Probing new physics with the kaon decays $K \to \pi\pi\not{E}$

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The latest search for the rare kaon decay $K^+ \to \pi^+\nu\bar{\nu}$ by the NA62 experiment has produced evidence for it with a branching fraction consistent with the prediction of the standard model. The new result implies that in this decay, with the $\nu\bar{\nu}$ pair appearing as missing energy ($\not{E}$), the room for possible new physics is no longer sizable and that therefore its contributions to underlying four-particle $s \to d\not{E}$ operators with parity-even ds quark bilinears have become significantly constrained. Nevertheless, we point out that appreciable manifestations from beyond the standard model induced by the corresponding operators with mainly parity-odd ds quark bilinears could still occur in $K \to \pi\pi\not{E}$ modes, on which there are only minimal empirical details at present. We find in particular that new physics of this kind may enhance the branching fraction of $K_L \to \pi^0\pi^0\not{E}$ to values reaching its current experimental upper limit and the branching fractions of $K^+ \to \pi^+\pi^0\not{E}$ and $K_L \to \pi^+\pi^-\not{E}$ to the levels of 10^{-7} and 10^{-6}, respectively. Thus, quests for these decays in existing kaon facilities such as KOTO and NA62 or future ones could provide valuable information complementary to that gained from $K \to \pi\not{E}$.

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I. INTRODUCTION

One of the potentially promising avenues to discover new physics (NP) beyond the standard model (SM) is to look for processes that are expected to be very rare in the SM. An observation of such a process having a rate much greater than what the SM predicts would then be a compelling indication of NP effects. Among places where this may be realized are the flavor-changing neutral current (FCNC) decays of light strange-flavored hadrons with missing energy ($\not{E}$). These reactions are known to be dominated by short-distance physics [1–8] and arise primarily from the quark transition $s \to d\not{E}$. In the SM, it proceeds from loop-suppressed diagrams [2] and the final state contains undetected neutrinos ($\nu\bar{\nu}$). Beyond the SM, there could be additional ingredients which alter the SM component and/or give rise to extra channels with one or more invisible nonstandard particles carrying away the missing energy.

Over the years hunts for $s \to d\not{E}$ have focused the kaon modes $K \to \pi\nu\bar{\nu}$, leading mostly to limits on their branching fractions [9–12]. The efforts are ongoing in the KOTO [10] and NA62 [12] experiments. The former [10] has set $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm KOTO} < 3.0\times10^{-9}$ at 90% confidence level (CL), exceeding but not far from the SM expectation [13] of $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} = (3.4\pm0.6)\times10^{-11}$. On the other hand, very recently NA62 [14] has preliminarily reported 3.5σ evidence for the charged channel with $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm NA62} = [11.0^{+4.0}_{-3.5}({\rm stat})\pm0.3({\rm syst})]\times10^{-11}$, which is in good agreement with the SM value [13] of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = (8.4\pm1.0)\times10^{-11}$ and more precise than the earlier E949 [9] finding of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm E949} = (17.3^{+11.5}_{-10.5})\times10^{-11}$. As these measurements, notably the K^+ ones, have moved increasingly close to their SM predictions, the room for NP in $K \to \pi\not{E}$ has become quite small.

As it turns out, of the possible underlying $s \to d\not{E}$ operators [7,15,16], these decays are sensitive to only a subset. Specifically, they can probe four-particle operators that have parity-even ds quark bilinears but are unaffected by those with exclusively parity-odd ds bilinears [6–8,16]. However, the latter operators can contribute to kaon reactions emitting no or two pions, namely $K \to \not{E}$ and $K \to \pi\pi\not{E}$, as well as to analogous decays in the hyperon sector [6–8]. This means that, since at the moment there are precious few data on these processes [17], searches for them might still come up with substantial manifestations of NP or at least yield useful information about it complementary to that supplied by $K \to \pi\not{E}$ measurements.

In this paper, we adopt a model-independent approach to explore how big the branching fractions of the various $K \to \pi\pi \not{E}$ modes might be, taking into account the available pertinent constraints. We assume especially that the invisibles comprise a pair of spin-1/2 fermions or spinless bosons, all of which are singlets under the SM gauge groups. It is hoped that the outcomes of our study will motivate renewed attempts to pursue these decays as NP tests.

The organization of the rest of the article is the following. In Sec. II, we describe the quark-level operators responsible for the interactions of interest. In Sec. III, we derive the amplitudes for the aforementioned kaon decay modes and calculate their rates. For the majority of them, we also write down the corresponding numerical branching fractions in terms of the coefficients of the operators. In Sec. IV, we compare the SM predictions for these transitions with their current data. In Sec. V, we address the allowed maximal branching fractions of $K \to \pi\pi \not{E}$ due to NP and present our conclusions. In all the instances we will discuss, we take the invisibles to be light enough that their masses can be neglected compared to those of the mesons, which helps maximize the kaon decay rates.

II. INTERACTIONS

Depending on the types of particles carrying away the missing energy, the effective $s \to d\not{E}$ operators are generally subject to different sets of restrictions. If the invisible particles are SM neutrinos, which have charged-lepton partners because of the SM $SU(2)_L$-gauge invariance, the operators would likely have to face stringent restraints from lepton-flavor violation data. Since these do not apply if the invisibles are SM-gauge singlets, hereafter we consider a couple of cases involving them.

The missing energy is carried away by a pair of spin-1/2 Dirac fermions, $\mathtt{f}$ and $\mathtt{f}'$, in the first scenario and by a pair of complex spin-0 bosons, ϕ and ϕ', in the second one.[1] At low energies, the relevant quark-level operators need to respect the strong and electromagnetic gauge symmetries and are mostly obtainable from the literature [7,15,16]. We can express the effective interaction Lagrangians as

$$\begin{aligned}\mathcal{L}_{\mathtt{ff}'} = -[&\bar{d}\gamma^\eta s \bar{\mathtt{f}}\gamma_\eta(\mathtt{C}^{\rm V}_{\mathtt{ff}'} + \gamma_5 \mathtt{C}^{\rm A}_{\mathtt{ff}'})\mathtt{f}' \\ &+ \bar{d}s\bar{\mathtt{f}}(\mathtt{C}^{\rm S}_{\mathtt{ff}'} + \gamma_5 \mathtt{C}^{\rm P}_{\mathtt{ff}'})\mathtt{f}' \\ &+ \bar{d}\sigma^{\eta\kappa}s\bar{\mathtt{f}}\sigma_{\eta\kappa}(\mathtt{C}^{\rm T}_{\mathtt{ff}'} + \gamma_5 \mathtt{C}^{\rm T'}_{\mathtt{ff}'})\mathtt{f}' \\ &+ \bar{d}\gamma^\eta\gamma_5 s\bar{\mathtt{f}}\gamma_\eta(\tilde{c}^{\rm V}_{\mathtt{ff}'} + \gamma_5 \tilde{c}^{\rm A}_{\mathtt{ff}'})\mathtt{f}' \\ &+ \bar{d}\gamma_5 s\bar{\mathtt{f}}(\tilde{c}^{\rm S}_{\mathtt{ff}'} + \gamma_5 \tilde{c}^{\rm P}_{\mathtt{ff}'})\mathtt{f}'] + \text{H.c.} \end{aligned} \tag{1}$$

and

$$\begin{aligned}\mathcal{L}_{\phi\phi'} = -[&(c^{\rm V}_{\phi\phi'}\bar{d}\gamma^\eta s + c^{\rm A}_{\phi\phi'}\bar{d}\gamma^\eta\gamma_5 s)i(\phi^\dagger\partial_\eta\phi' - \partial_\eta\phi^\dagger\phi') \\ &+ (c^{\rm S}_{\phi\phi'}\bar{d}s + c^{\rm P}_{\phi\phi'}\bar{d}\gamma_5 s)\phi^\dagger\phi'] + \text{H.c.}\end{aligned} \tag{2}$$

for the two scenarios, respectively, where $\sigma^{\eta\kappa} = i[\gamma^\eta, \gamma^\kappa]/2$ and the $\mathtt{C}$s, $\tilde{c}$s, and cs are in general complex coefficients which have the dimension of inverse squared mass, except for $c^{\rm S,P}_{\phi\phi'}$ which are of inverse-mass dimension. These are free parameters in our model-independent approach and will be treated phenomenologically in our numerical work later on. In Eq. (1) there are merely two tensor operators due to the identity $2i\sigma^{\alpha\omega}\gamma_5 = \epsilon^{\alpha\omega\beta\psi}\sigma_{\beta\psi}$. If $\mathtt{f}' \neq \mathtt{f}$ ($\phi' \neq \phi$), we implicitly also have another Lagrangian, $\mathcal{L}_{\mathtt{f}'\mathtt{f}}$ ($\mathcal{L}_{\phi'\phi}$), which is the same as $\mathcal{L}_{\mathtt{ff}'}$ ($\mathcal{L}_{\phi\phi'}$) but with $\mathtt{f}$ and $\mathtt{f}'$ (ϕ and ϕ') interchanged. We note that $\mathcal{L}_{\mathtt{ff}'}$ and $\mathcal{L}_{\phi\phi'}$ could originate from Lagrangians that are invariant under all the SM gauge groups [7,16].

III. DECAY AMPLITUDES AND RATES

To examine the amplitudes for the kaon decays of concern, we need the mesonic matrix elements of the quark portions of the operators in Eqs. (1) and (2). They can be estimated with the aid of flavor-SU(3) chiral perturbation theory at leading order [6,16,22]. For $K_{L,S} \to \not{E}$, the relevant hadronic matrix elements are

$$\begin{aligned}\langle 0|\bar{d}\gamma^\alpha\gamma_5 s|\bar{K}^0\rangle &= \langle 0|\bar{s}\gamma^\alpha\gamma_5 d|K^0\rangle = -if_K p_K^\alpha, \\ \langle 0|\bar{d}\gamma_5 s|\bar{K}^0\rangle &= \langle 0|\bar{s}\gamma_5 d|K^0\rangle = iB_0 f_K,\end{aligned} \tag{3}$$

with $f_K \simeq 156$ MeV [17] being the kaon decay constant and $B_0 = m_K^2/(\bar{m} + m_s) \simeq 2.0$ GeV involving the average kaon mass and the combination $\bar{m} + m_s \simeq 124$ MeV of light-quark masses at a renormalization scale of 1 GeV, while for $K \to \pi\not{E}$,

$$\begin{aligned}&\langle\pi^-|\bar{d}\gamma^\alpha s|K^-\rangle = p_K^\alpha + p_\pi^\alpha, \qquad \langle\pi^-|\bar{d}s|K^-\rangle = B_0, \\ &\langle\pi^-|\bar{d}\sigma^{\alpha\kappa}s|K^-\rangle = 2ia_T(p_\pi^\alpha p_K^\kappa - p_\pi^\kappa p_K^\alpha),\end{aligned} \tag{4}$$

where p_K and p_π denote the kaon and pion momenta, respectively, and a_T is a constant having the dimension of inverse mass. Assuming isospin symmetry and making use of charge conjugation, we further have $\langle\pi^0|\bar{d}(\gamma^\eta, 1, \sigma^{\eta\kappa})s|\bar{K}^0\rangle = \langle\pi^0|\bar{s}(-\gamma^\eta, 1, -\sigma^{\eta\kappa})d|K^0\rangle = -\langle\pi^-|\bar{d}(\gamma^\eta, 1, \sigma^{\eta\kappa})s|K^-\rangle/\sqrt{2}$. For $K \to \pi\pi\not{E}$, we find [6,16,20]

[1]In the recent literature covering the impact of NP on $K \to \pi\pi\not{E}$, there are other possibilities for what carries away the missing energy. In particular, it could alternatively be due to a single particle such as a massless dark photon [18–20] or an invisible axion [21].

$$\begin{aligned}
\langle\pi^0(p_0)\pi^-(p_-)|\bar d(\gamma^\eta,1)\gamma_5 s|K^-\rangle &= \frac{i\sqrt{2}}{f_K}\left[(p_0^\eta-p_-^\eta,0)+\frac{(p_0^\alpha-p_-^\alpha)\tilde q_\alpha}{m_K^2-\tilde q^2}(\tilde q^\eta,-B_0)\right],\\
\langle\pi^+(p_+)\pi^-(p_-)|\bar d(\gamma^\eta,1)\gamma_5 s|\bar K^0\rangle &= \frac{2i}{f_K}\left[(p_+^\eta,0)+\frac{p_+^\alpha\tilde q_\alpha}{m_K^2-\tilde q^2}(\tilde q^\eta,-B_0)\right],\\
\langle\pi^+(p_+)\pi^-(p_-)|\bar s(\gamma^\eta,1)\gamma_5 d|K^0\rangle &= \frac{2i}{f_K}\left[(p_-^\eta,0)+\frac{p_-^\alpha\tilde q_\alpha}{m_K^2-\tilde q^2}(\tilde q^\eta,-B_0)\right],\\
\langle\pi^0(p_1)\pi^0(p_2)|\bar d(\gamma^\eta,1)\gamma_5 s|\bar K^0\rangle &= \langle\pi^0(p_1)\pi^0(p_2)|\bar s(\gamma^\eta,1)\gamma_5 d|K^0\rangle\\
&= \frac{i}{f_K}\left[(p_1^\eta+p_2^\eta,0)+\frac{(p_1^\alpha+p_2^\alpha)\tilde q_\alpha}{m_K^2-\tilde q^2}(\tilde q^\eta,-B_0)\right],
\end{aligned}\tag{5}$$

$$\begin{aligned}
\langle\pi^0(p_0)\pi^-(p_-)|\bar d\sigma_{\eta\kappa}s|K^-\rangle &= \frac{i\sqrt{2}a_T}{f_K}\epsilon_{\eta\kappa\mu\tau}[4p_-^\mu p_0^\tau+(p_-^\mu-p_0^\mu)\tilde q^\tau],\\
\langle\pi^+(p_+)\pi^-(p_-)|\bar d\sigma_{\eta\kappa}s|\bar K^0\rangle &= \frac{2ia_T}{f_K}\epsilon_{\eta\kappa\mu\tau}(2p_-^\mu+\tilde q^\mu)p_+^\tau,\\
\langle\pi^+(p_+)\pi^-(p_-)|\bar s\sigma_{\eta\kappa}d|K^0\rangle &= \frac{2ia_T}{f_K}\epsilon_{\eta\kappa\mu\tau}p_-^\mu(2p_+^\tau+\tilde q^\tau),\\
\langle\pi^0(p_1)\pi^0(p_2)|\bar d\sigma_{\eta\kappa}s|\bar K^0\rangle &= -\langle\pi^0(p_1)\pi^0(p_2)|\bar s\sigma_{\eta\kappa}d|K^0\rangle = \frac{ia_T}{f_K}\epsilon_{\eta\kappa\mu\tau}\tilde q^\mu(p_1^\tau+p_2^\tau),
\end{aligned}\tag{6}$$

where $\tilde q = p_K-p_0-p_- = p_K-p_--p_+ = p_K-p_1-p_2$. Although generally the matrix elements in Eqs. (4)–(6) involve momentum-dependent form factors, to investigate the NP influence on these processes in this study we do not need a high degree of precision and therefore can disregard form-factor effects. We also ignore $\langle\pi^{0,-}\pi^+|\bar s\gamma^\eta d|K^{+,0}\rangle$, and their charge conjugates, as they arise from small contributions derived from the anomaly Lagrangian, which occurs at next-to-leading order in the chiral expansion [3,16].

We now apply these matrix elements to kaon decays induced by $\mathcal{L}_{\mathtt{ff}'}$ in Eq. (1) and take the $\mathtt{f}$ and $\mathtt{f}'$ masses to be negligible, *i.e.,* $m_{\mathtt{f},\mathtt{f}'}\simeq 0$. Thus, for $K_{L,S}\to \mathtt{f}\bar{\mathtt{f}}'$, with the approximate relations $\sqrt{2}K_{L,S}=K^0\pm\bar K^0$, we obtain the amplitudes to be

$$\mathcal{M}_{K_{L,S}\to\mathtt{f}\bar{\mathtt{f}}'} = \frac{i}{\sqrt{2}}B_0 f_K \bar u_{\mathtt{f}}(\tilde{\mathtt{S}}_{K_{L,S}\mathtt{ff}'}+\gamma_5\tilde{\mathtt{P}}_{K_{L,S}\mathtt{ff}'})v_{\bar{\mathtt{f}}'},\tag{7}$$

from which follow the decay rates

$$\Gamma_{K_{L,S}\to\mathtt{f}\bar{\mathtt{f}}'} = \frac{B_0^2 f_K^2 m_{K^0}}{16\pi}(|\tilde{\mathtt{S}}_{K_{L,S}\mathtt{ff}'}|^2+|\tilde{\mathtt{P}}_{K_{L,S}\mathtt{ff}'}|^2),\tag{8}$$

where

$$\begin{aligned}
\tilde{\mathtt{S}}_{K_L\mathtt{ff}'} &= \tilde c^{\mathrm S}_{\mathtt{ff}'}-\tilde c^{\mathrm S*}_{\mathtt{f}'\mathtt{f}}, \qquad \tilde{\mathtt{P}}_{K_L\mathtt{ff}'} = \tilde c^{\mathrm P}_{\mathtt{ff}'}+\tilde c^{\mathrm P*}_{\mathtt{f}'\mathtt{f}},\\
\tilde{\mathtt{S}}_{K_S\mathtt{ff}'} &= -\tilde c^{\mathrm S}_{\mathtt{ff}'}-\tilde c^{\mathrm S*}_{\mathtt{f}'\mathtt{f}}, \qquad \tilde{\mathtt{P}}_{K_S\mathtt{ff}'} = \tilde c^{\mathrm P*}_{\mathtt{f}'\mathtt{f}}-\tilde c^{\mathrm P}_{\mathtt{ff}'}.
\end{aligned}\tag{9}$$

We can see that $K_{L,S}\to\mathtt{f}\bar{\mathtt{f}}'$ are insensitive to $\mathtt{C}^{\mathrm{V,A,S,P,T,T'}}$ and $\tilde c^{\mathrm V}$ as well as $\tilde c^{\mathrm A}$ if $m_{\mathtt{f},\mathtt{f}'}=0$.

For $K\to\pi\mathtt{f}\bar{\mathtt{f}}'$, we express the amplitude as $\mathcal{M}_{K\to\pi\mathtt{f}\bar{\mathtt{f}}'}=\bar u_{\mathtt{f}}(S_{K\pi\mathtt{ff}'}+P_{K\pi\mathtt{ff}'}\gamma_5)v_{\bar{\mathtt{f}}'}$. The S and P terms for $K^-\to\pi^-\mathtt{f}\bar{\mathtt{f}}'$ and $K_L\to\pi^0\mathtt{f}\bar{\mathtt{f}}'$ are

$$\begin{aligned}
S_{K^-\pi^-\mathtt{ff}'} &= 2\not{p}_K\mathtt{C}^{\mathrm V}_{\mathtt{ff}'}+B_0\mathtt{C}^{\mathrm S}_{\mathtt{ff}'}+4a_T p_K\cdot(p_{\bar{\mathtt{f}}'}-p_{\mathtt{f}})\mathtt{C}^{\mathrm T}_{\mathtt{ff}'},\\
P_{K^-\pi^-\mathtt{ff}'} &= 2\not{p}_K\mathtt{C}^{\mathrm A}_{\mathtt{ff}'}+B_0\mathtt{C}^{\mathrm P}_{\mathtt{ff}'}+4a_T p_K\cdot(p_{\bar{\mathtt{f}}'}-p_{\mathtt{f}})\mathtt{C}^{\mathrm T'}_{\mathtt{ff}'},\\
S_{K_L\pi^0\mathtt{ff}'} &= (\mathtt{C}^{\mathrm V*}_{\mathtt{f}'\mathtt{f}}-\mathtt{C}^{\mathrm V}_{\mathtt{ff}'})\not{p}_K-\frac{1}{2}B_0(\mathtt{C}^{\mathrm S*}_{\mathtt{f}'\mathtt{f}}+\mathtt{C}^{\mathrm S}_{\mathtt{ff}'})\\
&\quad +2a_T p_K\cdot(p_{\bar{\mathtt{f}}'}-p_{\mathtt{f}})(\mathtt{C}^{\mathrm T*}_{\mathtt{f}'\mathtt{f}}-\mathtt{C}^{\mathrm T}_{\mathtt{ff}'}),\\
P_{K_L\pi^0\mathtt{ff}'} &= (\mathtt{C}^{\mathrm A*}_{\mathtt{f}'\mathtt{f}}-\mathtt{C}^{\mathrm A}_{\mathtt{ff}'})\not{p}_K+\frac{1}{2}B_0(\mathtt{C}^{\mathrm P*}_{\mathtt{f}'\mathtt{f}}-\mathtt{C}^{\mathrm P}_{\mathtt{ff}'})\\
&\quad -2a_T p_K\cdot(p_{\bar{\mathtt{f}}'}-p_{\mathtt{f}})(\mathtt{C}^{\mathrm T'*}_{\mathtt{f}'\mathtt{f}}+\mathtt{C}^{\mathrm T'}_{\mathtt{ff}'}).
\end{aligned}\tag{10}$$

These lead to the differential rates

$$\frac{d\Gamma_{K^-\to\pi^-\mathtt{f}\bar{\mathtt{f}}'}}{d\hat{s}} = \frac{\lambda_{K^-\pi^-}^{3/2}}{192\pi^3 m_{K^-}^3}\left[|\mathrm{C}^{\mathrm{V}}_{\mathtt{ff}'}|^2 + |\mathrm{C}^{\mathrm{A}}_{\mathtt{ff}'}|^2 + 3B_0^2\hat{s}\,\frac{|\mathrm{C}^{\mathrm{S}}_{\mathtt{ff}'}|^2 + |\mathrm{C}^{\mathrm{P}}_{\mathtt{ff}'}|^2}{2\lambda_{K^-\pi^-}} + 2a_T^2\big(|\mathrm{C}^{\mathrm{T}}_{\mathtt{ff}'}|^2 + |\mathrm{C}^{\mathrm{T}'}_{\mathtt{ff}'}|^2\big)\hat{s}\right],$$

$$\frac{d\Gamma_{K_L\to\pi^0\mathtt{f}\bar{\mathtt{f}}'}}{d\hat{s}} = \frac{\lambda_{K^0\pi^0}^{3/2}}{768\pi^3 m_{K^0}^3}\left[|\mathrm{C}^{\mathrm{V}}_{\mathtt{ff}'} - \mathrm{C}^{\mathrm{V}*}_{\mathtt{f}'\mathtt{f}}|^2 + |\mathrm{C}^{\mathrm{A}}_{\mathtt{ff}'} - \mathrm{C}^{\mathrm{A}*}_{\mathtt{f}'\mathtt{f}}|^2 + 3B_0^2\hat{s}\,\frac{|\mathrm{C}^{\mathrm{S}}_{\mathtt{ff}'} + \mathrm{C}^{\mathrm{S}*}_{\mathtt{f}'\mathtt{f}}|^2 + |\mathrm{C}^{\mathrm{P}}_{\mathtt{ff}'} - \mathrm{C}^{\mathrm{P}*}_{\mathtt{f}'\mathtt{f}}|^2}{2\lambda_{K^0\pi^0}}\right.$$
$$\left. + 2a_T^2\big(|\mathrm{C}^{\mathrm{T}}_{\mathtt{ff}'} - \mathrm{C}^{\mathrm{T}*}_{\mathtt{f}'\mathtt{f}}|^2 + |\mathrm{C}^{\mathrm{T}'}_{\mathtt{ff}'} + \mathrm{C}^{\mathrm{T}'*}_{\mathtt{f}'\mathtt{f}}|^2\big)\hat{s}\right], \tag{11}$$

where $\hat{s}$ represents the invariant mass squared of the $\mathtt{f}\bar{\mathtt{f}}'$ pair,

$$\lambda_{AB} = \mathcal{K}(m_A^2, m_B^2, \hat{s}), \quad \mathcal{K}(x,y,z) = (x-y-z)^2 - 4yz. \tag{12}$$

Evidently, $K\to\pi\mathtt{f}\bar{\mathtt{f}}'$, unlike $K\to\mathtt{f}\bar{\mathtt{f}}'$, can probe $\mathrm{C}^{\mathrm{V,A,S,P,T,T'}}$, but not $\tilde{c}^{\mathrm{V,A,S,P}}$.

For $K^-\to\pi^0\pi^-\mathtt{f}\bar{\mathtt{f}}'$ and $K_L\to(\pi^+\pi^-,\pi^0\pi^0)\mathtt{f}\bar{\mathtt{f}}'$, we find

$$\mathcal{M}_{K^-\to\pi^0\pi^-\mathtt{f}\bar{\mathtt{f}}'} = \frac{i\sqrt{2}}{f_K}\bar{u}_{\mathtt{f}}\Big\{(\not{p}_0 - \not{p}_-)(\tilde{c}^{\mathrm{V}}_{\mathtt{ff}'} + \gamma_5\tilde{c}^{\mathrm{A}}_{\mathtt{ff}'}) + \frac{B_0}{\tilde{\mathrm{K}}^2}(\tilde{c}^{\mathrm{S}}_{\mathtt{ff}'} + \gamma_5\tilde{c}^{\mathrm{P}}_{\mathtt{ff}'})(p_-^\alpha - p_0^\alpha)\hat{q}_\alpha$$
$$+ 2ia_T[4p_-^\alpha p_0^\tau + (p_-^\alpha - p_0^\alpha)\hat{q}^\tau]\sigma_{\alpha\tau}(\gamma_5\mathrm{C}^{\mathrm{T}}_{\mathtt{ff}'} + \mathrm{C}^{\mathrm{T}'}_{\mathtt{ff}'})\Big\}v_{\bar{\mathtt{f}}'},$$

$$\mathcal{M}_{K_L\to\pi^+\pi^-\mathtt{f}\bar{\mathtt{f}}'} = \frac{i\sqrt{2}}{f_K}\bar{u}_{\mathtt{f}}\Big[\not{p}_+(\tilde{c}^{\mathrm{V}}_{\mathtt{ff}'} + \gamma_5\tilde{c}^{\mathrm{A}}_{\mathtt{ff}'}) - \frac{B_0}{\tilde{\mathrm{K}}^2}(\tilde{c}^{\mathrm{S}}_{\mathtt{ff}'} + \gamma_5\tilde{c}^{\mathrm{P}}_{\mathtt{ff}'})p_+^\alpha\hat{q}_\alpha + 2ia_T(2p_-^\alpha + \hat{q}^\alpha)p_+^\tau\sigma_{\alpha\tau}(\gamma_5\mathrm{C}^{\mathrm{T}}_{\mathtt{ff}'} + \mathrm{C}^{\mathrm{T}'}_{\mathtt{ff}'})$$
$$+ \not{p}_-(\tilde{c}^{\mathrm{V}*}_{\mathtt{f}'\mathtt{f}} + \gamma_5\tilde{c}^{\mathrm{A}*}_{\mathtt{f}'\mathtt{f}}) + \frac{B_0}{\tilde{\mathrm{K}}^2}(\tilde{c}^{\mathrm{S}*}_{\mathtt{f}'\mathtt{f}} - \gamma_5\tilde{c}^{\mathrm{P}*}_{\mathtt{f}'\mathtt{f}})p_-^\alpha\hat{q}_\alpha + 2ia_T p_-^\alpha(2p_+^\tau + \hat{q}^\tau)\sigma_{\alpha\tau}(\gamma_5\mathrm{C}^{\mathrm{T}*}_{\mathtt{f}'\mathtt{f}} - \mathrm{C}^{\mathrm{T}'*}_{\mathtt{f}'\mathtt{f}})\Big]v_{\bar{\mathtt{f}}'},$$

$$\mathcal{M}_{K_L\to\pi^0\pi^0\mathtt{f}\bar{\mathtt{f}}'} = \frac{i}{\sqrt{2}f_K}\bar{u}_{\mathtt{f}}\Big\{\not{p}_K[\tilde{c}^{\mathrm{V}}_{\mathtt{ff}'} + \tilde{c}^{\mathrm{V}*}_{\mathtt{f}'\mathtt{f}} + \gamma_5(\tilde{c}^{\mathrm{A}}_{\mathtt{ff}'} + \tilde{c}^{\mathrm{A}*}_{\mathtt{f}'\mathtt{f}})] + \frac{B_0}{\tilde{\mathrm{K}}^2}[\tilde{c}^{\mathrm{S}}_{\mathtt{ff}'} - \tilde{c}^{\mathrm{S}*}_{\mathtt{f}'\mathtt{f}} + \gamma_5(\tilde{c}^{\mathrm{P}}_{\mathtt{ff}'} + \tilde{c}^{\mathrm{P}*}_{\mathtt{f}'\mathtt{f}})](\hat{s} - p_K^\alpha\hat{q}_\alpha)$$
$$+ 2ia_T\hat{q}^\alpha p_K^\tau\sigma_{\alpha\tau}[\gamma_5(\mathrm{C}^{\mathrm{T}}_{\mathtt{ff}'} - \mathrm{C}^{\mathrm{T}*}_{\mathtt{f}'\mathtt{f}}) + \mathrm{C}^{\mathrm{T}'}_{\mathtt{ff}'} + \mathrm{C}^{\mathrm{T}'*}_{\mathtt{f}'\mathtt{f}}]\Big\}v_{\bar{\mathtt{f}}}, \tag{13}$$

where

$$\hat{q} = p_{\mathtt{f}} + p_{\bar{\mathtt{f}}'}, \qquad \hat{s} = \hat{q}^2, \qquad \tilde{\mathrm{K}}^2 = m_K^2 - \hat{s}, \tag{14}$$

with m_K in $\tilde{\mathrm{K}}$ being the average kaon mass. We then arrive at the double differential rates

$$\frac{d^2\Gamma_{K^-\to\pi^0\pi^-\mathtt{f}\bar{\mathtt{f}}'}}{d\hat{s}\,d\hat{\varsigma}} = \frac{\beta_{\hat{\varsigma}}^3\tilde{\lambda}_{K^-}^{3/2}}{(4\pi)^5 f_K^2}\left\{\left(1 + \frac{12\hat{s}\,\hat{\varsigma}}{\tilde{\lambda}_{K^-}}\right)\frac{|\tilde{c}^{\mathrm{V}}_{\mathtt{ff}'}|^2 + |\tilde{c}^{\mathrm{A}}_{\mathtt{ff}'}|^2}{18m_{K^-}^3} + B_0^2\hat{s}\,\frac{|\tilde{c}^{\mathrm{S}}_{\mathtt{ff}'}|^2 + |\tilde{c}^{\mathrm{P}}_{\mathtt{ff}'}|^2}{12\tilde{\mathrm{K}}^4 m_{K^-}^3}\right.$$
$$\left. + a_T^2\left[\hat{s} + 4\hat{\varsigma} + \frac{12\hat{\varsigma}}{\tilde{\lambda}_{K^-}}(m_{K^-}^2 - \hat{\varsigma})^2\right]\frac{|\mathrm{C}^{\mathrm{T}}_{\mathtt{ff}'}|^2 + |\mathrm{C}^{\mathrm{T}'}_{\mathtt{ff}'}|^2}{9m_{K^-}^3}\right\}, \tag{15a}$$

$$\frac{d^2\Gamma_{K_L\to\pi^+\pi^-\mathtt{f}\bar{\mathtt{f}}'}}{d\hat{s}\,d\hat{\varsigma}} = \frac{\beta_{\hat{\varsigma}}^3\tilde{\lambda}_{K^0}^{3/2}}{4(4\pi)^5 f_K^2}\left\{\left(1 + \frac{12\hat{s}\,\hat{\varsigma}}{\tilde{\lambda}_{K^0}}\right)\frac{|\tilde{c}^{\mathrm{V}}_{\mathtt{ff}'} - \tilde{c}^{\mathrm{V}*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\mathrm{A}}_{\mathtt{ff}'} - \tilde{c}^{\mathrm{A}*}_{\mathtt{f}'\mathtt{f}}|^2}{18m_{K^0}^3}\right.$$
$$+ B_0^2\hat{s}\,\frac{|\tilde{c}^{\mathrm{S}}_{\mathtt{ff}'} + \tilde{c}^{\mathrm{S}*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\mathrm{P}}_{\mathtt{ff}'} - \tilde{c}^{\mathrm{P}*}_{\mathtt{f}'\mathtt{f}}|^2}{12\tilde{\mathrm{K}}^4 m_{K^0}^3} + \frac{|\tilde{c}^{\mathrm{V}}_{\mathtt{ff}'} + \tilde{c}^{\mathrm{V}*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\mathrm{A}}_{\mathtt{ff}'} + \tilde{c}^{\mathrm{A}*}_{\mathtt{f}'\mathtt{f}}|^2}{6\beta_{\hat{\varsigma}}^2 m_{K^0}^3}$$
$$+ \left(1 + \frac{4\hat{s}\,\hat{\varsigma}}{\tilde{\lambda}_{K^0}}\right)B_0^2\hat{s}\,\frac{|\tilde{c}^{\mathrm{S}}_{\mathtt{ff}'} - \tilde{c}^{\mathrm{S}*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\mathrm{P}}_{\mathtt{ff}'} + \tilde{c}^{\mathrm{P}*}_{\mathtt{f}'\mathtt{f}}|^2}{4\beta_{\hat{\varsigma}}^2\tilde{\mathrm{K}}^4 m_{K^0}^3} + a_T^2\left[\hat{s} + 4\hat{\varsigma} + \frac{12\hat{\varsigma}}{\tilde{\lambda}_{K^0}}(m_{K^0}^2 - \hat{\varsigma})^2\right]$$
$$\left.\times\frac{|\mathrm{C}^{\mathrm{T}}_{\mathtt{ff}'} + \mathrm{C}^{\mathrm{T}*}_{\mathtt{f}'\mathtt{f}}|^2 + |\mathrm{C}^{\mathrm{T}'}_{\mathtt{ff}'} - \mathrm{C}^{\mathrm{T}'*}_{\mathtt{f}'\mathtt{f}}|^2}{9m_{K^0}^3} + a_T^2\hat{s}\,\frac{|\mathrm{C}^{\mathrm{T}}_{\mathtt{ff}'} - \mathrm{C}^{\mathrm{T}'*}_{\mathtt{f}'\mathtt{f}}|^2 + |\mathrm{C}^{\mathrm{T}}_{\mathtt{ff}'} + \mathrm{C}^{\mathrm{T}'*}_{\mathtt{f}'\mathtt{f}}|^2}{3\beta_{\hat{\varsigma}}^2 m_{K^0}^3}\right\}, \tag{15b}$$

$$\frac{d^2\Gamma_{K_L\to\pi^0\pi^0\mathtt{f}\bar{\mathtt{f}}'}}{d\hat{s}\,d\hat{\varsigma}} = \frac{\beta_{\hat\varsigma}\tilde\lambda_{K^0}^{3/2}}{8(4\pi)^5 f_K^2}\left[\frac{|\tilde c^{\mathrm{V}}_{\mathtt{ff}'}+\tilde c^{\mathrm{V}*}_{\mathtt{f}'\mathtt{f}}|^2+|\tilde c^{\mathrm{A}}_{\mathtt{ff}'}+\tilde c^{\mathrm{A}*}_{\mathtt{f}'\mathtt{f}}|^2}{6m_{K^0}^3} + B_0^2\hat s\left(1+\frac{4\hat s\,\hat\varsigma}{\tilde\lambda_{K^0}}\right)\frac{|\tilde c^{\mathrm{S}}_{\mathtt{ff}'}-\tilde c^{\mathrm{S}*}_{\mathtt{f}'\mathtt{f}}|^2+|\tilde c^{\mathrm{P}}_{\mathtt{ff}'}+\tilde c^{\mathrm{P}*}_{\mathtt{f}'\mathtt{f}}|^2}{4\tilde{\mathrm{K}}^4 m_{K^0}^3} \right.$$
$$\left. + a_T^2\hat s\,\frac{|\mathrm{C}^{\mathrm{T}}_{\mathtt{ff}'}-\mathrm{C}^{\mathrm{T}*}_{\mathtt{f}'\mathtt{f}}|^2+|\mathrm{C}^{\mathrm{T}'}_{\mathtt{ff}'}+\mathrm{C}^{\mathrm{T}'*}_{\mathtt{f}'\mathtt{f}}|^2}{3m_{K^0}^3}\right], \tag{15c}$$

where $\hat\varsigma$ is the invariant mass squared of the pion pair,

$$\beta_{\hat\varsigma}=\sqrt{1-\frac{4m_\pi^2}{\hat\varsigma}},\quad \tilde\lambda_{\mathcal{P}}=\mathcal{K}(m_{\mathcal{P}}^2,\hat s,\hat\varsigma),\quad \mathcal{P}=K^-,K^0. \tag{16}$$

The $\hat s$ and $\hat\varsigma$ integration ranges for calculating the K^- and K_L partial rates from Eq. (15) are $0\le\hat s\le(m_{K^-,K^0}-2m_\pi)^2$ and $4m_\pi^2\le\hat\varsigma\le(m_{K^-,K^0}-\hat s^{1/2})^2$, respectively. For the mode with the $\pi^0\pi^-$ ($\pi^+\pi^-$ or $\pi^0\pi^0$) pair, m_π refers to the isospin-average (charged or neutral) pion mass. The expressions for the $K_S\to\pi\pi\mathtt{f}\bar{\mathtt{f}}'$ rates equal their K_L counterparts computed from Eq. (15) but with the signs of $\tilde c^{\mathrm{V,A,S,P}}_{\mathtt{ff}'}$ and $\mathrm{C}^{\mathrm{T,T}'}_{\mathtt{ff}'}$ flipped. Clearly, $K\to\pi\pi\mathtt{f}\bar{\mathtt{f}}'$, as opposed to $K\to\pi\mathtt{f}\bar{\mathtt{f}}'$, are sensitive to $\tilde c^{\mathrm{V,A,S,P}}$, besides $\mathrm{C}^{\mathrm{T,T}'}$, but not to $\mathrm{C}^{\mathrm{V,A,S,P}}$ in our approximation of the hadronic matrix elements. We remark that the $\mathtt{f}'=\mathtt{f}$ possibility has previously been considered in Refs. [6,16] and our formulas above applied to that case agree with those given therein in the $m_{\mathtt{f}}=0$ limit.

If $\mathtt{f}'\neq\mathtt{f}$, the extra channels $K\to(\pi^0,\pi^+\pi^-,\pi^0\pi^0)\mathtt{f}'\bar{\mathtt{f}}$ also occur, whose rates are obtainable from those of $K\to(\pi^0,\pi^+\pi^-,\pi^0\pi^0)\mathtt{f}\bar{\mathtt{f}}'$, respectively, by interchanging the labels $\mathtt{f}$ and $\mathtt{f}'$ of the coefficients. For $\mathtt{f}'\neq\mathtt{f}$, if $\tilde c^{\mathrm{V,A,S,P}}_{\mathtt{f}'\mathtt{f}}$ and $\mathrm{C}^{\mathrm{T,T}'}_{\mathtt{f}'\mathtt{f}}$ are not zero, they are generally independent from $\tilde c^{\mathrm{V,A,S,P}}_{\mathtt{ff}'}$ and $\mathrm{C}^{\mathrm{T,T}'}_{\mathtt{ff}'}$ and bring about $K^-\to\pi^0\pi^-\mathtt{f}'\bar{\mathtt{f}}$ as well.

Turning to the processes induced by $\mathcal{L}_{\phi\phi'}$ in Eq. (2), we again assume that the masses of the invisible particles, ϕ and ϕ', can be neglected, $m_{\phi,\phi'}\simeq 0$. It follows that the amplitudes for $K_{L,S}\to\phi\bar\phi'$ are

$$\mathcal{M}_{K_L\to\phi\bar\phi'}=\frac{i}{\sqrt2}(c^{\mathrm{P}}_{\phi\phi'}-c^{\mathrm{P}*}_{\phi'\phi})B_0 f_K,$$
$$\mathcal{M}_{K_S\to\phi\bar\phi'}=\frac{-i}{\sqrt2}(c^{\mathrm{P}}_{\phi\phi'}+c^{\mathrm{P}*}_{\phi'\phi})B_0 f_K, \tag{17}$$

which lead to the decay rates

$$\Gamma_{K_{L,S}\to\phi\bar\phi'}=\frac{B_0^2 f_K^2}{32\pi m_{K^0}}|c^{\mathrm{P}}_{\phi\phi'}\mp c^{\mathrm{P}*}_{\phi'\phi}|^2. \tag{18}$$

As for the two-body modes, we find

$$\frac{d\Gamma_{K^-\to\pi^-\phi\bar\phi'}}{d\hat s}=\frac{\lambda^{1/2}_{K^-\pi^-}}{768\pi^3 m_{K^-}^3}(\lambda_{K^-\pi^-}|c^{\mathrm{V}}_{\phi\phi'}|^2+3B_0^2|c^{\mathrm{S}}_{\phi\phi'}|^2),$$
$$\frac{d\Gamma_{K_L\to\pi^0\phi\bar\phi'}}{d\hat s}=\frac{\lambda^{1/2}_{K^0\pi^0}}{3072\pi^3 m_{K^0}^3}(\lambda_{K^0\pi^0}|c^{\mathrm{V}}_{\phi\phi'}-c^{\mathrm{V}*}_{\phi'\phi}|^2 + 3B_0^2|c^{\mathrm{S}}_{\phi\phi'}+c^{\mathrm{S}*}_{\phi'\phi}|^2), \tag{19}$$

where $\hat s=\hat{\mathtt{q}}^2$ with $\hat{\mathtt{q}}=\mathtt{p}+\bar{\mathtt{p}}$ being the sum of the momenta $\mathtt{p}$ and $\bar{\mathtt{p}}$ of ϕ and $\bar\phi'$, respectively. For $K\to\pi\pi\phi\bar\phi'$, we derive

$$\mathcal{M}_{K^-\to\pi^0\pi^-\phi\bar\phi'}=\frac{i\sqrt2(p_0^\tau-p_-^\tau)}{f_K}\left[c^{\mathrm{A}}_{\phi\phi'}(\mathtt{p}-\bar{\mathtt{p}})_\tau-\frac{B_0 c^{\mathrm{P}}_{\phi\phi'}\hat{\mathtt{q}}_\tau}{\tilde{\mathrm{K}}^2}\right],$$
$$\mathcal{M}_{K_L\to\pi^+\pi^-\phi\bar\phi'}=\frac{i\sqrt2}{f_K}\left[(c^{\mathrm{A}*}_{\phi'\phi}p_-^\tau+c^{\mathrm{A}}_{\phi\phi'}p_+^\tau)(\mathtt{p}-\bar{\mathtt{p}})_\tau + \frac{B_0}{\tilde{\mathrm{K}}^2}(c^{\mathrm{P}*}_{\phi'\phi}p_-^\tau-c^{\mathrm{P}}_{\phi\phi'}p_+^\tau)\hat{\mathtt{q}}_\tau\right],$$
$$\mathcal{M}_{K_L\to\pi^0\pi^0\phi\bar\phi'}=\frac{i(p_1^\tau+p_2^\tau)}{\sqrt2 f_K}\left[(c^{\mathrm{A}*}_{\phi'\phi}+c^{\mathrm{A}}_{\phi\phi'})(\mathtt{p}-\bar{\mathtt{p}})_\tau + \frac{B_0}{\tilde{\mathrm{K}}^2}(c^{\mathrm{P}*}_{\phi'\phi}-c^{\mathrm{P}}_{\phi\phi'})\hat{\mathtt{q}}_\tau\right], \tag{20}$$

from which we arrive at

$$\frac{d^2\Gamma_{K^-\to\pi^0\pi^-\phi\bar\phi'}}{d\hat s\,d\hat\varsigma}=\frac{4\beta^3_{\hat\varsigma}\tilde\lambda^{1/2}_{K^-}}{3(8\pi)^5 f_K^2}\left[\frac{\tilde\lambda_{K^-}+12\hat s\,\hat\varsigma}{3m_{K^-}^3}|c^{\mathrm{A}}_{\phi\phi'}|^2+\frac{\tilde\lambda_{K^-}B_0^2}{\tilde{\mathrm{K}}^4 m_{K^-}^3}|c^{\mathrm{P}}_{\phi\phi'}|^2\right],$$
$$\frac{d^2\Gamma_{K_L\to\pi^+\pi^-\phi\bar\phi'}}{d\hat s\,d\hat\varsigma}=\frac{\beta_{\hat\varsigma}\tilde\lambda^{1/2}_{K^0}}{3(8\pi)^5 f_K^2}\left[\beta^2_{\hat\varsigma}\frac{\tilde\lambda_{K^0}+12\hat s\,\hat\varsigma}{3m_{K^0}^3}|c^{\mathrm{A}*}_{\phi'\phi}-c^{\mathrm{A}}_{\phi\phi'}|^2+\frac{\beta^2_{\hat\varsigma}\tilde\lambda_{K^0}B_0^2}{\tilde{\mathrm{K}}^4 m_{K^0}^3}|c^{\mathrm{P}*}_{\phi'\phi}+c^{\mathrm{P}}_{\phi\phi'}|^2 + \frac{\tilde\lambda_{K^0}}{m_{K^0}^3}|c^{\mathrm{A}*}_{\phi'\phi}+c^{\mathrm{A}}_{\phi\phi'}|^2+\frac{3(\tilde\lambda_{K^0}+4\hat s\,\hat\varsigma)B_0^2}{\tilde{\mathrm{K}}^4 m_{K^0}^3}|c^{\mathrm{P}*}_{\phi'\phi}-c^{\mathrm{P}}_{\phi\phi'}|^2\right],$$
$$\frac{d^2\Gamma_{K_L\to\pi^0\pi^0\phi\bar\phi'}}{d\hat s\,d\hat\varsigma}=\frac{\beta_{\hat\varsigma}\tilde\lambda^{1/2}_{K^0}}{6(8\pi)^5 f_K^2}\left[\frac{\tilde\lambda_{K^0}}{m_{K^0}^3}|c^{\mathrm{A}*}_{\phi'\phi}+c^{\mathrm{A}}_{\phi\phi'}|^2+\frac{3(\tilde\lambda_{K^0}+4\hat s\,\hat\varsigma)B_0^2}{\tilde{\mathrm{K}}^4 m_{K^0}^3}|c^{\mathrm{P}*}_{\phi'\phi}-c^{\mathrm{P}}_{\phi\phi'}|^2\right]. \tag{21}$$

TABLE I. Summary of coefficients in Eqs. (1) and (2) contributing to the various FCNC kaon decays with missing energy carried away by Dirac spin-1/2 fermions, $\mathtt{f}\bar{\mathtt{f}}'$, or by spin-0 bosons, $\phi\bar{\phi}'$, if their masses are negligible, $m_{\mathtt{f},\mathtt{f}',\phi,\phi'} \simeq 0$.

Decay modes	$K \to \mathtt{f}\bar{\mathtt{f}}'$	$K \to \pi\mathtt{f}\bar{\mathtt{f}}'$	$K \to \pi\pi\mathtt{f}\bar{\mathtt{f}}'$	$K \to \phi\bar{\phi}'$	$K \to \pi\phi\bar{\phi}'$	$K \to \pi\pi\phi\bar{\phi}'$
Coefficients	$\tilde{c}^{\rm S}$, $\tilde{c}^{\rm P}$	$\mathtt{C}^{\rm V}, \mathtt{C}^{\rm A}, \mathtt{C}^{\rm S}, \mathtt{C}^{\rm P}, \mathtt{C}^{\rm T}, \mathtt{C}^{\rm T'}$	$\tilde{c}^{\rm V}, \tilde{c}^{\rm A}, \tilde{c}^{\rm S}, \tilde{c}^{\rm P}, \mathtt{C}^{\rm T}, \mathtt{C}^{\rm T'}$	$c^{\rm P}$	$c^{\rm V}$, $c^{\rm S}$	$c^{\rm A}$, $c^{\rm P}$

The expressions for the $K_S \to \pi\pi\phi\bar{\phi}'$ rates equal their K_L counterparts calculated from Eq. (21) except that the signs of $c_{\phi\phi'}^{\rm A,P}$ are flipped.

The last paragraph shows that $K \to \phi\bar{\phi}'$ are sensitive exclusively to $c^{\rm P}$, whereas $K \to \pi\pi\phi\bar{\phi}'$ can probe solely the parity-odd couplings, $c^{\rm A}$ and $c^{\rm P}$, in our approximation of the mesonic matrix elements. By contrast, $K \to \pi\phi\bar{\phi}'$ depend on the parity-even coefficients, $c^{\rm V}$ and $c^{\rm S}$, but not on $c^{\rm A,P}$. As in the fermionic scenario, if $\phi' \neq \phi$, the extra channels $K \to (\pi^0, \pi^+\pi^-, \pi^0\pi^0)\phi'\bar{\phi}$ also take place, as well as $K^- \to \pi^0\pi^-\phi'\bar{\phi}$ if $c_{\phi'\phi} \neq 0$. We comment that the $\phi' = \phi$ possibility has also been considered before in Refs. [8,16] and our formulas above applied to that case are consistent with those obtained therein for $m_\phi = 0$.

In Table I, we list the contributions of the different constants in Eqs. (1) and (2) to the kaon decays of interest according to the discussion above. We remark that in the $\mathtt{f}' = \mathtt{f}$ case for $\mathtt{f}$ having a Majorana nature, instead of Dirac one, $\bar{\mathtt{f}}\gamma^\eta\mathtt{f} = \bar{\mathtt{f}}\sigma^{\eta\kappa}\mathtt{f} = 0$, which causes the $\mathtt{C}_{\mathtt{ff}}^{\rm V,T,T'}$ and $\tilde{c}_{\mathtt{ff}}^{\rm V}$ parts to disappear. Moreover, for $\phi' = \phi$ being a real field, rather than complex one, the $c_{\phi\phi}^{\rm V,A}$ terms would be absent.

For later convenience, here we write down the numerical branching fractions of $K \to \not{E}$ and $K \to \pi\pi\not{E}$ in terms of the contributing coefficients, employing the central values of the measured kaon lifetimes and meson masses from Ref. [17] as well as $a_T = 0.658(23)/\text{GeV}$ from lattice QCD work [23]. Before doing so, in view of Eqs. (8) and (15) and the fact that $\mathtt{f}$ and $\mathtt{f}'$ are not detected in the searches, it is appropriate to define

$$\mathcal{B}(K \to (\pi\pi)\mathtt{f}\mathtt{f}') = \frac{\mathcal{B}(K \to (\pi\pi)\mathtt{f}\bar{\mathtt{f}}') + \mathcal{B}(K \to (\pi\pi)\mathtt{f}'\bar{\mathtt{f}})}{1 + \delta_{\mathtt{ff}'}}, \tag{22}$$

where the factor $1/(1 + \delta_{\mathtt{ff}'})$ with the Kronecker delta $\delta_{\mathtt{ff}'}$ has been added to prevent double counting when $\mathtt{f}' = \mathtt{f}$. Thus, Eq. (8) translates into

$$\begin{aligned}\mathcal{B}(K_L \to \mathtt{ff}') &= 1.45(|\tilde{c}^{\rm S}_{\mathtt{ff}'} - \tilde{c}^{\rm S*}_{\mathtt{f}'\mathtt{f}}|^2 \\ &\quad + |\tilde{c}^{\rm P}_{\mathtt{ff}'} + \tilde{c}^{\rm P*}_{\mathtt{f}'\mathtt{f}}|^2)\frac{10^{14}\ \text{GeV}^4}{1 + \delta_{\mathtt{ff}'}}, \\ \mathcal{B}(K_S \to \mathtt{ff}') &= 2.54(|\tilde{c}^{\rm S}_{\mathtt{ff}'} + \tilde{c}^{\rm S*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\rm P}_{\mathtt{ff}'} - \tilde{c}^{\rm P*}_{\mathtt{f}'\mathtt{f}}|^2) \\ &\quad \times \frac{10^{11}\ \text{GeV}^4}{1 + \delta_{\mathtt{ff}'}},\end{aligned} \tag{23}$$

and Eq. (15) yields

$$\begin{aligned}&\mathcal{B}(K^- \to \pi^0\pi^-\mathtt{ff}') \\ &= [6.28(|\tilde{c}^{\rm V}_{\mathtt{ff}'}|^2 + |\tilde{c}^{\rm A}_{\mathtt{ff}'}|^2 + |\tilde{c}^{\rm V}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\rm A}_{\mathtt{f}'\mathtt{f}}|^2) \\ &\quad + 2.01(|\tilde{c}^{\rm S}_{\mathtt{ff}'}|^2 + |\tilde{c}^{\rm P}_{\mathtt{ff}'}|^2 + |\tilde{c}^{\rm S}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\rm P}_{\mathtt{f}'\mathtt{f}}|^2) \\ &\quad + 6.59(|\mathtt{C}^{\rm T}_{\mathtt{ff}'}|^2 + |\mathtt{C}^{\rm T'}_{\mathtt{ff}'}|^2 + |\mathtt{C}^{\rm T}_{\mathtt{f}'\mathtt{f}}|^2 + |\mathtt{C}^{\rm T'}_{\mathtt{f}'\mathtt{f}}|^2)] \\ &\quad \times \frac{10^5\ \text{GeV}^4}{1 + \delta_{\mathtt{ff}'}},\end{aligned} \tag{24a}$$

$$\begin{aligned}\mathcal{B}(K_L \to \pi^+\pi^-\mathtt{ff}') = \{&13.4[|\tilde{c}^{\rm V}_{\mathtt{ff}'} - \tilde{c}^{\rm V*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\rm A}_{\mathtt{ff}'} - \tilde{c}^{\rm A*}_{\mathtt{f}'\mathtt{f}}|^2] + 67.7[|\tilde{c}^{\rm V}_{\mathtt{ff}'} + \tilde{c}^{\rm V*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\rm A}_{\mathtt{ff}'} + \tilde{c}^{\rm A*}_{\mathtt{f}'\mathtt{f}}|^2] \\ &+ 4.31[|\tilde{c}^{\rm S}_{\mathtt{ff}'} + \tilde{c}^{\rm S*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\rm P}_{\mathtt{ff}'} - \tilde{c}^{\rm P*}_{\mathtt{f}'\mathtt{f}}|^2] + 118[|\tilde{c}^{\rm S}_{\mathtt{ff}'} - \tilde{c}^{\rm S*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\rm P}_{\mathtt{ff}'} + \tilde{c}^{\rm P*}_{\mathtt{f}'\mathtt{f}}|^2] \\ &+ 14.3[|\mathtt{C}^{\rm T}_{\mathtt{ff}'} + \mathtt{C}^{\rm T*}_{\mathtt{f}'\mathtt{f}}|^2 + |\mathtt{C}^{\rm T'}_{\mathtt{ff}'} - \mathtt{C}^{\rm T'*}_{\mathtt{f}'\mathtt{f}}|^2] + 0.50[|\mathtt{C}^{\rm T}_{\mathtt{ff}'} - \mathtt{C}^{\rm T'*}_{\mathtt{f}'\mathtt{f}}|^2 + |\mathtt{C}^{\rm T}_{\mathtt{ff}'} + \mathtt{C}^{\rm T'*}_{\mathtt{f}'\mathtt{f}}|^2]\}\frac{10^5\ \text{GeV}^4}{1 + \delta_{\mathtt{ff}'}},\end{aligned} \tag{24b}$$

$$\begin{aligned}\mathcal{B}(K_L \to \pi^0\pi^0\mathtt{ff}') = \{&42.4[|\tilde{c}^{\rm V}_{\mathtt{ff}'} + \tilde{c}^{\rm V*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\rm A}_{\mathtt{ff}'} + \tilde{c}^{\rm A*}_{\mathtt{f}'\mathtt{f}}|^2] + 80.2[|\tilde{c}^{\rm S}_{\mathtt{ff}'} - \tilde{c}^{\rm S*}_{\mathtt{f}'\mathtt{f}}|^2 + |\tilde{c}^{\rm P}_{\mathtt{ff}'} + \tilde{c}^{\rm P*}_{\mathtt{f}'\mathtt{f}}|^2] \\ &+ 0.34[|\mathtt{C}^{\rm T}_{\mathtt{ff}'} - \mathtt{C}^{\rm T*}_{\mathtt{f}'\mathtt{f}}|^2 + |\mathtt{C}^{\rm T'}_{\mathtt{ff}'} + \mathtt{C}^{\rm T'*}_{\mathtt{f}'\mathtt{f}}|^2]\}\frac{10^5\ \text{GeV}^4}{1 + \delta_{\mathtt{ff}'}}.\end{aligned} \tag{24c}$$

In the invisible scalar case, analogously to Eq. (22), we express

$$\mathcal{B}(K \to (\pi\pi)\phi\phi') = \frac{\mathcal{B}(K \to (\pi\pi)\phi\bar{\phi}') + \mathcal{B}(K \to (\pi\pi)\phi'\bar{\phi})}{1 + \delta_{\phi\phi'}}. \tag{25}$$

From Eqs. (18) and (21) we then have

$$\mathcal{B}(K_L \to \phi\phi') = 2.93 \times 10^{14}\ \mathrm{GeV}^2 \frac{|c^{\mathrm{P}}_{\phi\phi'} - c^{\mathrm{P}*}_{\phi'\phi}|^2}{1+\delta_{\phi\phi'}},$$
$$\mathcal{B}(K_S \to \phi\phi') = 5.14 \times 10^{11}\ \mathrm{GeV}^2 \frac{|c^{\mathrm{P}}_{\phi\phi'} + c^{\mathrm{P}*}_{\phi'\phi}|^2}{1+\delta_{\phi\phi'}} \tag{26}$$

and

$$\mathcal{B}(K^- \to \pi^0\pi^-\phi\phi') = [0.0157(|c^{\mathrm{A}}_{\phi\phi'}|^2 + |c^{\mathrm{A}}_{\phi'\phi}|^2) + 1.38(|c^{\mathrm{P}}_{\phi\phi'}|^2 + |c^{\mathrm{P}}_{\phi'\phi}|^2)\mathrm{GeV}^{-2}]\frac{10^7\ \mathrm{GeV}^4}{1+\delta_{\phi\phi'}},$$
$$\mathcal{B}(K_L \to \pi^+\pi^-\phi\phi') = (0.0334|c^{\mathrm{A}*}_{\phi'\phi} - c^{\mathrm{A}}_{\phi\phi'}|^2 + 2.94|c^{\mathrm{P}*}_{\phi'\phi} + c^{\mathrm{P}}_{\phi\phi'}|^2\mathrm{GeV}^{-2} + 0.169|c^{\mathrm{A}*}_{\phi'\phi} + c^{\mathrm{A}}_{\phi\phi'}|^2 + 51.3|c^{\mathrm{P}*}_{\phi'\phi} - c^{\mathrm{P}}_{\phi\phi'}|^2\mathrm{GeV}^{-2})\frac{10^7\ \mathrm{GeV}^4}{1+\delta_{\phi\phi'}},$$
$$\mathcal{B}(K_L \to \pi^0\pi^0\phi\phi') = (0.106|c^{\mathrm{A}*}_{\phi'\phi} + c^{\mathrm{A}}_{\phi\phi'}|^2 + 32.1|c^{\mathrm{P}*}_{\phi'\phi} - c^{\mathrm{P}}_{\phi\phi'}|^2\mathrm{GeV}^{-2})\frac{10^7\ \mathrm{GeV}^4}{1+\delta_{\phi\phi'}}, \tag{27}$$

respectively.

IV. SM PREDICTIONS AND EMPIRICAL INFORMATION

As mentioned earlier, the latest NA62 measurement on $K^+ \to \pi^+\nu\bar{\nu}$ has turned up evidence for it that is fully consistent with the SM expectation [14]. In view of Table I, this implies that the couplings $\mathrm{c}^{\mathrm{V,A,S,P,T,T'}}_{\mathrm{f}}$ and $c^{\mathrm{V,S}}_{\phi}$ originating from possible NP cannot by sizable anymore.[2] To explore how much the other coefficients shown in Table I may be affected by NP to amplify the $K \to \not{E}$ and $K \to \pi\pi\not{E}$ rates with respect to their SM values, we need to know the latter.

In the SM, our processes of interest arise at short distance from effective $ds\nu_l\bar{\nu}_l$ interactions, with $l = e,\ \mu,\ \tau$, described by [2]

$$\mathcal{L}^{\mathrm{SM}}_{sd\nu\nu} = \frac{-\alpha_{\mathrm{e}} G_{\mathrm{F}}}{\sqrt{8}\pi \sin^2\theta_{\mathrm{W}}} \sum_{l=e,\mu,\tau} (V^*_{td}V_{ts}X_t + V^*_{cd}V_{cs}X^l_c) \times \bar{d}\gamma^\eta(1-\gamma_5)s\bar{\nu}_l\gamma_\eta(1-\gamma_5)\nu_l + \mathrm{H.c.}, \tag{28}$$

where $\alpha_{\mathrm{e}} = 1/128$, G_{F} is the Fermi constant, $\sin^2\theta_{\mathrm{W}} = 0.231$, $V_{qq'}$ are Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $X_t = 1.481$ from t-quark loops, and $X^e_c = X^\mu_c \simeq 1.2 \times 10^{-3}$ and $X^\tau_c \simeq 8 \times 10^{-4}$ are c-quark contributions [2]. Applying the notation of Eq. (1) to $\mathcal{L}^{\mathrm{SM}}_{sd\nu\nu}$, we then have $\mathrm{c}^{\mathrm{V}}_{\nu_l\nu_l} = -\mathrm{c}^{\mathrm{A}}_{\nu_l\nu_l} = -\tilde{c}^{\mathrm{V}}_{\nu_l\nu_l} = \tilde{c}^{\mathrm{A}}_{\nu_l\nu_l} = \alpha_{\mathrm{e}} G_{\mathrm{F}}(\lambda_t X_t + \lambda_c X^l_c)/(\sqrt{8}\pi\sin^2\theta_{\mathrm{W}}) \sim -(3+0.9i) \times 10^{-11}/\mathrm{GeV}^2$ and $\mathrm{c}^{\mathrm{S,P,T,T'}}_{\nu_l\nu_l} = \tilde{c}^{\mathrm{S,P}}_{\nu_l\nu_l} = 0$.

Accordingly, in light of Eqs. (8) and (9) we see that $\mathcal{B}(K_{L,S} \to \nu\bar{\nu})_{\mathrm{SM}} = 0$, given that the neutrinos are massless in the SM. However, supplementing it with nonzero neutrino masses and taking their biggest one from the direct limit $m^{\mathrm{exp}}_{\nu_\tau} < 18.2$ MeV [17] would instead lead to the maximal values [6] $\mathcal{B}(K_L \to \nu\bar{\nu})_{\mathrm{SM}} \simeq 1 \times 10^{-10}$ and $\mathcal{B}(K_S \to \nu\bar{\nu})_{\mathrm{SM}} \simeq 2 \times 10^{-14}$.

As for the four-body channels, employing Eq. (24) we get $\mathcal{B}(K^- \to \pi^0\pi^-\nu\bar{\nu})_{\mathrm{SM}} \sim 4 \times 10^{-15}$ and $\mathcal{B}(K_L \to (\pi^+\pi^-, \pi^0\pi^0)\nu\bar{\nu})_{\mathrm{SM}} \sim (8,5) \times 10^{-14}$. These are in rough agreement with more refined evaluations in the literature [3,4,28,29]:

$$\mathcal{B}(K^- \to \pi^0\pi^-\nu\bar{\nu})_{\mathrm{SM}} \sim 1.2 \times 10^{-14},$$
$$\mathcal{B}(K_L \to \pi^+\pi^-\nu\bar{\nu})_{\mathrm{SM}} \sim 2.8 \times 10^{-13},$$
$$\mathcal{B}(K_L \to \pi^0\pi^0\nu\bar{\nu})_{\mathrm{SM}} \sim 1.5 \times 10^{-13}. \tag{29}$$

with the latest CKM matrix elements [17]. The estimates for $K_S \to \pi\pi\nu\bar{\nu}$ are about three orders of magnitude less than their K_L counterparts. The two sets of K^- and K_L numbers above indicate the level of uncertainties in our $K \to \pi\pi\not{E}$ predictions in the next section.

On the experimental side, merely two of these modes have been looked for [17], with negative outcomes which led to the limits [30,31]

[2] A preliminary report from KOTO [24] has revealed that its most recent data contain a couple of $K_L \to \pi^0\nu\bar{\nu}$ events suggesting an anomalously high rate, which still needs confirmation from further measurements. If this anomaly persists in the future, it may be due to NP, as discussed in, e.g., [25–27] and the references therein, but its effects would not be large enough to modify our conclusions for $K \to \pi\pi\not{E}$.

$$\mathcal{B}(K^- \to \pi^0\pi^-\nu\bar{\nu})_{\rm exp} < 4.3\times10^{-5},$$
$$\mathcal{B}(K_L \to \pi^0\pi^0\nu\bar{\nu})_{\rm exp} < 8.1\times10^{-7} \quad (30)$$

both at 90% CL. These exceed the corresponding SM numbers in Eq. (29) by several orders of magnitude. As regards $K_{L,S} \to \not{E}$, there have been no direct searches for them yet. Nevertheless, from the existing data [17] on the visible decay channels of $K_{L,S}$ one can obtain indirect upper bounds on their invisible branching fractions [32]. Thus, one can infer [20]

$$\mathcal{B}(K_L \to \not{E}) < 1.8\times10^{-3}, \quad \mathcal{B}(K_S \to \not{E}) < 7.1\times10^{-4} \quad (31)$$

at the 2σ level, which are far away from the aforesaid $\mathcal{B}(K_{L,S} \to \nu\bar{\nu})_{\rm SM}$ values. Comparing Eqs. (30)–(31) with Eqs. (23)–(27), as well as Table I, we conclude that currently there remains potentially plenty of room for NP to boost the rates of these decays via $\tilde{c}^{\rm V,A,S,P}$ and $c^{\rm A,P}$.

V. NP EXPECTATIONS AND CONCLUSIONS

Based on the considerations made in the previous section, we hereafter entertain the possibility that, among the couplings listed in the table, NP manifests itself exclusively via those belonging to operators with parity-odd ds bilinears, namely $\tilde{c}^{\rm V,A,S,P}$ in Eq. (1) or $c^{\rm A,P}$ in Eq. (2), and demand that they fulfill the conditions

$$\mathcal{B}(K_L \to \not{E})_{\rm NP} < 1.8\times10^{-3},$$
$$\mathcal{B}(K_S \to \not{E})_{\rm NP} < 7.1\times10^{-4}, \quad (32)$$

$$\mathcal{B}(K^- \to \pi^0\pi^-\not{E})_{\rm NP} < 4.0\times10^{-5},$$
$$\mathcal{B}(K_L \to \pi^0\pi^0\not{E})_{\rm NP} < 8.0\times10^{-7}. \quad (33)$$

These $ds\mathtt{f}\mathtt{f}'$, or $ds\phi\phi'$, interactions additionally contribute to the mixing of neutral kaons via one-loop diagrams with $\mathtt{f}$ and $\mathtt{f}'$, or ϕ and ϕ', being in the loops and must therefore be compatible with its data. One can see that the resulting pertinent operators are of the form $\bar{d}(\gamma^\kappa)\gamma_5 s\bar{d}(\gamma_\kappa)\gamma_5 s$ and have coefficients proportional to linear combinations of $\tilde{c}^x_{\mathtt{ff}'}\tilde{c}^x_{\mathtt{f}'\mathtt{f}}$ with $x = \rm V, A, S, P$, or $c^x_{\phi\phi'}c^x_{\phi'\phi}$ with $x = \rm A, P$. As a consequence, these products can evade the kaon-mixing restrictions, which are stringent, if one of $\tilde{c}^x_{\mathtt{ff}'}$ and $\tilde{c}^x_{\mathtt{f}'\mathtt{f}}$ for $\mathtt{f}' \neq \mathtt{f}$, or one of $c^x_{\phi\phi'}$ and $c^x_{\phi'\phi}$ for $\phi' \neq \phi$, either vanishes or is sufficiently smaller than the other. To illustrate the ramifications that may arise for the various $K \to \pi\pi\not{E}$ modes if NP occurs in these couplings, we can look at several simple examples.

If it solely affects $\tilde{c}^{\rm S,P}_{\mathtt{ff}'}$ with $\mathtt{f}' \neq \mathtt{f}$, then $\tilde{c}^{\rm S,P}_{\mathtt{f}'\mathtt{f}}$ are absent, and so the kaon-mixing constraints are avoided. In this case, comparing Eqs. (23)–(24) to (32)–(33), we learn that the $K_L \to \not{E}$ requirement is the most significant and translates into $|\tilde{c}^{\rm S}_{\mathtt{ff}'}|^2 + |\tilde{c}^{\rm P}_{\mathtt{ff}'}|^2 < 1.2\times10^{-17}$ GeV^{-4}. Combining it with the branching fractions, we arrive at the maximal values

$$\mathcal{B}(K^- \to \pi^0\pi^-\mathtt{ff}') < 2.5\times10^{-12},$$
$$\mathcal{B}(K_L \to \pi^+\pi^-\mathtt{ff}') < 1.5\times10^{-10},$$
$$\mathcal{B}(K_L \to \pi^0\pi^0\mathtt{ff}') < 1.0\times10^{-10},$$
$$\mathcal{B}(K_S \to \pi^+\pi^-\mathtt{ff}') < 2.7\times10^{-13},$$
$$\mathcal{B}(K_S \to \pi^0\pi^0\mathtt{ff}') < 1.7\times10^{-13}. \quad (34)$$

Interchanging the roles of $\tilde{c}^{\rm S,P}_{\mathtt{ff}'}$ and $\tilde{c}^{\rm S,P}_{\mathtt{f}'\mathtt{f}}$ would not alter these numbers. They are considerably higher than the corresponding SM expectations quoted earlier but might not be high enough to be empirically testable any time soon.

If only $\tilde{c}^{\rm V,A}_{\mathtt{ff}'}$ with $\mathtt{f}' \neq \mathtt{f}$ are influenced by NP, hence $\tilde{c}^{\rm V,A}_{\mathtt{f}'\mathtt{f}} = 0$, it is clear from Eq. (23) that Eq. (32) is no longer relevant but Eq. (33) still matters, with the $K_L \to \pi^0\pi^0\not{E}$ restraint being the stronger and yielding $|\tilde{c}^{\rm V}_{\mathtt{ff}'}|^2 + |\tilde{c}^{\rm A}_{\mathtt{ff}'}|^2 < 1.9\times10^{-13}$ GeV^{-4}. With this, we obtain

$$\mathcal{B}(K^- \to \pi^0\pi^-\mathtt{ff}') < 1.2\times10^{-7},$$
$$\mathcal{B}(K_L \to \pi^+\pi^-\mathtt{ff}') < 1.5\times10^{-6},$$
$$\mathcal{B}(K_L \to \pi^0\pi^0\mathtt{ff}') < 8.0\times10^{-7},$$
$$\mathcal{B}(K_S \to \pi^+\pi^-\mathtt{ff}') < 2.7\times10^{-9},$$
$$\mathcal{B}(K_S \to \pi^0\pi^0\mathtt{ff}') < 1.4\times10^{-9}, \quad (35)$$

which greatly exceed their counterparts in Eq. (34) and some of which may be within the reach of ongoing or upcoming kaon factories. We would achieve the same results with $\tilde{c}^{\rm V,A}_{\mathtt{f}'\mathtt{f}}$ alone instead. It is worth pointing out that this kind of possibility can be realized in a scenario involving scalar leptoquarks plus light sterile neutrinos acting as the invisibles [7]. Furthermore, the model can also generate substantial enhancement in the rates of the aforementioned FCNC hyperon decays with missing energy [7], which are potentially detectable in the BESIII experiment [33,34].

If now NP enters exclusively through $c^{\rm P}_{\phi\phi'}$ with $\phi' \neq \phi$, implying $c^{\rm P}_{\phi'\phi} = 0$, then a comparison of Eqs. (26)–(27) and (32)–(33) reveals that the $K_L \to \not{E}$ requisite in Eq. (32) is the most important, from which we get $|c^{\rm P}_{\phi\phi'}|^2 < 6.1\times10^{-18}$ GeV^{-2}. This translates into

$$\mathcal{B}(K^- \to \pi^0\pi^-\phi\bar{\phi}) < 8.5\times10^{-11},$$
$$\mathcal{B}(K_L \to \pi^+\pi^-\phi\bar{\phi}) < 3.3\times10^{-9},$$
$$\mathcal{B}(K_L \to \pi^0\pi^0\phi\bar{\phi}) < 2.0\times10^{-9}$$
$$\mathcal{B}(K_S \to \pi^+\pi^-\phi\bar{\phi}) < 5.8\times10^{-12},$$
$$\mathcal{B}(K_S \to \pi^0\pi^0\phi\bar{\phi}) < 3.5\times10^{-12} \quad (36)$$

TABLE II. The maximal branching fractions of $K \to \pi\pi \not{E}$ due to NP being present in one or more of the coefficients $\tilde{c}^{\rm V,A,S,P}_{\rm ff',f'f}$ and $c^{\rm A,P}_{\phi\phi',\phi'\phi}$ as specified in the examples given in the main text.

Contributing coefficients	Decay modes				
	$K^- \to \pi^0\pi^- \not{E}$	$K_L \to \pi^+\pi^- \not{E}$	$K_L \to \pi^0\pi^0 \not{E}$	$K_S \to \pi^+\pi^- \not{E}$	$K_S \to \pi^0\pi^0 \not{E}$
$\tilde{c}^{\rm S,P}_{\rm ff'}$ or $\tilde{c}^{\rm S,P}_{\rm f'f}$	2.5×10^{-12}	1.5×10^{-10}	1.0×10^{-10}	2.7×10^{-13}	1.7×10^{-13}
$\tilde{c}^{\rm V,A}_{\rm ff'}$ or $\tilde{c}^{\rm V,A}_{\rm f'f}$	1.2×10^{-7}	1.5×10^{-6}	8.0×10^{-7}	2.7×10^{-9}	1.4×10^{-9}
$c^{\rm P}_{\phi\phi'}$ or $c^{\rm P}_{\phi'\phi}$	8.5×10^{-11}	3.3×10^{-9}	2.0×10^{-9}	5.8×10^{-12}	3.5×10^{-12}
$c^{\rm A}_{\phi\phi'}$ or $c^{\rm A}_{\phi'\phi}$	1.2×10^{-7}	1.5×10^{-6}	8.0×10^{-7}	2.7×10^{-9}	1.4×10^{-9}

which are larger than the corresponding values in Eq. (34) by roughly an order of magnitude. In contrast, if NP solely impacts $c^{\rm A}_{\phi\phi'}$ with $\phi' \neq \phi$, hence $c^{\rm A}_{\phi'\phi} = 0$, the situation turns out to be analogous to that reflected by Eq. (35). More explicitly, in view of Eqs. (27) and (33), from the K_L condition in the latter we extract $|c^{\rm A}_{\phi\phi'}|^2 < 7.5 \times 10^{-13}\ {\rm GeV}^{-4}$, which leads to

$$\begin{aligned} \mathcal{B}(K^- \to \pi^0\pi^-\phi\bar{\phi}) &< 1.2 \times 10^{-7}, \\ \mathcal{B}(K_L \to \pi^+\pi^-\phi\bar{\phi}) &< 1.5 \times 10^{-6}, \\ \mathcal{B}(K_L \to \pi^0\pi^0\phi\bar{\phi}) &< 8.0 \times 10^{-7} \\ \mathcal{B}(K_S \to \pi^+\pi^-\phi\bar{\phi}) &< 2.7 \times 10^{-9}, \\ \mathcal{B}(K_S \to \pi^0\pi^0\phi\bar{\phi}) &< 1.4 \times 10^{-9}. \end{aligned} \tag{37}$$

These numbers are identical to those in Eq. (35).

In Table II we collect our findings in Eqs. (34)–(37) and the associated coefficients. We note that, as alluded to in Sec. I and discussed in Refs. [6–8], in the cases seen in this table with high branching fractions the corresponding predictions for their hyperon counterparts are magnified in like manner and might therefore be within the sensitivity ranges of searches in the near future.

To conclude, motivated by the latest NA62 measurement on $K^+ \to \pi^+\nu\bar{\nu}$, which is in good agreement with the SM and consequently implies stringent constraints on NP which might be hiding in $K \to \pi\not{E}$, we have explored how other types of FCNC kaon decays with missing energy might shed additional light on potential NP in the underlying $s \to d\not{E}$ transition. Focusing on scenarios in which the missing energy is carried away by a pair of invisible new particles of spin 1/2 or 0, we have argued that there are four-particle operators contributing to $s \to d\not{E}$ which are not restricted by $K \to \pi\not{E}$ and accordingly could still significantly affect $K \to \not{E}$ and $K \to \pi\pi\not{E}$, on which the empirical details are currently meager. We have demonstrated especially that the branching fractions of $K \to \pi\pi\not{E}$ could yet be amplified far beyond their SM expectations, to levels which might be within the reach of ongoing experiments, specifically KOTO and NA62, or upcoming ones such as KLEVER [35]. Our results, which are illustrated with the instances summarized in Table II, will hopefully help stimulate new quests for these decays as NP probes. Last but not least, we have pointed out that similar kinds of enhancement would occur in the hyperon sector, which may be detectable by BESIII or future charm-tau factories [36,37], and thus it could offer complementary NP tests.

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