Studying $\mathcal{B}_1(\frac{1}{2}^+) \to \mathcal{B}_2(\frac{1}{2}^+)\ell^+\ell^-$ semileptonic weak baryon decays with the SU(3) flavor symmetry

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Motivated by recent anomalies in flavor changing neutral current $b \to s\ell^+\ell^-$ transitions, we study $\mathcal{B}_1 \to \mathcal{B}_2\ell^+\ell^-(\ell = e, \mu, \tau)$ semileptonic weak decays with the SU(3) flavor symmetry, where $\mathcal{B}_{1,2}$ are the spin-$\frac{1}{2}$ baryons of single bottomed antitriplet T_{b3}, single charmed antitriplet T_{c3}, or light baryons octet T_8. Using the SU(3) irreducible representation approach, we first obtain the amplitude relations among different decay modes and then predict the relevant not-yet measured observables of $T_{b3} \to T_8\ell^+\ell^-$, $T_{c3} \to T_8\ell^+\ell^-$, and $T_8 \to T'_8\ell^+\ell^-$ decays. (a) We calculate the branching ratios of the $T_{b3} \to T_8\mu^+\mu^-$ and $T_{b3} \to T_8\tau^+\tau^-$ decay modes in the whole q^2 region and in the different q^2 bins by the measurement of $\Lambda_b^0 \to \Lambda^0\mu^+\mu^-$. Many of them are obtained for the first time. In addition, the longitudinal polarization fractions and the leptonic forward-backward asymmetries of all $T_{b3} \to T_8\ell^+\ell^-$ decays are very similar to each other in certain q^2 bins due to the SU(3) flavor symmetry. (b) We analyze the upper limits of $\mathcal{B}(T_{c3} \to T_8\ell^+\ell^-)$ by using the experimental upper limits of $\mathcal{B}(\Lambda_c^+ \to p\mu^+\mu^-)$ and $\mathcal{B}(\Lambda_c^+ \to pe^+e^-)$, and find the experimental upper limit of $\mathcal{B}(\Lambda_c^+ \to p\mu^+\mu^-)$ giving the effective bounds on the relevant SU(3) flavor symmetry parameters. The predictions of $\mathcal{B}(\Xi_c^0 \to \Xi^0e^+e^-)$ and $\mathcal{B}(\Xi_c^0 \to \Xi^0\mu^+\mu^-)$ will be different between the single-quark transition dominant contributions and the W-exchange dominant ones. (c) As for $T_8 \to T'_8\ell^+\ell^-$ decays, we analyze the single-quark transition contributions and the W-exchange contributions by using the two experimental measurements of $\mathcal{B}(\Xi^0 \to \Lambda^0e^+e^-)$ and $\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)$, and give the branching ratio predictions by assuming either single-quark transition dominant contributions or the W-exchange dominant contributions. According to our predictions, some observables are accessible to the experiments at BESIII, LHCb and Belle-II.

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I. INTRODUCTION

Flavor changing neutral current (FCNC) processes, such as $b \to s\ell^+\ell^-$, give access to important tests of the standard model (SM) and searches for new physics beyond the SM. Recently, some discrepancies with the SM are reported in several observables in B meson decays, for example, the angular-distribution observable P'_5 of $B^0 \to K^{*0}\mu^+\mu^-$ [1–4] and the lepton flavor universality observables R_{K^+} and $R_{K^{*0}}$ with $R_M = \frac{\mathcal{B}(B\to M\mu^+\mu^-)}{\mathcal{B}(B\to Me^+e^-)}$ [5,6]. Semileptonic baryon decays are quite different to B, D, K meson ones; for instance, the initial baryons may be polarized, the transitions involve a diquark system as a spectator rather than a single-quark, and the W-exchange contributions of two-quark and three-quark transitions might appear in baryon decays. Therefore, the baryon decays provide the important additional tests of the SM predictions, which can be used to improve the understanding of recent anomalies in B meson decays. Recently, significant experimental progress has been achieved in studying rare Λ_b decays. The $\Lambda_b \to \Lambda\mu^+\mu^-$ baryon decay is the only one measured among the $T_{b3} \to T_8\ell^+\ell^-$ decays at present. $\mathcal{B}(\Lambda_b \to \Lambda\mu^+\mu^-)$ was first measured by the CDF Collaboration [7] and then greatly improved by LHCb [8,9]. For $T_{c3} \to T_8\ell^+\ell^-$ decays, only $\mathcal{B}(\Lambda_c \to pe^+e^-)$ and $\mathcal{B}(\Lambda_c \to p\mu^+\mu^-)$ have been upper limited by *BABAR* and LHCb [10,11]. As for $T_8 \to T'_8\ell^+\ell^-$ decays, $\Xi^0 \to \Lambda^0e^+e^-$ and $\Sigma^+ \to p\mu^+\mu^-$ have been measured by NA48 [12] and HyperCP [13], respectively. With the experiment

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development, some $\mathcal{B}_1 \to \mathcal{B}_2 \ell^+\ell^-$ decays will be improved or detected by the BESIII, LHCb, and Belle-II Collaborations in the near future, so it is necessary to study $\mathcal{B}_1 \to \mathcal{B}_2 \ell^+\ell^-$ decays theoretically.

The theoretical challenge in the study of $\mathcal{B}_1 \to \mathcal{B}_2 \ell^+\ell^-$ decays is calculating the hadronic $\mathcal{B}_1 \to \mathcal{B}_2$ form factors in the hadronic matrix elements. Form factors for $\Lambda_b \to \Lambda$ have been estimated in lattice QCD [14], QCD light cone sum rules [15], the soft-collinear effective theory [16], and perturbative QCD [17]. Form factors for $\Lambda_b \to n$ have been estimated in the relativistic quark diquark picture [18] and the context of light cone QCD sum rules [19]. Nevertheless, other form factors of $T_{b3} \to T_8 \ell^+\ell^-$, such as the ones for $\Xi_b^0 \to \Xi^0$, $\Xi_b^- \to \Xi^-$, $\Xi_b^0 \to \Lambda^0$, $\Xi_b^0 \to \Sigma^0$, and $\Xi_b^- \to \Sigma^-$, have not been calculated yet. Similarly in $T_{c3} \to T_8 \ell^+\ell^-$ and $T_8 \to T_8' \ell^+\ell^-$ decays, only some form factors are calculated, for example, ones for $\Lambda_c^+ \to p$ transition [19–22].

Theoretical calculations of the hadronic matrix elements are not well understood due to our poor understanding of QCD at low energy regions. The SU(3) flavor symmetry approach is independent of the detailed dynamics offering us an opportunity to relate different decay modes. Nevertheless, it cannot determine the sizes of the amplitudes by itself. However, if experimental data are enough, one may use the data to extract the amplitudes, which can be viewed as predictions based on symmetry. Although SU(3) flavor symmetry is only an approximate symmetry because u, d, and s quarks have different masses, it still provides some useful information about the decays. One popular way of predicting the SU(3) flavor symmetry is to construct the SU(3) irreducible representation amplitude by decomposing the effective Hamiltonian, in which one only focuses on the SU(3) flavor structure of the initial states and finial states but does not involve the details about the interaction dynamics.

Some $\mathcal{B}_1(\frac{1}{2}^+) \to \mathcal{B}_2(\frac{1}{2}^+)\ell^+\ell^-$ semileptonic baryon decays have been well studied, for instance, semileptonic Λ_b^0 decays in Refs. [14,23–28], semileptonic Λ_c^+ decays in Refs. [20,21,29], and semileptonic Σ^+ decays in Refs. [30–34]. In this work, we will study all weak $\mathcal{B}_1(\frac{1}{2}^+) \to \mathcal{B}_2(\frac{1}{2}^+)\ell^+\ell^-$ decays by using the SU(3) irreducible representation approach. We first obtain the amplitude relations among different decay modes then use the available data to extract the SU(3) irreducible amplitudes and finally, predict the not-yet-measured modes for further tests in experiments.

This paper is organized as follows. In Sec. II, we will collect the representations for the baryon multiplets of $\frac{1}{2}$-spin and the observable expressions of relevant baryon decays. In Sec. III, we will analyze the semileptonic weak decays of $T_{b3} \to T_8 \ell^+\ell^-$, $T_{c3} \to T_8 \ell^+\ell^-$, and $T_8 \to T_8' \ell^+\ell^-$. Our conclusions are given in Sec. IV.

II. THEORETICAL FRAME

A. Baryon multiplets with $\frac{1}{2}$ spin

The light baryons octet T_8 under the SU(3) flavor symmetry of u, d, s quarks can be written as

$$T_8 = \begin{pmatrix} \frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix}. \quad (1)$$

The single charmed antitriplet T_{c3} is given as

$$T_{c3} = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+). \quad (2)$$

The antitriplet T_{b3} with a heavy b quark is

$$T_{b3} = (\Xi_b^-, -\Xi_b^0, \Lambda_b^0). \quad (3)$$

B. Helicity amplitudes for semileptonic decays

In the SM, the low energy effective Hamiltonians for $b \to s/d\ell^+\ell^-$, $c \to u\ell^+\ell^-$, and $s \to d\ell^+\ell^-$ FCNC transitions have similar forms and can be written as [22,30,35–37]

$$\mathcal{H}(q_1 \to q_2\ell^+\ell^-) = -\frac{\alpha_e G_F}{\sqrt{2}\pi}\lambda_{q_1q_2}\Big(C_9^{\text{eff}}\bar{q}_2\gamma^\mu P_L q_1 \bar{\ell}\gamma_\mu\ell + C_{10}\bar{q}_2\gamma^\mu P_L q_1\bar{\ell}\gamma_\mu\gamma_5\ell - \frac{2m_{q_1}}{q^2}C_7^{\text{eff}}\bar{q}_2 i q_\nu\sigma^{\mu\nu}P_R q_1\bar{\ell}\gamma_\mu\ell\Big), \quad (4)$$

where G_F denotes the Fermi constant, the fine structure constant $\alpha_e = \frac{e^2}{4\pi}$, the chiral projection operators $P_{L,R} = (1 \mp \gamma_5)/2$, $\sigma_{\mu\nu} = \frac{i[\gamma_\mu,\gamma_\nu]}{2}$, $\lambda_{q_1q_2}$ denotes the Cabibbo-Kobayashi-Maskawa (CKM) elements, and C_i denote Wilson coefficients. For the $b \to s/d\ell^+\ell^-$ transitions via the $u\bar{u}$, $c\bar{c}$ loops, $\lambda_{bs(bd)} = V_{tb}V_{ts}^*\ (V_{tb}V_{td}^*)$, and the expressions of Wilson coefficients $C_{7,9}^{\text{eff}}$ and C_{10} are given in Ref. [37]. For the $c \to u\ell^+\ell^-$ transition via $d\bar{d}$, $s\bar{s}$ loops, $\lambda_{cu}C_{7,9}^{\text{eff}} = \frac{4\pi}{\alpha_s}[V_{cd}^*V_{ud}C_{7,9}^{\text{eff}(d)}(q^2) + V_{cs}^*V_{us}C_{7,9}^{\text{eff}(s)}(q^2)]$ as well as $C_{10} = 0$ and the expressions of $C_{7,9}^{\text{eff}(s,d)}(q^2)$ can be found in Refs. [22,35]. For the $s \to d\ell^+\ell^-$ transition via $u\bar{u}$ loop, $\lambda_{sd} = V_{us}V_{ud}^*$, $C_7^{\text{eff}} \approx \frac{V_{cs}V_{cd}^*}{2V_{us}V_{ud}^*}c_{7\gamma}^c$, $C_9^{\text{eff}} = (z_{7V} - \frac{V_{ts}V_{td}^*}{V_{us}V_{ud}^*}y_{7V})\frac{2\pi}{\alpha_e}$, and $C_{10} = -\frac{V_{ts}V_{td}^*}{V_{us}V_{ud}^*}\frac{2\pi}{\alpha_e}y_{7A}$ with $z_{7V} = -0.046\alpha_e$, $y_{7V} = 0.735\alpha_e$, $y_{7A} = -0.700\alpha_e$ as well as $c_{7\gamma}^c = 0.13\alpha_e$ from Refs. [30,36,37]. For $T_{b3} \to T_8\ell^+\ell^-$ and $T_{c3} \to T_8\ell^+\ell^-$ decays, the Wilson coefficient C_9^{eff} receives not only from the four quark operators but also from the long distance (LD) contributions coming from $c\bar{c}$ for $T_{b3} \to T_8\ell^+\ell^-$ and $d\bar{d}$, $s\bar{s}$ for $T_{c3} \to T_8\ell^+\ell^-$. Note that the $T_{c3} \to T_8\ell^+\ell^-$ decays are dominated by LD contributions. For $T_8 \to T'_8\ell^+\ell^-$ decays, the LD contribution arises mainly from the photon-mediated process $T_8 \to T'_8\gamma^* \to T'_8\ell^+\ell^-$ [31,34].

The helicity amplitudes for $\mathcal{B}_1 \to \mathcal{B}_2\ell^+\ell^-$ can be obtained from Eq. (4),

$$\mathcal{M}_{VA}^{\lambda_1,\lambda_2}(s_p,s_k) = -\frac{G_F}{\sqrt{2}}\frac{\alpha_e}{4\pi}\lambda_{q_1q_2}\sum_\lambda \eta_\lambda[H_{VA,\lambda}^{L,s_p,s_k}L_{L,\lambda}^{\lambda_1,\lambda_2} + H_{VA,\lambda}^{R,s_p,s_k}L_{R,\lambda}^{\lambda_1,\lambda_2}], \quad (5)$$

with

$$L_{L(R),\lambda}^{\lambda_1\lambda_2} = \bar{\epsilon}^\mu(\lambda)\langle\bar{\ell}(\lambda_1)\ell(\lambda_2)|\bar{\ell}\gamma_\mu(1\mp\gamma_5)\ell|0\rangle,$$
$$H_{VA,\lambda}^{L(R),s_p,s_k} = \bar{\epsilon}^*_\mu(\lambda)\left\langle \mathcal{B}_2(k,s_k)\middle|\left[(C_9^{\text{eff}} \mp C_{10})\bar{q}_2\gamma^\mu(1-\gamma_5)q_1 - \frac{2m_{q_1}}{q^2}C_7^{\text{eff}}\bar{q}_2 iq_\nu\sigma^{\mu\nu}(1+\gamma_5)q_1\right]\middle|\mathcal{B}_1(p,s_p)\right\rangle, \quad (6)$$

where $q^2 \equiv (p-k)^2$ bounded in physical region as $(2m_\ell)^2 \le q^2 \le (m_{\mathcal{B}_1} - m_{\mathcal{B}_2})^2$, the polarization of the gauge boson $\lambda = t, \pm1, 0$, the helicities of the final state leptons are $\lambda_{1,2}$, and $\eta_t = 1$, $\eta_{\pm1,0} = -1$.

The nonvanishing leptonic helicity amplitudes $L_{L(R),\lambda}^{\lambda_1\lambda_2}$ are

$$L_{L,+1}^{-\frac{1}{2}+\frac{1}{2}} = -L_{R,-1}^{+\frac{1}{2}-\frac{1}{2}} = \sqrt{\frac{q^2}{2}}(1+\beta_\ell)(1+\cos\theta_\ell),$$
$$L_{R,+1}^{+\frac{1}{2}-\frac{1}{2}} = -L_{L,-1}^{-\frac{1}{2}+\frac{1}{2}} = -\sqrt{\frac{q^2}{2}}(1+\beta_\ell)(1-\cos\theta_\ell),$$
$$L_{L,0}^{-\frac{1}{2}+\frac{1}{2}} = L_{R,0}^{+\frac{1}{2}-\frac{1}{2}} = \sqrt{q^2}(1+\beta_\ell)\sin\theta_\ell, \quad (7)$$

with $\beta_\ell = \sqrt{1-\frac{4m_\ell^2}{q^2}}$.

The $\mathcal{B}_1 \to \mathcal{B}_2$ hadronic matrix elements are calculated in the frameworks of soft-collinear effective theory [16] and lattice QCD [14]. The helicity-based definition of the form factors are presented as [14]

$$\langle\mathcal{B}_2(k,s_k)|\bar{q}_2\gamma^\mu q_1|\mathcal{B}_1\rangle = \bar{u}(k,s_k)\left[f_0(q^2)(m_{B_1}-m_{B_2})\frac{q^\mu}{q^2} + f_+(q^2)\frac{m_{B_1}+m_{B_2}}{s_+}\left\{p^\mu + k^\mu - \frac{q^\mu}{q^2}(m_{B_1}-m_{B_2})\right\} + f_\perp(q^2)\left\{\gamma^\mu - \frac{2m_{B_2}}{s_+}p^\mu - \frac{2m_{B_1}}{s_+}k^\mu\right\}\right]u(p,s_p), \quad (8)$$

$$\langle\mathcal{B}_2(k,s_k)|\bar{q}_2\gamma^\mu\gamma_5 q_1|\mathcal{B}_1(p,s_p)\rangle = -\bar{u}(k,s_k)\gamma_5\left[g_0(q^2)(m_{B_1}+m_{B_2})\frac{q^\mu}{q^2} + g_+(q^2)\frac{m_{B_1}-m_{B_2}}{s_-}\left\{p^\mu + k^\mu - \frac{q^\mu}{q^2}(m_{B_1}-m_{B_2})\right\} + g_\perp(q^2)\left\{\gamma^\mu + \frac{2m_{B_2}}{s_-}p^\mu - \frac{2m_{B_1}}{s_-}k^\mu\right\}\right]u(p,s_p), \quad (9)$$

$$\langle\mathcal{B}_2(p',s')|\bar{q}_2 i\sigma_{\mu\nu}q^\nu q_1|\mathcal{B}_1(p,s)\rangle = -\bar{u}_{\mathcal{B}_2}(p',s')\left\{h_+(q^2)\frac{q^2}{s_+}\left(p_\mu + p'_\mu - \frac{q_\mu}{q^2}(m_{\mathcal{B}_1}^2 - m_{\mathcal{B}_2}^2)\right) + (m_{\mathcal{B}_1}+m_{\mathcal{B}_2})h_\perp(q^2)\left(\gamma_\mu - \frac{2m_{\mathcal{B}_2}}{s_+}p_\mu - \frac{2m_{\mathcal{B}_1}}{s_+}p'_\mu\right)\right\}u_{\mathcal{B}_1}(p,s), \quad (10)$$

$$\langle\mathcal{B}_2(p',s')|\bar{q}_2 i\sigma_{\mu\nu}\gamma_5 q^\nu q_1|\mathcal{B}_1\rangle = -\bar{u}_{\mathcal{B}_2}(p',s')\gamma_5\left\{\tilde{h}_+(q^2)\frac{q^2}{s_-}\left(p_\mu + p'_\mu - \frac{q_\mu}{q^2}(m_{\mathcal{B}_1}^2 - m_{\mathcal{B}_2}^2)\right) + (m_{\mathcal{B}_1}-m_{\mathcal{B}_2})\tilde{h}_\perp(q^2)\left(\gamma_\mu + \frac{2m_{\mathcal{B}_2}}{s_-}p_\mu - \frac{2m_{\mathcal{B}_1}}{s_-}p'_\mu\right)\right\}u_{\mathcal{B}_1}(p,s), \quad (11)$$

where $s_\pm = (m_{\mathcal{B}_1} \pm m_{\mathcal{B}_2})^2 - q^2$ and $f_{0,\perp}^{V,A,T,T_5}$ are the form factors. And then we obtain the nonvanishing hadronic helicity amplitudes $H_{VA,\lambda}^{L(R),s_p,s_k}$,

$$H_{VA,0}^{L(R),+\frac{1}{2}+\frac{1}{2}} = f_+(q^2)(m_{\mathcal{B}_1}+m_{\mathcal{B}_2})\sqrt{\frac{s_-}{q^2}}\mathcal{C}_{VA}^{L(R)} - g_+(q^2)(m_{\mathcal{B}_1}-m_{\mathcal{B}_2})\sqrt{\frac{s_+}{q^2}}\mathcal{C}_{VA}^{L(R)} + \frac{2m_b}{q^2}\left(h_+(q^2)\sqrt{q^2s_-} - \tilde{h}_+(q^2)\sqrt{q^2s_+}\right)\mathcal{C}_7^{\text{eff}},$$
$$H_{VA,0}^{L(R),-\frac{1}{2}-\frac{1}{2}} = f_+(q^2)(m_{\mathcal{B}_1}+m_{\mathcal{B}_2})\sqrt{\frac{s_-}{q^2}}\mathcal{C}_{VA}^{L(R)} + g_+(q^2)(m_{\mathcal{B}_1}-m_{\mathcal{B}_2})\sqrt{\frac{s_+}{q^2}}\mathcal{C}_{VA}^{L(R)} + \frac{2m_b}{q^2}\left(h_+(q^2)\sqrt{q^2s_-} + \tilde{h}_+(q^2)\sqrt{q^2s_+}\right)\mathcal{C}_7^{\text{eff}},$$
$$H_{VA,+}^{L(R),-\frac{1}{2}+\frac{1}{2}} = -f_\perp(q^2)\sqrt{2s_-}\mathcal{C}_{VA}^{L(R)} + g_\perp(q^2)\sqrt{2s_+}\mathcal{C}_{VA}^{L(R)} - \frac{2m_b}{q^2}(h_\perp(q^2)(m_{\mathcal{B}_1}+m_{\mathcal{B}_2})\sqrt{2s_-} - \tilde{h}_\perp(q^2)(m_{\mathcal{B}_1}-m_{\mathcal{B}_2})\sqrt{2s_+})\mathcal{C}_7^{\text{eff}},$$
$$H_{VA,-}^{L(R),+\frac{1}{2}-\frac{1}{2}} = -f_\perp(q^2)\sqrt{2s_-}\mathcal{C}_{VA}^{L(R)} - g_\perp(q^2)\sqrt{2s_+}\mathcal{C}_{VA}^{L(R)} - \frac{2m_b}{q^2}(h_\perp(q^2)(m_{\mathcal{B}_1}+m_{\mathcal{B}_2})\sqrt{2s_-} + \tilde{h}_\perp(q^2)(m_{\mathcal{B}_1}-m_{\mathcal{B}_2})\sqrt{2s_+})\mathcal{C}_7^{\text{eff}}, \quad (12)$$

with $\mathcal{C}_{VA}^{L(R)} \equiv (\mathcal{C}_9^{\rm eff} \mp \mathcal{C}_{10})$.

In addition, in terms of the SU(3) flavor symmetry, baryon states and quark operators can be parametrized into SU(3) tensor forms, while the polarization vectors $\bar{\epsilon}^*(\lambda)$ and leptonic helicity amplitudes $L^{\lambda_1,\lambda_2}_{L(R),\lambda}$ are invariant under SU(3) flavor symmetry. The hadronic helicity amplitude relations of $\mathcal{B}_1 \to \mathcal{B}_2\ell^+\ell^-$ are similar to ones of $\mathcal{B}_1 \to \mathcal{B}_2\gamma$ as given in Ref. [38] and will be given in next section for convenience.

C. Observables for $\mathcal{B}_1 \to \mathcal{B}_2\ell^+\ell^-$

In the rest frame of the baryon $\mathcal{B}_1$, the double differential decay branching ratio is [39]

$$\begin{aligned}\frac{d\mathcal{B}(\mathcal{B}_1\to\mathcal{B}_2\ell^+\ell^-)}{dq^2 d\cos\theta_\ell} &= \frac{\tau_{\mathcal{B}_1}}{2m^3_{\Lambda_b}}\frac{2\sqrt{\lambda(m^2_{\mathcal{B}_1},m^2_{\mathcal{B}_2},q^2)}}{(8\pi)^3}\frac{1}{2s_p+1}\sum_{\lambda_1,\lambda_2}\sum_{s_p,s_k}|\mathcal{M}^{\lambda_1,\lambda_2}_{VA}(s_p,s_k)|^2\\
&= N^2(q^2)\Bigg[(1-\cos^2\theta_\ell)\left(|H^{L,+\frac{1}{2}+\frac{1}{2}}_{VA,0}|^2+|H^{L,-\frac{1}{2}-\frac{1}{2}}_{VA,0}|^2+|H^{R,+\frac{1}{2}+\frac{1}{2}}_{VA,0}|^2+|H^{R,-\frac{1}{2}-\frac{1}{2}}_{VA,0}|^2\right)\\
&\quad+\frac{1}{2}(1-\cos\theta_\ell)^2\left(|H^{L,+\frac{1}{2}-\frac{1}{2}}_{VA,-}|^2+|H^{R,-\frac{1}{2}+\frac{1}{2}}_{VA,+}|^2\right)\\
&\quad+\frac{1}{2}(1+\cos\theta_\ell)^2\left(|H^{R,+\frac{1}{2}-\frac{1}{2}}_{VA,-}|^2+|H^{L,-\frac{1}{2}+\frac{1}{2}}_{VA,+}|^2\right)\Bigg],\end{aligned} \tag{13}$$

with

$$N(q^2) = G_F\lambda_{q_1q_2}\alpha_e\left(1+\sqrt{1-\frac{4m^2_\ell}{q^2}}\right)\sqrt{\tau_{\mathcal{B}_1}\frac{q^2\sqrt{\lambda(m_{\mathcal{B}_1},m_{\mathcal{B}_2},q^2)}}{2^{15}m^3_{\mathcal{B}_1}\pi^5}}, \tag{14}$$

where $\lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca)$. And the differential decay branching ratio is

$$\frac{d\mathcal{B}(\mathcal{B}_1\to\mathcal{B}_2\ell^+\ell^-)}{dq^2} = \frac{4}{3}N^2(q^2)H_M(q^2), \tag{15}$$

with

$$\begin{aligned}H_M(q^2) &= \left(|H^{L,+\frac{1}{2}-\frac{1}{2}}_{VA,-}|^2+|H^{R,-\frac{1}{2}+\frac{1}{2}}_{VA,+}|^2\right)+\left(|H^{R,+\frac{1}{2}-\frac{1}{2}}_{VA,-}|^2+|H^{L,-\frac{1}{2}+\frac{1}{2}}_{VA,+}|^2\right)\\
&\quad+\left(|H^{L,+\frac{1}{2}+\frac{1}{2}}_{VA,0}|^2+|H^{L,-\frac{1}{2}-\frac{1}{2}}_{VA,0}|^2+|H^{R,+\frac{1}{2}+\frac{1}{2}}_{VA,0}|^2+|H^{R,-\frac{1}{2}-\frac{1}{2}}_{VA,0}|^2\right).\end{aligned} \tag{16}$$

The longitudinal polarization fraction can be obtained by Eq. (13),

$$F_L(q^2) = \frac{\int_{-1}^{+1}d\cos\theta_\ell(2-5\cos^2\theta_\ell)\frac{d^2\mathcal{B}}{dq^2d\cos\theta_\ell}}{\int_{-1}^{+1}d\cos\theta_\ell\frac{d^2\mathcal{B}}{dq^2d\cos\theta_\ell}}, \tag{17}$$

and the concrete expression is

$$F_L(q^2) = \left(|H^{L,+\frac{1}{2}+\frac{1}{2}}_{VA,0}|^2+|H^{L,-\frac{1}{2}-\frac{1}{2}}_{VA,0}|^2+|H^{R,+\frac{1}{2}+\frac{1}{2}}_{VA,0}|^2+|H^{R,-\frac{1}{2}-\frac{1}{2}}_{VA,0}|^2\right)[H_M(q^2)]^{-1}. \tag{18}$$

The leptonic forward-backward asymmetry,

$$A^\ell_{FB}(q^2) = \frac{\int_0^{+1}d\cos\theta_\ell\frac{d^2\mathcal{B}}{dq^2d\cos\theta_\ell}-\int_{-1}^{0}d\cos\theta_\ell\frac{d^2\mathcal{B}}{dq^2d\cos\theta_\ell}}{\int_{-1}^{0}d\cos\theta_\ell\frac{d^2\mathcal{B}}{dq^2d\cos\theta_\ell}+\int_0^{+1}d\cos\theta_\ell\frac{d^2\mathcal{B}}{dq^2d\cos\theta_\ell}}, \tag{19}$$

and the concrete expression is

$$A^{\ell}_{FB}(q^2) = \frac{3}{4}\left[\left(|H^{R,+\frac{1}{2}-\frac{1}{2}}_{VA,-}|^2 + |H^{L,-\frac{1}{2}+\frac{1}{2}}_{VA,+}|^2\right) - \left(|H^{L,+\frac{1}{2}-\frac{1}{2}}_{VA,-}|^2 + |H^{R,-\frac{1}{2}+\frac{1}{2}}_{VA,+}|^2\right)\right][H_M(q^2)]^{-1}. \quad (20)$$

The lepton flavor universality in baryon weak decays $T_{b3} \to T_8\ell^+\ell^-$ is defined in a manner identical $R_{K^{(*)}}$ as

$$R_{T_{b3}\to T_8} \equiv \frac{\int_{q_{\min}}^{q_{\max}} d\mathcal{B}(T_{b3} \to T_8\mu^+\mu^-)/ds}{\int_{q_{\min}}^{q_{\max}} d\mathcal{B}(T_{b3} \to T_8e^+e^-)/ds}. \quad (21)$$

For q^2 integration of $X(q^2) = F_L(q^2)$ and $A^{\ell}_{FB}(q^2)$, following Ref. [40], two ways of integration are considered. The normalized q^2-integrated observables $\langle X\rangle$ are calculated by separately integrating the numerators and denominators with the same q^2 bins. The "naively integrated" observables are obtained by

$$\bar{X} = \frac{1}{q^2_{\max} - q^2_{\min}} \int_{q^2_{\min}}^{q^2_{\max}} dq^2 X(q^2). \quad (22)$$

Note that, besides the single-quark transition contributions, the W-exchange contributions via the two-quark and three-quark transitions as well as the internal radiation transition, which contribute to the radiative baryon decays $\mathcal{B}_1 \to \mathcal{B}_2\gamma$ [41–43], may also contribute to the semileptonic baryon decays $\mathcal{B}_1 \to \mathcal{B}_2\ell^+\ell^-$. In some decays, for example, $\Sigma^+ \to p\ell^+\ell^-$ decays, the W-exchange contributions with the two-quark transition will play a major role [34]. So we will consider these W-exchange contributions in the later analysis of SU(3) flavor symmetry.

III. RESULTS AND ANALYSIS

The theoretical input parameters and the experimental data within the 1σ error from the Particle Data Group [44] will be used in our numerical results. To obtain SU(3) irreducible representation approach (IRA) amplitudes, one just needs to contract all upper and lower indices of the hadrons and the Hamiltonian to form all possible SU(3) singlets and associate each with a parameter which lumps up the Wilson coefficients and unknown hadronization effects [45]. These parameters can be determined theoretically and experimentally. In this work, we will determine these parameters by relevant experimental data and then give the predictions for other not-yet-measured decay modes. For T_{b3} semileptonic decays, there are enough phase spaces to allow for e^+e^-, $\mu^+\mu^-$, and $\tau^+\tau^-$ decays. T_{c3} and T_8 semileptonic decays only have enough phase spaces to allow for both e^+e^- and $\mu^+\mu^-$ decays. The results of $T_{b3} \to T_8e^+e^-$, $T_8\mu^+\mu^-$, $T_8\tau^+\tau^-$ decays, $T_{c3} \to T_8e^+e^-$, $T_8\mu^+\mu^-$ decays, and $T_8 \to T'_8e^+e^-$, $T'_8\mu^+\mu^-$ decays are given in the following subsections A, B, and C, respectively.

A. $T_{b3} \to T_8\ell^+\ell^-$ weak decays

For $T_{b3} \to T_8\ell^+\ell^-$ decays, the single-quark transition contributions are strongly dominant so that other contributions like the W-exchange contributions are usually omitted. The SU(3) flavor structure Hamiltonian with $b \to s, d$ transitions can been found, for instance, in Refs. [46–48], and the SU(3) IRA hadronic helicity amplitudes for $T_{b3} \to T_8\ell^+\ell^-$ via $b \to s/d\ell^+\ell^-$ can be parametrized as

$$H(T_{b3} \to T_8\ell^+\ell^-)^{L(R),s_p,s_k}_{VA,\lambda} = e_1(T_{b3})^{[ij]}T(\bar{3})^k(T_8)_{[ij]k} + e_2(T_{b3})^{[ij]}T(3)^k(T_8)_{[ik]j}, \quad (23)$$

which are similar to the decay amplitudes of corresponding $T_{b3} \to T_8\gamma$ modes in Ref. [49]. In Eq. (23), $T(\bar{3}) = (0,1,1)$ denoted the transition operators $(\bar{q}_2b)$ with $q_2 = s$, d, and the model as well as scale independent parameters $e_i \equiv (e_i)^{L(R),s_p,s_k}_{VA,\lambda}(q^2)$. The parameters e_i contain information about QCD dynamics and could include the long distance (LD) contributions from hadron resonances. The SU(3) IRA amplitudes of the $T_{b3} \to T_8\ell^+\ell^-$ weak decays are given in Table I, and for a better understanding, the information of relevant CKM matrix elements are also listed in Table I. From Table I, one can see that $\Lambda^0_b \to \Sigma^0\ell^+\ell^-$ decays are not allowed by the SU(3) flavor symmetry, and other decay modes via $b \to s/d\ell^+\ell^-$ can be related by only one parameter $E \equiv e_1 + e_2$.

Among $\Lambda^0_b \to \Lambda^0\ell^+\ell^-$, $\Xi^0_b \to \Xi^0\ell^+\ell^-$ $\Xi^-_b \to \Xi^-\ell^+\ell^-$, $\Lambda^0_b \to n\ell^+\ell^-$, $\Xi^0_b \to \Lambda^0\ell^+\ell^-$, $\Xi^0_b \to \Sigma^0\ell^+\ell^-$, and $\Xi^-_b \to \Sigma^-\ell^+\ell^-$ decays, only $\Lambda^0_b \to \Lambda^0\mu^+\mu^-$ decay has been

TABLE I. The SU(3) IRA amplitudes of the $T_{b3} \to T_8\ell^+\ell^-$ weak decays by the $b \to s/d\ell^+\ell^-$ transitions, and $E \equiv e_1 + e_2$.

Decay modes	$A(T_{b3} \to T_8\ell^+\ell^-)$
$T_{b3} \to T_8\ell^+\ell^-$ via the $b \to s\ell^+\ell^-$ transition:	
$\Lambda^0_b \to \Lambda^0\ell^+\ell^-$	$-2\lambda_{bs}E/\sqrt{6}$
$\Lambda^0_b \to \Sigma^0\ell^+\ell^-$	0
$\Xi^0_b \to \Xi^0\ell^+\ell^-$	$-\lambda_{bs}E$
$\Xi^-_b \to \Xi^-\ell^+\ell^-$	$\lambda_{bs}E$
$T_{b3} \to T_8\ell^+\ell^-$ via the $b \to d\ell^+\ell^-$ transition:	
$\Lambda^0_b \to n\ell^+\ell^-$	$\lambda_{bd}E$
$\Xi^0_b \to \Lambda^0\ell^+\ell^-$	$-\lambda_{bd}E/\sqrt{6}$
$\Xi^0_b \to \Sigma^0\ell^+\ell^-$	$-\lambda_{bd}E/\sqrt{2}$
$\Xi^-_b \to \Sigma^-\ell^+\ell^-$	$\lambda_{bd}E$

TABLE II. Branching ratios for $T_{b3} \to T_8 \ell^+ \ell^-$ decays with 1σ error in the whole q^2 region within S_1 and S_2 cases.

Decay modes	Experimental data [44]	Our results in S_1	Our results in S_2	Other predictions
$\mathcal{B}(\Lambda_b^0 \to \Lambda^0 \mu^+ \mu^-)(\times 10^{-6})$	1.08 ± 0.28	1.08 ± 0.28	1.08 ± 0.28	1.05 [50]
$\mathcal{B}(\Xi_b^0 \to \Xi^0 \mu^+ \mu^-)(\times 10^{-6})$	…	$1.55^{+0.45}_{-0.43}$	$1.77^{+0.49}_{-0.53}$	
$\mathcal{B}(\Xi_b^- \to \Xi^- \mu^+ \mu^-)(\times 10^{-6})$	…	$1.65^{+0.49}_{-0.46}$	$1.87^{+0.56}_{-0.54}$	
$\mathcal{B}(\Lambda_b^0 \to \Lambda^0 \tau^+ \tau^-)(\times 10^{-7})$	…	2.30 ± 0.60	$2.74^{+0.85}_{-0.71}$	2.60 [50]
$\mathcal{B}(\Xi_b^0 \to \Xi^0 \tau^+ \tau^-)(\times 10^{-7})$	…	$3.23^{+0.94}_{-0.89}$	$4.42^{+1.36}_{-1.21}$	
$\mathcal{B}(\Xi_b^- \to \Xi^- \tau^+ \tau^-)(\times 10^{-7})$	…	$3.42^{+1.01}_{-0.95}$	$4.76^{+1.44}_{-1.36}$	
$\mathcal{B}(\Lambda_b^0 \to n \mu^+ \mu^-)(\times 10^{-8})$	…	$8.15^{+2.44}_{-2.30}$	$7.77^{+2.42}_{-2.28}$	$(4.1^{+5.4}_{-1.2})$ 3.75 [50,51]
$\mathcal{B}(\Xi_b^0 \to \Lambda^0 \mu^+ \mu^-)(\times 10^{-8})$	…	$1.34^{+0.43}_{-0.39}$	$1.45^{+0.44}_{-0.45}$	
$\mathcal{B}(\Xi_b^0 \to \Sigma^0 \mu^+ \mu^-)(\times 10^{-8})$	…	$3.77^{+1.22}_{-1.10}$	$4.13^{+1.36}_{-1.24}$	
$\mathcal{B}(\Xi_b^- \to \Sigma^- \mu^+ \mu^-)(\times 10^{-8})$	…	$8.00^{+2.56}_{-2.40}$	$8.61^{+3.06}_{-2.52}$	
$\mathcal{B}(\Lambda_b^0 \to n \tau^+ \tau^-)(\times 10^{-8})$	…	$2.07^{+0.62}_{-0.58}$	$2.46^{+0.78}_{-0.70}$	$(2.9^{+3.7}_{-0.8})$ 1.21 [50,51]
$\mathcal{B}(\Xi_b^0 \to \Lambda^0 \tau^+ \tau^-)(\times 10^{-9})$	…	$3.42^{+1.11}_{-1.00}$	$4.63^{+1.71}_{-1.35}$	
$\mathcal{B}(\Xi_b^0 \to \Sigma^0 \tau^+ \tau^-)(\times 10^{-9})$	…	$8.97^{+2.91}_{-2.62}$	$12.23^{+4.12}_{-3.62}$	
$\mathcal{B}(\Xi_b^- \to \Sigma^- \tau^+ \tau^-)(\times 10^{-8})$	…	$1.91^{+0.61}_{-0.57}$	$2.60^{+0.87}_{-0.77}$	

measured, and its branching ratios in the whole q^2 region and in different q^2 bins are listed in Tables II and III, respectively. One can constrain the relevant SU(3) flavor parameters by the experimental data within 1σ error bar and then predict other not-yet-measured branching ratios. Two cases will be considered in our analysis of $T_{b3} \to T_8 \ell^+ \ell^-$ decays.

S_1: The SU(3) flavor symmetry parameters without the baryonic momentum-transfer q^2 dependence. We treat the SU(3) flavor parameters $(E)^{L(R),s_p,s_k}_{VA,\lambda}(q^2)$ as constants without q^2 dependence, which will lead $H_M(q^2)$ in Eq. (15) to a constant, too. We use the 1σ error experimental data of $\mathcal{B}(\Lambda_b^0 \to \Lambda^0 \mu^+ \mu^-)$ to constrain $H_M(q^2)$ (i.e., $|E|^2$) and then predict other $\mathcal{B}(T_{b3} \to T_8 \ell^+ \ell^-)$ by the amplitude relations in Table I.

S_2: The SU(3) flavor symmetry parameters with the baryonic momentum-transfer q^2 dependence. In order to obtain more precise predictions, we use the hadronic helicity amplitude expressions in Eq. (12), which are q^2 dependent and can be expressed by the Wilson coefficients and the form factors. The expressions of the Wilson coefficients without the LD contributions are taken from Ref. [52]. As for the q^2 dependent form factors involving the $T_{b3} \to T_8$ transitions, we use the recent lattice QCD results of $\Lambda_b^0 \to \Lambda^0$ [14], in which the form factors are parametrized by

$$f(q^2) = \frac{f(0)}{1 - q^2/(m^f_{\rm pole})^2}\left[1 + \frac{a_1^f}{a_0^f} z(q^2) + \frac{a_2^f}{a_0^f}[z(q^2)]^2\right], \tag{24}$$

where $f = f_+, f_\perp, f_0, g_+, g_\perp, g_0, h_+, h_\perp, \tilde{h}_+, \tilde{h}_\perp$, and the details of $z(q^2)$ and $m^f_{\rm pole}$ can be found in Ref. [14]. We keep $f_+(0)$ as an undetermined constant without q^2 dependence, and other $f(0)$ can be expressed as $\frac{a_0^f}{a_0^{f_+}} f_+(0)$. The central values of a_i^f in Table V of Ref. [14] will be used in our analysis. Since these form factors also preserve the SU(3) flavor symmetry, the same relations in Table I will be used for $f_+(0)$. We use the 1σ error experimental data of $\mathcal{B}(\Lambda_b^0 \to \Lambda^0 \mu^+ \mu^-)$ to constrain $f_+(0)$ and then predict other $\mathcal{B}(T_{b3} \to T_8 \ell^+ \ell^-)$.

Using the experimental data of $\mathcal{B}(\Lambda_b^0 \to \Lambda^0 \mu^+ \mu^-)$ in the whole q^2 region, one can obtain the branching ratios for $T_{b3} \to T_8 \mu^+ \mu^-$ and $T_{b3} \to T_8 \tau^+ \tau^-$ weak decays in the whole q^2 region, which are listed in the third and forth columns of Table II for S_1 case and S_2 case, respectively. Noted that, the amplitude relations listed in Table I are obtained from the SU(3) flavor symmetry; nevertheless, the different baryon masses in the same baryon multiplets are considered in the branching ratio predictions, and the below is same.

Previous predictions are also listed in the last column of Table II for comparing. Since the results of $T_{b3} \to T_8 e^+ e^-$ decays are quite similar to ones of $T_{b3} \to T_8 \mu^+ \mu^-$ decays, we only show $T_{b3} \to T_8 \mu^+ \mu^-$ in this work. We have the following remarks for the results in Table II.

(i) Comparing the branching ratios in S_1 and S_2 cases, one can see that the predictions are slightly different between S_1 and S_2 cases, which are mainly due to the q^2 dependence of the hadronic helicity amplitudes.

TABLE III. Branching ratios for $T_{b3} \to T_8\mu^+\mu^-$ weak decays in different q^2 bins with 1σ error in S_1 and S_2 cases (in the unit of 10^{-7}).

$[q^2_{\rm min}, q^2_{\rm max}]({\rm GeV}^2)$	[0.1, 2.0]	[2.0, 4.3]	[0.1, 4.3]	[4.0, 6.0]	[1.0, 6.0]	[6.0, 8.0]	[4.3, 8.68]	[10.09, 12.86]	[14.18, 16.0]	[0.1, 16.0]	[18.0, 20.0]	[15.0, 20.0]
$\mathcal{B}(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)^{\rm Exp.}$	0.71 ± 0.27	$0.28^{+0.28}_{-0.21}$	2.7 ± 2.7	$0.04^{+0.18}_{-0.02}$	$0.47^{+0.31}_{-0.27}$	$0.50^{+0.26}_{-0.24}$	0.5 ± 0.7	2.2 ± 0.6	1.7 ± 0.5	7.0 ± 2.9	2.44 ± 0.57	6.0 ± 1.3
$\mathcal{B}(\Xi_b^0 \to \Xi^0\mu^+\mu^-)_{S_1}$	$1.03^{+0.42}_{-0.41}$	$0.41^{+0.43}_{-0.31}$	$3.91^{+4.00}_{-3.85}$	$0.058^{+0.270}_{-0.030}$	$0.68^{+0.47}_{-0.40}$	$0.73^{+0.40}_{-0.36}$	$0.73^{+1.06}_{-1.02}$	$3.21^{+0.97}_{-0.92}$	$2.47^{+0.80}_{-0.77}$	$10.18^{+4.53}_{-4.35}$	$3.23^{+0.85}_{-0.81}$	$8.46^{+2.12}_{-2.01}$
$\mathcal{B}(\Xi_b^0 \to \Xi^0\mu^+\mu^-)_{S_2}$	$1.14^{+0.49}_{-0.46}$	$0.45^{+0.48}_{-0.34}$	$4.39^{+4.58}_{-4.32}$	$0.065^{+0.302}_{-0.033}$	$0.76^{+0.54}_{-0.45}$	$0.81^{+0.45}_{-0.39}$	$0.81^{+1.20}_{-0.81}$	$3.60^{+1.14}_{-1.06}$	$2.83^{+0.91}_{-0.89}$	$11.46^{+5.29}_{-4.96}$	$3.82^{+1.01}_{-1.03}$	$9.81^{+2.45}_{-2.55}$
$\mathcal{B}(\Xi_b^- \to \Xi^-\mu^+\mu^-)_{S_1}$	$1.09^{+0.46}_{-0.43}$	$0.43^{+0.45}_{-0.32}$	$4.15^{+4.28}_{-4.08}$	$0.062^{+0.289}_{-0.032}$	$0.72^{+0.51}_{-0.42}$	$0.77^{+0.43}_{-0.38}$	$0.77^{+1.13}_{-1.09}$	$3.40^{+1.04}_{-0.99}$	$2.62^{+0.86}_{-0.82}$	$10.80^{+4.89}_{-4.65}$	$3.40^{+0.91}_{-0.87}$	$8.94^{+2.30}_{-2.15}$
$\mathcal{B}(\Xi_b^- \to \Xi^-\mu^+\mu^-)_{S_2}$	$1.23^{+0.52}_{-0.49}$	$0.48^{+0.51}_{-0.37}$	$4.57^{+4.92}_{-4.50}$	$0.069^{+0.324}_{-0.035}$	$0.81^{+0.58}_{-0.47}$	$0.86^{+0.50}_{-0.42}$	$0.86^{+1.28}_{-0.86}$	$3.84^{+1.17}_{-1.13}$	$3.00^{+0.99}_{-0.94}$	$12.22^{+5.56}_{-5.38}$	$3.98^{+1.14}_{-1.03}$	$10.30^{+2.77}_{-2.56}$
$\mathcal{B}(\Lambda_b^0 \to n\mu^+\mu^-)_{S_1}$	$0.047^{+0.020}_{-0.019}$	$0.019^{+0.020}_{-0.014}$	$0.180^{+0.189}_{-0.177}$	$0.0027^{+0.0127}_{-0.0014}$	$0.031^{+0.022}_{-0.019}$	$0.034^{+0.019}_{-0.017}$	$0.034^{+0.050}_{-0.047}$	$0.152^{+0.048}_{-0.045}$	$0.123^{+0.041}_{-0.039}$	$0.482^{+0.225}_{-0.208}$	$0.236^{+0.065}_{-0.062}$	$0.491^{+0.128}_{-0.121}$
$\mathcal{B}(\Lambda_b^0 \to n\mu^+\mu^-)_{S_2}$	$0.044^{+0.020}_{-0.018}$	$0.017^{+0.019}_{-0.013}$	$0.169^{+0.176}_{-0.166}$	$0.0025^{+0.0116}_{-0.0012}$	$0.029^{+0.021}_{-0.017}$	$0.030^{+0.017}_{-0.015}$	$0.031^{+0.045}_{-0.031}$	$0.132^{+0.042}_{-0.040}$	$0.107^{+0.036}_{-0.034}$	$0.423^{+0.204}_{-0.181}$	$0.215^{+0.059}_{-0.056}$	$0.445^{+0.117}_{-0.114}$
$\mathcal{B}(\Xi_b^0 \to \Lambda^0\mu^+\mu^-)_{S_1}$	0.008 ± 0.003	0.003 ± 0.003	$0.029^{+0.031}_{-0.029}$	$0.0000^{+0.0025}_{-0.0000}$	$0.005^{+0.004}_{-0.003}$	0.005 ± 0.003	0.005 ± 0.008	0.025 ± 0.008	0.020 ± 0.007	$0.079^{+0.038}_{-0.035}$	$0.039^{+0.012}_{-0.011}$	$0.081^{+0.023}_{-0.021}$
$\mathcal{B}(\Xi_b^0 \to \Lambda^0\mu^+\mu^-)_{S_2}$	$0.008^{+0.004}_{-0.003}$	$0.003^{+0.003}_{-0.002}$	$0.030^{+0.034}_{-0.029}$	$0.0004^{+0.0021}_{-0.0002}$	$0.005^{+0.004}_{-0.003}$	0.005 ± 0.003	$0.005^{+0.008}_{-0.005}$	0.024 ± 0.008	$0.020^{+0.007}_{-0.006}$	$0.077^{+0.039}_{-0.034}$	$0.041^{+0.012}_{-0.011}$	$0.083^{+0.024}_{-0.022}$
$\mathcal{B}(\Xi_b^0 \to \Sigma^0\mu^+\mu^-)_{S_1}$	$0.023^{+0.010}_{-0.009}$	$0.009^{+0.010}_{-0.007}$	$0.087^{+0.093}_{-0.085}$	$0.0013^{+0.0062}_{-0.0013}$	$0.015^{+0.011}_{-0.009}$	$0.016^{+0.010}_{-0.008}$	$0.016^{+0.025}_{-0.023}$	$0.073^{+0.024}_{-0.023}$	$0.058^{+0.022}_{-0.019}$	$0.231^{+0.113}_{-0.103}$	$0.103^{+0.031}_{-0.028}$	$0.225^{+0.063}_{-0.057}$
$\mathcal{B}(\Xi_b^0 \to \Sigma^0\mu^+\mu^-)_{S_2}$	$0.024^{+0.011}_{-0.010}$	$0.010^{+0.011}_{-0.007}$	$0.093^{+0.101}_{-0.092}$	$0.0014^{+0.0065}_{-0.0007}$	$0.016^{+0.012}_{-0.009}$	$0.017^{+0.010}_{-0.009}$	$0.017^{+0.025}_{-0.017}$	$0.075^{+0.025}_{-0.023}$	$0.060^{+0.023}_{-0.020}$	$0.237^{+0.120}_{-0.103}$	$0.114^{+0.034}_{-0.032}$	$0.245^{+0.069}_{-0.064}$
$\mathcal{B}(\Xi_b^- \to \Sigma^-\mu^+\mu^-)_{S_1}$	$0.048^{+0.022}_{-0.020}$	$0.019^{+0.021}_{-0.014}$	$0.184^{+0.200}_{-0.181}$	$0.0027^{+0.0133}_{-0.0014}$	$0.032^{+0.024}_{-0.019}$	$0.034^{+0.021}_{-0.017}$	$0.034^{+0.053}_{-0.049}$	$0.155^{+0.053}_{-0.047}$	$0.124^{+0.045}_{-0.041}$	$0.491^{+0.240}_{-0.218}$	$0.220^{+0.066}_{-0.061}$	$0.478^{+0.140}_{-0.122}$
$\mathcal{B}(\Xi_b^- \to \Sigma^-\mu^+\mu^-)_{S_2}$	$0.051^{+0.024}_{-0.020}$	$0.020^{+0.023}_{-0.015}$	$0.196^{+0.213}_{-0.193}$	$0.0029^{+0.0140}_{-0.0015}$	$0.034^{+0.026}_{-0.020}$	$0.036^{+0.021}_{-0.018}$	$0.036^{+0.054}_{-0.036}$	$0.159^{+0.056}_{-0.049}$	$0.129^{+0.047}_{-0.043}$	$0.508^{+0.260}_{-0.225}$	$0.243^{+0.077}_{-0.066}$	$0.515^{+0.151}_{-0.130}$

(ii) Comparing our predictions for $\mathcal{B}(\Lambda_b^0 \to \Lambda^0\tau^+\tau^-)$, $\mathcal{B}(\Lambda_b^0 \to n\tau^+\tau^-)$ and $\mathcal{B}(\Lambda_b^0 \to n\mu^+\mu^-)$ with previous ones in the relativistic quark model [50] and the Bethe-Salpeter equation approach [51], one can see that our predicted $\mathcal{B}(\Lambda_b^0 \to \Lambda^0\tau^+\tau^-)$ and $\mathcal{B}(\Lambda_b^0 \to n\mu^+\mu^-)$ are quite consistent with previous ones,; nevertheless, our central value of $\mathcal{B}(\Lambda_b^0 \to n\tau^+\tau^-)$ is about 2 times larger than theirs.

(iii) Many branching ratio predictions for $T_{b3} \to T_8\ell^+\ell^-$ are obtained for the first time. The not-yet-measured $\mathcal{B}(T_{b3} \to T_8\ell^+\ell^-)$ are on the order of $\mathcal{O}(10^{-8}\text{–}10^{-6})$, and some of them could be reached by the LHCb or Belle-II experiments.

Using the experimental data of $\mathcal{B}(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)$ in different q^2 bins, one can get the branching ratios of $T_{b3} \to T_8\mu^+\mu^-$ and $T_{b3} \to T_8\tau^+\tau^-$ weak decays in different q^2 bins within S_1 and S_2 cases, which are collected for reference in Table III and Table IV, respectively.

The longitudinal polarization fractions and the leptonic forward-backward asymmetries with two ways of integration for $T_{b3} \to T_8\ell^+\ell^-$ decays could also be obtained in the S_2 case. As shown in Eq. (18) and Eq. (20), the $N(q^2)$ terms are canceled in the ratios; therefore, the longitudinal polarization fractions and the leptonic forward-backward asymmetries only depend on the hadronic helicity amplitudes, which preserve the SU(3) flavor symmetry in $T_{b3} \to T_8\ell^+\ell^-$ decays. So the longitudinal polarization fractions and the leptonic forward-backward asymmetries of all $T_{b3} \to T_8\mu^+\mu^-$ ($T_{b3} \to T_8\tau^+\tau^-$) decays are very similar to each other in certain q^2 bins. We take ones of $\Lambda_b^0 \to \Lambda^0\mu^+\mu^-$, $\Lambda^0\tau^+\tau^-$ as examples, which are given in Table V. Excepting in [0.1, 2.0], [0.1, 4.3], [0.1, 16.0], and the whole q^2 regions, $\overline{f_L}$ and $\langle f_L\rangle$ ($\overline{A^\ell_{FB}}$ and $\langle A^\ell_{FB}\rangle$) with different q^2 integration ways are quite similar in other certain q^2 bins. So the obvious differentiation between $\overline{f_L}$ and $\langle f_L\rangle$ ($\overline{A^\ell_{FB}}$ and $\langle A^\ell_{FB}\rangle$) mainly appears in the quite low q^2 regions. Note that the normalized longitudinal polarization fraction and normalized leptonic forward-backward asymmetry of $\Lambda_b^0 \to \Lambda^0\mu^+\mu^-$ in $q^2 \in [15, 20]$ GeV^2 have been measured by the LHCb experiment [44],

$$\langle f_L\rangle(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)_{[15,20]} = 0.61^{+0.11}_{-0.14},$$
$$\langle A^\ell_{FB}\rangle(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)_{[15,20]} = -0.39 \pm 0.04 \pm 0.01. \quad (25)$$

We do not impose the above experimental bounds but leave them as predictions. Comparing with the experimental results for $\langle f_L\rangle(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)_{[15,20]}$ and $\langle A^\ell_{FB}\rangle(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)_{[15,20]}$, our prediction of $\langle f_L\rangle(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)_{[15,20]}$ and $\langle A^\ell_{FB}\rangle(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)_{[15,20]}$ are agreeable with their experimental data within 1.5σ and 1σ error ranges, respectively.

In addition, the lepton flavor universality $R_{T_{b3}\to T_8}$ in three q^2 bins within the S_2 case are given in Table VI.

TABLE IV. Branching ratios for $T_{b3} \to T_8 \tau^+\tau^-$ weak decays in different q^2 bins with 1σ error in S_1 and S_2 cases (in the unit of 10^{-7}).

$[q^2_{\rm min}, q^2_{\rm max}]({\rm GeV}^2)$	[14.18, 16.0] in S_1	[14.18, 16.0] in S_2	[18.0, 20.0] in S_1	[18.0, 20.0] in S_2	[15.0, 20.0] in S_1	[15.0, 20.0] in S_2
$\mathcal{B}(\Lambda_b^0 \to \Lambda^0\tau^+\tau^-)$	0.83 ± 0.25	$0.84^{+0.27}_{-0.25}$	1.51 ± 0.36	1.52 ± 0.37	3.41 ± 0.76	$3.44^{+0.86}_{-0.80}$
$\mathcal{B}(\Xi_b^0 \to \Xi^0\tau^+\tau^-)$	$1.21^{+0.39}_{-0.38}$	$1.40^{+0.46}_{-0.45}$	$2.00^{+0.53}_{-0.50}$	$2.32^{+0.66}_{-0.59}$	$4.79^{+1.20}_{-1.14}$	$5.65^{+1.43}_{-1.44}$
$\mathcal{B}(\Xi_b^- \to \Xi^-\tau^+\tau^-)$	$1.29^{+0.42}_{-0.40}$	$1.49^{+0.50}_{-0.48}$	$2.11^{+0.57}_{-0.54}$	2.50 ± 0.67	$5.07^{+1.30}_{-1.22}$	$5.92^{+1.58}_{-1.48}$
$\mathcal{B}(\Lambda_b^0 \to n\tau^+\tau^-)$	$0.060^{+0.020}_{-0.019}$	0.053 ± 0.018	$0.147^{+0.041}_{-0.039}$	$0.133^{+0.038}_{-0.034}$	$0.282^{+0.074}_{-0.070}$	$0.257^{+0.068}_{-0.064}$
$\mathcal{B}(\Xi_b^0 \to \Lambda^0\tau^+\tau^-)$	$0.010^{+0.004}_{-0.003}$	$0.0095^{+0.0039}_{-0.0030}$	0.024 ± 0.007	0.026 ± 0.007	$0.047^{+0.013}_{-0.012}$	$0.048^{+0.014}_{-0.013}$
$\mathcal{B}(\Xi_b^0 \to \Sigma^0\tau^+\tau^-)$	$0.029^{+0.011}_{-0.009}$	$0.030^{+0.011}_{-0.010}$	$0.064^{+0.019}_{-0.017}$	$0.071^{+0.021}_{-0.019}$	$0.129^{+0.036}_{-0.033}$	$0.140^{+0.041}_{-0.037}$
$\mathcal{B}(\Xi_b^- \to \Sigma^-\tau^+\tau^-)$	$0.061^{+0.022}_{-0.020}$	$0.063^{+0.023}_{-0.021}$	$0.137^{+0.041}_{-0.038}$	$0.152^{+0.047}_{-0.041}$	$0.273^{+0.080}_{-0.070}$	$0.301^{+0.086}_{-0.078}$

TABLE V. Longitudinal polarization fractions and forward-backward asymmetries for $\Lambda_b^0 \to \Lambda^0\mu^+\mu^-$, $\Lambda^0\tau^+\tau^-$ decays in different q^2 bins with 1σ error in the S_2 case.

$[q^2_{\rm min}, q^2_{\rm max}]({\rm GeV}^2)$	[0.1, 2.0]	[2.0, 4.3]	[0.1, 4.3]	[4.0, 6.0]	[1.0, 6.0]	[6.0, 8.0]	[4.3, 8.68]	[10.09, 12.86]	[14.18, 16.0]	[0.1, 16.0]	[18.0, 20.0]	[15.0, 20.0]	whole q^2 regions
$\overline{f_L}(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)$	$0.64^{+0.04}_{-0.01}$	0.86 ± 0.01	$0.77^{+0.06}_{-0.03}$	0.81 ± 0.01	$0.83^{+0.02}_{-0.01}$	0.73 ± 0.01	0.77 ± 0.02	$0.57^{+0.00}_{-0.02}$	0.46 ± 0.01	$0.66^{+0.02}_{-0.03}$	0.36 ± 0.01	$0.39^{+0.02}_{-0.01}$	0.60 ± 0.02
$\langle f_L\rangle(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)$	$0.36^{+0.04}_{-0.02}$	$0.86^{+0.06}_{-0.04}$	$0.42^{+0.04}_{-0.02}$	$0.81^{+0.03}_{-0.02}$	$0.82^{+0.05}_{-0.02}$	0.73 ± 0.01	0.75 ± 0.03	0.56 ± 0.01	0.45 ± 0.01	$0.47^{+0.04}_{-0.02}$	0.36 ± 0.01	$0.40^{+0.01}_{-0.02}$	$0.34^{+0.03}_{-0.02}$
$\overline{A^\ell_{FB}}(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)$	0.12 ± 0.01	$0.05^{+0.00}_{-0.01}$	$0.08^{+0.00}_{-0.01}$	$-0.05^{+0.00}_{-0.01}$	$0.03^{+0.00}_{-0.01}$	$-0.15^{+0.00}_{-0.01}$	$-0.12^{+0.00}_{-0.01}$	-0.29 ± 0.01	-0.36 ± 0.00	-0.15 ± 0.01	$-0.31^{+0.01}_{-0.00}$	$-0.35^{+0.01}_{-0.02}$	$-0.19^{+0.00}_{-0.01}$
$\langle A^\ell_{FB}\rangle(\Lambda_b^0 \to \Lambda^0\mu^+\mu^-)$	$0.08^{+0.00}_{-0.01}$	$0.06^{+0.00}_{-0.01}$	0.07 ± 0.00	$-0.05^{+0.00}_{-0.01}$	$0.06^{+0.00}_{-0.01}$	$-0.15^{+0.00}_{-0.01}$	$-0.12^{+0.00}_{-0.01}$	$-0.29^{+0.01}_{-0.02}$	$-0.37^{+0.02}_{-0.01}$	$-0.03^{+0.00}_{-0.01}$	-0.30 ± 0.01	$-0.34^{+0.01}_{-0.02}$	$-0.04^{+0.00}_{-0.01}$
$\overline{f_L}(\Lambda_b^0 \to \Lambda^0\tau^+\tau^-)$									$0.46^{+0.00}_{-0.01}$		$0.36^{+0.01}_{-0.00}$	$0.39^{+0.02}_{-0.01}$	0.42 ± 0.01
$\langle f_L\rangle(\Lambda_b^0 \to \Lambda^0\tau^+\tau^-)$									0.46 ± 0.01		0.36 ± 0.01	$0.39^{+0.02}_{-0.01}$	$0.40^{+0.03}_{-0.01}$
$\overline{A^\ell_{FB}}(\Lambda_b^0 \to \Lambda^0\tau^+\tau^-)$									-0.36 ± 0.00		-0.31 ± 0.01	$-0.34^{+0.01}_{-0.02}$	-0.34 ± 0.01
$\langle A^\ell_{FB}\rangle(\Lambda_b^0 \to \Lambda^0\tau^+\tau^-)$									$-0.36^{+0.01}_{-0.02}$		-0.30 ± 0.01	$-0.34^{+0.01}_{-0.02}$	$-0.33^{+0.02}_{-0.01}$

TABLE VI. Lepton flavor universality of $T_{b3} \to T_8 \ell^+\ell^-$ baryon weak decays in different q^2 bins with 1σ error in the S_2 case.

$[q^2_{\rm min}, q^2_{\rm max}]({\rm GeV}^2)$	[1, 6]	[0.1, 16.0]	[15.0, 20.0]
$R_{\Lambda_b^0\to\Lambda^0}$	$0.99^{+0.05}_{-0.03}$	$0.98^{+0.07}_{-0.04}$	$1.02^{+0.04}_{-0.07}$
$R_{\Xi_b^0\to\Xi^0}$	0.99 ± 0.04	$0.99^{+0.06}_{-0.05}$	$0.99^{+0.08}_{-0.06}$
$R_{\Xi_b^-\to\Xi^-}$	0.99 ± 0.04	$0.99^{+0.06}_{-0.05}$	$1.03^{+0.03}_{-0.09}$
$R_{\Lambda_b^0\to n}$	1.00 ± 0.03	$0.98^{+0.07}_{-0.04}$	$1.01^{+0.03}_{-0.05}$
$R_{\Xi_b^0\to\Lambda^0}$	$1.01^{+0.02}_{-0.05}$	$1.01^{+0.04}_{-0.08}$	1.00 ± 0.04
$R_{\Xi_b^0\to\Sigma^0}$	$0.98^{+0.04}_{-0.03}$	$0.98^{+0.07}_{-0.04}$	1.00 ± 0.03
$R_{\Xi_b^-\to\Sigma^-}$	$1.00^{+0.03}_{-0.04}$	$1.03^{+0.02}_{-0.09}$	$1.00^{+0.03}_{-0.02}$

One can see that all predictions in three q^2 bins are virtually indistinguishable from unity; i.e., the lepton mass effects on all $R_{\mathcal{B}_1\to\mathcal{B}_2}$ are small in both the low-q^2 region and high-q^2 region in the SM.

B. T_{c3} semileptonic weak decays

Similar to $T_{c3,8} \to T'_8\gamma$ radiative decays [41–43,53], $T_{c3} \to T_8\ell^+\ell^-$ decays receive single-quark, two-quark, and three-quark transition contributions with the W-exchange and internal radiation contributions. The internal radiation contributions are suppressed by the two W propagators and can be safely neglected. The SU(3) IRA hadronic helicity amplitudes for $T_{c3} \to T_8\ell^+\ell^-$ may be parametrized as

$$\begin{aligned} H(T_{c3} \to T_8\ell^+\ell^-)^{L(R),s_p,s_k}_{VA,\lambda} &= f_1(T_{c3})^{[ij]}T'(\bar{3})^k(T_8)_{[ij]k} + f_2(T_{c3})^{[ij]}T'(3)^k(T_8)_{[ik]j} \\ &+ (\tilde{f}_1 H(\bar{6})^{lk}_j + \tilde{f}_4 H(15)^{lk}_j)(T_{c3})^{[ij]}(T_8)_{[il]k} \\ &+ (\tilde{f}_2 H(\bar{6})^{lk}_j + \tilde{f}_5 H(15)^{lk}_j)(T_{c3})^{[ij]}(T_8)_{[ik]l} \\ &+ (\tilde{f}_3 H(\bar{6})^{lk}_j + \tilde{f}_6 H(15)^{lk}_j)(T_{c3})^{[ij]}(T_8)_{[lk]i}, \end{aligned} \qquad (26)$$

where the SU(3) flavor symmetry parameters $f_i \equiv (f_i)^{L(R),s_p,s_k}_{VA,\lambda}$ and $\tilde{f}_i \equiv (\tilde{f}_i)^{L(R),s_p,s_k}_{VA,\lambda}$. The f_i terms in Eq. (26) and the later g_i terms in Eq. (29) denote the short distance (SD) and the LD contributions via the single-quark transitions. The $\tilde{f}_i$ terms in Eq. (26) and the later $\tilde{g}_i$ terms in Eq. (29) denote the W-exchange contributions of the two-quark and three-quark transitions. $T'(\bar{3}) = (1,0,0)$ denotes the transition operators $(\bar{q}_2 c)$ with $q_2 = u$, and $H(\bar{6})^{lk}_j$ ($H(15)^{lk}_j$) related to the $(\bar{q}_l q^j)(\bar{q}_k c)$ operator is antisymmetric (symmetric) in upper indices. The nonvanishing $H(\bar{6})^{lk}_j$ and $H(15)^{lk}_j$ for $c \to su\bar{d}, du\bar{s}, u\bar{d}d, u\bar{s}s$ transitions can be found in Ref. [54]. Using l,k antisymmetric in $H(\bar{6})^{lk}_j$ and l,k symmetric in $H(15)^{lk}_j$, we have

$$\tilde{f}_2 = -\tilde{f}_1, \qquad \tilde{f}_5 = \tilde{f}_4, \qquad \tilde{f}_6 = 0, \qquad (27)$$

which will be used in the following discussion.

For the W-exchange transitions, there are three kinds of charm quark decaying into light quarks,

$$\begin{aligned} &d + c \to u + s + \ell^+\ell^-, \\ &d + c \to u + d + \ell^+\ell^- (s + c \to u + s + \ell^+\ell^-), \\ &s + c \to u + d + \ell^+\ell^-, \end{aligned} \qquad (28)$$

which are related to $H(\bar{6},15)^{13}_2$, $H(\bar{6},15)^{12}_2[H(\bar{6},15)^{13}_3]$, and $H(\bar{6},15)^{12}_3$, and they are proportional to $V^*_{cs}V_{ud} \approx 1$, $V^*_{cd}V_{ud}(V^*_{cs}V_{us}) \approx -s_c(s_c)$, and $V^*_{cd}V_{us} \approx -s_c^2$ with $s_c \equiv \sin\theta_C \approx 0.22453$, respectively. So three kinds decays given in Eq. (28) are called Cabibbo allowed, singly Cabibbo suppressed, and doubly Cabibbo suppressed decays, respectively.

The SU(3) IRA amplitudes of the $T_{c3} \to T_8\ell^+\ell^-$ weak decays are given in the third column of Table VII, and for a better understanding, the information of relevant CKM matrix elements is also listed in this table.

TABLE VII. The SU(3) IRA amplitudes of the $T_{c3} \to T_8\ell^+\ell^-$ weak decays in the S_1 case, $F_1 \equiv f_1 + f_2$, $\tilde{F}_1 \equiv \tilde{f}_1 - \tilde{f}_3 + \tilde{f}_4$, $\tilde{F}_2 \equiv \tilde{f}_1 - \tilde{f}_3 - \tilde{f}_4$, and $\tilde{F} \equiv \tilde{f}_1 - \tilde{f}_3$.

Decay modes	$A(T_{c3} \to T_8\ell^+\ell^-)$	Approximative $A(T_{c3} \to T_8\ell^+\ell^-)$
Cabibbo allowed $T_{c3} \to T_8\ell^+\ell^-$:		
$\Lambda_c^+ \to \Sigma^+\ell^+\ell^-$	$-\tilde{F}_1$	$-\tilde{F}$
$\Xi_c^0 \to \Xi^0\ell^+\ell^-$	$-\tilde{F}_2$	$-\tilde{F}$
singly Cabibbo suppressed $T_{c3} \to T_8\ell^+\ell^-$:		
$\Lambda_c^+ \to p\ell^+\ell^-$	$[F_1 - (\frac{5}{8}\tilde{F}_1 - \frac{1}{8}\tilde{F}_2)]s_c$	$[F_1 - \frac{1}{2}\tilde{F}]s_c$
$\Xi_c^+ \to \Sigma^+\ell^+\ell^-$	$[-F_1 - (\frac{5}{8}\tilde{F}_1 - \frac{1}{8}\tilde{F}_2)]s_c$	$-[F_1 + \frac{1}{2}\tilde{F}]s_c$
$\Xi_c^0 \to \Lambda^0\ell^+\ell^-$	$[F_1 + 3(\frac{1}{8}\tilde{F}_1 - \frac{5}{8}\tilde{F}_2)]s_c/\sqrt{6}$	$[F_1 - \frac{3}{2}\tilde{F}]s_c/\sqrt{6}$
$\Xi_c^0 \to \Sigma^0\ell^+\ell^-$	$[-F_1 + (\frac{1}{8}\tilde{F}_1 - \frac{5}{8}\tilde{F}_2)]s_c/\sqrt{2}$	$-[F_1 + \frac{1}{2}\tilde{F}]s_c/\sqrt{2}$
doubly Cabibbo suppressed $T_{c3} \to T_8\ell^+\ell^-$:		
$\Xi_c^+ \to p\ell^+\ell^-$	$\tilde{F}_1 s_c^2$	$\tilde{F}s_c^2$
$\Xi_c^0 \to n\ell^+\ell^-$	$\tilde{F}_2 s_c^2$	$\tilde{F}s_c^2$

TABLE VIII. Branching ratios of $T_{c3} \to T_8 \ell^+\ell^-$ decays within 1σ theoretical error in the S_1 case.

Decay modes	Exp. UL [44]	Our predictions without F_1	Others without LD	Others with LD
$\mathcal{B}(\Lambda_c^+ \to \Sigma^+ e^+ e^-)(\times 10^{-6})$	…	≤ 2.63		
$\mathcal{B}(\Xi_c^0 \to \Xi^0 e^+ e^-)(\times 10^{-6})$	…	≤ 2.35		
$\mathcal{B}(\Lambda_c^+ \to p e^+ e^-)(\times 10^{-8})$	≤ 550	≤ 7.95	$(3.8\pm0.5)\times10^{-4}$ $(4.05\pm2.37)\times10^{-6}$ [20,21]	37 ± 8 420 ± 73 [20,21]
$\mathcal{B}(\Xi_c^+ \to \Sigma^+ e^+ e^-)(\times 10^{-7})$	…	≤ 1.29		
$\mathcal{B}(\Xi_c^0 \to \Lambda^0 e^+ e^-)(\times 10^{-8})$	…	≤ 8.69		
$\mathcal{B}(\Xi_c^0 \to \Sigma^0 e^+ e^-)(\times 10^{-8})$	…	≤ 2.22		
$\mathcal{B}(\Xi_c^+ \to p e^+ e^-)(\times 10^{-8})$	…	≤ 5.55		
$\mathcal{B}(\Xi_c^0 \to n e^+ e^-)(\times 10^{-8})$	…	≤ 1.92		
$\mathcal{B}(\Lambda_c^+ \to \Sigma^+ \mu^+ \mu^-)(\times 10^{-6})$	…	≤ 2.50		
$\mathcal{B}(\Xi_c^0 \to \Xi^0 \mu^+ \mu^-)(\times 10^{-6})$	…	≤ 2.25		
$\mathcal{B}(\Lambda_c^+ \to p \mu^+ \mu^-)(\times 10^{-8})$	≤ 7.7	≤ 7.7	$(2.8\pm0.4)\times10^{-4}$ $(3.77\pm2.28)\times10^{-6}$ [20,21]	37 ± 8 230 ± 66 [20,21]
$\mathcal{B}(\Xi_c^+ \to \Sigma^+ \mu^+ \mu^-)(\times 10^{-7})$	…	≤ 1.25		
$\mathcal{B}(\Xi_c^0 \to \Lambda^0 \mu^+ \mu^-)(\times 10^{-8})$	…	≤ 8.42		
$\mathcal{B}(\Xi_c^0 \to \Sigma^0 \mu^+ \mu^-)(\times 10^{-8})$	…	≤ 2.15		
$\mathcal{B}(\Xi_c^+ \to p \mu^+ \mu^-)(\times 10^{-8})$	…	≤ 5.41		
$\mathcal{B}(\Xi_c^0 \to n \mu^+ \mu^-)(\times 10^{-8})$	…	≤ 1.87		

From Table VII, one can see that singly Cabibbo suppressed $\Lambda_c^+ \to p\ell^+\ell^-$, $\Xi_c^+ \to \Sigma^+\ell^+\ell^-$, $\Xi_c^0 \to \Lambda^0\ell^+\ell^-$, $\Xi_c^0 \to \Sigma^0\ell^+\ell^-$ decays receive both the single-quark transition and the W-exchange contributions; nevertheless, Cabibbo allowed $\Lambda_c^+ \to \Sigma^+\ell^+\ell^-$, $\Xi_c^0 \to \Xi^0\ell^+\ell^-$ decays and doubly Cabibbo suppressed $\Xi_c^+ \to p\ell^+\ell^-$, $\Xi_c^0 \to n\ell^+\ell^-$ decays only receive the W-exchange contributions.

In addition, the contribution of $H(\bar{6})$ to the decay branching ratio is about 5.5 times larger than one of $H(15)$ due to the Wilson coefficient suppressed; for example, see Refs. [55,56]. If ignoring the Wilson coefficient suppressed $H(15)$ term contributions, there are only two parameters, F_1 and $\tilde{F} \equiv \tilde{f}_1 - \tilde{f}_3$, in the decay amplitudes of $T_{c3} \to T_8\ell^+\ell^-$. The simplified results are listed in the last column of Table VII. One can see that the Cabibbo allowed and doubly Cabibbo suppressed eight decay modes of $T_{c3} \to T_8\ell^+\ell^-$ are related by only one parameter $\tilde{F}$; nevertheless, the singly Cabibbo suppressed eight decays are related by two parameters $\tilde{F}$ and F_1. Moreover, there are amplitude relations $A(\Xi_c^+ \to \Sigma^+\ell^+\ell^-) = \sqrt{2}A(\Xi_c^0 \to \Sigma^0\ell^+\ell^-)$.

In these $T_{c3} \to T_8\ell^+\ell^-$ decays, only $\mathcal{B}(\Lambda_c^+ \to pe^+e^-)$ and $\mathcal{B}(\Lambda_c^+ \to p\mu^+\mu^-)$ are upper limited by experiment. We list their experimental upper limits (exp. UL) in the second column of Table VIII. Due to the lack of the experiment in $T_{c3} \to T_8\ell^+\ell^-$ decays and the complex amplitude expressions included the W-exchange contributions, we will only analyze the $T_{c3} \to T_8\ell^+\ell^-$ decays in the S_1 case. We assume that W-exchange contributions noted by $\tilde{F}$ or the single-quark transition contributions noted by F_1 play a dominant role in these decays. They are separately discussed as follows.

(i) Only considering the W-exchange contributions by setting $F_1 = 0$, all the $T_{c3} \to T_8\ell^+\ell^-$ decays are related by one parameter $\tilde{F}$ as shown in Table VII. The upper limit predictions of $T_{c3} \to T_8e^+e^-$ and $T_{c3} \to T_8\mu^+\mu^-$ decays in the S_1 case are listed in the third column of Table VIII. One can see that $\mathcal{B}(\Lambda_c^+ \to p\mu^+\mu^-)$ gives an effective constraint on these upper limit predictions of the branching ratios.

(ii) Only considering the single-quark transition contributions by setting $\tilde{F} = 0$, as shown in Table VII, all eight singly Cabibbo suppressed $T_{c3} \to T_8e^+e^-$ and $T_{c3} \to T_8\mu^+\mu^-$ decays are related by the parameter F_1. Then the six upper limit predictions of $\mathcal{B}(\Lambda_c^+ \to pe^+e^-)$, $\mathcal{B}(\Xi_c^+ \to \Sigma^+e^+e^-)$, $\mathcal{B}(\Xi_c^0 \to \Sigma^0e^+e^-)$, $\mathcal{B}(\Lambda_c^+ \to p\mu^+\mu^-)$, $\mathcal{B}(\Xi_c^+ \to \Sigma^+\mu^+\mu^-)$, and $\mathcal{B}(\Xi_c^0 \to \Sigma^0\mu^+\mu^-)$ in the S_1 case have the same predictions as the ones listed in the third column of Table VIII. Nevertheless, the predictions of other two singly Cabibbo suppressed $\mathcal{B}(\Xi_c^0 \to \Lambda^0e^+e^-)$ and $\mathcal{B}(\Xi_c^0 \to \Lambda^0\mu^+\mu^-)$ are different from the ones listed in Table VIII. We obtain that $\mathcal{B}(\Xi_c^0 \to \Lambda^0e^+e^-) \leq 9.66 \times 10^{-9}$ and $\mathcal{B}(\Xi_c^0 \to \Lambda^0\mu^+\mu^-) \leq 9.36 \times 10^{-9}$. The previous other predictions for $\mathcal{B}(\Lambda_c^+ \to pe^+e^-)$ and $\mathcal{B}(\Lambda_c^+ \to p\mu^+\mu^-)$, which considered the single-quark contributions with/without LD contributions, are also listed in the last two columns of Table VIII for comparing. The predicted upper limits of $\mathcal{B}(\Lambda_c^+ \to p\ell^+\ell^-)$ are about 5 orders larger than the predictions with the SD contributions, smaller than the LD contributing ones, which might mean that the W-exchange transitions cancel out with the LD contributions.

C. T_8 semileptonic weak decays

Similar to $T_8 \to T'_8\gamma$ radiative decays, $T_8 \to T'_8\ell^+\ell^-$ semileptonic decays receive the single-quark transition contributions and the W-exchange contributions [34]. The SD contributions come from the Z^0 and electromagnetic penguin diagrams as well as the Z^0 box diagrams, and the LD contributions arise from an intervirtual photon in the $T_8 \to T'_8\gamma^*$ processes. The LD contributions are much larger than the SD ones in the single-quark transition contributions in the $T_8 \to T'_8\ell^+\ell^-$ decays. So the LD contributions and the W-exchange contributions might play the major roles in $T_8 \to T'_8\ell^+\ell^-$ decays [30,34]. The SU (3) flavor structure of the $s \to d$ Hamiltonian can be found in Ref. [49]. The SU(3) IRA hadronic helicity amplitudes for $T_8 \to T'_8\ell^+\ell^-$ decays via $s \to d\ell^+\ell^-$ are

$$\begin{aligned} &H(T_8 \to T'_8\ell^+\ell^-)^{L(R),s_p,s_k}_{VA,\lambda} \\ &= g_1(T_8)^{[ij]n}T''(\bar{3})^k(T'_8)_{[ij]k} + g_2(T_8)^{[ij]n}T''(3)^k(T_8)_{[ik]j} \\ &\quad + g_3(T_8)^{[in]j}T''(\bar{3})^k(T'_8)_{[ij]k} + g_4(T_8)^{[in]j}T''(\bar{3})^k(T'_8)_{[ik]j} \\ &\quad + g_5(T_8)^{[in]j}T''(\bar{3})^k(T'_8)_{[jk]i} + \tilde{g}_1(T_8)^{[ij]n}(T'_8)_{[il]k}H(4)^{lk}_j \\ &\quad + \tilde{g}_2(T_8)^{[in]j}(T'_8)_{[il]k}H(4)^{lk}_j + \tilde{g}_3(T_8)^{[jn]i}(T'_8)_{[il]k}H(4)^{lk}_j, \end{aligned} \tag{29}$$

where the model and scale independent parameters $g_i \equiv (g_i)^{L(R),s_p,s_k}_{VA,\lambda}$ and $\tilde{g}_i \equiv (\tilde{g}_i)^{L(R),s_p,s_k}_{VA,\lambda}$, $T''(\bar{3}) = (0,1,0)$ related to the transition operator $(\bar{d}s)$, and $H(4)^{lk}_j$ related to $(\bar{q}_l q^j)(\bar{q}_k q^n)$ operator with $n \equiv 3$ for s quark is symmetric in upper indices [49]. The SU(3) IRA hadronic helicity amplitudes of $T_8 \to T'_8\ell^+\ell^-$ weak decays are given in Table IX, in which the information of the same CKM matrix elements $V_{us}V^*_{ud}$ is not shown.

There are four complex parameters G_1, G_2, $\tilde{G}_A$, and $\tilde{G}_B$ in $T_8 \to T'_8\ell^+\ell^-$ weak decays. Since the initial baryon Ξ^- does not contain u quark and the W-exchange contributions are canceled in $\Xi^0 \to \Lambda^0\ell^+\ell^-$ decays, the amplitudes of $\Xi^- \to \Sigma^-\ell^+\ell^-$ and $\Xi^0 \to \Lambda^0\ell^+\ell^-$ listed in Table IX only contain coefficients $G_{1,2}$, which means that the W-exchange transitions do not contribute to the $\Xi^- \to \Sigma^-\ell^+\ell^-$ and $\Xi^0 \to \Lambda^0\ell^+\ell^-$ decays. Therefore, $\Xi^- \to \Sigma^-\ell^+\ell^-$ and $\Xi^0 \to \Lambda^0\ell^+\ell^-$ decays could be used to explore the LD contributions. Other decay amplitudes contained both $G_{1,2}$ and $\tilde{G}_{A,B}$ could proceed from the LD contributions and the W-exchange contributions.

TABLE IX. The SU(3) IRA amplitudes of the $T_8 \to T'_8\ell^+\ell^-$ weak decays, $G_1 \equiv g_1 + g_2 + g_3 - g_5$, $G_2 \equiv g_4 + g_5$, $\tilde{G}_A \equiv \tilde{g}_1 - \tilde{g}_3$, and $\tilde{G}_B \equiv \tilde{g}_2 + \tilde{g}_3$.

Decay modes	$A(T_8 \to T'_8\ell^+\ell^-)$
$\Xi^- \to \Sigma^-\ell^+\ell^-$	G_1
$\Xi^0 \to \Lambda^0\ell^+\ell^-$	$(G_1 + 2G_2)/\sqrt{6}$
$\Xi^0 \to \Sigma^0\ell^+\ell^-$	$(G_1 + 2\tilde{G}_A)/\sqrt{2}$
$\Lambda^0 \to n\ell^+\ell^-$	$-[(G_1 + 2G_2) + (C_1 + 2\tilde{G}_A) - (G_2 - \tilde{G}_B)]/\sqrt{6}$
$\Sigma^0 \to n\ell^+\ell^-$	$-(G_2 - \tilde{G}_B)/\sqrt{2}$
$\Sigma^+ \to p\ell^+\ell^-$	$-(G_2 + \tilde{G}_B)$

Only two branching ratios of $\Xi^0 \to \Lambda^0 e^+e^-$ and $\Sigma^+ \to p\mu^+\mu^-$ decays have been measured at present, which are listed in the second column of Table X. We may constrain $|G_1 + 2G_2|$ and $|G_2 + \tilde{G}_B|$ from the experimental data of $\mathcal{B}(\Xi^0 \to \Lambda^0 e^+e^-)$ and $\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)$, respectively, and we obtain that $\frac{|G_1+2G_2|}{|G_2+\tilde{G}_B|} \approx 12$. $\mathcal{B}(\Sigma^+ \to pe^+e^-)$ is obtained by using the constrained $|G_2 + \tilde{G}_B|$,

$$\mathcal{B}(\Sigma^+ \to pe^+e^-) = (1.60^{+1.14}_{-0.87}) \times 10^{-7}, \tag{30}$$

which is 1 order smaller than its experimental upper limits $\mathcal{B}(\Sigma^+ \to pe^+e^-) \le 7 \times 10^{-6}$ at the 90% confidence level [44] and is also smaller than its SM predictions with the single-quark transition LD contributions, $9.1 \times 10^{-6} \le \mathcal{B}(\Sigma^+ \to pe^+e^-) \le 10.1 \times 10^{-6}$ in Ref. [30].

It is difficult to estimate which term gives the main contribution among G_1, G_2, $\tilde{G}_A$, and $\tilde{G}_B$ now. Nevertheless, the LD contributions noted by $G_{1,2}$ can not been entirely ignored via the experimental measurement of $\Xi^0 \to \Lambda^0\ell^+\ell^-$. So we will give the following discussions.

(i) If only considering the single-quark transition contributions, i.e., $\tilde{G}_A = \tilde{G}_B = 0$, one obtains $|\frac{G_1}{G2} + 2| \approx 12$; i.e., $G_1 \approx 10G_2$ or $-14G_2$. After ignoring the small G_2 terms in $G_1 + 2G_2$ and $2G_1 + G_2$, one gets all branching ratios of relevant $T_8 \to T'_8\ell^+\ell^-$ weak decays in S_1 case, which are given in the last column of Table X.

(ii) If $|\tilde{G}_B| \gg |G_2|$, $\frac{|G_1+2G_2|}{|G_2+\tilde{G}_B|} \approx \frac{|G_1|}{|\tilde{G}_B|} \approx 12$, the predictions of $\mathcal{B}(\Xi^- \to \Sigma^- e^+e^-)$, $\mathcal{B}(\Xi^0 \to \Lambda^0 e^+e^-)$, $\mathcal{B}(\Sigma^0 \to ne^+e^-)$, $\mathcal{B}(\Sigma^+ \to pe^+e^-)$, $\mathcal{B}(\Sigma^0 \to n\mu^+\mu^-)$, and $\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)$ are the same as given in Table X. Nevertheless, $B(\Xi^0 \to \Sigma^0\ell^+\ell^-)$ and $\mathcal{B}(\Lambda^0 \to n\ell^+\ell^-)$ can not be predicted in this case.

(iii) In the case of $\tilde{G}_B \approx G_2$, the predicted results are similar to above ones with $|\tilde{G}_B| \gg |G_2|$ except $\mathcal{B}(\Sigma^0 \to n\ell^+\ell^-) \approx 0$.

(iv) If $\tilde{G}_B \approx -G_2$, *i.e.*, the contributions between G_2 and $\tilde{G}_B$ are largely canceled in $G_2 + \tilde{G}_B$ term, the situations are complex, which is beyond the scope of this paper.

All predicted branching ratios except $\mathcal{B}(\Sigma^0 \to ne^+e^-)$ and $\mathcal{B}(\Sigma^0 \to n\mu^+\mu^-)$ in above first three cases are on the order of $\mathcal{O}(10^{-8}$–$10^{-5})$, some of them might be observed by BESIII and Belle-II experiments in the near future. The measurement of $\mathcal{B}(\Xi^- \to \Sigma^- e^+e^-)$ and $\mathcal{B}(\Xi^0 \to \Sigma^0 e^+e^-)$ in the future could further help us to understand the

TABLE X. Branching ratios of $T_8 \to T'_8 \ell^+\ell^-$ decays within 1σ theoretical error in the S_1 case.

Decay modes	Experimental data [44]	Our predictions without $\tilde{G}_{A,B}$ in S_1
$\mathcal{B}(\Xi^- \to \Sigma^- e^+ e^-)(\times 10^{-6})$	…	$2.49^{+0.31}_{-0.29}$
$\mathcal{B}(\Xi^0 \to \Lambda^0 e^+ e^-)(\times 10^{-6})$	7.6 ± 0.6	7.6 ± 0.6
$\mathcal{B}(\Xi^0 \to \Sigma^0 e^+ e^-)(\times 10^{-6})$	…	2.05 ± 0.17
$\mathcal{B}(\Lambda^0 \to n e^+ e^-)(\times 10^{-5})$	…	$2.06^{+0.25}_{-0.23}$
$\mathcal{B}(\Sigma^0 \to n e^+ e^-)(\times 10^{-17})$	…	$7.61^{+6.59}_{-4.44}$
$\mathcal{B}(\Sigma^+ \to p e^+ e^-)(\times 10^{-7})$	< 70	$1.60^{+1.14}_{-0.87}$
$\mathcal{B}(\Sigma^0 \to n \mu^+ \mu^-)(\times 10^{-17})$	…	$1.22^{+1.05}_{-0.71}$
$\mathcal{B}(\Sigma^+ \to p \mu^+ \mu^-)(\times 10^{-8})$	$2.4^{+1.7}_{-1.3}$	$2.40^{+1.70}_{-1.30}$

LD contributions and the W-exchange contributions, respectively.

IV. SUMMARY

Semileptonic baryon decays induced by flavor changing neutral current transitions play very important roles in testing the SM and probing new physics. We have studied the semileptonic decays of baryons with $\frac{1}{2}$ spin via the single-quark transitions $b \to s/d\ell^+\ell^-$, $c \to u\ell^+\ell^-$, and $s \to d\ell^+\ell^-$ as well as relevant W-exchange transitions by using the SU(3) flavor symmetry, which is a powerful tool to test the physics and to connect the physical quantities without knowing the underlying dynamics. Our main results can be summarized as follows.

(i) $T_{b3} \to T_8 \ell^+\ell^-$ decays: Decay $\Lambda_b^0 \to \Sigma^0 \ell^+\ell^-$ is not allowed by the SU(3) irreducible representation approach, and all other 21 decay amplitudes of the $T_{b3} \to T_8 \ell^+\ell^-$ decay modes via the $b \to s/d\ell^+\ell^-$ transitions could be related by only one SU(3) flavor symmetry parameter, which could be constrained by the present experimental data of $\mathcal{B}(\Lambda_b^0 \to \Lambda^0 \mu^+\mu^-)$. Using the constrained parameter, we have predicted the not-yet-measured observables in the whole q^2 region and in different q^2 bins within the S_1 and S_2 cases. The predicted branching ratios are on the order of $\mathcal{O}(10^{-8}\text{–}10^{-6})$, many of them are obtained for the first time, and some of them could be reached by the LHCb or Belle-II experiments. The longitudinal polarization fractions and the leptonic forward-backward asymmetries of all $T_{b3} \to T_8 \ell^+\ell^-$ decays are very similar to each other in certain q^2 bins by the SU(3) flavor symmetry. The predictions of $\langle f_L \rangle(\Lambda_b^0 \to \Lambda^0 \mu^+\mu^-)_{[15,20]}$ and $\langle A_{FB}^{\ell} \rangle(\Lambda_b^0 \to \Lambda^0 \mu^+\mu^-)_{[15,20]}$ are agreeable with their experimental data within 1.5σ and 1σ error ranges, respectively.

(ii) $T_{c3} \to T_8 \ell^+\ell^-$ decays: $T_{c3} \to T_8 \ell^+\ell^-$ decays are quite different from $T_{b3} \to T_8 \ell^+\ell^-$ decays, since the former may receive both the single-quark $c \to u\ell^+\ell^-$ transition contributions and the W-exchange contributions. After ignoring the Wilson coefficient suppressed $H(15)$ terms, all decay amplitudes of $T_{c3} \to T_8 \ell^+\ell^-$ have been related by two SU(3) flavor symmetry parameters. Using the 90% experimental upper limit of $\mathcal{B}(\Lambda_c^+ \to p\mu^+\mu^-)$, we have obtained the upper limit predictions of the not-yet-measured $\mathcal{B}(T_{c3} \to T_8 \ell^+\ell^-)$ by considering only one kind of dominant contributions from either single-quark transition LD contributions or the W-exchange contributions.

(iii) $T_8 \to T'_8 \ell^+\ell^-$ decays: Decays $T_8 \to T'_8 \ell^+\ell^-$ are more complicated than both $T_{b3} \to T_8 \ell^+\ell^-$ and $T_{c3} \to T_8 \ell^+\ell^-$ ones, since the quarks are anti symmetric in both the initial states T_8 and the final states T'_8. We have predicted $\mathcal{B}(\Sigma^+ \to p e^+ e^-)$ in the S_1 case. Moreover, we have analyzed the single-quark transition LD contributions and the W-exchange contributions, and found that $\mathcal{B}(\Xi^- \to \Sigma^- e^+ e^-)$, $\mathcal{B}(\Xi^0 \to \Sigma^0 e^+ e^-)$, and $\mathcal{B}(\Lambda^0 \to n e^+ e^-)$ are on the order of $\mathcal{O}(10^{-6}\text{–}10^{-5})$ in most cases except $\tilde{G}_B \approx -G_2$.

According to our predictions, many results in this work can be tested by the experiments at BESIII, LHCb, and Belle-II. And these results can be used to test SU(3) flavor symmetry approach in $T_{b3,c3,8} \to T'_8 \ell^+\ell^-$ by the future experiments.

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