Contributions of the kaon pair from $\rho(770)$ for the three-body decays $B \to DK\bar{K}$

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We study the contributions of the kaon pair originating from the resonance $\rho(770)$ for the three-body decays $B \to DK\bar{K}$ by employing the perturbative QCD approach. According to the predictions in this work, the contributions from the intermediate state $\rho(770)^0$ are relatively small for the three-body decays such as $B^0 \to \bar{D}^0 K^+K^-$, $B_s^0 \to \bar{D}^0 K^+K^-$, and $B^+ \to D_s^+ K^+K^-$, while about 20% of the total three-body branching fraction for $B^+ \to \bar{D}^0 K^+\bar{K}^0$ could possibly come from the subprocess $\rho(770)^+ \to K^+\bar{K}^0$. We also estimate the branching fractions for $\rho(770)^\pm$ decay into the kaon pair to be about 1%, and that for the neutral $\rho(770)$ decay into K^+K^- or $K^0\bar{K}^0$ to be about 0.5%, which will be tested by future experiments.

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I. INTRODUCTION

Three-body hadronic B meson decays are much more complicated than the two-body cases partly due to the entangled resonant and nonresonant contributions, but these decay processes provide us many advantages for the study of spectroscopy, the testing of factorization, and the extraction of the CKM angles from the CP asymmetries [1]. Attempts have been made to describe the whole region of the Dalitz plot for the three-body B decays [2–4], but more attention has been focused on the resonance contributions originating from the low-energy scalar, vector, and tensor intermediate states in the subprocesses of the three-body hadronic B-meson decays within different methods, such as the QCD factorization (QCDF) [5–20] and the perturbative QCD (PQCD) approach [21–37]. In addition, there are many works within the symmetries one can find in Refs. [38–49] dedicated to the relevant decay modes.

The decays of the B meson into a charmed D meson plus kaon pair, offering rich opportunities to study the resonant components in the DK or KK system, have been measured in the past two decades [50–55]. The analysis of the $B \to D^{(*)}K^-K^{0(*)}$ decays was performed for the first time by the Belle Collaboration with the detailed investigation of the invariant mass and the polarization distributions of the $K^-K^{0(*)}$ pair [50]. In Ref. [51], the *BABAR* Collaboration reported their measurement for the process $B^- \to D_s^+K^-K^-$. In the later study [52] by Belle, a significant deviation from the simple phase-space model in the D_sK invariant mass distribution was found. In the recent works by the LHCb Collaboration, observations of the decays $B^0 \to \bar{D}^0K^+K^-$ [53], $B_s^0 \to \bar{D}^0K^+K^-$ [54], and $B^+ \to D_s^+K^+K^-$ [55], together with the measurements of corresponding branching fractions, were presented. Moreover, the studies on the $B \to D\phi(1020)$ decays, where the $\phi(1020)$ meson was reconstructed through its decay to a K^+K^- pair, were performed in Refs. [55–60] by the CLEO, *BABAR*, and LHCb Collaborations.

The vector state $\phi(1020)$ in the $K\bar{K}$ invariant-mass spectrum for the three-body hadronic B-meson decays has attracted much attention [55,61–64], but one should note that the P-wave resonance contributions of the kaon pair can also come from $\rho(770)$, $\omega(782)$ and their excited states [65–67]. Besides these, the charged $\rho(770)$ and its excited states are the only possible sources of vector intermediate states for the $K^+\bar{K}^0$ or K^-K^0 system in the three-body B decays. Although the pole mass of $\rho(770)$ is below the threshold of the kaon pair, the virtual contribution [68–70] from the Breit-Wigner (BW) [71] tail of $\rho(770)$ for the $K\bar{K}$ was found indispensable for specific processes, such as $\pi^-p(n) \to K^-K^+n(p)$ [72,73], $e^+e^- \to K^+K^-$ [74–78], and $\pi\pi \to K\bar{K}$ scattering [79]. Recently, the component $\rho(1450)^0 \to K^+K^-$ in the decays $B^\pm \to \pi^\pm K^+K^-$ was reported by LHCb to be 30% of the total fit fraction and much larger than the fit fraction 0.3% from $\phi(1020)$ [64]. The subprocess $\rho(1450)^0 \to K^+K^-$ and the

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related topics for the decays $B^{\pm} \to \pi^{\pm} K^{+} K^{-}$ have been analyzed in Refs. [16,80,81] recently, and the contribution in these decays for $K^{+}K^{-}$ from $\rho(770)^{0}$, which has been ignored in the experimental and theoretical studies, was found to be of the same order of that from $\rho(1450)^{0}$ in Ref. [80].

In the previous works [70,82–89], the resonance contributions from various intermediate states for the three-body decays $B \to D h_1 h_2$ ($h_{1,2}$ stands for pion or kaon) have been studied within the PQCD approach based on the k_T factorization theorem [90–93]. In this work, we shall focus on the contributions of the subprocesses $\rho(770) \to K\bar{K}$ for the three-body decays $B \to DK\bar{K}$, where the symbol $\bar{K}$ means the kaons K^{+} and K^{0}, and the symbol K means the kaons K^{-} and $\bar{K}^{0}$. In view of the narrow decay width of $\omega(782)$ and the gap between its pole mass and the threshold of the kaon pair, the branching fractions for the decays with the subprocess $\omega(782) \to K\bar{K}$ are small and negligible compared with the contribution from $\rho(770) \to K\bar{K}$ in the same decay mode [80]. Meanwhile, there are still disparities between the fitted coefficients of the timelike form factors for kaons from currently known experimental results [65,66,94]; we will leave the possible subprocesses with those excited states of $\rho(770)$ and $\omega(782)$ decay into $K\bar{K}$ to future study.

The rest of this paper is organized as follows: In Sec. II, we give a brief review of the PQCD framework for the concerned decay processes. The numerical results and the phenomenological analyses are given in Sec. III. The summary of this work is presented in Sec. IV. The relevant quasi-two-body decay amplitudes are collected in the Appendix.

II. FRAMEWORK

In the light-cone coordinates, the momenta p_B, p, and p_3 for the B meson, the resonance ρ, and the final state D, respectively, are chosen as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_{\rm T}), \qquad p = \frac{m_B}{\sqrt{2}}(1 - r^2, \eta, 0_{\rm T}),$$
$$p_3 = \frac{m_B}{\sqrt{2}}(r^2, 1 - \eta, 0_{\rm T}), \tag{1}$$

where m_B denotes the mass for the B meson, the variable η is defined as $\eta = s/(m_B^2 - m_D^2)$, the invariant mass square $s = p^2 = m_{K\bar{K}}^2$ for the kaon pair, and the mass ratio $r = m_D/m_B$. The momenta of the light quark in the B meson, ρ, and the D meson are denoted as k_B, k, and k_3, with

$$k_B = \left(0, x_B \frac{m_B}{\sqrt{2}}, k_{B\rm T}\right), \quad k = \left(z \frac{(1-r^2) m_B}{\sqrt{2}}, 0, k_{\rm T}\right),$$
$$k_3 = \left(0, x_3 \frac{(1-\eta) m_B}{\sqrt{2}}, k_{3\rm T}\right), \tag{2}$$

where the momentum fractions x_B, z, and x_3 run between zero and unity.

The decay amplitude $\mathcal{A}$ for the quasi-two-body processes $B \to D\rho(770) \to DK\bar{K}$ in the PQCD approach can be expressed as the convolution of a hard kernel H containing one hard gluon exchange with the relevant hadron distribution amplitudes [21,95]

$$\mathcal{A} = \Phi_B \otimes H \otimes \Phi_D \otimes \Phi_{KK}, \tag{3}$$

where the distribution amplitudes Φ_B, Φ_D, and Φ_{KK} for the initial- and final-state mesons absorb the nonperturbative dynamics. In this work, we employ the same distribution amplitudes for B and D mesons as those widely adopted in the studies of the hadronic B-meson decays in the PQCD approach; one can find their explicit expressions and parameters in Ref. [82] and the references therein.

The P-wave $K\bar{K}$ system distribution amplitudes along with the subprocesses $\rho(770) \to K\bar{K}$ are defined as [37,80]

$$\phi_{KK}^{P\text{-wave}}(z,s) = \frac{-1}{\sqrt{2N_c}}[\sqrt{s}\not\epsilon_L \phi^0(z,s) + \not\epsilon_L \not p \phi^t(z,s) + \sqrt{s}\phi^s(z,s)], \tag{4}$$

where z is the momentum fraction for the spectator quark, s is the squared invariant mass of the kaon pair, and ϵ_L and p are the longitudinal polarization vector and momentum for the resonance. The twist-2 and twist-3 distribution amplitudes ϕ^0, ϕ^s, and ϕ^t are parameterized as [80]

$$\phi^0(z,s) = \frac{3F_K^{\rho}(s)}{\sqrt{aN_c}} z(1-z)[1 + a_2^0 C_2^{3/2}(1-2z)], \tag{5}$$

$$\phi^s(z,s) = \frac{3F_K^{s}(s)}{2\sqrt{aN_c}}(1-2z)[1 + a_2^s(1 - 10z + 10z^2)], \tag{6}$$

$$\phi^t(z,s) = \frac{3F_K^{t}(s)}{2\sqrt{aN_c}}(1-2z)^2[1 + a_2^t C_2^{3/2}(1-2z)], \tag{7}$$

with the Gegenbauer polynomial $C_2^{3/2}(t) = 3(5t^2 - 1)/2$, $F_K^{s,t}(s) \approx (f_{\rho}^T/f_{\rho})F_K^{\rho}(s)$ [24], $a = 1$ for $\rho(770)^0$, and $a = 2$ for $\rho(770)^{\pm}$. In the numerical calculation, we adopt $f_{\rho} = 0.216$ GeV [96,97] and $f_{\rho}^T = 0.184$ GeV [98]. The Gegenbauer moments $a_2^{0,s,t}$ are the same as those in the distribution amplitudes for the intermediate state $\rho(770)$ in Refs. [24,80]. The vector timelike form factors for kaons are written as [12]

$$F_{K^+K^-}^u = F_{K^0\bar{K}^0}^d = F_{\rho} + 3F_{\omega}, \tag{8}$$

$$F_{K^+K^-}^d = F_{K^0\bar{K}^0}^u = -F_{\rho} + 3F_{\omega}, \tag{9}$$

$$F_{K^+K^-}^s = F_{K^0\bar{K}^0}^s = -3F_{\phi}, \tag{10}$$

TABLE I. The fitted results for the coefficients $C^K_{\rho(770)}$ of the kaon form factors.

	Fit(1) [65]	Fit(2) [65]	Fit(1) [66]	Fit(2) [66]	Model I [94]	Model II [94]
$C^K_{\rho(770)}$	1.195 ± 0.009	1.139 ± 0.010	1.138 ± 0.011	1.120 ± 0.007	1.162 ± 0.005	1.067 ± 0.041

where F_ρ, F_ω, and F_ϕ come from the definitions of the electromagnetic form factors for the charged and neutral kaon [65,66]:

$$F^{I=1}_{K^+}(s) = +\frac{1}{2}\sum_\rho c^K_\rho \mathrm{BW}_\rho(s) + \frac{1}{6}\sum_\omega c^K_\omega \mathrm{BW}_\omega(s) + \frac{1}{3}\sum_\phi c^K_\phi \mathrm{BW}_\phi(s) = F_\rho + F_\omega + F_\phi, \tag{11}$$

$$F^{I=1}_{K^0}(s) = -\frac{1}{2}\sum_\rho c^K_\rho \mathrm{BW}_\rho(s) + \frac{1}{6}\sum_\omega c^K_\omega \mathrm{BW}_\omega(s) + \frac{1}{3}\sum_\phi c^K_\phi \mathrm{BW}_\phi(s) = -F_\rho + F_\omega + F_\phi. \tag{12}$$

The symbol $\sum$ means the summation for the resonances $\rho(770)$, $\omega(782)$, or $\phi(1020)$ and their corresponding excited states. The normalization factors c^K_V for resonances determined by fitting experimental data and the corresponding BW formula can be found in Refs. [65,66,94]. It is not difficult to find that the corresponding coefficients c^K_V for $\rho(770)$, $\omega(782)$, or $\phi(1020)$ are close to each other in Refs. [65,66,94], while it can be shown that those coefficients for the excited states have significant differences by comparing the fitted parameters in Table 2 in Refs. [65,66] and Table 1 in Ref. [94]. In this work, we concern ourselves only with the $\rho(770)$ components of the vector kaon timelike form factors; the fitted values for the coefficients $C^K_{\rho(770)}$ in the kaon form factors collected from Refs. [65,66,94] have been listed in Table I. The columns "Fit(1)," "Fit(2)" and "Model I," "Model II" represent the values parameterized with different constraints in each work. Due to the closeness of the coefficients $C^K_{\rho(770)}$ in Refs. [65,66,94], we choose the value of "Fit(1)" in Ref. [65] in our numerical calculation. The resonance shape for $\rho(770)$ is described by the KS version of the BW formula [65,99]:

$$\frac{m^2_\rho}{m^2_\rho - s - i\sqrt{s}\Gamma_{\mathrm{tot}}(s)}, \tag{13}$$

where the effective s-dependent width is given by

$$\Gamma_{\mathrm{tot}}(s) \approx \Gamma_{\rho\to 2\pi}(s) = \Gamma_\rho \frac{m^2_\rho}{s}\left(\frac{\beta(s, m_\pi)}{\beta(m^2_\rho, m_\pi)}\right)^3, \tag{14}$$

with $\beta(s,m) = \sqrt{1 - 4m^2/s}$. In addition, one has the timelike form factor for $K^+\bar{K}^0$ and K^-K^0 from the relation [65,67]

$$F_{K^+\bar{K}^0}(s) = -F_{K^-K^0}(s) = 2F^{I=1}_{K^+}(s) \tag{15}$$

and should keep only the ρ resonance contributions with isospin symmetry.

For the decays $B_{(s)} \to \bar{D}_{(s)}\rho(770) \to \bar{D}_{(s)}K\bar{K}$ and the CKM-suppressed decays $B_{(s)} \to D_{(s)}\rho(770) \to D_{(s)}K\bar{K}$, the effective Hamiltonian $\mathcal{H}_{\mathrm{eff}}$ can be expressed as

$$\mathcal{H}_{\mathrm{eff}} = \begin{cases} \frac{G_F}{\sqrt{2}} V^*_{cb} V_{ud(s)}[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)], & \text{for } B_{(s)} \to \bar{D}_{(s)}\rho(770) \to \bar{D}_{(s)}K\bar{K} \text{ decays}, \\ \frac{G_F}{\sqrt{2}} V^*_{ub} V_{cd(s)}[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)], & \text{for } B_{(s)} \to D_{(s)}\rho(770) \to D_{(s)}K\bar{K} \text{ decays}, \end{cases} \tag{16}$$

where $G_F = 1.16638 \times 10^{-5}$ GeV^{-2}, V_{ij} are the CKM matrix elements, $C_{1,2}(\mu)$ represent the Wilson coefficients at the renormalization scale μ, and $O_{1,2}$ are the local four-quark operators. According to the typical Feynman diagrams for the concerned decays as shown in Figs. 1 and 2, the decay amplitudes for $B_{(s)} \to \bar{D}_{(s)}\rho(770)$ with the subprocesses $\rho(770)^0 \to K^+K^-/K^0\bar{K}^0$ and $\rho(770)^+ \to K^+\bar{K}^0$ are given as follows:

$$\mathcal{A}(B^+ \to \bar{D}^0\rho^+) = \frac{G_F}{\sqrt{2}} V^*_{cb} V_{ud}[a_1 F^{LL}_{e\rho} + C_2 M^{LL}_{e\rho} + a_2 F^{LL}_{eD} + C_1 M^{LL}_{eD}], \tag{17}$$

$$\mathcal{A}(B^0 \to D^-\rho^+) = \frac{G_F}{\sqrt{2}} V^*_{cb} V_{ud}[a_1 F^{LL}_{a\rho} + C_2 M^{LL}_{a\rho} + a_2 F^{LL}_{eD} + C_1 M^{LL}_{eD}], \tag{18}$$

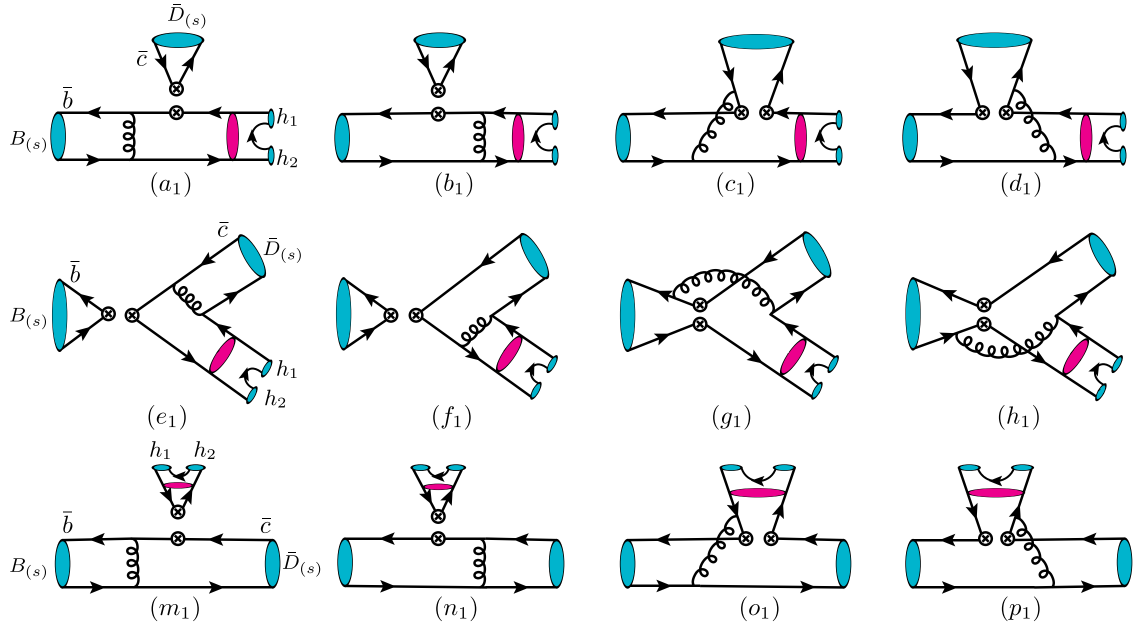

FIG. 1. The leading-order Feynman diagrams for the quasi-two-body decays $B_{(s)} \to \bar{D}_{(s)}\rho(770) \to \bar{D}_{(s)}K\bar{K}$. The label h_1h_2 denotes the kaon pair, and the pink ellipse represents the intermediate state $\rho(770)$.

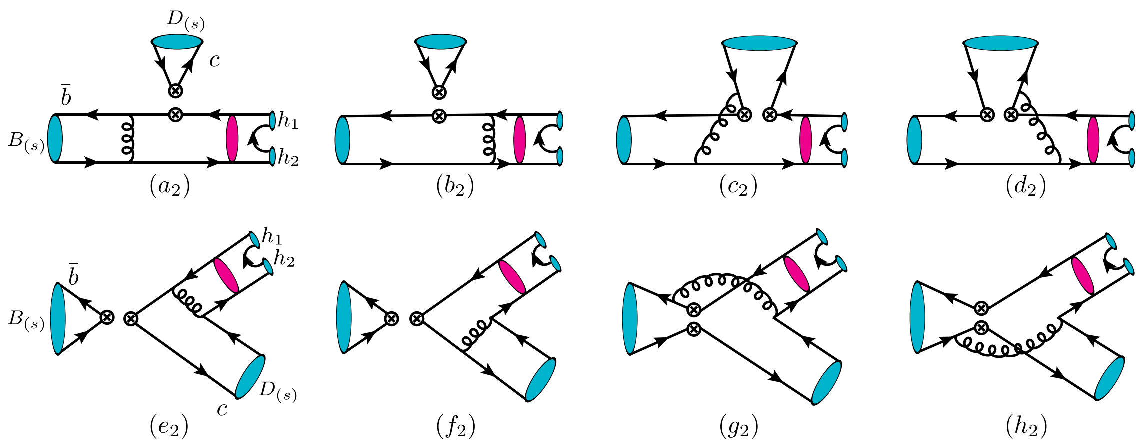

FIG. 2. The leading-order Feynman diagrams for the quasi-two-body decays $B_{(s)} \to D_{(s)}\rho(770) \to D_{(s)}K\bar{K}$. The label h_1h_2 denotes the kaon pair, and the pink ellipse represents the intermediate state $\rho(770)$.

$$\mathcal{A}(B^0 \to \bar{D^0}\rho^0) = \frac{G_F}{2} V^*_{cb} V_{ud}[a_1(-F^{LL}_{e\rho} + F^{LL}_{a\rho}) + C_2(-M^{LL}_{e\rho} + M^{LL}_{a\rho})], \tag{19}$$

$$\mathcal{A}(B^0_s \to D^-\rho^+) = \frac{G_F}{\sqrt{2}} V^*_{cb} V_{us}[a_1 F^{LL}_{a\rho} + C_2 M^{LL}_{a\rho}], \tag{20}$$

$$\mathcal{A}(B^0_s \to \bar{D^0}\rho^0) = \frac{G_F}{2} V^*_{cb} V_{us}[a_1 F^{LL}_{a\rho} + C_2 M^{LL}_{a\rho}], \tag{21}$$

$$\mathcal{A}(B^0_s \to D^-_s\rho^+) = \frac{G_F}{\sqrt{2}} V^*_{cb} V_{ud}[a_2 F^{LL}_{eD} + C_1 M^{LL}_{eD}], \tag{22}$$

while the decay amplitudes for $B_{(s)} \to D_{(s)}\rho(770)$ with the subprocesses $\rho(770)^0 \to K^+K^-/K^0\bar{K}^0$, $\rho(770)^+ \to K^+\bar{K}^0$, and $\rho(770)^- \to K^-K^0$ can be written as

$$\mathcal{A}(B^+ \to D^0\rho^+) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd}[a_1 F_{e\rho}^{LL} + C_2 M_{e\rho}^{LL} + a_2 F_{aD}^{LL} + C_1 M_{aD}^{LL}], \tag{23}$$

$$\mathcal{A}(B^+ \to D^+\rho^0) = \frac{G_F}{2} V_{ub}^* V_{cd}[a_2(F_{e\rho}^{LL} - F_{aD}^{LL}) + C_1(M_{e\rho}^{LL} - M_{aD}^{LL})], \tag{24}$$

$$\mathcal{A}(B^+ \to D_s^+\rho^0) = \frac{G_F}{2} V_{ub}^* V_{cs}[a_2 F_{e\rho}^{LL} + C_1 M_{e\rho}^{LL}], \tag{25}$$

$$\mathcal{A}(B^0 \to D^0\rho^0) = \frac{G_F}{2} V_{ub}^* V_{cd}[a_1(-F_{e\rho}^{LL} + F_{aD}^{LL}) + C_2(-M_{e\rho}^{LL} + M_{aD}^{LL})], \tag{26}$$

$$\mathcal{A}(B^0 \to D^+\rho^-) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd}[a_2 F_{e\rho}^{LL} + C_1 M_{e\rho}^{LL} + a_1 F_{aD}^{LL} + C_2 M_{aD}^{LL}], \tag{27}$$

$$\mathcal{A}(B^0 \to D_s^+\rho^-) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs}[a_2 F_{e\rho}^{LL} + C_1 M_{e\rho}^{LL}], \tag{28}$$

$$\mathcal{A}(B_s^0 \to D^0\rho^0) = \frac{G_F}{2} V_{ub}^* V_{cs}[a_1 F_{aD}^{LL} + C_2 M_{aD}^{LL}], \tag{29}$$

$$\mathcal{A}(B_s^0 \to D^+\rho^-) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs}[a_1 F_{aD}^{LL} + C_2 M_{aD}^{LL}], \tag{30}$$

with the Wilson coefficients $a_1 = C_1 + C_2/3$ and $a_2 = C_2 + C_1/3$. The explicit expressions of individual amplitude F and M for the factorizable and nonfactorizable Feynman diagrams can be found in the Appendix.

The differential branching fractions ($\mathcal{B}$) for the quasi-two-body decays $B \to D\rho(770) \to DK\bar{K}$ can be written as [7,37,80]

$$\frac{d\mathcal{B}}{d\eta} = \tau_B \frac{q^3 q_D^3}{12\pi^3 m_B^5} |\mathcal{A}|^2. \tag{31}$$

The magnitudes of the momenta for K and D in the center-of-mass frame of the kaon pair are written as

$$q = \frac{1}{2}\sqrt{s - 4m_K^2}, \tag{32}$$

$$q_D = \frac{1}{2\sqrt{s}}\sqrt{(m_B^2 - m_D^2)^2 - 2(m_B^2 + m_D^2)s + s^2}. \tag{33}$$

III. RESULTS

In the numerical calculations, the input parameters, such as masses and decay constants (in units of GeV) and B-meson lifetimes (in units of ps), are adopted as follows [100]:

$$\begin{aligned}
&m_{B^\pm} = 5.279, && m_{B^0} = 5.280, && m_{B_s^0} = 5.367, && m_{D^\pm} = 1.870, && m_{D^0} = 1.865,\\
&m_{D_s^\pm} = 1.968, && m_{K^\pm} = 0.494, && m_{K^0} = 0.498, && m_c = 1.27, && m_{\pi^\pm} = 0.140,\\
&m_{\pi^0} = 0.135, && f_B = 0.189, && f_{B_s} = 0.231, && f_D = 0.2126, && f_{D_s} = 0.2499,\\
&\tau_{B^\pm} = 1.638, && \tau_{B^0} = 1.519, && \tau_{B_s^0} = 1.515.
\end{aligned} \tag{34}$$

For the Wolfenstein parameters of the CKM mixing matrix, we use the values $A = 0.790^{+0.017}_{-0.012}$, $\lambda = 0.22650 \pm 0.00048$, $\bar{\rho} = 0.141^{+0.016}_{-0.017}$, and $\bar{\eta} = 0.357 \pm 0.011$ as listed in Ref. [100].

In Tables II and III, we list our numerical results for the branching fractions of the $B_{(s)} \to \bar{D}_{(s)}\rho(770) \to \bar{D}_{(s)}K\bar{K}$ decays and the CKM-suppressed $B_{(s)} \to D_{(s)}\rho(770) \to D_{(s)}K\bar{K}$ decays. The first error of these branching fractions comes from the uncertainty of the $B_{(s)}$ meson shape parameter $\omega_B = 0.40 \pm 0.04$ or $\omega_{B_s} = 0.50 \pm 0.05$; the second error is induced by the uncertainties of the Gegenbauer moments $a_2^0 = 0.25 \pm 0.10$, $a_2^s = 0.75 \pm 0.25$, and $a_2^t = -0.60 \pm 0.20$ in the kaon-kaon distribution amplitudes; the last one is due to $C_D = 0.5 \pm 0.1$ or $C_{D_s} = 0.4 \pm 0.1$ for the $D_{(s)}$ meson wave function. The errors that come from the uncertainties of other parameters are small and have been neglected. Since the concerned decay modes occur only through the tree-level quark diagrams, there are no direct CP asymmetries for these decays in the standard model.

The predictions for the branching fractions of the decays $B_{(s)} \to D_{(s)}\rho(770) \to D_{(s)}K\bar{K}$ in Table II are generally smaller than the corresponding results for the $B_{(s)} \to \bar{D}_{(s)}\rho(770) \to \bar{D}_{(s)}K\bar{K}$ decays in Table III due to the strong

TABLE II. The PQCD predictions of the branching fractions for the $B_{(s)} \to \bar{D}_{(s)}\rho(770) \to \bar{D}_{(s)}K\bar{K}$ decays. The decay mode with the subprocess $\rho(770)^0 \to K^0\bar{K}^0$ has the same branching fraction of its corresponding mode with $\rho(770)^0 \to K^+K^-$.

Decay modes	Unit	Quasi-two-body results
$B^+ \to \bar{D}^0\rho(770)^+ \to \bar{D}^0K^+\bar{K}^0$	(10^{-4})	$1.18^{+0.62}_{-0.40}(\omega_B)^{+0.09}_{-0.12}(a_2^0+a_2^s+a_2^t)^{+0.07}_{-0.09}(C_D)$
$B^0 \to D^-\rho(770)^+ \to D^-K^+\bar{K}^0$	(10^{-5})	$7.93^{+5.01}_{-2.93}(\omega_B)^{+0.32}_{-0.30}(a_2^0+a_2^s+a_2^t)^{+0.65}_{-0.63}(C_D)$
$B^0 \to \bar{D}^0\rho(770)^0 \to \bar{D}^0K^+K^-$	(10^{-6})	$1.07^{+0.46}_{-0.37}(\omega_B)^{+0.80}_{-0.58}(a_2^0+a_2^s+a_2^t)^{+0.01}_{-0.01}(C_D)$
$B_s^0 \to D^-\rho(770)^+ \to D^-K^+\bar{K}^0$	(10^{-8})	$4.22^{+0.58}_{-0.67}(\omega_B)^{+0.90}_{-0.65}(a_2^0+a_2^s+a_2^t)^{+0.40}_{-0.30}(C_D)$
$B_s^0 \to \bar{D}^0\rho(770)^0 \to \bar{D}^0K^+K^-$	(10^{-8})	$1.05^{+0.15}_{-0.17}(\omega_B)^{+0.23}_{-0.15}(a_2^0+a_2^s+a_2^t)^{+0.10}_{-0.07}(C_D)$
$B_s^0 \to D_s^-\rho(770)^+ \to D_s^-K^+\bar{K}^0$	(10^{-5})	$6.06^{+3.47}_{-2.06}(\omega_B)^{+0.04}_{-0.04}(a_2^0+a_2^s+a_2^t)^{+0.47}_{-0.45}(C_D)$

TABLE III. The PQCD predictions of the branching fractions for the CKM-suppressed $B_{(s)} \to D_{(s)}\rho(770) \to D_{(s)}K\bar{K}$ decays. The decay mode with the subprocess $\rho(770)^0 \to K^0\bar{K}^0$ has the same branching fraction of its corresponding decay with $\rho(770)^0 \to K^+K^-$.

Decay modes	Unit	Quasi-two-body results
$B^+ \to D^0\rho(770)^+ \to D^0K^+\bar{K}^0$	(10^{-10})	$5.27^{+1.23}_{-0.59}(\omega_B)^{+2.49}_{-1.68}(a_2^0+a_2^s+a_2^t)^{+0.33}_{-0.08}(C_D)$
$B^+ \to D^+\rho(770)^0 \to D^+K^+K^-$	(10^{-9})	$3.22^{+0.52}_{-0.45}(\omega_B)^{+0.86}_{-0.43}(a_2^0+a_2^s+a_2^t)^{+0.01}_{-0.01}(C_D)$
$B^+ \to D_s^+\rho(770)^0 \to D_s^+K^+K^-$	(10^{-8})	$6.26^{+1.69}_{-1.30}(\omega_B)^{+2.69}_{-0.92}(a_2^0+a_2^s+a_2^t)^{+0.03}_{-0.02}(C_D)$
$B^0 \to D^0\rho(770)^0 \to D^0K^+K^-$	(10^{-11})	$7.79^{+2.02}_{-1.33}(\omega_B)^{+4.63}_{-2.86}(a_2^0+a_2^s+a_2^t)^{+0.81}_{-0.61}(C_D)$
$B^0 \to D^+\rho(770)^- \to D^+K^0K^-$	(10^{-9})	$6.87^{+2.05}_{-1.60}(\omega_B)^{+3.30}_{-1.01}(a_2^0+a_2^s+a_2^t)^{+0.08}_{-0.08}(C_D)$
$B^0 \to D_s^+\rho(770)^- \to D_s^+K^0K^-$	(10^{-7})	$2.32^{+0.63}_{-0.48}(\omega_B)^{+1.00}_{-0.34}(a_2^0+a_2^s+a_2^t)^{+0.01}_{-0.01}(C_D)$
$B_s^0 \to D^0\rho(770)^0 \to D^0K^+K^-$	(10^{-9})	$1.85^{+0.36}_{-0.32}(\omega_B)^{+0.61}_{-0.45}(a_2^0+a_2^s+a_2^t)^{+0.09}_{-0.08}(C_D)$
$B_s^0 \to D^+\rho(770)^- \to D^+K^0K^-$	(10^{-9})	$7.47^{+1.49}_{-0.32}(\omega_B)^{+2.42}_{-1.83}(a_2^0+a_2^s+a_2^t)^{+0.40}_{-0.37}(C_D)$

CKM suppression factor $|\frac{V_{ub}^*V_{cd}}{V_{cb}^*V_{ud}}|^2$ or $|\frac{V_{ub}^*V_{cs}}{V_{cb}^*V_{us}}|^2$, as discussed in Ref. [82]. The central values for the PQCD-predicted branching fractions of the decays $B^0 \to \bar{D}^0\rho(770)^0 \to \bar{D}^0K^+K^-$ and $B_s^0 \to \bar{D}^0\rho(770)^0 \to \bar{D}^0K^+K^-$ are 0.18% and 0.019% of the experimental measurements $\mathcal{B}(B^0 \to \bar{D}^0K^+K^-) = (5.9 \pm 0.5) \times 10^{-4}$ and $\mathcal{B}(B_s^0 \to \bar{D}^0K^+K^-) = (5.5 \pm 0.8) \times 10^{-5}$, respectively, in the *Review of Particle Physics* (Ref. [100]), which have been averaged from the results in Refs. [53,54] presented by LHCb. However, with the branching ratio $\mathcal{B}(B^+ \to \bar{D}^0K^+\bar{K}^0) = (5.5 \pm 1.4 \pm 0.8) \times 10^{-4}$ presented by the Belle Collaboration [50], one has a sizable percent at 21.45% of the total branching fraction for the quasi-two-body decay $B^+ \to \bar{D}^0\rho(770)^+ \to \bar{D}^0K^+\bar{K}^0$. This tells us that the contributions from $\rho(770)^\pm \to K\bar{K}$ could be considerably large in the relevant three-body B-meson decays.

In Ref. [55], LHCb presented the first observation of the decay $B^+ \to D_s^+K^+K^-$, and the branching fraction was determined to be $(7.1 \pm 0.5 \pm 0.6 \pm 0.7) \times 10^{-6}$. Utilizing our prediction $\mathcal{B}(B^+ \to D_s^+\rho(770)^0 \to D_s^+K^+K^-) = (6.26^{+3.18}_{-1.59}) \times 10^{-8}$, where the individual errors have been added in quadrature, we obtain the ratio $\frac{\mathcal{B}(B^+\to D_s^+\rho(770)^0\to D_s^+K^+K^-)}{\mathcal{B}(B^+\to D_s^+K^+K^-)} = 0.88^{+0.47}_{-0.26}\%$, which is quite small, as expected. In addition, LHCb also gave a branching fraction for the $B^+ \to D_s^+\phi(1020)$ decay of $(1.2^{+1.6}_{-1.4} \pm 0.8 \pm 0.1) \times 10^{-7}$ and set an upper limit as $4.9(4.2) \times 10^{-7}$ at the 95% (90%) confidence level, which is roughly 1 order smaller than their previous result in Ref. [58]. By adopting $\mathcal{B}(\phi(1020) \to K^+K^-) = 0.492$ [100] and the relation between the quasi-body decay and the corresponding two-body decay

$$\mathcal{B}(B \to DR \to Dh_1h_2) \approx \mathcal{B}(B \to DR) \cdot \mathcal{B}(R \to h_1h_2), \quad (35)$$

we find that $\mathcal{B}(B^+ \to D_s^+\rho(770)^0 \to D_s^+K^+K^-)$ predicted in this work has the same magnitude as the branching ratio for $B^+ \to D_s^+\phi(1020) \to D_s^+K^+K^-$ measured by LHCb within large uncertainties, while $\mathcal{B}(B^+ \to D_s^+\phi(1020) \to D_s^+K^+K^-)$ was predicted to be $(1.53 \pm 0.23) \times 10^{-7}$ within the PQCD approach in Ref. [88].

In Fig. 3, we show the differential branching fraction of the decay mode $\mathcal{B}(B^+ \to D_s^+\rho(770)^0 \to D_s^+K^+K^-)$ with the invariant mass in the range of $[2m_K, 3\ \text{GeV}]$. The bump in the curve is caused by the strong depression of the phase-space factors q and q_D in Eqs. (32) and (33) near the K^+K^- threshold. This depression near the threshold, along with the similar mass between $K^\pm$ and K^0, $\bar{K}^0$, causes the decay channel with the subprocess $\rho(770)^0 \to K^0\bar{K}^0$ to have the

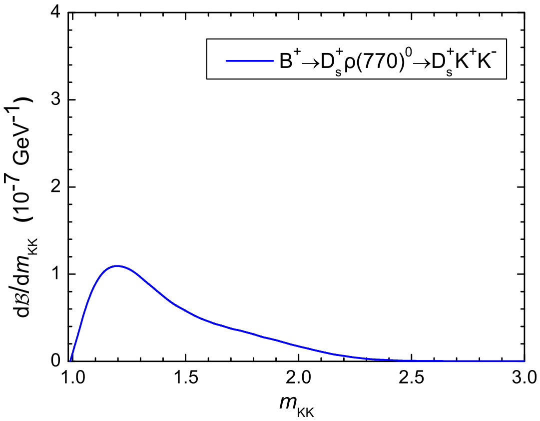

FIG. 3. The PQCD prediction for the differential branching ratio of the decay mode $B^+ \to D_s^+\rho(770)^0 \to D_s^+K^+K^-$, with the invariant mass ranges from $2m_K$ to 3 GeV.

same branching fraction as the corresponding decay mode with the subprocess $\rho(770)^0 \to K^+K^-$.

In principle, the applicability of the PQCD calculations in the high $K\bar{K}$ invariant mass region is apt to deteriorate because of the small energy release. Fortunately, the evolution of the kaon form factor $F_K(s)$ in the decay amplitude $\mathcal{A}$ will naturally suppress the resonant contribution from the region where the invariant mass of the kaon pair is far away from the pole mass of the resonant state $\rho(770)$. Taking the decay $B^+ \to D_s^+\rho(770)^0 \to D_s^+K^+K^-$ as an example, it is easy to check that the main portion of its branching ratio lies in the region around 1.2 GeV, as shown in Fig. 3. Numerically, the central values of its branching ratio are calculated as 4.08×10^{-8} and 5.85×10^{-8} after making the integration over the ranges of $m_{K\bar{K}}$ in [$2m_K$, 1.5 GeV] and [$2m_K$, 2 GeV], respectively, which amount to 65.18% and 93.45% of the value 6.26×10^{-8} accumulated in the mass range from $2m_K$ to $m_B - m_D$. Besides this, a ratio 91.04% for $B^0 \to D^+\rho(770)^- \to D^+K^0K^-$ can also be obtained by calculating the corresponding branching ratios in the ranges [$2m_K$, 2 GeV] and [$2m_K$, $m_B - m_D$]. These indicate that the PQCD predictions for the present processes are reasonable when considering that the current results still have large uncertainties.

TABLE IV. The comparison of the available experimental measurements for the branching fractions of the $B \to D\rho(770)$ decays and the PQCD predictions for the branching ratios of the relevant decay modes with the subprocess $\rho(770) \to K\bar{K}$.

Decay modes	$\mathcal{B}_{\rm exp}$ [100]	$\mathcal{B}_{th}$
$B^+ \to \bar{D}^0\rho(770)^+$	$(1.34\pm0.18)\times10^{-2}$	$(1.18^{+0.63}_{-0.43})\times10^{-4}$
$B^+ \to D_s^+\rho(770)^0$	$<3.0\times10^{-4}$	$(6.26^{+3.18}_{-1.59})\times10^{-8}$
$B^0 \to D^-\rho(770)^+$	$(7.6\pm1.2)\times10^{-3}$	$(7.93^{+5.06}_{-3.01})\times10^{-5}$
$B^0 \to D_s^+\rho(770)^-$	$<2.4\times10^{-5}$	$(2.32^{+1.80}_{-1.15})\times10^{-7}$
$B^0 \to \bar{D}^0\rho(770)^0$	$(3.21\pm0.21)\times10^{-4}$	$(1.07^{+0.92}_{-0.69})\times10^{-6}$
$B_s^0 \to D_s^-\rho(770)^+$	$(6.9\pm1.4)\times10^{-3}$	$(6.06^{+3.50}_{-2.11})\times10^{-5}$

For comparison, we list the available experimental measurements for the branching fractions of the two-body $B \to D\rho(770)$ decays from the *Review of Particle Physics* [100] in Table IV, together with the PQCD predictions for the branching ratios of the relevant decay modes with the subprocess $\rho(770) \to K\bar{K}$ shown in Tables II and III. The ratios between the relevant branching fractions are

$$\begin{aligned}
R_1 &= \frac{\mathcal{B}(B^+ \to \bar{D}^0\rho(770)^+ \to \bar{D}^0K^+\bar{K}^0)}{\mathcal{B}(B^+ \to \bar{D}^0\rho(770)^+)} = 0.0088^{+0.0048}_{-0.0034},\\
R_2 &= \frac{\mathcal{B}(B^0 \to D^-\rho(770)^+ \to D^-K^+\bar{K}^0)}{\mathcal{B}(B^0 \to D^-\rho(770)^+)} = 0.010^{+0.007}_{-0.004},\\
R_3 &= \frac{\mathcal{B}(B^0 \to \bar{D}^0\rho(770)^0 \to \bar{D}^0K^+K^-)}{\mathcal{B}(B^0 \to \bar{D}^0\rho(770)^0)} = 0.0033^{+0.0029}_{-0.0022},\\
R_4 &= \frac{\mathcal{B}(B_s^0 \to D_s^-\rho(770)^+ \to D_s^-K^+\bar{K}^0)}{\mathcal{B}(B_s^0 \to D_s^-\rho(770)^+)} = 0.0088^{+0.0054}_{-0.0035}.
\end{aligned} \tag{36}$$

Due to the suppression from the phase space, the predicted branching fractions of the quasi-two-body decays $B^+ \to \bar{D}^0\rho(770)^+ \to \bar{D}^0K^+\bar{K}^0$, $B^0 \to D^-\rho(770)^+ \to D^-K^+\bar{K}^0$, and $B_s^0 \to D_s^-\rho(770)^+ \to D_s^-K^+\bar{K}^0$ are around 0.9% of the experimental data for the corresponding two-body cases, while a ratio near 0.4% for $B^0 \to \bar{D}^0\rho(770)^0 \to \bar{D}^0K^+K^-$ is found.

With the relations [37,65]

$$|c_{\rho^0}| \approx \frac{f_{\rho(770)}|g_{\rho(770)^0K^+K^-}|}{\sqrt{2}m_{\rho(770)}}, \quad |c_{\rho^+}| \approx \frac{f_{\rho(770)}|g_{\rho(770)^+K^+\bar{K}^0}|}{m_{\rho(770)}},$$
$$|c_{\rho^-}| \approx \frac{f_{\rho(770)}|g_{\rho(770)^-K^0K^-}|}{m_{\rho(770)}} \tag{37}$$

and Eq. (15), one can obtain the relation between the strong couplings $|g_{\rho(770)^+K^+\bar{K}^0}| = |g_{\rho(770)^-K^0K^-}| \approx \sqrt{2}|g_{\rho(770)^0K^+K^-}|$, which leads to $\Gamma_{\rho(770)^+K^+\bar{K}^0} = \Gamma_{\rho(770)^-K^0K^-} \approx 2\Gamma_{\rho(770)^0K^+K^-}$. When considering $\Gamma_{\rho(770)^\pm\pi^\pm\pi^0} = \Gamma_{\rho(770)^0\pi^+\pi^-}$ and the relation in Eq. (35), we have

$$\begin{aligned}
&\frac{\mathcal{B}(B \to D\rho(770)^+ \to DK^+\bar{K}^0)}{\mathcal{B}(B \to D\rho(770)^+ \to \pi^+\pi^0)}\\
&\quad= \frac{\mathcal{B}(B \to D\rho(770)^- \to DK^-K^0)}{\mathcal{B}(B \to D\rho(770)^- \to \pi^-\pi^0)}\\
&\quad\approx 2\frac{\mathcal{B}(B \to D\rho(770)^0 \to DK^+K^-)}{\mathcal{B}(B \to D\rho(770)^0 \to \pi^+\pi^-)}.
\end{aligned} \tag{38}$$

Obviously, the above theoretical analysis is consistent with the numerical results based on the fact that most of the experimental data were measured by assuming

$\mathcal{B}(\rho(770)\to\pi\pi)\approx 100\%$. For the branching fractions of decays $B^+\to D_s^+\rho(770)^0$ and $B^0\to D_s^+\rho(770)^-$, no specific values but the upper limits of 3.0×10^{-4} and 2.4×10^{-5} at a 90% confidence level were given by the CLEO and *BABAR* Collaborations [101,102]. Utilizing the PQCD predictions $\mathcal{B}(B^+\to D_s^+\rho(770)^0)=1.52\times10^{-5}$ and $\mathcal{B}(B^0\to D_s^+\rho(770)^-)=2.82\times10^{-5}$ taken from our previous work in Ref. [82], and $\mathcal{B}(B^+\to D_s^+\rho^0\to D_s^+K^+K^-)=6.26\times10^{-8}$ and $\mathcal{B}(B^0\to D_s^+\rho(770)^-\to D_s^+K^0K^-)=2.32\times10^{-7}$ in this work, ratios around 0.5% and 1%, respectively, can be obtained. Also, from the comparison of the results in Ref. [82] and this work, we can find the similar ratios for other decay channels. Thus, we estimate the branching fractions $\mathcal{B}(\rho(770)^+\to K^+\bar{K}^0)=\mathcal{B}(\rho(770)^-\to K^-K^0)\approx1\%$ and $\mathcal{B}(\rho(770)^0\to K^+K^-)=\mathcal{B}(\rho(770)^0\to K^0\bar{K}^0)\approx0.5\%$. In consideration of the large uncertainties, more precise data from LHCb and Belle-II are expected to test our predictions.

IV. SUMMARY

In this work, we analyzed the contributions for the kaon pair originating from the intermediate state $\rho(770)$ for the three-body decays $B\to DK\bar{K}$ in the PQCD approach. By the numerical evaluations and the phenomenological analyses, we found the following points:

(i) The decay mode of $B\to D\rho(770)^0$ with the intermediate-state $\rho(770)^0$ decays into $K^0\bar{K}^0$ has the same branching fraction as the corresponding mode with the subprocess $\rho(770)^0\to K^+K^-$.

(ii) Our predictions for the corresponding branching fractions of the decay modes with the subprocess $\rho(770)^0\to K^+K^-$ are much less than the measured branching fractions for the three-body decays $B^0\to\bar{D}^0K^+K^-$, $B_s^0\to\bar{D}^0K^+K^-$, and $B^+\to D_s^+K^+K^-$, while the percentage at about 20% of the total three-body branching fraction for the quasi-two-body decay $B^+\to\bar{D}^0\rho(770)^+\to\bar{D}^0K^+\bar{K}^0$ was predicted in this work.

(iii) The branching ratio for the decay $B^+\to D_s^+\rho(770)^0\to D_s^+K^+K^-$ predicted in this work is of the same magnitude as that for $B^+\to D_s^+\phi(1020)^0\to D_s^+K^+K^-$ measured by LHCb within large uncertainties.

(iv) We estimate the branching fractions $\mathcal{B}(\rho(770)^+\to K^+\bar{K}^0)=\mathcal{B}(\rho(770)^-\to K^-K^0)\approx1\%$ and $\mathcal{B}(\rho(770)^0\to K^+K^-)=\mathcal{B}(\rho(770)^0\to K^0\bar{K}^0)\approx0.5\%$ by comparing the available experimental measurements and the PQCD predictions for the branching fractions of the $B\to D\rho(770)$ decays with the PQCD predicted branching ratios of the relevant decay modes $B\to D\rho(770)\to DK\bar{K}$ in this work.

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APPENDIX: DECAY AMPLITUDES

The expressions for amplitudes from diagrams (a_1–d_1) of Fig. 1 are written as

$$F_{e\rho}^{LL}=8\pi C_F m_B^4 f_D\int dx_B dz\int b_B db_B b db\phi_B\left\{\left[[-\bar{\eta}(1+z)+r^2(1+2\bar{\eta})z-r^4\bar{\eta}z]\phi_0\right.\right.$$
$$\left.-\sqrt{\eta(1-r^2)}[\bar{\eta}(1-2(1-r^2)z)(\phi_s+\phi_t)+r^2(\phi_s-\phi_t)]\right]E_e(t_a)h_a(x_B,z,b,b_B)S_t(z)$$
$$\left.-\left[(1-r^2)[\eta\bar{\eta}+r^2(x_B-\eta)]\phi_0+2\sqrt{\eta(1-r^2)}[\bar{\eta}-r^2(1-2\eta+x_B)]\phi_s\right]E_e(t_b)h_b(x_B,z,b_B,b)S_t(|x_B-\eta|)\right\},\quad\text{(A1)}$$

$$M_{e\rho}^{LL}=32\pi C_F m_B^4/\sqrt{6}\int dx_B dz dx_3\int b_B db_B b_3 db_3\phi_B\phi_D$$
$$\times\left\{\left[[(r^2(r^2-\eta)-\bar{\eta})(\bar{\eta}(1-x_3)-x_B-\eta z)+r(r_c(\bar{\eta}-r^2)+\eta r(\bar{\eta}+r^2))]\phi_0\right.\right.$$
$$\left.+\sqrt{\eta(1-r^2)}[-r^2(x_B+\bar{\eta}x_3)(\phi_s+\phi_t)+\bar{\eta}(1-r^2)z(\phi_s-\phi_t)+2r(\bar{\eta}r-2r_c)\phi_s]\right]E_n(t_c)h_c(x_B,z,x_3,b_B,b_3)$$
$$+\left[(r^2-\bar{\eta})(x_B-(1-r^2)z-\bar{\eta}x_3)\phi_0+\sqrt{\eta(1-r^2)}[r^2(x_B-\bar{\eta}x_3)(\phi_s-\phi_t)-\bar{\eta}(1-r^2)z(\phi_s+\phi_t)]\right]$$
$$\left.\times E_n(t_d)h_d(x_B,z,x_3,b_B,b_3)\right\}.\quad\text{(A2)}$$

The expressions for amplitudes from diagrams (e_1–h_1) of Fig. 1 are written as

$$F^{LL}_{a\rho}=8\pi C_F m_B^4 f_B\int dx_3dz\int b_3db_3bdb\phi_D\Big\{\Big[[-\bar{\eta}(1-r^2)^2z+(1-2rr_c)(\bar{\eta}-r^2)]\phi_0 + \sqrt{\eta(1-r^2)}[r_c\bar{\eta}(\phi_s+\phi_t)+r(2(1-r^2)z+rr_c)(\phi_s-\phi_t)-4r\phi_s]\Big]E_a(t_e)h_e(z,x_3,b,b_3)S_t(z) + \Big[[(r^2-1)((\bar{\eta}-r^2)\eta+\bar{\eta}^2x_3)]\phi_0+2r\sqrt{\eta(1-r^2)}[\bar{\eta}(1+x_3)+2\eta-r^2]\phi_s\Big]E_a(t_f)h_f(z,x_3,b_3,b)S_t(|\eta(x_3-1)-x_3|)\Big\}, \tag{A3}$$

$$M^{LL}_{a\rho}=32\pi C_F m_B^4/\sqrt{6}\int dx_Bdzdx_3\int b_Bdb_Bbdb\phi_B\phi_D\Big\{\Big[[\eta\bar{\eta}+r^2(r^2-1)+(\bar{\eta}+r^2(\eta-r^2))(x_B+\eta z+\bar{\eta}x_3)]\phi_0 + r\sqrt{\eta(1-r^2)}[((1-z)r^2+z)(\phi_s+\phi_t)+(-x_B+\bar{\eta}(1-x_3))(\phi_s-\phi_t)-4\phi_s]\Big]E_n(t_g)h_g(x_B,z,x_3,b,b_B) + \Big[[(\bar{\eta}-r^2)(-\bar{\eta}(1-z)+(-\eta+x_B+\bar{\eta}(1-z-x_3))r^2)]\phi_0+r\sqrt{\eta(1-r^2)} \times[(-x_B-\bar{\eta}(1-x_3))(\phi_s+\phi_t)-(z+(1-z)r^2)(\phi_s-\phi_t)+2\phi_s]\Big]E_n(t_h)h_h(x_B,z,x_3,b,b_B)\Big\}. \tag{A4}$$

The expressions for amplitudes from diagrams (m_1–p_1) of Fig. 1 are written as

$$F^{LL}_{eD}=8\pi C_F m_B^4F_K\int dx_Bdx_3\int b_Bdb_Bb_3db_3\phi_B\phi_D\{(1+r)[-\bar{\eta}-x_3+\eta^2(r-1)x_3+2\eta(r-1)^2x_3 + r(-2rx_3+r+3x_3)]E_e(t_m)h_m(x_B,x_3,b_3,b_B)S_t(x_3)+[\bar{\eta}(r_c+\eta x_B)+2r(-\eta x_B-\bar{\eta}(1+r_c)) + r^2(\bar{\eta}^2-r_c)+2r^3(1+r_c)-\bar{\eta}r^4)]E_e(t_n)h_n(x_B,x_3,b_B,b_3)S_t(x_B)\}, \tag{A5}$$

$$M^{LL}_{eD}=32\pi C_F m_B^4/\sqrt{6}\int dx_Bdzdx_3\int b_Bdb_Bbdb\phi_B\phi_D\phi_0\{[-\bar{\eta}^2(1-x_B-z)+rx_3+\eta r(x_B+z-x_3)+\bar{\eta}r^2 \times(\eta(z+x_3-2)-x_B-2z-x_3+2)-r^3(\eta z+\bar{\eta}x_3)-r^4(-\bar{\eta}(z+x_3)-2\eta+1)]E_n(t_o)h_o(x_B,z,x_3,b_B,b) + [(r-1)(\bar{\eta}+r)(x_B+(r^2-1)z)+\bar{\eta}(\bar{\eta}-(1+r-r^2)r)x_3]E_n(t_p)h_p(x_B,z,x_3,b_B,b)\}. \tag{A6}$$

The expressions for amplitudes from diagrams (a_2–d_2) of Fig. 2 are written as

$$F^{LL}_{e\rho}=8\pi C_F m_B^4 f_D\int dx_Bdz\int b_Bdb_Bbdb\phi_B\Big\{\Big[[-\bar{\eta}(1+z)+r^2(1+2\bar{\eta}z)-r^4\bar{\eta}z]\phi_0-\sqrt{\eta(1-r^2)} \times[\bar{\eta}(1-2(1-r^2)z)(\phi_s+\phi_t)+r^2(\phi_s-\phi_t)]\Big]E_e(t_a)h_a(x_B,z,b,b_B)S_t(z)-\Big[(1-r^2)[\eta\bar{\eta}+r^2(x_B-\eta)]\phi_0 + 2\sqrt{\eta(1-r^2)}[\bar{\eta}-r^2(1-2\eta+x_B)]\phi_s\Big]E_e(t_b)h_b(x_B,z,b_B,b)S_t(|x_B-\eta|)\Big\}, \tag{A7}$$

$$M^{LL}_{e\rho}=32\pi C_F m_B^4/\sqrt{6}\int dx_Bdzdx_3\int b_Bdb_Bb_3db_3\phi_B\phi_D\Big\{\Big[[(\bar{\eta}+r^2)(1-r^2)(x_B+\eta z-\bar{\eta}x_3)]\phi_0 + \sqrt{\eta(1-r^2)}[r^2(-x_B+\bar{\eta}x_3)(\phi_s+\phi_t)+\bar{\eta}(1-r^2)z(\phi_s-\phi_t)]\Big]E_n(t_c)h_c(x_B,z,x_3,b_B,b_3) + \Big[(-\bar{\eta}+r^2)[x_B-z+r(r(z-1)+r_c)-\bar{\eta}(1-x_3)]\phi_0+\sqrt{\eta(1-r^2)}[-\bar{\eta}(1-r^2)z(\phi_s+\phi_t) + r^2(\bar{\eta}x_3+x_B)(\phi_s-\phi_t)+2(2rr_c-\bar{\eta}r^2)\phi_s]\Big]E_n(t_d)h_d(x_B,z,x_3,b_B,b_3)\Big\}. \tag{A8}$$

The expressions for amplitudes from diagrams (e_2–h_2) of Fig. 2 are written as

$$F_{aD}^{LL} = 8\pi C_F m_B^4 f_B \int dx_3 dz \int b_3 db_3 b db \phi_D \Big\{ \Big[(r^2-1)[\eta(-\bar{\eta}+r^2) - \bar{\eta}^2 x_3]\phi_0 + 2r\sqrt{\eta(1-r^2)}[1+\eta+\bar{\eta}x_3 - r^2]\phi_s \Big] \times E_a(t_e) h_e(z,x_3,b_3,b) S_t(x_3) - \Big[[\bar{\eta}(r^4(z-1) + r^2(\bar{\eta}-2z) + z - 2rr_c) + 2r^3 r_c]\phi_0 + \sqrt{\eta(1-r^2)}[r(2z+2r^2(1-z) - rr_c)(\phi_s+\phi_t) + \bar{\eta}(2r-r_c)(\phi_s-\phi_t)] \Big] E_a(t_f) h_f(z,x_3,b,b_3) S_t(z) \Big\}, \tag{A9}$$

$$M_{aD}^{LL} = 32\pi C_F m_B^4/\sqrt{6} \int dx_B dz dx_3 \int b_B db_B b db \phi_B \phi_D \Big\{ \Big[(-\bar{\eta}+r^2)[\bar{\eta}(r^2(z-x_3) - x_B - z) + r^2 - \eta]\phi_0 + r\sqrt{\eta(1-r^2)}[(z(1-r^2)+x_B)(\phi_s+\phi_t) + \bar{\eta}x_3(\phi_s-\phi_t) + 2\phi_s] \Big] E_n(t_g) h_g(x_B,z,x_3,b,b_B) - \Big[(\bar{\eta}+r^2)[(1-r^2)(\bar{\eta}x_3 - \eta z) + x_B\eta]\phi_0 + r\sqrt{\eta(1-r^2)}[\bar{\eta}x_3(\phi_s+\phi_t) + ((1-r^2)z - x_B)(\phi_s-\phi_t)] \Big] \times E_n(t_h) h_h(x_B,z,x_3,b,b_B) \Big\}. \tag{A10}$$

In the formulas above, the symbol $\bar{\eta} = 1-\eta$, the mass ratio $r = \frac{m_D}{m_B}$, and $r_c = \frac{m_c}{m_B}$ are adopted. The values b_B, b, and b_3 are the conjugate variables of the transverse momenta of the light quarks in the B meson, resonance $\rho(770)$, and D meson. The explicit expressions for the hard functions h_i, the evolution factors $E(t_i)$, and the threshold resummation factor S_t can be found in Ref. [82].

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