Barrow holographic dark energy

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We formulate Barrow holographic dark energy, by applying the usual holographic principle at a cosmological framework, but using the Barrow entropy instead of the standard Bekenstein-Hawking one. The former is an extended black-hole entropy that arises due to quantum-gravitational effects which deform the black-hole surface by giving it an intricate, fractal form. We extract a simple differential equation for the evolution of the dark-energy density parameter, which possesses standard holographic dark energy as a limiting subcase, and we show that the scenario can describe the thermal history of the universe, with the sequence of matter and dark-energy eras. Additionally, the new Barrow exponent Δ significantly affects the dark-energy equation of state, and according to its value it can lead it to lie in the quintessence regime, in the phantom regime, or experience the phantom-divide crossing during the evolution.

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I. INTRODUCTION

Holographic dark energy is an interesting alternative scenario for the quantitative description of dark energy, originating from the holographic principle [1–5]. Starting from the connection between the largest length of a quantum field theory with its ultraviolet cutoff [6], one can result to a vacuum energy of holographic origin, which at cosmological scales form dark energy [7,8]. Holographic dark energy proves to lead to interesting cosmological behavior, both at its simple [7–18], as well as at its extended versions [19–44], and it is in agreement with observational data [45–53].

The important step in the application of holograpic principle at cosmological framework is that the universe horizon (i.e., largest distance) entropy is proportional to its area, similarly to the Bekenstein-Hawking entropy of a black hole. However, very recently Barrow was inspired by the COVID-19 virus illustrations and he showed that quantum-gravitational effects may introduce intricate, fractal features on the black-hole structure. This complex structure leads to finite volume but with infinite (or finite) area, and therefore to a deformed black-hole entropy expression [54]

$$S_B = \left(\frac{A}{A_0}\right)^{1+\Delta/2}, \tag{1}$$

where A is the standard horizon area and A_0 the Planck area. The quantum-gravitational deformation is therefore quantified by the new exponent Δ, with $\Delta = 0$ corresponding to the standard Bekenstein-Hawking entropy (simplest horizon structure), and with $\Delta = 1$ corresponding to the most intricate and fractal structure. Notice that the above quantum-gravitationally corrected entropy is different than the usual "quantum-corrected" entropy with logarithmic corrections [55,56]; however it resembles Tsallis nonextensive entropy [57–59]. Nevertheless the involved foundations and physical principles are completely different. Finally, note that the above effective fractal behavior does not arise from specific quantum gravity calculations, but from general simple physical principles, which adds to its plausibility and hence it is valid as a first approach on the subject [54].

In the present manuscript we are interested in constructing holographic dark energy, but using the extended, Barrow relation for the horizon entropy, instead of the usual Bekenstein-Hawking one. Barrow holographic dark energy possesses usual holographic dark energy as a limit in the $\Delta = 0$ case; however, in general it is a new scenario with richer structure and cosmological behavior.

II. BARROW HOLOGRAPHIC DARK ENERGY

In this section we construct the scenario of Barrow holographic dark energy. While standard holographic dark energy is given by the inequality $\rho_{\rm DE} L^4 \leq S$, where L is the horizon length, and under the imposition $S \propto A \propto L^2$ [8], the use of Barrow entropy (1) will lead to

$$\rho_{\rm DE} = C L^{\Delta-2}, \tag{2}$$

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with C a parameter with dimensions $[L]^{-2-\Delta}$. In the case where $\Delta = 0$, as expected, the above expression provides the standard holographic dark energy $\rho_{\rm DE} = 3c^2 M_p^2 L^{-2}$ (here M_p is the Planck mass), where $C = 3c^2 M_p^2$ and with c^2 the model parameter. However, in the case where the deformation effects quantified by Δ switch on, Barrow holographic dark energy will depart from the standard one, leading to different cosmological behavior.

We consider a flat Friedmann-Robertson-Walker (FRW) metric of the form

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{3}$$

where $a(t)$ is the scale factor. Concerning the largest length L which appears in the expression of any holographic dark energy, although there are many possible choices, the most common in the literature is to use the future event horizon [7]

$$R_h \equiv a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2}, \tag{4}$$

with $H \equiv \dot{a}/a$ the Hubble parameter. Hence, substituting L in (2) with R_h we obtain the energy density of Barrow holographic dark energy, namely

$$\rho_{\rm DE} = C R_h^{\Delta-2}. \tag{5}$$

We consider that the universe is filled with the usual matter perfect fluid, as well as with the above holographic dark energy. The two Friedmann equations are then written as

$$3M_p^2 H^2 = \rho_m + \rho_{\rm DE}, \tag{6}$$

$$-2M_p^2 \dot{H} = \rho_m + p_m + \rho_{\rm DE} + p_{\rm DE}, \tag{7}$$

where $p_{\rm DE}$ is the pressure of Barrow holographic dark energy, and ρ_m, p_m the energy density and pressure of matter, respectively. Additionally, for the matter sector we consider the standard conservation equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \tag{8}$$

Finally, we introduce the density parameters

$$\Omega_m \equiv \frac{1}{3M_p^2 H^2}\rho_m, \tag{9}$$

$$\Omega_{\rm DE} \equiv \frac{1}{3M_p^2 H^2}\rho_{\rm DE}. \tag{10}$$

Using the density parameters, expressions (4), (5) give

$$\int_x^\infty \frac{dx}{Ha} = \frac{1}{a}\left(\frac{C}{3M_p^2 H^2 \Omega_{\rm DE}}\right)^{\frac{1}{2-\Delta}}, \tag{11}$$

with $x \equiv \ln a$. Considering the matter to be dust ($p_m = 0$), from (8) we obtain $\rho_m = \rho_{m0}/a^3$, with ρ_{m0} the present matter energy density, namely at $a_0 = 1$ (in the following the subscript "0" denotes the value of a quantity at present). Thus, substituting into (9) leads to $\Omega_m = \Omega_{m0} H_0^2/(a^3 H^2)$, from which, using the Friedmann equation $\Omega_m + \Omega_{\rm DE} = 1$, we acquire

$$\frac{1}{Ha} = \frac{\sqrt{a(1-\Omega_{\rm DE})}}{H_0\sqrt{\Omega_{m0}}}. \tag{12}$$

Inserting (12) into expression (11) we get the useful relation

$$\int_x^\infty \frac{dx}{H_0\sqrt{\Omega_{m0}}}\sqrt{a(1-\Omega_{\rm DE})} = \frac{1}{a}\left(\frac{C}{3M_p^2 H^2 \Omega_{\rm DE}}\right)^{\frac{1}{2-\Delta}}. \tag{13}$$

Differentiating (13) with respect to $x = \ln a$ we get the result

$$\frac{\Omega'_{\rm DE}}{\Omega_{\rm DE}(1-\Omega_{\rm DE})} = \Delta + 1 + Q(1-\Omega_{\rm DE})^{\frac{\Delta}{2(\Delta-2)}} \cdot (\Omega_{\rm DE})^{\frac{1}{2-\Delta}} e^{\frac{3\Delta}{2(\Delta-2)}x}, \tag{14}$$

with

$$Q \equiv (2-\Delta)\left(\frac{C}{3M_p^2}\right)^{\frac{1}{\Delta-2}} (H_0\sqrt{\Omega_{m0}})^{\frac{\Delta}{2-\Delta}} \tag{15}$$

a dimensionless parameter and where primes denote derivatives with respect to x.

The above differential equation determines the evolution of Barrow holographic dark energy for dust matter in a flat universe. In the case $\Delta = 0$ it coincides with the usual holographic dark energy, i.e., $\Omega'_{\rm DE}|_{\Delta=0} = \Omega_{\rm DE}(1-\Omega_{\rm DE}) \times (1 + 2\sqrt{\frac{3M_p^2\Omega_{\rm DE}}{C}})$, which has an analytic solution in implicit form [7]. However, in the general case of Barrow exponent Δ, Eq. (14) presents an x dependence and it has to be elaborated numerically.

Using the above relations we can additionally calculate the equation-of-state parameter for Barrow holographic dark energy $w_{\rm DE} \equiv p_{\rm DE}/\rho_{\rm DE}$. Differentiation of (5) leads to $\dot{\rho}_{\rm DE} = (\Delta - 2) C R_h^{\Delta-3}\dot{R}_h$, with $\dot{R}_h$ calculated using (4) as $\dot{R}_h = HR_h - 1$, and where according to (5) R_h can be eliminated in terms of $\rho_{\rm DE}$ as $R_h = (\rho_{\rm DE}/C)^{1/(\Delta-2)}$. Inserting this into the dark-energy conservation equation $\dot{\rho}_{\rm DE} + 3H\rho_{\rm DE}(1 + w_{\rm DE}) = 0$ [which is a straightforward consequence of the matter conservation (8)], we acquire

$$(\Delta-2)C\left(\frac{\rho_{\rm DE}}{C}\right)^{\frac{\Delta-3}{\Delta-2}}\left[H\left(\frac{\rho_{\rm DE}}{C}\right)^{\frac{1}{\Delta-2}} - 1\right] + 3H\rho_{\rm DE}(1 + w_{\rm DE}) = 0. \tag{16}$$

Hence, inserting H from (12), and using (10) we finally obtain

$$w_{\rm DE} = -\frac{1+\Delta}{3} - \frac{Q}{3}(\Omega_{\rm DE})^{\frac{1}{2-\Delta}}(1-\Omega_{\rm DE})^{\frac{\Delta}{2(\Delta-2)}} e^{\frac{3\Delta}{2(2-\Delta)}x}. \qquad (17)$$

Therefore, the evolution of $w_{\rm DE}$ in terms of $x = \ln a$ is known, as long as $\Omega_{\rm DE}$ is known from (14). Lastly, in the standard case of $\Delta = 0$, expression (17) gives $w_{\rm DE}|_{\Delta=0} = -\frac{1}{3} - \frac{2}{3}\sqrt{\frac{3M_p^2\Omega_{\rm DE}}{C}}$, which is the usual holographic dark-energy result [8].

III. COSMOLOGICAL EVOLUTION

In this section we investigate in detail the cosmological evolution in the scenario of Barrow holographic dark energy. The dark-energy density parameter $\Omega_{\rm DE}$ is determined by Eq. (14), which can be solved analytically only in the standard case $\Delta = 0$ [7]. Nevertheless, we can extract its solution through numerical elaboration, and then find the redshift behavior knowing that $x \equiv \ln a = -\ln(1+z)$ (with $a_0 = 1$). Finally, concerning the initial conditions we impose $\Omega_m(x = -\ln(1+z) = 0) \equiv \Omega_{m0} \approx 0.3$ and thus $\Omega_{\rm DE}(x = -\ln(1+z) = 0) \equiv \Omega_{DE0} \approx 0.7$ in agreement with observations [60]. In Fig. 1 we depict the evolution of $\Omega_{\rm DE}(z)$ and $\Omega_m(z) = 1 - \Omega_{\rm DE}(z)$, as well as the corresponding evolution of $w_{\rm DE}(z)$ from (17). As we observe the scenario at hand can successfully describe the thermal history of the universe, with the sequence of matter and dark-energy epochs. Moreover, the value of $w_{\rm DE}$ at present is around -1 as required by observations.

Let us now investigate in more detail the equation-of-state parameter of Barrow holographic dark energy, and in particular examine how it is affected by the Barrow exponent Δ that quantifies the deviation from the usual scenario. In Fig. 2 we depict $w_{\rm DE}(z)$ for various values of Δ, including the standard value $\Delta = 0$. A general observation is that for increasing Δ the whole evolution of $w_{\rm DE}(z)$, as well as its current value $w_{\rm DE}(z=0) \equiv w_{DE0}$, tend to acquire lower values. We mention that for $\Delta \gtrsim 0.5$ the value of w_{DE0} lies in the phantom regime. This was expected, since expression (17) allows phantom values, which is a theoretical advantage of the scenario at hand and reveals its capabilities. Thus, as we see, according to the value of Δ, Barrow holographic dark energy can lie in the quintessence or in the phantom regime, or exhibit the phantom-divide crossing during the cosmological evolution.

We close this section by mentioning that the scenario of Barrow holographic dark energy has two parameters, i.e., the new Barrow exponent Δ, and the constant C (similar to the parameter c^2 of standard holographic dark energy) which incorporates the initial inequality validation. In the above analysis we preferred to fix $C = 3$, which is the value required if we desire the present scenario to have standard holographic dark energy as an exact limit for $\Delta = 0$, and we examined the pure role of Δ on the cosmological evolution. This was proved to be adequate for a successful description in agreement with observations, which serves as a significant advantage comparing to standard holographic dark energy, in which case one needs to adjust the value of the constant c^2 to fit the data. Definitely, varying the value of C too would lead to even more improved cosmological behavior, which reveals the capabilities of the scenario.

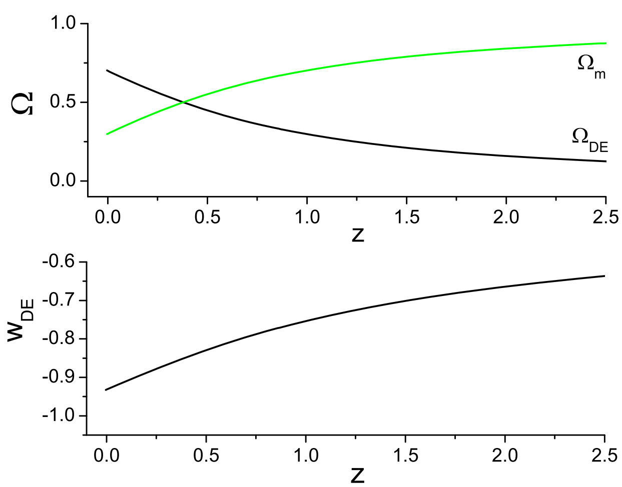

FIG. 1. Top: evolution of matter and of Barrow holographic dark-energy density parameters, as a function of the redshift z, for $\Delta = 0.2$ and $C = 3$, in units where $M_p^2 = 1$. Bottom: evolution of the corresponding dark-energy equation-of-state parameter $w_{\rm DE}$. We have imposed $\Omega_{\rm DE}(x = -\ln(1+z) = 0) \equiv \Omega_{DE0} \approx 0.7$ at present.

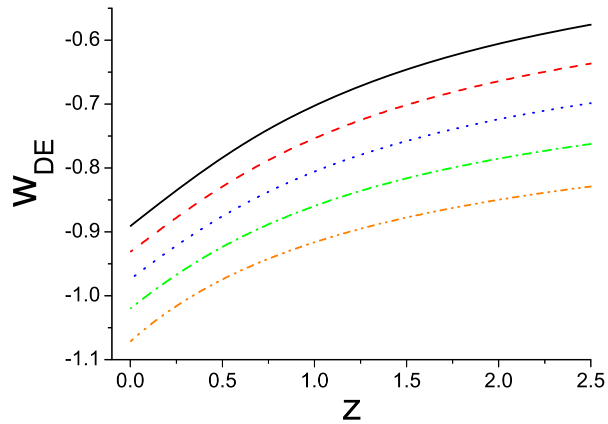

FIG. 2. The equation-of-state parameter $w_{\rm DE}$ of Barrow holographic dark energy, as a function of the redshift z, for $C = 3$, and for $\Delta = 0$ (black solid line), $\Delta = 0.2$ (red dashed line), $\Delta = 0.4$ (blue dotted line), $\Delta = 0.6$ (green dashed-dotted line), and $\Delta = 0.8$ (orange dashed-dot-dotted line), in units where $M_p^2 = 1$. We have imposed $\Omega_{\rm DE}(x = -\ln(1+z) = 0) \equiv \Omega_{DE0} \approx 0.7$ at present.

IV. CONCLUSIONS

We constructed Barrow holographic dark energy, by applying the usual holographic principle at a cosmological framework, but using the Barrow entropy, instead of the standard Bekenstein-Hawking one. Specifically, in a recent work Barrow proposed that quantum-gravitational effects may bring about intricate, fractal structure on the black-hole surface, and hence lead to a deformed black-hole entropy, quantified by a new exponent Δ [54]. Hence, the resulting Barrow holographic dark energy will possess the usual one as a limit, namely when $\Delta = 0$ which corresponds to the case where Barrow entropy becomes the standard one, but for $\Delta > 0$ and up to the maximal deformation for $\Delta = 1$ it gives rise to novel cosmological scenarios.

We extracted a simple differential equation for the evolution of the dark energy density parameter, and we presented the solution for the evolution of the corresponding dark energy equation-of-state parameter. As we showed, the scenario of Barrow holographic dark energy can describe the thermal history of the universe, with the sequence of matter and dark-energy eras. Additionally, the new Barrow exponent Δ significantly affects the dark-energy equation of state, and according to its value it can lead it to lie in the quintessence regime, in the phantom regime, or experience the phantom-divide crossing during the evolution. The above behaviors were obtained by changing only the value of Δ. Additional adjusting of the parameter C will enhance significantly the capabilities of the scenario.

We would like to mention here that the Barrow entropy proposal is just a first approximation on the subject of quantum gravitational implications on the black hole horizons. In reality the underlying spacetime foam deformation will be complex, wild, and dynamical. Nevertheless, as a first step the complexity of the phenomenon can effectively and coarse-grained be embedded in the new exponent, and thus the highly dynamical deformation of the black-hole surface can effectively be described by Δ, which is not fixed but it remains in an interval between extreme values. However, as a more realistic scenario which could incorporate the dynamics of spacetime foam, one could think of an exponent Δ that depends on time and scale, as it has already been done with Tsallis entropy exponent [61].

Barrow holographic dark energy exhibits more interesting and richer phenomenology comparing to the standard scenario, and thus it can be a candidate for the description of nature. It would be both necessary and interesting to perform a full observational analysis, confronting the scenario with observational data from Supernovae type Ia, baryonic acoustic oscillations, and cosmic microwave background probes, as well as with large scale structure (such as fσ8) data, in order to constrain the new parameter Δ. These necessary studies lie beyond the scope of the present work and are left for future investigation.

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