Supermassive gravitinos and giant primordial black holes

Krzysztof A. Meissner[1] and Hermann Nicolai[2]

[1]*Faculty of Physics, University of Warsaw Pasteura 5, 02-093 Warsaw, Poland*
[2]*Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut) Mühlenberg 1, D-14476 Potsdam, Germany*

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We argue that the stable (color-singlet) supermassive gravitinos proposed in our previous work can serve as seeds for giant primordial black holes. These seeds are hypothesized to start out as tightly bound states of fractionally charged gravitinos in the radiation-dominated era, whose formation is supported by the universally attractive combination of gravitational and electric forces between the gravitinos and antigravitinos (reflecting their "almost BPS-like" nature). When lumps of such bound states coalesce and undergo gravitational collapse, the resulting mini black holes can escape Hawking evaporation if the radiation temperature exceeds the Hawking temperature. Subsequently the black holes evolve according to an exact solution of Einstein's equations, to emerge as macroscopic black holes in the transition to the matter-dominated era, with masses on the order of the solar mass or larger. The presence of these seeds at such an early time provides ample time for further accretion of matter and radiation, and would imply the existence of black holes of almost any size in the Universe, up to the observed maximum.

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I. INTRODUCTION

The origin of large (galactic) black holes, present already in the early Universe has been a long-standing puzzle; see e.g., Ref. [1] for information on the most recently discovered behemoth black hole, [2] for a generally accessible update and overview, and Refs. [3–5] and references therein for more recent work. It seems generally agreed that such large black holes cannot form by the usual stellar processes (i.e., gravitational collapse of stars and subsequent accretion of mass), but must have originated from some other source. One possible explanation is that black holes were already present from the very beginning of the matter-dominated period, and in sufficient numbers and with sufficiently large masses to be able to grow further by accretion to very large sizes already a few hundred million years after the big bang. Various mechanisms have been proposed and discussed towards solving this problem, most of them based on extrapolations of known physics, such as e.g., large random density fluctuations in the early Universe; see Ref. [6] for a comprehensive recent review with many further references. That review also discusses different observational consequences and constraints, while emphasizing that "the limits are constantly changing as a result of both observational and theoretical developments". From a more theoretical perspective, a mechanism based on bubble formation during inflation was recently put forward in Refs. [7,8], but differs essentially from the one presented here, because there the substantive part of black hole growth must take place *before* the onset of the radiation phase. At any rate, the crucial question remains whether an explanation can be found in terms of known physics, or whether an explanation necessarily involves essentially new physics.

In this paper we present a new proposal towards addressing this problem which can complement existing proposals in that it does not rely on random processes, such as density fluctuations or bubble formation, but invokes *new* physics in the form of new elementary particles. Namely, it is based on the conjectured existence of certain supermassive particles (gravitinos) that allow for the formation of black holes already during the early radiation phase, well before decoupling. There are two necessary prerequisites for a mechanism based on the "condensation" of superheavy particles to work, namely

(1) the supermassive particles must be absolutely stable against decay into Standard Model matter; and
(2) they must be subject to sufficiently strong attractive forces to enable them to rapidly cluster in sufficient amounts to undergo gravitational collapse.

Although *Ansätze* towards fundamental physics, in particular Kaluza-Klein theory and string theory, abound in massive excitations that might serve as candidates for such a scenario, such excitations usually fail to meet the first requirement (with decay lifetimes on the order of the Planck time t_{Pl}), which is why they are often assumed to play no prominent role in the cosmology of the very early Universe. Here we will argue that, by contrast, the superheavy gravitinos proposed in our previous work [9,10] can meet both requirements. That the requisite particles should be gravitinos, rather than some other particle species, is perhaps unusual, so let us first explain the reasons for this claim.

Our proposal has its origin in our earlier attempt to understand the observed spin-$\frac{1}{2}$ fermion content of the Standard Model, with three generations of quarks and leptons (including three right-chiral neutrinos). It relies on a unification scenario based on a still hypothetical extension of maximally extended $N = 8$ supergravity involving the infinite-dimensional duality symmetries E_{10} and $K(E_{10})$ [9–11] (this proposal itself has its origins in much earlier work [12,13]). The enlargement of the known duality symmetries of supergravity and M-theory to the infinite-dimensional symmetries E_{10} and $K(E_{10})$ is absolutely essential here, because without this extension neither the charge assignments of the quarks and leptons, nor those of the gravitinos in Eq. (1) below could possibly work, and stability of the gravitinos against decay could not be achieved. A key feature of our proposal, and one that sets it apart from all other unification schemes, is that besides the 48 spin-$\frac{1}{2}$ fermions of the Standard Model, the *only* other fermions are the eight supermassive gravitinos corresponding to the spin-$\frac{3}{2}$ states of the $N = 8$ supermultiplet. It is thus a *prediction* that the spin-$\frac{1}{2}$ fermion content of the Standard Model will remain unaltered up to the Planck scale, a prediction that is (at least so far) supported by the absence of any signs of new physics from the LHC, and by the fact that the currently known Standard Model couplings can be consistently evolved all the way to the Planck scale. Indeed, the detection of any new fundamental spin-$\frac{1}{2}$ degree of freedom (such as a sterile fourth neutrino, or a fourth generation of quarks and leptons, or any of the "*-ino*" fermions predicted by low-energy supersymmetry) would immediately falsify the present scheme.

Evidence for infinite-dimensional duality symmetries of Kac-Moody type comes from an earlier Belinski-Khalatnikov-Lifshitz (BKL)-type analysis of cosmological singularities in general relativity [14,15]. This has led to the conjecture that M-theory in the "near singularity limit" is governed by the dynamics of an $E_{10}/K(E_{10})$ nonlinear σ model [16]. In this scenario space-time, and with it space-time-based quantum field theory and space-time symmetries would have to be emergent, in the sense that all the relevant information about space-time physics gets encoded in and "spread over" a hugely infinite-dimensional hyperbolic Kac-Moody algebra. In particular, this scheme goes *beyond* supergravity in that the infinite-dimensional E_{10} duality symmetry replaces, and quite possibly disposes of, supersymmetry as a guiding principle towards unification. The fermionic sector of the theory is then governed by the "maximal compact" (or more correctly, "involutory") subgroup $K(E_{10}) \subset E_{10}$, which can be regarded as an infinite-dimensional generalization of the usual R symmetries of extended supergravity theories. While an analysis of the bosonic sector of the $E_{10}/K(E_{10})$ model and its dynamics beyond the very first few levels is severely hampered by the fact that a full understanding of E_{10} remains out of reach, a remarkable property of its involutory subgroup $K(E_{10})$ is the existence of *finite*-dimensional (unfaithful) spinorial representations [17–19]. The combined spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ fermionic degrees of freedom at any given spatial point are then no longer viewed as fermionic members of the $N = 8$ supermultiplet, but rather as belonging to an (unfaithful) irreducible representation of the generalized R-symmetry $K(E_{10})$ [17–19]. The link with the physical fermion states is then made by identifying the known $K(E_{10})$ representation with the Standard Model fermions at a given spatial point, in the spirit of a BKL-type expansion in spatial gradients, as explained for the bosonic sector in Ref. [16].

A crucial feature is now that the gravitinos are predicted to participate in strong and electromagnetic interactions (unlike the sterile gravitinos of minimal supersymmetric Standard Model-like models with low-energy supersymmetry), and that they carry fractional charges. More precisely, as a consequence of the group-theoretic analysis in Refs. [9–11], the eight massive gravitinos are assigned to the following representations of the residual unbroken $SU(3)_c \times U(1)_{em}$ symmetry:

$$\left(\mathbf{3}_c, \frac{1}{3}\right) \oplus \left(\bar{\mathbf{3}}_c, -\frac{1}{3}\right) \oplus \left(\mathbf{1}_c, \frac{2}{3}\right) \oplus \left(\mathbf{1}_c, -\frac{2}{3}\right). \quad (1)$$

These assignments follow from an $SU(3) \times U(1) \subset SO(8)$ decomposition of the $N = 8$ supergravity gravitinos, *except* for the "spurion" shift of the U(1) charges by $\pm\frac{1}{6}$ that was originally introduced in Ref. [12] for the spin-$\frac{1}{2}$ members of the $N = 8$ supermultiplet, in order to make their electric charge assignments agree with those of three generations of quarks and leptons (including right-chiral neutrinos). As shown in Refs. [9–11], it is this latter shift which requires enlarging the R symmetry to $K(E_{10})$, and which takes the construction *beyond* $N = 8$ supergravity and *beyond* the confines of space-time-based field theory. All gravitinos are assumed to be superheavy, with masses just below the Planck mass. This assumption is plausible because in any scheme avoiding low-energy supersymmetry and in the absence of grand unification the Planck scale is the natural scale for symmetry breaking. Despite their large mass *all gravitinos are stable against decays into Standard Model matter*, as a consequence of their peculiar quantum numbers: there is simply no final state in the Standard Model into which they could possibly decay in compliance with Eq. (1) and the residual unbroken $SU(3)_c \times U(1)_{em}$ symmetry. This feature is essentially tied to the replacement of the usual R symmetry by $K(E_{10})$, because in a standard supergravity context a supermassive gravitino would not be protected against decay into other particles.

In the present paper we take a more pragmatic approach by simply proceeding with the assignments (1) as the starting point, but keeping in mind that this scheme is strongly motivated by unification and a possible explanation of the observed pattern of quark and lepton charge quantum numbers, and thus not based on *ad hoc* choices. In Refs. [20,21] we had already begun to explore the possible astrophysical implications of supermassive gravitinos with the above assignments. More specifically, in Ref. [20] we have proposed the color-singlet gravitinos as novel dark matter candidates, and discussed possible avenues to search for them (in fact, even within the present scenario, this proposal would hold up, in that the supermassive gravitinos could make up a large part, or even all of dark matter, via the black holes into which they would have been swallowed). In subsequent work [21] we showed that the color-triplet states in Eq. (1) can potentially explain the observed ultra-high-energy cosmic-ray events with energies of up to 10^{21} eV via gravitino-antigravitino annihilation in the crust of neutron stars. In this paper we now turn our attention again to the *color-singlet* gravitinos of charge $\pm\frac{2}{3}$, to argue that they can in addition play a key role in shedding light on the origin of giant black holes in the early Universe.

The structure of this paper, then is as follows. In Sec. II we show that quantum mechanically the wave function of a multigravitino bound state is highly unstable against gravitational collapse. In the following two sections we study the formation and evolution of mini black holes during the radiation era, also deriving numerical estimates. For the evolution we employ a generalization of the McVittie solution (on which there is already an ample literature; see e.g., Refs. [22–28] and references therein). In the last section we analyze the energy-momentum tensor for this solution, and show that it has the right form expected for a radiation-dominated universe. We also argue that the "blanket" surrounding the primordial black hole can further enhance the growth of massive black holes. These last two sections may be of interest in their own right, independently of the main line of development of this paper.

II. FORMATION OF MULTIGRAVITINO BOUND STATES

The main new feature of our proposal is that, as a result of the assumed large mass of the gravitinos, the combined gravitational and electric forces between any arrangement of gravitinos and antigravitinos is *universally attractive*. In natural units we define the Bogomol'nyi-Prasad-Sommerfield (BPS) mass $M_{\rm BPS}$ for the (anti)gravitino to be the one for which the electrostatic force between two gravitinos with charges $\pm Q_g$ equals their gravitational attraction (modulo sign)

$$Q_g^2 = GM_{\rm BPS}^2; \tag{2}$$

we refer to $M_{\rm BPS}$ as the "BPS mass" because it is the one relevant for extremal Reissner-Nordström solutions. This equality is written in units where $4\pi\epsilon_0 = \mu_0/(4\pi) = c = 1$ (here it is worthwhile to recall that these units, with the addition of $e = M_{\rm BPS} = 1$, were introduced already in 1881 by Stoney, probably the first physicist who seriously contemplated quantization of charge [29]; the electron was discovered only 16 years later, while Planck units were introduced 18 years later). As is well known, $M_{\rm BPS}$ is *not* the same as the (reduced) Planck mass $M_{\rm Pl}$, but differs from it by a factor of the fine-structure constant α (always with $c = 1$ from now on):

$$M_{\rm BPS}^2 = \frac{Q_g^2}{G} = \frac{Q_g^2}{\hbar}\cdot\frac{\hbar}{G} \equiv \alpha M_{\rm Pl}^2, \tag{3}$$

where α differs from the usual fine-structure constant α_{em} by a factor of $\frac{4}{9}$ because of the fractional charge (see below). We will assume that the gravitino mass lies between these two values, i.e.,

$$M_{\rm BPS} < M_g < M_{\rm Pl}. \tag{4}$$

The first of these inequalities is needed to ensure that the force between same-charge gravitinos remains attractive; for $M_g < M_{\rm BPS}$ we would have repulsion [because $(1-\beta^2)$ in Eq. (13) becomes negative], and the proposed mechanism would no longer work. Denoting the usual elementary charge by e we can thus write for the gravitino charges

$$Q_g = \pm\frac{2}{3}e = \pm\beta G^{\frac{1}{2}} M_g \tag{5}$$

with the "BPS parameter" β obeying $0 < \beta < \frac{2}{3}$; we will denote the (fixed) gravitino mass by M_g throughout this paper, whereas generic black hole masses will be designated by the letter m, where m can also vary with time. The total force between two (anti)gravitinos is thus determined by the combined electric and gravitational charges $(1 \pm \beta^2)GM_g^2 > 0$, so that even for like charges the force remains attractive because the gravitional attraction overwhelms the electrostatic repulsion (reflecting the "almost BPS-like" nature of the gravitinos). In this paper we hypothesize that it is this universal attraction that leads to the formation of multigravitino bound states inside the plasma of the radiation-dominated phase, starting from small inhomogeneities in analogy with cluster formation of galaxies. The main difference with the latter is that, prior to gravitational collapse, we are here initially dealing with a *quantum-mechanical* bound state, not one that can be understood in terms of Newtonian physics. For two gravitinos the bound state would be somewhat analogous to positronium, however with the crucial difference that "gravitinium" can be a longer-lived state because the annihilation cross section between two oppositely charged (color-singlet) gravitinos is very small, of the order $\sim M_g^{-2}$ (as follows from inspection of the standard tree-level Feynman diagram for annihilation into, say, a pair of gravitons, with one intermediate gravitino propagator). Note that in principle positronium can also be long lived, provided the bound state is formed in a state of very large radial quantum number [30] [see Eq. (19) below].

We wish to study the formation of bound states of gravitinos during the radiation era in the very early Universe. For a proper analysis, and as a first step, we would now have to go through a first quantized analysis of the massive Rarita-Schwinger equation in such a homogeneously and isotropically expanding background. This task is substantially simplified by our main assumption (4) which allows us to resort to the nonrelativistic limit, and by the fact that this inequality also implies

$$M_g > H(t) \tag{6}$$

for the Hubble parameter during the radiation era, whence we can also drop the usual friction term $\propto H(t) = \dot{a}(t)/a(t)$ that would normally have to be included in the equation of motion. It is therefore enough to consider the free Rarita-Schwinger equation for a massive spin-$\frac{3}{2}$ complex vector spinor, which reads

$$i\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho + M_g\gamma^{\mu\nu}\psi_\nu = 0. \tag{7}$$

From this one immediately deduces the Dirac and constraint equations

$$(i\gamma^\lambda\partial_\lambda - M_g)\psi_\mu = 0 \quad \text{and} \quad \gamma^\mu\psi_\mu = \partial^\mu\psi_\mu = 0 \tag{8}$$

(see e.g., Ref. [31] for a more complete account). The latter two equations imply a halving of the available degrees of freedom, and tell us that the vector spinor ψ_μ carries altogether four helicity degrees of freedom, with labels $\sigma, \tau \in \{\pm\frac{1}{2}, \pm\frac{3}{2}\}$ for both gravitino and antigravitino. The relevant expansion reads

$$\begin{aligned}\psi_\mu(x) = \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E(\mathbf{p})}} [&e^{ipx} f_\mu^+(p)u_+(p) \\ &+ e^{ipx} f_\mu^-(p)u_-(p) + e^{-ipx} g_\mu^+(p)v_+(p) \\ &+ e^{-ipx} g_\mu^-(p)v_-(p)]\end{aligned} \tag{9}$$

where, of course, $p^2 + M_g^2 = 0$, and $u_\pm(p)$ and $v_\pm(p)$ are the two positive- and negative-energy solutions of the Dirac equation. The last constraint equations are solved by

$$f_\mu^\pm(p) = \sum_{\mathrm{i}} b_{\mathrm{i}}^\pm(p)\varepsilon_\mu^{\mathrm{i}}(p), \qquad g_\mu^\pm(p) = \sum_{\mathrm{i}} d_{\mathrm{i}}^\pm(p)\varepsilon_\mu^{\mathrm{i}}(p) \tag{10}$$

with the three linearly independent polarization vectors $\varepsilon_\mu^{\mathrm{i}}(p)$ satisfying $p^\mu\varepsilon_\mu^{\mathrm{i}}(p) = 0$. For the other constraint equation we need to impose

$$\begin{aligned}&\sum_{\mathrm{i}} \gamma^\mu\varepsilon_\mu^{\mathrm{i}}(p)[b_{\mathrm{i}}^+(p)u_+(p) + b_{\mathrm{i}}^-(p)u_-(p)] \stackrel{!}{=} 0, \\ &\sum_{\mathrm{i}} \gamma^\mu\varepsilon_\mu^{\mathrm{i}}(p)[d_{\mathrm{i}}^+(p)v_+(p) + d_{\mathrm{i}}^-(p)v_-(p)] \stackrel{!}{=} 0,\end{aligned} \tag{11}$$

thus eliminating four out of the 12 free coefficients $b_{\mathrm{i}}^\pm(p)$ and $d_{\mathrm{i}}^\pm(p)$, respectively, leaving us with four helicity wave functions for gravitino and antigravitino each. As the spin interactions are not relevant for our approximation there is no need here to be any more specific about the parametrization of the helicity wave functions. However, each gravitino degree of freedom is exposed to the gravitational and electric background generated by the other gravitinos (as well as the surrounding plasma which we can neglect). In order to incorporate these interactions at lowest order, one performs the standard Foldy-Wouthuysen transformation on each component of ψ_μ, which yields a nonrelativistic one-particle Hamiltonian for each gravitino component.

The corresponding multiparticle Schrödinger Hamiltonian therefore reads

$$H = -\frac{\hbar^2}{2M_g}\sum_i(\triangle_{\mathbf{x}_i} + \triangle_{\mathbf{y}_i}) + V(\mathbf{x}, \mathbf{y}) \tag{12}$$

with the universally attractive potential (for $\beta^2 < 1$)

$$\begin{aligned}V(\mathbf{x}, \mathbf{y}) = &-(1-\beta^2)\left(\sum_{i\neq j}\frac{GM_g^2}{|\mathbf{x}_i - \mathbf{x}_j|} + \sum_{i\neq j}\frac{GM_g^2}{|\mathbf{y}_i - \mathbf{y}_j|}\right) \\ &-(1+\beta^2)\sum_{i,j}\frac{GM_g^2}{|\mathbf{x}_i - \mathbf{y}_j|}\end{aligned} \tag{13}$$

where the positions of the gravitinos and antigravitinos are designated by $\mathbf{x}_i$ and $\mathbf{y}_j$, respectively. This Hamiltonian acts on a fermionic wave function $\Psi(\mathbf{x}_1, \sigma_1, \ldots, \mathbf{x}_n, \sigma_p; \mathbf{y}_1, \tau_1, \ldots, \mathbf{y}_q, \tau_q)$ which is antisymmetric under simultaneous interchange of the position and spin labels of the gravitinos and antigravitinos, respectively. In writing this Hamiltonian we have also neglected the fluctuating external electric and magnetic fields in the radiation plasma. Likewise, as we already explained, we ignore subleading spin-orbit and spin-spin interactions that would follow from the Rarita-Schwinger equation in a fully relativistic treatment (and which would be very complicated). Finally, we can neglect the effect of the protons and electrons from the surrounding plasma (as well as all other Standard Model particles); for them, the gravitational interactions are governed by the factors $GM_gm_e, GM_gm_p, \ldots \ll GM_g^2$, whence their interactions are completely dominated by the purely electromagnetic forces. The latter are, however, screened out because of the overall electric neutrality of the plasma, and can thus be ignored.

Evidently the above considerations only apply to superheavy particles obeying Eqs. (4) and (6), and would not make any sense at all for ordinary (Standard Model) particles. For the latter all masses and binding energies are far below the temperature of the surrounding plasma, that is $m_e, m_p, \ldots \ll T_{\rm rad}$, and also below the Hubble parameter, $m_e, m_p, \ldots \ll H$. In that case, the stationary Schrödinger equation would have to be replaced by a relativistic equation in a time-dependent background, and the friction term involving the Hubble parameter H would lead to immediate decay of the wave function (as unitarity in the naive sense is violated in a time-dependent background).

We will not attempt here to investigate in any detail the multiparticle Schrödinger equation based on Eq. (12), which would amount to a quantum analog of the computations performed in connection with galaxy structure formation. Nevertheless, we can still make some rigorous statements relying on well-known estimates (see e.g., Ref. [32]). Namely, it is a rigorous result [33] that for a system of fermions (that is, particles obeying the Pauli principle with a fully antisymmetric wave function) the lowest-energy eigenvalue of the N-particle Hamiltonian

$$E_0(N) \coloneqq \inf_{||\Psi||=1} \langle\Psi|H|\Psi\rangle \tag{14}$$

(where N is the combined number of gravitinos and antigravitinos) is subject to the upper and lower bounds

$$\begin{aligned} &-AN(N-1)^{\frac{4}{3}}G^2M_g^5\hbar^{-2} \\ &\quad\le E_0(N) \le -BN^{\frac{1}{3}}(N-1)^2G^2M_g^5\hbar^{-2} \end{aligned} \tag{15}$$

with strictly positive constants $A > B > 0$. Consequently the lowest energy per particle $E_0(N)/N$ decreases as $\propto -N^{4/3}$ with N, signaling an instability. For a bosonic wave function the falloff would be even faster with $E_0(N)/N \propto -N^2$ [33]. Therefore the inclusion of spin degrees of freedom (where one combines a partially symmetric wave function in the space coordinates with an antisymmetric wave function in spin space) cannot improve the situation. The estimate (15) tells us that the system is unstable, and for sufficiently large N will thus undergo gravitational collapse, as the fermionic degeneracy pressure is not enough to sustain the system in a stable equilibrium. Because of Eq. (6) the basic instability estimate (15) is not affected by the cosmological expansion either.

Now if we consider a bound state of just two gravitinos (a hydrogen-like system) the associated "Bohr radius" is only a few orders of magnitude away from the Planck length, to wit

$$a_B \sim \frac{\hbar^2}{GM_g^3} \tag{16}$$

which is not too far from the Schwarzschild radius. If the formation of such bound states took place in vacuum, and the relaxation to the ground state proceeded too fast, the resulting mini black holes would immediately evaporate by Hawking radiation according to the well-known formula (see e.g., Ref. [34])

$$t_{evap} \sim t_{\rm Pl}\left(\frac{m}{M_{\rm Pl}}\right)^3 \sim 10^{-42}\ {\rm s}\left(\frac{m}{10^{-9}\ {\rm kg}}\right)^3 \tag{17}$$

which follows from the Stefan-Boltzmann law upon substitution of the Hawking temperature

$$T_{Hawking} = \frac{\hbar}{8\pi Gm}. \tag{18}$$

In order to prevent this from happening, and in order to create bigger black holes that can survive for longer and start growing, it is therefore necessary for the bound states to persist long enough to accrete a sufficiently large number of gravitinos *before* gravitational collapse. Metastability can be ensured if the initial energy of the bound state is much larger than $E_0(N)$, and consequently its overall extension stays well above its Schwarzschild radius for a sufficiently long time. Of course, the bound state will eventually relax to lower-lying bound states by the spontaneous emission of photons and gravitons, but this process will take some time. For instance, for positronium the average lifetime τ of a bound state as a function of the principal quantum number n scales as [see e.g., Ref. [35], Eqs. (7)–(9)]

$$\tau \sim n^4. \tag{19}$$

In comparison with positronium which has a large annihilation cross section, the mutual annihilation of (color-singlet) gravitinos and antigravitinos is further delayed by their small annihilation cross section $\sim M_g^{-2}$, which was already highlighted above. Extrapolating the above formula to the present case thus suggests that, with sufficiently large n at the time of formation, we can get lifetimes long enough to bind a large number of gravitinos into a metastable configuration before the collapse can occur. We also note that at this stage (that is, prior to the formation of a black hole) the absorption of protons and electrons from the ambient plasma plays no role, as these particles, unlike the gravitinos, will be only very weakly bound.

III. COLLAPSE OF GRAVITINO LUMPS AND MINI BLACK HOLES

At this point we have lumps, each corresponding to a quantum-mechanical multigravitino bound state, which are scattered throughout the radiation plasma. Because of the density fluctuations and inhomogeneities in the plasma, and as a result of their strong gravitional attraction these lumps will eventually coalesce before collapsing into small black holes, a microscopic analogue of the clumping of dust into galaxies and stellar matter. In a first approximation the ensemble of massive lumps can be treated classically (i.e., need not be considered as a single coherent wave function). In order to arrive at a rough estimate of the initial mass of the resulting black holes we first estimate the total number of gravitinos contained in a coalesced lump of gravitino matter. Treating them classically with an average kinetic energy per particle equal to the temperature of the plasma we have

$$\langle E_{kin}(t)\rangle \sim NT_{\rm rad}(t) = NT_{eq}\left(\frac{t_{eq}}{t}\right)^{1/2} \tag{20}$$

where $T_{eq} \sim 1$ eV and $t_{eq} \sim 40000$ yr $\sim 10^{12}$ s (we find it convenient to refer to all quantities in terms of equilibrium time t_{eq} rather than Planck units). The potential energy of N gravitinos and antigravitinos is given by

$$\langle E_{pot}(t)\rangle \sim -N^2 \frac{GM_g^2}{\langle d(t)\rangle} \tag{21}$$

where for numerical estimates we take $M_g \sim M_{\rm BPS}$. The average separation $\langle d(t)\rangle$ between gravitinos and antigravitinos at time t is given by

$$\langle d(t)\rangle \sim \left(\frac{M_g}{\rho(t)}\right)^{1/3} \sim (10^2\ {\rm m})\left(\frac{t}{t_{eq}}\right)^{1/2} \tag{22}$$

where we estimate the gravitino density $\rho(t)$ at time t by scaling back the known density at the equilibrium time t_{eq} (with $8\pi G\rho_{\rm rad} = 8\pi G\rho_{mat} \sim 4\times 10^{-25}\ {\rm s}^{-2}$), with the further assumption that at $t = t_{eq}$, most of the matter consisted of supermassive gravitinos, in line with our previous dark matter proposal [20]. For this estimate we also need to keep in mind that matter density scales as $a(t)^{-3}$ also during the radiation era [while the radiation density scales as $a(t)^{-4}$].

Gravitational collapse is expected to occur if the total energy is negative:

$$\langle E_{kin}(t)\rangle + \langle E_{pot}(t)\rangle < 0 \Rightarrow N > \frac{T_{eq}\cdot 10^2{\rm m}}{GM_g^2} \sim 10^{12}. \tag{23}$$

Importantly, the time t drops out of this relation because the temperature and the inverse average distance decrease in the same manner as a function of t during the radiation era. Let us stress that this is only a very rough estimate: if the bound state is metastable, the collapse can be delayed in such a way that a larger number of (anti)gravitinos can be accrued. With Eq. (23) the mass of the resulting mini black hole comes out to be

$$m_{\rm initial} \sim 10^{12} M_g \sim 10^{12} M_{\rm BPS} \sim 10^3\ {\rm kg}. \tag{24}$$

By Eq. (17) the Hawking evaporation time for a black hole of this mass would be

$$t_{evap}(m_{\rm initial}) \sim 10^{-7}{\rm s}. \tag{25}$$

However, it is important now that Hawking evaporation is not the only process that must be taken into account. There is a competing process which can in fact stabilize the mini black holes and their further evolution: it is the presence of the dense and hot plasma surrounding the black hole that can feed the growth of small black holes. More precisely, Hawking evaporation competes with accretion according to the following equation:

$$\frac{dm(t)}{dt} = C_0 G^2 \rho_{\rm rad}(t)\cdot m^2(t) - C_1 \frac{M_{\rm Pl}^3}{t_{\rm Pl}}\cdot\frac{1}{m^2(t)} \tag{26}$$

where C_0 and C_1 are constants of $\mathcal{O}(1)$. The first term on the rhs originates from the flux of the infalling radiation from the surrounding plasma, which is $\propto 4\pi R^2(t)\rho_{\rm rad}(t)c$ (with $c = 1$) for a (time-dependent) black hole of radius $R(t) = 2Gm(t)$ [a "fudge factor" $C_0 = \mathcal{O}(4\pi)$ can be included to account for the fact that not all the surrounding radiation falls in radially, but this is not essential for our argument]. The second term in Eq. (26) governs Hawking evaporation. For Hawking evaporation taking place in empty space we can ignore the first term on the rhs of Eq. (26), and Eq. (17) follows directly. In that case any microscopic black hole would disappear, and not be able to grow into a macroscopic black hole. The crucial difference with this standard scenario is embodied in the first term on the rhs of Eq. (26) (which is usually disregarded in discussions of Hawking evaporation). This term takes into account the fact that the decay takes place in an extremely hot surrounding plasma whose density varies with time as $8\pi G\rho_{\rm rad}(t) = 3/4t^2$. At the initially extremely high temperatures of the radiation era the accretion can thus outcompete Hawking evaporation *even for very small black holes*. In terms of temperature with the break-even point at $T_{\rm rad} = T_{Hawking}$ where the radiation temperature $T_{\rm rad}(t)$ at time t can be read off from Eq. (20). The simple criterion for black hole accretion to overcome the rate for Hawking radiation reads

$$T_{\rm rad} > T_{Hawking}. \tag{27}$$

This inequality is easy to achieve in the initially very dense and hot plasma where $T_{\rm rad} \sim 10^{17}$ GeV. Later, it is a delicate issue because from Eq. (26) it follows that $m(t)$ can run away in either direction. This can also be directly seen by setting to zero the rhs of Eq. (26): at time t the break-even point occurs for

$$m_0^4(t) \sim \frac{M_{\rm Pl}^3}{t_{\rm Pl}} \cdot \frac{1}{G^2 \rho_{\rm rad}(t)} \propto t^2 \tag{28}$$

where we have used $\rho_{\rm rad} = \frac{3}{32\pi G} t^{-2}$. Hence, a mini black hole of initial size $m_{\rm initial}(t) > m_0(t)$ will be able to survive and can start growing, whereas those of smaller mass decay. Consequently, the earlier the bound state is formed, the smaller its initial mass can be. From these considerations and the time-independent estimate (24) we can also derive a rough upper bound on the formation time, after which the radiation temperature is too low to stabilize mini black holes against Hawking evaporation. The maximal time $t_{\rm max}$ is found by equating $m_0(t_{\rm max}) \sim 10^3$ kg from Eq. (24), which yields the value

$$t_{\rm max} = 10^{-20}\ {\rm s}. \tag{29}$$

Mini black holes formed after this time can be expected to decay by Hawking radiation because $T_{\rm rad}(t) < T_{Hawking}$ for $t > t_{\rm max}$. In summary, the usual argument that small black holes would quickly decay via Eq. (17) no longer applies as long as the inequality (27) is obeyed.

Note that we invoke the "empirical" formula (26) mainly to argue that mini black holes can form in such a way as to remain stable against Hawking evaporation at early times. In fact, this reasoning can be made more quantitative by substituting $\rho_{\rm rad} = \frac{3}{32\pi G} t^{-2}$ into Eq. (26) which turns this equation into a simple differential equation that can be studied numerically. However, because this formula is only approximate, and once the stability of the mini black hole is ensured, we can switch to a classical description by means of an exact solution of Einstein's equations describing a Schwarzschild black hole in a radiatively expanding universe, to describe its further evolution. This will be explained in the next section.

IV. GROWTH OF BLACK HOLES IN RADIATION-DOMINATED UNIVERSE

Having motivated the assumption that small black holes stable against Hawking evaporation have formed in sufficient numbers early in the radiation-dominated era we can proceed to study their evolution in this background. For this purpose we employ an *exact* solution of the Einstein equations, rather than the "phenomenological" formula (26). This solution can be regarded as a variant of the so-called McVittie solution [22]; for more recent literature, see e.g., Refs. [23–28] and references therein. The solution that we require here is conveniently presented in terms of conformal coordinates, by starting from the general *Ansatz*

$$ds^2 = a(\eta)^2 \left[-C(\eta, r) d\eta^2 + \frac{dr^2}{C(\eta, r)} + r^2 d\Omega^2 \right] \tag{30}$$

where η is conformal time, which we use from now on as the time coordinate. $a(\eta)$ is the scale factor and $C = C(\eta, r)$ is some function to be specified. We will discuss the equations for the general *Ansatz* elsewhere, but for the present purposes it is enough to restrict to the special case, where C depends only on the radial coordinate, i.e., $C(\eta, r) \equiv C(r)$. Furthermore, since we are here mainly interested in perfect fluids, for which $a(t) \sim t^{2/3(w+1)} \sim \eta^{2/(3w+1)}$, and more specifically, a radiation-dominated universe, we right away specialize the scale factor to be

$$a(\eta) = A\eta \Leftrightarrow t = \frac{1}{2} A\eta^2, \tag{31}$$

where in our Universe $A \sim 4 \times 10^{-5}\ {\rm s}^{-1}$ [while $a(\eta)$ is dimensionless]. With these assumptions it is straightforward to compute the nonvanishing components of the Einstein tensor, and hence the components of the energy-momentum tensor, with the result

$$\begin{aligned}
&8\pi G T_{tt}(\eta, r) \\
&\quad = -\frac{1}{\eta^2 r^2} (C(r)C'(r) r\eta^2 + C^2(r)\eta^2 - C(r)\eta^2 - 3r^2), \\
&8\pi G T_{tr}(\eta, r) = \frac{C'(r)}{\eta C(r)}, \\
&8\pi G T_{rr}(\eta, r) \\
&\quad = \frac{1}{C(r)^2 r^2 \eta^2} (C(r)C'(r) r\eta^2 + C(r)^2\eta^2 - C(r)\eta^2 + r^2), \\
&8\pi G T_{\theta\theta}(\eta, r) \\
&\quad = \frac{1}{2C(r)\eta^2} (C(r)C''(r) r^2\eta^2 + 2C(r)C'(r) r\eta^2 + 2r^2), \\
&8\pi G T_{\varphi\varphi}(\eta, r) = 8\pi G \sin^2\theta\, T_{\theta\theta}(\eta, r)
\end{aligned} \tag{32}$$

where, of course, $C'(r) \equiv dC(r)/dr$, etc. At this point, this is just an identity (the so-called "Synge trick" [36]); in fact, such solutions trivially exist for *any* profile of the scale factor $a(\eta)$. The nontrivial part of the exercise is therefore in ascertaining that the energy-momentum tensor resulting from this calculation does make sense *physically*. The requisite condition for a radiation-dominated universe, stated in the most general and coordinate-independent way, is the vanishing of the trace of the energy-momentum tensor, viz.

$$T^\mu{}_\mu(\eta, r) = \frac{1}{A^2 \eta^2 r^2} \left[\frac{d^2}{dr^2} (r^2 C(r)) - 2 \right] \stackrel{!}{=} 0. \tag{33}$$

This condition is solved by

$$C(r) = 1 - \frac{2Gm}{r} + \frac{GQ^2}{r^2} \tag{34}$$

with two integration constants m (mass) and $\mathcal{Q}$ (charge). Remarkably, the metric (30) comes out to be conformal to the Reissner-Nordström metric not as a result of imposing the Einstein equations with an electromagnetic point charge source, but with the weaker and more general conformality constraint (33)! Taking $\mathcal{Q} = 0$ for simplicity (and also because we do not expect these black holes to carry significant amounts of electrical charge), the resulting solution describes the exterior region ($r > 2Gm$) of a Schwarzschild black hole in a radiation-dominated universe. We emphasize that there is absolutely no issue with the causal structure of this solution, because the conformal equivalence ensures that (for $\eta > 0$) the global structure of the space-time outside the would-be horizon $r = 2Gm$ is the same as for the Schwarzschild solution, and the tracelessness of the energy-momentum tensor holds right up to the would-be horizon (the black hole interpretation is also supported by the arguments in Refs. [26,27]). However, there are some subtleties (apart from issues related to de Sitter space and cosmological horizons discussed in Refs. [23–27], which are of no concern here) which have to do with the structure of the energy-momentum tensor. Namely, as we show in the following section, closer inspection reveals the existence of an apparent "superluminal barrier" surrounding the surface $r = 2Gm$, and shielding the would-be horizon from the outside observer.

For the physical mass of the black hole we take the formula

$$\frac{1}{2\pi}\int \mathrm{d}\theta a(\eta) r|_{r=2Gm} = 2Gma(\eta) \Rightarrow m(\eta) = ma(\eta) \quad (35)$$

keeping in mind that the observer at infinity will in addition measure the integrated matter density outside the apparent horizon, so the above formula is really a lower bound on the total mass accretion. The total mass therefore grows (at least) linearly with the scale factor, and this is also consistent with the fact that $T_{tr} \neq 0$. The formula (35) gives (with $\eta = \eta_{\mathrm{initial}}$)

$$m = \frac{m_{\mathrm{initial}}}{a_{\mathrm{initial}}} \quad (36)$$

where m_{initial} is any value compatible with the lower bound following from Eq. (28), and a_{initial} is the scale factor at the time when the black hole forms. The mass accretion described by Eq. (35) is also evident from the nonvanishing mixed component T_{tr} in Eq. (32) which states that there is energy flow into the black hole from the surrounding radiative medium. During the radiation era there is, in fact, an unlimited supply of "food" for the black hole to swallow. This supply will dry up only when inhomogeneities are formed, after which the accretion works in the more standard form.

Evolving the initial mass (4) with the formula (35) we calculate the final mass at the equilibrium time (assuming that $\eta_{\mathrm{initial}} \sim \eta_{\mathrm{Pl}}$)

$$m_{\mathrm{final}} \sim m_{\mathrm{initial}}\left(\frac{\eta_{eq}}{\eta_{\mathrm{Pl}}}\right) \sim 10^{30}\ \mathrm{kg} \sim M_{\odot} \quad (37)$$

with $\eta_{eq} \sim 2 \times 10^{8}$ s and $\eta_{\mathrm{Pl}} \sim 10^{-19}$ s. This estimate applies to mini black holes formed very early in the radiation era (for $\eta \ll \eta_{\max}$). The same calculation for a mini black hole at the latest possible time $\eta_{\max}$ given by Eq. (29) also yields a lower bound for the final mass of the primordial black hole upon exit from the radiation era,

$$\sqrt{m_{\mathrm{initial}} M_{\mathrm{Pl}}} < m_{\mathrm{final}} < M_{\mathrm{Pl}}\left(\frac{\eta_{eq}}{\eta_{\mathrm{Pl}}}\right) \quad (38)$$

or

$$10^{11}\ \mathrm{kg} < m_{\mathrm{final}} < M_{\odot}. \quad (39)$$

This inequality restricts the possible mass range for primordial black holes at the equilibrium time.

The above analysis can be repeated for matter-dominated and exponentially expanding universes, respectively. In this case we need the angular Killing vectors $k_{\theta}^{\mu}\partial_{\mu}$ and $k_{\varphi}^{\mu}\partial_{\mu}$ to state the pertinent conditions in a generally covariant way. In the matter-dominated era we have

$$a(\eta) = B^2\eta^2,$$
$$T_{\mu\nu}k_{\theta}^{\mu}k_{\theta}^{\nu} = T_{\mu\nu}k_{\phi}^{\mu}k_{\phi}^{\nu} = 0 \Rightarrow C(r) = 1 - \frac{2Gm}{r} \quad (40)$$

where we utilize the Killing vectors to state the condition of vanishing pressure. Because this solution does not allow for a nonvanishing charge, this provides another reason for setting $\mathcal{Q} = 0$ in Eq. (34), in order to allow for a smooth transition from the radiation-dominated to the matter-dominated phase. From this we see that the primordial black holes will continue to grow with the scale factor also in the early part of the matter-dominated phase, absorbing radiation *and* matter, as long as there are no significant inhomogeneities. After the distribution of matter develops inhomogeneities, the further evolution of black holes proceeds in the standard fashion. In other words, the range of mass values in Eq. (38) corresponding to time $t = t_{eq}$, only represent a lower limit, as the black holes will continue to accrete mass in significant amounts until inhomogeneities start forming.

Finally, for an exponentially expanding universe we have

$$a(\eta)=\frac{1}{H(\eta_\infty-\eta)},$$
$$T^\mu{}_\mu=2(T_{\mu\nu}k^\mu_\theta k^\nu_\theta+T_{\mu\nu}k^\mu_\phi k^\nu_\phi)\Rightarrow C(r)=1-\frac{2Gm}{r}-C_H r^2. \tag{41}$$

Please note that for $C_H\neq 0$ this is *not* the well-known Kottler solution (that is, de Sitter space in static coordinates). We stress again that for $C(\eta,r)=C(r)$ and with $\mathcal{Q}=0$ and $C_H=0$ the causal structure of the space-time is the same as for an ordinary black hole space-time, and only in this case we can have a smooth transition between all phases.

V. ENERGY-MOMENTUM TENSOR

To gain further insight into the physical properties of our solution let us examine the energy-momentum tensor (32) a bit more closely. Following Ref. [37] we parametrize the latter as

$$\begin{aligned}T_{\mu\nu}&=pg_{\mu\nu}+(p+\rho)u_\mu u_\nu-\Pi_{\mu\rho}Q^\rho u_\nu-\Pi_{\nu\rho}Q^\rho u_\mu\\&\quad-\zeta_1\Pi_\mu{}^\rho\Pi_\nu{}^\sigma\left(\nabla_\rho u_\sigma+\nabla_\sigma u_\rho-\frac{2}{3}g_{\rho\sigma}\nabla^\lambda u_\lambda\right)\\&\quad-\zeta_2\Pi_{\mu\nu}\nabla^\lambda u_\lambda\end{aligned} \tag{42}$$

where $u^\mu u_\mu=-1$, Q^μ is the heat flow, and ζ_1 and ζ_2 are the shear and bulk viscosity, respectively. All variables are assumed to depend on η and r only. The projector is defined by

$$\Pi_{\mu\nu}=g_{\mu\nu}+u_\mu u_\nu. \tag{43}$$

We will now match the energy-momentum tensor (32) to this formula. For simplicity we assume

$$\zeta_1=\zeta_2=0. \tag{44}$$

We also write $q_\mu\equiv\Pi_{\mu\nu}Q^\nu$ (so that $u^\mu q_\mu=0$), so that the energy-momentum tensor simplifies to

$$T_{\mu\nu}=pg_{\mu\nu}+(p+\rho)u_\mu u_\nu-q_\mu u_\nu-q_\nu u_\mu. \tag{45}$$

The assumption of vanishing viscosity coefficients (44) is certainly justified after baryogenesis (that is $t>10^{-12}$ s), when the number of photons by far exceeds the number of other particles in the plasma (for instance, $n_\gamma\sim 10^{10}\,n_b$). While the condition (33) leaves ζ_1 undetermined, we could in principle also admit a nonvanishing $\zeta_2\neq 0$, that is, self-interacting conformal matter (e.g., self-interacting massless scalar fields). In that case the relation $\rho=3p$ derived below would no longer hold even with vanishing $T^\mu{}_\mu$.

For the comparison we write out Eq. (32) explicitly for the solution (34) (with $\mathcal{Q}=0$), which gives

$$\begin{aligned}8\pi GT_{tt}&=\frac{3\dot a^2}{a^2}=\frac{3}{\eta^2},\\8\pi GT_{rr}&=\frac{r^2(-2a\ddot a+\dot a^2)}{a^2(r-2Gm)^2}=\frac{r^2}{\eta^2(r-2Gm)^2},\\8\pi GT_{rt}&=\frac{2Gm\dot a}{ar(r-2Gm)}=\frac{2Gm}{r\eta(r-2Gm)},\\T_{\theta\theta}&=r^2T_{rr},\qquad T_{\varphi\varphi}=r^2\sin^2\theta T_{rr}.\end{aligned} \tag{46}$$

Comparing Eqs. (46) and (45) we read off the unknown quantities on the rhs of Eq. (45); we find

$$\begin{aligned}u_\mu(\eta,r)&=A\eta\left(\sqrt{\frac{r-2Gm}{r}}\cosh\xi,\sqrt{\frac{r}{r-2Gm}}\sinh\xi,0,0\right),\\q_\mu(\eta,r)&=A\eta q(\eta,r)\left(\sqrt{\frac{r-2Gm}{r}}\sinh\xi,\sqrt{\frac{r}{r-2Gm}}\cosh\xi,0,0\right)\end{aligned} \tag{47}$$

where

$$\tanh\xi=\frac{Gm\eta}{r^2},\qquad(\Rightarrow\xi>0) \tag{48}$$

and

$$q(\eta,r)=2p(\eta,r)\tanh\xi \tag{49}$$

(with $m\equiv m_{\rm initial}$). The density and pressure are given by

$$\rho(\eta,r)=3p(\eta,r)\ \text{ with }\ p(\eta,r)=\frac{r}{A^2\eta^2(r-2Gm)} \tag{50}$$

as expected for a radiation-dominated universe. We stress that there are no pathologies here of the kind encountered in some of the previous literature on McVittie-type solutions. In particular, the energy density $\rho(\eta,r)$ is strictly positive for $r>2Gm$ and at all times $\eta>0$. Moreover, because q is positive from Eq. (49), the radial component of q^μ in Eq. (47) is also positive, which means that the radial heat flow is *inward directed*, explaining why the mass of the black hole *grows* with time.

To keep ξ real we must demand

$$\tanh \xi = \frac{Gm\eta}{r^2} < 1 \Rightarrow r > \sqrt{Gm\eta} \quad (>2Gm). \qquad (51)$$

For $r^2 \to Gm\eta$ the average velocity of the infalling matter reaches the speed of light, and the expansion (42) in powers of u_μ and its derivatives breaks down. Consequently, while the solution (30) remains valid down to $r = 2Gm$, the expressions (47), (49), and (50) become meaningless in the region $2Gm < r < \sqrt{Gm\eta}$ because of apparently superluminal propagation (similar conclusions regarding superluminality were already reached in Ref. [23]). Likewise the components of the heat flow q^μ diverge for $\tanh \xi \to 1$, indicating an apparent divergence of the temperature in this limit. This is also an unphysical feature in view of the breakdown of the expansion (42). Physically it is tempting to interpret this result as implying that the would-be horizon is shielded from the outside observer by a "blanket" at $r = \sqrt{Gm\eta}$, whose extension grows with cosmic time η. However, in recent work [38] it was argued that the gradient expansion (42) must be replaced by a different expansion; adapting these arguments to the present case we conclude that the solution can, in fact, remain meaningful all the way down to $r = 2Gm$. Because of the breakdown of the expansion (42), also the apparent "firewall" ($\equiv$ divergent energy density ρ) on the would-be horizon $r = 2Gm$ is an unphysical feature [we have checked that by reinstating the η dependence in the metric coefficient $C(\eta, r)$ and setting up an appropriate expansion near the would-be horizon one can eliminate this divergence]. This is just as well, because otherwise the total mass at infinity (which includes the integrated energy density for $r > 2Gm$) would diverge, as $\rho(\eta, r)$ has a nonintegrable singularity at $r = 2Gm$. At any rate these arguments show that the actual mass value for the black hole will exceed the estimated value (37) if the matter contributions outside the horizon are taken into account, thus further enhancing the growth of primordial black holes.

VI. CONCLUSIONS

In this paper we have proposed a new mechanism to explain the emergence of supermassive primordial black holes during the radiation period. The key element here is the conjectured existence of very massive particles stable against decay into Standard Model matter, that can "condense" into bound states sufficiently early in the radiation period which can subsequently collapse to black holes. Our proposal is chiefly motivated by the possible explanation of the observed spectrum of 48 spin-$\frac{1}{2}$ fermions in the Standard Model that was put forward in our previous work [9–11], and is thus subject to independent falsification if *any* new fundamental spin-$\frac{1}{2}$ fermions were to show up in future collider searches. In addition, we have derived a new solution of Einstein's equations describing the growth of black holes in a dense and hot plasma through inflow of radiation. This exact solution could be useful also in other contexts.

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