# Warm bounce in loop quantum cosmology and the prediction for the duration of inflation

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We study and estimate probabilistic predictions for the duration of the preinflationary and slow-roll phases after the bounce in loop quantum cosmology, determining how the presence of radiation in the prebounce phase affects these results. We present our analysis for different classes of inflationary potentials that include the monomial power-law chaotic type of potentials, namely, for the quadratic, quartic, and sextic potentials and also for a Higgs-like symmetry-breaking potential, considering different values for the vacuum expectation value in the latter case. We obtain the probability density function for the number of inflationary *e*-folds and for other relevant quantities for each model and produce probabilistic results drawn from these distributions. This study allows us to discuss under which conditions each model could either eventually lead to observable signatures in the spectrum of the cosmic microwave background, or be excluded for not predicting a sufficient amount of accelerated expansion. The effect of radiation on the predictions for each model is explicitly quantified. The obtained results indicate that the number of inflationary *e*-folds in loop quantum cosmology is not *a priori* an arbitrary number, but can in principle be a predictable quantity, even though the results are dependent on the model and the amount of radiation in the Universe prior to the start of the inflationary regime.

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## I. INTRODUCTION

Inflation is the current paradigm for the early Universe cosmology.[1] The inflationary scenario was developed before a majority of current data was recorded. Inflation is in good agreement with the predictions coming from the cosmic microwave background (CMB) spectrum and explains the origin of inhomogeneities present in the primordial Universe, which led to the formation of large-scale structures. Thus, although fine-tunings of the constants are necessary and appropriate choices of potentials have to be made, this is a very predictive scenario. Inflation is a good candidate for solving some of the puzzles in the standard big bang cosmology, such as the horizon and flatness problems[4–6]. Despite its success, the idea of inflation alone does not address the important issue of extending general relativity (GR) beyond its limit of applicability, which is associated with the big bang singularity problem. Apart from this problem, one should consider in the space of classic solutions for GR those solutions that exhibit sufficient inflation to account for the current observations [7–9]. This motivates an investigation of the probability of a sufficient amount of inflation in a cosmological model. In this endeavor one is plagued with problems, such as the difficulty in defining a measure to calculate probabilities in GR and finding the starting point for counting *e*-folds in the presence of a singularity [10,11]. These problems have received a lot of attention in recent years [12]. In order to better address these issues, we consider here a nonperturbative quantum gravity theory independent of the GR background, that is, loop quantum gravity (LQG) [13–18].

Loop quantum cosmology (LQC) is the reduced version of LQG [17], which uses the symmetries considered in cosmology. It uses the so-called Ashtekar variables and its quantization is obtained from holonomies of the connections and fluxes of the densitized triads. However, taking into account such quantum geometric effects in cosmological models, while Einstein's equations maintain an excellent degree of approximation at low curvature, in the Planck regime they undergo major changes. In LQC the big bang singularity is naturally resolved and replaced by a bounce due to repulsive quantum geometry effects [13,19].

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[1]Although inflation is the current paradigm for the early Universe cosmology, it is worth mentioning that there are alternative ideas [1–3], like several bouncing models, which can agree with current cosmological observations as well as inflation does.

In LQC, for matter that satisfies the normal conditions of energy, whenever a curvature invariant grows at the Planck scale the effects of quantum geometry dilute it, thus resolving the singularities of GR [13].

Within the community of LQC there is a lively debate on the naturalness of the emergence of an inflationary phase after the bounce, and following this line there is a search for the most probable number of inflationary $e$-folds predicted by a model [20]. First of all, in addressing this question the measure problem is something that requires quite some attention, given that there is no consensus on how to establish the initial conditions necessary to obtain the dynamics of the models and compute probabilities. Since there is no direct observational information from the initial conditions of the Universe, one has to consider all possible initial conditions to draw conclusions about the probability of an inflationary phase [21].

Beginning from the GR context, the possibility of using the Liouville measure as a candidate to calculate the probability was discussed by Gibbons *et al.* [7]. However, in the flat Friedmann-Lemaître-Robertson-Walker model this total Liouville measure is infinite, requiring a regularization scheme [10,11]. Besides that, there is a huge discrepancy between the probability estimated by Gibbons and Turok [11] and the results obtained, for example, by Linde [5].

In LQC, since the singularity of the big bang is solved and it is replaced by a (quantum) bounce [18,22,23], a regular surface can be used to introduce the structure needed to specify a Liouville measure (see also Refs. [21,24] for extensions of this approach). The problem of making a measurement present in GR [7] is naturally resolved in LQC [25]. In the absence of the singularity, an *a priori* probability for a sufficiently long slow-roll inflation phase can then be obtained. However, also in the context of LQC, different approaches have been advocated. Ashtekar and Sloan [26] argued that a natural measure can be implemented in LQC and proposed a Planck surface scale, with which probabilities can be calculated. The approach advocated in Ref. [26] does not agree with the one suggested in Refs. [27–30]. Despite the current debate, many works have consistently shown that in LQC models with a kinetic-energy-dominated bounce an inflationary phase almost inevitably sets in (see, e.g., Refs. [31–36]).

In addition to showing the naturalness of inflation, it is important to investigate the most probable number of inflationary $e$-folds predicted by these models. As it is well known [4], the inflationary phase must last at least around 60 or so $e$-folds in order to solve the main problems that inflation is expected to. On the other hand, another important question is whether the quantum bounce and subsequent preinflationary phase can leave observational signatures that can be observed in current and forthcoming experiments [37,38]. As shown in Ref. [37], the bounce and preinflationary dynamics leaves imprints on the spectrum of the CMB. In Ref. [31] it was shown that in LQC models, in order to be consistent with observations, the Universe must have expanded at least around 141 $e$-folds from the bounce until now. This is so because LQC can lead to scale-dependent features in the CMB, and the fact that we do not observe them today means that they must have been well diluted by the post-bounce expansion of the Universe. By comparing that total number of expansion of the Universe to the minimum number of inflationary $e$-folds required (added to the typical 60 $e$-folds from the end of inflation until today), this implies an extra number of inflationary $e$-folds in LQC, given by $\delta N \sim 21$ [31]. On the other hand, if the number of extra inflationary $e$-folds is much higher than this value the features imprinted in the CMB spectrum due to the LQC effects are too diluted, and in this case LQC cannot be directly tested even by forthcoming experiments. This motivates a deep investigation of the most probable number of $e$-folds in models of LQC. The most probable number of inflationary $e$-folds can be obtained with the calculation of a probability density function (PDF) [27,30], which can be performed with initial conditions defined during the bounce [26] or even in a contraction phase before the bounce [10]. In Refs. [32–36] different potentials were investigated in the context of LQC, including power-law potentials [32], monodromy potentials with a modulation term [34], alpha-attractor potentials [36], and chaotic and Starobinsky potentials in the framework of modified LQC models [35]. The duration of inflation was analyzed in all of these models by setting initial conditions at the bounce surface, providing very interesting results.

In this paper we are interested in obtaining the PDF for the number of inflationary $e$-folds in LQC by following the perspective adopted in Refs. [27–30], which suggests a natural quantity to which a flat prior can be assigned, providing the means to define initial conditions in a consistent way. Following this approach, we will define the set of initial conditions in the remote past of the contraction phase prior to the bounce, i.e., when the Universe is classic and well understood. In Refs. [27–30] studies were made of different forms of the inflationary potential, with the initial conditions taken far back in the contracting phase including only the energy density of the inflaton as the main ingredient of the early Universe and at the bounce.

The present paper extends the analysis performed in Refs. [27,29,30,32–36] by considering higher powers of the monomial potential and analyzing the duration of inflation with a Higgs-like potential as a function of the vacuum expectation value (VEV). These analyses provided us with a great comparison tool for the second part of our work, where we consider radiation as an additional ingredient of the energy density budget around the bounce, which is done for the first time. There are many good reasons for including radiation in these studies. First, it is not excluded at all that prior to inflation the Universe could have been radiation dominated. In fact, radiation has been claimed to be an important ingredient in setting appropriate initial conditions for inflation [39]. Dissipative effects are naturally expected in the early Universe, where radiation can be produced either by decaying processes involving the inflaton field through its coupling to other fields or through other fields not directly coupled to the inflaton. These processes—which can also lead to reheating at the end of cold inflation as the inflaton oscillates around its minimum—are similarly expected to occur in the prebounce phase, deep in the contracting phase, where the inflaton also displays oscillations. In fact, initial conditions in the contracting phase with inflaton oscillations are exactly the initial conditions advocated in Refs. [27,29,30]. In addition, radiation production may not even need strong breaking of adiabaticity caused by the inflaton oscillations but can also happen under quasiadiabatic conditions. An outstanding example of this is radiation production processes happening in the warm inflation picture [40] (for earlier studies of warm inflation in the context of LQC see, for example, Refs. [24,41–44]). There are also many other possible sources of radiation, including gravitational particle production mechanisms [45,46]. In particular, gravitational particle production has been shown to be very efficient in the bounce phase of several models [47–53] and we also expect the same to happen in LQC, as recently shown in Ref. [54]. The presence of radiation may adversely affect the predictions for inflation in LQC, and this provides the main motivation for the present work.

This paper is organized as follows. In Sec. II we briefly review the theoretical background about LQC and explain how radiation can be included in the system. In Sec. III we describe the different dynamic regimes expected in LQC, from the deep contracting phase prior to the bounce, up to the slow-roll phase in the expanding regime. In Sec. IV we describe the method used in our analysis and give the results obtained therein. In Sec. V we discuss additional effects neglected in our analysis that could contribute to the results. Finally, in Sec. VI we give our conclusions.

## II. THEORETICAL BACKGROUND

In this section we briefly review the background dynamics of LQC. We also discuss the generality of the inflationary phase that can be generated in LQC and how to obtain the most likely number of inflationary $e$-folds of a given model.

In LQC cosmological models are described using LQG principles. As discussed in Ref. [26], in LQC the spatial geometry is encoded in a variable $v$ proportional to the physical volume of a fixed, fiducial, cubic cell, in place of the scale factor $a$, i.e.,

$$v = -\frac{4\mathcal{V}_0 a^3 M_{\rm Pl}^2}{\gamma}, \tag{2.1}$$

where $\mathcal{V}_0$ is the comoving volume of the fiducial cell, $\gamma$ is the Barbero-Immirzi parameter obtained from the calculation of the black hole entropy in LQG (the typically value adopted in LQC is $\gamma \simeq 0.2375$ [55]), and $M_{\rm Pl} \equiv 1/\sqrt{8\pi G} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. The conjugate momentum to $v$ is denoted by $b$ and it is given by

$$b = -\frac{\gamma P_{(a)}}{6a^2 \mathcal{V}_0 M_{\rm Pl}^2}, \tag{2.2}$$

where $P_{(a)}$ is the conjugate momentum to the scale factor. Therefore, the pair $(v, b)$ is used in place of $(a, P_{(a)})$. These variables are related by the Poisson bracket $\{v, b\} = -2$. After solving the Einstein equations, $b$ is related to the Hubble parameter via $b = \gamma H$.

We are interested in the Friedmann equation modified in LQC. Hence, let us consider the equation of motion for $v$, which is given by [13]

$$\dot{v} = \frac{3}{\gamma\lambda} v \sin(\lambda b)\cos(\lambda b), \tag{2.3}$$

with $\lambda$ given by

$$\lambda^2 = \frac{\sqrt{3}\gamma}{2M_{\rm Pl}^2}. \tag{2.4}$$

LQC modifies the dynamics of the Einstein equations and, in terms of effective LQC solutions, the Hubble parameter can be written as

$$H = \frac{1}{2\gamma\lambda}\sin(2\lambda b), \tag{2.5}$$

where $b$ ranges over $(0, \pi/\lambda)$, and in the limit $\lambda \to 0$ GR is recovered. The energy density $\rho$ is related to the LQC variable $b$ through

$$\frac{\sin^2(\lambda b)}{\gamma^2\lambda^2} = \frac{\rho}{3M_{\rm Pl}^2}. \tag{2.6}$$

Thus, by combining Eqs. (2.6) and (2.5) the Friedmann equation in LQC assumes the form [26]

$$\frac{1}{9}\left(\frac{\dot{v}}{v}\right)^2 \equiv H^2 = \frac{\rho}{3M_{\rm Pl}^2}\left(1 - \frac{\rho}{\rho_{\rm cr}}\right), \tag{2.7}$$

where $\rho_{\rm cr} = 2\sqrt{3}M_{\rm Pl}^4/\gamma^3$.

Through the modified Friedmann equation (2.7) we can explicitly see the underlying quantum geometric effects [13], with the singularity replaced by a quantum bounce when $\rho = \rho_{\rm cr}$. For $\rho \ll \rho_{\rm cr}$ we recover GR, as expected. The above expression holds independently of the particular characteristics of the inflationary parameters when initial conditions for the Universe are assumed.

In a cosmological scenario where the Universe is dominated by the energy density of a scalar field $\phi$—the inflaton—the equation of motion for $\phi$ is simply

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \tag{2.8}$$

where $V_{,\phi} \equiv dV(\phi)/d\phi$ is the field derivative of the inflaton's potential. In the present work, we also include radiation as a main ingredient of the energy density. Radiation can be included by considering decaying processes involving the inflaton field, where part of its energy density is converted into radiation and parametrized through a dissipation term in Eq. (2.8), with dissipation coefficient $\Gamma$,

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V_{,\phi} = 0, \tag{2.9}$$

and supplemented by the equation for the evolution of the radiation energy density,[2]

$$\dot{\rho}_R + 4H\rho_R = \Gamma\dot{\phi}^2. \tag{2.10}$$

Note that by multiplying Eq. (2.9) by $\dot{\phi}$, adding it to Eq. (2.10), and using that $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$, $p_\phi = \dot{\phi}^2/2 - V(\phi)$ and $p_R = \rho_R/3$, we obtain

$$\dot{\rho}_{\rm total} + 3H(\rho_{\rm total} + p_{\rm total}) = 0, \tag{2.11}$$

which is the usual fluid equation for the total energy density, $\rho_{\rm total} = \rho_\phi + \rho_R$. This shows explicitly that Eqs. (2.9) and (2.10) are conservative with respect to the total energy density, as expected.

Alternatively to the approach adopted in Eqs. (2.9) and (2.10), we could also assume radiation to be already present in the system, at some early time, independent of explicitly relying on modifying the dynamical equations by the introduction of decay processes, e.g., directly affecting the inflaton field in Eq. (2.9). Radiation in this case could be due, for example, to the decay of other fields at some earlier times, or even through gravitational particle production mechanisms. In this case, at the time we set the initial conditions for the inflaton, there already can also be some nonvanishing early radiation energy density. In this work we consider both situations and show that our results remain unaltered and independent of the details of the radiation production mechanisms that might be at play. In either case, the total energy density is then given by $\rho = \dot{\phi}^2/2 + V(\phi) + \rho_R$, implying the modified Friedmann equation

$$H^2 = \frac{\dot{\phi}^2/2 + V(\phi) + \rho_R}{3M_{\rm Pl}^2}\left[1 - \frac{\dot{\phi}^2/2 + V(\phi) + \rho_R}{\rho_{\rm cr}}\right] \tag{2.12}$$

and its time derivative

$$\dot{H} = -\frac{3\dot{\phi}^2 + 4\rho_R}{6M_{\rm Pl}^2}\left[1 - 2\frac{\dot{\phi}^2/2 + V(\phi) + \rho_R}{\rho_{\rm cr}}\right]. \tag{2.13}$$

## III. PHASES OF LQC

Let us divide the dynamics of the Universe in LQC into that prior to and after the bounce.

### A. Pre-bounce regime

Let us consider a sufficient time back in the contracting phase where the inflaton is in an oscillatory regime. In this pre-bounce regime, where $H < 0$, $\phi$ and $\dot{\phi}$ are oscillating with increasing amplitudes or have damped oscillations, depending on whether the decay processes given by $\Gamma$ in Eq. (2.9) are present or absent ($\Gamma = 0$). Either way, we can characterize this regime by the conditions

$$\rho \ll \rho_{\rm cr}, \qquad H < 0, \qquad H^2 \ll |V_{,\phi\phi}|, \tag{3.1}$$

and when including $\Gamma$, with also the condition $\Gamma < 2\sqrt{|V_{,\phi\phi}|}$ such that the inflaton is still oscillating, albeit in an underdamped way. Following the proposal of Refs. [27,30], we define initial conditions for the Universe in this phase of an oscillating inflaton field in the contracting phase. In Ref. [27] it was suggested as a natural variable to assign initial conditions in this regime the phase $\delta$ of the field oscillations. Though this is a natural choice for the simple case of the quadratic inflaton potential, where both $\phi$ and $\dot{\phi}$ have simple oscillating (or, in the presence of $\Gamma$, underdamped) solutions in the regime of Eq. (3.1), for other types of potentials the expression for the field and its derivative in the contracting phase may not be that simple. Therefore, in our numerical analysis (which we describe below) we will assign initial conditions directly to the scalar field and its derivative by choosing appropriate values for the initial density ratio defined by $\alpha = \rho/\rho_{\rm cr}$, with $\alpha$ sufficiently small such that the conditions of Eq. (3.1) hold. Note that in the case where $\Gamma = 0$, but still including some initial radiation energy density, this will also entail some upper bound for the initial radiation energy density.

As we approach the bounce, starting from the point given by Eq. (3.1), there might be a phase of *slow-roll deflation*. This phase is the opposite of what happens in slow-roll inflation, as it is still in the contraction phase. This phase is

[2] Note that in the oscillating regime for the inflaton, we can also replace the term $\dot{\phi}^2$ in Eq. (2.10) by its average over an oscillation cycle [56], $\langle\dot{\phi}^2\rangle_{\rm cycle} = 2\rho_\phi$, which for Eq. (2.10) gives the more standard form used, e.g., in reheating studies.

characterized by an almost constant $\dot{\phi}$ and a linearly growing $|\phi|$. The conditions for slow-roll deflation are

$$\rho \ll \rho_{\rm cr}, \quad H < 0, \quad H^2 \gg |V_{,\phi\phi}|, \quad V(\phi) \gg \dot{\phi}^2/2, \rho_R. \tag{3.2}$$

However, the probability that this phase will occur is small since almost none of the possible paths that start at low energy in the contraction phase have an exponential contraction phase in the pre-bounce. Thus, the fraction of trajectories that have a significant contraction phase is very small, implying that the dynamics of these trajectories (for a high energy density) are strongly dominated by kinetic energy [30]. In the presence of radiation the probability of this phase gets even slimmer since, as one gets close to the bounce, the radiation energy density (which grows faster than the potential energy density in the contracting regime) will tend to dominate over $V(\phi)$.

Finally, just prior to the bounce, there is a phase of *superdeflation*. This phase, which occurs just before the bounce and thus still in the contracting phase when $H < 0$, lasts from the time when $\dot{H} = 0$ until $H = 0$ (i.e., already in the bounce). In this phase, we then have

$$H^2 \gg |V_{,\phi\phi}|, \qquad \dot{\phi}^2/2 \gg V(\phi), \qquad \rho_R. \tag{3.3}$$

We typically find that this phase of superdeflation happens very quickly, typically lasting less than a Planck time [31]. The presence of radiation can make it even shorter, as the radiation will tend to take a large portion of the energy density prior to the bounce.

### B. Post-bounce regime

Immediately after the bounce, if the energy density is mostly dominated by kinetic energy, we have a phase of *superinflation*. This phase, already at the beginning of the expansion, goes from just after the bounce (when $H = 0$, i.e., $\rho = \rho_{\rm cr}$) until the point where $\dot{H} = 0$. The conditions for superinflation are again the same as in Eq. (3.3), however, at the commencement of the expanding phase. This is also a very short phase, just like the superdeflation one, and radiation also tends to make it shorter.

After the bounce phase, the kinetic energy quickly decreases as $\dot{\phi}^2 \propto 1/a^6$ and the radiation decreases as $\rho_R \propto 1/a^4$, while the potential energy density $V(\phi)$ only changes slowly. The inflaton dynamics after the bounce and throughout the preinflationary phase is just monotonic, with no oscillations [31]; thus, we expect no significant radiation production in this phase. By also neglecting other possible sources of radiation in this phase, the potential energy of the inflaton will eventually dominate the energy content of the Universe and the standard slow-roll inflationary phase will set in, but with a duration that can be strongly affected by the radiation present already in the earlier phases, as we will see in the next section.

At the beginning of slow roll we have that $\rho \ll \rho_{\rm cr}$, the quantum corrections to the Friedmann equation are negligible, and the cosmological equations reduce to the usual ones of GR. Let us estimate the number of $e$-folds of expansion from the bounce to the beginning of slow-roll inflation. In the absence of radiation, the transition from the stiff matter kinetic-energy-dominated regime after the bounce to the slow-roll phase happens rather quickly, with the equation of state changing from $w \approx 1$ to $w \approx -1$ typically in less than one $e$-fold [31]. Depending on the amount of radiation present, we can have an intermediate radiation-dominated regime [24,54] where the equation of state at the bounce $w \simeq 1$ changes to $w \simeq 1/3$, before assuming the value $w \simeq -1$ at the start of inflation (which occurs when the equation of state becomes smaller than $-1/3$).

The number of $e$-folds during the preinflationary phase $N_{\rm preinfl}$, from the bounce to the start of slow roll, can be approximately estimated in the absence of radiation by assuming that around the start of slow roll, at time $t_{\rm sr}$, $\rho_{\rm kin}(t_{\rm sr}) \equiv \dot{\phi}^2(t_{\rm sr})/2 \sim \rho_V(t_{\rm sr})$, where $\rho_V \equiv V(\phi)$. By also recalling that the bounce is dominated by the kinetic energy, $\rho_{\rm kin}(t_{\rm bounce}) \simeq \rho_{\rm cr}$, we have that

$$\rho_{\rm kinetic}(t_{\rm sr}) \simeq \frac{\rho_{\rm cr}}{a^6(t_{\rm sr})} \sim \rho_V(t_{\rm sr}). \tag{3.4}$$

As an estimate for $\rho_V(t_{\rm sr})$ we can use the upper bound obtained by the *Planck* data on the scale of inflation when the pivot scale exits the Hubble radius [57], $V_* < (1.6 \times 10^{16}\ {\rm GeV})^4$. Using this result in Eq. (3.4), we obtain

$$N_{\rm preinfl} = \ln\left[\frac{a(t_{\rm sr})}{a(t_{\rm bounce})}\right] \sim \frac{1}{6}\ln\left(\frac{\rho_{\rm cr}}{V_*}\right) \sim 4.3. \tag{3.5}$$

Note that the estimate given by Eq. (3.5) is based on the value for the scale of inflation at around the time that the relevant wavelengths cross the Hubble radius during inflation, which happens at around 60 or so $e$-folds before the end of inflation. For inflation lasting much longer than the minimum, we do not expect a much higher value for the potential at the beginning of inflation as a consequence of the slow-roll conditions. As we will explicitly see for the different inflation models studied in the next section, despite the fact that each model predicts rather different values for the total number of $e$-folds of inflation, we always find that $N_{\rm preinf} \sim 4$. This shows that the estimate given by Eq. (3.5) is quite satisfactory when in the absence of radiation. The effect of radiation on the above estimate can be understood by the fact that it removes part of the energy density of the inflaton that would otherwise be available. Thus, it delays the start of inflation and $N_{\rm preinfl}$

increases when compared to the cases when radiation is absent. This effect will be explicitly seen in our numerical results. This result can also be understood analogously in terms of the scale of inflation in Eq. (3.5). Radiation not only delays the start of inflation, but also decreases $V_*$, thus increasing the estimate for $N_{\rm preinfl}$.

## IV. METHOD, NUMERICAL STRATEGY, AND RESULTS

As already mentioned, in this work we closely follow the procedure suggested in Refs. [27,30] to obtain the appropriate PDFs for the expected number of $e$-folds of inflationary expansion for the different models that we will analyze. The procedure can be summarized by the following steps:

(1) We consider an appropriate initial time deep in the contracting regime prior to the bounce. The initial energy density $\rho_0$ is such that $\rho_0 = \alpha\rho_{\rm cr}$ is small enough ($\alpha \ll 1$) so as to start the evolution early in the contracting phase with the inflaton field in the oscillatory regime defined in Eq. (3.1). For all of our numerical studies we consider in particular that $\alpha < 8 \times 10^{-17}$, while checking the consistency of the results for each potential as $\alpha$ was varied.

(2) For the considered initial energy density $\rho_0$ at the initial time $t_0$, we take random samples of initial values for the scalar field, which will be localized around the minimum of its potential with some dispersion $\Delta\phi$, such that $-\phi_0 - \Delta\phi \le \phi(t_0) \le \phi_0 + \Delta\phi$, where $\phi_0$ is the value of the inflaton field at the bottom of its potential. The radiation energy density can either be introduced through dissipative processes like in Eqs. (2.9) and (2.10), starting with $\rho_R(t_0) = 0$ with a fixed dissipation coefficient $\Gamma$, or we can set an initial radiation energy density $\rho_R(t_0) \neq 0$ and vanishing dissipation coefficient, as explained in the previous section. Finally, the time derivative of the inflaton field is then set as $\dot{\phi}(t_0) = \pm\sqrt{2}\sqrt{\rho_0 - V(\phi(t_0)) - \rho_R(t_0)}$, with a randomly chosen sign.

(3) We solve the dynamics with the produced initial conditions from the contracting branch to the end of slow-roll inflation in the expanding branch using the dynamical equations of motion given by Eqs. (2.9), (2.10), and (2.13), which are solved for the different inflationary models described by the potential $V(\phi)$. In the cases studied with radiation being produced in the contracting phase due to the inflaton's oscillations, we assume perturbative decay analogously to what can happen in the reheating phase after inflation [58,59], setting $\Gamma = 0$ when the inflaton stops oscillating, which happens right after the bounce. Due to the very short duration of the bounce phase ($\Delta t \sim t_{\rm Pl}$), we neglect any source of particle production during the bounce. Therefore, we can set $\Gamma = 0$ just after the bounce in the expanding phase. In a second approach, for comparison, we simply consider the presence of an already present initial amount of radiation energy density in the contracting phase at the beginning of our simulations and set $\Gamma = 0$ in Eqs. (2.9) and (2.10), and then evolve the system from the initial time $t_0$ to the end of inflation with the resulting equations.

(4) For each initial condition sampled we obtain the corresponding number of $e$-folds and produce the associated PDF, from which the appropriate statistical analysis can be performed. We work with samples ranging from 1000 to 5000 points for each model analyzed, which we find to be enough to obtain satisfactory statistics.

### A. Models

In this work we study two classes of inflation models with primordial potentials as given below.

#### 1. Power-law monomial potentials

In this class of models, we have $V(\phi)$ given by

$$V = \frac{V_0}{2n}\left(\frac{\phi}{M_{\rm Pl}}\right)^{2n}, \tag{4.1}$$

and we explicitly analyze the cases for the quadratic, quartic, and sextic forms of the potential (corresponding to the powers $n = 1$, 2, and 3, respectively). The model given by Eq. (4.1) covers the class of inflationary models corresponding to large-field models [60].

#### 2. The Higgs-like symmetry-breaking potential

The Higgs-like symmetry-breaking potential is given by the following expression:

$$V = V_0\left[1 - \left(\frac{\phi}{v}\right)^2\right]^2, \tag{4.2}$$

where $v$ denotes the VEV of the field. The Higgs-like symmetry-breaking potential can represent either a small-field inflation model if inflation starts (and ends) at the plateau part of the potential (i.e., for $|\phi| < |v|$), or a large-field model, for which inflation ends in the chaotic part of the potential ($|\phi| > |v|$). Throughout our analysis with this potential, we explicitly distinguish these two possibilities and produce results for both.

In all of the above potentials the constant $V_0$ is obtained from the normalization of the CMB spectrum, and this is how we define $V_0$ for each of the above potentials. For definiteness, we have fixed $V_0$ for each model as $V_0/M_{\rm Pl}^4 \simeq 3.41 \times 10^{-11}$ for the quadratic monomial potential, $V_0/M_{\rm Pl}^4 \simeq 1.37 \times 10^{-13}$ for the quartic monomial potential, and $V_0/M_{\rm Pl}^4 \simeq 1.82 \times 10^{-16}$ for the sextic monomial potential. Note that for the Higgs-like symmetry-breaking potential (4.2), the normalization of the spectrum implies that the value of $V_0$ will also have a dependence on the VEV of the inflaton, but for the VEVs we consider, $14 M_{\rm Pl} \leq v \leq 25 M_{\rm Pl}$, $V_0$ has values ranging from $V_0/M_{\rm Pl}^4 \simeq 1.72 \times 10^{-14}$ to $3.82 \times 10^{-14}$.[3]

Note that the monomial potentials like the ones we consider here are already ruled out in the simple scenarios of cold inflation, according to the *Planck* results [57]. The Higgs-like potential, on the other hand, can still be compatible with the observations for some ranges of the VEV. However, when radiation processes are present (most notably as is the case for these models when studied in the warm inflation context) all of these potentials can be shown to agree with the observations (see, e.g., Refs. [44,61–65]). Looking ahead at the possibility of extending the analysis presented here to warm inflation, this is why we consider the above potentials in particular, besides, of course, the fact that they are well motivated in the context of particle physics models in general.

## B. Results

Having explained the numerical strategy that we employ in our analysis, we now give the corresponding results obtained by using each of the primordial inflaton potential models defined by Eqs. (4.1) and (4.2). For comparative purposes, we first consider the case where radiation is absent throughout the evolution, from the contracting phase at the initial time $t_0$ to the end of inflation, and then consider explicitly how radiation influences these results.

### *1. Results in the absence of radiation*

In Fig. 1 we show the PDFs obtained for the total number of inflationary $e$-folds for the three cases considered for the monomial power-law potential (4.1), i.e., for the quadratic ($n = 1$), quartic ($n = 2$), and sextic ($n = 3$) potentials.

As we see from Fig. 1, as we increase the power $n$ of the potential the number of $e$-folds decreases. The PDFs for the three cases considered have a dispersion of around 20 $e$-folds from the peak of the distribution, and quickly vanish at the extrema. In particular, we obtain no more than about a total of 80 $e$-folds of inflationary expansion for the sextic potential. One recalls that from the results for the perturbation spectra in LQC, one typically requires at least around 80 $e$-folds of total expansion from the bounce in LQC to the end of inflation, such that the quantum effects on the primordial power spectra are sufficiently diluted [31]. On the contrary, if the total expansion lasts less than this minimum, the LQC effects on the spectra would already be visible. As the preinflationary expansion that starts from the bounce until the beginning of inflation does not last more than about 4 $e$-folds (see discussion at the end of Sec. III and also the explicit results on this given below), this already puts the sextic potential in strong tension with the observations and excludes all other higher-power monomial potentials ($n > 3$) when considering the predicted number of $e$-folds alone in LQC, even when these models are implemented in the warm inflation picture.[4] On the other hand, the quartic potential (and all other cases with $n < 3$) can most easily satisfy the required minimum amount of expansion from the bounce to the end of the inflationary phase. Finally, we note that the result we have obtained for the quadratic potential, which gives a $N_{\rm infl}$ of around 140, is in agreement with the previous results already obtained in Ref. [27] for this specific form of the inflationary potential. The results for the quartic and sextic forms of the potential are new.

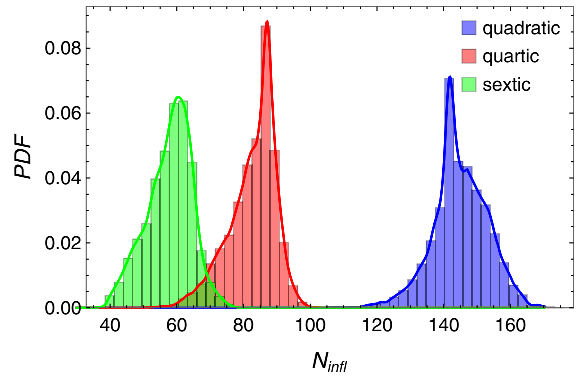



FIG. 1. PDF for the total number of inflationary $e$-folds for the monomial power-law potentials in LQC obtained when radiation is neglected throughout the evolution.

To complete our analysis for the monomials power-law potentials, in Fig. 2 we also show the results for the PDFs for the number of preinflationary $e$-folds, which considers the expansion from the bounce to the beginning of the slow-roll inflation. We note from the results shown in Fig. 2 that, despite the differences in the PDFs, the expected number of preinflationary $e$-folds is $N_{\rm preinfl} \sim 4$ for all three models, which agrees with the estimate given by Eq. (3.5).

For the Higgs-like symmetry-breaking potential (4.2) we analyze cases for different values of the VEV $v$. The results for the total number of $e$-folds of inflation as a function of $v$ are summarized in Fig. 3(a). Note that we have explicitly separated the cases of inflation happening in the plateau

[3]Note that, depending on the decay processes and the amount of radiation at the time that the CMB scales leave the Hubble radius during inflation, the normalization $V_0$ can change with respect to the vacuum values, as in, e.g., the case of warm inflation [61]. However, we do not consider these processes that can change the primordial power spectrum in the present study when fixing the value of $V_0$.

[4]Also, in standard cold inflation scenarios the monomial power-law potentials are strongly disfavored based on the values for the tensor-to-scalar ratio and/or the spectral tilt predicted by them [57].

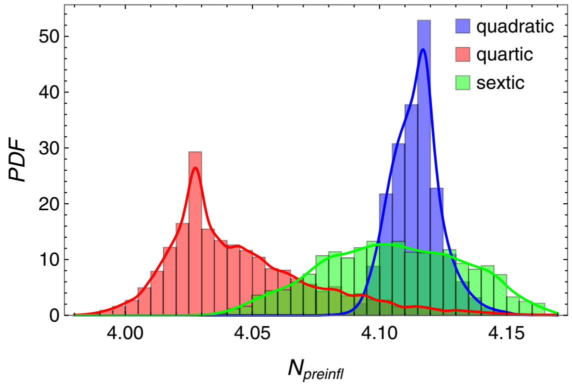
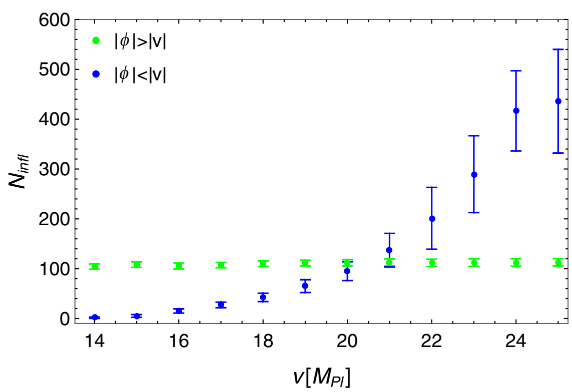
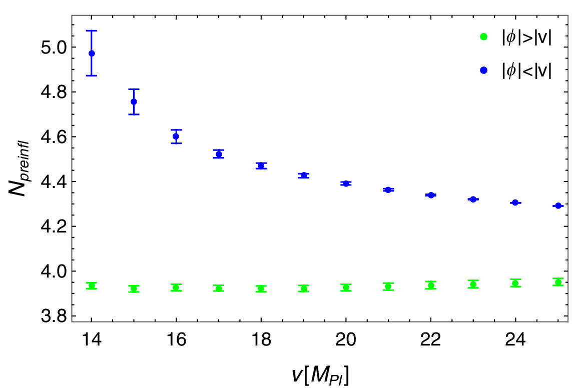



FIG. 2. Number of preinflationary $e$-folds for the power-law potentials in LQC.

part of the potential ($|\phi| < |v|$) from the cases of inflation happening in the chaotic part ($|\phi| > |v|$). We observe that the number of $e$-folds in the chaotic part of the potential is consistently slightly above 100 $e$-folds for the cases shown




(a)



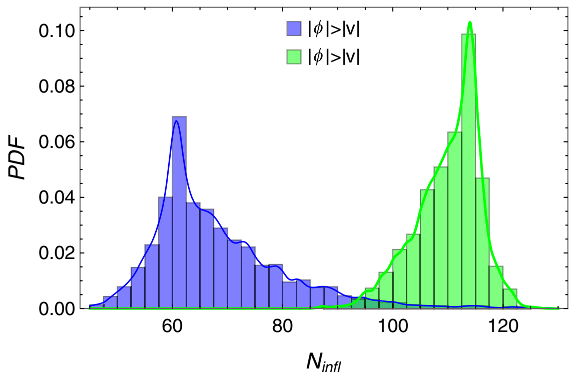

(b)

FIG. 3. (a) Number of total inflationary $e$-folds and (b) number of preinflationary $e$-folds for the Higgs-like symmetry-breaking potential in LQC as a function of the VEV. The errors bars in the plots indicate the $1\sigma$ standard deviation of the results from the median obtained from the respective PDFs. All cases were analyzed without radiation in the evolution.

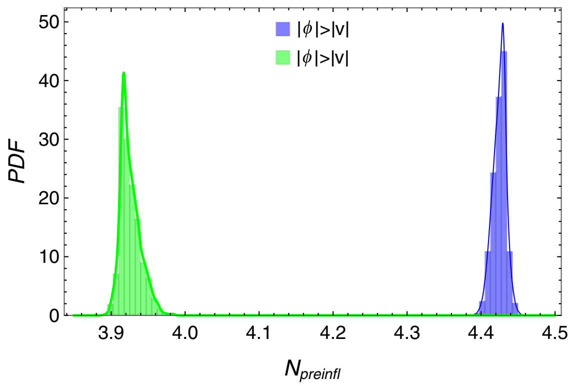
in Fig. 3(a). But we have also verified that when $|v| \lesssim 8M_{\rm Pl}$ (not shown in Fig. 3) the expected $N_{\rm infl}$ starts to approach the one seen for the quartic potential in the monomial case, as expected. We have also analyzed whether there would be any preference for inflation happening in either part of the potential. However, the results of our simulations do not show a significant preference for inflation to occur in the plateau or chaotic part of the potential. The probability for a given initial condition to end up leading to inflation in the plateau or chaotic region of the potential is always around 50%, with a slight oscillation around this value as $v$ is changed. But the results do show that for $|v| \lesssim 14M_{\rm Pl}$ there are essentially no more initial conditions leading to inflation starting and ending in the plateau region. Furthermore, for $|v| \lesssim 19M_{\rm Pl}$ the expected number of $e$-folds in the plateau part of the potential is already smaller than around 80 $e$-folds, and the discussion given above regarding the monomial potentials with $n \gtrsim 3$ applies here as well.

We note that inflation in the plateau region is subject to the well-known initial condition problem (see, e.g., Ref. [39] and references therein). In particular, the smaller the VEV in the Higgs-like potential, the less of an attractor the slow-roll trajectory becomes. Interestingly enough, in our results this initial condition problem for inflation in the plateau does not manifest in the number of initial conditions ending up in the plateau region, but instead in a reduction of the total number of inflationary $e$-folds as $v$ decreases. On the other hand, the larger the VEV, the larger the number of $e$-folds in the plateau region, which here is a manifestation of the increase of the attractor nature for the slow-roll trajectories on the plateau and as the plateau gets flatter as $v$ increases, hence leading to potentially more $e$-folds. In Fig. 3(b) we give the results for the predicted number of preinflationary $e$-folds for the Higgs-like potential. Once again, we have explicitly separated the cases of initial conditions leading to inflation in the plateau or in the chaotic parts of the potential. The results show that $N_{\rm preinfl}$ decreases with $v$ for the case of inflation occurring in the plateau and tends to converge towards $N_{\rm preinfl} \sim 4.3$ for $|v| > 24M_{\rm Pl}$. On the other hand, for inflation occurring in the chaotic part of the potential, we obtain that $N_{\rm preinfl}$ is almost independent of $v$, though the data shows a slow increase as $|v|$ increases and $N_{\rm preinfl}$ is slightly below 4, but still consistent with the estimate given by Eq. (3.5).

As a complement and example case extracted from the above results for the Higgs-like symmetry-breaking potential, in Fig. 4(a) we explicitly show the PDF for the number of inflationary $e$-folds, taking as an example the vacuum expectation value of the Higgs-like symmetry-breaking potential to be $v = 19M_{\rm Pl}$. Likewise, in Fig. 4(b) we also show the PDF for the number of preinflationary $e$-folds from the bounce to the beginning of the slow-roll inflation obtained for the same VEV.




(a)




(b)

FIG. 4. (a) PDF for the number of inflationary $e$-folds for the chaotic and plateau parts of the Higgs-like symmetry-breaking potential in LQC considering the value $v = 19M_{\rm Pl}$. (b) PDF for the number of preinflationary $e$-folds for the chaotic and plateau parts of the Higgs-like potential in LQC considering the same VEV. As in the previous figures, radiation is absent throughout the evolution.

Finally, for completeness we summarize our main results that can be extracted from all of the PDFs in the Table I, where we give the results for the median and standard deviation for $N_{\rm infl}$ and $N_{\rm preinfl}$ for each of the models studied when neglecting radiation effects. For the Higgs-like symmetry-breaking potential, we only give results obtained from the specific example shown in Fig. 4. For the other VEVs studied, see Fig. 3.

TABLE I. Values for the median and standard deviation ($1\sigma$) for the number of preinflationary and inflationary $e$-folds for the power-law and Higgs-like symmetry-breaking potentials in LQC in the absence of radiation effects.

| | Median and Standard Deviation | |
|---|---|---|
| Model | $N_{\rm preinf}$ | $N_{\rm infl}$ |
| Quadratic | $4.115 \pm 0.010$ | $144 \pm 8$ |
| Quartic | $4.038 \pm 0.030$ | $84 \pm 7$ |
| Sextic | $4.10 \pm 0.06$ | $59 \pm 7$ |
| Higgs ($v = 19M_{\rm Pl}$) plateau | $4.426 \pm 0.009$ | $65 \pm 13$ |
| Higgs ($v = 19M_{\rm Pl}$) chaotic | $3.923 \pm 0.014$ | $111 \pm 6$ |

### *2. Results in the presence of radiation*

Let us now study how the inclusion of radiation affects the above results. We start by considering Eqs. (2.9), (2.10), and (2.13) with the dissipation coefficient $\Gamma$. One notes that here $\Gamma$ parametrizes a radiation production process where part of the energy density of the inflaton is converted to radiation. As already pointed out in the previous section, there can be many other different processes at play generating radiation that are not directly related to the inflaton (e.g., the decay of spectator fields, gravitational particle production, etc.). Parametrizing radiation production like the perturbative decay of the inflaton might represent only one such process. However, as explained below, our results are only dependent on the amount of radiation prior to the bounce and much less on which particular process (or processes) might lead to it. This significantly simplifies our study, in addition to showing that our results should not be sensitive to the details of the dynamics of radiation production in the contracting phase. These are rather strong claims, and we justify them by considering as an example the case of the monomial quadratic inflaton potential.

In Fig. 5(a) we show the effect of the radiation production through $\Gamma$ on the expected number of $e$-folds of inflation for the monomial quadratic model. The larger the $\Gamma$, the smaller the number of $e$-folds expected for inflation later in the expanding region post-bounce. This result can also be correlated with the expected value for the inflaton field at the bounce time $t_B$, $\phi(t_B)$, as shown in Fig. 5(b). As seen in Fig. 5(b), the larger the $\Gamma$, the smaller the amplitude of the inflaton field at the bounce, and the smaller the resulting number of $e$-folds. Note that the smaller resulting potential energy density of the inflaton at the bounce cannot be compensated by a larger kinetic energy, since now part of the total energy density at the bounce comprising the critical density $\rho_c$ will be in the form of radiation energy density at the bounce $\rho_R(t_B)$, as can be seen in Fig. 5(c).

As explained in the previous section, these results are obtained from the PDFs that were generated for different values of $\Gamma$. In Fig. 5 we show the median and $1\sigma$ standard deviation (shown as error bars) derived from these PDFs. In this specific example, we consider in particular the fraction of total energy density at the initial time $t_0$ in the contracting phase as $\alpha \equiv \rho(t_0)/\rho_c = 10^{-19}$. We have added a subindex $\alpha$ to $\Gamma$ to explicitly point out that these results, when expressed in terms of the decay coefficient, should be interpreted as $\alpha$ dependent. This is understandable, since $\alpha$ specifies how far back in the contracting phase we initiate our simulations, and hence determines how many oscillations the inflaton will undergo during its evolution. Of course, the radiation energy density produced will be

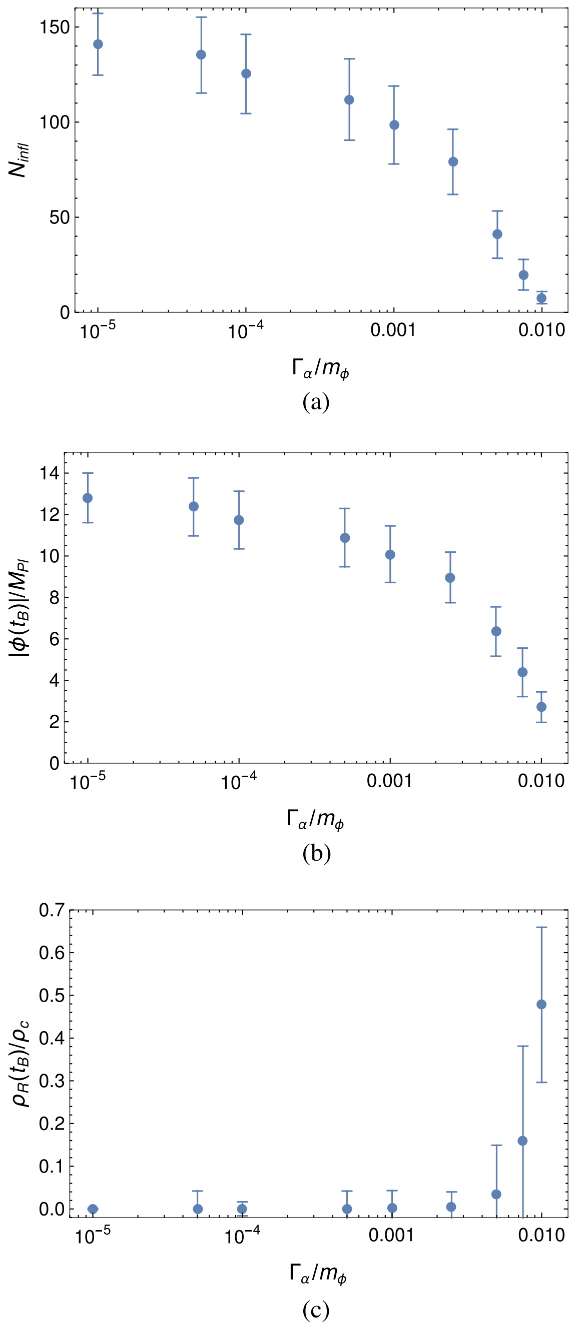
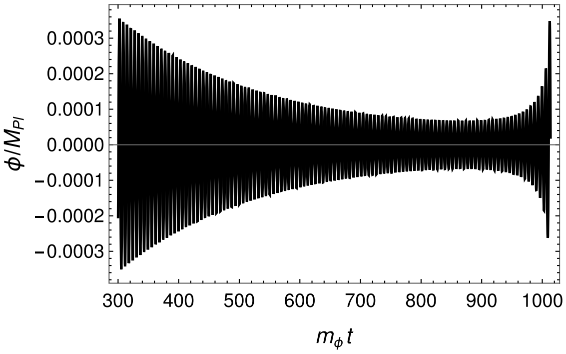



FIG. 5. (a) Number of total inflationary $e$-folds, (b) modulus of the amplitude of the inflaton at the bounce, and (c) radiation energy density fraction at the bounce as a function of the dissipation rate $\Gamma$, for the case of the monomial quadratic inflaton potential. The inflaton mass here is given by $m_\phi = V_0^{1/2}/M_{\rm Pl}$. The errors bars in the plots indicate the $1\sigma$ standard deviation of the results from the median obtained from the respective PDFs.

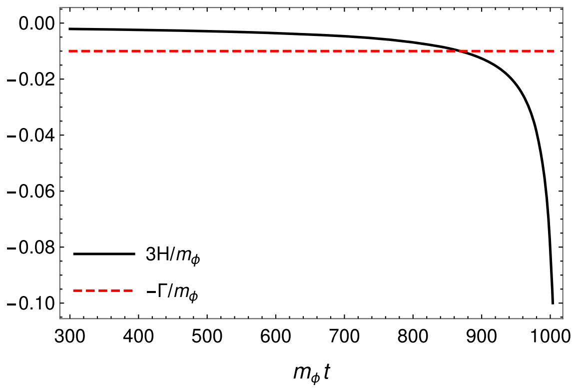
dependent on this evolution. Thus, for other values of $\alpha$ we will have a similar behavior as that shown in Fig. 5, though at different values of $\Gamma$. The important point to notice is that the Hubble parameter during the contracting phase increases in modulus (becoming more and more negative) before the bounce is approached. Therefore, even if we start the evolution with a $\Gamma > |H|$, at some point before the bounce we will necessarily have $\Gamma < |H|$. At this point the inflaton dynamics stops being damped with decreasing oscillations due to the presence of the dissipation term in Eq. (2.9) and starts to have oscillations with increasing amplitudes. In other words, the effect of $\Gamma$ on the dynamics is no longer relevant. In particular, note that radiation production is only efficient when $\Gamma > |H|$, similarly to what happens in perturbative reheating, and when $\Gamma < |H|$ radiation production becomes essentially ineffective. The radiation produced until that time will then evolve with the metric like $\rho_R \propto 1/a^4$ and increase towards the bounce time, while the inflaton still oscillates strongly.[5] Note that as we approach the bounce the modification of the Friedmann equation in LQC becomes important, and at some point we will again satisfy the condition $|H| < m_\phi$. However, the time interval of the bounce phase (when the correction to the Friedmann equation is important) is very short, typically of the order of a Planck time, such that the production of radiation due to $\Gamma$ is negligible during this short period. For this reason, we do not need to consider dissipation during the bounce phase. In Fig. 6(a) we explicitly show these expectations for the evolution of the inflaton field. The evolution of the Hubble parameter in the contracting phase is shown in Fig. 6(b). Note that when $\Gamma$ drops below $3|H|$ [which in the figure corresponds to the region where the red dashed line $(-\Gamma/m_\phi)$ is above the black solid line] is exactly the time when the damped oscillations of the inflaton turn into oscillations with increasing amplitudes, just as expected from Eq. (2.9) for the dynamics of the inflaton field in the contracting phase when $\Gamma = 0$. The resulting radiation energy density evolution times $a^4(t)$ is shown in Fig. 6(c). Once again, we see that at the same time that $\Gamma$ drops below $|H|$, i.e., the inflaton decouples from the radiation, the radiation production essentially stops and $\rho_R a^4 \sim cte$, i.e., the radiation evolves as expected had we started the evolution at that instant of decoupling $t_{\rm dec}$, with $\Gamma = 0$ and with the given radiation energy density at that instant $\rho_{\rm R}(t_{\rm dec})$ taken as its initial value. This is why both approaches—i.e., starting evolving the system of equation in the contracting phase with an explicit dissipation term in the equations at $t = t_0$ and with $\rho_{\rm R}(t_0) = 0$, or simply assuming the evolution starting at $t_{\rm dec} > t_0$ with an initial nonvanishing radiation energy density, $\rho_{{\rm R},i} \equiv \rho_{\rm R}(t_{\rm dec})$ at $t_{\rm dec}$, but with $\Gamma = 0$—turn out to be completely equivalent.

In our systematic analysis of how radiation affects the predictions for inflation in the models analyzed we still produce the PDFs starting with initial conditions in the contracting phase with either radiation being produced through a dissipation term in the evolution equations, or

[5] Recall that $|H| < m_\phi$ is the condition for the inflaton oscillations, while perturbative decay of the inflaton also requires $\Gamma \ll m_\phi$ [66].




(a)

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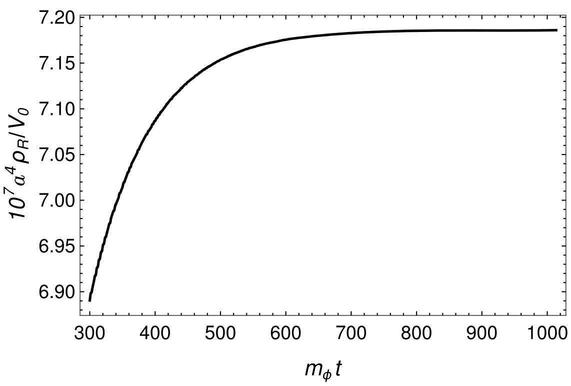
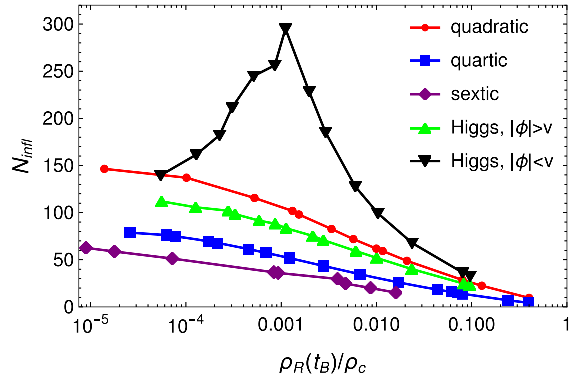
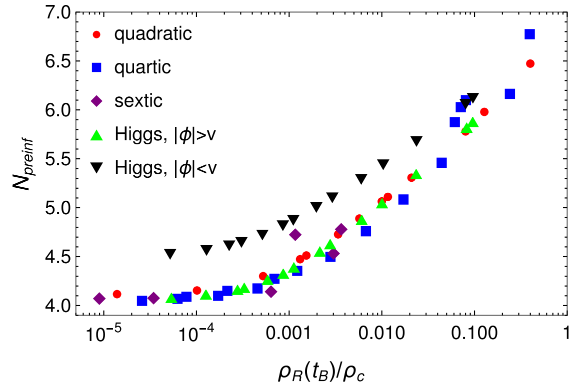



(c)

FIG. 6. One example of evolutions in the contracting phase, up to close to (but still below) the bounce instant $t_B$, for (a) the inflaton field, (b) the Hubble parameter, and (c) the radiation energy density times the fourth power of the scale factor at the bounce, $a^4\rho_R$, for the case of the monomial quadratic inflaton potential. These results were obtained for a dissipation rate $\Gamma/m_\phi = 0.01$ and a total energy density ratio at the initial time given by $\alpha \equiv \rho(t_0)/\rho_c = 10^{-19}$. Here, the bounce instant is $t_B \simeq 1018/m_\phi$.




(a)

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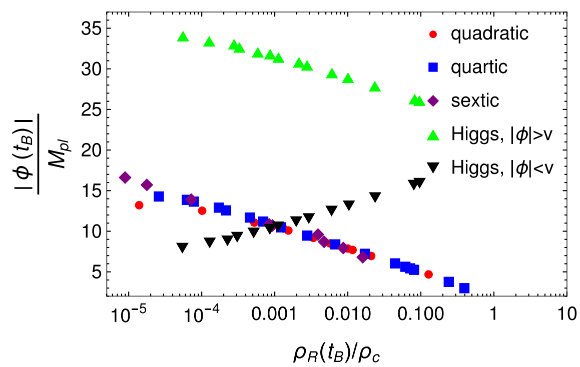


(c)

FIG. 7. (a) Duration of inflation, (b) duration of the preinflationary phase, and (c) the amplitude for the inflaton at the bounce as a function of the fraction of the radiation energy density at the bounce. All results refer to the medians extracted from the respective PDFs for each of the models studied. The results shown for the Higgs-like symmetry-breaking potential refer to the case with a VEV $v = 21M_{\rm Pl}$.

just assuming an initial radiation energy density but setting $\Gamma = 0$, as explained above. We have explicitly checked that the results post-bounce are independent of the approach used. In fact, we found that the results are better presented in a transparent way when they are expressed in terms of the fraction of the radiation energy density that will be present at the time of the bounce, $\rho_R(t_B)/\rho_c$. This way, the results are also expressed in a more general form, independent of the way the radiation production mechanisms are specified in the contracting phase.

Returning to the results for each of the inflaton potentials considered in this work and following the procedure explained above, in Figs. 7(a)–7(c) we show the results for the predicted number of $e$-folds of inflation, the number of preinflationary $e$-folds, and the value for the inflaton field amplitude at the bounce, respectively. To avoid crowding the figures, we do not show the $1\sigma$ standard deviation error bars for each of the data points (obtained from the medians of the respective PDFs for each model).

Analyzing the results shown in Fig. 7(a), a number of important features emerge as a result of including the effects of radiation. For the monomial potential we see the expected effect of radiation suppressing inflation according to the fraction of radiation that end up present at the bounce instant $t_B$ and that comes from the earlier evolution in the contracting phase. In particular, the larger the power $n$ in the monomial potential, the smaller the required fraction of radiation for the number of $e$-folds of inflation to drop to unsuitable values to account for the observations. For example, for the quadratic potential the number of $e$-folds drops below 50 when the fraction of radiation at the bounce is around 2%, for the quartic potential this fraction is around 0.13%, and for the sextic potential it is as small as 0.0073%. In the case of the symmetry-breaking Higgs-like potential, we have once again explicitly identified the regimes of inflation happening in the plateau region of the potential, with the inflaton amplitude at the beginning and end of inflation satisfying $|\phi| < v$, and the regime of inflation happening in the chaotic part of the potential, $|\phi| > v$. For the example shown in Fig. 7, we have chosen the case with a VEV $v = 21M_{\rm Pl}$, which in the absence of radiation produces approximately the same number of $e$-folds in the plateau and chaotic parts of the potential (see, e.g., Fig. 3), which gives $N_{\rm infl} = 118 \pm 21$ and $118 \pm 6$ for the expected number of $e$-folds for the plateau and chaotic parts of the potential, respectively. Thus, this particular VEV is better suited for comparative purposes to see the effects of radiation on the inflation dynamics when happening in one of the two branches of the potential. The behavior of $N_{\rm infl}$ as a function of radiation for the chaotic part of the potential exhibits a similar trend as the monomial potentials. It monotonically decreases with the amount of radiation that permeates the bounce and becomes less than 50 $e$-folds when the fraction of radiation at the bounce is around 1%. However, the behavior for the number of $e$-folds when inflation happens in the plateau region is quite peculiar. It instead shows a growing behavior with the increase of radiation up to a maximum value, and then decreases. This peculiar behavior can be explained by the fact that radiation takes up not only potential energy of the inflaton that it would otherwise have at the bounce instant, but also kinetic energy. There is then an increased chance for the initial conditions at the start of the slow-roll inflation to land close to the top of the potential, thus increasing the number of $e$-folds. However, as the radiation increases further beyond some value, the decrease in kinetic energy of the inflaton leads to less and less initial conditions reaching the top of the potential plateau, thus decreasing the number of $e$-folds. However, compared to the other cases we do see that inflation on the plateau is more resilient to an increase in radiation. The number of $e$-folds of inflation, for this particular VEV, only drops below 50 when the fraction of radiation at the bounce is larger than around 5%.

In Fig. 7(b) we see that the number of $e$-folds for the preinflationary phase increases with the fraction of radiation energy density. This behavior was already observed in Refs. [24,54] in the case of the quartic potential. Here we confirm that this is also a generic expectation for other forms of primordial inflaton potentials and it can be explained through the estimate for $N_{\rm preinfl}$ given in the previous section [Eq. (3.5)]. The presence of radiation will tend to lower the scale of inflation and, consequently, increase $N_{\rm preinfl}$. Furthermore, we see from the results in Fig. 7(b) that there is a certain universality of the results for the different potentials. The data points for the monomial potentials, along also the Higgs-like potential with inflation in the chaotic part of the potential, they all group together, thus having very similar behavior on how $N_{\rm preinfl}$ depends on the radiation energy density fraction at the bounce instant. In the case of the Higgs-like potential for the inflaton and with inflation happening along the plateau of the potential the behavior is similar, though shifted with respect to the other cases. This is also expected [and should also hold for other VEVs, as seen, for example, in Fig. 3(b)], given the different energy scales for inflation happening on the plateau or chaotic side of the potential.

Finally, a similar universality as that seen in Fig. 7(b) is also seen in Fig. 7(c), where we show how the (modulus of the) inflaton field amplitude at the bounce instant $t_B$ varies with the fraction of the radiation energy density. Note that all monomial potentials have data grouping together. The case of the Higgs-like inflation in the chaotic part of the potential is shifted from the monomial potentials by exactly the value of the VEV. Had we shifted the potential zero to the VEV point, $\phi \to \phi - v$, it would also be grouped with the results for the monomial potentials. Note that $|\phi(t_B)|$ decreases as the amount of radiation increases, thus leading to a smaller number of $e$-folds of inflation, consistent with what we see in Fig. 7(a). $|\phi(t_B)|$ on the plateau part of the potential can only increase towards the VEV, thus also decreasing the number of $e$-folds.

As a final remark concerning the results obtained for the Higgs-like potential, similarly to the case studied in the vacuum, we have found that the presence of radiation does not favor inflation happening either in the plateau (small-field) or chaotic (large-field) regions of the potential. We have essentially a 50/50 chance for some initial condition taken deep in the contracting phase to land in either part of the potential during the inflationary slow-roll phase. This is quite surprising in view of the fact that for inflation along the large-field part of the potential, like with any chaotic type of inflation, the slow-roll trajectory is a local attractor in the field phase space of initial conditions [67,68]. On the other hand, plateau inflaton potentials are known to suffer from the initial condition problem and have to be severely fine-tuned [69]. Though large VEVs for a Higgs-like symmetry-breaking potential can strongly alleviate this issue of the initial conditions, we have explicitly verified that the same trend also holds at small VEVs, though we are also led to a smaller number of $e$-folds, as seen in Fig. 3(a). It appears that this issue with small-field potentials in LQC turns out to manifest in the most likely (and sufficient) amount of inflation to happen than in a probability of a certain initial condition to land on either side of the potential. Surprisingly, as discussed in the case of the results shown in Fig. 7(a), there are also regimes where radiation ends up favoring a larger number of $e$-folds along the plateau part of the potential. (This is somewhat along the lines of the study done in Ref. [39] showing how a preinflationary phase dominated by radiation might end up favoring inflation by helping to localize the inflaton close to the plateau region of the potential.)

## V. ADDITIONAL EFFECTS AND FUTURE DIRECTIONS

It is important to discuss some issues that were not considered explicitly in this work but could lead to interesting effects. First, in order to make the analysis as general as possible, we did not consider any specific mechanism for the radiation production.

As discussed in the previous sections, we had simply assumed some *a priori* a particle decay process that leads to radiation production and acts in the contracting phase. That the dissipation term is added in the classical regime in the contracting phase is in particular quite convenient from a quantum field theory perspective in deriving these dissipation terms. In the classical regime, quantum gravity effects are negligible and a standard quantum field theory derivation for dissipation coefficients would apply. The quantum gravity effects would be important closer to the bounce. However, as explained in the previous section, the dissipation coefficient $\Gamma$ will in general become smaller than the (modulus of the) Hubble rate before the bounce is approached in the contracting phase, and from that point on the radiation production becomes inefficient. Thus, we do not have to deal with the details of how the quantum gravity effects would affect the radiation production (at least as far as a quantum field theory derivation for the inflaton dissipation coefficient to light fields is concerned). We could then think of decay rate terms involving, for instance, explicit interactions of the inflaton with some light fields, which can be either bosons or fermions, with interaction Lagrangian densities terms like, e.g., $\mathcal{L}_{\rm int} = -g\sigma\phi\chi^2$, with the inflaton coupled to some other scalar field $\chi$, or $\mathcal{L}_{\rm int} = -h\phi\bar{\psi}\psi$, for the case of couplings to fermions. Then, $\Gamma$ refers simply to the decay processes [70] (for $m_\phi > 2m_\chi, 2m_\psi$) $\Gamma_{\phi\to\chi\chi} = g^2\sigma^2/(8\pi m_\phi)$ and $\Gamma_{\phi\to\bar{\psi}\psi} = h^2 m_\phi/(8\pi)$, respectively, where $g$ and $h$ are two constants. Coupling other fields directly to the inflaton imposes constraints on the values for the respective couplings such that quantum corrections coming from these other fields do not spoil the required flatness of the inflaton potential. This typically requires small coupling constants, $g$, $h \ll 1$, thus leading to very small decay rates. This in turn would require a long evolution in the contracting phase such that sufficient radiation can be produced. However, there are other ways of having light fields (radiation) coupled to the inflaton and at the same time allowing for large couplings, provided the inflaton sector is protected by symmetries, like a shift symmetry in the case where the inflaton is a pseudo-Nambu-Goldstone boson, as in axionic inflation, or in the recent constructions involving the inflaton coupled directly to radiation fields, like in Refs. [63,65] in the context of warm inflation. These processes could also lead to strong dissipation mechanisms in the contracting phase and possibly be applicable in the context of the present paper. Additionally, we could also think in terms of gravitational particle production. However, these are in general very inefficient processes during the oscillatory regime of the inflaton in the pre-bounce phase. In this work, we have also not considered particle production from parametric resonance, similarly to what might happen in preheating after inflation [66], triggered by the oscillations of the inflaton. Parametric resonance is a very efficient particle production mechanism that can cause the energy density of the inflaton to quickly decrease. It would be interesting to investigate how parametric resonance could manifest itself due to the strong oscillations of the inflaton in the pre-bounce contraction phase. As we approach the bounce and the energy density approaches the Planck scale, we might also expect the opposite behavior to what we would see in the expansion regime post-inflation, probably with particle fusion happening efficiently, counterbalancing the evaporation of the inflaton condensate due to its decay during parametric resonance. In the high-energy regime close to the bounce, the energy transfer could then also target the inflaton field. Though quite interesting, a full study of the effects would certainly require a quantum kinetic study of bouncing cosmology in LQC, something beyond the scope of the present paper.

We have also neglected in our analysis the possible contribution of inhomogeneities encoded in the gradient terms, which could be important during the contraction. Even though one should not expect these terms to significantly change the PDFs we obtained, it could be important to study how these terms could affect the dynamics of the bounce phase in these models. In addition, although we have only studied the case of isotropic LQC, the presence of anisotropies could lead to important effects. In this context, the analysis made by the authors of Ref. [29] has shown that considering anisotropic effects the PDFs can be strongly affected, though we can still draw predictions from them, like for the number of $e$-folds of inflation. (In fact, the effects of anisotropies as studied in Ref. [29] have some similarities to the effects we have seen here due to radiation. By decreasing the energy density of the inflaton, we also expect a smaller number of $e$-folds for larger anisotropies.)

Our results can also affect the predictions for each model with respect to the changes radiation can impose on the power spectrum. The presence of radiation means that the initial state for which the primordial scalar curvature perturbations are evaluated is not the Bunch-Davis vacuum, but likely an excited state for the inflaton. In addition, if the radiation bath thermalizes, which in general requires that sufficient scattering happens among the radiation particles, then the formed thermal bath will be carried over into the preinflationary phase as well. Note that in general we require the condition that $\Gamma$ be larger than the expansion (contraction) rate of the Universe as a condition for thermalization [56]. As seen in the example discussed in the previous section and shown in Fig. 6(b), this condition is very likely to be satisfied during some time in the contracting phase. Even though the formed thermal bath can drop out of equilibrium after $\Gamma$ goes below $|H|$ before the bounce, the temperature of the thermal bath will simply evolve with the scale factor as $T \propto 1/a$ from that time onwards and be carried over into the post-bounce phase, even if no further particle/entropy production happens later on and before inflation. The presence of a thermal bath will lead to an enhancement of the power spectrum [71] and, consequently, to an enhancement of the power at the largest scales, i.e., for the smallest wave numbers. At the same time, the modification of mode functions due to the presence of radiation leads to a lowering of the quadrupole moment [72–74]. In LQC, the primordial scalar curvature power spectrum has also been shown to be modified [31,37], also causing an enhancement of the power at low multipoles. A recent study of these issues in the context of warm inflation [44] has shown how these different effects might counterbalance, easing the lower bound on the duration of inflation determined, e.g., in Ref. [31]. The results we have obtained in the present paper certainly call for a more detailed computation of the power spectrum in LQC whenever radiation might be present in the preinflationary phase.

## VI. CONCLUSIONS

Based on the proposal introduced by the authors of Ref. [27] on how some well-defined predictions can be made concerning the probability and duration of inflation in LQC, we have extended their analysis for other power-law monomial potentials, like the quadratic, quartic, and sextic potentials, and for the Higgs-like potential for the inflaton. In the latter model, we also investigated the results obtained for different values of the vacuum expectation value. While in the context of cold inflation the three power-law potentials are disadvantaged by the *Planck* data [57], warm inflation can rehabilitate them again due to the radiation production effects and this justifies using these potentials in the present study. Besides, as simple potential models, it is important to consider them for comparison purposes in general. Motivated by the warm inflation picture, where radiation can be present throughout the inflationary regime, in this work we investigated the effects of radiation on the predictions for inflation in LQC for all of the above-mentioned primordial inflation potential models.

Following the procedure detailed in Refs. [27,30], we obtained different PDFs for different relevant quantities including, for example, the number of $e$-folds of inflation, the number of preinflationary $e$-folds from the LQC bounce to the start of the slow-roll inflation, and the fraction of radiation energy density at the bounce, and drew statistical conclusions from them for each of the models studied here. We assumed initial conditions for the energy density in the remote past, well before the bounce, and evolved them considering also the radiation. For the cases studied and for the analysis performed for each of the resulting PDFs, we found that the number of $e$-folds of the preinflationary phase is approximately 4 $e$-folds in all of the models analyzed, and increases with the radiation energy density. On the other hand, the number of inflationary $e$-folds changes a lot between the models and also strongly depends on the radiation energy density present at the bounce time.

As already explained in previous studies (see, e.g., Refs. [29,30]), the approach of taking the initial conditions in the classical regime in the contracting phase leads to very different results than the other approach usually considered in the literature, i.e., taking the initial conditions at the bounce time. The reason for this difference can be understood as follows. In general, taking the initial conditions at the bounce time leads to a much larger number of $e$-folds, and a prediction for the duration of inflation is harder to obtain. This is understandable, since if we consider initial conditions at the bounce, i.e., where $\rho_{\rm total} = \rho_{\rm cr}$, we are allowed in principle to consider any value for the inflaton field amplitude up to the value for which $V(\phi) = \rho_{\rm cr}$, thus potentially leading to a very large number of $e$-folds. However, by taking initial conditions in the classical regime in the contracting phase, the amplitude of the inflaton at the bounce is always constrained and the bounce is essentially kinetic energy dominated, thus leading to a much smaller number of $e$-folds and allowing us to make predictions about the duration of the inflation. As explained, e.g., in Ref. [29], this is because a long deflation regime in the contracting regime (and before the bounce is reached) is strongly suppressed. (In fact, in all of our numerical simulations and for the different models we have considered, none reached such a regime of a long deflation.) This then prevents the inflaton from reaching large amplitudes and, consequently, the number of $e$-folds cannot be too large and remains constrained. We have seen this explicitly in all of our results.

We obtained that, among the power-law potentials analyzed, the sextic model in LQC is the one that predicts the lowest value for the number of inflationary $e$-folds $N_{\mathrm{infl}}$, implying a small probability of being consistent with the CMB data. The quartic potential, on the other hand, predicts the most likely $N_{\mathrm{infl}}$ to be around 80, in the absence of radiation, which suggests a very good possibility of leading to observable signatures from LQC in the CMB spectrum [31]. For the quadratic model, the most likely $N_{\mathrm{infl}}$ is around 140, in the absence of radiation, in agreement with the results obtained in Ref. [27]. With such high values of $N_{\mathrm{infl}}$, the effects from the quantum regime would probably be diluted to an unobservable level whenever there is no radiation present to affect the dynamics of expansion and the inflaton. For the Higgs-like symmetry-breaking potential we have shown that $N_{\mathrm{infl}}$ grows with the vacuum expectation value ($v$) for the case of inflation occurring in the plateau (small-field) region, while for inflation occurring in the chaotic (large-field) part of the potential $N_{\mathrm{infl}}$ is almost independent of $v$, being always around $N_{\mathrm{infl}} \sim 100$ in the absence of radiation effects. However, radiation has a strong influence on the number of $e$-folds in the plateau region of the potential. Instead of tending to suppress the duration of inflation in the plateau, it initially favors an increase of $N_{\mathrm{infl}}$, which can be by a large factor depending on the VEV and the available radiation energy density. This effect has been identified as a result of the fact that radiation production decreases the energy that would otherwise be available for the inflaton (both potential and kinetic energy). By having a smaller kinetic energy, the inflaton can then be better localized along the plateau and, hence, increase the duration of inflation.

We have also discussed the possible effects that the presence of a radiation bath might have on the primordial scalar curvature power spectrum in LQC, which also motivates further study in that direction.

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