Exploring scalar-photon interactions in energetic astrophysical events

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Scalar fields like dilaton appear in quantum field theory (QFT) due to scale symmetry breaking. Their appeal also extends to modified theories of gravity, like $F(R)$ gravity, Horva Lifshitz gravity etc. In unified theories they make their appearance through compactification of the extra dimension. Apart from resolving the issues of compactification scale and size, the particles of their fields can also turn out to be excellent candidate to solve the dark energy (DE) and dark matter (DM) problem of the universe. In this work we study their mixing dynamics with photons in a magnetized media, by incorporating the effect of parity violating part of the photon polarization tensor, evaluated in a finite density magnetized media. This piece, though in general is odd in the external magnetic field strength eB; in this work we however have retained terms to $O(eB)$. We are able to demonstrate in this work that, in magnetized medium a dilatonic scalar field (ϕ) can excite the two transverse degrees of freedom (DOF) of the photons. One due to direct coupling and the other indirectly through the parity violating term originating due to magnetized medium effects. This results in the mixing dynamics being governed by, 3×3 mixing matrices. This mixing results in making the underlying media optically active. In this work we focus on the spectro-polarimetric imprints of these particles, on the spectra of the electromagnetic (EM) fields of gamma ray bursters (GRB). Focusing on a range of parameters (i.e., magnetic field strength, plasma frequency (ω_p), size of the magnetized volume, coupling strength to photons and their mass) we make an attempt to point out how space-borne detectors should be designed to optimize their detection possibility.

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I. INTRODUCTION

The study of scale symmetry and its consequences on the dynamics of particles has drawn attention for some time now. The particles appearing as Goldstone bosons of a spontaneously broken scale symmetry [termed dilaton, $\phi(x)$] [1,2], have emerged from studies in QFT. There are many theories those predict the existence of dilatons. Apart from QFT, they appear in higher dimensional unified theories, for instance, in five dimensional Kaluza-Klein theory, they appear as the five-five component of the five dimensional metric, formulated to unify gravity with electromagnetism. In string and superstring theory they appear from compactification of the extra dimensions and are called string dilaton or moduli [3,4].

In some scale invariant extensions of standard model, they are made to communicate with the standard model sector via an underlying conformal sector, where they acquire mass due to breaking of the conformal invariance[5,6]. This physics of these models are phenomenologically rich with predicting power that can be tested in collider based experiments. On the other hand dilatons of unified theories, acquire their masses from the curvature of the extra dimension.

They couple to the standard model fields by the trace of their energy momentum tensor T^{μ}_{ν}, associated with the anomalous divergence of the dilaton 4-current.[1] Due to this, dilatons may induce other observable signatures, like dilatonic fifth force: [7–11]; bending of light [12], violation of equivalence principle [13], decay into two photons and optical activity [14–18] in external magnetic field (B). When the last two phenomena—that is decay of ϕ into two massless spin one photons and optical activity–follow from the interaction Lagrangian,

$$L_{\rm int} = -\frac{1}{4M}\phi F^{\mu\nu}F_{\mu\nu}. \quad (1.1)$$

In Eq. (1.1) $F^{\mu\nu}$ is the usual field strength tensor for EM field. And M is the symmetry breaking scale related to the inverse of the coupling constant $g_{\phi\gamma\gamma}$, between the quanta of scalar (ϕ) and photon (γ) fields. Equation (1.1) leads to their lifetime τ_ϕ against decay to two photons $\phi \rightarrow \gamma\gamma$, given by $\tau_\phi \sim \frac{1}{g^2_{\phi\gamma\gamma}m^3_\phi}$ [1]. If the life time of these particles for some values of $g_{\phi\gamma\gamma}$ and m_ϕ, turn out to be comparable to

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[1]The nonzero anomalous divergence, even for massless particles may realized due to scaling violation through radiative corrections.

the age of the universe, then the particles of the field ϕ will turn out to be excellent candidates for DM. Thus, simultaneously solving the two out standing problems of contemporary physics. There are other particles that produce similar signals that can be found in [19–46] but we will not discuss those in this work.

Given the state of our current understanding that, about 27% of the total matter-energy density is in the form of DM, it is possible to find what percentage of the total DM density is composed of ϕ, at some epoch t, as cosmological relic density $\rho(t)$, from [47]:

$$\rho(t) = \rho_d \left(\frac{\eta_d}{\eta}\right)^3 e^{-\frac{t}{\tau_\phi}} = \frac{\zeta(3)}{\pi^2} \frac{g^*(t)}{g_{*d}} T^3(t) e^{-\frac{t}{\tau_\phi}}. \tag{1.2}$$

In Eq. (1.2) ρ_d corresponds to the density and η_d the magnitude of the scale factor of Friedman-Robertson-Walker metric. And g_{*d} along with $g^*(t)$ are the number of DOF available at the time of decoupling and the same at the epoch t, respectively. The last line in Eq. (1.2) has been obtained demanding entropy conservation in the co-moving volume of the universe, from the time of decoupling to the epoch t. Since the same depends crucially on the lifetime τ_ϕ, that in turn depends on coupling constant $g_{\phi\gamma\gamma} = \frac{1}{4M}$ and the scalar mass m_ϕ—therefore the estimations of them are of utmost importance. In this study we focus on their estimation from the EM signals originating due to the energetic activities taking place in far away magnetized astrophysical objects. To that end, we have taken a spectro-polarimetric route in this paper, to estimate the parameters (mass and coupling constant) associated with these particles (dilatonic scalars) by studying their mixing dynamics in presence of magnetized plasma present in the GRB environments. We also indicate how such (spectro-polarimetric) analysis can be used to design the space-borne gamma-ray or x-ray detectors—optimally—to detect dilaton signatures through EM signals coming from GRB.

At this juncture we would like to digress a little, so as to pay attention to other possible physical sources those may contribute to the polarization of electromagnetic field, coming from far away sources. One of the possible sources that can contribute the polarization of electromagnetic beam is contribution from magnetized medium and the other can be coming due to the presence of pseudoscalar particles like axions or majorons, those couple to photons through mass dimension-5 operators. Rotation of the plane of polarization of light due to magnetized medium, that is referred usually in literature as Faraday effect, takes place in a material medium having nonzero chemical potential, and a magnetic field B when the magnetic field is oriented along the direction of propagation of the photon k. On the other hand the effect due to the scalar dilaton or pseudoscalar field, takes place, when the component of the magnetic field, is perpendicular to the wave vector k. So these two effects can be distinguished from each other by making the external magnetic field parallel or perpendicular to the propagation direction of the photons. It is also worth noting that, rate of rotation for plane of polarization for Faraday effect is inversely proportional to the square of the energy ω of the photons of light. Hence the same can also serve as a distinct feature to identify magnetized matter induced polarization effect from dilatons or axions. Moreover the degree of circular polarization associated with the beam of light passing through magnetized medium turns out to be zero. The details of these can be found in the Appendix A.

Now coming to the issue of distinguishing scalars (dilatons) from pseudoscalar axions, it should be noted that in magnetized vacuum, the dilaton mixes with polarized light having a plane of polarization oriented along the magnetic field and axion mixes with light having plane of polarization orthogonal to the magnetic field. This simple picture however gets complicated with the incorporation of magnetized matter effects. We will come back to this issue in a separate publication. Having taken these extra-dilaton sources, contributing to polarization of light from astrophysical sources, we turn our attention to the investigations carried out in this work.

During the course of this investigation, we have achieved few new things. They include the following (i) a better understanding of the unique nature of the set of basis vectors and the form factors, those appear in the description of the gauge fields (i.e., of the photons), for a system in an external magnetic field eB, and magnetized media. We provide the transformation properties of the EM form factors and other factors under charge conjugation C, parity P, and time reversal T and use them to justify the coupling between the different DOF available to the system, leading to a 3×3 mixing matrix; (ii) We next provide the analytical route to diagonalize this matrix exactly, using an unitary similarity transformation. (iii) The resulting equations of motion obtained thereby are also exact. (iv) The numerical estimates of the Stokes parameters obtained from the numerical estimates of the form factors, thus are also without any approximations.

The organization of this document is as follows: In Sec. II we introduce the details, about properties of the gauge fields (GF), the form factors those describe them (GF) and their transformation properties under C, P, and T. This is followed by the description of the photon polarization tensor in a magnetized medium in Sec. III, called inclusion of matter effects. In Sec. IV we move on to analyzing the equations of motion for $(\gamma - \phi)$ interacting system in a magnetized medium and demonstrate that the mixing matrix for $\phi F_{\mu\nu} F^{\mu\nu}$ interaction, turns out to be 3×3 instead of 2×2, that is usually encountered in magnetized vacuum, or unmagnetized plasma. We discuss the exact analytic diagonalization of the same in the following subsection. Subsequently we justify the same from discrete symmetry point of view. The solutions of the field equations followed by construction of the Stokes parameters is obtained in Sec. V. In Sec. VI we introduce a typical

GRB model and its environment that being used in this analysis and the polarization signals one would get from the same, for some bench mark values of $g_{\phi\gamma\gamma}$ and m_ϕ, the geometry of the GRB fireball and plasma frequency. The possible EM signatures of dilaton interaction from such environments is presented in Sec. VII. Section VIII houses a discussion on the relevance of our analysis to space-borne detectors. In Sec. IX, we conclude by providing an outlook for possible future directions of investigation. And lastly, we have provided an Appendix that deals with the details of polarization evaluation in a magnetized media. Few important details regarding discrete symmetry transformations and their effects on equations of motion can be found in the Supplemental Material [48] in a separate work, titled: "Supplementary materials for Exploring scalar-photon interactions in energetic astrophysical events."

II. ELECTROMAGNETIC FORM FACTORS FOR $A^\nu(\boldsymbol{k})$

In the standard formulation of the massless Abelian gauge theory, that describes the dynamics of photons, the action is written as,

$$S = -\frac{1}{4}\int F^{\mu\nu}F_{\mu\nu}d^4x \tag{2.1}$$

where in Eq. (2.1), the field strength tensor, $F^{\mu\nu} = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x)$, and $A^\nu(x)$ defines the gauge potentials having four DOF. The dynamics of these fields in vacuum are described by two transverse ± 1 helicity states.

In contrast to vacuum, photons in a medium, acquire one additional DOF, (the third) longitudinal DOF—in addition to the two (existing) transverse degrees of freedom. In a situation like this, if there exists an external magnetic field B too, then the gauge fields $A^\nu(k)$ corresponding to the in medium photons, can be expressed (in momentum space), in terms of four EM form factors: $A_\parallel(k)$, $A_\perp(k)$, $A_L(k)$, $A_{gf}(k)$ and four orthonormal four vectors $(b^{(1)\nu}, I^\nu, \tilde{u}^\nu, k^\nu)$ constructed out of the available 4-vectors and tensors for the system (in hand). They are given by:

$$A^\nu(k) = A_\parallel(k)N_1 b^{(1)\nu} + A_\perp(k)N_2 I^\nu + A_L(k)N_L\tilde{u}^\nu + N_k A_{gf}(k)k^\nu. \tag{2.2}$$

Here, N_is are the normalization constants. Rewriting $A^\nu(k)$ in terms of unit vectors; $\hat{b}^{(1)\nu} = N_1 b^{(1)\nu}$, $\hat{I}^\nu = N_2 I^\nu$, $\hat{\tilde{u}}^\nu = N_L b^{(1)\nu}$ and $\hat{k}^\nu = N_k k^\nu$ we can rewrite Eq. (2.2) in the following form,

$$A^\nu(k) = A_\parallel(k)\hat{b}^{(1)\nu} + A_\perp(k)\hat{I}^\nu + A_L(k)\hat{\tilde{u}}^\nu + A_{gf}(k)\hat{k}^\nu. \tag{2.3}$$

The vectors, introduced in Eq. (2.3), are defined as,

$$\hat{b}^{(1)\nu} = N_1 k_\mu \bar{F}^{\mu\nu}, \qquad \hat{I}^\nu = N_2\left(b^{(2)\nu} - \frac{(\tilde{u}^\mu b_\mu^{(2)})}{\tilde{u}^2}\tilde{u}^\nu\right),$$
$$\hat{\tilde{u}}^\nu = N_L\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)u_\mu,$$
$$b^{(2)\nu} = k_\mu \tilde{\bar{F}}^{\mu\nu}, \qquad \tilde{\bar{F}}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\bar{F}_{\lambda\rho}. \tag{2.4}$$

The normalisation constants, N_1, N_2, N_L, and N_k in Eq. (2.2) are given by,

$$N_1 = \frac{1}{\sqrt{-b_\mu^{(1)}b^{(1)\mu}}} = \frac{1}{K_\perp B}, \qquad N_2 = \frac{1}{\sqrt{-I_\mu I^\mu}} = \frac{K}{\omega K_\perp B},$$
$$N_L = \frac{1}{\sqrt{-\tilde{u}_\mu\tilde{u}^\mu}} = \frac{k^2}{|K|}, \quad \text{and} \quad N_k = \frac{1}{\sqrt{-k^2}} \tag{2.5}$$

where $K_\perp = (k_1^2 + k_2^2)^{\frac{1}{2}}$.

A. Degrees of freedom

In field theory, medium effects are incorporated into a system by adding a self-energy corrected effective Lagrangian to the tree level Lagrangian. For electromagnetic theory, this term, in momentum space has the form $A^\mu(-k)\Pi_{\mu\nu}(k,T,\mu)A^\nu(k)$; when, $\Pi_{\mu\nu}(k)$, the polarization tensor, can be expressed in terms of transverse and longitudinal form factors, $\Pi_T(k,T,\mu)$ and $\Pi_L(k,T,\mu)$ as,

$$\Pi_{\mu\nu}(k,T,\mu) = \Pi_T(k,T,\mu)[R_{\mu\nu} - Q_{\mu\nu}] + \Pi_L(k,T,\mu)Q_{\mu\nu}. \tag{2.6}$$

These form factors happen to be functions of finite temperature (T), finite chemical potential (μ) and scalars made out of photon four vector k^μ and center of mass four velocity of the medium u^μ individually, or as a combination such as $(k.u)$. Tensors $R_{\mu\nu}$ and $Q_{\mu\nu}$ are the transverse and longitudinal projection operators, constructed using the momentum and center of mass velocity four vector u^μ as,

$$R_{\mu\nu} = \tilde{g}_{\mu\nu}, \quad Q_{\mu\nu} = \frac{\tilde{u}_\mu\tilde{u}_\nu}{\tilde{u}_\mu\tilde{u}^\mu} \quad \text{and} \quad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}. \tag{2.7}$$

The transverse and longitudinal form factors have the property that, in the limit $\omega = 0$ and $k \to 0$, Π_T turns out to be zero and $\Pi_L \to \omega_p^2$ [49–52]. The limit $\omega = 0$ and $k \to 0$, also termed "the" infrared limit, remains an interesting one to study the long wavelength paradigm of the gauge field excitations in a finite density medium. One can analyse infrared dynamics of the system in configuration space by first taking the limit mentioned before, followed by replacing $k^i \to i\partial^i$ in Eq. (2.6). The addition of the background medium induced pieces to the effective Lagrangian may not change the number of degrees of freedom of the system always. For instance, in presence of a background external electromagnetic field, the number of physical degrees of freedom of quantum corrected U(1) gauge theory with fermions, remains the same as that of the free theory. However, the same may not be true when the medium induced quantum corrections are incorporated. The presence of unphysical degrees of freedom in a dynamical system can be inferred from the Hessian matrix of the same. If the Hessian matrix,

$$\frac{\partial^2 L_{\text{eff}}}{\partial \dot{A}_\mu \partial \dot{A}_\nu}, \tag{2.8}$$

is noninvertible, the system is constrained, i.e., the number of dynamical variables are more than the number of physical degrees of freedom present in the system. In that case, to analyze the dynamics of the system one needs to follow the procedures outlined in [53–56]. With finite density effects, incorporated (in the effective Lagrangian) this method, may get very complicated due to presence of higher derivative terms. However, once the procedures (of constraints analysis) are completed, one can find the number of physical degrees of freedom (N) from the equation,

$$N = \frac{N_{\text{psv}} - 2 \times n_1 - n_2}{2} \tag{2.9}$$

where, N_{psv} stands for number of phase space variables, n_1 stands for the number of first class constraints, and n_2 stands for number of second class constraints.

One can, however infer the number of physical degrees of freedom for a system made up of material medium, with lesser effort, if one considers taking the infrared limit we discussed earlier. In this limit the full in medium effective Lagrangian ($L_{\text{eff}(m)}$) takes the form,

$$L_{\text{eff}(m)} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \omega_p^2 A_0 A_0, \tag{2.10}$$

barring the Lorentz structure, this Lagrangian is very close to the Lagrangian of the Proca model. This model although is gauge noninvariant, but is known to produce correct number of degrees of freedom present in a massive theory. This system is known to have two second class constraints and no first class constraints. Therefore, the number of physical degrees of freedom, for the same, turns out to be three. Hence one needs to remove one of the four components of the gauge potential, of U(1) gauge theory, to describe the dynamics of the system. We expect the same to hold good in our case. The same is performed in the next paragraph.

B. Gauge fixing

In continuation to the discussion presented in the last paragraph, we consider A_{gf}, that appears in the equation for the gauge potential,

$$A^\nu(k) = A_\parallel(k)\hat{b}^{(1)\nu} + A_\perp(k)\hat{I}^\nu + A_L(k)\hat{\tilde{u}}^\nu + A_{gf}(k)\hat{k}^\nu, \tag{2.11}$$

to be a redundant DOF and consider it to be equal to zero. As a consequence, it turns out that $k_\nu A^\nu(k) = 0$ that happens to be the Lorentz gauge condition. We would like to point out at this stage that basis vectors $b^{(1)\nu}$, I^ν, and $\tilde{u}^\nu$ used in Eq. (2.2) to describe the gauge potential $A^\nu(k)$ can always be rotated to a set of new basis vectors, however the associated DOF in that basis may not be suitable for normal mode analysis of the system.

C. Some interesting observations

We further note here that Eqs. (2.4) and (2.5) offer some interesting possibilities: in terms of them one can further define an effective metric in the momentum space as:

$$G_{\mu\nu} = \frac{k_\mu k_\nu}{k^2} - \frac{b_\mu^{(1)} b_\nu^{(1)}}{b_\alpha^{(1)} b^{(1)\alpha}} - \frac{I_\mu I_\nu}{I_\alpha I^\alpha} - \frac{\tilde{u}_\mu \tilde{u}_\nu}{\tilde{u}_\alpha \tilde{u}^\alpha}; \tag{2.12}$$

using the orthogonal properties of the unit vectors it would be possible to raise or lower the index of any general four-vector in momentum space using Eq. (2.12). The other important consequence that follows from the definition of the vector potential given by Eq. (2.2) is that, the gauge fixed 4-vector potential is spacelike irrespective of the choice of the momentum.

D. Discrete symmetries

The discrete symmetry (C, P, and T) transformation properties of the tensors, four vectors and the EM form factors, used here are listed in Table I. These transformation laws (agrees with ones provided in [57–60]) can be

TABLE I. Transformation properties for the vectors, tensors and the EM form factors used to describe $A^\nu(k)$ in equation (2.2), under C, P, and T.

	$F_{\mu\nu}$	k_μ	u_μ	$\tilde{u}_\mu$	$b_\mu^{(1)}$	$b_\mu^{(2)}$	I_μ	$A_\parallel$	$A_\perp$	A_L	$f_{\mu\nu}$	i	$\epsilon_{\mu\nu\rho\sigma}$
C	$-F_{\mu\nu}$	k_μ	$-u_\mu$	$-\tilde{u}_\mu$	$-b_\mu^{(1)}$	$-b_\mu^{(2)}$	$-I_\mu$	$A_\parallel$	$A_\perp$	A_L	$-f_{\mu\nu}$	i	$\epsilon_{\mu\nu\rho\sigma}$
P	$F^{\mu\nu}$	k^μ	u^μ	$\tilde{u}^\mu$	$b^{(1)\mu}$	$b^{(2)\mu}$	I^μ	$A_\parallel$	$A_\perp$	A_L	$f_{\mu\nu}$	i	$-\epsilon_{\mu\nu\rho\sigma}$
T	$-F^{\mu\nu}$	k^μ	$-u^\mu$	$-\tilde{u}^\mu$	$-b^{(1)\mu}$	$-b^{(2)\mu}$	$-I^\mu$	$-A_\parallel$	$-A_\perp$	$-A_L$	$-f_{\mu\nu}$	$-i$	$-\epsilon_{\mu\nu\rho\sigma}$

obtained using standard QFT based arguments, except for u^μ (which can be obtained following the subtle principles of finite temperature field theory [20]).

The question that one would like to pose next is, how the interaction between charge neutral spin zero scalar with charge neutral spin one photon, does take place. The best way to address the same is to take a discrete symmetry based route, done elegantly by Raffelt and Stodolosky in [15], invoking CP symmetry based arguments. Where in the case of a pseudoscalar-photon (aγ) interacting system, it was shown that, in such situation, the CP asymmetric pseudoscalar (axion) would couple to the CP asymmetric part of the photon's four vector potential. The CP symmetric part of the photon would remain decoupled. That is the CP violating helicity state (HS) of the photon would couple with the CP violating axion field and evolve in space and time; and the CP preserving helicity state of the photon propagates freely. The dilaton photon dynamics, in similar situation, can be cast in a similar language as that of [15], exchanging the role of pseudoscalars with scalars and scalars with pseudoscalars.

Instead of following [15], we describe such a system here in terms of a set of EM form factors of the photon, described by Eq. (2.2), introduced originally in [61]. The proof of these transformations properties are provided in the Supplemental Material [48] of this article. Having these transformation laws in hand, we argue here, that, the equations of motion, when cast in terms of the electromagnetic form factors of Table I, follow a PT symmetry based coupling dynamics, instead of the CP symmetry based one of [15].

III. INCORPORATION OF MATTER EFFECTS

Matter effects are incorporated through the inclusion of a term of the following form, $A^\mu(k)\Pi_{\mu\nu}(T,\mu)A^\nu(k)$ in the effective Lagrangian ($L_{\rm eff}$) of the system [61–68]. This is a scalar made up by contracting in-medium photon self-energy tensor $\Pi_{\mu\nu}(T,\mu)$ with gauge fields. Parameters T and μ stand for temperature and chemical potential as arguments of $\Pi_{\mu\nu}(T,\mu)$. The contribution of the parity violating part of weakly magnetized matter effects is similarly taken into account by the inclusion of a term like $A^\mu(k)\Pi^p_{\mu\nu}(k,\mu,T,eB)A^\nu(k)$, when $\Pi^p_{\mu\nu}(k,\mu,T,eB)$ is the photon polarization tensor evaluated by incorporating the effects of the magnetic field to first order in the external field strength eB but exact to all orders in T, and chemical potential μ; using Schwinger's proper time propagator and formalism of finite temperature field theory [20,21,68]. The photon polarization tensor $\Pi^p_{\mu\nu}(k)$ can be parametrized in the following way:

$$\Pi^p_{\mu\nu}(k) = \Pi^p(k)P_{\mu\nu}, \quad \text{when } P_{\mu\nu} = i\epsilon_{\mu\nu\alpha\beta_\parallel}\frac{k^\alpha}{|k|}u^{\tilde\beta_\parallel}, \tag{3.1}$$

where $\epsilon_{\mu\nu\alpha\beta_\parallel}$ is Levi-Civita tensor and subscript $\beta_\parallel = 0, 3$. In the expression for $P_{\mu\nu}$, $\tilde\beta_\parallel$ is defined such that if $\beta_\parallel = 0$ then $\tilde\beta_\parallel = 3$ and vice-versa. We have discussed the C, P, and T symmetries of polarization tensors in the Supplemental Material [48] part of this article.

IV. EQUATIONS OF MOTION

Coupling between different DOF of the system, follows from the form of the effective Lagrangian. As can be verified from the Eq. (4.1) that, the couplings between PT violating $A_\parallel(k)$ and PT symmetric $\phi(k)$, is generated in the $\phi\gamma$ tree level Lagrangian (in external B field), through a multiplicative T violating factor i. Thus making the Lagrangian PT symmetric. The other two PT violating EM form factors $A_\perp(k)$ and $A_L(k)$ (see Table I) have no coupling with the scalar $\phi(k)$. Therefore, with the inclusion of $\Pi_{\mu\nu}(T,\mu)$, when $A_L(k)$ becomes nonzero, the mixing matrix for $\phi\gamma$ remains 2×2. But with the inclusion of the effective Lagrangian due to parity violating magnetized-media-induced photon self-energy term $\Pi^p(k)_{\mu\nu}$, $A_\parallel(k)$ gets coupled to $A_\perp(k)$, thus turning the mixing matrix 3×3 one.

In magnetized media the effective Lagrangian—for $\phi\gamma$ interactive system including the effective interaction Lagrangians due to photon self-energy terms—is given by:

$$L_{{\rm eff},\phi} = \frac{1}{2}\phi[k^2 - m_\phi^2]\phi - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} + \frac{1}{2}A^\mu\Pi_{\mu\nu}A^\nu + \frac{1}{2}A^\mu\Pi^p_{\mu\nu}A^\nu - \frac{1}{4}g_{\phi\gamma\gamma}\phi\bar F^{\mu\nu}f_{\mu\nu}. \tag{4.1}$$

In Eq. (4.1), the variable $f_{\mu\nu}$ stands for field strength for the dynamical photons, $\bar F_{\mu\nu}$ stands for the external field, $\Pi_{\mu\nu}$ photon polarization tensor in an isotropic medium, and $\Pi^p_{\mu\nu}$ the same in presence of magnetic field to $O(eB)$.

A. Mixing dynamics of $\phi\gamma$ interaction

The equations of motion for the EM form factors for the photon, in the notation of [61], turn out to be,

$$(k^2 - \Pi_T)A_\parallel(k) + i\Pi^p(k)N_1N_2\left[\epsilon_{\mu\nu\delta\beta}\frac{k^\beta}{|k|}u^{\tilde\delta_\parallel}b^{(1)\mu}I^\nu\right]A_\perp(k) = \frac{ig_{\phi\gamma\gamma}\phi(k)}{N_1}, \tag{4.2}$$

$$(k^2 - \Pi_T)A_\perp(k) - i\Pi^p(k)N_1N_2\left[\epsilon_{\mu\nu\delta\beta}\frac{k^\beta}{|k|}u^{\tilde\delta_\parallel}b^{(1)\mu}I^\nu\right]A_\parallel(k) = 0, \tag{4.3}$$

$$(k^2 - \Pi_L)A_L(k) = 0. \tag{4.4}$$

The three equations [i.e., (4.2)–(4.4)], describe the dynamics of the three DOF of the photon. And the equation of motion for $\phi(k)$ is given by,

$$(k^2 - m^2)\phi(k) = -\frac{ig_{\phi\gamma\gamma}A_{\parallel}(k)}{N_1}. \tag{4.5}$$

It can be checked that, left and right sides of the equations of motion above are PT symmetric. Once the discrete transformations P and T are applied on the variables on both side including background field ($\bar{F}^{\mu\nu}$), EM form factors of the photons and the scalar field $\phi(k)$, then it would show that equations of motion (4.2) to (4.5) remain invariant. For notational convenience, we next introduce the new variables F and G, defined as,

$$F = N_1 N_2 \Pi^p(k)\left[\epsilon_{\mu\nu\delta\beta}\frac{k^\beta}{|k|}u^{\tilde{\delta}_{\parallel}}b^{(1)\mu}I^\nu\right] \text{ and } G = \frac{g_{\phi\gamma\gamma}}{N_1}. \tag{4.6}$$

and would be using them when necessary.

As stated already, in the long wavelength limit, we consider $\Pi_T = \omega_p^2$, where $\omega_p = \sqrt{\frac{4\pi\alpha n_e}{m_e}}$, is the plasma frequency, α is EM coupling constant and n_e is the density of electrons. Other terms, F and G introduced in Eq. (4.6) can be simplified to yield $F = \frac{\omega_p^2}{\omega}\frac{eB\cos\theta}{m_e}$ and $G = -g_{\phi\gamma\gamma}B\sin\theta\omega$, where θ is angle between the photon propagation vector $\vec{k}$ and the magnetic field B. Here m_e is the mass of electron and e is the electronic charge. The equations of motion can now be cast in terms of a compact 4×4 matrix $\mathbf{M}'$ as:

$$[k^2\mathbf{I} - \mathbf{M}']\begin{pmatrix} A_{\parallel}(k) \\ A_{\perp}(k) \\ A_L(k) \\ \phi(k) \end{pmatrix} = 0, \tag{4.7}$$

As we have already mentioned that for $\phi\gamma$ system in magnetized media, the longitudinal DOF of the photon does not mix with the others, therefore we will exclude this term from the mixing matrix, and upon doing so, the 4×4 mixing matrix $\mathbf{M}'$, reduces to a 3×3 mixing matrix denoted by $\mathbf{M}$. For the sake of brevity, using shorthand notations the equations of motions now can be written as follows:

$$[k^2\mathbf{I} - \mathbf{M}]\begin{pmatrix} A_{\parallel}(k) \\ A_{\perp}(k) \\ \phi(k) \end{pmatrix} = 0, \tag{4.8}$$

where the 3×3 mixing matrix for scalar-photon interaction is given as:

$$\mathbf{M} = \begin{pmatrix} \omega_p^2 & iF & -iG \\ -iF & \omega_p^2 & 0 \\ iG & 0 & m_\phi^2 \end{pmatrix}. \tag{4.9}$$

In order to get the Stokes parameters $\mathbf{I}(\omega,z)$, $\mathbf{Q}(\omega,z)$, $\mathbf{U}(\omega,z)$, and $\mathbf{V}(\omega,z)$ for the EM radiation, we need to get the solutions of the field equations; this can be achieved by diagonalizing $\mathbf{M}$.

B. Diagonalizing the 3×3 mixing matrix

Our objective here is to obtain the analytical expression for the unitary matrix $\mathbf{U}$, that would diagonalize the Hermitian matrix $\mathbf{M}$. We express the elements of the same ($\mathbf{U}$), using analytic algebraic expressions. Using this matrix we carry out the numerical operations, to estimates of observables, maintaining a numerical accuracy of the order $\sim 10^{-9}$ or more for the identities those the various intermediate expressions of interest need to satisfy during the numerical evaluation of the form factors.

In order to obtain the elements of the matrix $\mathbf{U}$, we need to solve the characteristic equation, obtained from Det $(\mathbf{M} - \lambda_j\mathbf{I}) = 0$, and find the eigenvalues (roots), i.e., λ_j ($j = 1$, 2, and 3) of $\mathbf{M}$. Then use the same to find the corresponding eigenvectors. Finally, using the eigenvectors, construct the unitary matrix $\mathbf{U}$ that would diagonalize $\mathbf{M}$. The characteristic equation for this 3×3 Hermitian matrix $\mathbf{M}$, for obvious reasons turns out to be a cubic equation, having real roots. The cubic equation, that follows from the characteristic equation can be written, in terms of parameters b, c, and d as,

$$\lambda_j^3 + b\lambda_j^2 + c\lambda_j + d = 0, \tag{4.10}$$

where the parameters b, c, and d are functions of the elements of mixing matrix $\mathbf{M}$, denoted by:

$$b = -(2\omega_p^2 + m_\phi^2) \tag{4.11}$$

$$c = \omega_p^4 + 2\omega_p^2 m_\phi^2 - \left(\frac{eB_{\parallel}}{m_e}\frac{\omega_p^2}{\omega}\right)^2 - (g_{\phi\gamma\gamma}B_{\perp}\omega)^2 \tag{4.12}$$

$$d = -\left[\omega_p^4 m_\phi^2 - \left(\frac{eB_{\parallel}}{m_e}\frac{\omega_p^2}{\omega}\right)^2 m_\phi^2 - (g_{\phi\gamma\gamma}B_{\perp}\omega)^2\omega_p^2\right]. \tag{4.13}$$

Next we introduce the variables P and Q, when $P = (\frac{3c-b^2}{9})$ and $2Q = (\frac{2b^3}{27} - \frac{bc}{3} + d)$; in terms of them, the roots turn out to be,

$$\begin{aligned} \lambda_1 &= \mathbf{R}\cos\alpha + \sqrt{3}\mathbf{R}\sin\alpha - b/3, \\ \lambda_2 &= \mathbf{R}\cos\alpha - \sqrt{3}\mathbf{R}\sin\alpha - b/3, \\ \lambda_3 &= -2\mathbf{R}\cos\alpha - b/3, \\ &\text{with } \begin{cases} \alpha = \frac{1}{3}\cos^{-1}(\frac{Q}{\mathbf{R}^3}) \\ \mathbf{R} = \sqrt{(-P)}\,\mathbf{sgn}(Q). \end{cases} \end{aligned} \tag{4.14}$$

It should be noted that, in principle $\mathbf{R}$ can be equal to $+\sqrt{(-P)}$ or $-\sqrt{(-P)}$. However, the ratio $(\frac{Q}{\mathbf{R}^3})$ should be positive. Hence to maintain the same, the factor of $\mathbf{sgn}(Q)$ is introduced in the definition of $\mathbf{R}$. The orthonormal eigenvectors $\mathbf{X}_j$ of $\mathbf{M}$ are to be found from the matrix relation, $[\mathbf{M}-\lambda_j\mathbf{I}][\mathbf{X}_j]=0$. In terms of its elements, the normalized column vector $[\mathbf{X}_j]$, can be denoted as,

$$[\mathbf{X}_j] = \begin{bmatrix} \bar{u}_j \\ \bar{v}_j \\ \bar{w}_j \end{bmatrix}.$$

Following standard methods, one can evaluate these elements in terms of the roots λ_j and elements of the mixing matrix $\mathbf{M}$. The same, once evaluated turns out to be,

$$\begin{aligned} \bar{u}_j &= (\omega_p^2-\lambda_j)(m_\phi^2-\lambda_j)\times\mathcal{N}_{vn}^{(j)}, \\ \bar{v}_j &= i\frac{eB_\parallel}{m_e}\frac{\omega_p^2}{\omega}(m_\phi^2-\lambda_j)\times\mathcal{N}_{vn}^{(j)}, \\ \bar{w}_j &= ig_{\phi\gamma\gamma}B_\perp\omega(\omega_p^2-\lambda_j)\times\mathcal{N}_{vn}^{(j)}, \\ &\text{when } \left\{\mathcal{N}_{vn}^{(j)} = \frac{1}{\sqrt{[|\bar{u}_j|^2+|\bar{v}_j|^2+|\bar{w}_j|^2]}}\right.. \end{aligned} \tag{4.15}$$

Here $\mathcal{N}_{vn}^{(j)}$ is normalization constant and j can take values from 1 to 3. Using these eigenvectors, the unitary matrix $\mathbf{U}$ turns out to be

$$\mathbf{U} = \begin{pmatrix} (\omega_p^2-\lambda_1)(m_\phi^2-\lambda_1)\mathcal{N}_{vn}^{(1)} & (\omega_p^2-\lambda_2)(m_\phi^2-\lambda_2)\mathcal{N}_{vn}^{(2)} & (\omega_p^2-\lambda_3)(m_\phi^2-\lambda_3)\mathcal{N}_{vn}^{(3)} \\ i\frac{eB_\parallel}{m_e}\frac{\omega_p^2}{\omega}(m_\phi^2-\lambda_1)\mathcal{N}_{vn}^{(1)} & i\frac{eB_\parallel}{m_e}\frac{\omega_p^2}{\omega}(m_\phi^2-\lambda_2)\mathcal{N}_{vn}^{(2)} & i\frac{eB_\parallel}{m_e}\frac{\omega_p^2}{\omega}(m_\phi^2-\lambda_3)\mathcal{N}_{vn}^{(3)} \\ ig_{\phi\gamma\gamma}B_\perp\omega(\omega_p^2-\lambda_1)\mathcal{N}_{vn}^{(1)} & ig_{\phi\gamma\gamma}B_\perp\omega(\omega_p^2-\lambda_2)\mathcal{N}_{vn}^{(2)} & ig_{\phi\gamma\gamma}B_\perp\omega(\omega_p^2-\lambda_3)\mathcal{N}_{vn}^{(3)} \end{pmatrix}. \tag{4.16}$$

The unitary matrix given by (4.16) is the one that diagonalizes the 3×3 mixing matrix $\mathbf{M}$.

C. Field equation: Solutions

In order to obtain the solutions of the coupled equation (4.8), one can multiply the same by inverse of the matrix given by Eq. (4.16), i.e., $\mathbf{U}^{-1}$ from left and use the property of unitary matrices $\mathbf{U}\mathbf{U}^{-1}=\mathbf{1}$ in the same equation [i.e., (4.8)], and arrive at,

$$\begin{aligned} \mathbf{U}^{-1}[k^2\mathbf{I}-\mathbf{M}]\mathbf{U}\mathbf{U}^{-1}\begin{pmatrix} A_\parallel(k) \\ A_\perp(k) \\ \phi(k) \end{pmatrix} &= [k^2\mathbf{I}-\mathbf{M}_D]\begin{pmatrix} A'_\parallel(k) \\ A'_\perp(k) \\ \phi'(k) \end{pmatrix} \\ &= 0. \end{aligned} \tag{4.17}$$

In order to arrive at Eq. (4.17), we have used the following notation,

$$\begin{pmatrix} A'_\parallel(k) \\ A'_\perp(k) \\ \phi'(k) \end{pmatrix} = \mathbf{U}^{-1}\begin{pmatrix} A_\parallel(k) \\ A_\perp(k) \\ \phi(k) \end{pmatrix}. \tag{4.18}$$

The matrix $\mathbf{M}_D$ is the diagonal matrix given by $\mathbf{M}_D = \mathbf{U}^{-1}\mathbf{M}\mathbf{U}$ that has eigenvalues of the matrix $\mathbf{M}$ as diagonal elements. For a propagating beam of photons in the z direction, k_3 can be retransformed back to z by taking the inverse Fourier transform. Furthermore one can express $k^2\approx 2\omega(\omega-i\partial_z)$, without much loss of generality, to use in the equations of motion. As a result of these (algebraic manipulations), Eq. (4.17) assumes the following form,

$$\left[(\omega-i\partial_z)\mathbf{I} - \begin{bmatrix} \frac{\lambda_1}{2\omega} & 0 & 0 \\ 0 & \frac{\lambda_2}{2\omega} & 0 \\ 0 & 0 & \frac{\lambda_3}{2\omega} \end{bmatrix}\right]\begin{bmatrix} A'_\parallel(z) \\ A'_\perp(z) \\ \phi'(z) \end{bmatrix} = 0. \tag{4.19}$$

It is now easy to solve the matrix equation (4.19) by introducing the variables, $\Omega_\parallel = (\omega-\frac{\lambda_1}{2\omega})$, $\Omega_\perp = (\omega-\frac{\lambda_2}{2\omega})$ and $\Omega_\phi = (\omega-\frac{\lambda_3}{2\omega})$. Instead of going into details, we can now directly write down the solutions for the column vector $[\mathbf{A}(z),\phi(z)]^T$; in matrix form, they are given by:

$$\begin{bmatrix} A_\parallel(z) \\ A_\perp(z) \\ \phi(z) \end{bmatrix} = \mathbf{U}\begin{bmatrix} e^{-i\Omega_\parallel z} & 0 & 0 \\ 0 & e^{-i\Omega_\perp z} & 0 \\ 0 & 0 & e^{-i\Omega_\phi z} \end{bmatrix}\mathbf{U}^{-1}\begin{bmatrix} A_\parallel(0) \\ A_\perp(0) \\ \phi(0) \end{bmatrix}. \tag{4.20}$$

The magnitudes of the elements of column vector $[\mathbf{A}(0),\phi(0)]^T$ in Eq. (4.20), are subject to the boundary conditions appropriate for the physical situations assumed to be prevailing at the origin. Using the same initial conditions, one can write down the solution of Eq. (4.20) for $\mathrm{A}_\parallel(\omega,z)$, and it is

$$A_{\parallel}(\omega,z) = (e^{-i\Omega_{\parallel}z}\bar{u}_1\bar{u}_1^* + e^{-i\Omega_{\perp}z}\bar{u}_2\bar{u}_2^* + e^{-i\Omega_{\phi}z}\bar{u}_3\bar{u}_3^*)A_{\parallel}(\omega,0) + (e^{-i\Omega_{\parallel}z}\bar{u}_1\bar{v}_1^* + e^{-i\Omega_{\perp}z}\bar{u}_2\bar{v}_2^* + e^{-i\Omega_{\phi}z}\bar{u}_3\bar{v}_3^*) \times A_{\perp}(\omega,0). \tag{4.21}$$

Similarly the perpendicular component $A_{\perp}(\omega, z)$, turns out to be,

$$A_{\perp}(\omega,z) = (e^{-i\Omega_{\parallel}z}\bar{v}_1\bar{u}_1^* + e^{-i\Omega_{\perp}z}\bar{v}_2\bar{u}_2^* + e^{-i\Omega_{\phi}z}\bar{v}_3\bar{u}_3^*)A_{\parallel}(\omega,0) + (e^{-i\Omega_{\parallel}z}\bar{v}_1\bar{v}_1^* + e^{-i\Omega_{\perp}z}\bar{v}_2\bar{v}_2^* + e^{-i\Omega_{\phi}z}\bar{v}_3\bar{v}_3^*) \times A_{\perp}(\omega,0). \tag{4.22}$$

Here, the parameters $A_{\perp}(\omega, 0)$ and $A_{\parallel}(\omega, 0)$ present in Eqs. (4.21) and (4.22) are $A_{\perp}(\omega, z)$ and $A_{\parallel}(\omega, z)$ respectively under the boundary conditions as mentioned above.

V. POLARIMETRIC OBSERVABLES

From the coherency matrix, Stokes parameters, can be obtained. In terms of the solutions of the field equations they can be expressed as:

$$\begin{aligned}
\mathbf{I}(\omega,z) &= \langle A_{\parallel}{}^*(\omega,z)A_{\parallel}(\omega,z)\rangle + \langle A_{\perp}^*(\omega,z)A_{\perp}(\omega,z)\rangle,\\
\mathbf{Q}(\omega,z) &= \langle A_{\parallel}{}^*(\omega,z)A_{\parallel}(\omega,z)\rangle - \langle A_{\perp}^*(\omega,z)A_{\perp}(\omega,z)\rangle,\\
\mathbf{U}(\omega,z) &= 2\mathrm{Re}\langle A_{\parallel}{}^*(\omega,z)A_{\perp}(\omega,z)\rangle,\\
\mathbf{V}(\omega,z) &= 2\mathrm{Im}\langle A_{\parallel}{}^*(\omega,z)A_{\perp}(\omega,z)\rangle.
\end{aligned} \tag{5.1}$$

It should be noted that $\mathbf{V}(\omega, z)$ in the Eq. (5.1) is a measure of circular polarization. Other polarimetric observables, i.e., degree of linear polarization, ellipticity angle, polarization angle, follows from the expressions of $\mathbf{I}(\omega, z)$, $\mathbf{Q}(\omega, z)$, $\mathbf{U}(\omega, z)$, and $\mathbf{V}(\omega, z)$. The degree of linear polarization (represented as P_L) along with Π, are given by,

$$P_L = \frac{\sqrt{\mathbf{Q}^2(\omega,z) + \mathbf{U}^2(\omega,z)}}{\mathbf{I}(\omega,z)}, \tag{5.2}$$

$$\Pi = \frac{\mathbf{Q}(\omega,z)}{\mathbf{I}(\omega,z)} \tag{5.3}$$

The polarization angle (represented by Ψ_p), is defined in terms of $\mathbf{U}$ and $\mathbf{Q}$, as:,

$$\tan(2\Psi_p) = \frac{\mathbf{U}(\omega,z)}{\mathbf{Q}(\omega,z)}. \tag{5.4}$$

The ellipticity angle (denoted by χ), is defined as:

$$\tan(2\chi) = \frac{\mathbf{V}(\omega,z)}{\sqrt{\mathbf{Q}^2(\omega,z) + \mathbf{U}^2(\omega,z)}}. \tag{5.5}$$

A. Stokes $\mathbf{I}(\omega,z)$ and $\mathbf{Q}(\omega,z)$

To find out the expressions of Stokes parameters $\mathbf{I}(\omega, z)$ and $\mathbf{Q}(\omega, z)$ for scalar-photon mixing, we need to evaluate $|A_{\parallel}(\omega, z)|^2$ and $|A_{\perp}(\omega, z)|^2$, using Eqs. (4.21) and (4.22). Introducing the new variables, $\mathbb{P} = |\bar{u}_1||\bar{v}_1|$, $\mathbb{Q} = |\bar{u}_2||\bar{v}_2|$ and $\mathbb{R} = |\bar{u}_3||\bar{v}_3|$, the expression for $|A_{\parallel}(\omega, z)|^2$ in terms of them, turns out to be,

$$\begin{aligned}
|A_{\parallel}(\omega,z)|^2 = {}& \left[1 - 4|\bar{u}_1|^2|\bar{u}_2|^2\sin^2\left(\frac{(\Omega_{\parallel}-\Omega_{\perp})z}{2}\right) - 4|\bar{u}_2|^2|\bar{u}_3|^2\sin^2\left(\frac{(\Omega_{\perp}-\Omega_{\phi})z}{2}\right)\right.\\
&\left. - 4|\bar{u}_3|^2|\bar{u}_1|^2\sin^2\left(\frac{(\Omega_{\phi}-\Omega_{\parallel})z}{2}\right)\right] \times |A_{\parallel}(\omega,0)|^2\\
& - 4\left[\mathbb{P}\mathbb{Q}\,\sin^2\left(\frac{(\Omega_{\parallel}-\Omega_{\perp})z}{2}\right) + \mathbb{Q}\mathbb{R}\,\sin^2\left(\frac{(\Omega_{\perp}-\Omega_{\phi})z}{2}\right) + \mathbb{R}\mathbb{P}\,\sin^2\left(\frac{(\Omega_{\phi}-\Omega_{\parallel})z}{2}\right)\right] \times |A_{\perp}(\omega,0)|^2\\
& + [|\bar{u}_1\bar{u}_2|(|\bar{u}_1\bar{v}_2| - |\bar{u}_2\bar{v}_1|)\sin((\Omega_{\parallel}-\Omega_{\perp})z) + |\bar{u}_2\bar{u}_3|(|\bar{u}_2\bar{v}_3| - |\bar{u}_3\bar{v}_2|)\sin((\Omega_{\perp}-\Omega_{\phi})z)\\
& + |\bar{u}_3\bar{u}_1|(|\bar{u}_3\bar{v}_1| - |\bar{u}_1\bar{v}_3|)\sin((\Omega_{\phi}-\Omega_{\parallel})z)] \times 2|A_{\parallel}(\omega,0)||A_{\perp}(\omega,0)|.
\end{aligned} \tag{5.6}$$

Similarly, we can find $|A_{\perp}(\omega, z)|^2$. The expression for the same, in terms of the variables introduced earlier, turns out to be,

$$
\begin{aligned}
|A_\perp(\omega,z)|^2 = &-4\left[\mathbb{P}\mathbb{Q}\,\sin^2\left(\frac{(\Omega_\parallel-\Omega_\perp)z}{2}\right)+\mathbb{Q}\mathbb{R}\,\sin^2\left(\frac{(\Omega_\perp-\Omega_\phi)z}{2}\right)+\mathbb{R}\mathbb{P}\,\sin^2\left(\frac{(\Omega_\phi-\Omega_\parallel)z}{2}\right)\right]\times|A_\parallel(\omega,0)|^2\\
&+\left[1-4|\bar{v}_1|^2|\bar{v}_2|^2\sin^2\left(\frac{(\Omega_\parallel-\Omega_\perp)z}{2}\right)-4|\bar{v}_2|^2|\bar{v}_3|^2\sin^2\left(\frac{(\Omega_\perp-\Omega_\phi)z}{2}\right)\right.\\
&\left.-4|\bar{v}_3|^2|\bar{v}_1|^2\sin^2\left(\frac{(\Omega_\phi-\Omega_\parallel)z}{2}\right)\right]\times|A_\perp(\omega,0)|^2\\
&+[|\bar{v}_1\bar{v}_2|(|\bar{u}_1\bar{v}_2|-|\bar{u}_2\bar{v}_1|)\sin((\Omega_\parallel-\Omega_\perp)z)+|\bar{v}_2\bar{v}_3|(|\bar{u}_2\bar{v}_3|-|\bar{u}_3\bar{v}_2|)\sin((\Omega_\perp-\Omega_\phi)z)\\
&+|\bar{v}_3\bar{v}_1|(|\bar{u}_3\bar{v}_1|-|\bar{u}_1\bar{v}_3|)\sin((\Omega_\phi-\Omega_\parallel)z)]\times2|A_\parallel(\omega,0)||A_\perp(\omega,0)|. \qquad (5.7)
\end{aligned}
$$

Using Eqs. (5.6) and (5.7), one can get the expressions for $\mathbf{I}(\omega,z)$ and $\mathbf{Q}(\omega,z)$ as follows:

$$\mathbf{I}(\omega,z)=|A_\parallel(\omega,z)|^2+|A_\perp(\omega,z)|^2 \qquad (5.8)$$

$$\mathbf{Q}(\omega,z)=|A_\parallel(\omega,z)|^2-|A_\perp(\omega,z)|^2. \qquad (5.9)$$

B. Stokes $\mathbf{U}(\omega,z)$ and $\mathbf{V}(\omega,z)$

The expressions for $\mathbf{U}(\omega,z)$ and $\mathbf{V}(\omega,z)$ can be written in terms of $\mathcal{U}_\parallel,\mathcal{U}_\perp,\mathcal{U}_{\parallel\perp}$ and $\mathcal{V}_\parallel$, $\mathcal{V}_\perp$, $\mathcal{V}_{\parallel\perp}$. For $\mathbf{U}(\omega,z)$ they are given by:

$$
\begin{aligned}
\mathcal{U}_\parallel = &|\bar{u}_1\bar{u}_2|(|\bar{v}_1\bar{u}_2|-|\bar{u}_1\bar{v}_2|)\sin((\Omega_\parallel-\Omega_\perp)z)+|\bar{u}_2\bar{u}_3|(|\bar{v}_2\bar{u}_3|-|\bar{u}_2\bar{v}_3|)\sin((\Omega_\perp-\Omega_\phi)z)\\
&+|\bar{u}_3\bar{u}_1|(|\bar{v}_3\bar{u}_1|-|\bar{u}_3\bar{v}_1|)\sin((\Omega_\phi-\Omega_\parallel)z). \qquad (5.10)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{U}_\perp = &|\bar{v}_1\bar{v}_2|(|\bar{v}_1\bar{u}_2|-|\bar{u}_1\bar{v}_2|)\sin((\Omega_\parallel-\Omega_\perp)z)+|\bar{v}_2\bar{v}_3|(|\bar{v}_2\bar{u}_3|-|\bar{u}_2\bar{v}_3|)\sin((\Omega_\perp-\Omega_\phi)z)\\
&+|\bar{v}_3\bar{v}_1|(|\bar{v}_3\bar{u}_1|-|\bar{u}_3\bar{v}_1|)\sin((\Omega_\phi-\Omega_\parallel)z). \qquad (5.11)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{U}_{\parallel\perp} = &(|\bar{v}_1\bar{u}_2|-|\bar{u}_1\bar{v}_2|)^2\cos((\Omega_\parallel-\Omega_\perp)z)+(|\bar{v}_2\bar{u}_3|-|\bar{u}_2\bar{v}_3|)^2\cos((\Omega_\perp-\Omega_\phi)z)\\
&+(|\bar{v}_3\bar{u}_1|-|\bar{u}_3\bar{v}_1|)^2\cos((\Omega_\phi-\Omega_\parallel)z). \qquad (5.12)
\end{aligned}
$$

And similarly, the expressions for $\mathcal{V}_\parallel$, $\mathcal{V}_\perp$, and $\mathcal{V}_{\parallel\perp}$, introduced to express the measure of circular polarization $\mathbf{V}(\omega,z)$ are

$$
\begin{aligned}
\mathcal{V}_\parallel = &[(|\bar{u}_1\bar{v}_1||\bar{u}_1|^2+|\bar{u}_2\bar{v}_2||\bar{u}_2|^2+|\bar{u}_3\bar{v}_3||\bar{u}_3|^2)+|\bar{u}_1\bar{u}_2|(|\bar{v}_1\bar{u}_2|+|\bar{u}_1\bar{v}_2|)\cos((\Omega_\parallel-\Omega_\perp)z)\\
&+|\bar{u}_2\bar{u}_3|(|\bar{v}_2\bar{u}_3|+|\bar{u}_2\bar{v}_3|)\cos((\Omega_\perp-\Omega_\phi)z)+|\bar{u}_3\bar{u}_1|(|\bar{v}_3\bar{u}_1|+|\bar{u}_3\bar{v}_1|)\cos((\Omega_\phi-\Omega_\parallel)z)]. \qquad (5.13)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}_\perp = &[(|\bar{u}_1\bar{v}_1||\bar{v}_1|^2+|\bar{u}_2\bar{v}_2||\bar{v}_2|^2+|\bar{u}_3\bar{v}_3||\bar{v}_3|^2)+|\bar{v}_1\bar{v}_2|(|\bar{u}_1\bar{v}_2|+|\bar{v}_1\bar{u}_2|)\cos((\Omega_\parallel-\Omega_\perp)z)\\
&+\bar{v}_2\bar{v}_3|(|\bar{u}_2\bar{v}_3|+|\bar{u}_3\bar{v}_2|)\cos((\Omega_\perp-\Omega_\phi)z)+|\bar{v}_3\bar{v}_1|(|\bar{u}_3\bar{v}_1|+|\bar{u}_1\bar{v}_3|)\cos((\Omega_\phi-\Omega_\parallel)z)]. \qquad (5.14)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}_{\parallel\perp} = &[(|\bar{u}_1|^2|\bar{v}_2|^2-|\bar{u}_2|^2|\bar{v}_1|^2)\sin((\Omega_\parallel-\Omega_\perp)z)+(|\bar{u}_2|^2|\bar{v}_3|^2-|\bar{u}_3|^2|\bar{v}_2|^2)\sin((\Omega_\perp-\Omega_\phi)z)\\
&+(|\bar{u}_3|^2|\bar{v}_1|^2-|\bar{u}_1|^2|\bar{v}_3|^2)\sin((\Omega_\phi-\Omega_\parallel)z)]. \qquad (5.15)
\end{aligned}
$$

Using them, one can write the expressions for $\mathbf{U}(\omega,z)$ and $\mathbf{V}(\omega,z)$ as follows:

$$\mathbf{U}(\omega,z)=\mathcal{U}_\parallel\times2|A_\parallel(\omega,0)|^2+\mathcal{U}_\perp\times2|A_\perp(\omega,0)|^2+\mathcal{U}_{\parallel\perp}\times2|A_\perp(\omega,0)||A_\parallel(\omega,0)| \qquad (5.16)$$

$$\mathbf{V}(\omega,z)=\mathcal{V}_\parallel\times2|A_\parallel(\omega,0)|^2+\mathcal{V}_\perp\times2|A_\perp(\omega,0)|^2+\mathcal{V}_{\parallel\perp}\times2|A_\perp(\omega,0)||A_\parallel(\omega,0)|. \qquad (5.17)$$

The measures of linear and circular polarization $\mathbf{Q}(\omega,z)$, $\mathbf{U}(\omega,z)$ and polarization angle Ψ_p are plotted as a function of energy in Figures 1. The discussion about them is provided below.

VI. POSSIBLE PHYSICAL SITUATION

Gamma ray bursts (GRB) are stellar sized explosions (those usually found with an associated luminosity of about $\sim 10^{51}$ ergs/ sec), observed to occur isotropically at a redshifts $z = 1$ or $z = 2$ with the EM radiation in the form of x-ray or gamma-ray, with fluence of the order of 10^{-6} ergs/cm^2, as observed from earth [69]. Their size is usually estimated from the timescale variability of their light curve, estimated to be around 0.1 sec [70].

Accordingly their size is estimated to be $\sim 10^9$ cm. The energy is believed to be injected from an object, like neutron star, of radius 10^6 cm. The high the degree of linear polarization associated with the EM spectrum (e.g., $\Pi = 27 \pm 11\%$ in GRBICO826A), associated with them, makes one infer, a strong magnetic field $\sim 10^9$ Gauss to be associated with them. The plasma frequency ω_p associated with them are believed to lie above, $\omega_p > 10^{-17}$ GeV [71–73]. The mechanism behind this explosion is currently under scrutiny; one believes that the polarization studies with intensity of the spectra holds the key to understanding the geometry of the magnetic field at the source and the energy release mechanism. Though these aspects can be studied using classical physics, one needs to be cautious, since factors like the presence of ALPs also potentially contribute to the modification of these observables. Therefore any anomaly in these signatures may provide a clue to the existence of ALPs.

VII. RESULTS

Assuming a global magnetic field of magnitude $\sim 10^9$ Gauss, to be existing in a GRB fireball—over a path length of $\sim 10^9$ cm, the stokes parameters **Q**, **U** and the polarization angle Ψ_p were estimated using Eqs. (5.1) and (5.4) numerically—for scalar-photon coupling constant $g_{\phi\gamma\gamma} \sim 10^{-11}$ GeV^{-1} with $\omega_p \sim 10^{-13}$ GeV, and $m_\phi \sim 10^{-2}$ eV. The plots of the same are provided in Fig. 1.

Since the degree of linear polarization Π, turns out to be equal to **Q**, when **U** is neglected (as is evident from the respective definitions of the same), we can get the information about the amplitude variation for **Q**, **Π** and $\mathbf{\Psi}_p$ over the energy (ω) interval $1.0\times 10^{-6}\,\text{GeV} < \omega < 1.0\times 10^{-5}\,\text{GeV}$ from the plots provided in Fig. 1.

In most of the astrophysically viable, satellite based experiments, the detectors are hardly line sensitive, they usually operate over a broad energy range; hence the parameters (of Stokes) are usually estimated by adding the signal strengths over an energy range (according to the detector under consideration), followed by an averaging of the signal—over that same energy range.

Now if we look into these plots, we notice the existence of the highly oscillating part for the estimates of **Q** in the energy interval, 1.0×10^{-6} GeV $< \omega < 3.0 \times 10^{-6}$ GeV and a similar pattern also for **U** in the interval 1.0×10^{-6} GeV $< \omega < 2.0 \times 10^{-6}$ GeV. This is followed by an monotonically increasing or decreasing pattern (left panel of Fig. 1). Similar effect, is identifiable in the plot for $\mathbf{\Psi}_p$ vs energy in the right panel of the same figure (i.e., Fig. 1).

Therefore if the standard satellite based experiment dictated data extraction prescriptions are followed—while extracting signals from the datum like that producing figure [1]-it would lead to the generation of an extremely low strength unphysical signal. This happens due to a major cancellation of the contributions due to strong oscillations of the actual signals around zero.

Thus, in order to get a realistic and statistically significant estimate of a signal, from detectors operating over a broad energy range, one must explore the available parameter space, over which the variations of the signals are stable with energy.

A collection of data sets for the spectro-polarimetric observables, those (unlike the ones of Fig. 1) looked stable under the variation with respect to ω), were found in the 1–10 KeV range, when the numerical size of the parameters ω_p, and m_ϕ were considered at around $\sim 10^{-15}$ GeV, while

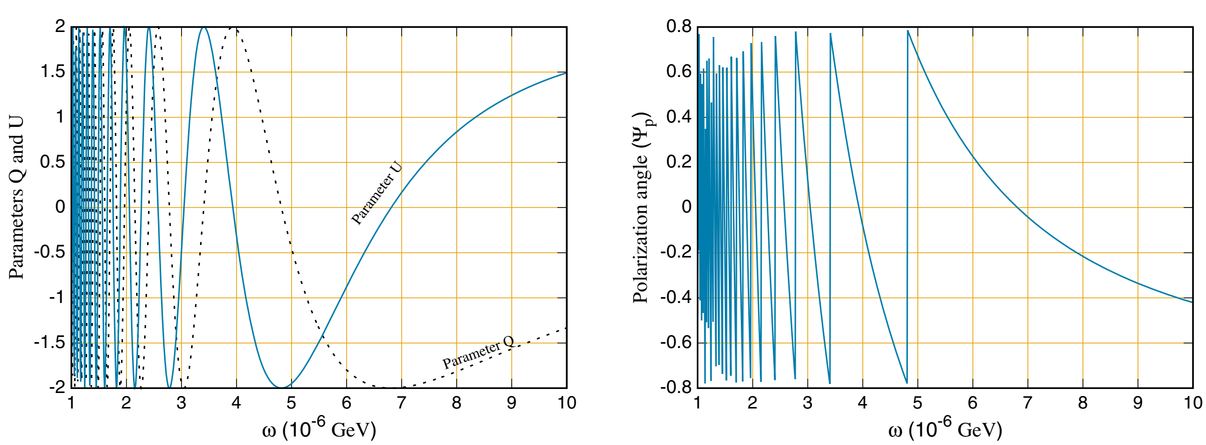

FIG. 1. (In left), plot of Stokes parameter **Q** and **U** vs energy (ω). (In right), plot of polarization angle (Ψ_p) vs energy (ω). Here, $m_\phi = 1.0 \times 10^{-12}$ GeV and $\omega_p = 3.7 \times 10^{-13}$ GeV).

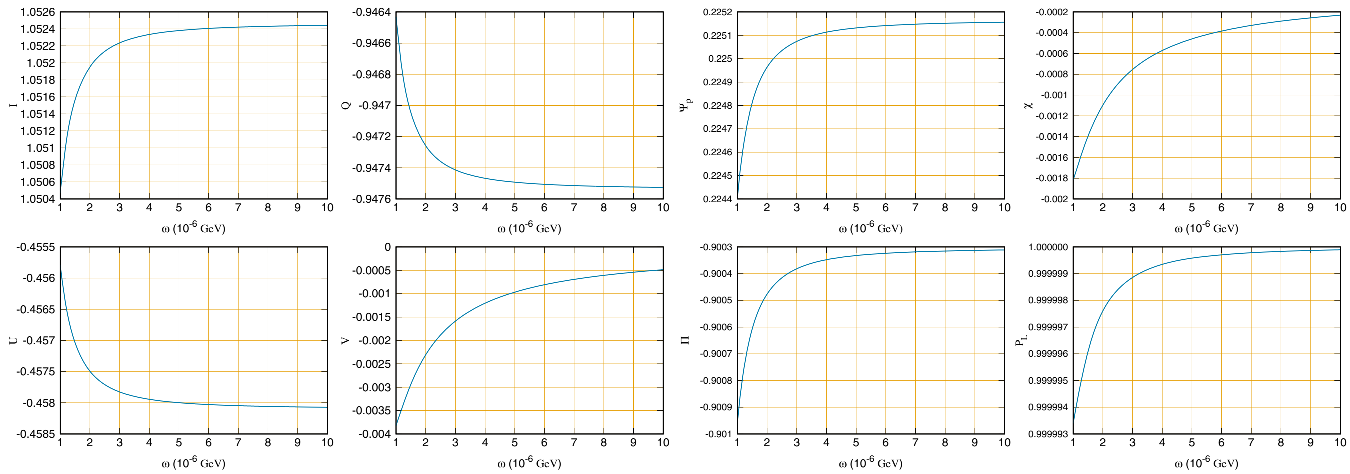

FIG. 2. Left: plots of Stokes parameters ($\mathbf{I}, \mathbf{Q}, \mathbf{U}, \mathbf{V}$) vs energy ($\omega$). Right: plots of polarization angle [Ψ_p, ellipticity angle (χ), degree of polarization (Π), and linear polarization (P_L) vs energy (ω)]. Here, $m_\phi = 1.0 \times 10^{-15}$ GeV and $\omega_p = 3.7 \times 10^{-15}$ GeV.

the path length(z), magnetic field strength(eB) and scalar photon coupling strength ($g_{\phi\gamma\gamma}$) were maintained at 10^6 cm, $eB \sim 10^9$ Gauss and, $g_{\phi\gamma\gamma} \sim 10^{-11}$ GeV^{-1}, respectively.

The plots of the spectro-polarimetric observables, such as linear polarization (P_L), polarization angle Ψ_p, ellipticity angle χ, and degree of polarization Π, estimated with these parameters, are displayed in figure [2], for an energy interval–in the 1–10 KeV range. Interestingly enough, the estimates of maximum linear polarization, for these parameters and energy range, is about 99% and the polarization angle is about 12°; those lie well within the range of the observed linear-polarization and polarization angle—estimated from satellite borne astrophysical observations, for the GRBs, occurring across the sky.

This little interesting exercise could have been used to project out a possible operating energy range for the space borne detectors, to explore the existence of ALPs in the parameter range considered above; had it not have faced a constraint (veto) from the fifth-force experiments due to dilatons, that we elaborate below.

A. Fifth force constraints

The scale symmetry, thats being investigated in this work, is an approximate one. This is due to the presence of massive dilatons. In the field theory context, it can happen through the incorporation of higher dimensional conformal symmetry breaking operators (having scaling dimension other than four) or by breaking the symmetry spontaneously as was done in [6].

And in unified theories of extra dimension, this breaking may be due to nonzero curvature associated with the extra dimension. Whatever may be the origin, their existence would modify, the predictions of gravitational force of Einstein gravity with dilatonic fifth force. Experiments performed with torsion-balance, to find the range of dilatonic fifth force, puts an upper bound on dilaton mass to be $\sim O(10^{-11})$ GeV. Hence, one needs to consider the value of $m_\phi \sim 10^{-11}$ GeV, for any realistic search for ϕ.

Although we had seen earlier, that, the change in the spectro-polarimetric observables with energy (ω) were marginal when the mass of m_ϕ and plasma frequency ω_p were of the order of $\sim 10^{-15}$ GeV; but due to the reasons explained above those results are unrealistic.

In the light of this, (i.e., for $m_\phi \geq 10^{-2}$ eV), identifying the favourable regions in the parameter variables (i.e., estimates of ω_p eB and $g_{\phi\gamma\gamma}$ etc.) over which the signals would remain stabilized is a time consuming task. A way of bypassing this apparent difficulty, is by scaling these variables with m_ϕ and finding the magnitude of the changed variables, such that, the scaled ones remain constant. These new modified values are expected to provide a suitable parameter space, that one is searching for. We had performed this scaling exercise and found the following modified values for the variables $\omega_p = 3.7 \times 10^{-10}$ GeV, $eB \sim 10^4$ Gauss, and $g_{\phi\gamma\gamma} = 1.0 \times 10^{-14}$ GeV, rest being the same. The results are provided in Fig. 3. The numerical exercise confirms the existence of stable signals in the energy range 1.0×10^{-5} GeV $< \omega < 1.0 \times 10^{-4}$ GeV, for the scaled variables, chosen here.

However, in the process, the original parameters, chosen from the generally accepted models of gamma ray bursts, got modified. So one is left with the option of exploring refined models of GRB emission, like synchrotron emission based models or hydrodynamic instability based models or streaming instability based models etc., as described in [[74–82]].

VIII. DISCUSSION

For obvious reasons, detecting x-ray or gamma- ray signals from earth surface is difficult. Hence spaceborne

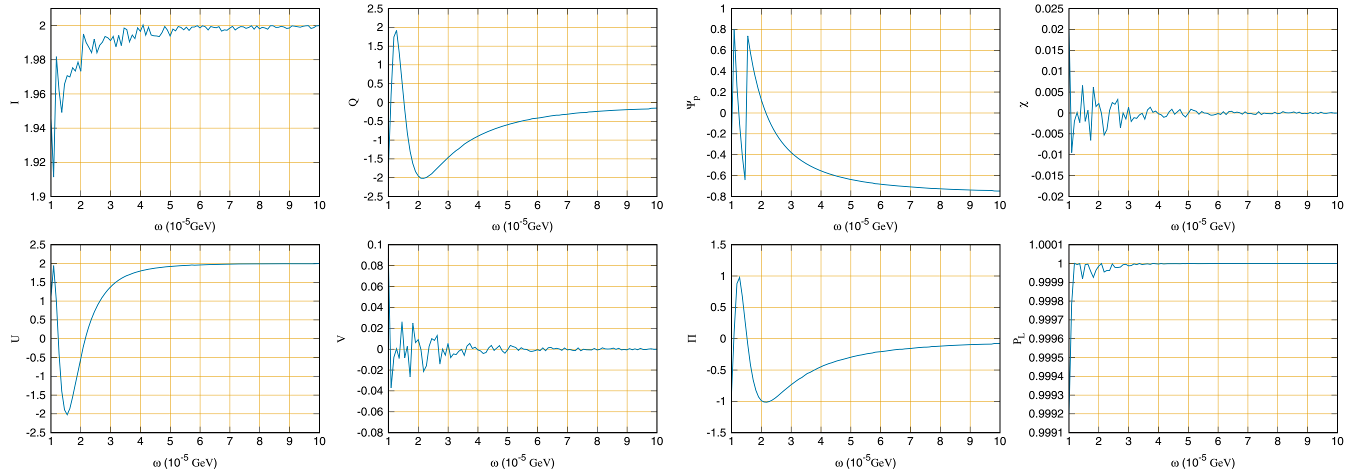

FIG. 3. Left: plots of Stokes parameters (**I**, **Q**, **U**, **V**) vs energy (ω). Right: plots of polarization angle (Ψ_p, ellipticity angle (χ), degree of polarization (Π), and linear polarization (P_L) vs energy (ω). Parameters considered here, i.e., $m_\phi = 1.0 \times 10^{-11}$ GeV and $\omega_p = 3.7 \times 10^{-10}$ GeV, respecting fifth force constraints.

detectors (e.g., GAP) have been developed and used to detect the same [72]. They happen to be sensitive to a wide energy range; for instance GAP is sensitive to an energy range 70–300 KeV. The Ranaty High Energy Solar Spectrometer Imager (RHESSI) [75] had an operational window of 0.15–2 MeV, the same for SWIFT satellite [76] happen to be 300 KeV–10 MeV etc. On the other hand our numerical estimates show that, the signals may undergo huge amount of oscillations in these operational windows. A crucial assumption that goes in the detection of polarization signal is additivity of the Stokes parameters over an energy range. And the difference in the polarization angles of the GRB EM beam, at two ends of the band, to be less than ninety degrees, i.e., [73]

$$|\Delta(E_2, z) - \Delta(E_1, z)| \leq \frac{\pi}{2}. \tag{8.1}$$

This may not always be the case, as our numerical estimates show. Since many of the polarization data–GRB or others are used to extract estimates about $g_{\phi\gamma\gamma}$ and m_ϕ, assuming Eq. (8.1) to be valid, when in principle, that may not be the case; therefore those estimates of the coupling constant and dilaton mass become questionable.

IX. CONCLUSION

To conclude, in this work we have analyzed the mixing pattern of dilatons with photons in a magnetized medium. Our analysis establishes that the mixing matrix for $\gamma\phi$ system is 3×3 not 2×2. We also found that, in a magnetized medium, the longitudinal DOF associated with the photon, does not get excited, by the dilatons; the same happens only for axions, as was rightly pointed out in [77]. Using analytical techniques we have solved for the equations of motion of this system exactly. Following that we have estimated the strength of the polarimetric signals that is expected for a typical GRB geometry.

What we find is, there are regions in the parameter space over which, the signals are stable for some energy range, and there are regions for which the same may not be true (for instance see Fig. 2, where the signal (Ψ_p) is seen to stable over photon energy range $1.0 \times 10^{-6}\,\text{GeV} < \omega < 1 \times 10^{-5}\,\text{GeV}$).

Given the fact that there are quite a few proposed satellite borne experiments in line (e.g., [83–87]), one needs to be careful while designing the detectors for them. The operational window of the energy range over which these detectors should work may be decided after taking into account the lessons those emerged from the investigations like ours, in the respective simulations those usually considered for the purpose of detectors design.

Furthermore, the estimates of other observables like the spectral evolutions over energy, the magnitude of the fluence, the intensity of the spectrum in each polarization channel etc., those usually emerge from an analysis like ours and should also be taken into account while designing the detectors (i.e., their energy sensitivity, operational energy range, dead time fixation and physical dimension etc.). Once these are taken into account, it would help in getting better quality data—for understanding the nature of the ALPs and their parameters.

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APPENDIX: POLARIMETRY WITH MAGNETIZED MEDIA

In this part of the work, we discuss the polarization effects[2] on an electromagnetic field due to magnetized media. The description of photon propagation in a magnetized media are provided by the set of equations describing the evolution of the respective degrees of freedom, one of them having plane of polarization along the magnetic field called $A_\parallel$ and the other one having plane of polarization orthogonal to the same. This set is given by

$$(k^2 - \Pi_T^2)A_\parallel(\omega, z) + iFA_\perp(\omega, z) = 0, \tag{A1}$$

$$(k^2 - \Pi_T^2)A_\perp(\omega, z) - iFA_\parallel(\omega, z) = 0. \tag{A2}$$

In Eqs. (A1) and (A2), the strength of magnetization is carried by the variable F and the same is given by $F = \frac{\omega_p^2 eB\cos\theta}{\omega m_e}$. Plasma frequency is denoted by ω_p, ω is the energy of photon, eB is the magnetic field strength. Defining, $\Omega^2 = (\frac{\omega_p^2}{2\omega} - \omega)$. The Eqs. (A1) and (A2) can further be written terms of a mixing matrix $\mathbf{M}$ causing mixing between the components of polarization $A_\parallel$ and $A_\perp$, in the following fashion,

$$i\partial_z\left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \Omega^2 & -\frac{iF}{2\omega} \\ -\frac{iF}{2\omega} & \Omega^2 \end{bmatrix}\right]\begin{bmatrix} A_\parallel(\omega, z) \\ A_\perp(\omega, z) \end{bmatrix} = 0. \tag{A3}$$

To find the solutions of $A_\parallel$ and $A_\perp$, we need eigenvalues and the eigenvectors for matrix $\mathbf{M}$. Using the eigenvectors and eigenvalues λ_+ and λ_-, the unitary matrices turn out to be,

$$\tilde{U} = \frac{1}{\sqrt{2}}\begin{bmatrix} i & 1 \\ 1 & i \end{bmatrix} \quad \text{and,} \quad \tilde{U}^\dagger = \frac{1}{\sqrt{2}}\begin{bmatrix} -i & 1 \\ 1 & -i \end{bmatrix}. \tag{A4}$$

Using them, we finally arrive at the solutions of $A_\parallel$ and $A_\perp$ in terms of the initial conditions $A_\parallel(\omega, 0)A_\perp(\omega, 0)$ the solutions are,

$$\begin{aligned} A_\parallel(\omega, z) = &\left[\cos(\Omega^2 z)\cos\left(\frac{Fz}{2\omega}\right)A_\parallel(\omega, 0) - \cos(\Omega^2 z)\sin\left(\frac{Fz}{2\omega}\right)A_\perp(\omega, 0)\right] \\ &+ i\left[\sin(\Omega^2 z)\cos\left(\frac{Fz}{2\omega}\right)A_\parallel(\omega, 0) - \sin(\Omega^2 z)\sin\left(\frac{Fz}{2\omega}\right)A_\perp(\omega, 0)\right], \end{aligned} \tag{A5}$$

$$\begin{aligned} A_\perp(\omega, z) = &\left[\cos(\Omega^2 z)\cos\left(\frac{Fz}{2\omega}\right)A_\perp(\omega, 0) + \cos(\Omega^2 z)\sin\left(\frac{Fz}{2\omega}\right)A_\parallel(\omega, 0)\right] \\ &+ i\left[\sin(\Omega^2 z)\sin\left(\frac{Fz}{2\omega}\right)A_\parallel(\omega, 0) + \sin(\Omega^2 z)\cos\left(\frac{Fz}{2\omega}\right)A_\perp(\omega, 0)\right]. \end{aligned} \tag{A6}$$

It is easy to check the consistency of the solutions, by noting that for $F = 0$, there is no mixing between $A_\parallel$ and $A_\perp$. Now we make use of the solutions to obtain the Stokes parameters I, Q, U, and V and following that can evaluate the polarization angle $\mathbf{\Psi}$ and ellipticity angle χ from them. The expressions for I, Q, U, and V are given as follows,

$$I = A_\parallel^2(\omega, 0) + A_\perp^2(\omega, 0), \tag{A7}$$

$$\begin{aligned} Q = &\cos\left(\frac{Fz}{\omega}\right)[A_\parallel^2(\omega, 0) - A_\perp^2(\omega, z)] \\ &- 2\sin\left(\frac{Fz}{\omega}\right)A_\perp(\omega, 0)A_\parallel(\omega, 0), \end{aligned} \tag{A8}$$

$$\begin{aligned} U = &\sin\left(\frac{Fz}{\omega}\right)[A_\parallel^2(\omega, 0) - A_\perp^2(\omega, 0)] \\ &+ 2\cos\left(\frac{Fz}{\omega}\right)A_\perp(\omega, 0)A_\parallel(\omega, 0). \end{aligned} \tag{A9}$$

It turns out that, in this case the Stokes parameter V describing circular polarization is equal to zero. It is to be noted that the Stokes parameter I is now independent of the path length z, which provides the consistency check of energy conservation of the system. Also, the rate of rotation of the plane of polarization, with change of path length distance, is proportional to the inverse square of energy of photon, i.e.,

$$\frac{d\Psi}{dz} = \frac{\omega_p^2 eB\cos\theta}{2\omega^2 m_e}. \tag{A10}$$

The ω dependence of this result matches with the same reported in [68]. Therefore one can state that the effect of

[2]For more details on discrete symmetries of different mediums and their effects on propagation of EM signals, one can go through [88] and [89].

magnetized medium alone on the polarimetric signature of light can be predicted by studying the state of circular polarization of light along with energy dependence of rate of rotation of the polarization angle per unit length by studying the system when the magnetic field $\vec{B}$ is along $\vec{k}$. On the other hand when the angle between $\vec{k}$ and $\vec{B}$ is $\frac{\pi}{2}$ then the effect of magnetized media of polarization of the electromagnetic beam vanishes and the variation of polarization with ω can be found in [45]. Similar studies only for scalars or pseudoscalars have been performed in [63], that show different outcome.

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