1/2-BPS membrane instantons in $AdS_4 \times S^7/Z_k$

Jaemo Park* and Hyeonjoon Shin†

Department of Physics & Center for Theoretical Physics, POSTECH, Pohang, Gyeongbuk 37673, South Korea and Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 37673, South Korea

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According to the covariant open superstring description of D-branes in the $AdS_4 \times \mathbf{CP}^3$ background, 1/2-BPS D2-branes are purely instantonic. Based on this and by taking the eleven dimensional viewpoint, we identify the 1/2-BPS instantonic M2-brane configurations in the $AdS_4 \times S^7/\mathbf{Z}_k$ background, which reduces to the $AdS_4 \times \mathbf{CP}^3$ under the large k limit, and evaluate their action values. We also consider the previously known 1/2-BPS instantonic objects in ten dimensions from the M2-brane viewpoint to compare with our results.

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I. INTRODUCTION

The covariant open superstring description of D-branes [1,2] is a useful tool in classifying supersymmetric D-branes, especially 1/2-BPS D-branes, in a given supersymmetric background. It has been successfully applied to some important backgrounds in superstring theories such as the flat spacetime [1], IIB plane wave [2], IIA plane wave [3], $AdS_5 \times S^5$ [4–7], and $AdS_4 \times \mathbf{CP}^3$ [8] backgrounds. The data obtained after the classification of supersymmetric D-branes are however "primitive" in a sense that they do not tell us about which configuration of a given D-brane is really supersymmetric or which part of the background supersymmetry is preserved on the D-brane world volume. Nevertheless, the classification provides us a good guideline for further exploration of supersymmetric D-branes. Indeed this has been illustrated for the $AdS_5 \times S^5$ background in [9].

In our previous work [8], we have obtained the data about 1/2-BPS D-branes in the $AdS_4 \times \mathbf{CP}^3$ background. One interesting point from the data is that 1/2-BPS D2-brane is purely instantonic and there is no 1/2-BPS Lorentzian D2-brane in contrast to other D-branes of different dimensionalities.

As is well known, the type IIA superstring theory in the $AdS_4 \times \mathbf{CP}^3$ background is dual to the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory [10]. Under this correspondence, the study on the nonperturbative aspects of the ABJM theory has been a fascinating research subject, in which some instantonic objects have played important roles.[1] Basically, there are two types of instantonic objects called world sheet and membrane instantons. Both of them are known to be 1/2-BPS and, in the bulk side, are identified as string [12] and D2-brane [13] instantons wrapping certain subspaces of $\mathbf{CP}^3$. By the way, the result of [8] indicates that, in addition to these instantons, there is a possibility to have other 1/2-BPS instantonic D2-brane configurations.

In the present work, we try to identify such 1/2-BPS D2-brane configurations purely based on the data obtained in [8] and evaluate their action values. These 1/2-BPS D2 brane configurations can be lifted to membrane (M2-brane) configurations. Then one can extrapolate 1/2 BPS M2-brane configurations in the $AdS_4 \times S^7/\mathbf{Z}_k$ background for all finite k, which reduces to the $AdS_4 \times \mathbf{CP}^3$ in the large k limit. We also consider the ten dimensional instantons of [12,13] and comment on the corresponding the M2-brane configurations.

In the next section, we give a brief description of the $AdS_4 \times S^7/\mathbf{Z}_k$ background. The possible 1/2-BPS instantonic M2-brane configurations are identified in Sec. III, and their action values are evaluated in Sec. IV. Finally, the conclusion follows in Sec. V.

II. $AdS_4 \times S^7/Z_k$ BACKGROUND

The $AdS_4 \times S^7/\mathbf{Z}_k$ background originated from the near horizon limit of M2-brane supergravity solution is composed of the $AdS_4 \times S^7/\mathbf{Z}_k$ geometry

*jaemo@postech.ac.kr
†nonchiral@gmail.com

[1] For comprehensive review, see [11].

$$ds^2 = \frac{R^2}{4} ds^2_{\mathrm{AdS}_4} + R^2 ds^2_{S^7/Z_k} \tag{1}$$

and the four-form field strength

$$F_4 = \frac{3}{8} R^3 \epsilon_{\mathrm{AdS}_4}, \tag{2}$$

where R is the radius of S^7 given by

$$R = \ell_p (2^5 \pi^2 N k)^{1/6} \tag{3}$$

with the eleven dimensional Planck length ℓ_p and the number of M2-branes Nk, and $\epsilon_{\mathrm{AdS}_4}$ is the volume form of AdS_4 space. The metric of AdS_4 is, in the global coordinates,

$$ds^2_{\mathrm{AdS}_4} = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho(d\theta^2 + \sin^2\theta d\psi^2). \tag{4}$$

As for the geometry of $\mathrm{S}^7/\mathbf{Z}_k$, it is natural to express it through the $U(1)$ Hopf fibration over $\mathbf{CP}^3$ for its obvious connection with the type IIA background [14]:

$$ds^2_{S^7/Z_k} = ds^2_{\mathbf{CP}^3} + \frac{1}{k^2}(dy + A)^2, \tag{5}$$

where y is the $U(1)$ fiber coordinate with the period $y \sim y + 2\pi$, A is a one-form given by

$$A = \frac{k}{2}\left(\cos\alpha d\chi + \cos^2\frac{\alpha}{2}\cos\theta_1 d\phi_1 + \sin^2\frac{\alpha}{2}\cos\theta_2 d\phi_2\right), \tag{6}$$

and the $\mathbf{CP}^3$ geometry is as follows:

$$\begin{aligned} ds^2_{\mathbf{CP}^3} = &\frac{1}{4}d\alpha^2 + \frac{1}{4}\cos^2\frac{\alpha}{2}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) \\ &+ \frac{1}{4}\sin^2\frac{\alpha}{2}(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2) \\ &+ \frac{1}{4}\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}(2d\chi + \cos\theta_1 d\phi_1 - \cos\theta_2 d\phi_2)^2, \end{aligned} \tag{7}$$

with the ranges of angles, $0 \le \alpha, \theta_1, \theta_2 \le \pi$ and $0 \le \chi, \phi_1, \phi_2 \le 2\pi$.

From the metric of (5), we choose the elfbeine for $\mathrm{S}^7/\mathbf{Z}_k$ as[2]

$$\begin{aligned} e^4 &= \frac{1}{2}d\alpha, \\ e^5 &= \frac{1}{2}\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}(2d\chi + \cos\theta_1 d\phi_1 - \cos\theta_2 d\phi_2), \\ e^6 &= \frac{1}{2}\cos\frac{\alpha}{2}d\theta_1, \qquad e^7 = \frac{1}{2}\cos\frac{\alpha}{2}\sin\theta_1 d\phi_1, \\ e^8 &= \frac{1}{2}\sin\frac{\alpha}{2}d\theta_2, \qquad e^9 = \frac{1}{2}\sin\frac{\alpha}{2}\sin\theta_2 d\phi_2, \\ e^\natural &= -\frac{1}{k}(dy + A). \end{aligned} \tag{8}$$

The reason for this choice is to make the Kähler structure of $\mathbf{CP}^3$ in a canonical form. The Kähler structure itself can be read off from one property of one-form A, $dA = 2kJ$, where J is the Kähler two-form of $\mathbf{CP}^3$. If we compute dA using (6), we obtain

$$dA = -2k(e^4 \wedge e^5 + e^6 \wedge e^7 + e^8 \wedge e^9), \tag{9}$$

which indeed shows that J has the block diagonal structure, that is, the canonical form and thus the above choice for the elfbeine is a proper one.

III. 1/2-BPS M2-BRANE INSTANTONS

For the investigation of 1/2-BPS M2-brane configurations, we need the Killing spinor ϵ of the $\mathrm{AdS}_4 \times \mathrm{S}^7/\mathbf{Z}_k$ background. It has been obtained in [13,15,16] by solving the Killing spinor equation and has the following form.

$$\epsilon = \mathcal{M}\epsilon_0, \tag{10}$$

where ϵ_0 is a 32 component constant spinor[3] and $\mathcal{M}$ is given by

$$\begin{aligned} \mathcal{M} = &e^{\frac{\alpha}{4}(\hat{\gamma}\gamma_4 - \gamma_{5\natural})} e^{\frac{\theta_1}{4}(\hat{\gamma}\gamma_6 - \gamma_{7\natural})} e^{\frac{\theta_2}{4}(\gamma_{59} + \gamma_{48})} e^{-\frac{\xi_1}{2}\hat{\gamma}\gamma_\natural} e^{-\frac{\xi_2}{2}\gamma_{67}} e^{-\frac{\xi_3}{2}\gamma_{45}} e^{-\frac{\xi_4}{2}\gamma_{89}} \\ &\times e^{\frac{\rho}{2}\hat{\gamma}\gamma_1} e^{\frac{t}{2}\hat{\gamma}\gamma_0} e^{\frac{\theta}{2}\gamma_{12}} e^{\frac{\psi}{2}\gamma_{23}} \end{aligned} \tag{11}$$

with $\hat{\gamma} = \gamma^{0123}$ and a set of coordinate combinations,

$$\begin{aligned} \xi_1 &= \frac{1}{2}(\phi_1 + \chi + 2y), \qquad \xi_2 = \frac{1}{2}(-\phi_1 + \chi + 2y), \\ \xi_3 &= \frac{1}{2}(\phi_2 - \chi + 2y), \qquad \xi_4 = \frac{1}{2}(-\phi_2 - \chi + 2y). \end{aligned} \tag{12}$$

The Killing spinor ϵ is obtained in the Lorentzian signature. Since we are concerned about the instantonic configuration for which the spacetime is taken to be Euclidean, we should recast ϵ in a way to respect the Euclidean nature. However, we will not care much about it because the time coordinate

[2] $e^{0,1,2,3}$ are for the AdS_4 space. Although their explicit form is not necessary in this work, if we follow the choice of [13], they are $e^0 = \frac{1}{2}\cosh\rho dt$, $e^1 = \frac{1}{2}d\rho$, $e^2 = \frac{1}{2}\sinh\rho d\theta$, and $e^3 = \frac{1}{2}\sinh\rho\sin\theta d\psi$.

[3] The constant spinor ϵ_0 has 24 free components for $k > 2$. Otherwise, that is, for $k = 1, 2$, it has 32 free components. For more detailed discussion, see Refs. [13,15,16].

will be set to zero and our interest is the consistent projection operators acting on ϵ (strictly speaking ϵ_0) which identify the 1/2-BPS configurations.

Having the Killing spinor, the 1/2-BPS instantonic M2-brane configurations can be considered by using the usual equation

$$\Gamma\epsilon = \epsilon, \tag{13}$$

which is obtained by combining the spacetime supersymmetry and κ-symmetry transformation. The symbol Γ is the spinorial matrix appearing in the κ-symmetry projector and satisfies $\Gamma^2 = 1$ and $\mathrm{Tr}\Gamma = 0$. The explicit expression of Γ for M2-brane is

$$\Gamma = \frac{i}{3!\sqrt{g}}\epsilon^{ijk}\Pi_i^a\Pi_j^b\Pi_k^c\Gamma_{abc} \tag{14}$$

where $\Pi_i^a = \partial_i X^\mu e_\mu^a$ and g is the determinant of the induced metric on M2-brane, $g_{ij} = \Pi_i^a\Pi_j^b\eta_{ab}$. The indices i, j, k are those of the M2-brane world volume, and $\mu, \nu, \ldots$ $(a, b, \ldots)$ are the curved (tangential) spacetime indices. Now, by using the Killing spinor (10), the Eq. (13) can be rewritten as

$$\tilde{\Gamma}\epsilon_0 = \epsilon_0, \tag{15}$$

where we have defined

$$\tilde{\Gamma} \equiv \mathcal{M}^{-1}\Gamma\mathcal{M}. \tag{16}$$

Here $\tilde{\Gamma}^2 = 1$ is guaranteed because $\Gamma^2 = 1$. For a given M2-brane configuration, Γ and $\mathcal{M}$ have the corresponding expressions by which $\tilde{\Gamma}$ is determined. If the resulting $\tilde{\Gamma}$ does not depend on any world volume coordinate, then the M2-brane configuration is confirmed to be 1/2-BPS.

For specifying a M2-brane configuration, let us introduce a notation

$$[X, Y, Z], \tag{17}$$

which means that the M2 world volume coordinates (ζ^1, ζ^2, ζ^3) are identified as $\zeta^1 = X$, $\zeta^2 = Y$, and $\zeta^3 = Z$. In other words, the notation represents a static gauge for the world volume reparametrization. Except for the coordinates along which M2-brane spans, all other coordinates in $\mathcal{M}$ which are transverse to M2-brane are set to zero. These are the natural generalizations of the previous results of 1/2-BPS D2-brane configurations, which can be easily verified in M theory setting. However, if a polar coordinate among α, θ_1, θ_2 is taken to be transverse one, it is kept to be an arbitrary constant.

Before investigating the instantonic M2-brane configurations based on the covariant open string description of 1/2-BPS D-branes, we briefly reconsider the previously explored string world sheet instanton [12] and D2-brane instanton [13] from the viewpoint of eleven dimensional M2-brane. Both of them have been studied in the context of ten dimensional IIA string theory and turned out to be 1/2-BPS. As for the string world sheet instanton, it wraps $\mathbf{CP}^1$ ($\subset \mathbf{CP}^3$) parametrized by α and χ. Since a string is nothing but the M2-brane wrapping the M-theory circle direction y, the corresponding M2-brane configuration is given by $[\alpha, \chi, y]$. The expression of Γ (14) becomes simply $\Gamma = -i\gamma_{45\natural}$ and $\tilde{\Gamma}$ of Eq. (16) is evaluated as

$$\tilde{\Gamma} = -i\gamma_{45\natural}. \tag{18}$$

By the way, since there are two more two spheres within $\mathbf{CP}^3$ parametrized by (θ_1, ϕ_1) and (θ_2, ϕ_2), one may be curious about whether the configurations $[\theta_1, \phi_1, y]$ and $[\theta_2, \phi_2, y]$ are also 1/2-BPS. Actually, they are but in a restricted sense. As for $[\theta_1, \phi_1, y]$, it is not difficult to check that the configuration is 1/2-BPS only when its transverse position in α is zero, $\alpha = 0$.[4] In this case, $\Gamma = -i\gamma_{67\natural}$ and $\tilde{\Gamma}$ simply becomes $-i\gamma_{67\natural}$. As for another configuration $[\theta_2, \phi_2, y]$, it is 1/2-BPS only when $\alpha = \pi$. For this, we get $\Gamma = -i\gamma_{89\natural}$ and the corresponding $\tilde{\Gamma}$ as $i\hat{\gamma}\gamma_{4589} = -i\gamma_{67\natural}$.[5]

On the other hand, the D2-brane instanton spans the Lagrangian submanifold $\mathbf{RP}^3$ within $\mathbf{CP}^3$ which is parametrized by the coordinates (χ, ϑ, φ) with the identifications $\vartheta = \theta_1 = \theta_2$ and $\varphi = \phi_1 = -\phi_2$. Since a D2-brane in ten dimensions is just an M2-brane in eleven dimensions, the M2-brane configuration corresponding to the D2-brane instanton is simply $[\chi, \vartheta, \varphi]$. Then the Γ of (14) for this configuration becomes $\Gamma = -i\gamma_{67\natural}e^{\frac{\alpha}{2}(2\gamma_{5\natural}+\gamma_{68}-\gamma_{79})}$ and the evaluation of $\tilde{\Gamma}$ of (16) leads to

$$\tilde{\Gamma} = -i\gamma_{67\natural}e^{\frac{\alpha}{2}(\hat{\gamma}\gamma_4+\gamma_{5\natural}+\gamma_{68}-\gamma_{79})}. \tag{19}$$

Obviously, the above two $\tilde{\Gamma}$'s, (18) and (19), are independent of any of the world volume coordinates. Thus, from Eq. (15), it is confirmed that the previously known M2-brane configurations are 1/2-BPS as they should be.

Now let us turn to our main concern and investigate the 1/2-BPS M2-brane instantonic configurations based on the covariant open string description of D-branes. According to the open string description [8], purely instantonic D2-brane can be 1/2-BPS under the following condition: only one world volume direction is allowed to extend in each of three two dimensional subspaces of $\mathbf{CP}^3$. The three subspaces are realized by looking at the Kähler structure of $\mathbf{CP}^3$ (9). Following this condition, we see that there are eight candidates for the 1/2-BPS M2-brane instantonic

[4]One can check that the resulting two sphere has maximal radius $R/2$ when $\alpha = 0$.

[5]An identity $\hat{\gamma}\gamma_{456789} = \gamma_\natural$ has been used.

configurations which are $[\alpha,\theta_1,\theta_2]$, $[\alpha,\theta_1,\phi_2]$, $[\alpha,\phi_1,\theta_2]$, $[\alpha,\phi_1,\phi_2]$, $[\chi,\theta_1,\theta_2]$, $[\chi,\theta_1,\phi_2]$, $[\chi,\phi_1,\theta_2]$, and $[\chi,\phi_1,\phi_2]$.

We have investigated all the candidates and found that only four configurations in which α is a world volume direction are 1/2-BPS. In what follows, we describe the 1/2-BPS configurations in sequence. First of all, for the configuration $[\alpha,\theta_1,\theta_2]$, the Γ of (14) becomes $\Gamma = i\gamma_{468}$, and the corresponding $\tilde{\Gamma}$ of (16) is evaluated as

$$\tilde{\Gamma} = i\gamma_{468}. \tag{20}$$

Second, the configuration $[\alpha,\theta_1,\phi_2]$ with fixed θ_2 leads to $\Gamma = i\gamma_{456} e^{-\frac{\alpha}{4}\gamma_{5\natural}} e^{-\theta_2\gamma_{59}} e^{-\frac{\alpha}{4}\gamma_{5\natural}}$, and we obtain the corresponding $\tilde{\Gamma}$ as

$$\tilde{\Gamma} = i\gamma_{456} e^{\frac{\theta_2}{2}(\gamma_{48}-\gamma_{59})}. \tag{21}$$

The third configuration $[\alpha,\phi_1,\theta_2]$ is equivalent to $[\alpha,\theta_1,\phi_2]$. So we just move on to the last configuration $[\alpha,\phi_1,\phi_2]$ with fixed θ_1 and θ_2. For this configuration, we have $\Gamma = -i\gamma_{45\natural} e^{-\frac{\alpha}{4}\gamma_{5\natural}} e^{\theta_1\gamma_{7\natural}} e^{-\theta_2\gamma_{59}} e^{\frac{\alpha}{4}\gamma_{5\natural}}$, and the corresponding $\tilde{\Gamma}$ is obtained as

$$\tilde{\Gamma} = -i\gamma_{45\natural} e^{\frac{\theta_1}{2}(\hat{\gamma}\gamma_6+\gamma_{7\natural})} e^{\frac{\theta_2}{2}(\gamma_{48}-\gamma_{59})}. \tag{22}$$

We see that all the resulting $\tilde{\Gamma}$'s in (20), (21), and (22) do not depend on any of the worldvolume coordinates. Therefore Eq. (15) shows that half of the spacetime supersymmetry $(1+\tilde{\Gamma})\epsilon_0$ is preserved on the M2-brane world volume.

At this point, one may ask why the other four configurations in which χ is a world volume direction are not 1/2-BPS. Although we have checked all of them, we just take one of them as an example and briefly illustrate the reason. Consider $[\chi,\theta_1,\theta_2]$. For this configuration, Γ of (14) is simply $\Gamma = -i\gamma_{68\natural}$ and $\mathcal{M}$ of (11) becomes $\mathcal{M} = e^{\frac{\theta_1}{4}(\hat{\gamma}\gamma_6-\gamma_{7\natural})} e^{\frac{\theta_2}{4}(\gamma_{59}+\gamma_{48})} e^{-\frac{\chi}{4}(\hat{\gamma}\gamma_{\natural}+\gamma_{67}-\gamma_{45}-\gamma_{89})}$. The evaluation of $\tilde{\Gamma}$ of (16) is the process of pushing Γ to the left of $\mathcal{M}^{-1}$ (or to the right of $\mathcal{M}$). But, unlike the previous cases, this process makes us face with a problem from the beginning. That is, what we get for the θ_1 dependent part is

$$e^{-\frac{\theta_1}{4}(\hat{\gamma}\gamma_6-\gamma_{7\natural})}\gamma_{68\natural} e^{\frac{\theta_1}{4}(\hat{\gamma}\gamma_6-\gamma_{7\natural})} = \gamma_{68\natural} e^{-\frac{\theta_1}{2}\gamma_{7\natural}}, \tag{23}$$

which means that $\tilde{\Gamma}$ depends on the world volume coordinate θ_1. There is no way to eliminate the θ_1 dependence. This is also the case for other world volume coordinates, θ_2 and χ. Thus $\tilde{\Gamma}$ depends on the world volume coordinates implying that we have different projection operators at different world volume positions. This inconsistency explicitly shows that the M2-brane configuration $[\chi,\theta_1,\theta_2]$ is not 1/2-BPS. Of course, the configuration may be less supersymmetric by imposing other projection conditions. However, since here we are interested in 1/2-BPS objects, it will not be considered further.

IV. INSTANTON ACTION

For 1/2-BPS M2-brane instanton configurations considered in the last section, let us evaluate their classical action values. The basic purpose is to compare the resulting values with those for the string world sheet instanton and D2-brane instanton obtained in [12,13], respectively.

The bosonic part of the Euclidean M2-brane action is given by

$$S = T_2 \int_{\mathcal{M}_3} d^3\zeta \sqrt{g} + iT_2 \int_{\mathcal{M}_4(\mathcal{M}_3=\partial\mathcal{M}_4)} F_4, \tag{24}$$

where T_2 is the M2-brane tension,

$$T_2 = \frac{1}{4\pi^2 \ell_p^3}, \tag{25}$$

and g is the determinant of the induced metric on the M2-brane world volume. Since the M2-brane instantons we have considered are placed within $S^7/\mathbf{Z}_k$ while the background four-form field strength F_4 is turned on in the AdS_4 space as one can see from Eq. (2), the Wess-Zumino term including F_4 does not contribute to the action. Thus we only need to evaluate the Nambu-Goto type kinetic term.

We first consider the configuration $[\alpha,\phi_1,\phi_2]$ with fixed θ_1 and θ_2. For this, $\sqrt{g}$ is computed as $\frac{R^3}{8}\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}$. We see that there is no dependence on the transverse coordinates θ_1 and θ_2, and hence the action value will always be the same regardless of the transverse position of the M2-brane configuration. Having the expression for $\sqrt{g}$, it is straightforward to evaluate the action, which is obtained as follows:

$$\begin{aligned} S &= 2T_2 \int_0^\pi d\alpha \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \frac{R^3}{8}\cos\frac{\alpha}{2}\sin\frac{\alpha}{2} \\ &= \pi^2 T_2 R^3 \\ &= \pi k\sqrt{\frac{2N}{k}}, \end{aligned} \tag{26}$$

where Eqs. (3) and (25) have been used in the last step. We note that the α integration has doubled in the first line as $\int_0^\pi d\alpha \to 2\int_0^\pi d\alpha$ basically because the nature of α is a polar angle. The two angles α and χ parametrize a sphere as one notices from Eq. (7). One world volume direction of the present M2-brane wraps this sphere along α with fixed χ. But, since α is a polar angle, the world volume direction covers not a circle but half the circle geometrically. Thus, in order to get the correct action value, we should double the integration over α.

By following the same way, the action values for other configurations $[\alpha,\theta_1,\theta_2]$, $[\alpha,\theta_1,\phi_2]$, and $[\alpha,\phi_1,\theta_2]$ can be evaluated. The resulting values are exactly the same with that of $[\alpha, \phi_1, \phi_2]$ obtained in Eq. (26). We note that the action value $\pi k\sqrt{2N/k}$ agrees with that for the D2-brane instanton wrapping the Lagrangian submanifold $\mathbf{RP}^3$ within $\mathbf{CP}^3$ computed in [13]. Although the D2-brane result of [13] has been obtained in ten dimensional IIA theory, it is not difficult to check that it holds even in eleven dimensions without any modification by considering the M2-brane configuration corresponding to the D2-brane wrapping the $\mathbf{RP}^3$. Thus it seems plausible that all the 1/2-BPS M2-brane instantons which become D2-brane instantons in ten dimensions are characterized by the action value $\pi k\sqrt{2N/k}$.

A noteworthy point of the above result is that the action value is valid even at finite k in contrast to the ten dimensional case where $k \gg 1$ is assumed for the validity of type IIA $AdS_4 \times \mathbf{CP}^3$ background. Actually, this situation continues to hold even for the case of world sheet instanton for which the action value has been obtained as [12]

$$S = 2\pi\sqrt{\frac{2N}{k}}. \tag{27}$$

If we evaluate the corresponding M2-brane configuration for the world sheet instanton, that is, $[\alpha, \chi, y]$, then we get the same result $2\pi\sqrt{2N/k}$ yet valid at finite k.[6] Thus we see that the action values of the ten dimensional 1/2-BPS instantons are not modified at finite k.

V. CONCLUSION

Based on the data [8] obtained from the covariant open superstring description of 1/2-BPS D-branes in type IIA $AdS_4 \times \mathbf{CP}^3$ background, we have explored the possible 1/2-BPS M2-brane instanton configurations from the eleven dimensional viewpoint. It has been shown that there exist four additional 1/2-BPS M2-brane instanton configurations in addition to the previously known ones. All of them, which are interpreted as D2-brane instantons in ten dimensions, have the same action value $\pi k\sqrt{2N/k}$ which is identical to that of D2-brane instanton studied in [13]. One important result is that the action value is valid even at finite k. Actually, this is also the case for the 1/2-BPS M2-brane instanton configuration corresponding to the world sheet instanton [12]. We speculate that the validity of action values at finite k is due to large amount of supersymmetry.

A supersymmetric brane configuration means that the theory on its world volume is supersymmetric. This in turn implies that there is no tachyonic mode on the world volume signaling instability of the configuration. Here, since we have considered the Euclidean theory, it would be appropriate to view a supersymmetric configuration as a minimal action configuration rather than stable one. Let us now read off the quadratic parts of small bosonic transverse deformations for each supersymmetric configuration based on the M2-brane action (24). What we obtain is that all the transverse modes are massless for all the supersymmetric configurations and their actions are of the form $\int d^3\zeta\sqrt{h}h^{ij}\partial_i\delta\varphi\partial_j\delta\varphi$ where h^{ij} is the induced world volume metric for a given supersymmetric configuration and $\delta\varphi$ represents the small transverse deformation. Since the quadratic action is positive definite, the transverse deformations tend to increase the action value. This shows that the supersymmetric instantons explored in this work are action minimizing ones. It has been known that for the supergravity background with nontrivial fluxes, the supersymmetric cycle satisfies the generalized calibration condition, which satisfies action minimizing condition. Under the generalized calibration condition, it has been shown that one can have stable and supersymmetric cycles even though cycles are not carrying any topological charges [17]. Note that three cycles on $\mathbf{CP}^3$ do not carry integer charge since the third integer homology is at most torsion. However since the instantonic configurations we have considered are supersymmetric and therefore satisfy the generalized calibration condition, they are nevertheless favored ones.

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[6]We note that the configurations $[\theta_1, \phi_1, y]$ at $\alpha = 0$ and $[\theta_2, \phi_2, y]$ at $\alpha = \pi$ considered in the last section also have the same action value.

[1] N. Lambert and P. C. West, D-branes in the Green-Schwarz formalism, Phys. Lett. B **459**, 515 (1999).
[2] P. Bain, K. Peeters, and M. Zamaklar, D-branes in a plane wave from covariant open strings, Phys. Rev. D **67**, 066001 (2003).
[3] S. Hyun, J. Park, and H. Shin, Covariant description of D-branes in IIA plane wave background, Phys. Lett. B **559**, 80 (2003).
[4] M. Sakaguchi and K. Yoshida, D-branes of covariant AdS superstrings, Nucl. Phys. **B684**, 100 (2004).
[5] M. Sakaguchi and K. Yoshida, Notes on D-branes of type IIB string on AdS(5) x S**5, Phys. Lett. B **591**, 318 (2004).
[6] E. Chang-Young, H. Nakajima, and H. Shin, Consistent bilinear Wess-Zumino term for open AdS superstring, J. High Energy Phys. 10 (2012) 048.
[7] S. Hanazawa and M. Sakaguchi, D-branes from pure spinor superstring in $AdS_5 \times S^5$ background, Nucl. Phys. **B914**, 234 (2017).
[8] J. Park and H. Shin, $1/2$-BPS D-branes from covariant open superstring in $AdS_4 \times CP^3$ background, J. High Energy Phys. 05 (2018) 158.
[9] J. Park and H. Shin, Notes on world volume supersymmetries for D-branes on $AdS_5 \times S^5$ background, J. High Energy Phys. 09 (2017) 022.
[10] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, $N = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals, J. High Energy Phys. 10 (2008) 091.
[11] M. Marino, Localization at large N in ChernSimons-matter theories, J. Phys. A **50**, 443007 (2017).
[12] A. Cagnazzo, D. Sorokin, and L. Wulff, String instanton in AdS(4) x CP**3, J. High Energy Phys. 05 (2010) 009.
[13] N. Drukker, M. Marino, and P. Putrov, Nonperturbative aspects of ABJM theory, J. High Energy Phys. 11 (2011) 141.
[14] B. E. W. Nilsson and C. N. Pope, Hopf fibration of eleven-dimensional supergravity, Classical Quantum Gravity **1**, 499 (1984).
[15] T. Nishioka and T. Takayanagi, Fuzzy ring from M2-brane giant torus, J. High Energy Phys. 10 (2008) 082.
[16] N. Drukker, J. Plefka, and D. Young, Wilson loops in 3-dimensional $N = 6$ supersymmetric Chern-Simons Theory and their string theory duals, J. High Energy Phys. 11 (2008) 019.
[17] J. F. G. Cascales and A. M. Uranga, Branes on generalized calibrated submanifolds, J. High Energy Phys. 11 (2004) 083.