Pionium as a source of false events in the $K \to \pi\nu\bar{\nu}$ decays

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We suggest that the decay modes of kaons with a pion and a pionium ($\pi^+\pi^-$ atom) in the final state can constitute a not yet considered background to the very rare decay $K \to \pi\nu\bar{\nu}$. In fact, a part of pioniums may escape the decay region before decaying into two π^0s (or to $\pi^0\pi^0\gamma$ in the case of excited pionium). To illustrate the importance of this background, we show that it may even explain, under some assumptions, the unexpected K_L decay events that appeared in the KOTO experiment.

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Two important experiments investigating the rare kaon decays in flight are currently running. The main aim of both is to test the Standard Model and to constrain new physics theories by precisely measuring the very rare kaon decays into a pion and two neutrinos. The NA62 experiment at the CERN Super Proton Synchrotron [1–4] deals with positively charged kaons K^+ and aims to collect enough $K^+ \to \pi^+\nu\bar{\nu}$ events to get a signal to background ratio of 10:1. The Standard Model predicts the branching fraction [5] for this decay [6],

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = (8.4 \pm 1.0) \times 10^{-11}. \tag{1}$$

The KOTO experiment [7] is being conducted at the Hadron Experimental Facility at the Japan Proton Accelerator Research Complex. It was designed to observe the decay $K_L \to \pi^0\nu\bar{\nu}$ of long-lived neutral kaons. The theoretical branching fraction [6] is

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) = (3.4 \pm 0.6) \times 10^{-11}. \tag{2}$$

Until recently, both rare kaon decay experiments proceeded as expected, slowly pushing down the upper bounds of branching fractions. However, in September 2019, Satoshi Shinohara (on behalf of the KOTO Collaboration) [8] announced the presence of four events in the signal region in the situation where mere 0.10 ± 0.02 events were expected. Very soon, several papers appeared, e.g., Refs. [9–11], aimed at finding new physics interpretations of this surprising result. The analysis performed in Ref. [9] shows that if the four events in the KOTO signal region were real, it would mean a branching fraction of the underlying mechanism equal to

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\mathrm{KOTO}} = 2.1^{+2.0(+4.1)}_{-1.1(-1.7)} \times 10^{-9} \tag{3}$$

at the 68(95)% confidence level.

The $\pi^+\pi^-$ atom, pionium (usually denoted as $A_{2\pi}$), was discovered in 1993 at the Institute of High Energy Physics at Serpukhov, Russia [12] and intensively studied in the Dimeson Relativistic Atomic Complex (DIRAC) experiment [13] at the CERN Proton Synchrotron. In these experiments, the pioniums were produced by the proton beam impinging on a target. In the same target, the pioniums broke-up into their constituents with approximately equal energies and small relative momenta.

The decay of pionium to two neutral pions is dominant and the measured lifetime is [13]

$$\tau_{1\mathrm{s}} = 3.15^{+0.28}_{-0.26} \times 10^{-15}\ \mathrm{s}. \tag{4}$$

The NA48/2 Collaboration at the CERN SPS [14] studied $K^\pm \to \pi^\pm\pi^0\pi^0$ decays and found an anomaly in the $\pi^0\pi^0$ invariant mass distribution in the vicinity of $2m_{\pi^+}$ that can be interpreted as the production of pioniums in the kaon decays and their subsequent two-π^0 decay. For our later considerations, it is important that the NA48/2 Collaboration in their seminal paper [14] determined the branching ratio

$$R = \frac{\Gamma(K^\pm \to \pi^\pm A_{2\pi})}{\Gamma(K^\pm \to \pi^\pm\pi^+\pi^-)} = (1.61 \pm 0.66) \times 10^{-5}. \tag{5}$$

Another important discovery concerns the exited pionium. The DIRAC Collaboration observed [15] so-called long-lived $\pi^+\pi^-$ atoms, which are the 2p atomic states ($A'_{2\pi}$) with quantum numbers $J^{PC} = 1^{--}$. Its lifetime was measured in Ref. [15] with the result

$$\tau_{2p} = 0.45^{+1.08}_{-0.30} \times 10^{-11}\ \mathrm{s}. \quad (6)$$

Such a long lifetime is caused by the fact that the decay modes to the positive C-parity states $\pi^0\pi^0$ and $\gamma\gamma$ are now forbidden and the slow 2p → 1s transition dominates. After reaching the 1s state, a decay to two π^0s quickly follows: $A'_{2\pi} \to A_{2\pi}\gamma \to \pi^0\pi^0\gamma$.

The decay of the charged kaons into excited pionium $A'_{2\pi}$ has not been reported yet. Neither has been the neutral kaon decay into any pionium.

In order to show that the pioniums may contribute to the background in the $K \to \pi\nu\bar{\nu}$ experiments by important amount, we will estimate it in the KOTO experiment. To this end, we need at least a crude estimate of the branching fraction of decay $K_L \to \pi^0 A_{2\pi}$. We assume that the branching ratio

$$\tilde{R} = \frac{\Gamma(K_L \to \pi^0 A_{2\pi})}{\Gamma(K_L \to \pi^0\pi^+\pi^-)}$$

has the same value as that for charged kaons (5). Then we can write the branching-fraction estimate

$$\mathcal{B}(K_L \to \pi^0 A_{2\pi}) = R \times \mathcal{B}(K_L \to \pi^0\pi^+\pi^-) \approx 2 \times 10^{-6}, \quad (7)$$

where we have also consulted Ref. [16].

We will also consider possibility that the pionium that appears in the kaon decays is not the ground state (1s) pionium, but its excited (2p) partner and that the corresponding branching fraction are the same,

$$\mathcal{B}(K_L \to \pi^0 A'_{2\pi}) \approx 2 \times 10^{-6}. \quad (8)$$

In what follows, we will pursue those two alternatives in parallel.

To simplify the reasoning, we will ignore the momentum spread of the K_L beam in the KOTO experiment and will use its peak value $P = 1.4$ GeV/c [7]. The length of the KOTO signal region is $L = 1.7$ m. Another quantity that enters the game is the pionium mass. It is given by $m_a = 2m_{\pi^+} - b$, where b is the binding energy. We will take its coulombic value, which can be calculated from the hydrogen-atom-like formula. One obtains $b = 1.86$ keV for the pionium ground state and $b = 0.464$ keV for the excited 2p state.

The laboratory energy of pionium with mass m_a is uniformly distributed in an interval, the bounds of which are given by the formula

$$E_{a\pm} = \frac{1}{M}(EE^*_a \pm Pp^*_a),$$

where M, E, and P are the mass, energy, and momentum of the K_L, respectively, $E^*_a = (M^2 + m_a^2)/(2M)$, $p^*_a = (M^2 - m_a^2)/(2M)$. Numerically, $E_{a-} = 0.497$ GeV and $E_{a+} = 1.456$ GeV.

To simplify the consideration further [17], we will assume that all pioniums have the same laboratory momentum, given as the mean value,

$$p_a = \frac{1}{E_{a+} - E_{a-}} \int_{E_{a-}}^{E_{a+}} \sqrt{E^2 - m_a^2}\mathrm{d}E = \frac{1}{2(E_{a+} - E_{a-})} \times \left[E_{a+}p_{a+} - E_{a-}p_{a-} - m_a^2 \log\frac{E_{a+} + p_{a+}}{E_{a-} + p_{a-}}\right], \quad (9)$$

where $p_{a\pm} = \sqrt{E^2_{a\pm} - m_a^2}$. Numerically, $p_a = 0.880$ GeV/c.

The probability that pionium travels the path s without decaying is given by

$$S(s) = \exp\{-s/l\},$$

where l is the mean decay length of pionium,

$$l = \frac{p_a\tau}{m_a}, \quad (10)$$

τ being the pionium mean lifetime. For the two types of pioniums, we get $l_{1s} = 2.98^{+0.27}_{-0.25}\ \mu$m and $l_{2p} = 4.2^{+10.2}_{-2.8}$ mm.

If we denote the length of the signal region as L, the mean survival probability of pionium at the point where it leaves the signal region is

$$\bar{S} = \frac{1}{L}\int_0^L S(L-z)\mathrm{d}z = \frac{l}{L}[1 - \exp\{-L/l\}]. \quad (11)$$

Numerical values for two kinds of pionium are [18]

$$\bar{S}_{1s} = 1.75^{+0.16}_{-0.15} \times 10^{-6},$$
$$\bar{S}_{2p} = 2.5^{+6.0}_{-1.7} \times 10^{-3}.$$

Multiplying these numbers by the assumed branching fractions (7) and (8), we obtain the branching fractions of $K_L \to \pi^0 A_{2\pi}$ and $K_L \to \pi^0 A'_{2\pi}$ events that look like the $K_L \to \pi^0\nu\bar{\nu}$ events because pioniums left the KOTO decay volume undecayed,

$$\mathcal{B}_{1s} = 3.50^{+0.32}_{-0.30} \times 10^{-12}, \quad (12)$$

$$\mathcal{B}_{2p} = 5.0^{+12.0}_{-3.4} \times 10^{-9}. \quad (13)$$

Branching fraction (12) is by 3 orders of magnitude smaller than branching fraction (3), which characterizes the presence of unexpected events in the KOTO signal region.

Therefore, we will not follow the 1s pionium option any longer.

A comparison of (13) with (3) suggests that the anomalous events in the KOTO experiment could be explained as undecayed 2p pioniums. But to make a realistic comparison, we must take into account that the experimental efficiencies for the $K_L \to \pi^0 \nu\bar{\nu}$ and $K_L \to \pi^0 A'_{2\pi}$ are different. The right quantity that should be compared with (3) is

$$\mathcal{B}'_{2p} = R \times \mathcal{B}_{2p}, \tag{14}$$

where R is the ratio of efficiencies,

$$R = \frac{\epsilon_{\pi^0 A'_{2\pi}}}{\epsilon_{\pi^0 \nu\bar{\nu}}}. \tag{15}$$

Recently, the authors of Ref. [9] considered a similar ratio,

$$R(m_{X^0}) = \frac{\epsilon_{\pi^0 X^0}}{\epsilon_{\pi^0 \nu\bar{\nu}}}, \tag{16}$$

where X_0 is an invisible boson. They performed a Monte Carlo simulation taking into account the experimental cuts, acceptances, and resolutions. The dependence of ratio (16) on the boson mass m_{X^0} can be obtained from the curve in the left pane of Fig. 2 [9]. In principle, it should be possible to use their results and get ratio (15) by setting m_{X^0} to $m_{A'_{2\pi}}$. Unfortunately, the value for $m_{A'_{2\pi}}$, which is close to 280 MeV, cannot be read from the curve. We will therefore make our own crude estimate of the efficiency ratio (15).

The efficiencies are most influenced by the p_T cut imposed in the KOTO experiment. To suppress the events from the $K_L \to \pi^+\pi^-\pi^0$, a p_T momentum of π^0 greater than 130 MeV/c is required over the majority of the signal region; at the downstream edge, the cut is even higher (up to 150 MeV/c). This cut also suppresses the events with two-body final states containing, besides the π^0, a massive particle X^0. It may be an invisible light boson, as in Ref. [9], or a pionium. In these events, the maximum transverse momentum of π^0 is equal to the momentum of the outgoing decay products in the K_L rest frame,

$$p_{\mathrm{T,max}} = \frac{1}{2m_K}\sqrt{\lambda(m_K^2, m_\pi^2, m_{X^0}^2)}, \tag{17}$$

where we use the usual notation

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.$$

The maximum transverse momentum of π^0 in the $K_L \to \pi^0 \nu\bar{\nu}$ events is higher because the lowest invariant mass of the two-neutrino system is zero (neutrinos' masses neglected), namely,

$$p_{\mathrm{Tmax},\nu\nu} = \frac{m_K^2 - m_\pi^2}{2m_K} = 230.5\ \mathrm{MeV}/c.$$

Given a p_T cut, we can restrict the masses of X^0s that can be detected as

$$m_{X^0} < \sqrt{m_K^2 + m_\pi^2 - 2m_K\sqrt{m_\pi^2 + p_{\mathrm{T,max}}^2}}.$$

For $p_{\mathrm{T,max}} = 130$ MeV/c, it implies $m_{X^0} < 281$ MeV. It is obvious that $\epsilon_{\pi^0 X^0}$ should go rapidly to zero with m_{X^0} approaching 281 MeV. Figure 2 in Ref. [9] confirms that.

In order to get estimates of the efficiencies, we first randomly generate the particle momenta within the $\pi^0 \nu\bar{\nu}$ system in the K_L rest frame using the GENBOD program from the old CERN Program Library [19]. Counting the events with the π^0 transverse momentum greater than the KOTO cut of 130 MeV/c, we get an efficiency $\epsilon_{\pi^0\nu\bar{\nu}}$ of 22%.

Next, we consider the 2p pionium with the Coulombic binding energy, the mass of which is $m_a = 279.14$ MeV. Using the formula (17) we get $p_{\mathrm{Tmax},\pi^0} = 132.0$ MeV/c. Assuming isotropic decay in the K_L rest frame, we can estimate the portion of events with p_{T,π^0} greater than 130 MeV/c as $\epsilon_{\pi^0 A'} = (132 - 130)/132 = 1.5\%$. It means an efficiency ratio (15) of

$$R(\mathrm{Coul.}) = 0.069$$

and an efficiency corrected branching ratio

$$\mathcal{B}'_{2p}(\mathrm{Coul.}) = 0.34^{+0.83}_{-0.23} \times 10^{-9}.$$

A comparison with (3) shows that if the experimental cut on transverse momentum of π^0 is taken into account, the 2p pioniums with the Coulombic binding energy cannot be a source of the unexpected events in the KOTO experiment.

Yet, no direct measurement of the 2p pionium mass or binding energy exists. From its decay chain, we know that its mass must be greater than $2m_{\pi^0}$. The binding energy b should thus be smaller than $2(m_{\pi^+} - m_{\pi^0}) \approx$ 9.19 MeV. It is possible [20] that the binding energy of $A'_{2\pi}$ may be around 9 MeV [21]. In that case, we have $m_a = 270.14$ MeV, $p_{\mathrm{Tmax,A'}} = 139.0$ MeV/c, $\epsilon_{\pi^0 A'} = (139 - 130)/139 = 6.5\%$, and

$$R(b = 9\ \mathrm{MeV}) = 0.29.$$

The efficiency corrected branching fraction now comes out as

$$\mathcal{B}'_{2p}(b = 9\ \mathrm{MeV}) = 1.5^{+3.5}_{-1.0} \times 10^{-9},$$

which is, within the errors, compatible with the branching fraction (3) characterizing the unexpected KOTO events.

The KOTO Collaboration continues in analyzing its older data and taking new ones [22]. One of the four events reported in [8] has been shown to be due to a mistake, so only three good candidate events remain. The new background estimate is 1.05 ± 0.28 events. The number of expected Standard Model events is 0.04.

It is interesting that the KOTO experiment sees more $K^{\pm}$ than expected. It may be worth remaining that the $K_L \to K^{\pm} e^{\mp} \bar{\nu}(\nu)$ decays [23] have not been observed yet. Nevertheless, the KOTO Collaboration considered them [24] as a source of background and varied the branching fraction in reasonable bounds. A negligible contribution was found.

To conclude, we have suggested that events in which the kaon decays into pionium, which then leaves the decay region without decaying, may contribute to the background in the $K \to \pi\nu\bar{\nu}$ experiments. In our opinion, this source of background should be taken into account when analyzing the existing experiments or planning new ones [25]. However, to get reliable estimates of this background, the discovery and measurement of the following decays will be very valuable: $K_L \to \pi^0 A_{2\pi}$ (a neutral kaon partner of the charged kaon decay already observed [14]), $K_L \to \pi^0 A'_{2\pi}$, and $K^{\pm} \to \pi^{\pm} A'_{2\pi}$.

As an illustration of the idea, we have shown that, under some assumptions, the production of 2p pioniums $A'_{2\pi}$ in the K_L decays and their subsequent escaping from the signal region may explain the production of unexpected events reported by the KOTO Collaboration [8]. Our key assumptions, which may or may not be correct, have been as follows: (i) main contribution comes from pioniums that are in the 2p state, which lives much longer than the ground state; (ii) the branching ratio of $K_L \to \pi^0 A'_{2\pi}$ to $K_L \to \pi^0\pi^+\pi^-$ is the same as that of $K^+ \to \pi^+ A_{2\pi}$ to $K^+ \to \pi^+\pi^+\pi^-$, which was determined by the NA48/2 Collaboration [14]; (iii) the $A'_{2\pi}$ binding energy is larger than the Coulombic one, which helps pioniums to overcome the experimental p_T cut.

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