Quark sea flavor asymmetries in the spin $-\frac{3}{2}^{+}$ decuplet baryons

Harleen Dahiya

Department of Physics, Dr. B.R. Ambedkar National Institute of Technology, Jalandhar 144011, India

(Received 23 November 2020; accepted 23 December 2020; published 13 January 2021)

The implications of the quark sea generation through the chiral fluctuations have been studied in the context of quark flavor distribution functions for the spin $-(3/2)^{+}$ decuplet baryons. The chiral constituent quark model allows a simple and intuitive method to investigate the principle features of the hadron structure and is able to qualitatively generate the requisite amount of quark sea. It is also known to provide a satisfactory justification for the chiral symmetry breaking and SU(3) symmetry breaking. In light of the recent developments to test the mechanism for the quark sea generation, the quark sea asymmetries $\bar{d}-\bar{u}$, $\bar{d}/\bar{u}$ and the fraction of a particular quark $F_q^{B^*} = (q^{B^*} + \bar{q}^{B^*}/\sum_q(q^{B^*} + \bar{q}^{B^*}))$ have been studied for the spin $-(3/2)^{+}$ decuplet baryons. The suppression factors estimating the strange quark content with respect to the nonstrange quarks in the Δ baryons and the suppression factors estimating the u and d quark contents with respect to the strange quarks in Ω^- baryons have also been discussed.

DOI: 10.1103/PhysRevD.103.014016

I. INTRODUCTION

In the present day, the study of composition of baryons is one of the active areas in hadronic physics and is primarily the study of the generation of quark sea and is dynamics. Due to its complexity, it still remains to be a major unresolved issue in high energy physics. After the first major surprise of presence of pointlike constituents in the nucleon, revealed in the deep inelastic scattering (DIS) experiments [1], several efforts have been made to probe the proton structure. The DIS results surprisingly [2–5] indicated that the constituent quarks of the proton carry only about 30% of its spin which was in contradiction with the structure envisaged in the naive constituent quark model (NQM) [6–8]. Several experiments were conducted to probe the structure and all of them indicated the presence of indistinct sea of quark-antiquark pairs apart from the constituent quarks. Recently, experiments from the elastic scattering of electrons have been performed in SAMPLE at MIT-Bates [9], G0 at JLab [10], PVA4 at MAMI [11], and HAPPEX at JLab [12] and they have clearly indicated the role played by the nonconstituent quarks in understanding the charge, current, and spin structure of the nucleon.

A major finding from the famous DIS experiment by the New Muon Collaboration (NMC) in 1991 [13] further established this fact by measuring the violation of the Gottfried sum rule (GSR) ($\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$) [14] clearly indicating excess of $\bar{d}$ over $\bar{u}$ quarks which are otherwise absent in the baryons. The quark sea asymmetry of the unpolarized quarks leading to excess of $\bar{d}$ over $\bar{u}$ was subsequently confirmed by other experiments performed in E866 at Fermilab [15], NA51 [16], and HERMES [17]. These experiments measured the different quark sea asymmetries like $\bar{d}/\bar{u}$, $\bar{d}-\bar{u}\neq 0$, $\frac{\bar{d}-\bar{u}}{u-d}$ and reinforced the fact that the structure of the nucleon is not limited to just u and d quarks. One possibility to account for the observed quark sea asymmetry is through the perturbative production of the quark-antiquark pairs by gluons. However, this process produces nearly equal numbers of $\bar{u}$ and $\bar{d}$ and therefore the sea quarks were expected to be nonperturbative in nature.

The observed quark sea asymmetry has been attempted to be explained using the pion cloud mechanism [18]. In this case, the quark sea is believed to originate from process such as virtual pion production. In the deep inelastic lepton-nucleon scattering, the lepton probe is also suggested to scatter off the pion cloud surrounding the target proton. Therefore, the pion cloud idea needs to be improved upon by adopting a mechanism which operates in the *interior* of the hadron. This is intrinsically a nonperturbative phenomenon and because of confinement, it is still a big challenge to perform these calculations from the first principles of quantum chromodynamics (QCD).

The meson-baryon component of the proton arises from the two processes: $p\to\pi^+n$ and $p\to\pi^0p$. In $p\to\pi^+n$, $\pi^+(\bar{d}u)$ naturally contains an excess of $\bar{d}$ whereas in $p\to\pi^0p$, $\pi^0(\bar{u}u,\bar{d}d)$ produces equal numbers of $\bar{u}$ and $\bar{d}$. However, $p\to\pi^0p$ does not affect the $\bar{d}$ and $\bar{u}$ asymmetry since it is suppressed by a factor of 2 by isospin couplings as compared to $p\to\pi^+n$. Further, the excess of $\bar{d}$ could be significantly reduced by the emissions such as $p\to\Delta^{++}+\pi^-$ with $\pi^-(\bar{u}d)$ cloud having excess of $\bar{u}$ quarks but again the $N\to\pi N$ dominates over the $N\to\pi\Delta$ process and a significant $\bar{d}>\bar{u}$ asymmetry can be accounted for. It would be interesting to mention here that for the case of Δ baryons, the processes $\Delta \to \pi\Delta$ and $\Delta \to \pi N$ both favor the production of π^+ over π^- and π^0 hence has excess of $\bar{d}$ as compared to $\bar{u}$.

There has been considerable progress in the past few years to understand the origin of the sea quark flavor structure for the case of nucleon and other octet baryons [19–30]; however, the different models differ in the assumptions and there is no consensus regarding the origin of quark sea. Further, the information on the explicit strange quark sea is obtained from the neutrino-induced DIS experiments [31] where it has been emphasized that the region $x > 0.3$ is dominated by the constituent quark distributions whereas the sea quarks dominate in the region $x < 0.3$. This has been further endorsed by CDHS [32], CCFR [33,34], CHARMII [35], NOMAD [36,37], NuTeV [38], and CHORUS [39]. These experiments clearly point out the need for additional data toward extended kinematic range and more accuracy of the quark sea asymmetry which are being currently studied at the ongoing Drell-Yan experiment at Fermilab [40] and a proposed experiment at J-PARC facility [41]. In view of the above experimental developments, it becomes desirable to measure the quark sea asymmetry for the case of Δ baryons as well as the other baryons in the spin $-\frac{3}{2}^+$ decuplet which would undoubtedly provide vital clues to the nonperturbative aspects of QCD.

This motivates us to use the chiral constituent quark model (χCQM) [42] which is based on the idea that chiral symmetry breaking takes place at a distance scale much smaller than the confinement scale. This is an effective interaction Lagrangian approach of the strong interactions where the effective degrees of freedom are the constituent quarks and the internal Goldstone bosons (GBs) which are coupled to the constituent quarks [43–46]. The χCQM successfully explains the "proton spin problem" [46], magnetic moments of octet and decuplet baryons including their transitions and the Coleman-Glashow sum rule [47], hyperon β decay parameters [48], magnetic moments of octet baryon resonances [49], magnetic moments of Λ resonances [50], charge radii and quadrupole moment [51], quark sea asymmetry for the case of octet baryons [52], etc.

The purpose of the present communication is to study the implications of the quark sea generation through the chiral fluctuations and estimate the quark flavor distribution functions for the spin $-\frac{3}{2}^+$ decuplet baryons. We will use the chiral χCQM which allows a simple and intuitive method to investigate these quantities and is also one of the most successful phenomenological models which provides a satisfactory justification for the chiral symmetry breaking and SU(3) symmetry breaking. In particular, after the recent developments to test the mechanism for the quark sea generation, we would like to understand in detail the quark sea asymmetries $\bar{d} - \bar{u}$, quark sea ratios $\bar{d}/\bar{u}$, and the fraction of a particular quark $F_q^{B^*} = \frac{q^{B^*} + \bar{q}^{B^*}}{\sum_q (q^{B^*} + \bar{q}^{B^*})}$ in the spin $-\frac{3}{2}^+$ decuplet baryons. Further, it would be interesting to extend the calculations to predict the suppression factors corresponding to the strange quark content with respect to the nonstrange quarks in the Δ baryons and the suppression factors estimating the u and d quark contents with respect to the strange quarks in Ω^- baryons. These results will explicitly describe the role of nonconstituent degrees of freedom and also provide important constraints on the future experiments to measure the structure of spin $-\frac{3}{2}^+$ decuplet baryons.

II. CHIRAL CONSTITUENT QUARK MODEL

The dynamics of light quarks (u, d, and s) and the internal structure of the baryons can be described using the QCD Lagrangian. However, under the chiral transformation for the quark fields $\psi \to \gamma^5\psi$, the Lagrangian does not remain invariant and changes sign because of the mass terms. If the mass terms in the QCD Lagrangian are neglected, it will have global chiral symmetry of the $SU(3)_L \times SU(3)_R$ group. At a scale of around 1 GeV, this chiral symmetry is spontaneously broken as $SU(3)_L \times SU(3)_R \to SU(3)_{L+R}$ which results in creating a set of massless GBs identified as π, K, η mesons. A ninth GB η' also exists as the QCD Lagrangian is also invariant under the axial $U(1)$ symmetry. Within the region of QCD confinement scale ($\Lambda_{\rm QCD} \simeq 0.1$–$0.3$ GeV) and the chiral symmetry breaking scale $\Lambda_{\chi SB}$, the constituent quarks and the nonet of GBs form the appropriate degrees of freedom.

Weinberg introduced the χCQM which was further developed by Manohar and Georgi [42]. The underlying idea of χCQM is the fluctuation process where the GBs couple directly to the constituent quarks in the hadron interior as

$$q^{\pm} \to \mathrm{GB} + q'^{\mp} \to (q\bar{q}') + q'^{\mp}, \tag{1}$$

where $q\bar{q}' + q'$ constitute the "quark sea" [43,44,46]. The effective Lagrangian describing interaction between quarks and a nonet of GBs forms the basis for the χCQM which, in the leading order, can be expressed as

$$\mathcal{L}_{\rm int} = -\frac{g_A}{f_\pi}\bar{\psi}\partial_\mu\Phi\gamma^\mu\gamma^5\psi \tag{2}$$

and further simplified using the Dirac equation $(i\gamma^\mu\partial_\mu - m_q)q = 0$ to

$$\mathcal{L}_{\rm int} \approx i \sum_{q=u,d,s} \frac{m_q + m_{q'}}{f_\pi}\bar{q}'\Phi\gamma^5 q = i \sum_{q=u,d,s} P_\pi \bar{q}'\Phi\gamma^5 q. \tag{3}$$

Here g_A is the axial-vector coupling constant and $P_\pi(=\frac{m_q+m_{q'}}{f_\pi})$ is the coupling constant for octet of GBs and m_q ($m_{q'}$) is the quark mass parameter. The Lagrangian of the quark-GB interaction, suppressing all the space-time structure to the lowest order, can now be expressed as

$$\mathcal{L}_{\rm int} = P_\pi \bar{\psi}\Phi\psi. \tag{4}$$

The QCD Lagrangian is also invariant under the axial $U(1)$ symmetry, which would imply the existence of ninth GB. This breaking symmetry picks the η' as the ninth GB. The effective Lagrangian describing interaction between quarks and a nonet of GBs, consisting of octet and a singlet, can now be expressed as

$$\mathcal{L}_{\rm int} = P_\pi \bar{\psi}\left(\Phi + P_{\eta'}\frac{\eta'}{\sqrt{3}}I\right)\psi = P_\pi \bar{\psi}(\Phi')\psi. \tag{5}$$

Here $P_{\eta'}$ is the ratio of the coupling constants for the singlet and octet GBs.

The chiral fluctuations possible from Eq. (1) are given as

$$\begin{aligned} u &\rightleftharpoons (d+\pi^+) + (s+K^+) + (u+\pi^0,\eta,\eta'),\\ d &\rightleftharpoons (u+\pi^-) + (s+K^0) + (d+\pi^0,\eta,\eta'),\\ s &\rightleftharpoons (u+K^-) + (d+\bar{K}^0) + (s+\eta,\eta'). \end{aligned} \tag{6}$$

The chiral fluctuations to π, K, η, and η' are, respectively, determined from the transition probabilities P_π^2, P_K^2, P_η^2, and $P_{\eta'}^2$ [43,44,46]. These probabilities quantify the extent to which the quark sea contributes to the structure of the baryon. The transition probability P_π^2 is fixed by considering the strange and nonstrange quark masses to be nondegenerate $M_s > M_{u,d}$, the transition probabilities P_K^2 and P_η^2 are fixed by considering GB masses of K, η, and π to be nondegenerate $M_{K,\eta} > M_\pi$, and finally the transition probability $P_{\eta'}^2$ is fixed by considering GB masses η', K, and η to be nondegenerate $M_{\eta'} > M_{K,\eta}$.

The GB field Φ' can be expressed in terms of the GBs and their transition probabilities as

$$\Phi' = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + P_\eta\frac{\eta}{\sqrt{6}} + P_{\eta'}\frac{\eta'}{\sqrt{3}} & \pi^+ & P_K K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + P_\eta\frac{\eta}{\sqrt{6}} + P_{\eta'}\frac{\eta'}{\sqrt{3}} & P_K K^o \\ P_K K^- & P_K\bar{K}^0 & -P_\eta\frac{2\eta}{\sqrt{6}} + P_{\eta'}\frac{\eta'}{\sqrt{3}} \end{pmatrix}. \tag{7}$$

The transition probability of the emission of a GB from any of the q quark, $P(q \to GB)$, can now be expressed in terms of the transition probabilities P_π^2, P_K^2, P_η^2, and $P_{\eta'}^2$. We have

$$P(u\to GB) = P(d\to GB) = \frac{P_\pi^2}{6}(9 + 6P_K^2 + P_\eta^2 + 2P_{\eta'}^2), \tag{8}$$

$$P(s\to GB) = \frac{P_\pi^2}{3}(6P_K^2 + 2P_\eta^2 + P_{\eta'}^2). \tag{9}$$

The transition probability of the emission of a q constituent quark to all the possible $q' = u$, d, s quarks along with GBs $(q\bar{q}')$, $P(q\to q\bar{q}' + q')$, as calculated from the Lagrangian, can be expressed as

$$\begin{aligned} P(u\to u\bar{q}' + q') = {}& \frac{P_\pi^2}{36}[63 + 36P_K^2 + 6P_\eta + 7P_\eta^2 + 4P_\eta P_{\eta'} + 12P_{\eta'} + 16P_{\eta'}^2]u \\ & + \frac{P_\pi^2}{36}[9 + 6P_\eta + P_\eta^2 + 4P_\eta P_{\eta'} + 12P_{\eta'} + 4P_{\eta'}^2]\bar{u} \\ & + \frac{P_\pi^2}{36}[45 - 6P_\eta + P_\eta^2 + 4P_\eta P_{\eta'} - 12P_{\eta'} + 4P_{\eta'}^2](d+\bar{d}) \\ & + \frac{P_\pi^2}{9}[9P_K^2 + P_\eta^2 - 2P_\eta P_{\eta'} + P_{\eta'}^2](s+\bar{s}), \end{aligned} \tag{10}$$

$$
\begin{aligned}
P(d \to d\bar{q}' + q') = {} & \frac{P_\pi^2}{36}[45 - 6P_\eta + P_\eta^2 + 4P_\eta P_{\eta'} - 12P_{\eta'} + 4P_{\eta'}^2](u + \bar{u}) \\
& + \frac{P_\pi^2}{36}[63 + 36P_K^2 + 6P_\eta + 7P_\eta^2 + 4P_\eta P_{\eta'} + 12P_{\eta'} + 16P_{\eta'}^2]d \\
& + \frac{P_\pi^2}{36}[9 + 6P_\eta + P_\eta^2 + 4P_\eta P_{\eta'} + 12P_{\eta'} + 4P_{\eta'}^2]\bar{d} \\
& + \frac{P_\pi^2}{9}[9P_K^2 + P_\eta^2 - 2P_\eta P_{\eta'} + P_{\eta'}^2](s + \bar{s}), \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
P(s \to s\bar{q}' + q') = {} & \frac{P_\pi^2}{9}[9P_K^2 + P_\eta^2 - 2P_\eta P_{\eta'} + P_{\eta'}^2](u + \bar{u} + d + \bar{d}) \\
& + \frac{P_\pi^2}{9}[18P_K^2 + 10P_\eta^2 + 4P_\eta P_{\eta'} + 4P_{\eta'}^2]s \\
& + \frac{P_\pi^2}{9}[4P_\eta^2 + 4P_\eta P_{\eta'} + P_{\eta'}^2]\bar{s}. \qquad (12)
\end{aligned}
$$

The flavor structures for the spin $-\frac{3}{2}^+$ decuplet baryons are expressed as

$$
\begin{aligned}
\Delta^{++}(uuu) &= 3P(u \to GB)u + 3P(u \to u\bar{q}' + q'), \\
\Delta^{+}(uud) &= 2P(u \to GB)u + P(d \to GB)d + 2P(u \to u\bar{q}' + q') + P(d \to d\bar{q}' + q'), \\
\Delta^{0}(udd) &= P(u \to GB)u + 2P(d \to GB)d + P(u \to u\bar{q}' + q') + 2P(d \to d\bar{q}' + q'), \\
\Delta^{-}(ddd) &= 3P(d \to GB)d + 3P(d \to d\bar{q}' + q'), \\
\Sigma^{*+}(uus) &= 2P(u \to GB)u + P(s \to GB)s + 2P(u \to u\bar{q}' + q') + P(s \to s\bar{q}' + q'), \\
\Sigma^{*0}(uds) &= P(u \to GB)u + P(d \to GB)d + P(s \to GB)s \\
&\quad + P(u \to u\bar{q}' + q') + P(d \to d\bar{q}' + q') + P(s \to s\bar{q}' + q'), \\
\Sigma^{*-}(dds) &= 2P(d \to GB)d + P(s \to GB)s + 2P(d \to d\bar{q}' + q') + P(s \to s\bar{q}' + q'), \\
\Xi^{*0}(uss) &= P(u \to GB)u + 2P(s \to GB)s + P(u \to u\bar{q}' + q') + 2P(s \to s\bar{q}' + q'), \\
\Xi^{*-}(dss) &= P(d \to GB)d + 2P(s \to GB)s + P(d \to d\bar{q}' + q') + 2P(s \to s\bar{q}' + q'), \\
\Omega^{-}(sss) &= 3P(s \to GB)s + 3P(s \to s\bar{q}' + q'). \qquad (13)
\end{aligned}
$$

III. QUARK FLAVOR DISTRIBUTIONS

The quark flavor distributions of the spin $-\frac{3}{2}^+$ decuplet baryons can be evaluated using the scalar matrix elements which are in general defined as follows [43]:

$$
\widehat{B^{*\frac{3}{2}^+}} \equiv \langle B^{*\frac{3}{2}^+} | \mathcal{N}_{q^+q^-} | B^{*\frac{3}{2}^+} \rangle, \qquad (14)
$$

where $|B^{*\frac{3}{2}^+}\rangle$ is the SU(6) wave function (detailed in Ref. [53]) and $\mathcal{N}_{q\bar{q}}$ is the number operator measuring the sum of the quark and antiquark numbers,

$$
\begin{aligned}
\mathcal{N}_{q\bar{q}} &= \sum_{q=u,d,s} (n_q q + n_{\bar{q}} \bar{q}) \\
&= n_u u + n_{\bar{u}} \bar{u} + n_d d + n_{\bar{d}} \bar{d} + n_s s + n_{\bar{s}} \bar{s}, \qquad (15)
\end{aligned}
$$

with the coefficients $n_{q(\bar{q})}$ being the number of $q(\bar{q})$ quarks with electric charge $e_q(e_{\bar{q}})$.

The quark flavor distributions receive contribution from the constituent as well as the sea quark distributions as follows:

$$
\widehat{B^{*\frac{3}{2}^+}} = q_{\rm Const}^{B^*} + q_{\rm Sea}^{B^*}. \qquad (16)
$$

The sea quarks coming from the GB fluctuation process comprise quark as well as antiquark distributions. We can therefore express the sea quark distributions as the sum of quark and antiquark distributions as

$$
\widehat{B^{*\frac{3}{2}^+}} = q_{\rm Const}^{B^*} + [q^{B^*} + \bar{q}^{B^*}]_{\rm Sea}. \qquad (17)
$$

TABLE I. The normalization conditions for the constituent quark distributions of the spin $-\frac{3}{2}^{+}$ decuplet baryons.

Baryon	$\int_0^1 u_{\text{Const}}^{B^*}(x)dx$	$\int_0^1 d_{\text{Const}}^{B^*}(x)dx$	$\int_0^1 s_{\text{Const}}^{B^*}(x)dx$
$\Delta^{++}(uuu)$	3	0	0
$\Delta^{+}(uud)$	2	1	0
$\Delta^{0}(udd)$	1	2	0
$\Delta^{-}(ddd)$	0	3	0
$\Sigma^{*+}(uus)$	2	0	1
$\Sigma^{*0}(uds)$	1	1	1
$\Sigma^{*-}(dds)$	0	2	1
$\Xi^{*0}(uss)$	1	0	2
$\Xi^{*-}(dss)$	0	1	2
$\Omega^{-}(sss)$	0	0	3

The quark flavor distributions now have mainly two parts: (i) q quark contribution from the constituent quarks as well as q quark contribution from the sea quarks and (ii) $\bar{q}$ antiquark contribution purely from the sea quarks. We have

$$\widehat{B^{*\frac{3}{2}^{+}}} = q^{B^*} + \bar{q}^{B^*}. \tag{18}$$

The normalization conditions integrated over the Bjorken variable x for the constituent quark distribution functions of the spin $-\frac{3}{2}^{+}$ decuplet baryons can be summarized in Table I.

The antiquark densities of the spin $-\frac{3}{2}^{+}$ decuplet baryons, which are basically the coefficients $n_{\bar{q}}$ of the number of $\bar{q}$ quarks, can easily be calculated using Eqs. (8)–(13). The results have been presented for $\Delta^{++}(uuu)$, $\Delta^{+}(uud)$, $\Delta^{0}(udd)$, $\Delta^{-}(ddd)$, $\Sigma^{*+}(uus)$, $\Sigma^{*0}(uds)$, $\Sigma^{*-}(dds)$, $\Xi^{*0}(uss)$, $\Xi^{*-}(dss)$, and $\Omega^{-}(sss)$ in Table II.

Another important quantity which is relevant to understand the flavor structure of the spin $-\frac{3}{2}^{+}$ decuplet baryons is the fraction of particular quark and antiquark present in a baryon relative to the total number of the quarks and antiquarks. It determines the explicit amount of sea quarks present in the baryon in comparison to the constituent quarks. We have

$$F_q^{B^*} = \frac{q^{B^*} + \bar{q}^{B^*}}{\sum(q^{B^*} + \bar{q}^{B^*})}, \tag{19}$$

where q^{B^*} and $\bar{q}^{B^*}$ are the number of quarks and antiquarks for the decuplet baryons, and $\sum(q^{B^*} + \bar{q}^{B^*})$ is the sum of all the quarks and antiquarks present in a particular baryon. Further, suppression factors (ρ^{B^*} and κ^{B^*}) which give the strange quark content present with respect to the non-strange quarks are important, particularly in the case of Δ baryons, as the strange quarks are otherwise not present in the Δ baryons. They come only from the quark sea and are hence important to understand the internal structure. We have

TABLE II. The sea quark (antiquark) distribution functions for the spin $-\frac{3}{2}^{+}$ decuplet baryons.

Baryon	$\bar{u}_S^{B^*}$	$\bar{d}_S^{B^*}$	$\bar{s}_S^{B^*}$
$\Delta^{++}(uuu)$	$\frac{P_\pi^2}{12}[9+6P_\eta+P_\eta^2+4P_\eta P_{\eta'}+12P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{12}[45-6P_\eta+P_\eta^2+4P_\eta P_{\eta'}-12P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{3}[9P_K^2+P_\eta^2-2P_\eta P_{\eta'}+P_{\eta'}^2]$
$\Delta^{+}(uud)$	$\frac{P_\pi^2}{12}[21+2P_\eta+P_\eta^2+4P_\eta P_{\eta'}+4P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{12}[33-2P_\eta+P_\eta^2+4P_\eta P_{\eta'}-4P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{3}[9P_K^2+P_\eta^2-2P_\eta P_{\eta'}+P_{\eta'}^2]$
$\Delta^{0}(udd)$	$\frac{P_\pi^2}{12}[33-2P_\eta+P_\eta^2+4P_\eta P_{\eta'}-4P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{12}[21+2P_\eta+P_\eta^2+4P_\eta P_{\eta'}+4P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{3}[9P_K^2+P_\eta^2-2P_\eta P_{\eta'}+P_{\eta'}^2]$
$\Delta^{-}(ddd)$	$\frac{P_\pi^2}{12}[45-6P_\eta+P_\eta^2+4P_\eta P_{\eta'}-12P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{12}[9+6P_\eta+P_\eta^2+4P_\eta P_{\eta'}+12P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{3}[9P_K^2+P_\eta^2-2P_\eta P_{\eta'}+P_{\eta'}^2]$
$\Sigma^{*+}(uus)$	$\frac{P_\pi^2}{6}[3+6P_K^2+2P_\eta+P_\eta^2+4P_{\eta'}+2P_{\eta'}^2]$	$\frac{P_\pi^2}{6}[15+6P_K^2-2P_\eta+P_\eta^2-4P_{\eta'}+2P_{\eta'}^2]$	$\frac{P_\pi^2}{3}[6P_K^2+2P_\eta^2+P_{\eta'}^2]$
$\Sigma^{*0}(uds)$	$\frac{P_\pi^2}{6}[9+6P_K^2+P_\eta^2+2P_{\eta'}^2]$	$\frac{P_\pi^2}{6}[9+6P_K^2+P_\eta^2+2P_{\eta'}^2]$	$\frac{P_\pi^2}{3}[6P_K^2+2P_\eta^2+P_{\eta'}^2]$
$\Sigma^{*-}(dds)$	$\frac{P_\pi^2}{6}[15+6P_K^2-2P_\eta+P_\eta^2-4P_{\eta'}+2P_{\eta'}^2]$	$\frac{P_\pi^2}{6}[3+6P_K^2+2P_\eta+P_\eta^2+4P_{\eta'}+2P_{\eta'}^2]$	$\frac{P_\pi^2}{3}[6P_K^2+2P_\eta^2+P_{\eta'}^2]$
$\Xi^{*0}(uss)$	$\frac{P_\pi^2}{12}[3+24P_K^2+2P_\eta+3P_\eta^2-4P_\eta P_{\eta'}+4P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{12}[15+24P_K^2-2P_\eta+3P_\eta^2-4P_\eta P_{\eta'}-4P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{3}[3P_K^2+3P_\eta^2+2P_\eta P_{\eta'}+P_{\eta'}^2]$
$\Xi^{*-}(dss)$	$\frac{P_\pi^2}{12}[15+24P_K^2-2P_\eta+3P_\eta^2-4P_\eta P_{\eta'}-4P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{12}[3+24P_K^2+2P_\eta+3P_\eta^2-4P_\eta P_{\eta'}+4P_{\eta'}+4P_{\eta'}^2]$	$\frac{P_\pi^2}{3}[3P_K^2+3P_\eta^2+2P_\eta P_{\eta'}+P_{\eta'}^2]$
$\Omega^{-}(sss)$	$\frac{P_\pi^2}{9}[9P_K^2+P_\eta^2-2P_\eta P_{\eta'}+P_{\eta'}^2]$	$\frac{P_\pi^2}{9}[9P_K^2+P_\eta^2-2P_\eta P_{\eta'}+P_{\eta'}^2]$	$\frac{P_\pi^2}{9}[4P_\eta^2+4P_\eta P_{\eta'}+P_{\eta'}^2]$

$$\rho_s^{B^*} = \frac{s^{B^*} + \bar{s}^{B^*}}{u^{B^*} + d^{B^*}},$$
$$\kappa_s^{B^*} = \frac{s^{B^*} + \bar{s}^{B^*}}{\bar{u}^{B^*} + \bar{d}^{B^*}}. \tag{20}$$

To numerically calculate the phenomenological quantities pertaining to the quark flavor distribution functions, we have to first fix the probabilities of fluctuations to pions, K, η, η' which have already been defined in χCQM. These parameters can be fixed by taking into account the physical considerations involving the GBs. To start with, it is important to fix a hierarchy for the transition probabilities. Based on the scaling of the quark contributions $\frac{1}{M_q^2}$, a constraint can be fixed as

$$P_\pi^2 > P_\pi^2 P_K^2 > P_\pi^2 P_\eta^2 > P_\pi^2 P_{\eta'}^2. \tag{21}$$

Since no experimental data are available for the case of spin $-\frac{3}{2}^+$ decuplet baryons, we will use the same set of parameters as used for the case of flavor distribution functions of the nucleon [54]. The input parameters used in the present work are

$$P_\pi^2 = 0.114, \qquad P_K^2 = 0.202,$$
$$P_\eta^2 = 0.202, \qquad P_{\eta'}^2 = 0.562. \tag{22}$$

Using the above set of parameters, the sea quark flavor distribution functions and related flavor-dependent functions for the case of $\Delta^{++}(uuu)$, $\Delta^{+}(uud)$, $\Delta^{0}(udd)$, $\Delta^{-}(ddd)$, $\Sigma^{*+}(uus)$, $\Sigma^{*0}(uds)$, $\Sigma^{*-}(dds)$, $\Xi^{*0}(uss)$, $\Xi^{*-}(dss)$, and $\Omega^{-}(sss)$ baryons have been presented in Table III.

In order to study the results in χCQM and its advantage over the naive quark models, we first need to compare the results for the quantities in the NQM where only the constituent quarks contribute. NQM, which is quite successful in explaining a good deal of low energy data [6–8], has the following predictions for the above mentioned quantities:

$$\bar{u}^{B^*} - \bar{d}^{B^*} = 0,$$
$$\bar{u}^{B^*}/\bar{d}^{B^*} = -,$$
$$F_s^{\Delta} = 0,$$
$$\rho_s^{\Delta} = 0,$$
$$\kappa_s^{\Delta} = 0,$$
$$F_u^{\Omega} = F_d^{\Omega} = 0,$$
$$\rho_u^{\Omega} = \rho_d^{\Omega} = 0,$$
$$\kappa_u^{\Omega} = \kappa_d^{\Omega} = 0. \tag{23}$$

The results for the χCQM given in Table III clearly indicate that the Δ^{++} and Δ^{+} quark sea contains more number of $\bar{d}^{\Delta}$ quarks than the $\bar{u}^{\Delta}$ quarks. The results for Δ^{-} and Δ^{0} are reversed as expected from isospin asymmetry. The ratio $\bar{u}^{\Delta}/\bar{d}^{\Delta}$ for Δ^{++}, Δ^{+}, Δ^{0}, and Δ^{-} is 0.660, 0.872, 1.146, and 1.515, respectively. Since the $\bar{u}^{\Delta}$ ($\bar{d}^{\Delta}$) sea quarks increase (decrease) with the decreasing (increasing) u (d) constituent quarks in the Δ baryons, the ratio therefore increases as the u constituent quark decreases. This is because in the case of Δ baryons the production of π^+ is favored over π^- and hence the $\bar{u}^{\Delta} - \bar{d}^{\Delta}$ asymmetry. Similarly, in the case of Σ^{*+}, Σ^{*0}, and Σ^{*-}, the ratio is 0.692, 1.000, and 1.445, respectively. For Ξ^{*0} and Ξ^{*-}, it is 0.759 and 1.317, respectively. Future experiments to measure the quark content of the spin $-\frac{3}{2}^+$ decuplet baryons would not only justify the quark sea asymmetry

TABLE III. The χCQM results for the sea quark flavor distribution functions and related flavor-dependent functions for the spin $-\frac{3}{2}^+$ decuplet baryons.

	B^*									
Quantity	$\Delta^{++}(uuu)$	$\Delta^{+}(uud)$	$\Delta^{0}(udd)$	$\Delta^{-}(ddd)$	$\Sigma^{*+}(uus)$	$\Sigma^{*0}(uds)$	$\Sigma^{*-}(dds)$	$\Xi^{*0}(uss)$	$\Xi^{*-}(dss)$	$\Omega^{-}(sss)$
$\bar{u}^{B^*}$	0.232	0.272	0.312	0.352	0.179	0.219	0.259	0.126	0.166	0.024
$\bar{d}^{B^*}$	0.352	0.312	0.272	0.232	0.259	0.219	0.179	0.166	0.126	0.024
$\bar{s}^{B^*}$	0.136	0.136	0.136	0.136	0.083	0.083	0.083	0.093	0.093	0.034
$\bar{u}^{B^*}/\bar{d}^{B^*}$	0.660	0.872	1.146	1.515	0.692	1	1.445	0.759	1.317	1
$\bar{u}^{B^*} - \bar{d}^{B^*}$	−0.120	−0.040	0.040	0.120	−0.080	0	0.080	−0.040	0.040	0
$F_u^{B^*} = \frac{u^{B^*}+\bar{u}^{B^*}}{\sum_q (q^{B^*}+\bar{q}^{B^*})}$	0.868	0.611	0.352	0.095	0.618	0.346	0.073	0.332	0.049	0.008
$F_d^{B^*} = \frac{d^{B^*}+\bar{d}^{B^*}}{\sum_q (q^{B^*}+\bar{q}^{B^*})}$	0.095	0.352	0.611	0.868	0.073	0.346	0.618	0.049	0.332	0.008
$F_s^{B^*} = \frac{s^{B^*}+\bar{s}^{B^*}}{\sum_q (q^{B^*}+\bar{q}^{B^*})}$	0.037	0.037	0.037	0.037	0.307	0.307	0.307	0.618	0.618	0.984
$\rho_s^{B^*} = \frac{s^{B^*}+\bar{s}^{B^*}}{u^{B^*}+d^{B^*}}$	0.076	0.076	0.076	0.076	0.339	0.339	0.339	0.664	0.664	1.006
$\kappa_s^{B^*} = \frac{s^{B^*}+\bar{s}^{B^*}}{\bar{u}^{B^*}+\bar{d}^{B^*}}$	0.467	0.467	0.467	0.467	2.658	2.568	2.568	7.493	7.493	63.485

present but also strengthen the qualitative and quantitative roles of the sea quarks in understanding the dynamics of the constituents of the baryons.

For the case of fraction of a particular quark present in the baryon $F_q^\Delta = \frac{q^\Delta + \bar{q}^\Delta}{\sum_q (q^\Delta + \bar{q}^\Delta)}$, we find that F_u^Δ is maximum for Δ^{++} which has maximum number of u quarks in the constituent structure and F_d^Δ is maximum for Δ^- which has maximum number of d quarks in the constituent structure. The strange quark fraction F_s^Δ is however same for all the Δ baryons which is evident from the production of $\bar{s}$ sea quarks. The strange quark fraction is zero in the case of NQM. Similarly, the other quantities giving the strange quark content with respect to nonstrange quarks are ρ_s^Δ and κ_s^Δ. These are predicted to be zero in NQM, but in χCQM we have their values as 0.076 and 0.467, respectively. Similar conclusions can be drawn for the case of Σ^{*+}, Σ^{*0}, Σ^{*-}, Ξ^{*0}, and Ξ^{*-} baryons. However, the case of $\Omega^-(sss)$ is also interesting as it contains only the strange quarks. It is evident from the results that $F_s^{\Omega^-}$ dominates and also $\rho_s^{\Omega^-}$ and $\kappa_s^{\Omega^-}$ are very large as compared to the other baryons. In fact, the study of u and d quarks would be interesting in this case. Whether these predicted values are correct can be tested from future measurements as well as by studying the implications chiral symmetry breaking and SU(3) symmetry breaking. This will help us to understand the nonperturbative features of QCD and its subtle features.

IV. FLAVOR STRUCTURE FUNCTIONS AND THE GOTTFRIED INTEGRAL

The quark sea asymmetry of the unpolarized quarks can be established by measuring the deviation from the Gottfried sum rule [14] which can further be obtained through the Gottfried integral (I_G). This integral can be obtained from the flavor structure functions F_1 and F_2 which can be defined as

$$F_2(x) = x\sum_{u,d,s} e_q^2[q(x) + \bar{q}(x)], \tag{24}$$

$$F_1(x) = \frac{1}{2x} F_2(x). \tag{25}$$

The x dependence cannot be incorporated in χCQM in a straightforward manner using an *ab initio* approach; however, it can be done phenomenologically [7,8,55]. The deviation from the GSR obtained from the structure functions of decuplet baryons can be measured through the Gottfried integral for the spin $-\frac{3}{2}^+$ decuplet baryons $I_G^{\Delta^{++}\Delta^-}$, $I_G^{\Delta^+\Delta^0}$, $I_G^{\Sigma^{*+}\Sigma^{*-}}$, and $I_G^{\Xi^{*0}\Xi^{*-}}$ giving the asymmetry between the $\bar{u}$ and $\bar{d}$ sea quarks for the case of decuplet baryons.

The structure function F_2 for Δ^{++}, Δ^+, Δ^0, Δ^-, Σ^{*+}, Σ^{*-}, Ξ^{*0}, and Ξ^{*-} can be expressed using Eq. (18) and we have

$$\begin{aligned}
F_2^{\Delta^{++}}(x) &= \frac{4}{9}x(u^{\Delta^{++}}(x) + \bar{u}^{\Delta^{++}}(x)) \\
&\quad + \frac{1}{9}x(d^{\Delta^{++}}(x) + \bar{d}^{\Delta^{++}}(x) + s^{\Delta^{++}}(x) + \bar{s}^{\Delta^{++}}(x)), \\
F_2^{\Delta^{+}}(x) &= \frac{4}{9}x(u^{\Delta^{+}}(x) + \bar{u}^{\Delta^{+}}(x)) \\
&\quad + \frac{1}{9}x(d^{\Delta^{+}}(x) + \bar{d}^{\Delta^{+}}(x) + s^{\Delta^{+}}(x) + \bar{s}^{\Delta^{+}}(x)), \\
F_2^{\Delta^{0}}(x) &= \frac{4}{9}x(u^{\Delta^{0}}(x) + \bar{u}^{\Delta^{0}}(x)) \\
&\quad + \frac{1}{9}x(d^{\Delta^{0}}(x) + \bar{d}^{\Delta^{0}}(x) + s^{\Delta^{0}}(x) + \bar{s}^{\Delta^{0}}(x)), \\
F_2^{\Delta^{-}}(x) &= \frac{4}{9}x(u^{\Delta^{-}}(x) + \bar{u}^{\Delta^{-}}(x)) \\
&\quad + \frac{1}{9}x(d^{\Delta^{-}}(x) + \bar{d}^{\Delta^{-}}(x) + s^{\Delta^{-}}(x) + \bar{s}^{\Delta^{-}}(x)), \\
F_2^{\Sigma^{*+}}(x) &= \frac{4}{9}x(u^{\Sigma^{*+}}(x) + \bar{u}^{\Sigma^{*+}}(x)) \\
&\quad + \frac{1}{9}x(d^{\Sigma^{*+}}(x) + \bar{d}^{\Sigma^{*+}}(x) + s^{\Sigma^{*+}}(x) + \bar{s}^{\Sigma^{*+}}(x)), \\
F_2^{\Sigma^{*-}}(x) &= \frac{4}{9}x(u^{\Sigma^{*-}}(x) + \bar{u}^{\Sigma^{*-}}(x)) \\
&\quad + \frac{1}{9}x(d^{\Sigma^{*-}}(x) + \bar{d}^{\Sigma^{*-}}(x) + s^{\Sigma^{*-}}(x) + \bar{s}^{\Sigma^{*-}}(x)), \\
F_2^{\Xi^{*0}}(x) &= \frac{4}{9}x(u^{\Xi^{*0}}(x) + \bar{u}^{\Xi^{*0}}(x)) \\
&\quad + \frac{1}{9}x(d^{\Xi^{*0}}(x) + \bar{d}^{\Xi^{*0}}(x) + s^{\Xi^{*0}}(x) + \bar{s}^{\Xi^{*0}}(x)), \\
F_2^{\Xi^{*-}}(x) &= \frac{4}{9}x(u^{\Xi^{*-}}(x) + \bar{u}^{\Xi^{*-}}(x)) \\
&\quad + \frac{1}{9}x(d^{\Xi^{*-}}(x) + \bar{d}^{\Xi^{*-}}(x) + s^{\Xi^{*-}}(x) + \bar{s}^{\Xi^{*-}}(x)).
\end{aligned} \tag{26}$$

The Gottfried integrals for $\Delta^{++}\Delta^-$, $\Delta^+\Delta^0$, $\Sigma^{*+}\Sigma^{*-}$, and $\Xi^{*0}\Xi^{*-}$ can be expressed as follows:

$$\begin{aligned}
I_G^{\Delta^{++}\Delta^-} &\equiv \int_0^1 \frac{F_2^{\Delta^{++}}(x) - F_2^{\Delta^-}(x)}{x} dx = 1 + \frac{1}{3}[\bar{u}^{\Delta^{++}} - \bar{d}^{\Delta^{++}}], \\
I_G^{\Delta^{+}\Delta^0} &\equiv \int_0^1 \frac{F_2^{\Delta^{+}}(x) - F_2^{\Delta^0}(x)}{x} dx = \frac{1}{3} + \frac{1}{3}[\bar{u}^{\Delta^{+}} - \bar{d}^{\Delta^{+}}], \\
I_G^{\Sigma^{*+}\Sigma^{*-}} &\equiv \int_0^1 \frac{F_2^{\Sigma^{*+}}(x) - F_2^{\Sigma^{*-}}(x)}{x} dx = \frac{2}{3} + \frac{1}{3}[\bar{u}^{\Sigma^{*+}} - \bar{d}^{\Sigma^{*+}}], \\
I_G^{\Xi^{*0}\Xi^{*-}} &\equiv \int_0^1 \frac{F_2^{\Xi^{*0}}(x) - F_2^{\Xi^{*-}}(x)}{x} dx = \frac{1}{3} + \frac{1}{3}[\bar{u}^{\Xi^{*0}} - \bar{d}^{\Xi^{*0}}].
\end{aligned} \tag{27}$$

The normalization conditions for the valence quarks used to derive the above equations have been taken from Table I, whereas the sea quark contributions corresponding to each baryon obey the following normalization conditions:

$$\int_0^1 \bar{d}^{\Delta^{++}}(x)dx = \int_0^1 \bar{u}^{\Delta^-}(x)dx,$$
$$\int_0^1 \bar{u}^{\Delta^{++}}(x)dx = \int_0^1 \bar{d}^{\Delta^-}(x)dx,$$
$$\int_0^1 \bar{d}^{\Delta^{+}}(x)dx = \int_0^1 \bar{u}^{\Delta^0}(x)dx,$$
$$\int_0^1 \bar{u}^{\Delta^{+}}(x)dx = \int_0^1 \bar{d}^{\Delta^0}(x)dx,$$
$$\int_0^1 \bar{d}^{\Sigma^{*+}}(x)dx = \int_0^1 \bar{u}^{\Sigma^{*-}}(x)dx,$$
$$\int_0^1 \bar{u}^{\Sigma^{*+}}(x)dx = \int_0^1 \bar{d}^{\Sigma^{*-}}(x)dx,$$
$$\int_0^1 \bar{d}^{\Xi^{*0}}(x)dx = \int_0^1 \bar{u}^{\Xi^{*-}}(x)dx,$$
$$\int_0^1 \bar{u}^{\Xi^{*0}}(x)dx = \int_0^1 \bar{d}^{\Xi^{*-}}(x)dx. \tag{28}$$

The numerical values by taking into account the $\bar{d}(x) - \bar{u}(x)$ asymmetry are given as follows:

$$I_G^{\Delta^{++}\Delta^-} = 0.96,$$
$$I_G^{\Delta^{+}\Delta^0} = 0.32,$$
$$I_G^{\Sigma^{*+}\Sigma^{*-}} = 0.64,$$
$$I_G^{\Xi^{*0}\Xi^{*-}} = 0.32. \tag{29}$$

The quark sea asymmetry $\int_0^1(\bar{d}(x) - \bar{u}(x))dx$ has been measured for the case of nucleon in the NMC and E866 experiments [13,15], and the results of χCQM are in very good agreement with the experiments [52]. The measurement for quark sea asymmetry and the violation of Gottfried sum rule in future experiments for the case of Δ, Σ, and Ξ baryons are needed for profound understanding of not only the nonperturbative properties of QCD but also to understand the important role of the sea quarks at low value of x.

V. SUMMARY AND CONCLUSIONS

To summarize, in an attempt to understand the dynamics and the constituents of the baryon from the DIS results, the quark flavor distribution functions of the spin $-\frac{3}{2}^+$ Δ baryons [$\Delta^{++}(uuu)$, $\Delta^{+}(uud)$, $\Delta^{0}(udd)$, $\Delta^{-}(ddd)$, $\Sigma^{*+}(uus)$, $\Sigma^{*0}(uds)$, $\Sigma^{*-}(dds)$, $\Xi^{*0}(uss)$, $\Xi^{*-}(dss)$, and $\Omega^{-}(sss)$] have been phenomenologically estimated in the chiral χCQM. The internal structure of the baryons constitutes a major challenge for any model trying to explain the nonperturbative regime of QCD, and χCQM is one of the most successful phenomenological models which allows a simple and intuitive method to investigate these quantities. The quark sea generation through the chiral fluctuations has important implications for the sea quark contributions, chiral symmetry breaking, as well as SU(3) symmetry breaking. The explicit contributions of the constituent and sea quark flavor distribution functions have been computed for each spin $-\frac{3}{2}^+$ decuplet baryon to test the mechanism for the quark sea generation and to quantitatively understand the role of sea quarks in a baryon. The implications of this model have been studied for the quark sea asymmetries $\bar{d} - \bar{u}$ and quark sea ratios $\bar{d}/\bar{u}$. The quark sea asymmetry is clearly evident from the results which is because of the favored production of π^+ over π^-. For the case of fraction of a particular quark present in the baryon $F_q^{B^*} = \frac{q^{B^*} + \bar{q}^{B^*}}{\sum_q (q^{B^*} + \bar{q}^{B^*})}$, we find that the fraction is directly dependent on the constituent quark structure. The strange quark fraction $F_s^{B^*}$ is however same for each set of the decuplet baryon (Δ, Σ, Ξ and Ω) which is evident from the production of $\bar{s}$ sea quarks. Further, the calculations have been extended to predict the suppression factors corresponding to the strange quark content with respect to the nonstrange quarks $\rho_s^{B^*}$ and $\kappa_s^{B^*}$. These are predicted to be zero in NQM for the case of Δ baryons, but in χCQM they have significant contributions. On the other hand, these quantities are very large for the case of Ω baryons as there are only s quarks in the constituent structure. These quantities are important as they explicitly describe the role of nonconstituent degrees of freedom and also provide important constraints on the future experiments to measure the structure of decuplet baryons. Future experiments to measure the quark content of the spin $-\frac{3}{2}^+$ decuplet baryons would not only justify the quark sea asymmetry present but also strengthen the qualitative and quantitative roles of the sea quarks in understanding the dynamics of the constituents of the baryons and the subtle features of χCQM which include chiral symmetry breaking and the weakly interacting Goldstone bosons as the appropriate degrees of freedom. This will help us to understand the nonperturbative features of QCD and its subtle features.

ACKNOWLEDGMENTS

H. D. would like to thank the Department of Science and Technology (Ref. No. MTR/2019/000003) Government of India for financial support.

[1] E. D. Bloom *et al.*, Phys. Rev. Lett. **23**, 930 (1969); M. Breidenbach, J. I. Friedman, H. W. Kendall, E. D. Bloom, D. H. Coward, H. DeStaebler, J. Drees, L. W. Mo, and R. E. Taylor, Phys. Rev. Lett. **23**, 935 (1969).
[2] J. Ashman *et al.* (EMC Collaboration), Phys. Lett. B **206**, 364 (1988); Nucl. Phys. **B328**, 1 (1989).
[3] B. Adeva *et al.* (SMC Collaboration), Phys. Rev. D **58**, 112001 (1998).
[4] P. Adams *et al.*, Phys. Rev. D **56**, 5330 (1997); P. L. Anthony *et al.* (E142 Collaboration), Phys. Rev. Lett. **71**, 959 (1993); K. Abe *et al.* (E143 Collaboration), Phys. Rev. Lett. **76**, 587 (1996); K. Abe *et al.* (E154 Collaboration), Phys. Rev. Lett. **79**, 26 (1997).
[5] A. Airapetian *et al.* (HERMES Collaboration), Phys. Rev. D **71**, 012003 (2005).
[6] A. De Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).
[7] N. Isgur, G. Karl, and R. Koniuk, Phys. Rev. Lett. **41**, 1269 (1978); N. Isgur and G. Karl, Phys. Rev. D **21**, 3175 (1980); N. Isgur, G. Karl, and J. Soffer, Phys. Rev. D **35**, 1665 (1987); P. Geiger and N. Isgur, Phys. Rev. D **55**, 299 (1997); N. Isgur, Phys. Rev. D **59**, 034013 (1999).
[8] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Phys. Rev. D **12**, 2137 (1975); **15**, 844 (1977).
[9] D. T. Spayde *et al.* (SAMPLE Collaboration), Phys. Lett. B **583**, 79 (2004).
[10] D. Armstrong *et al.* (G0 Collaboration), Phys. Rev. Lett. **95**, 092001 (2005). D. Androić *et al.* (G0 Collaboration), Phys. Rev. Lett. **104**, 012001 (2010).
[11] F. E. Maas *et al.* (PVA4 Collaboration), Phys. Rev. Lett. **93**, 022002 (2004); **94**, 152001 (2005).
[12] K. A. Aniol *et al.* (HAPPEX Collaboration), Phys. Rev. C **69**, 065501 (2004); Phys. Rev. Lett. **98**, 032301 (2007); Eur. Phys. J. A **31**, 597 (2007); Z. Ahmed *et al.* (HAPPEX Collaboration), Phys. Rev. Lett. **108**, 102001 (2012).
[13] P. Amaudruz *et al.* (New Muon Collaboration), Phys. Rev. Lett. **66**, 2712 (1991); M. Arneodo *et al.* (New Muon Collaboration), Phys. Rev. D **50**, R1 (1994).
[14] K. Gottfried, Phys. Rev. Lett. **18**, 1174 (1967).
[15] E. A. Hawker *et al.* (E866/NuSea Collaboration), Phys. Rev. Lett. **80**, 3715 (1998); J. C. Peng *et al.* (E866/NuSea Collaboration), Phys. Rev. D **58**, 092004 (1998); R. S. Towell *et al.* (E866/NuSea Collaboration), Phys. Rev. D **64**, 052002 (2001).
[16] A. Baldit *et al.* (NA51 Collaboration), Phys. Lett. B **253**, 252 (1994).
[17] K. Ackerstaff *et al.* (HERMES Collaboration), Phys. Rev. Lett. **81**, 5519 (1998).
[18] J. D. Sullivan, Phys. Rev. D **5**, 1732 (1972); A. W. Thomas, Phys. Lett. **126B**, 97 (1983); J. Magnin and H. R. Christiansen, Phys. Rev. D **61**, 054006 (2000).
[19] S. J. Brodsky, J. R. Ellis, and M. Karliner, Phys. Lett. B **206**, 309 (1988).
[20] R. Alkofer, H. Reinhardt, and H. Weigel, Phys. Rep. **265**, 139 (1996).
[21] K. Goeke, C. V. Christov, and A. Blotz, Prog. Part. Nucl. Phys. **36**, 207 (1996); C. V. Christov, A. Blotz, H.-C. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, and K. Goeke, Prog. Part. Nucl. Phys. **37**, 91 (1996).
[22] D. Diakonov, V. Yu. Petrov, P. V. Pobylitsa, M. V. Polyakov, and C. Weiss, Phys. Rev. D **56**, 4069 (1997); **58**, 038502 (1998).
[23] M. Alberg, E. M. Henley, and G. A. Miller, Phys. Lett. B **471**, 396 (2000); S. Kumano and M. Miyama, Phys. Rev. D **65**, 034012 (2002); F.-G. Cao and A. I. Signal, Phys. Rev. D **68**, 074002 (2003); F. Huang, R.-G. Xu, and B.-Q. Ma, Phys. Lett. B **602**, 67 (2004); B. Pasquini and S. Boffi, Nucl. Phys. **A782**, 86 (2007).
[24] M. Wakamatsu, Phys. Rev. D **44**, R2631 (1991); **46**, 3762 (1992); H. Weigel, L. Gamberg, and H. Reinhardt, Phys. Rev. D **55**, 6910 (1997); M. Wakamatsu and T. Kubota, Phys. Rev. D **57**, 5755 (1998); M. Wakamatsu, Phys. Rev. D **67**, 034005 (2003).
[25] Y. Ding, R.-G. Xu, and B.-Q. Ma, Phys. Rev. D **71**, 094014 (2005); L. Shao, Y.-J. Zhang, and B.-Q. Ma, Phys. Lett. B **686**, 136 (2010).
[26] L. A. Trevisan, C. Mirez, T. Frederico, and L. Tomio, Eur. Phys. J. C **56**, 221 (2008); Y. Zhang, L. Shao, and B.-Q. Ma, Phys. Lett. B **671**, 30 (2009); Nucl. Phys. **A828**, 390 (2009).
[27] A. I. Signal and A. W. Thomas, Phys. Rev. D **40**, 2832 (1989).
[28] J. Alwall and G. Ingelman, Phys. Rev. D **71**, 094015 (2005).
[29] M. Gluck, E. Reya, and A. Vogt, Z. Phys. C **67**, 433 (1995); M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D **53**, 4775 (1996); D. de Florian, C. A. Garcia Canal, and R. Sassot, Nucl. Phys. **B470**, 195 (1996).
[30] J.-C. Peng, W.-C. Chang, H.-Y. Cheng, T.-J. Hou, K.-F. Liu, and J.-W. Qiu, Phys. Lett. B **736**, 411 (2014); W.-C. Chang and J.-C. Peng, Prog. Part. Nucl. Phys. **79**, 95 (2014).
[31] W. M. Alberico, S. M. Bilenky, and C. Maieron, Phys. Rep. **358**, 227 (2002); U. Dore, Eur. Phys. J. H **37**, 115 (2012).
[32] H. Abramowicz, J. G. H. de Groot, J. Knobloch, J. May, P. Palazzi, A. Para, F. Ranjard, and J. Rothberg *et al.*, Z. Phys. C **15**, 19 (1982); H. Abramowicz *et al.*, Z. Phys. C **17**, 283 (1983); G. Costa, J. Ellis, G. L. Fogli, D. V. Nanopoulos, and F. Zwirner, Nucl. Phys. **B297**, 244 (1988).
[33] S. A. Rabinowitz, C. Arroyo, K. T. Bachmann, A. O. Bazarko, T. Bolton, C. Foudas, B. J. King, and W. Lefmann *et al.*, Phys. Rev. Lett. **70**, 134 (1993).
[34] A. O. Bazarko *et al.* (CCFR and NuTeV Collaborations), Z. Phys. C **65**, 189 (1995).
[35] P. Vilain *et al.* (CHARM II Collaboration), Eur. Phys. J. C **11**, 19 (1999).
[36] P. Astier *et al.* (NOMAD Collaboration), Phys. Lett. B **486**, 35 (2000).
[37] O. Samoylov *et al.* (NOMAD Collaboration), Nucl. Phys. **B876**, 339 (2013).
[38] M. Goncharov *et al.* (NuTeV Collaboration), Phys. Rev. D **64**, 112006 (2001); G. P. Zeller *et al.*, Phys. Rev. Lett. **88**, 091802 (2002); Phys. Rev. D **65**, 111103 (2002); D. Mason *et al.*, Phys. Rev. Lett. **99**, 192001 (2007).
[39] A. Kayis-Topaksu *et al.* (CHORUS Collaboration), Nucl. Phys. **B798**, 1 (2008); A. Kayis-Topaksu *et al.*, New J. Phys. **13**, 093002 (2011).
[40] Fermilab E906 proposal, Spokespersons: D. Geesaman and P. Reimer.
[41] J-PARC P04 proposal, Spokespersons: J. C. Peng and S. Sawada.

[42] S. Weinberg, Physica (Amsterdam) **96A**, 327 (1979); A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).

[43] T. P. Cheng and L. F. Li, Phys. Rev. Lett. **74**, 2872 (1995); Phys. Rev. D **57**, 344 (1998); Phys. Rev. Lett. **80**, 2789 (1998).

[44] J. Linde, T. Ohlsson, and H. Snellman, Phys. Rev. D **57**, 452 (1998); **57**, 5916 (1998).

[45] X. Song, J. S. McCarthy, and H. J. Weber, Phys. Rev. D **55**, 2624 (1997); X. Song, Phys. Rev. D **57**, 4114 (1998).

[46] H. Dahiya and M. Gupta, Phys. Rev. D **64**, 014013 (2001); Int. J. Mod. Phys. A **19**, 5027 (2004); H. Dahiya, M. Gupta, and J. M. S. Rana, Int. J. Mod. Phys. A **21**, 4255 (2006); H. Dahiya and M. Gupta, Phys. Rev. D **78**, 014001 (2008); H. Dahiya and M. Randhawa, Phys. Rev. D **90**, 074001 (2014).

[47] H. Dahiya and M. Gupta, Phys. Rev. D **66**, 051501(R) (2002); **67**, 114015 (2003).

[48] N. Sharma, H. Dahiya, P. K. Chatley, and M. Gupta, Phys. Rev. D **79**, 077503 (2009); N. Sharma, H. Dahiya, and P. K. Chatley, Eur. Phys. J. A **44**, 125 (2010).

[49] N. Sharma, A. M. Torres, K. P. Khemchandani, and H. Dahiya, Eur. J. Phys. A **49**, 11 (2013).

[50] A. M. Torres, K. P. Khemchandani, N. Sharma, and H. Dahiya, Eur. J. Phys. A **48**, 185 (2012).

[51] N. Sharma and H. Dahiya, Pramana **81**, 449 (2013); **80**, 237 (2013).

[52] N. Sharma and H. Dahiya, Phys. Rev. D **81**, 114003 (2010).

[53] A. Le Yaouanc *et al.*, *Hadron Transitions in the Quark Model* (Gordon and Breach, New York, 1988).

[54] H. Dahiya, Phys. Rev. D **91**, 094010 (2015).

[55] E. J. Eichten, I. Hinchliffe, and C. Quigg, Phys. Rev. D **45**, 2269 (1992).