Interpretation of structure in the di-J/ψ spectrum

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(Received 30 October 2020; accepted 30 November 2020; published 29 December 2020)

Structure in the di-J/ψ mass spectrum observed by the LHCb experiment around 6.9 and 7.2 GeV is interpreted in terms of $J^{PC} = 0^{++}$ and 2^{++} resonances between a cc diquark and a $\bar{c}\bar{c}$ antidiquark, using a recently confirmed string-junction picture to calculate tetraquark masses. The main peak around 6.9 GeV is likely dominated by the $0^{++}(2S)$ state, a radial excitation of the $cc - \bar{c}\bar{c}$ tetraquark, which we predict at 6.871 ± 0.025 GeV. The dip around 6.75 GeV is ascribed to the opening of the S-wave di-χ_{c0} channel, while the dip around 7.2 GeV could be correlated with the opening of the di-$\eta_c(2S)$ or $\Xi_{cc}\bar{\Xi}_{cc}$ channel. The low-mass part of the di-J/ψ structure appears to require a broad resonance consistent with a predicted $2^{++}(1S)$ state with invariant mass around $M_{\text{inv}} = 6400$ MeV. Implications for $bb\bar{b}\bar{b}$ tetraquarks are discussed.

DOI: 10.1103/PhysRevD.102.114039

I. INTRODUCTION

The picture of hadrons as bound states of colored quarks described the observed mesons as $q\bar{q}$ and baryons as qqq states but also could accommodate more complicated color-singlet combinations such as $qq\bar{q}\bar{q}$ (tetraquarks) or $q^4\bar{q}$ (pentaquarks). Since 2003, experimental evidence has accumulated for such combinations, but it has not been clear whether they are genuine bound states with equal roles for all constituents or loosely bound "molecules" of two mesons or a meson and a baryon, with quarks mainly belonging to one hadron or the other. There is, however, fairly robust theoretical evidence for a deeply bound genuine $bb\bar{u}\bar{d}$ tetraquark [1,2].

Recently the LHCb Collaboration at CERN has presented evidence for structure in the spectrum of a pair of J/ψ mesons, $M_{\text{inv}}(\text{di} - J/\psi)$ [3], interpreted as a narrow structure around 6.9 GeV and a broad structure just above twice the J/ψ mass. A dip in $M_{\text{inv}}(\text{di} - J/\psi)$ around 6.75 GeV suggests interference with nonresonant behavior in a channel with the same J^{PC}. Such behavior is difficult to regard from a molecular standpoint but is compatible with a picture of a compact $cc\bar{c}\bar{c}$ state. Many theoretical interpretations of the LHCb data take this point of view [4–22].

In this paper we adopt the compact tetraquark point of view (see [23,24] for lists of related predictions) and point out a feature in the data which is characteristic of many processes. We note that the position of the dip roughly coincides with twice the mass of $\chi_{c0}(3415)$. If the major resonant di-J/ψ activity is in the $J^{PC} = 0^{++}$ channel, a pair of $\chi_{c0}(3415)$ charmonia can be produced in an S wave as soon as $M_{\text{inv}}(\text{di} - J/\psi)$ exceeds 6829 MeV. Unitarity then can induce a *dip* in the production channel. (See also [11].)

In Sec. II we recall a number of instances in which the opening of an S-wave channel induces a dip in the production channel. We apply similar methods to the S-wave process $J/\psi J/\psi \to \chi_{c0}\chi_{c0}$ and $J/\psi J/\psi \to \eta_c(2S)\eta_c(2S)$ in Sec. III and discuss implications for $cc\bar{c}\bar{c}$ tetraquarks in Sec. IV and for $bb\bar{b}\bar{b}$ tetraquarks in Sec. V, concluding in Sec. VI. An Appendix contains details of resonance fitting.

II. DIPS AND CUSPS IN S-WAVE PRODUCTION CHANNELS

Dips or cusps in the cross section for a number of S-wave processes occur when a new S-wave threshold is crossed. Here we review several such cases. More details and references may be found in Ref. [25].

A. $\pi\pi I = J = 0$ amplitude at $K\bar{K}$ threshold

The rapid drop in the magnitude of the $I = 0$ S-wave $\pi\pi$ scattering amplitude near a center-of-mass energy $E_{\text{cm}} \simeq 1$ GeV is associated with the rapid passage of the elastic phase shift through 180°. (See Ref. [26] for a recent parametrization.) This behavior is correlated with the

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opening of the $K\bar{K}$ threshold, forcing the $I = J = 0$ $\pi\pi$ amplitude to become highly inelastic [27]. It also reflects the effect of a narrow resonance $f_0(980)$ [28] coupling to both $\pi\pi$ and $K\bar{K}$. For more details see [29,30]. A related discussion applies to the S-wave $\pi\eta$ channel near the $I = 1$, $J = 0$ $K\bar{K}$ threshold [31].

B. Cusp in $\pi^0\pi^0$ spectrum at $\pi^+\pi^-$ threshold

The $\pi^0\pi^0$ S-wave scattering amplitude is expected to have a cusp at $\pi^+\pi^-$ threshold [32,33]. This behavior can be studied in the decay $K^+ \to \pi^+\pi^0\pi^0$, where the contribution from the $\pi^+\pi^+\pi^-$ intermediate state allows one to study the charge-exchange reaction $\pi^+\pi^- \to \pi^0\pi^0$ and thus to measure the $\pi\pi S$-wave scattering length difference $a_0 - a_2$ [34]. The CERN NA48 Collaboration has performed such a measurement, finding results [35] in remarkable agreement with the prediction [34]. One can also study this effect in $\pi^+\pi^-$ atoms [36].

C. Hadron production by e^+e^- collisions around 4.26 GeV

The value of $R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ drops sharply just below threshold for production of $D(1865)^0\bar{D}_1(2420)^0$ + c.c. [37], which is the lowest-mass $c\bar{c}$ channel accessible in an S wave from a virtual photon. If this behavior is not coincidental, the drop in R should be confined to the $c\bar{c}$ final states.

D. Six-pion diffractive photoproduction

The diffractive photoproduction of $3\pi^+3\pi^-$ leads to a spectrum with a pronounced dip near 1.9 GeV$/c^2$ [38,39]. This is just the threshold for production of a proton-antiproton pair in the $^3S_1$ channel. This dip also occurs in the $3\pi^+3\pi^-$ spectrum produced in radiative return in higher-energy e^+e^- collisions, i.e., in $e^+e^- \to \gamma 3\pi^+3\pi^-$, observed by the *BABAR* Collaboration at SLAC [40]. The feature can be reproduced by a 1^{--} resonance with $M = 1.91 \pm 0.01$ GeV$/c^2$ and width $\Gamma = 37 \pm 13$ MeV interfering destructively with a broader 1^{--} resonance at lower mass [38,39].

E. Greater generality

The vanishing of an S-wave amplitude when its elastic phase shift goes through 180° is not confined to particle physics. The Ramsauer-Townsend effect represents similar behavior in atomic physics [41]. Cusps in S-wave scattering cross sections occur at thresholds for *any* new channels [42,43]. Monochromatic neutrons may be produced by utilizing the vanishing absorption cross sections of neutrons of certain energies on specific nuclei [44].

F. A cautionary note

Although the rapid passage of the $I = J = 0$ $\pi\pi$ phase shift through 180° near $K\bar{K}$ threshold can be ascribed to the nearby $f_0(980)$ resonance, one cannot conclude that similar behavior in other of the above cases (or many more examined in [25]) is due to nearby poles in the scattering amplitude [43]. As in the case of diffractive six-pion production mentioned above, unitarity alone will cause a suppression of the input channel at the expense of the newly open channel. *The ability to fit the amplitude with a resonance does not guarantee its existence.*

III. DIPS IN $M_{\rm inv}$(di-J/ψ) AT DI-CHARMONIUM THRESHOLDS

The spectrum of $M_{\rm inv}$(di-J/ψ) receives contributions from both single-parton scattering (SPS) and double-parton scattering (DPS). We assume, along with [3], that only the former process contributes to resonant di-J/ψ structure and subtract the latter (from [3]) before fitting the observed spectrum. We allow a fraction α of the SPS amplitude to interfere with the resonances, which are introduced by Breit-Wigner amplitudes with arbitrary phases with respect to the nonresonant SPS (NRSPS) amplitude, assumed real. In Fig. 1 we show the spectrum together with a fit to data in the range 6.2–7.5 GeV using the sum of three resonances with masses M_i, widths Γ_i, normalizations η_i, and phases ϕ_i ($i = 1$, 2, 3). Signal normalization, background normalization, and background shape are described by parameters C_i defined in the Appendix. The results of this fit are shown in Table I.

The shapes of the peaks around 6.9 and 7.2 GeV suggest destructive interference between signal and background on the low-mass side of both peaks. The sudden rise following a dip is characteristic of an S-wave amplitude, as illustrated in the previous section. It was associated with the opening of a nearby threshold. In the case of the 6.9 GeV peak, we note that $2M(\chi_{c0}) = 6829$ MeV, so we can ascribe the

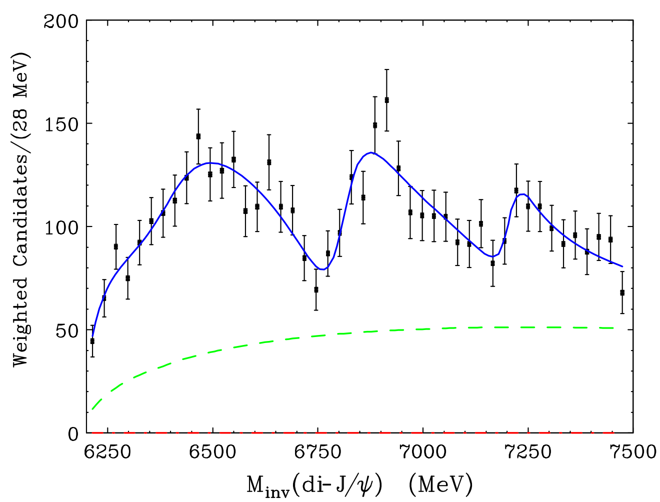

FIG. 1. Spectrum of J/ψ pairs reported by the LHCb experiment [3], together with our best fit to data (blue line), as given in Table I and described in the Appendix. The green dashed line denotes the DPS contribution, subtracted before fitting.

TABLE I. Parameters in best fit to data (see the Appendix for definitions) with $\chi^2 = 25.855$ for 31 degrees of freedom (d.o.f.). Masses M_i and widths Γ_i are in MeV. Constants C_i describe signal normalization, background normalization, and background shape, respectively. Parameters η_i ($\eta_1 \equiv 1$) and ϕ_i (in degrees) describe normalizations and phases of ith Breit-Wigner amplitudes.

Peak i	$i = 1$	$i = 2$	$i = 3$
M_i	6377.1	6808.6	7208.1
Γ_i	277.3	138.0	82.96
C_i	5.057	25.74	1.184
η_i	1.000[a]	1.445	0.7754
ϕ_i	−26.62	−34.78	−4.995
α	1.000	Coherence factor	

[a]Input.

steep behavior between about 6750 and 6900 GeV as associated with opening of the di-χ_{c0} channel. [Reference [21] ascribes cusplike behavior to the opening of the $J/\psi\psi(2S)$ channel at 6783 MeV.]

If the di-χ_{c0} channel is in an S wave, as implied by its sudden onset, the S-wave behavior in the di-J/ψ channel requires the two J/ψ mesons to be in a state of $J^{PC} = 0^{++}$. An initial state of two J/ψ mesons consists of two $c\bar{c}$ pairs, each in a $^3S_1$ state. A χ_{c0} is a P-wave charmonium state with the quarks' spins coupled to 1 and spin coupled with $L = 1$ to give $J = 0$. The final state with two $^3P_0$ states in a relative S wave can be reached from the initial state by orbital excitation of each spin-triplet state.

Detection of the presence of the two χ_{c0} states is challenging in view of the small branching fractions of χ_{c0} to observable final states. The only branching fractions of χ_{c0} that exceed a percent are given in Table II [28]. With sufficient mass resolution, one could combine the modes with all charged tracks to get an effective branching fraction of a bit above 5%. The total width of χ_{c0} is 10.8 ± 0.6 MeV. The experimental mass resolution in other LHCb analyses (see, e.g., [45,46]) is somewhat greater and thus dominates the sensitivity to a signal. An explicit simulation would be helpful. If the cusplike behavior is due to the opening of the $J/\psi\psi(2S)$ channel [21], that final state should be easier to detect, consisting of two J/ψ plus a $\pi^+\pi^-$ pair.

TABLE II. Branching fractions of $\chi_{c0}(3415)$ exceeding a percent.

Mode	Percent
$2(\pi^+\pi^-)$	2.34 ± 0.18
$\pi^+\pi^-\pi^0\pi^0$	3.3 ± 0.4
$\pi^+\pi^-K^+K^-$	1.81 ± 0.14
$K^+\pi^-\bar{K}^0\pi^0$ + c.c.	2.49 ± 0.33
$3(\pi^+\pi^-)$	1.20 ± 0.18
$\gamma J/\psi$	1.40 ± 0.05

Similar behavior is apparent on the low-$M_{\rm inv}$ shoulder of the peak at 7.2 GeV. The only nearby di-charmonium threshold is associated with a pair of $\eta_c(2S)$ mesons, with $2M[\eta_c(2S)] = 7275$ MeV. If this threshold plays an important role in the line shape of the peak, one should see decay products of two $\eta_c(2S)$ mesons on the high-$M_{\rm inv}$ side of this peak. This, of course, is even more challenging than detecting a pair of χ_{c0} mesons. (References [12,13] draw attention to the slightly lower $\Xi_{cc}\bar{\Xi}_{cc}$ threshold at 7242 MeV, which we shall discuss further at the end of Sec. IV.)

We initially sought evidence for a di-$\eta_c(1S)$ threshold at $2M[\eta_c(1S)] = 5968$ MeV and inserted a corresponding pole below di-J/ψ threshold into our fitting amplitude. The expectation was that this would contribute a needed enhancement of the spectrum between $M_{\rm inv} \simeq 6.2$ and 6.6 GeV. The fitting program (see the Appendix) instead preferred a much higher-mass pole, as one sees for M_1 in Table I. Thus the predicted 1S candidate [23] with mass $M[T(cc\bar{c}\bar{c})] = 6191.5 \pm 25$ MeV for the lightest all-charm tetraquark remains to be observed. The value of M_1 in Table I is more consistent with that of a $2^{++}(1S)$ state predicted in Ref. [20] to lie at 6367 MeV. These authors predict a mass of 6190 MeV for the 1S state, consistent with ours.

Although we do not predict a tetraquark resonance near di-$\eta_c(1S)$ threshold, it would be worth examining channels that couple to a pair of $\eta_c(1S)$ to see if they exhibit cusps in S-wave amplitudes near $M_{\rm inv} = 5968$ MeV. Examples of such channels include $D\bar{D}$ and $D^*\bar{D}^*$ [8,14].

IV. IMPLICATIONS FOR $cc\bar{c}\bar{c}$ TETRAQUARKS

The treatment of the doubly charmed diquark as pointlike is an approximation but a fairly accurate one. It has been tested in the successful prediction [47] of the mass of the doubly charmed baryon Ξ_{cc} as 3627 ± 12 MeV to be compared with the experimental value [48,49] $M(\Xi_{cc}^{++}) = 3621.55 \pm 0.23({\rm stat}) \pm 0.30({\rm syst})$ MeV. In Ref. [23], using a diquark-antidiquark picture, we predicted the ground state $T(cc\bar{c}\bar{c})$ mass to be 6191.5 ± 25 MeV. This error is taken to be twice that obtained when fitting nonexotic mesons and baryons in the string-junction picture (see also [12]), recently confirmed by the successful prediction of the mass of a $T(cs\bar{u}\bar{d})$ tetraquark [50] and which we are assuming here [23]. This would be the $0^{++}(1S)$ state of the spin-1 color-antitriplet diquark and the spin-1 color-triplet antidiquark. The ingredients of the prediction included a term $2S = 2(165.1)$ MeV for two QCD string junctions, $2(M_{cc}) = 2(3204.1)$ MeV for the masses of two diquarks, an interpolated binding energy of the cc diquark with the $\bar{c}\bar{c}$ antidiquark of -388.3 MeV, and a hyperfine term of -158.5 MeV. The predicted mass is just below $2M(J/\psi) = 6194$ MeV but above $2M(\eta_c(1S) = 5968$ MeV, so strong decay to a pair of

$\eta_c(1S)$ is favored. Here and subsequently we use the latest Particle Data Group masses [28].

The above discussion is based on S-wave cc diquarks in a color 3^* state, with spin 1. There should also be states involving color 6 diquarks, with spin zero. There should be an additional spinless tetraquark made of a 6 in an S-wave state with a 6^*. Estimates, for example in Ref. [16], of its mass are not far from that of the $1S\ 3^* \times 3$ state, and the two may mix with one another.

The above estimate concerns the ground state 0^{++} mass. One estimates the ground state 2^{++} mass by noting that the hyperfine terms for a pair of spin-1 particles in states of $J = 0,\ 1,\ 2$ are in the ratio $(1/2)[J(J+1) - 4] = -2, -1, 1$, so the hyperfine term for the lowest 2^{++} state is 79.3 MeV and the mass of the $2^{++}(1S)$ state is 6429 ± 25 MeV, 238 MeV above the $0^{++}(1S)$ and well above $2M(J/\psi)$ threshold. This is the state predicted in Ref. [20] to lie at 6367 MeV. It could be some or all of the low-M_{inv} di-J/ψ peak with $M_1 = 6377$ MeV, in Table I, allowing the 0^{++} state to lie at our predicted value of 6191.5 ± 25 MeV. A spin-parity analysis should be able to distinguish between 0^{++} and 2^{++} components of the amplitude.

The $1S$-$2S$ splittings of the charmonium and bottomonium systems are almost the same. The spin-weighted average ($1S$, $2S$) masses are (3068.65, 3673.95) MeV for charmonium and (9444.9, 10017.2) MeV for bottomonium, so $M(2S) - M(1S) = (605.3, 572.3)$ MeV for $(c\bar{c}, b\bar{b})$. They would be equal for a logarithmic interquark potential, providing a convenient interpolation between short-distance and long-distance QCD for these systems [51]. The cc diquark mass is intermediate between m_c and m_b: using the values from [23],

$$m_c = 1655.6\ \mathrm{MeV}, \qquad m_b = 4988.6\ \mathrm{MeV},$$
$$m_{cc} = 3204.1\ \mathrm{MeV}, \tag{1}$$

a power-law interpolation between m_c and m_b of the form $M(2S) - M(1S) = am^p$ with $a = 882.22m^{-0.050826}$ gives the $1S$-$2S$ splitting for a cc diquark and a $\bar{c}\bar{c}$ antidiquark to be 585.3 MeV.

The hyperfine splittings $M(^3S_1) - M(^1S_0)$ are in the ratio $\Delta M(2S)/\Delta M(1S) = 0.430 \pm 0.005$ for $c\bar{c}$ and 0.390 ± 0.066 for $b\bar{b}$. Interpolating these central values in terms of a power law in masses (1) we find $\Delta M(2S)/\Delta M(1S) = 0.4053$ for the bound states of the cc diquark and the $\bar{c}\bar{c}$ antidiquark. This means that for the $2S$ system, we replace the $1S$ hyperfine term of -158.5 MeV by -64.2 MeV, a change of 94.3 MeV. The mass of the $0^{++}(2S)$ state is then $6192 + 585 + 94 = 6871$ MeV, close to the peak claimed by LHCb. (See also [9,13].) The $2^{++}(2S)$ state is then $(0.4053)(237.8) = 96$ MeV higher, at 6967 MeV. This state could also be contributing to the LHCb signal.

We have not discussed 1^{++} states of cc diquark and $\bar{c}\bar{c}$ antidiquark decaying to a pair of J/ψ in an S wave. Two identical spin-1 bosons in an S wave are forbidden by Bose statistics to have total angular momentum $J = 1$. Various predictions have been made for the mass of a 1^{+-} state, which, however, does not couple to a pair of J/ψ. The S-wave threshold amplitude for $\chi_{c0}\chi_{c0}$ production, starting at $2M(\chi_{c0}) = 6829$ MeV, thus interferes primarily with the $0^{++}(2S)\ 2J/\psi$ resonant amplitude.

The peak around 7200 MeV is in approximately the right place for a $3S$ state of $(cc)_{3^*}(\bar{c}\bar{c})_3$. The flavor threshold for charmonium lies just above the $2S$ level, while that for bottomonium lies just below the $4S$ level. As a system with reduced mass intermediate between that of charmonium and that of bottomonium, the di-J/ψ system can be expected to have a flavor threshold around the $3S$ level (see Fig. 1 of [52]). This estimate is based on the observation [53,54] that flavor threshold in a quarkonium system always occurs at a universal length of the QCD string connecting the two heavy constituents. Indeed, the first open-flavor state in which a QCD string connecting $(cc)_{3^*}$ with $(\bar{c}\bar{c})_3$ breaks is that in which a light $q\bar{q}$ pair is produced, giving $\Xi_{cc}\bar{\Xi}_{cc}$ with threshold 7242 MeV [12,13].

V. IMPLICATIONS FOR $bb\bar{b}\bar{b}$ TETRAQUARKS

Some attention to the question of fully heavy tetraquarks was drawn by an unpublished report by the CMS Collaboration at CERN [55] of an exotic structure in the four-lepton channel at $18.4 \pm 0.1 \pm 0.2$ GeV, an excess with a global significance of 3.6σ. CMS reported 38 ± 7 events of $\Upsilon(1S)$ pairs produced with an integrated luminosity of 20.7 fb^{-1} at $\sqrt{s} = 8$ TeV, each decaying to μ pairs [56]. There is no published confirmation of the structure [57,58], but in view of the di-J/ψ structure it is worth updating and extending the predictions of Ref. [23] for $bb\bar{b}\bar{b}$ tetraquarks. (Note should be taken of the absence of observed narrow resonances between 17.5 and 19.5 GeV in searches by LHCb [59] and CMS [60].) The predicted mass of the $2^{++}(1S)$ $bb\bar{b}\bar{b}$ state is only 36 ± 25 MeV above the di-$\Upsilon(1S)$ threshold and thus could be a narrow state due to the phase space suppression of its decay rate.

In Ref. [23] we predicted the ground state $T(bb\bar{b}\bar{b})$ mass to be 18826 ± 25 MeV, just above $2M[\eta_b(1S)] = 18797$ MeV, so its main decay will likely be to two η_b's. It would be the 0^{++} state of a color-antitriplet spin-1 bb diquark and the corresponding antidiquark. One predicts

$$\begin{aligned} M[T(bb\bar{b}\bar{b})(0^{++})] &\\ &= 2S + 2M(bb, 3^*) + B_{(bb)(\bar{b}\bar{b})} + \Delta M_{HF} \\ &= [2(165.1) + 2(9718.9) - 855.7 - 86.7]\ \mathrm{MeV} \\ &= 18825.6\ \mathrm{MeV}, \end{aligned} \tag{2}$$

where S is the contribution of a QCD string junction, $B_{(bb)(\bar{b}\bar{b})}$ is the binding energy between the bb diquark and the $\bar{b}\bar{b}$ antidiquark, and ΔM_{HF} is the hyperfine interaction between the diquark and the antidiquark. An error of ± 25 MeV was assigned to this prediction, which we will assume applies to the other predictions in this section.

The hyperfine term for the 2^{++} state is $(-1/2)(-86.7) = 43.4$ MeV, so the $2^{++}(1S)$ state is 130.1 MeV higher than the $0^{++}(1S)$ state, or 18955.7 MeV. This lies above $2M(\Upsilon(1S)) = 2(9460.3) = 18920.6$ MeV so it can decay to a pair of $\Upsilon(1S)$.

In order to estimate the $1S$-$2S$ splitting for $T(bb\bar{b}\bar{b})$, we use the power-law dependence of the previous section, $\Delta M = 882.22 m^{-0.050826}$ (units in MeV) with $m = 9718.9$ MeV, to predict $M(2S) - M(1S) = 553.2$ MeV. To estimate the $2S$ hyperfine splitting we extrapolate the ratio $\Delta M_{HF}(2S)/\Delta M_{HF}(1S) = 0.83232 m^{-0.089428}$ to obtain $\Delta M_{HF}(2S)/\Delta M_{HF}(1S) = 0.3671$. The hyperfine terms for $(0^{++}, 2^{++})(2S)$ are then $(-31.8, 15.9)$ MeV, resulting in the predictions $M(0^{++}, 2^{++})(2S) = (19433.6, 19481.4)$ MeV.

The radially excited $0^{++}(2S)$ bb-$\bar{b}\bar{b}$ tetraquark at 19.434 ± 0.025 GeV is the bottom analog of the $0^{++}(2S)$ excited cc-$\bar{c}\bar{c}$ tetraquark at 6.871 ± 0.025 GeV, proposed here as the main component of the peak near 6.9 GeV reported by LHCb [3].

The predicted $0^{++}(2S)$ mass is large enough to imply a substantial partial width into a pair of $\Upsilon(1S)$. It lies below the $\chi_{b0}\chi_{b0}$ threshold, which is $2(9859.44) = 19718.9$ MeV, so its interference with the 0^{++} state will depend on the width of that state and should exhibit a different pattern from the $T(cc\bar{c}\bar{c})$ case, where the $\chi_{c0}\chi_{c0}$ threshold roughly coincides with the $0^{++}(2S)$ resonance mass. We should also keep in mind the $\Xi_{bb}\bar{\Xi}_{bb}$ threshold at $2(10162 \pm 12) = 20324 \pm 25$ MeV, where we have used the prediction [47] $M(\Xi_{bb}) = 10162 \pm 12$ MeV, in analogy with the $\Xi_{cc}\bar{\Xi}_{cc}$ threshold mentioned earlier.

VI. CONCLUSIONS

We have interpreted the structure in the di-J/ψ mass spectrum observed by LHCb in terms of a diquark-antidiquark picture [23], with the predicted masses in Table III. The irregular structure is seen to be due to the rapidly opening $\chi_{c0}\chi_{c0}$ S-wave channel at 6829 MeV, interfering primarily with the 0^{++} $2S$ state. Another possibility is the opening of the J/ψ $\psi(2S)$ threshold at 6783 MeV [21], which should show up in the $2\mu^+ 2\mu^- \pi^+ \pi^-$ final state. We have also updated and extended our prediction [23] for the tetraquark $T(bb\bar{b}\bar{b})$, with the results shown in Table IV. The relative position of the $2\chi_{b0}$ threshold with respect to the predicted $0^{++}(2S)$ state is different from that in the charm case, implying a structure in invariant mass of different shape.

TABLE III. Predicted masses of lowest-lying bound states of a color-antitriplet spin-1 cc diquark and a color-triplet spin-1 $\bar{c}\bar{c}$ antidiquark. The $\chi_{c0}\chi_{c0}$ threshold is 6829 MeV.

	$M(1S)$ (MeV)	$M(2S)$ (MeV)
$J^{PC} = 0^{++}$	6192 ± 25	6871 ± 25
$J^{PC} = 2^{++}$	6429 ± 25	6967 ± 25

TABLE IV. Predicted masses of lowest-lying bound states of a color-antitriplet spin-1 bb diquark and a color-triplet spin-1 $\bar{b}\bar{b}$ antidiquark. The $\chi_{b0}\chi_{b0}$ threshold is 19719 MeV.

	$M(1S)$ (MeV)	$M(2S)$ (MeV)
$J^{PC} = 0^{++}$	18826 ± 25	19434 ± 25
$J^{PC} = 2^{++}$	18956 ± 25	19481 ± 25

ACKNOWLEDGMENTS

We thank Liupan An for discussion of the LHCb data following the LHC seminar and for directing us to the supplemental files with numerical values and errors in Ref. [3]. We also thank Richard Lebed, Mikhail Mikhasenko, Tomasz Skwarnicki, and Changzheng Yuan for useful discussions. The research of M. K. was supported in part by the Natural Science Foundation of China and Israel Science Foundation (NSFC-ISF) Grant No. 3423/19.

APPENDIX: DETAILS OF DATA FITTING

We assume the di-J/ψ spectrum is due to a smooth background with proper threshold behavior:

$$B(M_{\rm inv}) = -C_2 q \exp[(2M(J/\psi) - M_{\rm inv})({\rm GeV})C_3],$$
$$q \equiv (M_{\rm inv}^2/4 - [M(J/\psi)]^2)^{1/2}, \tag{3}$$

of which an amplitude fraction α is added coherently to the sum of three Breit-Wigner resonances each of the form

$$A_i = N_i/D_i, \qquad N_i = C_1 e^{i\phi_i} \eta_i M_{\rm inv} \Gamma_i,$$
$$D_i = M_i^2 - M_{\rm inv}^2 - i M_{\rm inv} \Gamma_i \qquad (i = 1, 2, 3), \tag{4}$$

where M_i and Γ_i are the mass and width, respectively, of the ith resonance. The best fit is obtained for $\alpha = 1$, consistent with the assumption in model II of Ref. [3]. We set $\eta_1 \equiv 1$ and absorb normalization of resonance 1 into the constant C_1. The constants C_2 and C_3 parametrize background normalization and shape, respectively. The observed number of events per 28 MeV bin is then

$$N(M_{\rm inv}) = |T(M_{\rm inv})|^2, \qquad T \equiv B + \sum_1^3 A_i. \tag{5}$$

The numerical data $N \pm dN$ are those in Fig. 3(a) of Ref. [3], restricted to the range $6200 \le M_{\rm inv} \le 7488$ MeV (our choice of upper bound; the data are quoted up

TABLE V. Parameters in alternative fits to data with $\alpha = 0.7156$ (left) and 0 (right). Notation as in Table I.

	$\alpha = 0.7156$			$\alpha = 0$		
Peak i	$i = 1$	$i = 2$	$i = 3$	$i = 1$	$i = 2$	$i = 3$
M_i	6377.9	6898.4	7208.4	6500.0	6811.3	7211.8
Γ_i	286.3	138.8	81.85	729.2	155.9	92.21
C_i	24.65	25.66	1.245	19.25	16.62	7.019
η_i	1.000[a]	0.5227	0.1902	1.000[a]	0.4666	0.2097
ϕ_i	−74.37	−95.02	−51.39	37.61	−46.87	9.263

[a]Input.

to 8000 MeV). We minimize $\chi^2 \equiv \sum_j \{[N_j(\mathrm{fit}) - N_j(\mathrm{data})]/dN_j\}^2$, the sum over 46 28-MeV-wide bins centered on from 6214 to 7474 MeV.

Some parameters are not well determined by the χ^2 criterion and must be regarded as only representative values. To illustrate this, we present in Table V the best fits for $\alpha = 0.7156$ (a local χ^2 minimum with $\chi^2 = 25.86787$ for 32 d.o.f.) and $\alpha = 0$ (giving the largest global χ^2 minimum, $\chi^2 = 26.19538$, for any fixed value of α between 0 and 1).

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