Pion gravitational form factors in a relativistic theory of composite particles

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We extend our relativistic theory of electroweak properties of composite systems to describe simultaneously the gravitational form factors of hadrons. The approach is based on a version of the instant-form relativistic quantum mechanics and makes use of the modified impulse approximation. We exploit the general method of the relativistic invariant parametrizaton of local operators to write the energy-momentum tensor of a particle with an arbitrary spin. We use the obtained results to calculate the gravitational form factors of the pion assuming pointlike constituent quarks. All but one parameters of our first-principle model were fixed previously in works on electromagnetic form factors. The only free parameter, D_q, is a characteristic of the gravitational form factor of a constituent quark. The derived form factors of the pion satisfy the constraints given by the general principles of the quantum field theory of hadron structure. The calculated gravitational form factors and gravitational mean-square radius are in a reasonable agreement with known results.

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I. INTRODUCTION

The probably most fundamental information about a particle is contained in the matrix elements of its energy-momentum tensor (EMT). So, it is clear that the gravitational form factors (GFFs) of hadrons that enter the EMT matrix elements and their dependence on the square of the momentum transfer t are in the focus of investigations (see, e.g., [1–5] and references therein). These form factors contain the information about the distribution of mass, spin, and internal forces inside the hadron. These forces are connected with the additional global characteristic of a particle, the so-called D term of the EMT matrix. The study [6] of the structure of the elements of the EMT matrix and of their Lorentz-covariant decomposition in terms of the form factors gives the limitations at $t \to 0$ that mean that the form factor $D(t)$ is not constrained (not even at $t = 0$) by general principles and the value $D = D(0)$ is therefore not known for (nearly) any particle (see also [7,8]).

At present one obtains the information about the GFF mainly from the hard-exclusive processes described in terms of unpolarized generalized parton distribution (GPD). Particularly, in Ref. [9] (see also Ref. [10]) it is shown that GPD, derived from processes, gives the information on the space distribution of strong forces that act on quarks and gluons inside hadrons. The link of gravitational form factors with GPD gives a possibility of obtaining the data on these form factors using the hard-exclusive processes. The first results for nucleon GFFs were obtained through the analysis of JLab data [5]. The data for the pion form factors were extracted from the experiment of the collaboration Belle at KEKB [11,12].

It is worth noting that the model-independent extraction of GPD from the experimental data is a difficult long-term problem. So, today the theoretical estimation of the GFF, including D-term, is usually obtained in the framework of different model approaches. We mention here only the publications that are strictly related to the present paper while more general information can be found, for example, in the reviews [1–4].

The pion electromagnetic and gravitational form factors are obtained from GPD in the Nambu–Jona-Lasino (NJL) model [13,14]. The authors show, in particular, that the light-cone mass radii for the pion are almost twice smaller than the light-cone charge radii (see also [15]). The calculated light-cone mass radius agrees with the value obtained through a phenomenological extraction from KEKB data [12]. Note that the NJL model as well as the model of our approach contains gluons only implicitly.

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Different approaches based on various forms of dispersion relations (see, e.g., [16,17] and references therein), that is on quite general principles of the quantum-field theory, can be considered as "maximal model-independent" approaches. The D term was calculated using unsubtracted t-channel dispersion relations for the deeply virtual Compton scattering amplitudes [16].

In connection with the first experimental results for the GFFs of nucleons [5], the gravitational characteristics of these particles were calculated. The dependence of EMT on the long-ranged electromagnetic interaction was investigated in Ref. [18]. Using a simple model it was shown that in the case of the long-ranged forces in the proton one needs a sophisticated theory of the D-term construction. There is a possibility that the D-term is ill defined and even singular. They propose the exploiting of the fixed-t dispersion relations for deep virtual Compton scattering as in Ref. [10] to avoid this difficulty. It is of interest that in the free-field model, as was shown in Refs. [19,20], the D-term for the fermion with spin $1/2$ is of a dynamical nature and vanishes for the free fermion. The interaction inclusion gives rise to the D-term of a fermion with an internal structure, the nucleon. Recently [21] the results of [3] have been extended to the different frames where the nucleon has a nonvanishing average momentum.

The authors of [22] use the Skyrme model which respects the chiral symmetry and provides a practical realization of the large-N_c picture of baryons described as solitons of mesonic fields. The EMT form factors are consistently described (see, e.g., [23] and references therein) in the bag model in the large-N_c limit. It is important to mention the recent results for the GFFs as obtaibed from the lattice QCD (see, e.g., [24,25] and the references therein), and the study using an approach based on the light-cone sum rules [26], and in the light-cone quark model [27].

Different theoretical approaches to the GFFs of hadrons give different results and the absence of the model-independent extracted data makes it impossible to choose between them. We believe that in such a situation, a theory which is intrinsically self-consistent theory and describes as large set of the physical characteristics and systems as possible is welcome. This motivates the extension of the relativistic theory of the electromagnetic properties of composite systems developed previously to the calculation of the GFF of the pion.

The goal of the present paper is twofold. First, we extend our relativistic model of the electromagnetic structure of composite systems to include their gravitational characteristics. Second, we derive the pion GFFs using our previous calculations of electroweak properties of hadrons.

The model [28,29] was successfully used for various composite two-particle systems, namely, the deuteron [30], the pion [31–34], the ρ meson [35–37] and the kaon [38]. This model had predicted, with surprising accuracy, the values of the form factor $F_\pi(Q^2)$, which were measured later in JLab experiments (see the discussion in Ref. [33]): all new measurements followed the predicted curve. Another advantage of the approach is matching with the QCD predictions in the ultraviolet limit, when constituent-quark masses are switched off, as expected at high energies. The model reproduces correctly not only the functional form of the QCD asymptotics, but also the numerical coefficient; see Refs. [32,34,38] for details. The method allows for an analytic continuation of the pion electromagnetic form factor from the spacelike region to the complex plane of momentum transfers and gives good results for the pion form factor in the timelike region [39].

Now we show that besides electroweak properties of composite systems, our approach can be used to calculate their gravitational characteristics. Even in a simple version of our approach (with the pointlike quarks and the two-particle wave functions of the harmonic oscillator) the results agree well with other calculations and with scarce measurements. The only free parameter that we add to the model is the constituent-quark $D(0) = D_q$. This parameter is constrained from the pion mean-square radius. Despite uncertainties in the latter, D_q is fixed to a narrow interval which makes it possible to predict the GFFs at nonzero momentum transfers. Using the obtained results we calculate the values of the static gravitational characteristics of the pion and obtain A and D form factors as functions of momentum transfer up to $1\ \mathrm{GeV}^2$. Note that the new parameter is not used in the calculation of the A term, its value is a direct prediction of our previous approach. The form factors calculated through our nonperturbative method satisfy all the constraints given by the general principles of the quantum-field theory of hadron structure [6–8].

The approach that we use is a particular variant of the theory based on the classical paper by P. Dirac [40], so-called relativistic Hamiltonian dynamics or relativistic quantum mechanics (RQM). It can be formulated in different ways or in different forms of dynamics. The main forms are the instant form, point form and light-front dynamics. Here we are dealing with instant-form (IF) RQM. The properties of different forms of RQM dynamics are discussed in the reviews [41–44]. Today the theory is largely used as a basis of the nonperturbative approaches to the particles structure.

The presentation of the matrix elements of EMT in terms of form factors, the invariant parametrization, is an important part of model approaches to gravitational characteristics of particles. The majority of authors use the parametrization given by Pagels [45] (see also [46]). The parametrization [45] was constructed in an almost phenomenological way using an analogy with the investigations performed in connection with the self-stress of the electron. It is valid for the simplest cases of spin 0 and $1/2$ and cannot be directly extended to systems with higher spins, when more general special methods are needed (see [4,47]). We use the general method of the relativistic invariant parametrization of the matrix elements of the local operators established in Ref. [47]. The parametrization is written in the canonical basis, so it is natural to call it the canonical parametrization. Certainly, this method of obtaining a parametrization is not unique (see the discussion in Ref. [4]). With the use of the method [47] the invariant parametrization was obtained for systems with arbitrary spin in the cases with diagonal [28,29,48] and nondiagonal [49] total angular momenta. In the present paper we give the general formulas, although for the actual calculation we use only the form factors for systems of spin 0(the pion), spin $1/2$ (the constituent quark) and for the system of two free constituent quarks. The form factors obtained by the canonical parametrization can be expressed in terms of the largely accepted one-particle GFFs [3,45].

As an important part, our approach contains the construction of the EMT matrix element of the system of two free particles with spins $1/2$, momenta $\vec{p}_1, \vec{p}_2$ and spin projections m_1, m_2, that is the two-particle system having quantum numbers of the pion. We construct the EMT in the basis with separated center-of-mass motion [50] $|\vec{P}, \sqrt{s}\rangle$, where $P = p_1 + p_2, s = P^2$ is the invariant mass squared and P, p_1 and p_2 are 4-vectors. We refer to the corresponding form factors as to the free two-particle GFFs. These form factors are the functions of the invariant masses of the two-particle system in the initial and the final states and depend on the momentum-transfer square as a parameter. They are the regular generalized functions, the distributions corresponding to the functionals given by the two-dimensional integrals over the invariant masses [28].

To construct the pion GFFs we use a modified impulse approximation (MIA) (see Refs. [28,29] and the review [44]). In contrast to the baseline impulse approximation, MIA is formulated in terms of the form factors and not in terms of the EMT operator itself. So, in MIA the pion GFFs are presented as functionals given by the free two-particle form factors on the set of the two-quark wave functions of the pion. *The necessity* of using the distributions was justified in the case of the electroweak interaction in [28,29,48] (see also [6,44]).

The rest of the paper is organized as follows. In Sec. II we construct the matrix elements of the EMT of the particle with an arbitrary spin and, in particular, with spins 0 and $1/2$. Section III presents the construction of the EMT matrix element for the system of two free spin $1/2$ particles with total quantum numbers of the pion and the explicit forms of corresponding form factors. In Sec. IV we give a brief account of RQM and MIA and derive the formulae for the pion GFFs. In Sec. V we discuss the important role of the relativistic effects in the pion GFFs behavior. We calculate the static limits of the GFFs and of their derivatives, obtain the mean square radius and the values of the form factors up to 1 GeV^2. We briefly conclude and discuss the results in Sec. VI.

II. THE ENERGY-MOMENTUM TENSOR MATRIX ELEMENTS FOR A PARTICLE WITH AN ARBITRARY SPIN

In this section we describe the general procedure of parametrization of the EMT matrix element for a particle with mass M and spin j. To write EMT in terms of gravitational form factors we make use of the method [47]. Because of translational invariance it is sufficient to consider only the following matrix element:

$$\langle \vec{p}, m|T_{\mu\nu}(0)|\vec{p}\,', m'\rangle, \tag{1}$$

where $\vec{p}\,', \vec{p}$ are the particle moments, m', m are the spin projections in the initial and final states, respectively; $p'^2 = p^2 = M^2$.

The normalization condition for the state vectors in (1) is

$$\langle \vec{p}, m|\vec{p}\,', m'\rangle = 2p_0\delta(\vec{p} - \vec{p}\,')\delta_{mm'}, \tag{2}$$

with $p_0 = \sqrt{M^2 + \vec{p}^2}$. We have exploited the general method of parametrization of matrix elements of local operators developed in [47] to construct the matrix elements of the operator of the electromagnetic current (see, e.g., [28,29,44]). Upon formulation of this method the canonical basis in the Hilbert space was used. From the point of view of group theory the parametrization procedure represents the realization of the known Wigner-Eckart theorem on the Poincaré group [48]. The parametrization represents the procedure of separation of the reduced matrix elements (form factors) which are invariant with respect to transformations of the Poincaré group. The main idea of the canonical parametrization can be formulated as follows. Objects of two types should be constructed from the variables in the vectors in the Hilbert space in (1):

(1) The set of linearly independent matrices in spin projections in the initial and final states. At the same time this set represents the set of linearly independent Lorentz scalars (scalars and pseudoscalars). This set describes the EMT matrix elements nondiagonal with respect to m, m' and the behavior of the matrix elements under discrete space-time transformations.
(2) The set of linearly independent objects with the same tensor dimension as the operator. In our case (1) this is a 4-tensor of the rank two. This set describes the behavior of the matrix elements under Lorentz transformations.

The matrix element of the operator is written as the sum of all possible products of objects of the first type and objects of the second type. The coefficients of the elements of this sum are the desired reduced matrix elements, that is form factors. The obtained linear combination is modified if additional constraints, for example, conservation laws, are imposed on the EMT operator.

To construct a Lorentz-invariant matrix in spin projections we use the well-known 4-pseudovector of (see, e.g., [51]):

$$\Gamma_0(p) = (\vec{p}\,\vec{j}), \qquad \vec{\Gamma}(p) = M\vec{j} + \frac{\vec{p}(\vec{p}\,\vec{j})}{p_0 + M},$$
$$\Gamma^2 = -M^2 j(j+1). \tag{3}$$

Under the Lorentz transformations $p^\mu = \Lambda^\mu_\nu p'^\nu$, the operator of the 4-spin (3) is transformed according to the representation of the small group:

$$\Gamma^\mu(p) = \Lambda^\mu_\nu D^j_w(p,p')\Gamma^\nu(p')D^j_w(p',p), \tag{4}$$

where Λ^μ_ν is the matrix of a Lorentz transformation and $D^j_w(p,p')$ is the transformation operator from the small group, the matrix of three-dimensional rotation. The Lorentz-transformation matrix in our case is of the form

$$\Lambda^\mu_\nu = \delta^\mu_\nu + \frac{2}{M^2}p^\mu p'_\nu - \frac{(p^\mu + p'^\mu)(p_\nu + p'_\nu)}{M^2 + p^\lambda p'_\lambda}. \tag{5}$$

It can be shown using (4) that matrix elements of the operator $D^j_w(p,p')\Gamma^\mu(p')$ transform as the 4-pseudovector and matrix elements of the operators $D^j_w(p,p')p_\mu\Gamma^\mu(p')$ and $p'_\mu\Gamma^\mu(p)D^j_w(p,p')$ as 4-pseudoscalars. Thus, the set of linearly independent scalars composed of the vectors p^μ, p'^μ and the pseudovector $\Gamma^\mu(p')$ contains not only diagonal (with respect to spin projections) terms, but nondiagonal terms, too. Note, that the pseudovector $\Gamma^\mu(p)D^j_w(p,p')$ does not enter the set of scalars. Its linear dependence can be shown if we use relation (4) and the explicit form of the matrix Λ^μ_ν (5). After simple calculations we obtain

$$\Gamma^\mu(p)D^j_w(p,p') = D^j_w(p,p')\left[\Gamma^\mu(p') - \frac{p^\mu + p'^\mu}{M^2 + p_\mu p'^\mu}[p_\nu\Gamma^\nu(p')]\right]. \tag{6}$$

Since ${p'}_\mu\Gamma^\mu(p') = 0$, the desired set of linear independent matrices (that is the set of independent Lorentz scalars) is given by $2j+1$ elements

$$D^j_w(p,p')(ip_\mu\Gamma^\mu(p'))^n, \quad n = 0,1,2\ldots,2j. \tag{7}$$

The imaginary unit $i^2 = -1$ is introduced for self-adjointness of the obtained scalar operators (7). The self-adjointness property can be proved using the relation following from (6):

$$p'_\mu\Gamma^\mu(p)D^j_w(p,p') = -D^j_w(p,p')p_\mu\Gamma^\mu(p'). \tag{8}$$

The number of linearly independent scalars in (7) is limited by the fact that the product containing more than $2j$ numbers of factors $\Gamma^\mu(p')$ is reduced to the products of smaller number of factors, i.e., is not linearly independent. For even n the obtained objects in (7) are scalars, and for odd n they are pseudoscalars.

In the decomposition of the matrix element (1) we make use of the metric pseudotensor $g_{\mu\nu}$ and the rank 2 tensors, that should be constructed from the variables on which the state vectors in (1) do depend. Using the available variables in the state vectors of the particle, it is possible to construct one pseudovector $\Gamma^\mu(p')$ (3) and three independent vectors:

$$K_\mu = (p - p')_\mu, \qquad K'_\mu = (p + p')_\mu,$$
$$R_\mu = \epsilon_{\mu\nu\lambda\rho}p^\nu p'^\lambda\Gamma^\rho(p'). \tag{9}$$

Here $\epsilon_{\mu\nu\lambda\rho}$ is the absolutely antisymmetric pseudotensor of rank 4, $\epsilon_{0123} = -1$. For the matrix elements of the operators in (9) to transform as the 4-vector, it is necessary to multiply them by $D^j_w(p,p')$ from the left [in analogy with (7)].

The matrix element (1) is written in terms of all possible products of vectors (9), pseudovector $\Gamma_\mu(p')$, and pseudotensor $g_{\mu\nu}$. Each of these objects is multiplied by a sum of linearly independent scalars (7). The coefficients in such a decomposition are just form factors, or reduced matrix elements.

Taking into account the symmetry properties of the EMT the parametrization of the matrix element (1) can be written in the form:

$$\langle\vec{p},m|T^{(\pi)}_{\mu\nu}(0)|\vec{p}',m'\rangle = \sum_{m''}\langle m|D^j_w(p,p')|m''\rangle \times \langle m''|\tau_{\mu\nu}(0)|m'\rangle, \tag{10}$$

where

$$\begin{aligned}\tau_{\mu\nu}(0) = {} & G_1K'_\mu K'_\nu + G_2\Gamma_\mu\Gamma_\nu + G_3(K'_\mu\Gamma_\nu + \Gamma_\mu K'_\nu) \\ & + G_4(K'_\mu R_\nu + R_\mu K'_\nu) + G_5(R_\mu\Gamma_\nu + \Gamma_\mu R_\nu) \\ & + G_6K_\mu K_\nu + G_7g_{\mu\nu} + G_8(K_\mu\Gamma_\nu + \Gamma_\mu K_\nu) \\ & + G_9(K'_\mu K_\nu + K_\mu K'_\nu) + G_{10}(K_\mu R_\nu + R_\mu K_\nu),\end{aligned} \tag{11}$$

$$G_i = \sum_n g_{\text{in}}(t)(ip_\mu\Gamma^\mu(p'))^n. \tag{12}$$

In (12), $g_{\text{in}}(t)$ are the invariant coefficients, form factors, $t = K^2$ is momentum-transfer square and $\Gamma_\mu = \Gamma_\mu(p')$.

Let us impose some additional physical conditions on the operator (10).

(1) The requirement of self-adjointness. It is easy to show, making use of (8), that the self-adjointness for

the right-hand side (rhs) of (10) requires a modification of the pseudovector Γ_μ with the help of the quantities introduced by (7), (9); namely:

$$\Gamma^\mu \to \tilde{\Gamma}_\mu = \Gamma^\mu(p') - \left(\frac{K^\mu}{K^2} + \frac{K'^\mu}{K'^2}\right)[p_\mu \Gamma^\mu(p')]. \quad (13)$$

Note that this modification (13) does not affect the Lorentz scalars (7). The requirement of self-adjointness also results in the multiplication of the terms containing G_4, G_5, G_8, G_9 by the imaginary unit.

(2) The conservation law for EMT, $T_{\mu\nu}K^\mu = 0$, gives the following conditions to be imposed on the Lorentz scalars,

$$G_8 = G_9 = G_{10} = 0. \quad (14)$$

The conservation law requires also the following changes:

$$G_6 \to -G_6, \qquad G_7 \to tG_6 \quad (15)$$

(3) The parity-conservation condition gives limitations for the summation in (7). Namely, in G_1, G_2, G_4, G_6 the values of n are even while in G_3, G_5 they are odd. The limits of summations are the following: for G_1, G_6 they are $0 \le n \le 2j$; for G_3, G_4 they are $0 \le n \le 2j - 1$; for G_2, G_5 they are $0 \le n \le 2j - 2$. Summing is limited by the fact that each term in the decomposition (11) contains no more than $2j$ factors $\Gamma(p')$.

So, the most general parameterization of the matrix element (10) has the following form if the above constraints are taken into account:

$$\begin{aligned}\tau_{\mu\nu}(0) = &\frac{1}{2}G_1 K'_\mu K'_\nu + G_2\tilde{\Gamma}_\mu\tilde{\Gamma}_\nu + G_3(K'_\mu\tilde{\Gamma}_\nu + \tilde{\Gamma}_\mu K'_\nu)\\ &+ iG_4(K'_\mu R_\nu + R_\mu K'_\nu) + iG_5(R_\mu\tilde{\Gamma}_\nu + \tilde{\Gamma}_\mu R_\nu)\\ &+ G_6(tg_{\mu\nu} - K_\mu K_\nu), \end{aligned} \quad (16)$$

where the summation is limited as is pointed above and the factor $1/2$ before G_1 is a result of the normalization condition: the static limit of EMT should be equal to the mass.

Let us use the obtained general parametrization in the case of spin 0. Now for the pion EMT we have:

$$\begin{aligned}\langle \vec{p}|T^{(\pi)}_{\mu\nu}(0)|\vec{p}'\rangle = &\frac{1}{2}G^{(\pi)}_{10}(t)K'_\mu K'_\nu\\ &+ G^{(\pi)}_{60}(t)[tg_{\mu\nu} - K_\mu K_\nu], \end{aligned} \quad (17)$$

The pion GFFs in canonical parametrization (17) are connected with commonly used (see, e.g., [3]) by the following relations:

$$G^{(\pi)}_{10}(t) = A^{(\pi)}(t), \qquad G^{(\pi)}_{60}(t) = -\frac{1}{2}D^{(\pi)}(t). \quad (18)$$

In the case of spin $1/2$, Eq. (16) gives the following result which we will use below as the constituent-quark EMT canonical parametrization:

$$\begin{aligned}\langle p, m|T^{(q)}_{\mu\nu}(0)|p', m'\rangle = &\sum_{m''}\langle m|D^{1/2}_w(p,p')|m''\rangle\\ &\times\langle m''|(1/2)g^{(q)}_{10}(t)K'_\mu K'_\nu\\ &+ ig^{(q)}_{40}(t)[K'_\mu R_\nu + R_\mu K'_\nu]\\ &+ g^{(q)}_{60}(t)[tg_{\mu\nu} - K_\mu K_\nu]|m'\rangle, \end{aligned} \quad (19)$$

These GFFs in the canonical parametrization (19) can be written in terms of commonly used GFFs for particles of spin $1/2$ in the form

$$\begin{aligned}g^{(q)}_{10}(t) = &\frac{1}{\sqrt{1 - t/4M^2}}\\ &\times\left[\left(1 - \frac{t}{4M^2}\right)A^{(q)}(t) + 2\frac{t}{4M^2}J^{(q)}(t)\right], \end{aligned} \quad (20)$$

$$g^{(q)}_{40}(t) = -\frac{1}{M^2}\frac{J^{(q)}(t)}{\sqrt{1 - t/4M^2}}\Bigg), \quad (21)$$

$$g^{(q)}_{60}(t) = -\frac{1}{2}\sqrt{1 - \frac{t}{4M^2}}D^{(q)}(t). \quad (22)$$

In the following section we generalize the method of construction of the EMT matrix elements, given above, to composite systems.

III. THE EMT MATRIX ELEMENTS FOR A SYSTEM OF TWO FREE PARTICLES WITH PION QUANTUM NUMBERS

Our relativistic approach to form factors of composite systems of interacting components makes use of form factors of corresponding free systems. So, to obtain GFFs of a composite system we need to construct GFFs that describe the gravitational properties of a system of two free constituents, the two-particle system as a whole having quantum numbers of the composite system under consideration. We call the GFFs of the two-particle system without interaction the free gravitational two-particle form factors. The form factors of a composite system of two interacting particles are written in our approach in terms of free two-particle form factors and wave functions exploiting modified impulse approximation (MIA). This approximation was first formulated in the case of electroweak properties of hadrons in our papers [28,29] (see also the review [44]).

EMT operator $T^{(0)}_{\mu\nu}(0)$ for a system of two free particles is of the form

$$T^{(0)}_{\mu\nu}(0) = T_{1\mu\nu} \otimes I^{(2)} \oplus T_{2\mu\nu} \otimes I^{(1)}. \qquad (23)$$

Here $T_{1,2\mu\nu}$ are EMTs of the particles, and $I^{(1,2)}$ are the identity operators in one-particle Hilbert-state spaces of the particles. The following set of two-particle vectors can be chosen as the basis:

$$|\vec{p}_1, m_1; \vec{p}_2, m_2\rangle = |\vec{p}_1 m_1\rangle \otimes |\vec{p}_2 m_2\rangle, \qquad (24)$$

where $\vec{p}_1, \vec{p}_2$ are the 3-momenta of particles, m_1, m_2 are the projections of spins to the z axis, the normalization of one-particle vectors is given in (2). In terms of matrix elements in the basis (24), the relation (23) is rewritten as the sum of matrix elements of one-particle EMT operators

$$\langle\vec{p}_1, m_1; \vec{p}_2, m_2|T^{(0)}_{\mu\nu}(0)|\vec{p}\,'_1, m'_1; \vec{p}\,'_2, m'_2\rangle$$
$$= \langle\vec{p}_1, m_1|\vec{p}\,'_1, m'_1\rangle\langle\vec{p}_2, m_2|T_{2\mu\nu}(0)|\vec{p}\,'_2, m'_2\rangle$$
$$+ (1 \leftrightarrow 2). \qquad (25)$$

Each of the matrix elements of the one-particle EMT in (25) can be written in terms of GFFs (19) (see, e.g., [28,29,50]).

Along with this basis (24), we consider the basis in which the motion of the center of mass of two particles is separated ([28,29,50]):

$$|\vec{P}, \sqrt{s}, J, l, S, m_J\rangle,$$
$$\langle\vec{P}, \sqrt{s}, J, l, S, m_J|\vec{P}', \sqrt{s'}, J', l', S', m_{J'}\rangle$$
$$= N_{CG}\delta^{(3)}(\vec{P} - \vec{P}')\delta(\sqrt{s} - \sqrt{s'})$$
$$\times \delta_{JJ'}\delta_{ll'}\delta_{SS'}\delta_{m_J m_{J'}},$$
$$N_{CG} = \frac{(2P_0)^2}{8k\sqrt{s}}, \qquad k = \frac{\sqrt{\lambda(s, M^2, M^2)}}{2\sqrt{s}}, \qquad (26)$$

where $P_\mu = (p_1 + p_2)_\mu$, $P^2_\mu = s$, $\sqrt{s}$ is the invariant mass of the system of two particles, l is the orbital momentum in the center-of-mass system (c.m.s.), $\vec{S}^2 = (\vec{S}_1 + \vec{S}_2)^2 = S(S + 1)$, S is the total spin in c.m.s., J is the total angular momentum, m_J is the projection of the total angular momentum, M is the constituent mass, and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. The basis (26) is related to the basis (24) by the Clebsch-Gordan decomposition for the Poincaré group. The corresponding decomposition of a direct product (24) of two irreducible representations of the Poincaré group into irreducible representations (26) for particles with spin 1/2 has the form [50] (see also [44]):

$$|\vec{p}_1, m_1; \vec{p}_2, m_2\rangle$$
$$= \sum |\vec{P}, \sqrt{s}, J, l, S, m_J\rangle$$
$$\times \langle Jm_J|Slm_S m_l\rangle Y^*_{lm_l}(\vartheta, \varphi)\langle Sm_S|1/2 1/2 \tilde{m}_1\tilde{m}_2\rangle$$
$$\times \langle\tilde{m}_1|D^{1/2}_w(P, p_1)|m_1\rangle\langle\tilde{m}_2|D^{1/2}_w(P, p_2)|m_2\rangle, \qquad (27)$$

where $\vec{p} = (\vec{p}_1 - \vec{p}_2)/2$, $p = |\vec{p}|$, ϑ, φ are the spherical angles of the vector $\vec{p}$ in c.m.s., Y_{lm_l} is the spherical function, $\langle Sm_S|1/2 1/2 \tilde{m}_1\tilde{m}_2\rangle$ and $\langle Jm_J|Slm_S m_l\rangle$ are the Clebsh-Gordan coefficients of the group $SU(2)$, $\langle\tilde{m}|D^{1/2}_w(P, p)|m\rangle$ is the matrix of the three-dimensional spin rotation, that is necessary for the relativistic invariant summation of the particle spins. The sums go over all discrete variables, $\tilde{m}_1, \tilde{m}_2, m_l, m_S, l, S, J, m_J$. To obtain the basis where the center-of-mass motion is separated we invert the decomposition (27):

$$|\vec{P}, \sqrt{s}, J, l, S, m_J\rangle$$
$$= \sum_{m_1 m_2} \int \frac{d\vec{p}_1}{2p_{10}}\frac{d\vec{p}_2}{2p_{20}}|\vec{p}_1, m_1; \vec{p}_2, m_2\rangle$$
$$\times \langle\vec{p}_1, m_1; \vec{p}_2, m_2|\vec{P}, \sqrt{s}, J, l, S, m_J\rangle, \qquad (28)$$

with the Clebsh-Gordan coefficient

$$\langle\vec{p}_1, m_1; \vec{p}_2, m_2|\vec{P}, \sqrt{s}, J, l, S, m_J\rangle$$
$$= \sqrt{2s}[\lambda(s, M^2, M^2)]^{-1/2} 2P_0\delta(P - p_1 - p_2)$$
$$\times \sum\langle m_1|D^{1/2}_w(p_1, P)|\tilde{m}_1\rangle\langle m_2|D^{1/2}_w(p_2, P)|\tilde{m}_2\rangle$$
$$\times \langle 1/2 1/2 \tilde{m}_1\tilde{m}_2|Sm_S\rangle Y_{lm_l}(\vartheta, \varphi)\langle Slm_S m_l|Jm_J\rangle,$$

the sum being over $\tilde{m}_1, \tilde{m}_2, m_l, m_S$.

We use below the basis (28) with pion quantum numbers $J = l = S = 0$:

$$|\vec{P}, \sqrt{s}, 0, 0, 0, 0\rangle = |\vec{P}, \sqrt{s}\rangle. \qquad (29)$$

We construct the EMT matrix element in the basis (28) for quantum numbers given above using the general method of parametrization of Sec. II. Using (10)–(12), (16) we obtain the parametrization which is analogous to that for zero spin (17):

$$\langle P, \sqrt{s}|T^{(0)}_{\mu\nu}(0)|P', \sqrt{s'}\rangle$$
$$= \frac{1}{2}G^{(0)}_{10}(s, t, s')A'_\mu A'_\nu + G^{(0)}_{60}(s, t, s')[tg_{\mu\nu} - A_\mu A_\nu], \qquad (30)$$

where $G^{(0)}_{i0}(s, t, s')$, $i = 1, 6$ are free two-particle GFFs,

$$A_\mu = (P-P')_\mu, \qquad A^2 = t,$$
$$A'_\mu = \frac{1}{(-t)}[(s-s'-t)P_\mu + (s'-s-t)P'_\mu].$$

It is easy to show that all the imposed constraints are satisfied.

It is possible to derive the equations analogous to (30) for free two-particle systems with different quantum numbers. Such constructions were obtained in [28,29,35,50] in the context of the parametrization of the matrix elements of electroweak currents.

Note that the objects $G^{(0)}_{i0}(s,t,s'), i=1, 6$, in general, are generalized functions (distributions), defined on a space of test functions (see, e.g., [52], and also [6,28,29,48]), and so the static limits at $t\to 0$ are to be understood in a weak sense. The functionals generated by free two-particle form factors on the space of the two-quark wave functions of pion give the corresponding pion GFFs in MIA (see Sec. IV below).

The free two-particle form factors in (30) can be written in terms of one-particle GFFs realizing the parametrization of the matrix elements (25), namely, in terms of the form factors of constituent quarks (19), (20)–(22). Using the decomposition (28), we obtain the matrix element (30) in the following form,

$$\begin{aligned}&\langle P,\sqrt{s}|T^{(0)}_{\mu\nu}(0)|P',\sqrt{s'}\rangle\\ &\quad=\sum\int\frac{d\vec p_1}{2p_{10}}\frac{d\vec p_2}{2p_{20}}\frac{d\vec p\,'_1}{2p'_{10}}\frac{d\vec p\,'_2}{2p'_{20}}\langle P,\sqrt{s}|\vec p_1,m_1;\vec p_2,m_2\rangle\\ &\qquad\times[\langle\vec p_1,m_1|\vec p\,'_1,m'_1\rangle\langle p_2,m_2|T^{(2)}_{\mu\nu}(0)|p'_2,m'_2\rangle\\ &\qquad+\langle\vec p_2,m_2|\vec p\,'_2,m'_2\rangle\langle p_1,m_1|T^{(1)}_{\mu\nu}(0)|p'_1,m'_1\rangle]\\ &\qquad\times\langle\vec p\,'_1,m'_1;\vec p\,'_2,m'_2|P',\sqrt{s'}\rangle,\end{aligned}\tag{31}$$

where the sums are over the variables m_1, m_2, m'_1, m'_2.

The substitution of (30), one-particle matrix elements of EMT (19), and the Clebsh-Gordan coefficiemts (28) for $J=l=S=0$ in (31) gives the desired free two-particle GFFs. The integrals in (31) are written in the coordinate frame with $\vec P' = 0, \vec P = (0,0,P)$. The D_w-functions for spin 1/2 are of the form [53]:

$$\begin{aligned}D^{1/2}_w(p_1,p_2) &= \cos(\omega/2) - 2i(\vec k\,\hat{\vec j})\sin(\omega/2),\\ \vec k &= \frac{[\vec p_1\vec p_2]}{|[\vec p_1\vec p_2]|},\\ \omega &= 2\arctan\frac{|[\vec p_1\vec p_2]|}{(p_{10}+M_1)(p_{20}+M_2)-(\vec p_1\vec p_2)},\end{aligned}\tag{32}$$

where $\hat{\vec j}$ is the operator of the particle spin written in terms of the Pauli matrices. In the chosen coordinate system, two D_w-functions in the rhs of (31) become unity matrices and the other are written with the use of (32). The sum of the rotations around the same axis is obtained following the prescription:

$$D^{1/2}_w(\omega_1)D^{1/2}_w(\omega_2) = D^{1/2}_w(\omega_1+\omega_2).\tag{33}$$

After performing the convolution of both sides, first, with the tensor $A'^\mu A'^\nu$, second, with $g^{\mu\nu}$, and the integrations and summations, we obtain the system of two algebraic equations for the free form factors $G^{(0)}_{i0}(s,t,s'), i=1, 6$:

$$\begin{aligned}&\frac12 G^{(0)}_{10}\left[\frac{\lambda(s,t,s')}{t}\right]^2 - \lambda(s,t,s')G^{(0)}_{60}\\ &=\frac12 A\left\{\frac12\left[g^{(u)}_{10}(t)+g^{(\bar d)}_{10}(t)\right](s+s'-t)^2\cos(\omega_1+\omega_2)\right.\\ &\quad - M\left[g^{(u)}_{40}(t)+g^{(\bar d)}_{40}(t)\right]\xi(s,t,s')(s+s'-t)\sin(\omega_1+\omega_2)\\ &\quad\left.-\left[g^{(u)}_{60}(t)+g^{(\bar d)}_{60}(t)\right]\lambda(s,t,s')\cos(\omega_1+\omega_2)\right\},\end{aligned}\tag{34}$$

$$\begin{aligned}&\frac12 G^{(0)}_{10}\left[\frac{\lambda(s,t,s')}{(-t)}\right] + 3tG^{(0)}_{60}\\ &\quad=\frac12 A\left\{\frac12\left[g^{(u)}_{10}(t)+g^{(\bar d)}_{10}(t)\right](4M^2-t)\cos(\omega_1+\omega_2)\right.\\ &\qquad\left.+3t\left[g^{(u)}_{60}(t)+g^{(\bar d)}_{60}(t)\right]\cos(\omega_1+\omega_2)\right\},\end{aligned}\tag{35}$$

where $A = A(s,t,s') = 2R(s,t,s')\lambda(s,t,s')$,

$$\begin{aligned}R(s,t,s') &= \frac{(s+s'-t)}{2\sqrt{(s-4M^2)(s'-4M^2)}}\\ &\quad\times\frac{\vartheta(s,t,s')}{[\lambda(s,t,s')]^{3/2}},\\ \xi(s,t,s') &= \sqrt{-(M^2\lambda(s,t,s')+ss't)},\end{aligned}$$

ω_1 and ω_2 are the Wigner spin-rotation parameters:

$$\begin{aligned}\omega_1 &= \arctan\frac{\xi(s,t,s')}{M[(\sqrt{s}+\sqrt{s'})^2-t]+\sqrt{ss'}(\sqrt{s}+\sqrt{s'})},\\ \omega_2 &= \arctan\frac{\alpha(s,s')\xi(s,t,s')}{M(s+s'-t)\alpha(s,s')+\sqrt{ss'}(4M^2-t)},\end{aligned}$$

$\alpha(s,s') = 2M+\sqrt{s}+\sqrt{s'}$, $\vartheta(s,t,s') = \theta(s'-s_1) - \theta(s'-s_2)$, θ is the Heaviside function.

$$\begin{aligned}s_{1,2} &= 2M^2 + \frac{1}{2M^2}(2M^2-t)(s-2M^2)\\ &\quad\mp\frac{1}{2M^2}\sqrt{(-t)(4M^2-t)s(s-4M^2)},\end{aligned}$$

$g_{i0}^{(u,\bar{d})}(t), i = 1,\ 4,\ 6$ the GFFs of u- and $\bar{d}$- quarks, respectively. The cutting off by the Heaviside functions in (34), (35) gives the kinematically available region in the plane of invariant variables (s, s') (see, e.g., [44]).

The formal solution of the system (34), (35) is of the form:

$$G_{10}^{(0)}(s,t,s') = \frac{R(s,t,s')t}{\lambda(s,t,s')} \times \left\{\frac{1}{2}\left[g_{10}^{(u)}(t) + g_{10}^{(\bar{d})}(t)\right][(4M^2 - t)\lambda(s,t,s') + 3t(s+s'-t)^2]\cos(\omega_1+\omega_2) - 3Mt\left[g_{40}^{(u)}(t) + g_{40}^{(\bar{d})}(t)\right] \times \xi(s,t,s')(s+s'-t)\sin(\omega_1+\omega_2)\right\}, \tag{36}$$

$$G_{60}^{(0)}(s,t,s') = \frac{1}{2}R(s,t,s') \times \left\{\frac{1}{2}\left[g_{10}^{(u)}(t) + g_{10}^{(\bar{d})}(t)\right][(s+s'-t)^2 + (4M^2-t)\lambda(s,t,s')/t]\cos(\omega_1+\omega_2) - M\left[g_{40}^{(u)}(t) + g_{40}^{(\bar{d})}(t)\right] \times \xi(s,t,s')(s+s'-t)\sin(\omega_1+\omega_2) + 2\left[g_{60}^{(u)}(t) + g_{60}^{(\bar{d})}(t)\right]\lambda(s,t,s')\cos(\omega_1+\omega_2)\right\}. \tag{37}$$

Note that the system (34), (35) is ill-defined at $t \to 0$: the corresponding determinant is zero for $t = 0$. So, the solution for the form factor (37) does not exist at $t = 0$, and the weak limit at $t \to 0$ of the form factor (37), considered as a regular generalized function on the space of the test functions, is infinite. The singularity $\sim 1/t$ is contained in the term with quark form factors $g_{10}^{(u)}(t) + g_{10}^{(\bar{d})}(t)$.

Let us argue that the occurrence of this singularity does not discard the approach but rather puts it into the general trend. To clarify the physical meaning of the singularity in the case of the free two-particle system, we consider the expression for the mean-square mechanical radius [3] in the following form:

$$\langle r^2\rangle_{\rm mech} = \lim_{R\to\infty}\frac{\int_0^R d^3r\, r^2(\frac{2}{3}s(r) + p(r))}{\int_0^R d^3r(\frac{2}{3}s(r) + p(r))}, \tag{38}$$

where $s(r)$ and $p(r)$ are the longitudinal and transverse mechanical stresses in a system, correspondingly.

The fact that the form factors (36), (37) describe the properties of a system of two pointlike particles without interaction between them, means that there are no mechanical stresses in the system. If we let $s(r)$ and $p(r)$ in (38) be constant, and then let these constants vanish, we would obtain that the mechanical MSR in the system is infinite. On the other hand, this MSR is of the form [3],

$$\langle r^2\rangle_{\rm mech} = \frac{6}{D(t)}\frac{dD}{dt}\bigg|_{t=0}, \tag{39}$$

where $D(t)$ is a functional generated by a regular distribution (37) on the suitable space of test functions.

The Eq. (39), in analogy to (38), gives the infinity for the value of the mechanical MSR if the functional $D(t)$, and, consequently, the distribution (37), are singular at the point $t = 0$. So, we conclude that it is adequate to use the free two-particle form factor with singularity (37) for calculations.

Now we include the interaction in this system and derive the pion D form factor using MIA (see Sec. IV). To obtain a finite mechanical MSR of the pion, we need to regularize (37) in the vicinity of the point $t = 0$, that is to find a closely related nonsingular function which gives similar physical results. Here we choose to appeal to the nonrelativistic case. For the simplest variant of our relativistic composite model that we use in the present paper it is sufficient to present an *ansatz* for constructing the free two-particle form factor in a small neighbourhood of $t = 0$ and to show that the construction has a narrow range of choice.

The main point is the fact that in the nonrelativistic limit of (37) the first two terms vanish. So, the limit does not contain the singularity and is defined by the third term with quark form factors $g_{60}^{(u)}(t) + g_{60}^{(\bar{d})}(t)$, having the following form,

$$G_{60nr}^{(0)}(k,t,k') = 2\left[g_{60}^{(u)}(t) + g_{60}^{(\bar{d})}(t)\right]\frac{\vartheta(k,t,k')}{kk'\sqrt{(-t)}},$$
$$\vartheta(k,t,k') = \theta(k' - |k - \sqrt{(-t)}/2|) - \theta(k' - (k + \sqrt{(-t)}/2)). \tag{40}$$

The quantity $G_{60nr}^{(0)}(k,t,k')$ is, in fact, a free nonrelativistic two-particle form factor, the nonrelativistic analog of the form factor (30), (37). The weak limit of the form factor (40) at $t \to 0$ is finite.

We require, first, that the nonrelativistic limit of the regularized construction for the free two-particle form factor near $t = 0$ coincides with (40). Further, we take into account the fact that, usually, nonrelativistic models give reasonable results at low momentum transfer. So, we require, secondly, that our relativistic *ansatz* gives in MIA for pion at low t the results close to nonrelativistic results. This requirement means, in particular, that the assumed construction depends on the quark form factors $g_{60}^{(u)}(t)+g_{60}^{(\bar{d})}(t)$ only. In this case, if $g_{60}^{(u)}(t)+g_{60}^{(\bar{d})}(t)\to 0$, then the construction, as well as the nonrelativistic expression (40), goes to zero. Both these requirements are satisfied by the third term in (40). So, we choose our *ansatz* in the form,

$$G_{60}^{(0)}(s,t,s')=G_{60}^{(0a)}(s,t,s')=R(s,t,s')\lambda(s,t,s') \times[g_{60}^{(u)}(t)+g_{60}^{(\bar{d})}(t)]\cos(\omega_1+\omega_2), \quad (41)$$

where $G^{(0a)}$ denotes the functions that appear as a result of the supposition.

It is clear that even for the chosen assumption, based on the nonrelativistic limit, the actual proposed form is not unique. It is possible to add to (41) some arbitrary functions which do not change its nonrelativistic limit (40). However, the second condition requires that the contribution of these functions near $t=0$ is small: the results have to be close to the nonrelativistic case. This means that qualitatively the added terms must not change the result. So, in what follows we use (37) for the pion D-form factor at finite values of t, but in the vicinity of the point $t=0$ we make use of (41).

IV. GRAVITATIONAL FORM FACTORS OF PION IN MODIFIED IMPULSE APPROXIMATION

To obtain the GFFs of pion we use instant form (IF) of RQM ([40–43]). The details of our version for composite systems can be found in the review ([44]). In RQM the interaction operator is included in the generators of the Poincaré group, the commutation relations of the algebra being preserved. We include the interaction in the algebra of the Poincaré group following the procedure of [54]:

$$\hat{M}_0\to\hat{M}_I=\hat{M}_0+\hat{V}, \quad (42)$$

here $\hat{M}_0$ is the operator of the invariant mass for a free system, $\hat{V}$ is interaction operator, and $\hat{M}_I$ the mass operator for the system with interaction.

The wave function of the system of interacting particles in IF RQM is defined as the eigenfunction of the following complete set of the operators:

$$\hat{M}_I^2(\text{or}\,\hat{M}_I),\quad \hat{J}^2,\quad \hat{J}_3,\quad \hat{\vec{P}}, \quad (43)$$

here $\hat{J}^2$ is the operator of the square of the total angular moment, $\hat{J}_3$ is the operator of the projection of the total angular moment on the z axis and $\hat{\vec{P}}$ is the operator of the total momentum.

In the IF RQM the operators $\hat{J}^2,\hat{J}_3,\hat{\vec{P}}$ coincide with corresponding operators for the composite system without interaction and only the term $\hat{M}_I^2(\hat{M}_I)$ is interaction depending. The two-quark wave function of pion in the basis given by the complete set of vectors (26), (28), (29) diagonalizes (43) and has the form:

$$\langle\vec{P},\sqrt{s}|\vec{p}\rangle=N_C\delta(\vec{P}-\vec{p})\varphi(k),\qquad N_C=\sqrt{2p_0}\sqrt{\frac{N_{CG}}{4k}}, \quad (44)$$

The wave function of intrinsic motion is the eigenfunction of the operator $\hat{M}_I^2(\hat{M}_I)$ and in the case of two particles of equal masses is

$$\varphi(k(s))=\sqrt[4]{s}u(k)k,\qquad \int u^2(k)k^2dk=1, \quad (45)$$

The normalization factors in (45) correspond to the transition to the relativistic density of states

$$k^2dk\to\frac{k^2dk}{2\sqrt{k^2+M^2}}. \quad (46)$$

The decomposition of the matrix element (17) of the pion EMT in terms of the complete set of the vectors (26), (28), (29) is

$$\langle\vec{p}|T_{\mu\nu}^{(\pi)}(0)|\vec{p}'\rangle =\int\frac{d\vec{P}d\vec{P}'}{N_{CG}N_{CG}'}d\sqrt{s}d\sqrt{s'}\langle\vec{p}|\vec{P},\sqrt{s}\rangle \times\langle\vec{P},\sqrt{s}|T_{\mu\nu}^{(\pi)}(0)|\vec{P}',\sqrt{s'}\rangle\langle\vec{P}',\sqrt{s'}|\vec{p}'\rangle, \quad (47)$$

where $\langle\vec{P}',\sqrt{s'}|\vec{p}'\rangle$ is the wave function in the sense of IF RQM (44). We obtain

$$\langle\vec{p}|T_{\mu\nu}^{(\pi)}(0)|\vec{p}'\rangle =\int\frac{N_CN_C'}{N_{CG}N_{CG'}}d\sqrt{s}d\sqrt{s'}\varphi(s) \times\langle\vec{p},\sqrt{s}|T_{\mu\nu}^{(\pi)}(0)|\vec{p}',\sqrt{s'}\rangle\varphi(s'). \quad (48)$$

The matrix element of the tensor in (48) is to be considered as a Lorentz-covariant generalized function [28,29,48,52]), that has a meaning only under the integral. The integral itself presents a functional giving a regular distribution. The decomposition of the tensor in the integral in terms of tensors which were used in the decomposition in the left-hand side (lhs) of (48) entering (17) is

$$\frac{N_CN_C'}{N_{CG}N_{CG}'}\langle\vec{p},\sqrt{s}|T_{\mu\nu}^{(\pi)}(0)|\vec{p}',\sqrt{s'}\rangle =\frac{1}{2}\tilde{G}_{10}(s,t,s')K_\mu'K_\nu'+\tilde{G}_{60}(s,t,s')[tg_{\mu\nu}-K_\mu K_\nu], \quad (49)$$

here $\tilde{G}_{i0}(s,t,s')$, $i=1,6$ are the Lorentz-invariant regular distributions. A rigorous proof of the importance of distributions in the interpretation of the decomposition analogous to (49) in the case of electromagnetic current was given in [28,29]. After substituting of (17) and (49) in the lhs and rhs of (48), respectively, we obtain pion GFFs in the form of functionals:

$$G^{(\pi)}_{i0}(t) = \int d\sqrt{s}d\sqrt{s'}\varphi(s)\tilde{G}_{i0}(s,t,s')\varphi(s'),\quad i=1,6. \tag{50}$$

The main point now is the calculation of the function $\tilde{G}_{i0}(s,t,s')$. To obtain similar form factors describing electroweak structure of composite hadrons it is customary exploit the so-called impulse approximation (IA) (see, e.g., the review [42]). Let us demonstrate the meaning of IA extending the approach to GFF. These form factors characterize the scattering cross section of a projectile by a composite system in the process of graviton exchange.

So, the EMT in this case can be written in the following form:

$$T = \sum_k T^{(k)} + \sum_{k\langle m} T^{(km)} + \cdots, \tag{51}$$

where the first term presents the sum of one-particle EMTs, the second term presents the sum of two-particle EMT, and so on. The first sum describes the scattering of a projectile by each independent constituent, the second sum describes the scattering by two constituents simultaneously and so on. The standard IA leaves in (51) only the first term:

$$T \approx \sum_k T^{(k)}. \tag{52}$$

Note that in the approximation (52) the operators in the instant form RQM does not satisfy the Lorentz-covariance conditions and the conservation law [42].

To study the electroweak structure of hadrons, we had proposed [28,29] the modified impulse approximation (MIA). Constructing MIA for GFFs we change the form factors $\tilde{G}_{i0}(s,t,s')$ in (50) for free two-particle GFFs (30): in the invariant part of the decomposition (48), (49) we throw off the contribution of the simultaneous scattering by two and more constituents and take into account only scattering by free two-constituent system.

The covariant part of the decomposition (48)–(50) is not changed by MIA and so, the Lorentz-covariance conditions and the conservation law for the EMT matrix element (48) are not broken. This happens because in MIA the contribution of the second term in (51) is partially taken into account in a self-consistent way.

In MIA, the pion GFFs (50) are written in the form:

$$G^{(\pi)}_{i0}(t) = \int d\sqrt{s}d\sqrt{s'}\varphi(s)G^{(0)}_{i0}(s,t,s')\varphi(s'),\quad i=1,6, \tag{53}$$

where $G^{(0)}_{i0}(s,t,s')$ are free two-particle form factors (30), given by (36), (37), (41).

In the following section, the details of calculation of pion GFFs using (53) and the corresponding results are given.

V. RESULTS OF CALCULATIONS

In what follows we use the conventional notations of A, J, and D form factors (see, e.g., [3]) and the linking relations (18), (20)–(22).

To obtain the pion form factor A we use directly the Eqs. (18), (53), (36) while in the case of the form factor D, which is ill defined, we involve an *anzatz* described in detail in Sec. III. We obtain the pion D form factor in the vicinity of $t=0$ using (18), (53) and assuming (41) [the form factor $D^{(\pi a)}(t)$]. For the overall description of the pion D form factor we need to join smoothly this function $D^{(\pi a)}(t)$ with the solution for finite values of t given by (18), (53), (37). We describe this procedure in detail later.

Let us list first the relativistic effects contained in (53). The contributions of the J form factors of the constituent quarks ($g^{(q)}_{40}(t)$) to the pion A ($G^{(\pi)}_{10}(t)$) and D ($G^{(\pi)}_{60}(t)$) form factors are a consequence of pure relativistic effect of spin rotation. These contributions vanish if we set $\omega_{1,2}$ in (36), (37) equal to zero. The contribution of quark A form factor $g^{(q)}_{10}(t)$ to pion D form factor (37) is of relativistic origin, too.

To obtain numerical results for pion GFFs in our model (53), (36), (37), we need some parameters to be used as an input. We suppose that u- and d- quarks have one and the same gravitational structure and so, we have to set three quark GFFs as functions of momentum transfer square. It is also necessary to choose a model two-quark wave function of pion (45), and to fix the mass of light quark, M.

In the present work we consider the simplest case, that of pointlike constituent quarks. This means that instead of quark form factors, we use their standard static moments:

$$A^{(q)}(t) = A^{(q)}(0) = 1,\qquad J^{(q)}(t) = J^{(q)}(0) = \frac{1}{2},$$
$$D^{(q)}(t) = D^{(q)}(0) = D_q,\qquad q = u,\bar{d}, \tag{54}$$

where D_q is the D-term of the constituent quark.

We had shown [31] that the results of calculations for electromagnetic form factors depend weakly on the actual form of the two-quark wave function in pion. Here we choose for (45) the wave function of the ground state of harmonic oscillator which ensures square-law quark confinement,

$$u(k) = \left(\frac{4}{\sqrt{\pi}b^3}\right)^{1/2} \exp\left(-\frac{k^2}{2b^2}\right). \tag{55}$$

Here b is the parameter of the model along with the quark-mass M. The best results for electroweak properties of light mesons [31–39] were obtained for the following values of these parameters:

$$M = 0.22\ \text{GeV}, \qquad b = 0.35\ \text{GeV}. \tag{56}$$

In what follows we fix these values also for the calculation of pion GFFs. So, to derive the pion GFFs we need to fix only the constituent-quark D-term (54).

The mean-square radius (MSR) of pion we define as follows (see [3] and the original paper [47]):

$$\langle r_\pi^2 \rangle = 6A^{(\pi)\prime}(0) - \frac{3}{2M_\pi^2} D^{(\pi)}(0), \tag{57}$$

where $M_\pi = 0.13957$ GeV is the pion mass. Note that the standard condition $A^{(\pi)}(0) = 1$ is fulfilled automatically.

For the interval of possible data for the pion MSR we adopt the interval that can be calculated using the results listed in the review [3], namely:

$$\langle r_\pi^2 \rangle_{\min} = 65.38\ \text{GeV}^{-2},$$
$$\langle r_\pi^2 \rangle_{\max} = 69.52\ \text{GeV}^{-2}. \tag{58}$$

To obtain this interval of MSR values in our approach, we require in addition the parameter D_q (54) to be in the following region of approximately the same relative spread

$$D_q = -0.1435 \pm 0.0045. \tag{59}$$

The interval of values of the pion D-term corresponding to the chosen interval of the quark D-term (59) is

$$D^{(\pi)}(0)_{\min} = -0.905, \qquad D^{(\pi)}(0)_{\max} = -0.851. \tag{60}$$

The equations for the form factor $A^{(\pi)}(t)$ do not contain the parameter D_q. So, the derivative of the A form factor of pion at $t = 0$ is defined by the parameters (56) fixed in our model approach to the pion electroweak form factors and has a predictive nature. This value is obtained numerically using (18), (53):

$$A^{(\pi)\prime}(0) = 0.0408\ \text{GeV}^{-2}. \tag{61}$$

The results of calculation of the pion A form factor are presented in Figs. 1 and 2. Note that Fig. 1 demonstrates, in particular, that the relativistic spin rotation effect gives an essential contribution to A form factor. This effect is purely kinematical and thus takes place for any model wave function. The effect changes essentially the slope of A form factor at zero t and, as a consequence, the value of the pion gravitational radius (57), (61). This fact emphasizes the importance of the corresponding theory to be essentially relativistic.

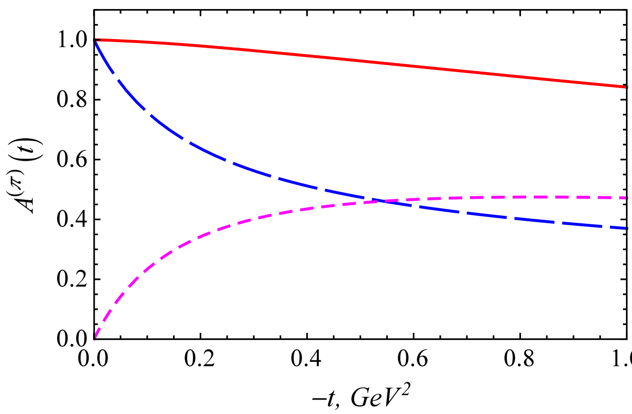

FIG. 1. Gravitational A form factor of pion. Full line (red)—the full result; dashed line (blue)—the contribution of the A form factors $g_{10}^{(q)}$ of the constituent quarks; short-dashed line (magenta)—the contribution of the quark J form factors $g_{40}^{(q)}$ (relativistic spin rotation effect).

As we have mentioned above recently the data on the pion GFFs was extracted from the experiment [11] for the first time in [12]. In Fig. 2 we compare our results for pion A form factor with those given in [12]. The results are in a qualitative agreement, however the slope of our A form factor is smaller. Note that we choose here the simplest variant of the model confining ourselves to pointlike constituent quarks. If we depart from this condition, the quark form factors would give the additional decreasing of the pion form factor and would ameliorate the agreement.

To calculate the pion D form factor we need first to join smoothly the function $D^{(\pi a)}(t)$ defined in the vicinity of

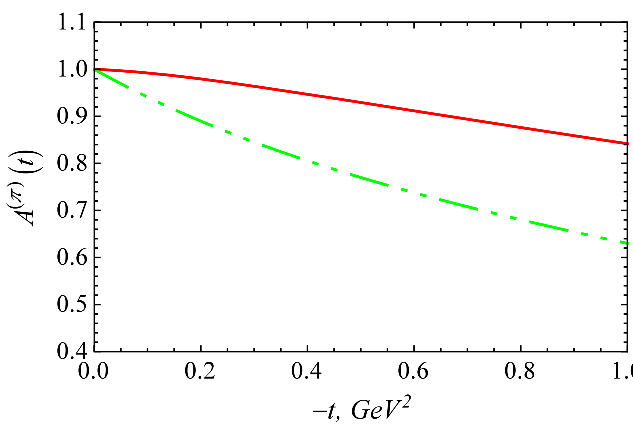

FIG. 2. Our A form factor of pion [full line (red)] in comparison with the result of the authors of [12]. Their A form factor of pion is normalized to its value at zero t (double-dot-dashed, green) line.

$t = 0$ by (18), (53) and the suggestion (41) with the solution for finite values of t given by (18), (53), (37). The smooth joint is possible because the function $D^{(\pi a)}(t)$ in the vicinity of zero is defined up to order of $\sim t$. Let us give some details of the procedure. First, we add to $D^{(\pi a)}(t)$ a cubic polynomial, which vanish at $t \to 0$, with the coefficients a, b, c that are to be defined by the joint conditions:

$$D^{(\pi a)}(t) + a(-t) + b(-t)^2 + c(-t)^3. \qquad (62)$$

We require the form factor (62) to be joint smoothly with the form factor for finite t at a point $t = t_c$. The coefficients a, b, c and the point t_c can be calculated unambiguously if the following conditions are satisfied.

(1) The derivative of the function (62) satisfies the following constraints obtained in [12] (see also [3]):

$$\frac{D^{(\pi a)\prime}(0)}{D^{(\pi a)}(0)} = 2.88 \sim 3.31\ \mathrm{GeV}^{-2}. \qquad (63)$$

(2) The values of the two functions coincide at the point $t = t_c$, as well as the values of their first derivatives.

(3) The form factor (62) satisfies the condition $D^{(\pi a)}(t) < 0$ that ensures the mechanical stability of the pion. Note, that the D form factor defined for finite t does satisfy this condition.

(4) For t_c we choose among all possible points satisfying the conditions 1–3 the point of maximal absolute value $|t_c|$. This is necessary for the contribution of singular term $\sim 1/t$ be minimized at small values of t.

We demonstrate the procedure in Fig. 3, using for the calculation the minimal value from the interval (63) and $D_q = -0.1435$.

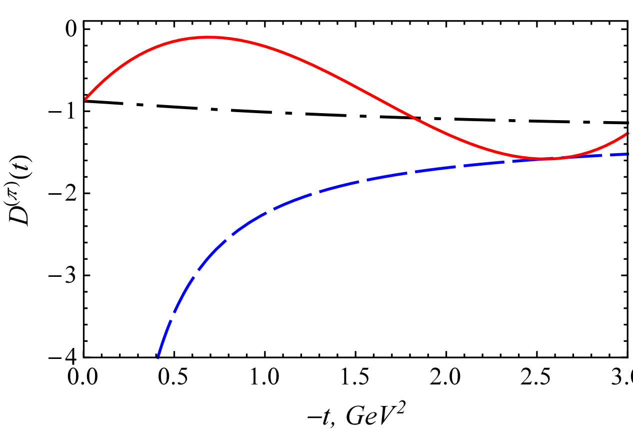

FIG. 3. The joint of two functions for the pion D form factor for the middle value from (59) and the minimal value from (63). The full line (red)—the D form factor (62); the dashed line (blue)—the solution of (18), (37), (53); the dot-dashed line (black)—$D^{(\pi a)}(t)$; $(-t_c) = 2.53$ GeV^2.

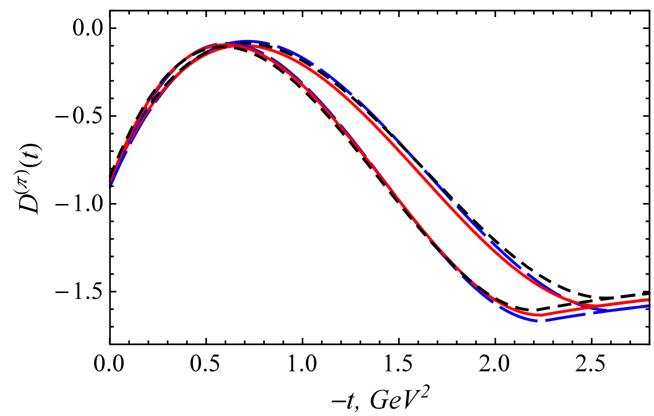

FIG. 4. The dependence of the joint D form factor of pion procedure on the values of the parameter D_q (59) and on the values from the interval (63). The full line (red)—$D_q = -0.1435$, long-dashed line (blue)—$D_q = -0.148$, short-dashed line (magenta)—$D_q = -0.139$. The upper set of curves at $(-t) \sim 1.5$ GeV^2—for the minimal value from (63), the lower set—for the maximal value from (63).

We present in Fig. 4 the dependence of the procedure on the values of the parameter D_q (59) and on the values from the interval (63). Figure 4 demonstrates the stability of the procedure. As can be seen in Fig. 4 the result of joining depends weakly on the value of the quark D-term from (59). However, the parameters in (62) and the point of joint t_c do depend on the value from (63).

The results of calculation of the pion D form factor using the Eqs. (18), (37), (53), $D^{(\pi a)}(t)$ and the separate contributions of the quark A, J, and D form factors are

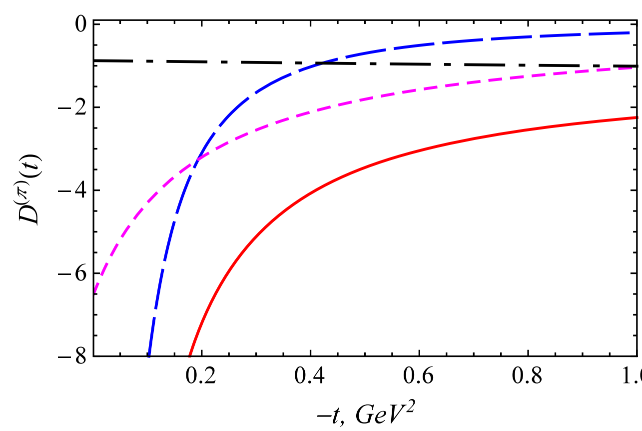

FIG. 5. The pion gravitational D form factor calculated for the parameters (54), (56) and $D_q = -0.1435$ (59). The full line (red)—the total values obtained using (18), (37), (53). Long-dashed line (blue)—the contribution of quarks A form factors $g_{10}^{(q)}$. Short-dashed line (magenta)—the spin rotation effect (the contribution of the quark J form factor $g_{40}^{(q)}$). Dot-dashed line (black)—the contribution of the quark D form factor $g_{60}^{(q)}$; this curve coincides with $D^{(\pi a)}(t)$.

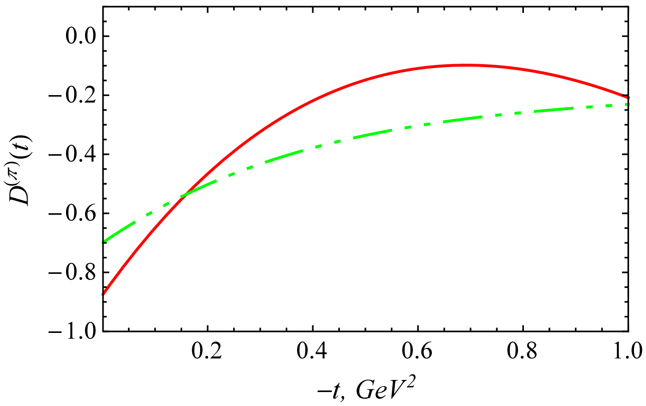

FIG. 6. The pion D form factor calculated for the quark parameters (54), (56), $D_q = -0.1435$ (59) and the minimal value of (63) in comparison with the results of the paper [12]. The full line (red)—our result, double-dot-dashed line (green)—the pion D form factor from [12].

presented in Fig. 5. One can see from Fig. 5 that the singularity in the pion D form factor at the point $t = 0$ is caused by the term containing the A form factors of the constituent quarks $g_{10}^{(q)}$ (20), (37). Note also, that, as well as the pion A form factor, the pion D form factor contains large contribution of the relativistic spin rotation effect through the contribution of J form factors of the constituent quarks $g_{40}^{(q)}$ (21). It is seen that the condition of mechanical stability of pion $D^{(\pi)}(t) < 0$ is fulfilled.

The pion D form factor calculated with the use of (62) for $D_q = -0.1435$ and the minimal value of (63) are compared with the results of the paper [12] in Fig. 6.

Using the results for the pion GFFs we calculate the mass radius of pion

$$\langle r^2\rangle_{\mathrm{mass}} = 6\frac{dA^{(\pi)}}{dt}\bigg|_{t=0},$$

and its mechanical radius defined by (39). To calculate the mass radius we need only the parameters (54), (56) and so obtain the strictly fixed by our previous results value $\sqrt{\langle r^2\rangle_{\mathrm{mass}}} = 0.1$ fm. The chosen interval for D_q gives for the pion mechanical radius the interval of values $\sqrt{\langle r^2\rangle_{\mathrm{mech}}} = 0.82$–$0.88$ fm. It is highly probable that the model with nonpointlike quarks will give larger values for the radii.

Note, that the slopes of the form factors at $t = 0$ in Fig. 6 are different. Nevertheless our result for the mechanical MSR as defined above coincides with that of [12].

Let us make some remarks concerning a possibility of comparing our results with experimental data.

First, we use an extremely rough approximation—the pointlike constituent quarks. As it was pointed out and argued in detail in [55], the accounting for the quark structure, the full quark form factor, is a necessary part of efficient describing of the electromagnetic form factors of hadrons. We use here the simplest model aiming to demonstrate that relativistic invariant canonical parametrization together with MIA in the framework of IF RQM does give a real possibility of obtaining the pion GFFs. The obtained results are reasonable and satisfy all standard constraints.

Second, today there are no trustworthy results on pion GFFs unambiguously extracted from precise experimental data. Although the pion GFFs and gravitational radii were estimated [12], the errors of the Belle measurements are large (even at current stage), and the obtained results can be affected by the experimental errors. Belle II began data taking with the much higher luminosity SuperKEKB in 2018, and the precise measurements of $\gamma^*\gamma \to \pi^0\pi^0$ can be expected since the statistic errors are much larger than the systematic errors in the previous Belle data [56]. One may expect more quantitative insights from experiments CLAS at Jefferson Lab [57], COMPASS at CERN [58] and the envisioned future Electron-Ion-Collider [59].

VI. CONCLUSION

In this work we extend our relativistic theory of electroweak properties of composite systems, developed previously, to describe simultaneously the gravitational structure of hadrons. The approach is based on a version of the instant-form relativistic quantum mechanics and makes use of the modified impulse approximation. We use the general method of the relativistic invariant parametrizaton of local operators to write the energy-momentum tensor of particle with an arbitrary spin. From the point of view of group theory the parametrization procedure represents the realization of the known Wigner-Eckart theorem on the Poincaré group. We give general formulas and use for the actual calculation those for systems of spin 0(the pion), spin 1/2 (the constituent quark) and for the free two-quark system with total quantum numbers of pion.

To construct the pion GFFs we use the modified impulse approximation which, in contrast to the baseline impulse approximation, is formulated in terms of the form factors and not in terms of the EMT operator itself. The pion GFFs are presented as functionals given by the free two-particle form factors on the set of the two-quark wave functions of the pion.

We calculate the pion GFFs assuming that the quarks are pointlike. For the two-quark wave function we take the ground-state wave function of the harmonic oscillator. All but one parameters of our first-principle model were fixed previously in works on electromagnetic form factors. The only free parameter, D_q, is a characteristic of gravitational form factor of constituent quark, the quark D-term. This parameter is constrained from the pion mean-square radius despite large uncertainties in the extraction of the latter from the experimental data through a phenomenological approach. We calculate the values of the static gravitational characteristics of the pion and obtain A and D form factors as functions of momentum transfer up to 1 GeV^2. Note that the new parameter is not used in the calculation of the A form factor, its value is a direct prediction of our previous approach. In the calculation of the $D(t)$ form factor we use the new parameter and also exploit a special procedure (based on an *ansatz*) to get rid of a singularity at $t = 0$. The important role of the relativistic effects in the pion gravitational characteristics is discussed in detail. The calculated gravitational form factors and gravitational mean-square radii are in a reasonable agreement with the known results.

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