

Chiral crossover transition from the Dyson-Schwinger equations in a sphere

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Within the framework of Dyson-Schwinger equations of QCD, we study the effect of finite volume on the chiral phase transition in a sphere with the MIT boundary condition. We find that the chiral quark condensate $\langle\bar{\psi}\psi\rangle$ and pseudotransition temperature T_{pc} of the crossover decreases as the volume decreases, until there is no chiral crossover transition at last. We find that the system for $R = \infty$ fm is indistinguishable from $R = 10$ fm and there is a significant decrease in T_{pc} with R as $R < 4$ fm. When $R < 1.5$ fm, there is no chiral transition in the system.

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I. INTRODUCTION

The quantum chromodynamics (QCD) as a underlying theory describing strong interactions exhibits two fascinating aspects: confinement and dynamical chiral symmetry breaking (DCSB). As the temperature increases, the strongly interacting matter will undergo a phase transition from hadronic matter to quark-gluon plasma (QGP) with deconfinement and chiral restoration. QCD phase transitions are experimental and theoretical frontiers, which have been studied in relativistic heavy-ion collisions (HICs) at CERN (France/Switzerland), BNL (USA), and GSI (Germany) [1,2]. However, most of the theoretical calculations are based on the thermodynamic limit (namely, the volume of the system $V \rightarrow \infty$). It is worth bearing in mind that the QGP system produced in HICs always has a finite volume, depending on the collision nuclei, the center of mass energy, and the centrality. According to the UrQMD transport approach [3], the volume of Au-Au and Pb-Pb collisions before freeze-out is about 50–250 fm³ [4]. It is believed that the radii of possible quark gluon plasma are estimated to be 2–10 fm. Therefore, there is a problem we need to consider: does the size and shape of QGP system produced in HICs affect the phase transition?

There has been a lot of theoretical studies for the effect of finite volume on QGP phase transition, within the Nambu-Jona-Lasinio (NJL) model [5–7], Polyakov-Nambu-Jona-Lasinio model [8–10], quark-meson model [11–13], and

Dyson-Schwinger equations (DSEs) [14–19]. In most existing theoretical studies, for the sake of convenience, the systems are usually treated as a cube, and antiperiodic boundary condition (APBC) is used. However, when the volume of the fireball produced in collision is small enough, not only its size but also its shape have a non-negligible effect on the QCD phase transition. In order to simulate more realistic condition such as the fireball expected to arise in HICs, the authors of Ref. [20] consider a sphere with the MIT boundary condition under the framework of NJL model for the first time. However, it should be pointed out that the NJL model is a non-renormalizable theory, in which the confinement property is not preserved. Meanwhile, the gauge sector of QCD, i.e., the gluon degrees of freedom, is lost. This led us to consider a more realistic approach to study QGP phase transitions in a sphere with the MIT boundary condition.

In this work, we employ the framework of Dyson-Schwinger equations to deal with the finite size effects in a sphere. DSEs has been widely used in studying strongly interacting phenomena in vacuum and in heat bath [21–32]. It is capable of simultaneously implementing color confinement and expressing DCSB [30,33–38]. Recently, as mentioned before, it is used to study the effects of finite volume on QGP crossover transition. However, those studies were all for cubic systems, i.e., APBC. In this work, we study finite size effects with MIT boundary condition for the first time.

This paper is organized as follows: In Sec. II, we introduce the DSEs at finite temperature within the MIT boundary condition. In Sec. III, chiral quark sation and chiral susceptibility of a spherical system at different radii

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are defined and calculated. On this basis, the influence of system volume and shape on chiral crossover transition temperature is discussed, and the results are compared with those obtained by traditional APBC based on cubic systems. In Sec. IV, we present a brief summary.

II. DYSONSCHWINGER EQUATIONS IN A FINITE VOLUME

The formulation of DSEs at nonzero temperature is described in Refs. [21,36]. The $T \neq 0$ dressed-quark propagator is obtained from the following gap equation:

$$\begin{aligned} S(\tilde{\omega}_n, \vec{p})^{-1} &= Z_2(i\vec{\gamma} \cdot \vec{p} + i\gamma_4\tilde{\omega}_n + Z_m m) \\ &+ Z_1 T \sum_l \int \frac{d^3 p}{(2\pi)^3} g^2 D_{\mu\nu}(k_\Omega) \frac{\lambda_a}{2} \gamma_\mu S(\tilde{\omega}_l, \vec{q}) \frac{\lambda_a}{2} \gamma_\nu, \end{aligned} \quad (1)$$

where $\tilde{\omega}_n = (2n+1)\pi T$ are the fermionic Matsubara frequencies; m is the current-quark mass and $m = 0$ defines the chiral limit. $Z_{1,2,m}$ are the vertex, quark field, and mass renormalization constants, respectively. As we employ an ultraviolet-finite model, renormalization is unnecessary, i.e., $Z_{1,2,m} = 1$. $D_{\mu\nu}(k_\Omega)$ is the dressed-gluon propagator which has the form

$$g^2 D_{\mu\nu}(k_\Omega) = P_{\mu\nu}^T(k_\Omega) \mathcal{D}(k_\Omega^2) + P_{\mu\nu}^L(k_\Omega) \mathcal{D}(k_\Omega^2 + m_g^2), \quad (2)$$

where $k_\Omega = (\tilde{\omega}_n - \tilde{\omega}_l, \vec{p} - \vec{q})$, and $m_g = (16/5)\pi^2 T^2$ is the gluon Debye mass. Since the temperature breaks the Lorentz symmetry, the tensor structure of gluon has both transverse and longitudinal parts, where $P_{\mu\nu}^{T,L}$ are transverse and longitudinal projection operators, respectively,

$$P_{\mu\nu}^T(k_\Omega) := \begin{cases} 0, & \mu \text{ and/or } \nu = 4 \\ \delta_{ij} - \frac{k_i k_j}{k^2}, & \mu, \nu = i, j = 1, 2, 3 \end{cases}, \quad (3)$$

$$P_{\mu\nu}^L(k_\Omega) + P_{\mu\nu}^T(k_{\Omega_k}) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}. \quad (4)$$

The choice of interaction kernel is not unique. In this work, we use a simplified form of Maris-Tandy model [32],

$$\mathcal{D}(k_\Omega^2) = D_0 \frac{4\pi^2}{\omega^6} k_\Omega^2 e^{-k_\Omega^2/\omega^2}. \quad (5)$$

The parameters D_0 and ω are not independent: a change in D_0 can be compensated by an alteration of ω [38]. In this paper, we choose a typical value $\omega = 0.5$ GeV with $D_0\omega = (0.8 \text{ GeV})^3$ [32].

The gap equation's solution can be generally expressed as

$$\begin{aligned} S(\tilde{\omega}_n, \vec{p})^{-1} &= i\vec{\gamma} \cdot \vec{p} A(\vec{p}^2, \tilde{\omega}_n^2) + B(\vec{p}^2, \tilde{\omega}_n^2) \\ &+ i\gamma_4 \tilde{\omega}_n C(\vec{p}^2, \tilde{\omega}_n^2) + \vec{\gamma} \cdot \vec{p} \gamma_4 \tilde{\omega}_n D(\vec{p}^2, \tilde{\omega}_n^2), \end{aligned} \quad (6)$$

with the four scalar dressing functions A, B, C, D. The dressing function D, however, is power law suppressed in the UV [21] and does not contribute in all cases investigated here.

The mass function of quarks can be defined as [39,40]

$$M(\tilde{\omega}_n^2, \vec{p}^2) = \frac{B(\tilde{\omega}_n^2, \vec{p}^2)}{C(\tilde{\omega}_n^2, \vec{p}^2)} \quad (7)$$

and the Euclidean constituent mass $M_E := \{p^2 | p^2 > 0, p^2 = M(\tilde{\omega}_0^2, \vec{p}^2)\}$, which provides a realistic estimate of the quarks active quasiparticle mass [41].

For finite size system, Eq. (1) should be modified. Three momenta will be discretized by the boundary condition

$$\int \frac{d^3 \vec{p}}{(2\pi)^3} \rightarrow \frac{1}{V} \sum_{p_k}. \quad (8)$$

The allowed values of momentum modes depend on the selection of boundary conditions. For APBC, we have

$$\vec{p}_k = \sum_{k_i=\pm 1, \pm 3, \dots} \frac{k_i \pi}{L} \hat{e}_i, \quad (9)$$

with L is the size of cubic box. Another boundary condition is multiple reflection expansion (MRE), which introduces an IR cutoff in the momentum space and modifies the density of states [19].

However, APBC works on a cubic box and MRE becomes invalid for very small volume. In this work, we use MIT boundary condition. Under spherical MIT boundary condition, the allowed momentum values are given by the following eigen equations:

$$j_{l_\kappa}(pR) = -\text{sgn}(\kappa) \frac{P}{E+M} \bar{j}_{l_\kappa}(pR), \quad (10)$$

where

$$l_\kappa = \begin{cases} -\kappa - 1 & \text{for } \kappa < 0 \\ \kappa & \text{for } \kappa > 0 \end{cases}$$

$$\bar{l}_\kappa = \begin{cases} -\kappa & \text{for } \kappa < 0 \\ \kappa - 1 & \text{for } \kappa > 0 \end{cases},$$

$\kappa = \pm 1, \pm 2, \dots$ and $j_l(x)$ is the l th ordered spherical Bessel function. R is the radius of sphere, p is the allowed momentum value. In the MIT bag model, the quark inside the bag has current-quark mass, and the equation of motion is the free Dirac equation with the MIT boundary condition.

So the mass which appears in the eigen equation is the current-quark mass. In Ref. [20], the NJL model is constrained inside a sphere by the MIT boundary condition. Then one takes the homogeneous mean field approximation, and the NJL model becomes a quasiparticle model with the current-quark mass becoming constituent quark mass. The equation of motion in this case is still the free Dirac equation. Eventually, the mass appearing in the eigen equation is the constituent mass in Ref. [20]. However, in the DSEs, the dressed-quark mass function is dependent on the ω_n and \vec{p} and there is no constant constituent mass as in NJL model. So, we choose a mass similar to the constituent mass in NJL model within the DSEs framework, i.e., the Euclidean constituent mass M_E whose magnitude is typical of that employed in constituent-quark models [41–44], where $E = \sqrt{M_E^2 + \vec{p}^2}$ is the quasiparticle energy. Our treatment therefore serves as an approximation at this stage.

III. FINITE VOLUME EFFECTS ON THE CHIRAL PHASE TRANSITION

Solving the DSEs at finite temperature and finite size, we obtain the numerical results of fully dressed-quark propagator. We then study the chiral phase transition temperature T_c . The corresponding order parameter is the chiral condensate. In the chiral limit, we have [14]

$$-\langle \bar{\psi}\psi \rangle^0 = 4N_c T \sum_{n=-\infty}^{\infty} \text{tr}_D \int \frac{d^3 p}{(2\pi)^3} S(\vec{p}, \tilde{\omega}_n). \quad (11)$$

When $m \neq 0$, Eq. (11) diverges, we do not have a well-defined chiral condensate. Hence, we employ the renormalized chiral condensate defined as [23,45]

$$-\langle \bar{\psi}\psi \rangle = 4N_c T \sum_{n=-\infty}^{\infty} \text{tr}_D \int \frac{d^3 p}{(2\pi)^3} [S(\vec{p}, \tilde{\omega}_n) - S_0(\vec{p}, \tilde{\omega}_n)]. \quad (12)$$

For finite size system, it becomes

$$-\langle \bar{\psi}\psi \rangle_V = 4N_c \frac{T}{V} \sum_{k,n} \text{tr}_D [S(\vec{p}_k, \omega_n) - S_0(\vec{p}_k, \omega_n)], \quad (13)$$

with $S_0(\vec{p}, \tilde{\omega}_n)$ being free quark propagator and $S(\vec{p}_k, \omega_n)$ being the fully dressed-quark propagator. However, it should be noted that this renormalization prescription only works for simple interaction models without logarithmic running at large momenta [such as Eq. (5)]. More elaborate versions of the model and renormalization scheme have been developed [46–53].

A direct computation of finite volume DSEs, i.e., Eqs. (1) and (8), is quite difficult. The reason is obvious: in general, for the simplified Maris-Tandy model, the ultraviolet cutoff of the three momentum integral is

$O(10)$ GeV and the summation of Matsubara frequency should be consistent with it. As the volume increases and the temperature decreases, the number of allowed values of three momentum and Matsubara frequency is so large that it is difficult to calculate numerically. Some people approximate the momentum modes summation using an integral with an infrared cutoff [8,15]. However, this approximation is known to get worse as the system size decreases. In this work, we adopt a more elegant approximation, i.e., we rewrite Eq. (8) as

$$\int \frac{d^3 \vec{p}}{(2\pi)^3} \rightarrow \frac{1}{V} \sum_{|\vec{p}_k| < \lambda} + \int_{|\vec{p}_k| > \lambda} \frac{d^3 \vec{p}}{(2\pi)^3}. \quad (14)$$

As the high momentum modes are generally denser than low momentum modes, we therefore approximate the summation by an integral for high momentum modes. Here λ is an adjustable parameter, depending on volume and temperature. We can keep increasing it until the numerical result is stable.

For the summation of Matsubara frequency, symmetry of quark propagator can help us reduce the computation effort,

$$\mathcal{F}(\tilde{\omega}_n, \vec{p}_k; T) = \mathcal{F}^*(\tilde{\omega}_{-n}, \vec{p}_k; T), \quad (15)$$

with $\mathcal{F} = A, B$ or C . At low temperature, the number of Matsubara frequency is of $\sim O(10^2)$. For this reason, the numerical calculation is very difficult within DSEs framework when $T < 0.1$ GeV. However, when n is large enough, \mathcal{F} is very smooth, so in the iteration we can reduce computing complexity by interpolation. The technique goes as follows: we take all the low Matsubara frequencies and keep only a few high frequencies by means of a mapping

$$n' = \text{Int}[n' \cdot a + n \cdot (1 - a)], \quad (16)$$

where $n = 1, 2, 3, \dots, N$; $a = \frac{-N' + N}{-N' + (N')^\gamma}$ and $N(N')$ is the number of elements in array $n(n')$. By controlling the scaling factor γ , we can map the evenly distributed array n to an array n' that gets sparse as n' enlarges. We then solve the scalar functions \mathcal{F} at frequencies n' , while at the rest frequencies, \mathcal{F} are obtained by the cubic spline interpolation.

When the chiral condensate is obtained, we can further study the chiral susceptibility [54–57]

$$\chi_V^m(T) = -\frac{\partial}{\partial m} \langle \bar{\psi}\psi \rangle_V. \quad (17)$$

In the chiral limit, chiral symmetry is restored via a second-order transition at T_c in which chiral susceptibility diverges. At nonzero current mass, the chiral symmetry restoration transition is replaced by a crossover. The pseudocritical temperature T_{pc} is obtained as the maxima of the chiral susceptibility with respect to temperature.

In this work, we choose the current-quark mass $m = 4.8$ MeV, which is obtained by fitting the pion mass ($m_\pi = 0.135$ GeV) with the Bethe-Salpeter equation. Accordingly, we can obtain the pion decay constant ($f_\pi = 0.095$ GeV) close to the experimental value based on the $D_0\omega$ that we chose in Eq. (5). When the volume of the system is infinite, we have $T_{pc} = 137$ MeV, close to the recent lattice QCD simulation value $T_{pc} = 156.5 \pm 1.5$ MeV [58]. The numerical results show that T_{pc} gradually decreases and the curve of chiral susceptibility $\chi_V^m(T)$ becomes flat while the system size dwindling (see Fig. 1).

When the volume is small enough, the influence of the shape of the system on T_c cannot be ignored. Our earlier study shows that in a cubic box, when L is greater than 3 fm ($V \sim 27$ fm³), the size of the system can be regarded as infinite [15–17,20]. As Fischer and co-workers pointed out in Ref. [14], at zero temperature, the finite size will have large effects when cubic side length goes below $L = 1.8$ fm and chiral symmetry will get restored at small volumes with antiperiodic boundary condition. However, this work shows that, in contrast to a cubic system with APBC, the QGP phase transition in a spherical system using MIT condition is more sensitive to volume change

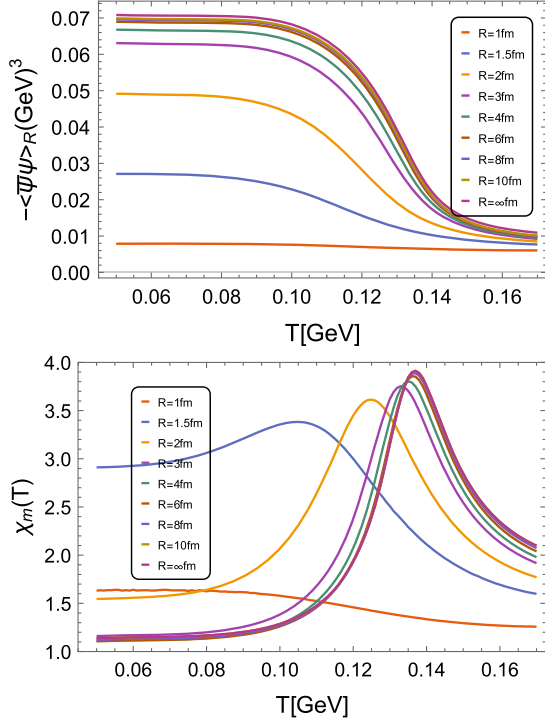


FIG. 1. Upper panel: chiral condensate at different volumes. Lower panel: chiral susceptibility at different volumes. For a spherical system, when $R > 10$ fm, the size of the system can be regarded as infinite. The volume change of the system starts to have a significant effect when the $R < 4$ fm and there is no phase transition in the system when $R < 1.5$ fm.

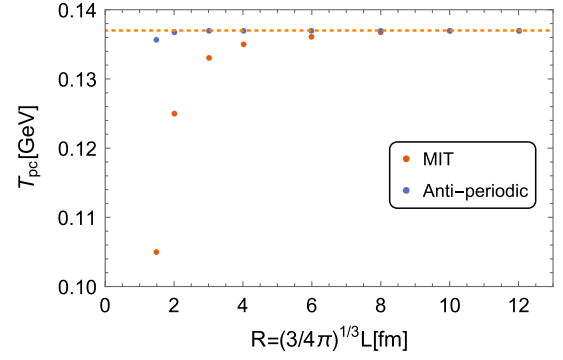


FIG. 2. This figure shows the effect of the system shape on the T_{pc} . For a spherical system, the chiral crossover transition is more sensitive to the size of the system.

(see Fig. 2). When the radius is less than 10 fm ($V \sim 4000$ fm³), the finite volume effect of the system emerges. The size change of the system starts to have a significant effect on T_{pc} when the radius is decreased to 4 fm ($V \sim 270$ fm³). We argue that there is no chiral crossover transition in the system when the radius is less than 1.5 fm ($V \sim 14$ fm³). In other words, it is meaningless to discuss the chiral crossover transition in a very small space size. Consider that DCSB can only occur in infinitely large systems in principle, chiral symmetry has restored when the size is small enough.

We note that similar results were found in Ref. [20]. Therein the NJL model study with MIT boundary condition shows that a system whose radius is above 14 fm can be regarded as an infinitely large system, while in a cubic the marginal size is $L = 3$ fm. Hence, our results are closer to Ref. [20] than to DSEs with APBC [15–17]. This indicates that the boundary condition potentially plays an important role when we want to simulate the realistic fireball produced in HICs.

IV. SUMMARY AND PERSPECTIVE

Based on the DSEs formalism, we consider the influence of the finite volume on the chiral transition of QCD at finite temperature in a cubic and in a spherical. For the cubic volume, we use the widely adopted antiperiodic boundary condition and for the spherical volume we choose MIT boundary condition which had been used in NJL model [20]. While a cubic system within APBC could be regarded as an infinite system if the volume was greater than 27 fm³ ($L \sim 3$ fm), this calculation shows that the finite volume effects do not vanish until the sphere radius gets larger than 10 fm. Since the QGP system generated by the collision is closer to a sphere, we believe that the MIT boundary condition is closer to the real physics than the antiperiodic boundary condition. Our calculation further suggests that when the volume of the fireball produced in collision is

less than 270 fm^3 ($R \sim 4 \text{ fm}$), the volume effect becomes significant. And for $R < 1.5 \text{ fm}$, the chiral symmetry of system has restored. This result also verifies Weinbergs view: symmetry breaking can only occur in systems with a certain large size [59]. The study of QGP chiral transition is under way at the Relativistic Heavy Ion Collider and is planned at several future facilities; we expect that our results will be useful for recent and future experiments [60–64].

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- [1] J. Adams *et al.*, *Nucl. Phys.* **A757**, 102 (2005).
 [2] E. Shuryak, *Prog. Part. Nucl. Phys.* **62**, 48 (2009).
 [3] S. Bass, *Prog. Part. Nucl. Phys.* **41**, 255 (1998).
 [4] G. Grf, M. Bleicher, and Q. Li, *Phys. Rev. C* **85**, 044901 (2012).
 [5] Q.-W. Wang, Y. Xia, and H.-S. Zong, *Mod. Phys. Lett. A* **33**, 1850232 (2018).
 [6] L. M. Abreu, E. B. Corrêa, C. A. Linhares, and A. P. Malbouisson, *Phys. Rev. D* **99**, 076001 (2019).
 [7] Y. Xia, Q. Wang, H. Feng, and H. Zong, *Chin. Phys. C* **43**, 034101 (2019).
 [8] A. Bhattacharyya, P. Deb, S. K. Ghosh, R. Ray, and S. Sur, *Phys. Rev. D* **87**, 054009 (2013).
 [9] Z. Pan, Z.-F. Cui, C.-H. Chang, and H.-S. Zong, *Int. J. Mod. Phys. A* **32**, 1750067 (2017).
 [10] R.-L. Liu, M.-Y. Lai, C. Shi, and H.-S. Zong, *Phys. Rev. D* **102**, 014014 (2020).
 [11] R.-A. Tripolt, J. Braun, B. Klein, and B.-J. Schaefer, *Phys. Rev. D* **90**, 054012 (2014).
 [12] B. Klein, *Phys. Rep.* **707–708**, 1 (2017).
 [13] N. Magdy, *Universe* **5**, 94 (2019).
 [14] J. Luecker, C. S. Fischer, and R. Williams, *Phys. Rev. D* **81**, 094005 (2010).
 [15] B.-L. Li, Z.-F. Cui, B.-W. Zhou, S. An, L.-P. Zhang, and H.-S. Zong, *Nucl. Phys.* **B938**, 298 (2019).
 [16] C. Shi, Y. Xia, W. Jia, and H. Zong, *Sci. Chin. Phys. Mech. Astron.* **61**, 082021 (2018).
 [17] C. Shi, W. Jia, A. Sun, L. Zhang, and H. Zong, *Chin. Phys. C* **42**, 023101 (2018).
 [18] C. Shi, X.-T. He, W.-B. Jia, Q.-W. Wang, S.-S. Xu, and H.-S. Zong, *J. High Energy Phys.* **06** (2020) 122.
 [19] Y.-P. Zhao, R.-R. Zhang, H. Zhang, and H.-S. Zong, *Chin. Phys. C* **43**, 063101 (2019).
 [20] Z. Zhang, C. Shi, and H.-S. Zong, *Phys. Rev. D* **101**, 043006 (2020).
 [21] C. Roberts and S. Schmidt, *Prog. Part. Nucl. Phys.* **45**, S1 (2000).
 [22] R. S. Hayano and T. Hatsuda, *Rev. Mod. Phys.* **82**, 2949 (2010).
 [23] P. Maris and C. D. Roberts, *Phys. Rev. C* **56**, 3369 (1997).
 [24] P. Maris and P. C. Tandy, *Phys. Rev. C* **60**, 055214 (1999).
 [25] C. S. Fischer, S. Kubrak, and R. Williams, *Eur. Phys. J. A* **51**, 10 (2015).
 [26] T. Hilger, M. Gomez-Rocha, and A. Krassnigg, *Eur. Phys. J. C* **77**, 625 (2017).
 [27] A. Höll, A. Krassnigg, P. Maris, C. D. Roberts, and S. V. Wright, *Phys. Rev. C* **71**, 065204 (2005).
 [28] M. S. Bhagwat and P. Maris, *Phys. Rev. C* **77**, 025203 (2008).
 [29] P. Maris and P. C. Tandy, *Phys. Rev. C* **61**, 045202 (2000).
 [30] C. Roberts, *Prog. Part. Nucl. Phys.* **61**, 50 (2008).
 [31] Y.-Z. Xu, D. Binosi, Z.-F. Cui, B.-L. Li, C. Roberts, S.-S. Xu, and H.-S. Zong, *Phys. Rev. D* **100**, 114038 (2019).
 [32] S.-x. Qin, L. Chang, H. Chen, Y.-x. Liu, and C. D. Roberts, *Phys. Rev. Lett.* **106**, 172301 (2011).
 [33] C. D. Roberts and A. G. Williams, *Prog. Part. Nucl. Phys.* **33**, 477 (1994).
 [34] C. D. Roberts, M. S. Bhagwat, A. Hill, and S. V. Wright, *Eur. Phys. J. Special Topics* **140**, 53 (2007).
 [35] L. McLerran and R. D. Pisarski, *Nucl. Phys.* **A796**, 83 (2007).
 [36] A. Bashir, L. Chang, I. C. Cloet, B. El-Bennich, Y.-X. Liu, C. D. Roberts, and P. C. Tandy, *Commun. Theor. Phys.* **58**, 79 (2012).
 [37] L. Chang, C. D. Roberts, and P. C. Tandy, arXiv:1107.4003.
 [38] P. Maris and C. D. Roberts, *Int. J. Mod. Phys. E* **12**, 297 (2003).
 [39] J. A. Müller, A Dyson-Schwinger approach to finite temperature QCD, Ph.D. thesis, Darmstadt, Tech. Hochsch., 2011.
 [40] S.-x. Qin and D. H. Rischke, *Phys. Lett. B* **734**, 157 (2014).
 [41] M. Hecht, C. D. Roberts, and S. Schmidt, *Phys. Rev. C* **63**, 025213 (2001).
 [42] M. Hecht, C. D. Roberts, and S. Schmidt, *Phys. Rev. C* **64**, 025204 (2001).
 [43] C. Burden, L. Qian, C. D. Roberts, P. Tandy, and M. J. Thomson, *Phys. Rev. C* **55**, 2649 (1997).
 [44] Y. Kalinovsky, K. Mitchell, and C. D. Roberts, *Phys. Lett. B* **399**, 22 (1997).
 [45] H.-s. Zong, J.-l. Ping, H.-t. Yang, X.-f. Lu, and F. Wang, *Phys. Rev. D* **67**, 074004 (2003).
 [46] C. S. Fischer, *Prog. Part. Nucl. Phys.* **105**, 1 (2019).
 [47] R. Williams, C. Fischer, and M. Pennington, *Phys. Lett. B* **645**, 167 (2007).
 [48] C. Welzbacher, Quarks and gluons in the phase diagram of quantum chromodynamics, Ph.D. thesis, Giessen University, 2016.
 [49] C. S. Fischer, *Phys. Rev. Lett.* **103**, 052003 (2009).
 [50] M. Blank and A. Krassnigg, *Phys. Rev. D* **82**, 034006 (2010).

- [51] C. S. Fischer, J. Luecker, and C. A. Welzbacher, *Phys. Rev. D* **90**, 034022 (2014).
- [52] R. Contant and M. Q. Huber, *Phys. Rev. D* **96**, 074002 (2017).
- [53] F. Gao and Y.-x. Liu, *Phys. Rev. D* **94**, 076009 (2016).
- [54] Z.-F. Cui, J.-L. Zhang, and H.-S. Zong, *Sci. Rep.* **7**, 45937 (2017).
- [55] Z.-F. Cui, F.-Y. Hou, Y.-M. Shi, Y.-L. Wang, and H.-S. Zong, *Ann. Phys. (Amsterdam)* **358**, 172 (2015).
- [56] P. Chakraborty, M. G. Mustafa, and M. H. Thoma, *Phys. Rev. D* **67**, 114004 (2003).
- [57] A. Smilga and J. J. M. Verbaarschot, *Phys. Rev. D* **54**, 1087 (1996).
- [58] A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, N. Karthik, E. Laermann, A. Lahiri, R. Larsen, S.-T. Li, S. Mukherjee, H. Ohno, P. Petreczky, H. Sandmeyer, C. Schmidt, S. Sharma, and P. Steinbrecher, *Phys. Lett. B* **795**, 15 (2019).
- [59] S. Weinberg, *The Quantum Theory of Fields. Vol. 2: Modern Applications* (Cambridge University Press, Cambridge, United Kingdom, 2013).
- [60] C. Loizides, in *28th International Conference on Ultra-relativistic Nucleus-Nucleus Collisions* (2020) [arXiv:2007.00710].
- [61] P. Senger, *JPS Conf. Proc.* **32**, 010092 (2020).
- [62] M. Harrison, T. Ludlam, and S. Ozaki, *Nucl. Instrum. Methods Phys. Res., Sect. A* **499**, 235 (2003).
- [63] M. Aggarwal *et al.* (STAR Collaboration), arXiv:1007.2613.
- [64] A. Dainese *et al.*, *CERN Yellow Rep.* **3**, 635 (2017).