Perturbative proximity between supersymmetric and nonsupersymmetric theories

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I argue that a certain perturbative proximity exists between some supersymmetric and nonsupersymmetric theories (namely, pure Yang-Mills and adjoint QCD with two flavors, adjQCD$_{N_f=2}$). I start with $\mathcal{N} = 2$ super–Yang-Mills theory built of two $\mathcal{N} = 1$ superfields: vector and chiral. In $\mathcal{N} = 1$ language, the latter presents matter in the adjoint representation of SU(N). Then, I convert the matter superfield into a *phantom* one (in analogy with ghosts), breaking $\mathcal{N} = 2$ down to $\mathcal{N} = 1$. The global SU(2) acting between two gluinos in the original theory becomes graded. Exact results in thus deformed theory allow one to obtain insights in certain aspects of nonsupersymmetric gluodynamics. In particular, it becomes clear how the splitting of the β function coefficients in pure gluodynamics, $\beta_1 = (4 - \frac{1}{3})N$ and $\beta_2 = (6 - \frac{1}{3})N^2$, occurs. Here, the first terms in the braces (4 and 6, always integers) are geometry related, while the second terms ($-\frac{1}{3}$ in both cases) are *bona fide* quantum effects. In the same sense, adjQCD$_{N_f=2}$ is close to $\mathcal{N} = 2$ SYM. Thus, I establish a certain proximity between pure gluodynamics and adjQCD$_{N_f=2}$ with supersymmetric theories. (Of course, in both cases, we loose all features related to flat directions and Higgs/Coulomb branches in $\mathcal{N} = 2$.) As a warmup exercise, I use this idea in the two-dimensional CP(1) sigma model with $\mathcal{N} = (2, 2)$ supersymmetry, through the minimal heterotic $\mathcal{N} = (0, 2) \rightarrow$ bosonic CP(1).

I. INTRODUCTION

It has long been known that the behavior of Yang-Mills theories is unique in the sense that, unlike others, they possess asymptotic freedom. It is also known that the first coefficient of the β function has a peculiar form,

$$\beta_1 = \frac{11}{3}N \equiv N\left(4 - \frac{1}{3}\right), \qquad (1)$$

where the coefficients $\beta_{1,2}$ are defined as

$$\beta(\alpha) = \partial_L \alpha = -\beta_1 \frac{\alpha^2}{2\pi} - \beta_2 \frac{\alpha^3}{4\pi^2} + \cdots \quad \partial_L \equiv \frac{\partial}{\partial \log \mu}. \qquad (2)$$

The first term in the parentheses in (1) presents the famous *antiscreening*, while the second is the conventional screening, as in QED. This was first noted by Khriplovich, who calculated [1] the Yang-Mills coupling constant

renormalization [for SU(2)] in the Coulomb gauge[1] in 1969. In his calculation, the distinction in the origin of 4 vs $-\frac{1}{3}$ is transparent: the graph determining the first term in the braces does not have imaginary part and hence can—and in fact does—produce antiscreening; see Fig. 1.

The same 4 vs $-\frac{1}{3}$ split as in (1) is also seen in instanton calculus and in calculation based on the background field method [3]. In the first case, the term 4 emerges from the zero modes and has a geometrical meaning of the number of symmetries nontrivially realized on the instanton (see below). Hence, it is necessarily integer. This part of β_1 is in essence classical. Bona fide quantum corrections due to nonzero modes yield $-\frac{1}{3}$.

In the background field method, 4 vs $-\frac{1}{3}$ split in (1) emerges as an interplay between the magnetic (spin) vs electric (charge) parts of the gluon vertex.

Below, we will discuss whether a similar interpretation exists for the second coefficient in the β function in gluodynamics (nonsupersymmetric Yang-Mills). To this end, I will start from the simplest $\mathcal{N} = 2$ super–Yang-Mills theory, which is built of two $\mathcal{N} = 1$ superfields: vector and

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[1]See also 1977 papers in Ref. [2] devoted to the same issue. Their authors apparently were unaware of Khriplovich's publication [1].
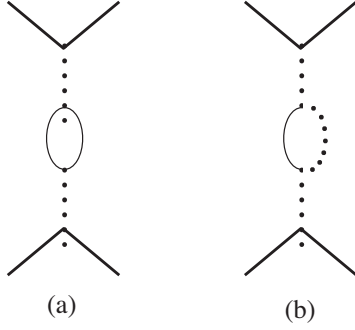
FIG. 1.   Feynman graphs for the interaction of two (infinitely) heavy probe quarks denoted by bold strait lines were calculated by Khriplovich in the Coulomb gauge. The dotted lines stand for the (instantaneous) Coulomb interaction. Thin solid lines depict transverse gluons. In part a, a pair of transverse gluons is produced. This graph has an imaginary part seen by cutting the loop. As in QED, this pair produces screening. In part b, a similar cut in the loop is absent since it would go through a transverse gluon *and* the Coulomb dotted line. This graph is responsible for antiscreening.

chiral. In the $\mathcal{N} = 1$ language, the latter presents matter in the adjoint representation of SU(N). The central point is that I convert the matter superfield into a "phantom" one (in analogy with ghosts), i.e., replace the corresponding superdeterminant by 1/superdeterminant. Then, $\mathcal{N} = 2$ is broken down to $\mathcal{N} = 1$. The global SU(2) acting between two gluinos in the original $\mathcal{N} = 2$ theory becomes graded. Exact results in thus deformed theory will allow me to obtain insights in *nonsupersymmetric gluodynamics*.

II. SETTING THE STAGE

Supersymmetric Yang-Mills (SYM) was the first *four-dimensional* theory in which exact results had been obtained. In what follows, I will use one of them, namely, the so-called Novikov-Shifman-Vainshtein-Zakharov (NSVZ) beta function [3] in SYM theory (reviewed in Ref. [4]). Without matter fields, we have the following result general for $\mathcal{N} = 1$, $\mathcal{N} = 2$, and $\mathcal{N} = 4$ theories,

$$\beta(\alpha) = -\left(n_b - \frac{n_f}{2}\right)\frac{\alpha^2}{2\pi}\left[1 - \frac{(n_b - n_f)\alpha}{4\pi}\right]^{-1}, \quad (3)$$

where n_b and n_f count the gluon and gluino zero modes, respectively. For $\mathcal{N} = 1$, these numbers are

$$n_b = 2n_f = 4T_G = 4N \quad (4)$$

[I limit myself to the SU(N) gauge group theory in this paper]. For $\mathcal{N} = 2$, one obtains

$$n_b = n_f = 4T_G = 4N. \quad (5)$$

Finally, for For $\mathcal{N} = 4$,

$$n_b = \frac{n_f}{2} = 4T_G = 4N. \quad (6)$$

As it follows from (5), the $\mathcal{N} = 2$ β function reduces to one-loop ($n_b = n_f$).

The main lesson obtained in Ref. [3] was as follows. Equation (3) makes explicit that *all* coefficients of the β functions in *pure* super–Yang-Mills theories have a geometric origin since they are in one-to-one correspondence with the number of symmetries nontrivially realized on instantons.

I will also need the extension of (3) including $\mathcal{N} = 1$ matter fields. We will consider one extra $\mathcal{N} = 1$ chiral matter superfield in the *adjoint* representation of SU(N); then,

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\frac{3T_G - T_G(1 - \gamma)}{1 - \frac{T_G\alpha}{2\pi}}, \quad (7)$$

where γ is the anomalous dimension of the corresponding matter field.[2] This so-called NSVZ formula appeared first in Ref. [3] and shortly after in a somewhat more general form in Ref. [5].[3]

Needless to say, in *nonsupersymmetric* Yang-Mills theory (gluodynamics), exact β function determination is impossible. Moreover, only the first two coefficients in the β function are scheme independent. In supersymmetric theories, the all-order results mentioned above are valid in a special scheme (usually called NSVZ) recently developed also in perturbation theory in Refs. [6,7]. So far, no analog of this special scheme exists in nonsupersymmetric theories. Therefore, in discussing below geometry-related terms in the β function coefficients in pure Yang-Mills theory, I will limit myself to β_1 and β_2, which are scheme independent. One can think of extending these results to higher loops in the future.

III. SIMPLE MODEL TO BEGIN WITH

A pedagogical example of the model where the β function similar to (7) appears is the $\mathcal{N} = (0, 2)$ heterotic CP(1) model in two dimensions [8].[4] Since Ref. [8] remains

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[2]For the adjoint matter superfield, $\gamma = -\frac{N\alpha}{\pi}$.

[3]Quite recently, derivation of the NSVZ β function in perturbation theory was completed in Ref. [6]. This work also completes construction of the NSVZ scheme. It contains an extensive list of references, including those published after Ref. [7].

[4]Of course, in CP($N - 1$) models, even nonsupersymmetric, all coefficients have a geometric meaning; see e.g., Ref. [9]. This is because such models themselves are defined through target space geometry. We will use $\mathcal{N} = (0, 2)$ heterotic CP(1) model as a toy model, a warmup before addressing Yang-Mills. Note also that minimal heterotic models of the type (8), (11) do not exist for CP($N - 1$) with $N > 2$ because of the anomaly [10].

relatively unknown, I will first briefly describe it in terms on $\mathcal{N} = (0, 2)$ superfields.

The Lagrangian of the $\mathcal{N} = (0, 2)$ model in two dimensions analogous to that of $\mathcal{N} = 1$ 4D SYM is

$$\mathcal{L}_A = \frac{1}{g^2} \int d^2\theta_R \frac{A^\dagger i \overset{\leftrightarrow}{\partial}_{RR} A}{1 + A^\dagger A}, \tag{8}$$

where A is an $\mathcal{N} = (0, 2)$ bosonic chiral superfield,

$$A(x, \theta_R^\dagger, \theta_R) = \phi(x) + \sqrt{2}\theta_R \psi_L(x) + i\theta_R^\dagger \theta_R \partial_{LL} \phi. \tag{9}$$

Here, ϕ is a complex scalar, and ψ_L is a left-moving Weyl fermion in two dimensions. Furthermore, the matter term is introduced through another $\mathcal{N} = (0, 2)$ superfield B,

$$B_i(x, \theta_R, \theta_R^\dagger) = \psi_{R,i}(x) + \sqrt{2}\theta_R F_i(x) + i\theta_R^\dagger \theta_R \partial_{LL} \psi_{R,i}(x), \tag{10}$$

where i is the flavor index, $i = 1, 2, ..., n_f$, and

$$\mathcal{L}_{\text{matter}} = \int d^2\theta_R \sum_i \frac{1}{2} \frac{B_i^\dagger B_i}{(1 + A^\dagger A)^2},$$

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_{\text{matter}}. \tag{11}$$

Note that the superfield B contains only one physical (dynamic) field ψ_R, with no bosonic counterpart. This is only possible in two dimensions.

In the minimal model (8) without matter, the exact beta function [8] takes the form

$$\beta(g^2) = -\frac{g^4}{2\pi} \frac{1}{1 - \frac{g^2}{4\pi}}, \tag{12}$$

while including matter, we arrive at

$$\beta(g^2) = -\frac{g^4}{2\pi} \frac{1 + \frac{n_f \gamma}{2}}{1 - \frac{g^2}{4\pi}}, \tag{13}$$

to be compared with (7). Equation (12) must be compared with (7) with the second term in the parentheses omitted. The parallel is apparent. There are two minor but technically important distinctions, however. First, in the two-dimensional CP(1) model, unlike four-dimensional SYM, the fermion contribution appears only in the second loop. Second, as was mentioned, the matter superfield B contains only one physical degree of freedom, namely, ψ_R. In super–Yang-Mills theory, the matter superfield contains both components, bosonic and fermionic.

Now, if $n_f = 1$ in the model under consideration, supersymmetry is extended to $\mathcal{N} = (2, 2)$. In other words, in this case, we deal with nonchiral $\mathcal{N} = 2$ CP(1) which, as

is well known, has only one-loop beta function. From this fact, we conclude that

$$\gamma = -\frac{g^2}{2\pi}. \tag{14}$$

Let us ask the following question: Is this information sufficient to find the first and second coefficients in the nonsupersymmetric (purely bosonic) CP(1) model without actual calculation of relevant Feynman graphs?

Surprising though it is, the answer is positive. Indeed, let us make the B superfield "phantom," i.e., quantize ψ_R as a *bosonic* field. In other words, we will treat ψ_R as a fermion ghost field. In other words, the extra minus sign is needed in the ψ_R loop. Then, the contributions of $\psi_L \in A$ and $\psi_R \in B$ exactly cancel each other in the two-loop β function (see Fig. 2; neither ψ_L nor ψ_R appears at one loop). At this stage, $\mathcal{N} = (0, 2)$ supersymmetry is preserved. The cancellation of $\psi_{R,L}$ contributions to β function occurs despite the fact that one of the fermions is a left mover and the other is a right mover.

Then, with the phantom B superfield, we can take Eq. (13) and formally put

$$n_f = -1.$$

As a result, we end up the with the following two-loop answer in the bosonic CP(1) model:

$$\beta_{\text{CP(1) bosonic}} = -\frac{g^4}{2\pi} \left(1 + \frac{g^2}{2\pi}\right). \tag{15}$$

With satisfaction, we observe that the above formula coincides with the standard answer [9].

IV. SU(N) YANG-MILLS THEORIES

Now, I return to gauge theories with the goal of analyzing the second coefficient of the β function in pure gluodynamics,
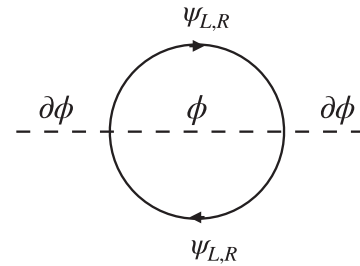

FIG. 2. The phantom field ψ_R cancels the contribution of ψ_L in the two-loop beta function. This is the only diagram to be considered at small ϕ.

$$\beta_{\text{pure YM}} = -\frac{11}{3}N\frac{\alpha^2}{2\pi} - \frac{17}{3}N^2\frac{\alpha^3}{4\pi^2} + \cdots \qquad (16)$$

We will see that the second coefficient can be represented as follows,

$$\beta_2 = N^2\frac{17}{3} = N^2\left(6 - \frac{1}{3}\right), \qquad (17)$$

to be compared with the first coefficient in Eq. (1).[5] The term 6 in the parentheses of (17) is again related to the number of instanton zero modes in $\mathcal{N} = 1$ SYM. Thus, it has a geometrical meaning. The second term $-\frac{1}{3}$ has a bona fide quantum origin.

The line of reasoning will be the same as in Sec. III.

Let us start from $\mathcal{N} = 1$ SYM without matter. The corresponding expression is given by (7) with $\sum_i T_G(1 - \gamma) = 0$ or, which is the same, in Eqs. (3) and (4). Then, we add one chiral superfield in the adjoint representation of SU(N). By the same token as in Sec. III, supersymmetry of the model with the $\mathcal{N} = 1$ adjoint matter included is automatically extended from $\mathcal{N} = 1$ to $\mathcal{N} = 2$. In the latter, the β function is given by Eqs. (3) and (5); i.e., it reduces to one loop.

The next step is to declare the chiral adjoint matter superfield to be phantom in $\mathcal{N} = 2$ super–Yang-Mills theory. At this point, we brake the global SU(2) acting on the doublet

$$\begin{pmatrix} \lambda \\ \tilde{\lambda} \end{pmatrix}. \qquad (18)$$

More exactly, we transform it into a graded $su(2)$ algebra with the generators $T^{\pm}$ becoming odd elements, while T^0 remains even. In Eq. (18), λ is the first gluino, i.e., the one from the vector superfield, while $\tilde{\lambda}$ is the second gluino belonging to the matter superfield.

Declaring the chiral superfield to be phantom amounts to replacing the instanton superdeterminant for $\mathcal{N} = 1$ matter by its reverse, or, diagrammatically, we must change the sign of the matter contribution in Eq. (7), namely,

$$-T_G(1 - \gamma) \rightarrow +T_G(1 - \gamma) \qquad (19)$$

From the one-loop condition for $\mathcal{N} = 2$ SYM β function using Eq. (7), we derive

$$\gamma = -T_G\frac{\alpha}{\pi}. \qquad (20)$$

Equation (20) in combination with (19) is to be substituted into (7). After this is done, the β function in the "phantomized" theory takes the form [after expanding the denominator in (7)]

[5]For the definition, see Eq. (2).

$$\begin{aligned}
\beta_{\text{ph}} &= -\frac{\alpha^2}{2\pi}4N\left(1 + \frac{N\alpha}{4\pi}\right)\left(1 + \frac{N\alpha}{2\pi}\right) \\
&= -\frac{\alpha^2}{2\pi}4N\left(1 + \frac{3N\alpha}{4\pi}\right) + \cdots \\
&\leftrightarrow -\frac{\alpha^2}{2\pi}n_b\left[1 + \left(n_b - \frac{n_f}{2}\right)\frac{\alpha}{4\pi}\right] \\
&= -\frac{\alpha^2}{2\pi}\left[4N + 6N^2\frac{\alpha}{2\pi}\right]. \qquad (21)
\end{aligned}$$

Here, β carries a subscript ph (standing for phantom)—it does not refer to any physical theory. Exactly the same formula emerges from the instanton calculation in which the instanton measure is adjusted to reflect "phantomization" of the matter field. Since $\mathcal{N} = 1$ is unbroken, all nonzero modes still cancel. At this level, the first and the second coefficients in β_{ph} are related to geometry and are integers (see the second line above). Moreover, n_b and n_f in (21) refer to $\mathcal{N} = 1$ theory; see (4).

This is not the end of the story, however. Unlike the situation in Sec. III, in the Yang-Mills case, phantomizing the theory is not enough to pass to nonsupersymmetric gluodynamics.

Let us ask ourself what diagrams present in SYM but absent in nonsupersymmetric gluodynamics are canceled by phantom matter. The answer is obvious and is depicted in Fig. 3. Any number of gluons (with possible λ, $\tilde{\lambda}$ insertions) can be drawn inside the λ, $\tilde{\lambda}$ loops in Fig. 3; cancellation still persists.

What does not cancel? It is obvious that all diagrams with the adjoint scalar field ϕ^a from the matter $\mathcal{N} = 1$ superfield still reside in Eq. (21) and must be subtracted in passing to nonsupersymmetric gluodynamics. The simplest example of such a graph is presented in Fig. 4. In fact, this graph is the only one to be dealt with at one loop. One should not forget that in (21) the above diagram refers to the phantom ϕ^a field. Hence, its subtraction is equivalent to addition of the regular (unphantomized) ϕ^a loop. As for two-loop diagrams—for brevity, I will present them omitting background field legs—they are shown in Fig. 5. The situation with Fig. 5(a) is exactly the same as with that in Fig. 4. Both Figs. 5(a) and 5(b), taken together, present the phantom ϕ^a field contribution, which must be subtracted
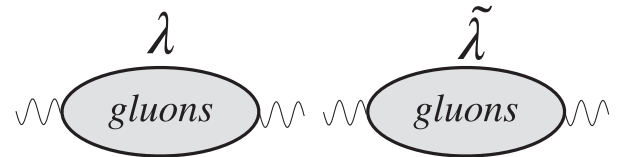

FIG. 3. Two classes of graphs canceling each other. Here, λ marks the gluino lines, while $\tilde{\lambda}$ is the adjoint matter fermion from the phantom matter field. The gray shading indicates all possible gluon insertion. The wavy line is the gluon background field.
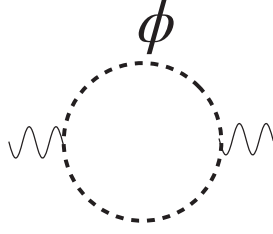
FIG. 4. The one-loop ϕ^a contribution to be subtracted in passing from Eq. (21) to nonsupersymmetric Yang-Mills.
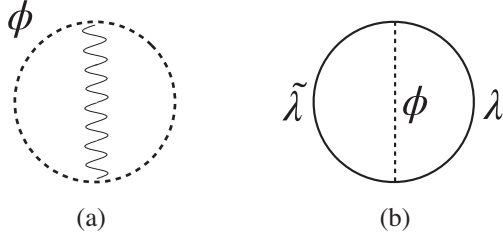

(a)   (b)

FIG. 5. Two-loop graphs to be subtracted in passing from Eq. (21) to non-SUSY Yang-Mills. Here, λ marks the gluino lines, while $\tilde{\lambda}$ is the adjoint matter fermion from the phantom superfield. The diagram a combined with that in Fig. 4 gives the ϕ^a contribution to two-loop β function in QCD with scalar "quark" (i.e., ϕ^a is to be considered as scalar quark in the adjoint representation.)

from β_{ph} if we want to pass to pure gluodynamics. In fact, because of their phantom sign, to pass to pure gluodynamics, we must add these graphs as a normal (non-phantom) contribution to β_{ph}.

In the Appendix, I check that the impact of the quantum corrections associated with Figs. 4 and 5 on the phantom β function in (21) is as follows,

$$\beta_{\text{pure YM}} = \beta_{\text{ph}} + \delta_{\text{sc}}\beta, \qquad (22)$$

where

$$\delta_{\text{sc}}\beta = \frac{1}{3}\left(N\frac{\alpha^2}{2\pi} + N^2\frac{\alpha^3}{4\pi^2} \right), \qquad (23)$$

which perfectly coincides with Eqs. (1), (2), (16), and (17). Needless to say, the sign of these corrections corresponds to screening.

V. $\mathcal{N} = 2$ SYM AND ADJOINT QCD WITH TWO FLAVORS

Recently, a renewed interest in adjoint QCD led to some unexpected results (see e.g., Ref. [11] and an old but useful for my present purposes review [12]). In this section, I compare $\mathcal{N} = 2$ SYM with adjoint QCD with two quarks (two adjoint Weyl or Majorana fields).

As is well known, in $\mathcal{N} = 2$ SYM, the β function is exhausted by one loop; see Eq. (3) and (5). The first coefficient in this β function is

$$(\beta_{\mathcal{N}=2})_1 = n_b - \frac{1}{2}n_f = 2N. \qquad (24)$$

The geometric origin of this coefficient is obvious.

What should be changed in $\mathcal{N} = 2$ SYM to convert this theory into adjoint QCD with $N_f = 2$? The answer is clear—one should subtract the same graphs in Figs. 4 and 5, which were added in Eq. (22),

$$\beta_{\text{adj QCD}} = \beta_{\mathcal{N}=2} - \delta_{\text{sc}}\beta$$
$$= -\left(2 + \frac{1}{3}\right)N\frac{\alpha^2}{2\pi} - \frac{N^2}{3}\frac{\alpha^3}{4\pi^2}. \qquad (25)$$

Equation (25) coincides with the known β function in adjoint QCD [12].

VI. RENORMALONS AND ADIABATIC CONTINUITY

Renormalons introduced by 't Hooft [13] emerge from a specific narrows class of multiloop diagrams,[6] the so-called bubble chains; see Fig. 6. *Formally,*[7] this chain produces a factorially divergent perturbative series

$$\sim \sum_n \left(\frac{\beta_1 \alpha_s}{8\pi} \right)^n n!. \qquad (26)$$

In the Borel plane, the above factorial divergence manifests itself as a singularity at

$$\frac{8\pi}{\beta_1} = \frac{2\pi}{N}\frac{12}{11}$$

in pure gluodynamics. At the same time, in adjoint $\text{QCD}_{N_f=2}$ discussed in Sec. V, the renormalon-induced singularity in the Borel plane is at

$$\frac{8\pi}{\beta_1} = \frac{4\pi}{N}\frac{6}{7}.$$

The both cases are depicted by crosses in Fig. 7.

Simultaneously, this figure shows (by closed circles) the would-be positions of the renormalon singularities if small quantum terms $\frac{1}{3}$ in Eqs. (1) and (25) were neglected. If measured in the units of $2\pi/N$, the latter are integers.

Why this is important? The answer to this question is associated with the program due to Ünsal launched some

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[6]For a recent brief review, see Ref. [14].
[7]"Formally" means that in order to stay within the limits of applicability of perturbation theory we have to cut off the sum in Eq. (26) at a certain value of $n = n_*$.
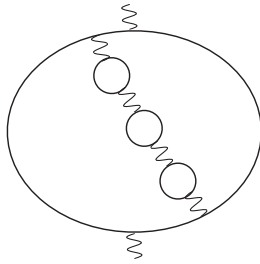
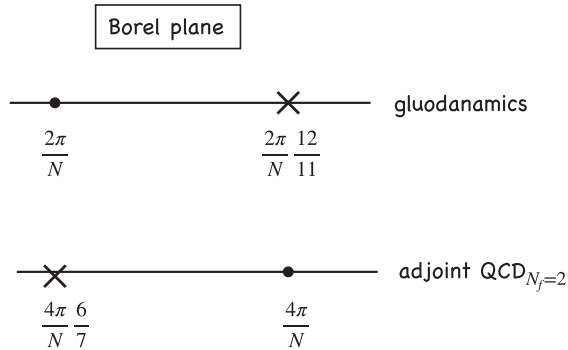FIG. 6. 'T Hooft's bubble-chain diagrams representing renormalons.

FIG. 7. Leading renormalon-related singularities in the Borel plane (marked by crosses) for pure gluodynamics and adjoint $QCD_{N_f=2}$, respectively. The closed circles mark the would-be positions of the renormalon singularities if small quantum terms $\frac{1}{3}$ in Eqs. (1) and (25) were ignored.

time ago [15] aimed at developing a quasiclassical picture on $R_3 \times S_1$ at sufficiently small radius $r(S_1)$. This program later was supplemented by the idea of *adiabatic continuity* stating that tending $r(S_1) \to \infty$ (i.e., returning to R_4) one does not encounter phase transitions on the way. Another crucial observation is that the renormalon singularity must conspire with operator product expansion (OPE). In both theories outlined in Fig. 7, the leading term in OPE is due to the operator $G_{\mu\nu} G^{\mu\nu}$ (the gluon condensate).

Now, within the program [15] a large number of new saddle-point configurations were discovered, the so-called monopole instantons, also known as bions, both magnetic and neutral. Their action is $2\pi/N$, i.e., N times smaller than that of instantons. In other words, in this picture, a single instanton can be viewed as a composite state of N bions.

Then, it becomes clear why a single bion saturates the gluon condensate in gluodynamics. Moreover, in adjoint $QCD_{N_f=2}$, similar saturation is due to an bion-antibion pair which is tied up because of the existence of the fermion zero modes. The zero modes produced by bion and those from antibion must be contracted to give rise to the gluon condensate. In the Borel plane (Fig. 7), the single-bion contribution is shown by a closed circle in the upper graph, while the bion pair's contribution is depicted in the lower

graph. We see that the adiabatic matching would be *perfect* if we could ignore the small quantum terms $\frac{1}{3}$ in Eqs. (1) and (25). The adiabatic matching is perfect in the phantom theory of Sec. IV.

VII. CONCLUSION

Starting from $\mathcal{N} = 2$ super–Yang-Mills and phantomizing the $\mathcal{N} = 1$ matter superfield, I arrived at a fully geometric β_{ph} function. This proves the statement I made in Sec. I that the integer part of the first and second coefficients in $\beta_{\mathrm{pure\,YM}}$ count the number of certain symmetry generators, both bosonic and fermionic, in $\mathcal{N} = 1$ SYM. The latter theory becomes relevant because my phantomization procedure breaks $\mathcal{N} = 2 \to \mathcal{N} = 1$. Relatively small noninteger additions represent bona fide quantum corrections, which do not appear in $\mathcal{N} = 1$ SYM because of the Bose-Fermi cancellations. At the moment, one might think that this idea could shed light on some other intricate aspects of gauge theories, for instance on nuances of renormalons.

In particular, the geometric integers 4 and 6 appearing in (1) and (17) are the dimensions of the lowest-dimension gluon operators, quadratic and cubic in the gluon field strength tensor, respectively. The standard renormalon wisdom says that the renormalon singularities in the Borel plane conspire with these operators in the operator product expansion.

At the same time, the current understanding of renormalons in SYM theory continues to be incomplete (see Refs. [14,16]). The gluon condensate vanishes in supersymmetric gluodynamics. The leading renormalons have nothing to conspire with. Are there undiscovered cancellations? This suggestion does not seem likely, but we cannot avoid providing a definite answer any longer.

Summarizing, I established a certain proximity between pure gluodynamics and an $\mathcal{N} = 1$ theory. Moreover, in the same sense, *adjoint* $QCD_{N_f=2}$ is close to $\mathcal{N} = 2$ SYM. Conceptually, this is similar to the proximity of pure gluodynamics to fundamental QCD in the limit $N \to \infty$. In the latter case, the parameter governing the proximity is adjustable, $1/N$. In the cases considered in this paper, it is rather a numerical parameter whose origin is still unclear but is quite apparent in Eqs. (1) and (17). It seems to be related with the dominance of magnetic interactions of gluons over their charge interactions. If we could tend this parameter to zero, we would be able to say more about the adiabatic continuity.

I would like to add a few remarks on questions relevant for the future studies.

Remark 1.—Since pure gluodynamics is close to $\mathcal{N} = 1$ phantom theory, the gluon condensate in the former must be suppressed to a certain extent since it is forbidden in $\mathcal{N} = 1$ supersymmetry.

Remark 2.—My present consideration entangles renormalons, their conspiracy with OPE, the quasiclassical treatment on $R_3 \times S_1$, and adiabatic continuity all in one junction both in pure gluodynamics and adjoint $QCD_{N_f=2}$. Can this line of studies be continued?

Remark 3.—$\mathcal{N} = 1$ phantom SYM theory I have discussed in this paper calls for further investigations. For instance, I believe that, despite the presence of nonunitary contributions, if we consider amplitudes with only gluonic external legs, their inclusive imaginary parts will be positive. In this narrow sense, they will preserve unitarity.

ACKNOWLEDGMENTS

APPENDIX: COMPLEX SCALAR ADJOINT QUARKS

First, let us analyze diagrams in Figs. 4 and 5(a). As was mentioned, they present the "scalar quark" contributions in QCD. We can extract them from the known result for the QCD β function, by changing the appropriate Casimir coefficients from the fundamental representation to the adjoint.[8]

The QCD β function with one adjoint scalar quark extracted from Refs. [17,18] in my notation reduces to

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[8] I thank K. Chetyrkin, A. Kataev, and K. Stepanyants for help on this point.

$$\beta_{\text{YM+SQ}} = -\left(\frac{11}{3}N - \frac{1}{3}N\right)\frac{\alpha^2}{2\pi} - \left(\frac{17}{3}N^2 \underbrace{-2N^2 - \frac{1}{3}N^2}_{\text{Fig. 5a}}\right)\frac{\alpha^3}{4\pi^2}$$
$$+ \cdots \tag{A1}$$

The scalar quark contribution is underlined by underbraces. This is not the end of the story, however. In addition, I have to take into account the graph depicted in Fig. 5(b).

The fastest and most efficient method of such calculations is the background field method; see Ref. [19]. Background field emission can occur either from the λ, $\tilde{\lambda}$ lines or from the ϕ line. In the first case, the relevant propagator is

$$S(x, 0) = \frac{1}{2\pi^2}\frac{\hat{x}}{x^4} - \frac{1}{8\pi^2}\frac{x_\alpha}{x^2}\tilde{G}_{\alpha\varphi}(0)\gamma^\varphi\gamma^5 + \cdots, \tag{A2}$$

where the ellipses denote irrelevant terms in the expansion and, moreover,

$$\tilde{G}_{\alpha\varphi} = \frac{1}{2}\varepsilon_{\alpha\varphi\beta\rho}G^{\beta\rho}.$$

In the second case, the relevant propagator is that of ϕ (see Ref. [20]),

$$G(x, 0) = \frac{i}{4\pi^2}\frac{1}{x^2} + \frac{i}{512\pi^2}x^2G^2(0) + \cdots \tag{A3}$$

The result of calculation of the diagram in Fig. 5(b) is also known in the literature. We can borrow it from Sec. 5 of Ref. [20],

$$\Delta\beta|_{\text{Fig. 5b}} = -2N^2\frac{\alpha^3}{4\pi^2}. \tag{A4}$$

Combining (A4) and (A1), we arrive at $\delta_{\text{sc}}\beta$ given in Eq. (23).

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