Masses of the $QQ\bar{Q}\bar{Q}$ tetraquarks in the relativistic diquark–antidiquark picture

R. N. Faustov and V. O. Galkin

Institute of Cybernetics and Informatics in Education, Federal Research Center "Computer Science and Control," Russian Academy of Sciences, Vavilov Street 40, 119333 Moscow, Russia

E. M. Savchenko

Faculty of Physics, M. V. Lomonosov Moscow State University, 119991 Moscow, Russia

(Received 29 September 2020; accepted 2 December 2020; published 21 December 2020)

Masses of the ground-state tetraquarks composed from heavy c and b quarks and antiquarks are calculated in the diquark-antidiquark picture in the framework of the relativistic quark model based on the quasipotential approach. The quasipotentials of the quark-quark and diquark-antidiquark interactions are constructed similarly to the previous consideration of mesons and baryons. It is assumed that the diquark and antidiquark interact in the tetraquark as a whole, and the internal structure of the diquarks is taken into account. All such tetraquarks are found above the thresholds of decays to two heavy quarkonia. This is a result of the consideration of the diquark not to be a pointlike object. Therefore, such tetraquarks can be observed as broad structures decaying dominantly to quarkonia. The broad structure next to the di-J/ψ mass threshold, recently observed by the LHCb Collaboration, can correspond to the ground 2^{++}-state tetraquark consisting of four charm quarks.

DOI: 10.1103/PhysRevD.102.114030

I. INTRODUCTION

Theoretical and experimental investigations of the properties of exotic hadrons have attracted substantial interest, especially in last two decades. This subject became a hot topic since the first explicit experimental evidence of the existence of hadrons with compositions different from the usual $q\bar{q}$ for mesons and qqq for baryons became available (for recent reviews, see Refs. [1–3] and references therein). Candidates for both the exotic tetraquark $qq\bar{q}\bar{q}$ and pentaquark $qqqq\bar{q}$ states were found. However, in the literature there is no consensus about the composition of these states [1–3]. For example, significantly different interpretations for the $qq\bar{q}\bar{q}$ candidates were proposed: molecules composed from two mesons loosely bound by the meson exchange, compact tetraquarks composed of a diquark and antidiquark bound by strong forces, hadroquarkonia composed of a heavy quarkonium embedded in a light meson, kinematic cusps, etc. Discrimination between different approaches is a very complicated experimental task.

The investigation of exotic $QQ\bar{Q}\bar{Q}$ states consisting of heavy quarks ($Q = c$ and/or b) is of special interest, since their nature can be determined more easily. They should be predominantly compact tetraquarks. Indeed, a molecular configuration is unlikely. Only heavy $Q\bar{Q}$ mesons can be exchanged between constituents in such a molecule, and the arising Yukawa-type potential is not strong enough to provide binding. Soft gluons can be exchanged between two heavy quarkonia, leading to the so-called QCD van der Waals force. Such a force is known to be attractive, though whether it is strong enough to form a bound state remains unclear. The hadroquarkonium picture is not applicable. Thus, the diquark (QQ)–antidiquark ($\bar{Q}\bar{Q}$) configuration is preferable.

The CMS [4] and LHCb [5] Collaborations searched for the tetraquark states composed only of bottom quarks in the Υ-pair production. No evidence of such states was found. Very recently, the LHCb Collaboration [6] reported results of the study of the J/ψ-pair invariant mass spectrum in proton-proton collision data at the center-of-mass energies of $\sqrt{s} = 7.8$ and 13 TeV. A narrow structure around 6.9 GeV and a broad structure just above twice the J/ψ mass were observed. This discovery caused considerable theoretical activity interpreting these data (see Refs. [7–10] and references therein).

In this paper, we calculate masses of the ground-state $QQ\bar{Q}\bar{Q}$ tetraquarks in the framework of the relativistic quark model based on the quasipotential approach. It is assumed that such tetraquarks are composed from the doubly heavy diquark (QQ) and antidiquark ($\bar{Q}\bar{Q}$). Such approximation significantly simplifies calculations, since instead of the very complicated relativistic four-body problem, we need to solve two more simple relativistic two-body problems. First, masses and wave functions of diquarks (antidiquarks) are obtained by solving the relativistic quark-quark (antiquark-antiquark) quasipotential equation. Second, masses of tetraquarks are calculated by considering them to be the diquark-antidiquark bound states. The quasipotentials of the corresponding interactions are constructed using the same assumptions about their structure and parameters which were previously used for the investigation of the different properties of mesons and baryons [11–15]. The spin-independent and spin-dependent relativistic contributions to the quasipotentials of the QQ interaction in a diquark d and the $d\bar{d}$ interaction in a tetraquark are considered nonperturbatively. It is assumed that a diquark and antidiquark in a tetraquark interact as a whole; thus interactions between quarks from a diquark with antiquarks from an antidiquark are not considered. It is important to point out that diquarks and antidiquarks are not the pointlike objects. Their short-distance interaction with gluons is smeared by the form factors which are calculated in terms of the overlap integrals of the diquark wave functions. Such an approach was previously applied for the calculation of the masses of heavy ($qQ\bar{q}\bar{Q}$, $QQ\bar{q}\bar{q}$, $Qq\bar{q}\bar{q}$) and light ($qq\bar{q}\bar{q}$) tetraquarks [16–20].

This paper is organized as follows: In Sec. II, we describe our relativistic quark model. The quasipotentials of the QQ and $d\bar{d}$ interactions are presented. The masses of the doubly heavy diquarks and the form factors of their interaction with gluons are obtained. In Sec. III, the masses of the $QQ\bar{Q}\bar{Q}$ tetraquarks are calculated. They are confronted with the lowest thresholds for the fall-apart decays to two heavy quarkonia. Detailed comparisons with previous theoretical predictions within different approaches are given. Finally, we present our conclusions and summary of the obtained results in Sec. IV.

II. RELATIVISTIC DIQUARK-ANTIDIQUARK MODEL

For the calculation of the masses of tetraquarks, we use the relativistic quark model based on the quasipotential approach and the diquark-antidiquark picture of tetraquarks. First, we calculate the masses and wave functions (Ψ_d) of the doubly heavy diquarks as the bound quark-quark states. Second, the masses of the tetraquarks and their wave functions (Ψ_T) are obtained for the bound diquark-antidiquark states. These wave functions are solutions of the Schrödinger-type quasipotential equations [18]

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,T}(\mathbf{p}) = \int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_{d,T}(\mathbf{q}), \tag{1}$$

with the on-mass-shell relative momentum squared given by

$$b^2(M) = \frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2} \tag{2}$$

and the relativistic reduced mass

$$\mu_R = \frac{E_1E_2}{E_1+E_2} = \frac{M^4-(m_1^2-m_2^2)^2}{4M^3}. \tag{3}$$

The on-mass-shell energies E_1, E_2 are defined as follows:

$$E_1 = \frac{M^2-m_2^2+m_1^2}{2M}, \qquad E_2 = \frac{M^2-m_1^2+m_2^2}{2M}. \tag{4}$$

The bound-state masses of a diquark or a tetraquark are $M = E_1+E_2$, where $m_{1,2}$ are the masses of the quarks (Q_1 and Q_2) which form the diquark, or of the diquark (d) and antidiquark ($\bar{d}'$) which form the heavy tetraquark (T), while $\mathbf{p}$ is their relative momentum.

The quasipotential operator $V(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. The quark-quark (QQ') interaction quasipotential[1] is considered to be 1/2 of the quark-antiquark ($Q\bar{Q}'$) interaction and is given by [11]

$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q), \tag{5}$$

with

$$\mathcal{V}(\mathbf{p},\mathbf{q};M) = \frac{1}{2}\left[\frac{4}{3}\alpha_sD_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k}) + V^S_{\rm conf}(\mathbf{k})\right].$$

Here, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge, $u(p)$ are the Dirac spinors, and α_s is the running QCD coupling constant with freezing

$$\alpha_s(\mu^2) = \frac{4\pi}{(11-\frac{2}{3}n_f)\ln\frac{\mu^2+M_B^2}{\Lambda^2}}, \tag{6}$$

where the scale μ is chosen to be equal to $2m_1m_2/(m_1+m_2)$, the background mass is $M_B = 2.24\sqrt{A} = 0.95$ GeV, and n_f is the number of flavors. The effective

[1] We consider diquarks in a tetraquark, as in a baryon, to be in the color triplet state, since in the color sextet there is a repulsion between two quarks.

TABLE I. Masses M and form factor parameters of heavy QQ' diquarks. S and A denote scalar and axial-vector diquarks, antisymmetric $[Q,Q']$ and symmetric $\{Q,Q'\}$ in flavor, respectively.

		$Q=c$			$Q=b$		
Quark content	Diquark type	M (MeV)	ξ (GeV)	ζ (GeV^2)	M (MeV)	ξ (GeV)	ζ (GeV^2)
$[Q,c]$	S				6519	1.50	0.59
$\{Q,c\}$	A	3226	1.30	0.42	6526	1.50	0.59
$\{Q,b\}$	A	6526	1.50	0.59	9778	1.30	1.60

long-range vector vertex contains both Dirac and Pauli terms [11]:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}\tilde{k}^\nu, \qquad \tilde{k} = (0,\mathbf{k}), \tag{7}$$

where κ is the long-range anomalous chromomagnetic moment. In the nonrelativistic limit, the vector and scalar confining potentials in configuration space have the form

$$V^V_{\rm conf}(r) = (1-\varepsilon)(Ar+B), \qquad V^S_{\rm conf}(r) = \varepsilon(Ar+B),$$
$$V_{\rm conf}(r) = V^V_{\rm conf}(r) + V^S_{\rm conf}(r) = Ar+B, \tag{8}$$

where ε is the mixing coefficient. Therefore, in the nonrelativistic limit the QQ' quasipotential reduces to

$$V^{\rm NR}_{QQ'}(r) = \frac{1}{2}V^{\rm NR}_{Q\bar{Q}'}(r) = \frac{1}{2}\left(-\frac{4}{3}\frac{\alpha_s}{r} + Ar + B\right), \tag{9}$$

reproducing the usual Cornel potential. Thus, our quasipotential can be viewed as its relativistic generalization. It contains both spin-independent and spin-dependent relativistic contributions.

Constructing the diquark-antidiquark ($d\bar{d}'$) quasipotential, we use the same assumptions about the structure of the short- and long-range interactions. Taking into account the integer spin of a diquark in the color triplet state, the quasipotential is given by [17,18]

$$V(\mathbf{p},\mathbf{q};M) = \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d E_d}}\frac{4}{3}\alpha_s D^{\mu\nu}(\mathbf{k})\frac{\langle d'(P')|J_\nu|d'(Q')\rangle}{2\sqrt{E_{d'}E_{d'}}}$$
$$+\psi^*_d(P)\psi^*_{d'}(P')[J_{d;\mu}J^\mu_{d'}V^V_{\rm conf}(\mathbf{k}) + V^S_{\rm conf}(\mathbf{k})]\psi_d(Q)\psi_{d'}(Q'), \tag{10}$$

where $\psi_d(p)$ and $J_{d;\mu}$ are the wave function and effective long-range vector vertex of the diquark, respectively. The vertex of the diquark-gluon interaction $\langle d(P)|J_\mu|d(Q)\rangle$ accounts for the internal structure of the diquark and leads to emergence of the form factor $F(r)$ smearing the one-gluon exchange potential.

All parameters of the model were fixed previously [11–14] from the consideration of meson and baryon properties. They are as follows: The constituent heavy quark masses are $m_b = 4.88$ GeV, $m_c = 1.55$ GeV. The parameters of the quasipotential are $A = 0.18$ GeV^2, $B = -0.3$ GeV, $\Lambda = 413$ MeV. The mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$, and the universal Pauli interaction constant $\kappa = -1$.

The resulting diquark-antidiquark potential for the tetraquark ground states (the orbital momentum $L=0$), where quark energies $\epsilon_{1,2}(p)$ were replaced by the on-shell energies $E_{1,2}$ (4) to remove the nonlocality, is given by [18]:

$$V(r) = \hat{V}_{\rm Coul}(r) + V_{\rm conf}(r) + \frac{1}{E_1E_2}\Big\{\mathbf{p}[\hat{V}_{\rm Coul}(r) + V^V_{\rm conf}(r)]\mathbf{p} - \frac{1}{4}\Delta V^V_{\rm conf}(r) + \frac{2}{3}\Delta\hat{V}_{\rm Coul}(r)\mathbf{S}_1\cdot\mathbf{S}_2\Big\}. \tag{11}$$

Here

$$\hat{V}_{\rm Coul}(r) = -\frac{4}{3}\alpha_s\frac{F_1(r)F_2(r)}{r}$$

is the Coulomb-like one-gluon exchange potential which takes into account the finite sizes of the diquark and antidiquark through corresponding form factors $F_{1,2}(r)$. $\mathbf{S}_{1,2}$ are the diquark and antidiquark spins. The numerical analysis shows that this form factor can be approximated with high accuracy by the expression

$$F(r) = 1 - e^{-\xi r - \zeta r^2}. \tag{12}$$

Such a form factor smears the one-gluon exchange potential and removes spurious singularities in the local relativistic quasipotential, thus allowing one to use it nonperturbatively to find the numerical solution of the quasipotential equation. The masses and parameters of heavy diquarks are the same as in the doubly heavy baryons [12] and are given in Table I.

III. MASSES OF $QQ\bar{Q}\bar{Q}$ TETRAQUARKS

We substitute the quasipotential [Eq. (11)] in the quasipotential equation (1) and solve the resulting differential equation numerically. The calculated masses M of the neutral $QQ'\bar{Q}\bar{Q}'$ tetraquarks composed of the heavy diquark (QQ', $Q = b, c$) and heavy antidiquark ($\bar{Q}\bar{Q}'$) are given in Table II. The masses of the charged heavy $QQ'\bar{Q}\bar{Q}'$ tetraquarks are presented in Table III. In these tables, we give the values of the lowest thresholds T for decays into two corresponding heavy mesons [$(Q\bar{Q}')$], which were calculated using the measured masses of these mesons [21]. We also show values of the difference of the tetraquark and threshold masses, $\Delta = M - T$. If this quantity is negative, then the tetraquark lies below the threshold of the fall-apart decay into two mesons and thus should be a narrow state. The states with small positive values of Δ could be also observed as resonances, since their decay rates will be suppressed by the phase space. All other states are expected to be broad and thus difficult to observe.

From these tables, we see that the predicted masses of almost all $QQ\bar{Q}\bar{Q}$ tetraquarks lie significantly higher than the thresholds of the fall-apart decays to the lowest allowed two-quarkonium states. All these states should be broad, since they can decay to corresponding quarkonium states through quark and antiquark rearrangements, and these decays are not suppressed either dynamically or kinematically. This conclusion is in accord with the current experimental data. Indeed, the CMS [4] and LHCb [5] Collaborations have not observed narrow beautiful tetraquarks in the $\Upsilon(1S)$-pair production. Note that the lattice nonrelativistic QCD [22] calculations did not find a signal for the $bb\bar{b}\bar{b}$ tetraquarks below the lowest noninteracting two-bottomonium threshold. On the other hand, the broad structure near the di-J/ψ mass threshold very recently observed by the LHCb [6] can correspond to the 2^{++} state of the $cc\bar{c}\bar{c}$ tetraquark, with a mass predicted to be 6367 MeV. The narrow structure, $X(6900)$ [6], could be the orbital or radial excitation of this tetraquark. Such excited states can be narrow despite the large phase space, since it will be necessary in the fall-apart process to overcome the suppression either due to the centrifugal barrier for the orbital excitations or due to the presence of the nodes in the wave function of the radially excited state.

In Tables IV–VII, we compare our predictions for the masses of $QQ\bar{Q}\bar{Q}$ tetraquarks with the results of previous calculations [9,23–42]. The nonrelativistic

TABLE II. Masses M of the neutral heavy diquark (QQ')-antidiquark $(\bar{Q}\bar{Q}')$ states. T is the threshold for the decays into two heavy-$(Q\bar{Q}')$ mesons, and $\Delta = M - T$. All values are given in MeV.

Composition	$d\bar{d}$	J^{PC}	M	Threshold	T	Δ
$cc\bar{c}\bar{c}$	$A\bar{A}$	0^{++}	6190	$\eta_c(1S)\eta_c(1S)$	5968	222
				$J/\psi(1S)J/\psi(1S)$	6194	−4
		1^{+-}	6271	$\eta_c(1S)J/\psi(1S)$	6081	190
		2^{++}	6367	$J/\psi(1S)J/\psi(1S)$	6194	173
$cb\bar{c}\bar{b}$	$A\bar{A}$	0^{++}	12 813	$\eta_c(1S)\eta_b(1S)$	12 383	430
				$J/\psi(1S)\Upsilon(1S)$	12 557	256
				$B_c^{\pm}B_c^{\mp}$	12 550	263
				$B_c^{*\pm}B_c^{*\mp}$	12 666	147
		1^{+-}	12 826	$\eta_c(1S)\Upsilon(1S)$	12 444	382
				$J/\psi(1S)\eta_b(1S)$	12 496	330
				$B_c^{\pm}B_c^{*\mp}$	12 608	218
				$B_c^{*\pm}B_c^{*\mp}$	12 666	160
		2^{++}	12 849	$J/\psi(1S)\Upsilon(1S)$	12 557	292
				$B_c^{*\pm}B_c^{*\mp}$	12 666	183
	$\frac{1}{\sqrt{2}}(A\bar{S} \pm S\bar{A})$	1^{++}	12 831	$J/\psi(1S)\Upsilon(1S)$	12 557	274
				$B_c^{\pm}B_c^{*\mp}$	12 608	223
				$B_c^{*\pm}B_c^{*\mp}$	12 666	165
		1^{+-}	12 831	$\eta_c(1S)\Upsilon(1S)$	12 444	387
				$J/\psi(1S)\eta_b(1S)$	12 496	335
				$B_c^{\pm}B_c^{*\mp}$	12 608	223
				$B_c^{*\pm}B_c^{*\mp}$	12 666	165
	$S\bar{S}$	0^{++}	12 824	$\eta_c(1S)\eta_b(1S)$	12 383	441
				$J/\psi(1S)\Upsilon(1S)$	12 557	267
				$B_c^{\pm}B_c^{\mp}$	12 550	274
				$B_c^{*\pm}B_c^{*\mp}$	12 666	158
$bb\bar{b}\bar{b}$	$A\bar{A}$	0^{++}	19 314	$\eta_b(1S)\eta_b(1S)$	18 797	517
				$\Upsilon(1S)\Upsilon(1S)$	18 920	394
		1^{+-}	19 320	$\eta_b(1S)\Upsilon(1S)$	18 859	461
		2^{++}	19 330	$\Upsilon(1S)\Upsilon(1S)$	18 920	410

TABLE III. Masses M of the charged heavy diquark-antidiquark states. T is the threshold for the decays into two heavy $(Q\bar{Q}')$ mesons, and $\Delta = M - T$. All values are given in MeV.

Composition	$d\bar{d}$	J^P	M	Threshold	T	Δ
$cc\bar{c}\bar{b}$, $cb\bar{c}\bar{c}$	$A\bar{A}$	0^+	9572	$\eta_c(1S)B_c^{\pm}$	9259	313
				$J/\psi(1S)B_c^{*\pm}$	9430	142
		1^+	9602	$\eta_c(1S)B_c^{*\pm}$	9317	285
				$J/\psi(1S)B_c^{\pm}$	9372	230
				$J/\psi(1S)B_c^{*\pm}$	9430	172
		2^+	9647	$J/\psi(1S)B_c^{*\pm}$	9430	217
	$A\bar{S}$, $S\bar{A}$	1^+	9619	$\eta_c(1S)B_c^{*\pm}$	9317	302
				$J/\psi(1S)B_c^{\pm}$	9372	247
				$J/\psi(1S)B_c^{*\pm}$	9430	189
$cc\bar{b}\bar{b}$, $bb\bar{c}\bar{c}$	$A\bar{A}$	0^+	12 846	$B_c^{\pm}B_c^{\pm}$	12 550	296
				$B_c^{*\pm}B_c^{*\pm}$	12 666	180
		1^+	12 859	$B_c^{\pm}B_c^{*\pm}$	12 608	251
				$B_c^{*\pm}B_c^{*\pm}$	12 666	193
		2^+	12 883	$B_c^{*\pm}B_c^{*\pm}$	12 666	217
$cb\bar{b}\bar{b}$, $bb\bar{c}\bar{b}$	$A\bar{A}$	0^+	16 109	$B_c^{\pm}\eta_b(1S)$	15 674	435
				$B_c^{*\pm}\Upsilon(1S)$	15 793	316
		1^+	16 117	$B_c^{\pm}\Upsilon(1S)$	15 735	382
				$B_c^{*\pm}\eta_b(1S)$	15 732	385
				$B_c^{*\pm}\Upsilon(1S)$	15 793	324
		2^+	16 132	$B_c^{*\pm}\Upsilon(1S)$	15 793	339
	$S\bar{A}$, $A\bar{S}$	1^+	16 117	$B_c^{\pm}\Upsilon(1S)$	15 735	382
				$B_c^{*\pm}\eta_b(1S)$	15 732	385
				$B_c^{*\pm}\Upsilon(1S)$	15 793	324

quark model and diquark-antidiquark structure of tetraquarks was employed in Refs. [23,24], while the authors of Refs. [9,26] used for the calculations the string-junction picture and the constituent diquark-antidiquark model. References [25,27,28] present results obtained in different versions of the QCD sum rules. A simple constituent quark model with the color-magnetic interaction was applied in Ref. [29]. The relativized diquark-antidiquark model and variational method with harmonic oscillator trial wave functions were employed in Refs. [30,35]; mass inequality

TABLE IV. Comparison of theoretical predictions for the masses of the neutral $(QQ)(\bar{Q}\bar{Q})$ tetraquarks composed from the same flavor heavy quarks and antiquarks (in MeV).

	$cc\bar{c}\bar{c}$			$bb\bar{b}\bar{b}$		
Reference	0^{++}	1^{+-}	2^{++}	0^{++}	1^{+-}	2^{++}
This paper	6190	6271	6367	19 314	19 320	19 330
[23,24]	5966	6051	6223	18 754	18 808	18 916
[25]	6460–6470	6370–6510	6370–6510	18 460–18 490	18 320–18 540	18 320–18 530
[9,26]	6192 ± 25		6429 ± 25	18826 ± 25		18956 ± 25
[27,28]	5990 ± 80	6050 ± 80	6090 ± 80	18840 ± 90	18840 ± 90	18850 ± 90
[29]	6797	6899	6956	20 155	20 212	20 243
[30]	< 6140			18 750		
[31,32]	5969	6021	6115			
[33,34]	6487	6500	6524	19 322	19 329	19 341
[35]	5883	6120	6246	18 748	18 828	18 900
[36]				18690 ± 30		
[37]	6425	6425	6432	19 247	19 247	19 249
[38]	6407	6463	6486	19 329	19 373	19 387
[39]				19 178	19 226	19 236
[40]	6314	6375	6407	19 237	19 264	19 279
[41]	6542	6515	6543	19 255	19 251	19 262

TABLE V. Comparison of theoretical predictions for the masses of the $(cb)(\bar{c}\bar{b})$ tetraquarks (in MeV).

	$A\bar{A}$			$\frac{1}{\sqrt{2}}(A\bar{S} \pm S\bar{A})$		$S\bar{S}$
Reference	0^{++}	1^{+-}	2^{++}	1^{++}	1^{+-}	0^{++}
This paper	12 813	12 826	12 849	12 831	12 831	12 824
[23]	12 359	12 424	12 566	12 485	12 488	12 471
[29]	13 483	13 520	13 590	13 510	13 592	13 553
[30]	< 12 620					
[33]	13 035	13 047	13 070	13 056	13 052	13 050
[35]	12 374	12 491	12 576	12 533	12 533	12 521
[38]	12 829	12 881	12 925			
[42]	12 746	12 804	12 809		12 776	

relations among tetraquarks and heavy quarkonia were also obtained [30]. Different versions of the nonrelativistic quark model and diquark-antidiquark picture were used in Refs. [31–34,37]. The diffusion Monte Carlo method was applied to solve the nonrelativistic four-body problem for the $bb\bar{b}\bar{b}$ tetraquark in Ref. [36]. In Ref. [38], the multiquark color flux-tube model was employed. The meson-meson and diquark-antidiquark structures were considered: in the nonrelativistic chiral quark model using the Gaussian expansion method in Refs. [39,42], in the nonrelativistic quark delocaliztion color screening model using the resonating group method for bound states [40], and in the extended relativized quark model using a variational approach with Gaussian wave functions in Ref. [41]. The diquark-antidiquark picture with the potential taken from lattice calculations was studied in Ref. [43], and the simplifying dynamical assumptions were investigated: whether color-sextet diquark couplings are suppressed, and whether spin couplings between the diquark and antidiquark are suppressed. Note that we and authors of Refs. [9,23,24,26,30–32,35] consider diquarks and antidiquarks only in the color triplet and antitriplet color states,

TABLE VI. Comparison of theoretical predictions for the masses of the charged $(QQ)(\bar{Q}\bar{Q}')$ tetraquarks (in MeV).

	$A\bar{A}$			$A\bar{S}, S\bar{A}$
Reference	0^+	1^+	2^+	1^+
		$cc\bar{c}\bar{b}$, $cb\bar{c}\bar{c}$		
This paper	9572	9602	9647	9619
[29]	10 144	10 282	10 273	10 174
[30]	< 9390			
[33]	9740	9749	9768	9746
[38]	9670	9683	9732	
		$bb\bar{c}\bar{b}$, $cb\bar{b}\bar{b}$		
This paper	16 109	16 117	16 132	16 117
[29]	16 823	16 840	16 917	16 915
[30]	< 15 770			
[33]	16 158	16 164	16 176	16 157
[38]	16 126	16 130	16 182	

TABLE VII. Comparison of theoretical predictions for the masses of the $cc\bar{b}\bar{b}$, $bb\bar{c}\bar{c}$ tetraquarks (in MeV).

	$A\bar{A}$		
Reference	0^+	1^+	2^+
This paper	12 846	12 859	12 883
[29]	13 496	13 560	13 595
[30]	< 12 580		
[33]	12 953	12 960	12 972
[35]	12 445	12 536	12 614
[37]	12 866	12 864	12 868
[38]	12 906	12 945	12 960
[42]	12 892	12 898	12 905

while the color sextet and antisextet configurations and their mixing are accounted for in Refs. [29,33,34,37–42]. In most of the previous calculations, diquarks and antidiquarks were considered to be pointlike. Our calculation shows that the account of the diquark structure (size) weakens the Coulomb-like one-gluon exchange potential, thus increasing tetraquark masses and reducing spin-spin splittings. We can see from Tables IV–VII that there are significant disagreements between different theoretical approaches. Indeed, Refs. [23,24,27,28,31,32,35,36] predict heavy tetraquark masses that are below or slightly above the thresholds of the decays to two quarkonia, and thus, stable or significantly suppressed against fall-apart decays with a very narrow decay width. On the other hand, our model and other approaches predict tetraquark masses significantly above these thresholds, and thus they can be observed only as broad resonances. Note that the arguments that these tetraquarks should be unbound were also given on the basis of the hyperspherical harmonic expansion [44], the string dynamics [45], and the Hall-Post inequalities [46].

IV. CONCLUSIONS

We calculated the masses of ground-state tetraquarks composed only of heavy (b and/or c) quarks and antiquarks in the framework of the diquark-antidiquark picture and relativistic quark model based on the quasipotential approach. It was assumed that two heavy quarks and two heavy antiquarks will form a doubly heavy diquark and antidiquark, respectively. The dynamics of quarks in the diquark is governed by the relativistic QQ quasipotential, which is one half of the $Q\bar{Q}$ potential in the heavy quarkonium. Masses and wave functions of diquarks were calculated by the numerical solution of the quasipotential equation. The obtained diquark wave functions were used for the evaluation of the form factors of the diquark-gluon interaction $F(r)$. Then, the $QQ\bar{Q}\bar{Q}$ tetraquark was considered as a bound diquark-antidiquark system. It was assumed that diquarks and antidiquarks interact as a whole. Constructing the quasipotential of the d-$\bar{d}$ interaction, the same assumptions about the structure of the long-range confining interaction were used with the correction to the integer spin of the diquark. In the potential of the one-gluon exchange between the diquark and antidiquark, the form factors $F(r)$ of the diquark-gluon interaction were introduced. They are expressed as the overlap integrals of the diquark wave functions and take into account the internal structure (finite size) of the diquarks and antidiquarks. These form factors significantly weaken the Coulomb-like potential, thus increasing the masses of the tetraquarks and reducing spin splittings. This effect is especially pronounced for the $bb\bar{b}\bar{b}$ tetraquarks, since they have a larger Coulomb contribution due to their smaller size. Note that the approaches with a pointlike diquark substantially underestimate the mass of the doubly charmed baryon Ξ_{cc}, while our model correctly predicted its mass [12] long before its experimental discovery.

It was found that the predicted masses of all ground-state $QQ\bar{Q}\bar{Q}$ tetraquarks are above the thresholds for decays into two heavy $(Q\bar{Q})$ mesons. Therefore, they should rapidly fall apart into the two lowest allowed quarkonium states. Such decays proceed through quark rearrangements and are not suppressed dynamically or kinematically. These states should be broad, and are thus difficult to observe experimentally. The states, with masses predicted to be less than 200 MeV higher than the lowest allowed thresholds, are the 1^{+-} and 2^{++} states of the $cc\bar{c}\bar{c}$ tetraquark. They have the smallest phase space for the decay to two charmonium states. The former one decays mainly to $\eta_c J/\psi$, while the latter one decays to $J/\psi J/\psi$. The 2^{++} $cc\bar{c}\bar{c}$ state with the predicted mass 6367 MeV can correspond to the broad structure recently observed by the LHCb Collaboration [6] in the mass spectrum of J/ψ pairs produced in proton-proton collisions. On the other hand, all ground-state $bb\bar{b}\bar{b}$ tetraquarks have masses significantly (400–500 MeV) higher than corresponding thresholds, and thus should be very broad. This agrees well with the absence of the narrow beautiful tetraquarks in the Υ-pair production reported by the CMS [4] and LHCb [5] Collaborations.

ACKNOWLEDGMENTS

We are grateful to A. Berezhnoy and D. Ebert for support and valuable discussions.

[1] Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Pentaquark and tetraquark states, Prog. Part. Nucl. Phys. **107**, 237 (2019).
[2] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, The *XYZ* states: experimental and theoretical status and perspectives, Phys. Rep. **873**, 1 (2020).
[3] G. Yang, J. Ping, and J. Segovia, Tetra- and penta-quark structures in the constituent quark model, Symmetry **12**, 1869 (2020).
[4] A. M. Sirunyan *et al.* (CMS Collaboration), Measurement of the $\Upsilon(1S)$ pair production cross section and search for resonances decaying to $\Upsilon(1S)\mu^+\mu^-$ in proton-proton collisions at $\sqrt{s} = 13$ TeV, Phys. Lett. B **808**, 135578 (2020).
[5] R. Aaij *et al.* (LHCb Collaboration), Search for beautiful tetraquarks in the $\Upsilon(1S)\mu^+\mu^-$ invariant-mass spectrum, J. High Energy Phys. 10 (2018) 086.
[6] R. Aaij *et al.* (LHCb Collaboration), Observation of structure in the J/ψ-pair mass spectrum, Sci. Bull. **65**, 1983 (2020).
[7] L. Maiani, J/ψ-pair resonances by LHCb: A new revolution? arXiv:2008.01637.
[8] J.-M. Richard, About the $J/\psi J/\psi$ peak of LHCb: Fully-charmed tetraquark? arXiv:2008.01962.
[9] M. Karliner and J. L. Rosner, Interpretation of structure in the di-J/ψ spectrum, arXiv:2009.04429 [Phys. Rev. D (to be published)].
[10] X.-K. Dong, V. Baru, F.-K. Guo, C. Hanhart, and A. Nefediev, Coupled-channel interpretation of the LHCb double-J/ψ spectrum and hints of a new state near $J/\psi J/\psi$ threshold, arXiv:2009.07795.
[11] D. Ebert, R. N. Faustov, and V. O. Galkin, Properties of heavy quarkonia and B_c mesons in the relativistic quark model, Phys. Rev. D **67**, 014027 (2003).
[12] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, Mass spectra of doubly heavy baryons in the relativistic quark model, Phys. Rev. D **66**, 014008 (2002).
[13] D. Ebert, R. N. Faustov, and V. O. Galkin, Spectroscopy and Regge trajectories of heavy quarkonia and B_c mesons, Eur. Phys. J. C **71**, 1825 (2011).
[14] D. Ebert, R. N. Faustov, and V. O. Galkin, Spectroscopy and Regge trajectories of heavy baryons in the relativistic quark-diquark picture, Phys. Rev. D **84**, 014025 (2011).
[15] R. Faustov and V. Galkin, Strange baryon spectroscopy in the relativistic quark model, Phys. Rev. D **92**, 054005 (2015).
[16] D. Ebert, R. Faustov, and V. Galkin, Masses of heavy tetraquarks in the relativistic quark model, Phys. Lett. B **634**, 214 (2006).
[17] D. Ebert, R. N. Faustov, and V. O. Galkin, Excited heavy tetraquarks with hidden charm, Eur. Phys. J. C **58**, 399 (2008).
[18] D. Ebert, R. N. Faustov, V. O. Galkin, and W. Lucha, Masses of tetraquarks with two heavy quarks in the relativistic quark model, Phys. Rev. D **76**, 114015 (2007).
[19] D. Ebert, R. Faustov, and V. Galkin, Masses of light tetraquarks and scalar mesons in the relativistic quark model, Eur. Phys. J. C **60**, 273 (2009).

[20] D. Ebert, R. Faustov, and V. Galkin, Masses of tetraquarks with open charm and bottom, Phys. Lett. B **696**, 241 (2011).
[21] M. Tanabashi *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **98**, 030001 (2018).
[22] C. Hughes, E. Eichten, and C. T. H. Davies, Searching for beauty-fully bound tetraquarks using lattice nonrelativistic QCD, Phys. Rev. D **97**, 054505 (2018).
[23] A. V. Berezhnoy, A. V. Luchinsky, and A. A. Novoselov, Heavy tetraquarks production at the LHC, Phys. Rev. D **86**, 034004 (2012).
[24] A. Berezhnoy, A. Likhoded, A. Luchinsky, and A. Novoselov, Formation of a $4c$ tetraquark in J/ψ-meson pair production at LHC, Phys. At. Nucl. **75**, 1006 (2012).
[25] W. Chen, H.-X. Chen, X. Liu, T. Steele, and S.-L. Zhu, Hunting for exotic doubly hidden-charm/bottom tetraquark states, Phys. Lett. B **773**, 247 (2017).
[26] M. Karliner, S. Nussinov, and J. L. Rosner, $QQ\bar{Q}\bar{Q}$ states: Masses, production, and decays, Phys. Rev. D **95**, 034011 (2017).
[27] Z.-G. Wang, Analysis of the $QQ\bar{Q}\bar{Q}$ tetraquark states with QCD sum rules, Eur. Phys. J. C **77**, 432 (2017).
[28] Z.-G. Wang and Z.-Y. Di, Analysis of the vector and axialvector $QQ\bar{Q}\bar{Q}$ tetraquark states with QCD sum rules, Acta Phys. Pol. B **50**, 1335 (2019).
[29] J. Wu, Y.-R. Liu, K. Chen, X. Liu, and S.-L. Zhu, Heavy-flavored tetraquark states with the $QQ\bar{Q}\bar{Q}$ configuration, Phys. Rev. D **97**, 094015 (2018).
[30] M. N. Anwar, J. Ferretti, F.-K. Guo, E. Santopinto, and B.-S. Zou, Spectroscopy and decays of the fully-heavy tetraquarks, Eur. Phys. J. C **78**, 647 (2018).
[31] V. R. Debastiani and F. S. Navarra, Spectroscopy of the all-charm tetraquark, *Proc. Sci.*, Hadron2017 (2018) 238.
[32] V. R. Debastiani and F. S. Navarra, A non-relativistic model for the $[cc][\bar{c}\bar{c}]$ tetraquark, Chin. Phys. C **43**, 013105 (2019).
[33] M.-S. Liu, Q.-F. Lu, X.-H. Zhong, and Q. Zhao, All-heavy tetraquarks, Phys. Rev. D **100**, 016006 (2019).
[34] M.-S. Liu, F.-X. Liu, X.-H. Zhong, and Q. Zhao, Full-heavy tetraquark states and their evidences in the LHCb di-J/ψ spectrum, arXiv:2006.11952.
[35] M. A. Bedolla, J. Ferretti, C. D. Roberts, and E. Santopinto, Spectrum of fully-heavy tetraquarks from a diquark + antidiquark perspective, Eur. Phys. J. C **80**, 1004 (2020).
[36] Y. Bai, S. Lu, and J. Osborne, Beauty-full Tetraquarks, Phys. Lett. B **798**, 134930 (2019).
[37] G.-J. Wang, L. Meng, and S.-L. Zhu, Spectrum of the fully-heavy tetraquark state $QQ\bar{Q}'\bar{Q}'$, Phys. Rev. D **100**, 096013 (2019).
[38] C. Deng, H. Chen, and J. Ping, Towards the understanding of fully-heavy tetraquark states from various models, arXiv:2003.05154 [Phys. Rev. D (to be published)].
[39] X. Chen, Analysis of hidden-bottom $bb\bar{b}\bar{b}$ states, Eur. Phys. J. A **55**, 106 (2019).
[40] X. Jin, Y. Xue, H. Huang, and J. Ping, Full-heavy tetraquarks in constituent quark models, Eur. Phys. J. C **80**, 1083 (2020).
[41] Q.-F. Lü, D.-Y. Chen, and Y.-B. Dong, Masses of fully heavy tetraquarks $QQ\bar{Q}\bar{Q}$ in an extended relativized quark model, Eur. Phys. J. C **80**, 871 (2020).
[42] X. Chen, Fully-heavy tetraquarks: $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$, Phys. Rev. D **100**, 094009 (2019).
[43] J. F. Giron and R. F. Lebed, The simple spectrum of $c\bar{c}c\bar{c}$ states in the dynamical diquark model, Phys. Rev. D **102**, 074003 (2020).
[44] J. Vijande, N. Barnea, and A. Valcarce, Hyperspherical harmonic study of identical-flavor four-quark systems, Nucl. Phys. **A790**, 542 (2007).
[45] J.-M. Richard, A. Valcarce, and J. Vijande, String dynamics and metastability of all-heavy tetraquarks, Phys. Rev. D **95**, 054019 (2017).
[46] J.-M. Richard, A. Valcarce, and J. Vijande, Hall-Post inequalities: Review and application to molecules and tetraquarks, Ann. Phys. (Amsterdam) **412**, 168009 (2020).