

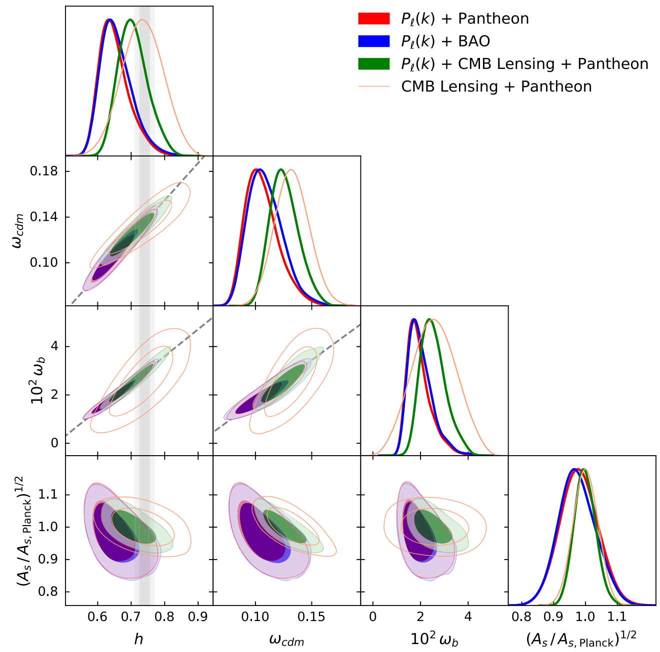

FIG. 1. Parameter constraints from analyses of BOSS DR12 power spectra and *Planck* lensing (as in Ref. [19]). Datasets are combined with either Pantheon supernovae or uncalibrated BAO to provide Ω_m information, but no ω_b prior is assumed, such that the H_0 constraints do not derive information from the sound horizon (as evidenced by subsequent figures). Dashed lines show linear relationships, and vertical bands give the SH0ES H_0 constraint [1]. We omit the posteriors for n_s, $\sum m_\nu$, and the 12 nuisance parameters for clarity. Following the caption ordering, 68%H_0 confidence intervals are $65.1^{+3.0}_{-5.4}$, $65.6^{+3.4}_{-5.5}$, $70.6^{+3.7}_{-5.0}$ and 73.4 ± 6.1, respectively, in km s^{-1} Mpc^{-1} units.

Using uncalibrated BAO instead of the Pantheon sample gives a similar posterior; $H_0 = 65.6^{+3.4}_{-5.5}$ km s^{-1} Mpc^{-1}. This is unsurprising; the sound horizon rescaling parameter, α_{r_d}, removes the H_0 information, and the marginalized Ω_m constraint from BAO alone is similar to Pantheon. Interestingly, the α_{r_d} posterior is 0.993 ± 0.016; the combination of equality-based power spectra and (independent) uncalibrated BAO prefer a sound horizon consistent with ΛCDM.

Combination with *Planck* lensing shifts the H_0 posterior to larger values, with a marginalized limit of $70.6^{+3.7}_{-5.0}$ km s^{-1} Mpc^{-1}. Because of the addition of galaxy information, this is somewhat tighter than the lensing-only constraints of Ref. [19], though there is no improvement relative to the $P_\ell(k)$ posteriors, due to the broad error bars on the CMB lensing measurements and similar degeneracy directions. Note that the lensing-only constraint

[11]Technically, some correlation will be present since the probes partially overlap. Since the lensing kernel is much broader in redshift space than the BOSS selection function, and CMB lensing is only sensitive to modes that are perpendicular to the line of sight, we expect this to be small.

[12]This corresponds to $k_{\rm eq} = (1.40^{+0.10}_{-0.14}) \times 10^{-2} h$ Mpc^{-1}.

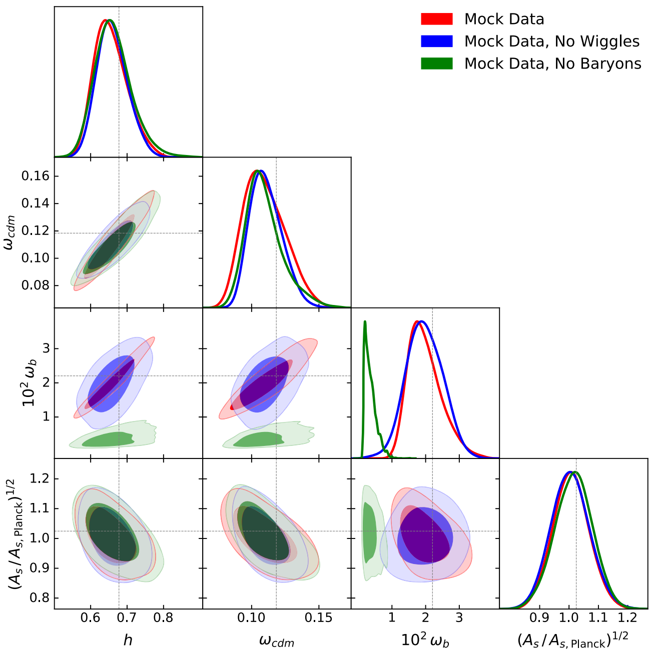

FIG. 2. As Fig. 1, but for three mock datasets: fiducial, with BAO wiggles removed, and with ω_b reduced by a factor of 10. All analyses include Pantheon priors on Ω_m. Thin lines show the true parameter values, and the 1σ constraints on H_0 are $65.6^{+3.7}_{-5.3}$, $65.9^{+3.7}_{-4.6}$, and $66.2^{+4.1}_{-5.6}$ in $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ units. The insensitivity of H_0 constraints to the removal of BAO wiggles and baryon damping implies that information is not being sourced from the sound horizon scale.

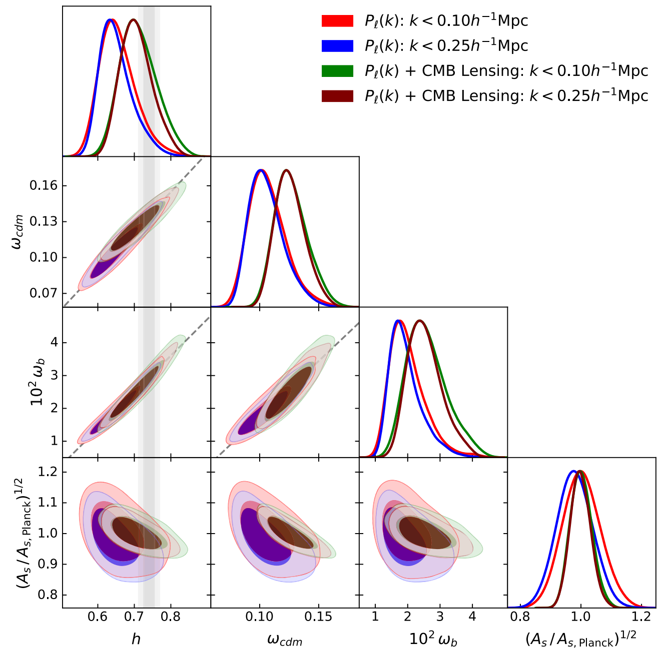

FIG. 3. As Fig. 1, but restricting the k range of the BOSS power spectrum analysis from the fiducial value of $k_{\max} = 0.25h\,\mathrm{Mpc}^{-1}$. All analyses include the Pantheon Ω_m prior. For the $k_{\max} = 0.1h\,\mathrm{Mpc}^{-1}$ datasets, H_0 constraints are $65.6^{+3.8}_{-5.8}$ (BOSS) and $71.2^{+4.3}_{-5.7}$ (BOSS + *Planck*) in $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ units. The lack of dependence of H_0 on $k_{\max}$ again indicates that information is not arising from the BAO scale.

($H_0 = 73.4 \pm 6.1\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$) is shifted somewhat from that of Ref. [19], due to slightly different prior choices.

B. Dependence on r_d

We now demonstrate that our H_0 constraints do not receive significant information from the sound horizon, using three tests: repeating the analysis on mock datasets without baryon oscillations and damping, employing scale cuts, and performing Fisher forecasts, where we can explicitly marginalize over r_d.

First, we turn to the synthetic datasets discussed above. As shown in Fig. 2, our H_0 constraints are negligibly impacted by removing BAO wiggles or reducing baryonic damping. Since the mock data are generated to match the BOSS spectra, this is a strong indication that our H_0 constraints are independent of sound horizon physics.[13] Note that the best-fit values of ω_{cdm} and h are shifted by approximately 0.5σ from the truth; this indicates a (modest) prior-volume effect due to non-Gaussianity of the high-dimensional posterior, confirmed by its removal when reanalyzing the data with a covariance appropriate for a 10 times larger survey. While this could be ameliorated by stricter nuisance parameter priors, given that the offsets are small, we do not include these. The mocks also highlight the importance of Ω_m priors; since the FS likelihood sources ω_{cdm} information from BAO wiggles, the no-wiggle constraints on H_0 would degrade if an external prior were not present.

Scale cuts provide further evidence to support our conclusions. Figure 3 shows the effect of reducing $k_{\max}$ from $0.25h\,\mathrm{Mpc}^{-1}$ to $0.10h\,\mathrm{Mpc}^{-1}$, which, if information were coming from BAO wiggles, would be expected to significantly inflate the H_0 posterior (e.g., Ref. [33]). Notably, the reduction in constraining power is slight (approximately 10%), though the nuisance parameters of the one-loop model suffer significant posterior inflation. Again, this indicates that the primary information is sourced by k_{eq}^{-1} rather than r_d (and thus relatively large scales, though we note the lowest k modes have limited impact due to their large statistical error).

Finally, we consider a simplified Fisher analysis in which r_d can be marginalized over exactly. This is made possible by using the Eisenstein-Hu transfer function [25] in CLASS-PT, rather than the usual output from CLASS. In addition to the five cosmological parameters $\{h, \omega_{cdm}, \omega_b, A_s/A_{s,\mathrm{Planck}}, n_s\}$ (the Eisenstein-Hu

[13] That the constraints are not affected by the removal of *all* baryon information also indicates that the weak prior placed on ω_b in the real analysis is not affecting the H_0 posterior in the fiducial analysis.

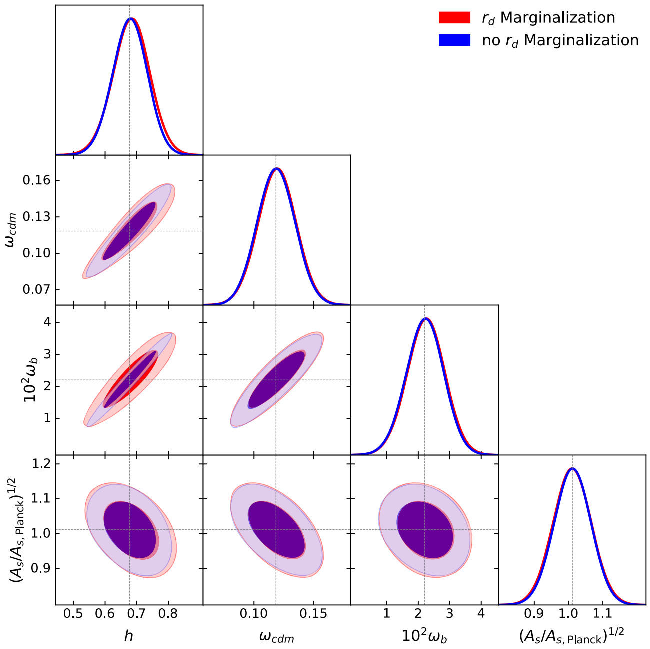

FIG. 4. Posteriors from a simplified Fisher forecast mimicking the BOSS + Pantheon results, including explicit marginalization over the sound horizon, r_d, using the Eisenstein-Hu transfer function [25]. We obtain $\sigma_{H_0} = 5.9$ (5.5) $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ with (without) r_d marginalization; the small size of this degradation supplies further evidence that our constraints are insensitive to the sound horizon scale.

approximation does not allow for massive neutrinos), we vary a sound horizon rescaling parameter β_{r_d}, which rescales r_d within the transfer function. For simplicity, a single redshift bin (centered at $z = 0.51$) with the total BOSS volume is used, and window function effects are ignored. As seen in Fig. 4, the H_0 posteriors are broader than those found in Fig. 1; this is due to the assumptions of an Eisenstein-Hu model and exclusion of redshift evolution, yet the model retains qualitative utility. Marginalization over r_d has little effect, reducing σ_{H_0} by less than 10%, reinforcing our conclusions that the H_0 constraints are insensitive to r_d. Without an Ω_m prior, the marginalization gives significant degradation, with $\sigma_{H_0} = 11\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$; in this case, Ω_m information enters from the BAO wiggle *amplitudes*, which are washed out by the marginalization.

The Fisher formalism may also be used to test the dependence of the H_0 constraints on the range of k modes included in the analysis. Given that data systematics are concentrated in the first few k bins, this is a useful probe of our sensitivity to such effects. Rerunning the above forecast (without marginalization over β_{r_d}) increasing k_{min} from $0.01h\,\mathrm{Mpc^{-1}}$ to $0.03h\,\mathrm{Mpc^{-1}}$ ($0.05h\,\mathrm{Mpc^{-1}}$) gives an H_0 posterior inflated by 14% (35%). We thus stress that the constraints found in this work have greater dependence on large-scale modes (and thus any present systematics) than for most BAO-only analyses. This is as expected since most of the k_{eq} information is wave numbers in the linear regime, yet large enough to avoid excessive cosmic variance. We also note that the constraints are not affected by removal of the weak ω_b prior for $k_{\mathrm{min}} = 0.01h\,\mathrm{Mpc^{-1}}$ and $0.03h\,\mathrm{Mpc^{-1}}$ but suffer approximately 10% inflation if $k_{\mathrm{min}} = 0.05h\,\mathrm{Mpc^{-1}}$. This indicates that the results are prior limited only if most of the large-scale power is removed.

C. Forecasting for future surveys

To estimate the potential of future surveys to constrain H_0 without the sound horizon, we perform a simplistic Fisher analysis, similar to that presented above. In particular, we consider a Euclid-like survey in eight redshift bins, taking the volumes and fiducial bias parameters from the forecast of Ref. [50]. For consistency, we slightly expand our k range up to $k_{\mathrm{max}} = 0.3h\,\mathrm{Mpc^{-1}}$ and do not impose nuisance parameter priors. Adopting the A_s and Ω_m priors of this work, and marginalizing over r_d, we obtain $\sigma_{H_0} \sim 1.7\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$; this tightens to approximately $1.6\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ with the more optimistic $\sigma_{\Omega_m} = 0.012$ prior of Ref. [19]. For future surveys, it is unclear whether removing the ω_b prior will be sufficient to ensure r_d independence; this will be discussed in future work alongside a more complete forecast.

V. CONCLUSION

In the past decade, galaxy surveys have focused on measuring BAO feature. In this work, we make use of the fact that an additional standard ruler is present; the horizon size at matter-radiation equality, k_{eq}^{-1}. Combining galaxy power spectra from BOSS with cosmological priors on Ω_m gives equality-based constraints of $H_0 = 65.1^{+3.0}_{-5.4}\ \mathrm{s^{-1}\,Mpc^{-1}}$ (power spectrum only) and $70.6^{+3.7}_{-5.0}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (adding *Planck* lensing). For BOSS, such a measurement can be obtained simply by analyzing the data without use of an informative ω_b prior; we demonstrate this using mock catalogs, scale cuts, and Fisher forecasts. For the next generation of surveys, simple forecasts indicate that sound horizon independent constraints of $\sigma_{H_0} \simeq 1.6\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ should be possible; more sophisticated techniques may be required to remove r_d information, however.

To close, we consider implications for the "Hubble tension." Most proposed mechanisms for its resolution rely on modifying the sound horizon at recombination, and thus altering the BAO scale. Given that equality-based measurements are sensitive to higher redshifts than BAO measurements, H_0 constraints anchored at z_d and z_{eq} may differ if new physics is at work, making this a valuable test of new physics prior to recombination. Here, we find good agreement between galaxy-only H_0 measurements derived from the sound horizon (e.g., Refs. [33,37]) and equality scales, both of which favor lower values than those of SH0ES. If this consistency holds to much higher precision, it will place strong bounds on many beyond-ΛCDM resolutions of the Hubble tension.

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