Can a nonextremal black hole be a particle accelerator?

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We consider particle collisions in the background of a nonextremal black hole. Two particles fall from infinity, particle 1 is fine-tuned (critical), collision occurs in its turning point. The first example is the Reissner-Nordström (RN) one. If the energy at infinity E_1 is big enough, the turning point is close to the horizon. Then, we derive a simple formula according to which $E_{\mathrm{c.m.}} \sim E_1 \kappa^{-1/2}$, where κ is a surface gravity. Thus significant growth of $E_{\mathrm{c.m.}}$ is possible if (i) particle 1 is ultrarelativistic (if both particles are ultrarelativistic, this gives no gain as compared to collisions in flat space-time), (ii) a black hole is near-extremal (small κ). In the scenario of multiple collisions the energy $E_{\mathrm{c.m.}}$ is finite in each individual collision. However, it can grow in subsequent collisions, provided new near-critical particles are heavy enough. For neutral rotating black holes, in case (i) a turning point remains far from the horizon but large $E_{\mathrm{c.m.}}$ is still possible. Case (ii) is similar to that for collisions in the RN metric. We develop a general theoretical scheme, direct astrophysical applications can be a next step to be studied.

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I. INTRODUCTION

During the last decade, a lot of effort was devoted to high energy processes near black holes. A large series of papers was triggered by the observation made by Bañados, Silk and West [1]. They noticed that if two particles fall toward a black hole and one of particles is fine-tuned (critical), the energy $E_{\mathrm{c.m.}}$ can grow unbounded (this is the so-called BSW effect, after the names of its authors). It is important that a black hole was supposed to be extremal. Moreover, in [2] the impossibility of astrophysical black holes to be exactly extremal was considered as an obstacle to the realization of this effect. This was repeated many times in subsequent works. The main objection against the counterpart of the BSW effect for nonextremal black holes consists in that the critical particle cannot approach the horizon in this case. But if both particles are not fine-tuned (they are called "usual"), $E_{\mathrm{c.m.}}$ remains modest.

Meanwhile, in [3] an important observation was made. Let one particle be not exactly critical but, instead, near-critical. Then, one can adjust the deviation from the critical state to the proximity of the point of collision to the horizon in such a way, that $E_{\mathrm{c.m.}}$ becomes unbounded. However, one difficulty remains for nonextremal black holes. The most physically interesting situation arises when both particles fall from infinity. This can be realized for extremal black holes. But for nonextremal ones, the potential barrier prevents a near-critical particle from reaching the horizon in the same manner as this happens for an exactly critical one.

To overcome this difficulty, the scenario of multiple scattering was proposed in [3]. According to it, particles 1 and 2 come from infinity and collide close to the horizon, creating particles 3 and 4. In doing so, particle 3 is almost critical. Afterwards, a new particle 5 coming from infinity collides with particle 3 producing an indefinitely large $E_{\mathrm{c.m.}}$. However, straightforward application of the multiple scattering scenario is not fruitful. At first glance, one can obtain finally unbounded $E_{\mathrm{c.m.}}(3,5)$ in this way (arguments in parentheses indicate particle numbers). The problem is, however, that if particles 1 and 2 are both usual, particle 3 cannot be near-critical. Indeed, $E_{\mathrm{c.m.}}(1,2) = E_{\mathrm{c.m.}}(3,4)$. Meanwhile, it follows from general principles [1,3,4] that near-horizon collision of two usual (or two near-critical) particles 1 and 2 with finite individual energies leads to bounded $E_{\mathrm{c.m.}}(1,2)$ while collision between the critical and usual particles gives unbounded $E_{\mathrm{c.m.}}(3,4)$. Thus we have a contradiction, so particle 3 with desired properties cannot appear as a result of previous collision between particles arrived from infinity. A special case arises when particle 3 is not a critical in the standard sense but simply has small individual energy E [5]. However, careful analysis shows that such a particle cannot be obtained as a result of a precedent collision too [6], so the same problem remains.

In [7] the authors categorically claimed that nonextremal black holes cannot be accelerators, provided initial particles come from infinity and have finite individual energies E.

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Meanwhile, details of dynamics of collision were not taken into account in [7] and this leaves some potential gaps and questions. After the first collision, the second one can occur much more close to the horizon. Can it lead to unbounded $E_{\mathrm{c.m.}}$? The main obstacle against obtaining very high $E_{\mathrm{c.m.}}$ is related to the fact that the critical particle cannot overcome the potential barrier on its way to the horizon and bounces back in the turning point. But what happens if the turning point itself becomes closer and closer to the horizon? It was pointed out in [7] that indefinitely large $E_{\mathrm{c.m.}}$ entails an indefinitely large individual energy E. Meanwhile, the fact that $E \to \infty$ is required does not destroy the value of a black hole as a particle accelerator since one can compare $E_{\mathrm{c.m.}}$ with a similar quantity $(E_{\mathrm{c.m.}})_\infty$, had collision would have occurred at flat infinity. If $(E_{\mathrm{c.m.}})_\infty$ is modest for such collision but $E_{\mathrm{c.m.}} \gg (E_{\mathrm{c.m.}})_\infty$, this can be considered as some kind of accelerator even despite large initial E. We would also like to remind a reader that collisions with very large $E_{\mathrm{c.m.}}$ were found to be possible if (i) a corresponding nonextremal black is near-extremal, (ii) this includes particles on the circular orbits [8,9].

In the present work, we consider the result of collisions when both particles come from infinity and collide in the turning point of the critical particle. We discuss these effects for charged static black holes and rotating neutral ones separately. As we will see, this leaves some possibility of nonextremal black holes to serve as particle accelerators, although with some reservations. In doing so, the effect is achieved at the first collision, whereas the second collision does not bring new features, so the scenario of multiple collisions is, typically, irrelevant in the situations under considerations. Nonetheless, there is a special alternative. If superheavy particles can be created in collisions, this can significantly increase the energy gain. We develop a general scheme that enables us to understand potential possibilities of nonextremal black holes but refrain from concrete astrophysical applications.

One reservation is in order. In papers [10–12] indefinitely large $E_{\mathrm{c.m.}}$ was obtained irrespective of whether the horizon is extremal or nonextremal. Moreover, fine-tuning of a particle was not required there. However, head-on collisions described by the first line in Eq. (2.57) of [12] correspond to white holes (with one of particles moving away from the horizon) rather to black holes (when both particles move to the horizon). Such a scenario is possible but it is beyond of scope of our work.

In what follows, we use the geometric system of units in which fundamental constants $G = c = 1$.

II. EQUATIONS OF MOTION

We begin with the spherically symmetric case since it is rather simple and admits a number of exact results. Let us consider the black hole metric

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2, \tag{1}$$

where $d\omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, $f = f(r)$. For the Reissner-Nordström (RN) metric,

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right), \tag{2}$$

where M is the mass, Q being the electric charge of a nonextremal black hole. Here, $r_+ = M + \sqrt{M^2 - Q^2}$, is the event horizon radius, $r_- = M - \sqrt{M^2 - Q^2}$ is the Cauchy horizon radius, $M > Q$, $r_+ > r_-$.

The electric potential equals

$$\varphi = \frac{Q}{r}. \tag{3}$$

If a particle with the mass m and electric charge q moves in this background and other external forces are absent, the equations of motion give us

$$m\dot{t} = \frac{X}{f}, \tag{4}$$

$$m\dot{\phi} = \frac{L}{r^2}, \tag{5}$$

$$X = E - q\varphi = E - \frac{qQ}{r}, \tag{6}$$

$$m\dot{r} = \sigma P, \qquad P = \sqrt{U}, \qquad U = X^2 - f\tilde{m}^2, \tag{7}$$

$$\tilde{m}^2 = m^2 + \frac{L^2}{r^2}, \tag{8}$$

E is the energy, L being the angular momentum, dot denotes derivative with respect to the proper time, $\sigma = \pm 1$. The forward-in-time condition $\dot{t} > 0$ entails

$$X \geq 0. \tag{9}$$

We use the standard classification. If $X_H > 0$ is separated from zero, we call a particle usual. If $X_H = 0$, it is called critical. If $X_H = O(\sqrt{f})$ near the horizon is small, it is called near-critical. Here, X_H is the value of X on the horizon.

III. PARTICLE COLLISIONS

Let particles 1 and 2 collide. One can define the energy in the center of mass frame $E_{\mathrm{c.m.}}$ according to the relation

$$\begin{aligned} E_{\mathrm{c.m.}}^2 &= -(m_1 u_{1\mu} + m_2 u_{2\mu})(m_1 u_1^\mu + m_2 u_2^\mu) \\ &= m_1^2 + m_2^2 + 2m_1 m_2 \gamma, \end{aligned} \tag{10}$$

where $\gamma = -u_{1\mu}u^{2\mu}$ is the Lorentz factor of relative motion, u^μ is the four-velocity. We consider pure radial motion of particles in the RN background, so $L_1 = L_2 = 0$. From equations of motion (4)–(7) one finds

$$m_1 m_2 \gamma = \frac{X_1 X_2 - P_1 P_2}{f}, \tag{11}$$

where we assumed that both particle move toward a black hole, so $\sigma_1 = \sigma_2 = -1$. In particular, if collision occurs in the turning point for one of particles (say, particle 1),

$$m_1 m_2 \gamma = \frac{X_1 X_2}{f}. \tag{12}$$

To simplify formulas, we assume that particle 2 is neutral. This also enables us to avoid the question about the direct electric interaction between particles.

In what follows, we also assume for simplicity that $m_1 = m_2 \equiv m$. Then, for collision in the turning point where $P_1 = 0$, Eqs. (10), (11) give us

$$E^2_{\mathrm{c.m.}} = 2m^2 + \frac{2X_1 E_2}{f}, \tag{13}$$

where the right-hand side is taken in the turning point.

IV. FLAT SPACE-TIME

Before discussion of collisions in the RN metric, it is instructive to list the main formulas for the flat space-time. They are quite trivial by themselves, but in what follows we will need to compare with them the results of collision in the black hole background to check, whether collision in the turning point gives some enhancement as compared to the collision at infinity.

If $E_1 \sim E_2 \sim m$ it is obvious that $E_{\mathrm{c.m.}} \sim m$ as well. If $E_2 = m$,

$$(E^2_{\mathrm{c.m.}})_{\mathrm{flat}} = 2m^2 + 2E_1 m. \tag{14}$$

Thus if E_1 grows, $(E^2_{\mathrm{c.m.}})_{\mathrm{flat}}$ grows as well.

If $E_1 = E_2 = E \gg m$, it follows from (10) and (11) with $f = 1$ that

$$(E^2_{\mathrm{c.m.}})_{\mathrm{flat}} \approx 4m^2 \tag{15}$$

is finite.

V. ALLOWED ZONE OF MOTION

Now, we return to the RN metric. The motion is possible where $U \geq 0$. This condition gives us

$$\left(E - \frac{qQ}{r}\right)^2 \geq \left(m^2 + \frac{L^2}{r^2}\right)\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right). \tag{16}$$

We assume that $Qq > 0$ (say, $Q > 0$, $q > 0$) since it is this case that potentially gives us unbounded $E_{\mathrm{c.m.}}$ [13].

In the turning points $U = 0$. If $L = 0$, we can find the turning point analytically:

$$r_{1,2} = \frac{1}{\varepsilon^2 - 1}(\varepsilon\tilde{q}Q - M \pm \sqrt{D}), \tag{17}$$

where $\varepsilon = \frac{E}{m}$, $\tilde{q} = \frac{q}{m}$, $r_1 \leq r_2$.

$$D = Q^2(\tilde{q}^2 + \varepsilon^2 - 1) - 2M\varepsilon\tilde{q}Q + M^2. \tag{18}$$

As a particle falls from infinity, where $E \geq m$, we have $\varepsilon^2 \geq 1$. Turning points outside the horizon are absent if $D < 0$ or

$$r_2 < r_+. \tag{19}$$

In what follows we will consider the case when particle 1 is critical and particle 2 is neutral, $q_2 = 0$. This means that the turning point r_1 is absent for particle 2,

$$r_2 = \frac{\sqrt{D} - M}{\varepsilon^2 - 1}, \tag{20}$$

where

$$D = M^2 + Q^2(\varepsilon^2 - 1). \tag{21}$$

It is easy to check that (19) is satisfied, so the point r_2 is absent too. Thus particle 2 comes from infinity and reaches the horizon.

VI. CRITICAL PARTICLE

For the critical particle, $X_H = 0$, so we have from (6) that

$$E = \frac{qQ}{r_+}, \tag{22}$$

$$D = M^2 - Q^2, \tag{23}$$

$$r_1 = r_+, \tag{24}$$

$$r_2 = r_+ + \frac{2\sqrt{M^2 - Q^2}}{\varepsilon^2 - 1} = r_+\left(1 + \frac{2\kappa r_+}{\varepsilon^2 - 1}\right), \tag{25}$$

$$X(r_2) = E\left(1 - \frac{r_+}{r_2}\right) = \frac{2\kappa r_+^2 E}{r_2(\varepsilon^2 - 1)}. \tag{26}$$

Here, $\kappa = \frac{1}{2}f'(r_+)$ is the surface gravity,

$$\kappa = \frac{1}{2r_+}\left(1 - \frac{r_-}{r_+}\right) = \frac{\sqrt{M^2 - Q^2}}{r_+^2}. \tag{27}$$

A special case arises if $\varepsilon = 1$. Then, $\tilde{q}Q = r_+$, and for $r \to \infty$ we have $U \approx \frac{2m^2(M - r_+)}{r} < 0$. Such a particle cannot move at infinity, so to avoid this case, we assume $\varepsilon > 1$ in what follows.

VII. COLLISION BETWEEN THE CRITICAL AND NEUTRAL PARTICLES

If particles fall from infinity and collide in point $r = r_2$, it follows from (13) that

$$E_{\mathrm{c.m.}}^2 = 2m^2 + \frac{2E_1E_2(1-\frac{r_+}{r_2})}{f(r_2)}. \tag{28}$$

In the region $r_+ < r < r_2$, motion of particle 1 is forbidden since U becomes negative there.

The only hope to obtain unbounded $E_{\mathrm{c.m.}}^2$ is to arrange collision near the horizon, where $f \to 0$. So, now we examine, whether or not this gives the unbounded $E_{\mathrm{c.m.}}^2$.

The condition $f(r_2) \ll 1$ requires $r_2 \to r_+$. As we see it from (25), this happens if the second term in parentheses is small, so

$$\frac{\kappa r_+}{\varepsilon^2 - 1} \ll 1. \tag{29}$$

There are two typical cases here.

A. $\kappa r_+ = O(1)$, $\varepsilon \to \infty$

Then,

$$f(r_2) \approx 2\kappa(r_2 - r_+) \approx \frac{4\kappa^2 r_+^2}{\varepsilon^2}, \tag{30}$$

taking into account (26) we obtain

$$E_{\mathrm{c.m.}}^2 \approx 2m^2 + \frac{E_1E_2}{\kappa r_+}. \tag{31}$$

If $E_2 = m$, there is no energy gain as compared to the flat case (14). However, if not only $E_1 \gg m$, but also $E_2 \gg m$, collision near the horizon is much more effective due to the factor E_1E_2 that is absent in (15).

B. $\varepsilon = O(1)$, $\kappa r_+ \ll 1$

This means that our black hole is near-extremal. Then, we must retain in the expansion for the function $f(r)$ also the next term:

$$f \approx 2\kappa(r - r_+) + \frac{(r - r_+)^2}{r_+^2}, \tag{32}$$

so

$$f(r_2) \approx 4\kappa^2 r_+^2 \frac{\varepsilon^2}{(\varepsilon^2 - 1)^2} = 4\kappa^2 r_+^2 \frac{E_1^2 m^2}{(E_1^2 - m^2)^2}. \tag{33}$$

Taking into account (28), we obtain

$$E_{\mathrm{c.m.}}^2 \approx \frac{E_2(E_1^2 - m^2)}{\kappa r_+ E_1}. \tag{34}$$

Independently of E_1 and E_2, we obtain formally unbounded growth when $\kappa \to 0$.

And, the combined case $\varepsilon \gg 1$, $\kappa r_+ \ll 1$ is possible as well. Then, (34) turns into (31).

VIII. NONZERO ANGULAR MOMENTUM

Let us consider now the case, when $L \neq 0$ for particle 1. Then, if particle 1 is critical, we have for it

$$U = \left(1 - \frac{r_+}{r}\right)\left[E^2\left(1 - \frac{r_+}{r}\right) - \left(1 - \frac{r_-}{r}\right)\tilde{m}^2\right]. \tag{35}$$

We are interested in the situation when the turning point r_2 is close to the horizon. Assuming

$$\kappa r_+ \frac{\tilde{m}_1^2(r_+)}{E^2 - \tilde{m}_1^2(r_+)} \ll 1 \tag{36}$$

and repeating simple calculations step by step, we obtain that if

$$E \gg \tilde{m}(r_+) \tag{37}$$

is satisfied, then (31) holds.

If $\kappa r_+ \ll 1$,

$$\frac{r_2 - r_+}{r_+} \approx \frac{\tilde{m}_1^2(r_+)}{E^2 - \tilde{m}^2(r_+)}\left(1 - \frac{r_-}{r_+}\right) = 2\kappa r_+ \frac{\tilde{m}_1^2(r_+)}{E^2 - \tilde{m}_1^2(r_+)}. \tag{38}$$

Then we have, instead of (34),

$$E_{\mathrm{c.m.}}^2 \approx \frac{E_2[E_1^2 - \tilde{m}^2(r_+)]}{\kappa r_+ E_1}, \tag{39}$$

where now the case $E_1 \gtrsim \tilde{m}(r_+)$ is allowed.

The only difference as compared to the case $L = 0$ consists in the fact that the quantity $\tilde{m}(r_+)$ appears in some formulas instead of m.

IX. MULTIPLE COLLISIONS

We see that indeed $E_{\mathrm{c.m.}}^2$ can become large due to big E_1 or small κ. Now, we want to elucidate, is it possible to improve the result (39) and increase $E_{\mathrm{c.m.}}$? To this end, we consider the following realization of multiple scattering scenario [3]. Particle 1 and 2 collide creating particles 3 and 4. We want to achieve X_3 as small as possible. Then, in the case of success, collision between particle 3 and particle 5 coming from infinity can give large $E_{\mathrm{c.m.}}$. Then, we can take advantage of the results of analysis already carried out in [14]. Although the corresponding equations are derived in [14] for the rotating case whereas now a black hole is static, the general formulas look the same. For simplicity, again $m_1 = m_2 = m$, also $m_3 = m_4$ and all angular momenta $L_i = 0$. Then, given parameters of particles 1 and 2, in the point of collision (where subscript "c" will be used) one has from Eqs. (19), (20) of [14] (this can also be reobtained directly form the conservation laws)

$$(X_3)_c = \frac{1}{2}\left(X_0 - P_0\sqrt{1 - 4\frac{m_3^2}{m_0^2}}\right)_c, \tag{40}$$

$$(X_4)_c = \frac{1}{2}\left(X_0 + P_0\sqrt{1 - 4\frac{m_3^2}{m_0^2}}\right)_c, \tag{41}$$

$$P_0 = \sqrt{X_0^2 - m_0^2 f}, \tag{42}$$

where $m_0 = E_{\mathrm{c.m.}}$, $X_0 = X_1 + X_2$. As before, particle 1 is critical, particle 2 is usual. Let $\varepsilon \gg 1$ with $\kappa r_+ \sim 1$.

According to (30) and (31), in the point of collision near the horizon $f = O(\frac{1}{\varepsilon^2})$, $m_0^2 = O(\varepsilon)$, $X_0 \approx E_2 = m$,

$$P_0 \approx X_0 - \frac{m_0^2 f}{2X_0}. \tag{43}$$

Then,

$$(X_3)_c \approx \frac{\kappa r_+ (m^2 + m_3^2)}{\varepsilon m}. \tag{44}$$

Let $q_1 = q_3 = q$, $q_2 = q_4 = 0$. Then, it follows from (6), (25) that

$$X_3 = (X_3)_c + qQ\left(\frac{1}{r_2} - \frac{1}{r}\right). \tag{45}$$

In particular,

$$X_3(r_+) \approx (X_3)_c - \frac{2\kappa q Q}{\varepsilon^2}. \tag{46}$$

The first term in (45) has the order ε^{-1} and dominates everywhere between r_+ and r_2. Thus, in the main approximation, the second term can be neglected and $X_3 \approx (X_3)_c$. It is convenient to make the substitution

$$r - r_+ = \frac{(X_3)_c^2}{2\kappa m_3^2} y. \tag{47}$$

Then, for $f \approx 2\kappa(r - r_+)$ we have

$$f \approx \frac{(X_3)_c^2}{m_3^2} y. \tag{48}$$

Correspondingly,

$$P_3^2 \approx (X_3)_c^2 - m_3^2 f = (X_3)_c^2 (1 - y). \tag{49}$$

The collision between particles 1 and 2 occurred in the point $r = r_2$, for which the corresponding value $y = y_1$ follows from (25), (47):

$$y_1 = \frac{4m^2 m_3^2}{(m_3^2 + m^2)^2}. \tag{50}$$

After this collision, a new particle 3 can move either toward the horizon with $\sigma_3 = -1$ or reach a new turning point where $r = \tilde{r}_2$, $y = 1$. In the second case, it bounces back there and moves further toward the horizon with $\sigma_3 = -1$.

From (47) one can find a location of a new turning point:

$$\tilde{r}_2 - r_+ = \frac{(X_3)_c^2}{2\kappa m_3^2} = \frac{\kappa r_+^2 (m^2 + m_3^2)^2}{2\varepsilon^2 m_3^2 m^2}. \tag{51}$$

If $m_3 = m$, this coincides with (25). Then, a particle either has $\sigma_3 = -1$ or bounces back and changes σ_3 to -1 immediately. In general,

$$\frac{\tilde{r}_2 - r_+}{r_2 - r_+} = \frac{(m_3^2 + m^2)^2}{4m_3^2 m^2} \geq 1. \tag{52}$$

Let a usual particle 5 with the energy $E_5 = m_5 = m$ and $q = 0$ fall from infinity, $\sigma_5 = -1$. If collision occurs when $\sigma_3 = +1$, we have from (10), (11), (44) that

$$E_{\mathrm{c.m.}}^2 \approx \frac{2E_1 m_3^2 m F_+(y)}{\kappa r_+ (m^2 + m_3^2)}, \tag{53}$$

$$F_+(y) = \frac{1 + \sqrt{1 - y}}{y}, \tag{54}$$

where $y \geq y_1$, This function is monotonically decreasing with y, so it attains the maximum value at $y = y_1$, where

$$(E_{\mathrm{c.m.}}^2)_{\max} \approx \frac{2E_1 m_3^2 m F_+(y_1)}{\kappa r_+ (m^2 + m_3^2)}. \tag{55}$$

The most "profitable" case corresponds to head-on collision in the point $y = y_1$. This implies that the 2nd collision occurs in the same point as the first one. If $m \ll m_3$, $y_1 \approx \frac{4m^2}{m_3^2} \ll 1$. Then, $F_+(y_1) \approx \frac{2}{y_1}$,

$$(E^2_{\rm c.m.})_{\max} \approx \frac{E_1 m_3^2}{\kappa r_+ m}. \quad (56)$$

But, if $\kappa r_+ = O(1)$, $E^2_{\rm c.m.}$ remains limited.

If collision occurs when $\sigma_3 = -1$, we have $\sigma_5\sigma_3 = +1$. Then, in the same manner we obtain

$$E^2_{\rm c.m.} \approx \frac{2E_1 m_3^2 m F_-(y)}{\kappa r_+(m^2 + m_3^2)}, \quad (57)$$

$$F_-(y) = \frac{1-\sqrt{1-y}}{y} = \frac{1}{1+\sqrt{1-y}}. \quad (58)$$

Here, F is monotonically increasing bounded function, $F(0) = \frac{1}{2}$, $F(1) = 1$. Thus if the second collision occurs at $y = 1$, the result for $E^2_{\rm c.m.}$ is as twice as many as compared to the collision on the horizon. This is quite similar to the observation made for the nonextremal Kerr metric in [15] (see discussion after Eq. (31) there) and generalized in Sec. 2.2. of [9]. Thus a second collision does not lead to unbounded $E_{\rm c.m.}$.

We can compare $(E^2_{\rm c.m.})_2$ after the 2nd collision with a similar quantity $(E^2_{\rm c.m.})_1$ (31) after the 1st collision. Taking into account (56), we obtain

$$\frac{(E^2_{\rm c.m.})_2}{(E^2_{\rm c.m.})_1} \approx \frac{m_3^2}{m^2}. \quad (59)$$

If all masses have the same order m, there is no big gain. However, if, say, $m_5 = m$ but $m_3 \gg m$, $\frac{(E^2_{\rm c.m.})_2}{(E^2_{\rm c.m.})_1} \gg 1$. Meanwhile, there is an upper bound here. As $(E_{\rm c.m.})_1 \geq 2m_3$, there is a bound

$$\frac{(E^2_{\rm c.m.})_2}{(E^2_{\rm c.m.})_1} \leq \frac{(E^2_{\rm c.m.})_1}{4m^2} \approx \frac{E_1}{4m\kappa r_+}, \quad (60)$$

where (31) with $E_2 = m$ was used again.

One can repeat the procedure. Let a new particle 6 with $m_6 = m$ is sent from infinity. It collides with particle 3 and produces a new near-critical particle 7. Repeating derivation, we obtain in the new point of collision (44) with m_3 replaced with m_7. In Eq. (59) m_3 should be replaced with m_7.

We can imagine a scenario in which initially a (near) critical particle 1 with $E_1 \gg m$ is sent from infinity together with particle 2 having $E_2 = m$. They collide, create a near-critical particle with m_3 that collides with a new particle having $E = m$ and coming from infinity, etc. If, for simplicity, all new near-critical particles have the same mass m_3 and falling particles have the same mass m, each time $E_{\rm c.m.}$ can acquire an additional factor $(\frac{m_3}{m})$ that results in $(\frac{m_3}{m})^n$, where n is the number of additional collisions. It can be quite big, provided new near-critical particles are heavy enough. In this scenario, a big energy E_1 is pumped into the system but this is done only one time. It is worth noting that in the multiple scenario suggested in [3], only the angular momentum changes due to collisions. Meanwhile, now parameters of a near-critical particle are fixed, the effect of big $E_{\rm c.m.}$ is achieved due to the relation between masses of a near-critical and usual particles.

Thus if we want to obtain big $E_{\rm c.m.}$, the near-critical particle should be superheavy. In this sense, there is some analogy between collisions in our scenario and collisions near extremal charged black holes. Namely, it was shown in [16] that in the scenario denoted there OUT+, there is no upper bound on m_3 and, instead, there is a lower bound. The similar result was obtained in somewhat different approach in [17]. In this sense, the collisional Penrose process with ultrahigh $E_{\rm c.m.}$ can be accompanied with ultrahigh m_3. Meanwhile, there is also difference between the process under discussion in the present work and that considered in [16,17]. In the extremal case, $E_{\rm c.m.}$ and m_3 are independent, so it can happen that $E_{\rm c.m.}$ is ultrahigh whereas m_3 is modest (although restricted from below). Meanwhile, for nonextremal black holes, big m_3 is a necessary condition for obtaining high $E_{\rm c.m.}$.

There is also a counterpart of the phenomenon of interrelation between $E_{\rm c.m.}$ and m_3 for rotating black holes. It was found in [18] for the Kerr metric and was generalized in [19]. Then, although there is an upper bound on $E_{\rm c.m.}$, significant increase in $E_{\rm c.m.}$ occurs when a created particle is superheavy.

One additional remark is in order. As it is clear from the method of derivation, it is not important, whether the new particle will have parameters close to the criticality condition $X_H \approx 0$ due to the compensation between E and $q\varphi$ or simply it has $q = 0$ and small energy [5,6].

X. ROTATING CASE

It is the case of rotating black holes that we now turn to. In doing so, we assume no electric interaction between particles and a black hole. It means that either particles or a black hole are electrically neutral (or both a black hole and particles). As consideration of collisions runs along the same line, we give only brief description. The metric has the form

$$ds^2 = -N^2 dt^2 + g_\phi(d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2, \quad (61)$$

where for shortness $g_\phi \equiv g_{\phi\phi}$ and $g_\theta \equiv g_{\theta\theta}$. We assume that the metric coefficients do not depend on t and ϕ and possess symmetry because of which motion within the plane $\theta = \frac{\pi}{2}$ is possible. In this plane, we can redefine the radial coordinate to have $N^2 = A$. Then, the equations of motion for a free particle have the form

$$m\dot{t} = \frac{X}{N^2}, \quad (62)$$

$$m\dot{\phi} = \frac{L}{g_\phi} + \frac{\omega X}{N^2}, \tag{63}$$

$$m\dot{r} = \sigma P, \qquad P = \sqrt{U} \tag{64}$$

with

$$U = X^2 - \tilde{m}^2 N^2, \tag{65}$$

$$X = E - \omega L, \tag{66}$$

$$\tilde{m}^2 = m^2 + \frac{L^2}{g_\phi}. \tag{67}$$

The main difference with respect to the RN case consists in that the critical particle has

$$L = \frac{E}{\omega_H}, \tag{68}$$

so for it, E and L are not independent parameters any longer.

The rotational counterpart of Eq. (13) for collision in the turning point r_t of particle 1 now reads

$$E^2_{\rm c.m.} = 2m^2 + \frac{2X_1(r_t)X_2(r_t)}{N^2(r_t)} - \frac{2L_1L_2}{g_{\phi(}r_t)}. \tag{69}$$

A. Ultrarelativistic particles

It turns out that even for ultrarelativistic particles with (37), the turning point does not approach the horizon. Indeed, if $N \ll 1$, $\frac{r-r_+}{r_+} \ll 1$ and the Taylor expansion has the form

$$\omega = \omega_H - B_1(r - r_+) + \cdots \tag{70}$$

where B_1 is some constant. For the critical particle,

$$X = \frac{E}{\omega_H} B_1(r - r_+) + \cdots \tag{71}$$

The first term in U has the order $(r - r_+)^2$ whereas the second negative one has the order $N^2 \sim (r - r_+)$, so $U < 0$. For the RN metric, we were able to choose a large energy of the particle to achieve proximity of the turning point to the horizon since large E compensated small $r - r_+$. But now this is impossible since the negative contribution in U has the same factor E^2 as a positive one due to condition (68).

Thus the turning point is located in some intermediate region where $N \sim 1$. Now, the type of particle is irrelevant at all. Let, for simplicity, both particles be usual with $L_1 = L_2 = 0$, so $X_1 = E_1$. $X_2 = E_2$. Then, in the turning point $r = r_t$

$$E^2_{\rm c.m.} = 2m^2 + \frac{2E_1E_2}{N(r_t)}. \tag{72}$$

By itself, $E^2_{\rm c.m.}$ is finite. However, one can obtain a significant energy gain as compared to collision in the flat space-time (15) even in this "trivial" scenario, provided both particles are ultrarelativistic, $E_1 \gg m$, $E_2 \gg m$. The corresponding additional factor equals $\frac{E_1E_2}{m^2}$. As now $\frac{E_1}{m}$ and $\frac{E_2}{m}$ are free parameters, we can formally increase the energy gain without a limit. The only difficulty is that we must have ultrarelativistic particles from the very beginning. (To some extent, that resembles the "energy feeding problem" discussed in Sec. IV C1 of [20] for another scenario of collision in the extremal black hole background, when particles move along the axis. Now, a similar problem reveals itself for nonextremal ones and equatorial particle motion.)

B. Near-extremal black holes

Now, let us consider the limit $\kappa \to 0$. If κ is small,

$$N^2 \approx 2\kappa(r - r_+) + H(r - r_+)^2, \tag{73}$$

where H is the model-dependent coefficient. Then, the position of the turning point r_t for the critical particle is determined by equation $U = 0$. Taking into account (71), we obtain from (65)

$$(r_t - r_+)C \approx 2\kappa\left(m^2 + \frac{E^2}{\omega_H^2(g_\phi)_H}\right), \tag{74}$$

where

$$C = \frac{E^2}{\omega_H^2}\left(B_1^2 - \frac{H}{(g_\phi)_H}\right) - Hm^2 \tag{75}$$

and it is assumed that $C > 0$, subscript "H" refers to quantities calculated on the horizon. Bearing in mind that $E \geq m$, it is sufficient to require that $B_1^2 > H(\omega_H^2 + \frac{1}{(g_\phi)_H})$.

Then,

$$r_t - r_+ \approx \frac{2\kappa\tilde{m}^2(r_+)}{C}. \tag{76}$$

If $\kappa r_+ \sim 1$ and $\frac{E}{m} \to \infty$, the numerator has the same order as the denominator, so $r_t - r_+$ does not become small in accordance with what is said after Eq. (71). However, for $\kappa \to 0$ we see that indeed $r_t \to r_+$.

For the critical particle 1, it follows from (71) and (76) that

$$X \approx \frac{E}{\omega_H} B_1 \frac{2\kappa\tilde{m}^2(r_+)}{C}. \tag{77}$$

Equation (73) gives us

$$N^2(r_t) \approx 4\frac{\kappa^2 \tilde{m}^2(r_+)}{C}\left[1 + H\frac{\tilde{m}^2(r_+)}{C}\right]. \qquad (78)$$

Then, it follows from (69) that

$$E^2_{\text{c.m.}} \approx \frac{E_1 (X_2)_H B_1 C}{\kappa \omega_H [C + H\tilde{m}^2(r_+)]}. \qquad (79)$$

Thus again

$$E^2_{\text{c.m.}} \sim \frac{1}{\kappa} \qquad (80)$$

can grow unbounded if $\kappa \to 0$.

For the near-extremal Kerr metric,

$$(\kappa r_+)^{-1} \approx \frac{2}{\eta}, \qquad \eta \approx \sqrt{1 - a_*^2}, \qquad (81)$$

where $a_* = \frac{a}{M}$, a being the standard parameter characterizing an angular momentum, M black hole mass.

In principle, there are three different scenarios: (i) the near-critical particle is created already near the horizon [3], (ii) collisions involve a particle moving on a circular orbit near a black hole [8], (iii) both particles come from infinity and collide in the turning point. Thus in all three cases there is only one small parameter that is able to increase $E^2_{\text{c.m.}}$ significantly. It is the same for all three scenarios and depends on the properties of a black hole. This is the surface gravity κ or, equivalently, η. If one takes the astrophysically relevant limit $a_* = 0,998$ [21] one obtains that $\eta^{-1} \approx 22.361$. There are also numeric factors depending on the scenario but we omit such details. We see that there exists enhancement of the energy $E^2_{\text{c.m.}}$, although it remains bounded. Meanwhile, the scenario under discussion gives some additional factors that can improve the situation. It consists in the process that include superheavy particles. We considered it in detail for the RN black hole but a similar phenomenon should happen also for the Kerr one. Then, the ratio $\frac{m_3}{m}$ in each additional collision can somehow increase the energy gain. Whether and how this can be realized in a realistic astrophysical context is a separate interesting question beyond the scope of the present paper.

XI. DISCUSSION AND CONCLUSIONS

Thus we considered two types of nonextremal black holes: charged static and neutral rotating ones. In both cases, we considered scenarios in which the critical and usual particles come from infinity and collide in the turning point of the critical particle. Under some conditions, the location of this point turns out to be close to the horizon. For the RN black hole, there are two different factors that make it possible: either critical particle 1 is ultralativistic or a black hole is near-extremal (or both factors are valid). Then, $E^2_{\text{c.m.}} \sim \frac{E_1}{\kappa}$. On the first glance, the necessity to have large E_1 from the very beginning, depreciates the ability of a black hole to serve as a particle accelerator [7]. However, this is not so. One can compare, say, the scenario under discussion to collision of two ultrarelativistic particles at flat infinity. Then, we have significant gain in the energy of collisions if it happens near the horizon. Also, for a moderate Killing energy $E_{1.2} \sim m$ the energy of collision becomes indefinitely large if the surface gravity κ is as small as we like. This is a counterpart of collisions on near-circular orbits in the background of near-extremal black holes. There exist two versions of the corresponding collisions in which $E^2_{\text{c.m.}} \sim \kappa^{-1}$ similarly to our case or $E^2_{\text{c.m.}} \sim \kappa^{-2/3}$ for two different types of scenarios [8,9]. But now, the scenario has nothing to do with the circular orbits, both particles come from infinity.

We also saw that if, after the first collision, new particle 3 collides again with some particle that arrived from infinity, the energy $E_{\text{c.m.}}$ remains bounded in each individual collisions. However, if new created near-critical particles are heavy enough with $m_3 \gg m$, the process can be repeated giving a growing factor proportional to (m_3/m) for each new collision (where for simplicity we assumed that new near-critical particles have the same mass m_3). Only an initial particle with big E_1 is required, afterwards it is sufficient to send from infinity particles with modest energy of the order m.

As far as the neutral rotating black hole is concerned, near-extremal black holes with $\kappa \to 0$ are relevant in this context with the same result $E^2_{\text{c.m.}} \sim \kappa^{-1}$.

To summarize, there are two different types of accelerators connected with black holes. The first type is presented by extremal black holes, where the presence of the horizon reveals itself directly. It is the proximity of a point of collision to the horizon (together with the fine-tuning of parameters of one particle) that matters [1], while the mass of colliding particles are of secondary importance. Choosing this point close enough to the horizon, one can obtain $E_{\text{c.m.}}$ as large as one likes already in the first collision. For nonextremal black holes this is impossible. But, nonetheless, nonextremal black hole can indeed be particle accelerators, although with a number of restrictions described above. In doing so, the relation between masses of particles that are created near the horizon and those coming from infinity plays a crucial role in the scenario of multiple collisions. It is able to enhance the initial gain in the energy of collision. It would be interesting to consider more realistic astrophysically relevant scenarios on the basis of the results obtained in the present work.

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