Implication of hidden sub-GeV bosons for the $(g-2)_\mu$, ^{8}Be-^{4}He anomaly, proton charge radius, EDM of fermions, and dark axion portal

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We discuss new physics phenomenology of hidden scalar (S), pseudoscalar (P), vector (V) and axial-vector (A) particles coupled to nucleons and leptons, which could give contributions to proton charge radius, $(g-2)_\mu$, ^{8}Be-^{4}He anomaly and electric dipole moment (EDM) of Standard Model (SM) particles. In particular, we estimate sensitivity of NA64μ experiment to observe muon missing energy events involving hidden scalar and vector particles. That analysis is based on GEANT4 Monte Carlo simulation of the signal process of muon scattering off target nuclei $\mu N \to \mu N S(V)$ followed by invisible boson decay into dark matter (DM) particles, $S(V) \to \chi\chi$. The existence of light sub-GeV bosons could possibly explain the muon $(g-2)$ anomaly observed. We also summarize existing bounds on ATOMKI $X17(J^P = 0^-, 1^\pm)$ boson coupling with neutron, proton, and electron. We implement these constraints to estimate the contribution of P, V. and A particles to proton charge radius via direct 1-loop calculation of Sachs form factors. The analysis reveals the corresponding contribution is negligible. We also calculate bounds on dark axion portal couplings of dimension-five operators, which contribute to the EDMs of leptons and neutron.

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I. INTRODUCTION

The measurement of anomalous magnetic moment of muon provides the potential signal of new physics. Indeed, the value of $(g-2)_\mu$ measured at the Brookheaven National Laboratory [1] differs from the prediction of Standard Model (SM) at the level of 3.7 standard deviations [2,3], $\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{th} = (279 \pm 76) \times 10^{-11}$. The existence of light and weakly coupled hidden bosons [4–7] could be a possible beyond SM explanations of that discrepancy [8,9]. In particular, BELLEII experiment [10] has been already put constraints on hidden vector boson Z' coupled with muons, which can contribute to $(g-2)_\mu$ anomaly. In addition, M^3 compact muon missing momentum experiment [11] has been proposed recently at Fermilab to examine $(g-2)_\mu$ puzzle. Moreover, muon (electron) fixed target NA64μ (NA64e) experiment at CERN SPS [12] plans to collect data after CERN Long Shutdown (LS2) in 2021 to test sub-GeV boson contribution into muon and electron $(g-2)$. In particular, the NA64 experiment at the CERN SPS combines the active target and missing energy techniques to search for rare events.

The processes accompanied by the emission and decay of hypothetical hidden boson [13,14] provide an additional evidence toward the weakly coupled particle interactions beyond SM [15–19]. Namely, ATOMKI Collaboration has been reported recently the ~6.8σ and ~7.2σ anomalies of e^+e^- pair excess from electromagnetically transition in ^{8}Be [13] and ^{4}He [14], respectively. The relevant ^{8}Be data have been explained as creation and decay of $X17$ boson particle with mass $m_X = 16.70 \pm 0.35 \pm 0.50$ MeV. Furthermore, most favored candidates, that could play the role of the $X17$ boson [16–18] have spin-parity $J^P = 1^+$, $J^P = 0^-$, and $J^P = 1^-$. In particular, in order to explain ^{8}Be anomaly, authors of Ref. [16] provided an analysis for excited ^{8}Be states and presented anomaly-free extension of SM

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that contains gauge boson with experimentally favored couplings [19–28]. In addition, in Ref. [17] light pseudoscalar state from Higgs extended sector was suggested to describe relevant e^+e^- excess in ^{8}Be transition with coupling that satisfies existing constraints [29–31]. Moreover, authors of Ref. [18] investigated the production of vector boson with primarily axial couplings to quarks that is consistent with experimental data [32–34], such that new axial field has a mass $m_X \simeq 16.7$ MeV (see e.g., Refs. [35–37] for a recent review) and describes comprehensively nuclear properties of the $^8\mathrm{Be}(1^+) \to {}^8\mathrm{Be}(0^+)$ anomalous transition.

However, in [38] authors provide dedicated analysis of e^+e^- pair emission anisotropy in nuclear transitions of ^{8}Be, which has a possible relevance to that anomaly. Another analysis of ^{8}Be anomaly not involving beyond Standard Model explanation was carried out recently in Ref. [39]. In particular, author provides a hint that 17 MeV excess in the experiment with ^{8}Be [13] and ^{8}He [14] can be associated with the quantum phase transition in the α-like nuclei of ^{8}Be, ^{4}He, ^{12}C, and ^{16}O.

It is worth mentioning that electron fixed target NA64e facility at the CERN SPS [19,20] has excellent opportunity of probing ^{8}Be anomaly due to its dedicated searching sensitivity for short-lived hidden particles, $\tau_X \lesssim 10^{-12}$ s. In particular, we expect that NA64e active target facility will be able to probe hidden pseudoscalar $X17$ boson after CERN LS2 in 2021.

Precise determination of the proton charge radius r_p^E, one of the fundamental quantities of hadron physics, remains unsolved problem for many years. There are three methods of measurement of the proton charge radius from study: (1) cross section of elastic lepton-proton scattering, (2) Lamb shift in atomic hydrogen, and (3) Lamb shift in muonic hydrogen.

The most recent and precise result for the r_p^E extracted from the elastic electron scattering off proton was obtained by the A1 Collaboration at MAMI [40]: $r_p^E = 0.879 \pm 0.005 \pm 0.006$ fm. It is in a good agreement with the 2014 CODATA recommended value $r_p^E = 0.8751 \pm 0.0061$ fm [41]. However, these results are in a sizable disagreement (by 5.6 standard deviations) with most accurate result for the $r_p^E = 0.84087 \pm 0.00026 \pm 0.00029$ fm obtained from Lamb shift in μp atom by the CREMA Collaboration at PSI [42,43]. In 2019 the proton radius was deduce from measurement of the electronic hydrogen Lamb shift: $r_p^E = 0.833 \pm 0.010$ fm [44], which led to a conclusion that the electron- and muon-based measurements of the r_p^E finally agrees with each other. Recently, the PRad Collaboration at JLab [45] reported on improved measurement of the proton charge radius from an electron-proton scattering experiment: $r_p^E = 0.831 \pm 0.007(\mathrm{stat}) \pm 0.012(\mathrm{syst})$ fm. As stressed in Ref. [45], this prediction is smaller than the most recent high-accuracy predictions based on ep elastic scattering and very close to the results of the precise muonic hydrogen experiments [42,43]. Also it was noticed in [45] that their prediction is 2.7 standard deviations smaller than the average of all ep experimental results [41]. We note that an independent and a highly-precise measurement proposed by the COMPASS++/AMBER at the M2 beam line of the CERN SPS [46] has very strong physical motivation as independent and complimentary experiment to recent observation done by the PRad Collaboration [45]. On the other hand, the use of the muon beam in the planned COMPASS++/AMBER experiment [46] gives a unique opportunity to test electron-muon universality and to reduce systematic uncertainties and radiative corrections. For discussion of future experiments and overview on proton radius see, e.g., Refs. [47–49].

One should stress that from theoretical point new particles with different spin-parity assignments could contribute to resolving of puzzles in particle phenomenology and to more precise determination of their properties. E.g., one can imagine existence of new particles with different spin-parity assignments, e.g., scalar ($J^P = 0^+$), pseudoscalar ($J^P = 0^-$), vector ($J^P = 1^+$), and axial ($J^P = 1^-$) particles. Also one can analyze a possible contribution of these states to the $(g-2)_\mu$ anomaly. Note that effects of scalar, pseudoscalar, and vector particles on the Lamb shift in lepton-hydrogen and $(g-2)_\mu$ anomaly have been already discussed and estimated in literature [50–55]. We noticed that one can also estimate the relative contribution of new particles (S, P, V, and A) to the proton charge radius via direct 1-loop calculation of Sachs form factors. From our preliminary analysis it follows that contribution of these particles to the charge radius of proton is negligible.

However, it is instructive to collect existing bounds on $X17$ boson coupling with SM fermions and calculate contribution of $X17$ to EDMs of leptons and neutron. The relevant coupling terms originate from dimension-five operators (see, e.g., Eq. (3) below). These interactions are motivated by dark-axion portal scenarios, involving couplings of photon, dark photon, and axionlike particle (for details, see, e.g., Refs. [56–61]). In addition, several well-motivated scenarios of new physics involving the light hidden sector and EDMs are discussed in Ref. [62–65].

Our paper is structured as follows. In Sec. II we consider effective couplings of sub-GeV bosons with SM fermions. In Sec. III we estimate sensitivity of NA64μ muon active target experiment to probe sub-GeV vector and scalar mediator of DM by using comprehensive GEANT4 MC simulation. These bosons can possibly explain $(g-2)_\mu$ anomaly. In Sec. IV we summarize existing constraints on ^{8}Be anomaly for hidden $X17(J^P = 0^-, 1^\pm)$ bosons. In Sec. VI we estimate contribution of $X17(J^P = 0^-, 1^\pm)$ bosons to proton charge radius directly from Sachs form factors. We conclude, that current information on new particles suggests that their contribution to the charge radius of proton is negligible. In Sec. VI we also set constraints on dimension-five operator couplings of light bosons which can contribute to EDM of SM fermions. That analysis is motivated by dark axion portal study. Finally, in Sec. VII, we summarize our results.

II. EFFECTIVE LAGRANGIAN

We consider entirely phenomenological couplings of light bosons to SM particles, which are based on an effective theory approach. Namely, new physics (NP) Lagrangian involving coupling of nucleons and leptons with scalar S, pseudoscalar P, vector V, and axial A bosons, which could contribute to the proton radius, muon magnetic moment, and electric dipole moments of electron (muon) and neutron can be written as follows

$$\mathcal{L}_{\rm NP} = \sum_H \mathcal{L}_H + \sum_{H_1 H_2} \mathcal{L}_{\gamma H_1 H_2}, \tag{1}$$

where $H = S,\ P,\ V,\ A$ and $H_1 H_2 = SV, PV, PA$. Here $\mathcal{L}_H = H J_H$, where J_H is the fermionic currents including effects of P-parity violation. They are composed of nucleons and fermions as

$$J_H = \sum_{N=p,n} \bar{N}(g_H^N \Gamma_H + f_H^N \tilde{\Gamma}_H) N + \sum_{\ell = e,\mu,\tau} \bar{\ell}(g_H^\ell \Gamma_H + f_H^\ell \tilde{\Gamma}_H)\ell, \tag{2}$$

where $\Gamma_S = \tilde{\Gamma}_P = I,\ \Gamma_P = \tilde{\Gamma}_S = i\gamma^5,\ \Gamma_V = \tilde{\Gamma}_A = \gamma^\mu$, and $\Gamma_A = \tilde{\Gamma}_V = \gamma^\mu \gamma^5$ are the Dirac spin matrices. Second term in Lagrangian (1) describes the coupling of new particles with photon (here we list only the terms which contribute to the electric dipole moment):

$$\begin{aligned} \mathcal{L}_{\gamma SV} &= \frac{e}{4M_p} g_{\gamma SV} F^{\mu\nu} V_{\mu\nu} S, \\ \mathcal{L}_{\gamma PA} &= \frac{e}{4M_p} g_{\gamma PA} F^{\mu\nu} A_{\mu\nu} P, \\ \mathcal{L}_{\gamma PV} &= \frac{e}{4M_p} f_{\gamma PV} F^{\mu\nu} V_{\mu\nu} P. \end{aligned} \tag{3}$$

$g_H^{N(\ell)}$, $g_{\gamma H_1 H_2}$ and $f_H^{N(\ell)}$, $f_{\gamma H_1 H_2}$ are the sets of P-parity even and P-parity odd couplings, respectively. In Appendix we list the expressions for the contributions of new particles to the muon magnetic moment and proton charge radius including both P-even and P-odd couplings, while in numerical analysis, for simplicity we will neglect by the P-odd couplings. Later, we derive the constraints of combinations of P-even and P-odd couplings of new particles using data on electric dipole moments of leptons and neutron. However, we note that constraints on (3) couplings can be motivated by dark axion-portal study [56–61].

III. NA64μ EXPERIMENT FOR PROBING $(g-2)_\mu$ ANOMALY

The NA64μ is upcoming experimental facility at CERN SPS [12,66–68], which aims to examine light hidden sector particles weakly coupled to muons. It will utilize a muon beam at CERN SPS to search for missing energy signatures in the bremsstrahlung process on the active target, $\mu N \to \mu N E_{\rm miss}$. That process can be associated with sub-GeV hidden vector boson V invisibly decaying into light dark matter particles, $V \to \chi\chi$, or neutrinos, $V \to \bar{\nu}\nu$. That vector particle is referred to Z'-boson, which interacts mainly with $L_\mu - L_\tau$ currents of SM. In addition, it can serve a sub-GeV vector mediator between SM and DM sector due to the mechanism of relic DM abundance [11,69–72]. We note however that there are several other well-motivated scenarios of Z' boson which are based on hidden Abelian symmetries, say $U(1)_{B-L}$ or $U(1)_{B-3L_e}$ (for recent review see, e.g., Refs. [73,74]).

Furthermore, in Refs. [11,70,75] authors considered a scenarios with muon-specific scalar mediator between visible and hidden matter in order to resolve $(g-2)_\mu$ anomaly and DM puzzle. One can expect that relevant scalar originates from UV completed models with vector-like fermions and Higgs extended sector [76,77].

In Refs. [66–68] probing of the new dark boson V and hidden scalar S at NA64μ was discussed in the light of explanation of the muon magnetic moment anomaly. In this section we extend the analysis of V and S implication to NA64μ [66–68]. In particular, there are two general extensions of the analysis discussed in Ref. [66–68]. First, we calculate the exact-tree-level production cross-section of hidden neutral boson S and V at NA64μ. That analysis is based on the result of Refs. [78,79] and our previous study [80] for dark photon production at NA64e without using Weizsaecker-Williams approximation in the cross sections of hidden bosons. Second, we calculate the expected sensitivity curves of NA64μ for muon-specific couplings of sub-GeV vector and scalar hidden particles $L \supset g_S^\mu S\bar{\mu}\mu + g_V^\mu V_\nu \bar{\mu}\gamma^\nu \mu$ by using GEANT4 Monte-Carlo (MC) simulation.

In Fig. 1 the expected limits of NA64μ detector are shown for hidden scalar and vector boson, we also set benchmark assumption, $g_P^\mu = g_A^\mu = 0$, such that pseudo-scalar and axial vector coupling admixtures don't contribute to $(g-2)_\mu$ anomaly. The expected sensitivity of NA64μ was calculated by using GEANT4 MC simulation of missing energy signal of $E_0 = 100$ GeV muon scattering on target with heavy nuclei $\mu N \to \mu N S(V)$. The number of produced light bosons can be approximated as follows

$$N_{S(V)} \simeq {\rm MOT} \times \frac{\rho N_A}{A} \times L_T \times \sigma_{S(V)}, \tag{4}$$

where MOT is a number of muons accumulated on target, A is a atomic weight of target medium, N_A is Avogadro's number, ρ designates the target density, $L_T \simeq 40 X_0$ is a

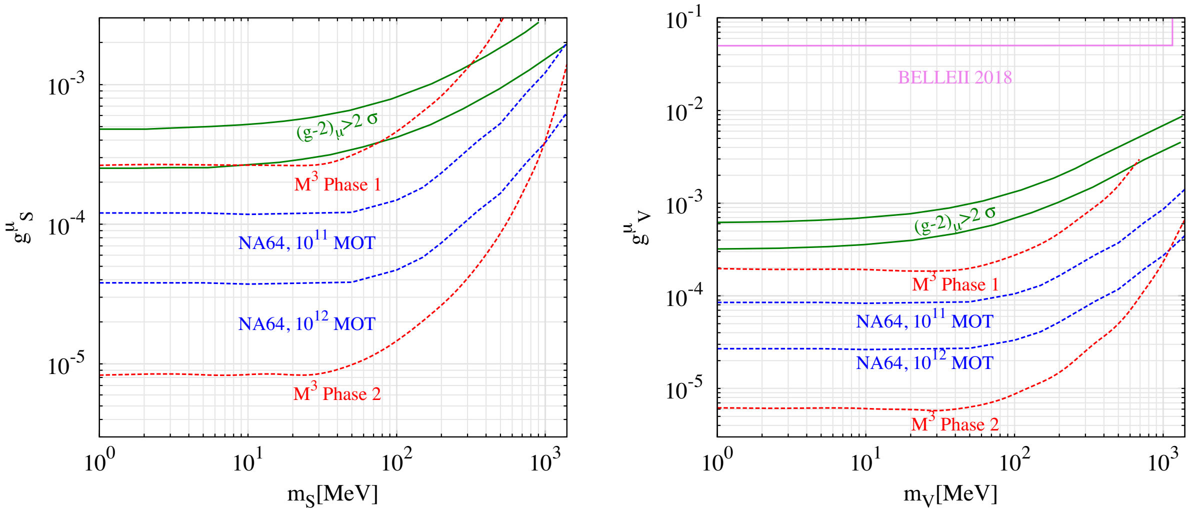

FIG. 1. Left plot: dashed blue lines are dark scalar expected sensitivities of NA64μ for MOT $= 10^{11}$ and MOT $= 10^{12}$. Right plot: dashed blue lines are dark vector expected sensitivities of NA64μ for MOT $= 10^{11}$ and MOT $= 10^{12}$. Corresponding dashed red lines are expected limits of M^3 experiment [11] for Phase 1 and Phase 2. Pink line shows recent constraints of BELLEII experiment [10] from data collected in 2018. Green lines represent the bounds which correspond to resolving of $(g-2)_\mu$ anomaly at 2σ level for both scalar and vector particles. In particular, we use the following inequalities, $|\delta a^\mu_{V(S)} - \delta a^{c.}_\mu| < 2\sigma_{\delta a_\mu}$, where $\delta a^{c.}_\mu = 27.9 \times 10^{-10}$ and $\sigma_{\delta a_\mu} = 7.6 \times 10^{-10}$ are taken from Ref. [3].

typical distance that are passed by muon before producing $S(V)$ with the energy of $E_{S(V)} \gtrsim E_0/2$ in the active lead target of NA64μ ($X_0 \simeq 0.5$ cm), $\sigma_{S(V)}$ is a total exact-tree-level production cross section of light bosons (for details see, e.g., Ref. [78–80]).

For $m_{S(V)} \lesssim M_\mu$ that production rate can be approximated in bremsstrahlunglike limit as $\sigma_{S(V)} \sim (g^\mu_{S(V)})^2/M^2_\mu$. Which implies that relevant sensitivity curves in Fig. 2 have a plateau in the light mass region. In Fig. 2 we require $N_{S(V)} > 2.3$, which corresponds to 90% CL exclusion bound on $g^\mu_{S(V)}$ coupling for the background free case. In particular, a preliminary hadron contamination analysis and study of the detector Hermiticity with muon beam [12] show

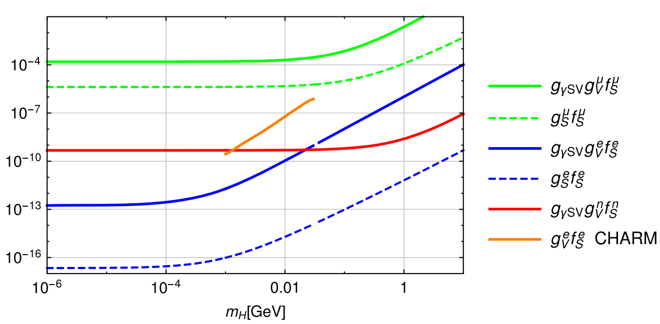

FIG. 2. 90% CL constrains on coupling combinations from EDM of SM fermions. Solid (dashed) green line shows bound on $g_{\gamma SV} g^\mu_V f^\mu_S$ ($g^\mu_S f^\mu_S$) coupling from EDM of muon. Solid (dashed) blue line shows bound on $g_{\gamma SV} g^e_V f^e_S$ ($g^e_S f^e_S$) coupling from EDM of electron. Solid red line shows bound on $g_{\gamma SV} g^n_V f^n_S$ coupling from EDM of neutron. Solid orange line represents combined limits on $g^e_V f^e_S$ from electron EDM and CHARM bounds.

that total background to be at the level $\lesssim 10^{-12}$. It is worth mentioning that muon energy losses in the lead target can be neglected [75], since the muon energy attenuation is small for typical beam energy, $\langle dE_\mu/dz \rangle \simeq 12.7 \times 10^{-3}$ GeV/cm.

In the NA64μ experiment one assumes to utilize two, upstream and downstream, magnetic spectrometers. These spectrometers, will provide a precise measurements of initial and final muon energies [12]. We suppose that $S(V)$ being produced by muons in the target escapes the NA64μ detector without interaction decaying invisibly into DM particles.

Indeed, let us estimate the absorption length of light scalar and vector in the medium of the NA64 detector. Hidden bosons should be sterile enough to avoid energy deposition in ECAL and HCAL due to their absorption. In high energy limit, $E_{V(S)} \gg 10$ GeV, the number QCD resonances of $\sim$100 MeV produced due to the boson absorption by the protons will be negligible. So that, the leading process of boson attenuation in calorimeters is inelastic scattering $V(S) + p \to p + \text{jet}$. For relatively light bosons, $m_{V(S)} \lesssim 100$ MeV, one can estimate the absorption cross section in the high-energy regime, $E_{V(S)} = E_H \gg m_p$, as $\sigma_{abs} \simeq Z\alpha_s (g^p_H)^2/(4s)$, where $s \simeq 2E_H m_p$ is a center of mass energy squared. Here we denote $H = (V, S)$. For estimate we take $g^p_{V(S)} \simeq 10^{-4}$ as a typical benchmark coupling of hidden bosons to proton (see, e.g., Table I below). Therefore for $\alpha_s(M_Z) = 0.12$ one has

$$\sigma_{abs} \simeq 1.2 \times 10^{-31}\ \text{cm}^2 (g^p_H)^2 Z \left(\frac{E_H}{100\ \text{GeV}}\right)^{-1}. \quad (5)$$

TABLE I. Favored couplings for $X17(J^P = 1^{\pm}, 0^-)$.

Coupling	Neutron	Proton	Electron
$g_P/e = f_S/e$	$\lesssim 1.8 \times 10^{-3}$ from Ref. [17]	$\lesssim 8.3 \times 10^{-3}$ from Ref. [17]	$\gtrsim 3.0 \times 10^{-5}$ from Refs. [17,29]
$g_V/e = f_A/e$	$\lesssim (2-10) \times 10^{-3}$ from Ref. [15,16]	$\lesssim 1.2 \times 10^{-3}$ from Ref. [15,16]	$\lesssim 1.4 \times 10^{-3}$ from Ref. [22], $\gtrsim 6.8 \times 10^{-4}$ from Ref. [19]
$g_A/e = f_V/e$	$\lesssim 3.3 \times (10^{-5} - 10^{-4})$ from Ref. [18]	$\lesssim 3.3 \times (10^{-5} - 10^{-4})$ from Ref. [18]	$\gtrsim 6.8 \times 10^{-4}$ from Ref. [19]

Now we can estimate the absorption length as $\lambda_{abs} \simeq (n\sigma_{abs})^{-1}$, where $n = \rho N_A/A$ is a typical density number of atoms in the medium of the target. For iron medium of hadronic calorimeter, $\rho = 7.87$ g/cm^3, $A = 56$ g/mole, $Z = 26$, $E_H = 100$ GeV, $g_H^p = 10^{-4}$, one obtains $n^{Fe} \simeq 2.2 \times 10^{24}$ cm^{-3} and $\lambda_{abs} \simeq 5 \times 10^{13}$ cm. This means that light bosons produced by muon beam in target will pass the hadronic calorimeter module without energy deposition.

IV. ^{8}Be ANOMALY CONSTRAINTS

It is worth mentioning that nucleon terms in Lagrangian (2) can be referred to hadron-$X17$ boson couplings [15,16] for the case of parity-violating interaction [37]. In particular, authors of [15,16] provide a rough estimate of P-even hadronic couplings of $X17$ boson as $|f_A^p/e| \simeq |g_V^p/e| \lesssim 1.2 \times 10^{-3}$ and $|f_A^n/e| \simeq |g_V^n/e| \lesssim (2-10) \times 10^{-3}$ from null result of $\pi^0 \to \gamma(X17 \to e^+e^-)$ decay at NA48/2 [21,22] and best fit of $X17$ decay in the ATOMKI experiment [13]. For P-odd hadronic couplings of $X17$ vector boson one can expect them to be proportional to quark axial couplings $g_A^{n(p)} \simeq f_V^{n(p)} \sim g_q^A$ in a manner of Ref. [18]. Namely, a comprehensive analysis of [18] for both, enhanced isoscalar, $^8\mathrm{Be}^{*\prime}(J^P = 1^+; T = 0) \to {}^8\mathrm{Be}^*(J^P = 0^+; T = 0) + X17$, and suppressed isovector, $^8\mathrm{Be}^{*\prime}(J^P = 1^+; T = 1) \to {}^8\mathrm{Be}^*(J^P = 0^+; T = 0) + X17$, nuclear transitions implies a conservative bounds $|g_A^{n(p)}| \simeq |f_V^{n(p)}| \lesssim 10^{-5} - 10^{-4}$. The hadronic terms in the Lagrangian (2) involving hidden scalar and pseudoscalar particles can be originated from extended Higgs sector of SM [81]. In particular, light pseudoscalar can be a valid candidate for ^{8}Be anomaly explanation [17]. The relevant Lagrangian reads

$$\mathcal{L} \supset \sum_{q=u,d} \xi_p \frac{m_q}{v} P\bar{q}i\gamma_5 q, \qquad (6)$$

where $v = 246$ GeV is the Higgs vacuum expectation value. This implies [17] that the resulting Yukawa-like couplings of P to up and down type quarks are $\xi_u \simeq \xi_d \simeq 0.3$, with ξ_u and ξ_d being a linear combination of nucleus couplings, such that

$$g_P^p \simeq \frac{M_p}{v}(-0.40\xi_u - 1.71\xi_d), \qquad (7)$$

$$g_P^n \simeq \frac{M_n}{v}(-0.40\xi_u + 0.85\xi_d). \qquad (8)$$

Therefore, one has conservative limits, $|g_P^p| \lesssim 2.5 \times 10^{-3}$ and $|g_P^n| \lesssim 5.5 \times 10^{-4}$, which, however depend on nuclear shell model of isospin transition [17]. We note, that Lagrangian (6) does not respect gauge symmetry of SM unbroken gauge group, and therefore can be considered as effective interaction of UV completed model [82].

Now let us consider ^{8}Be constraints for light hidden boson from lepton sector, which is described by the second term in the Lagrangian (2). A numerous well motivated scenarios [83–97] have been suggested recently for explaining the ATOMKI e^+e^- anomaly, which involve neutral vector boson interacting with leptons. That vector particle decays visibly via e^+e^- pair, with $\mathrm{Br}(V \to e^+e^-) \simeq 1$, since its mass does not exceed the masses of any hadronic states. The dominant constraints on vector coupling to electron come from NA48/2 data on $\pi^0 \to \gamma V(V \to e^+e-)$ decay and from NA64e data on $eN \to eNV(V \to e^+e^-)$ bremsstrahlung e^+e^- pair emission. In particular, NA48/2 experimental facility provides best upper limit on $X17(J^P = 1^+)$ mixing with electrons, $\mathcal{L} \supset g_V^e V_\mu \bar{e}\gamma^\mu e$, such that the allowed values of coupling are $g_V^e/e \lesssim 1.4 \times 10^{-3}$ at 90% CL. NA64e experiment has been recently set the lower limit on the relevant coupling at 90% CL [19]. Therefore, the existence of $X17$ vector boson favors the following values of electron mixing $g_V^e/e \gtrsim 6.8 \times 10^{-4}$. The former bound can be rescaled for the case of axial-vector coupling admixture, $\mathcal{L} \supset V_\mu \bar{e}\gamma^\mu(g_V^e + \gamma_5 g_A^e)e$, as $\sqrt{(g_V^e)^2 + (g_A^e)^2}/e \gtrsim 6.8 \times 10^{-4}$.

It is worth mentioning that one can estimate the projected sensitivity of the NA64 to probe pseudoscalar particle $X17(J^P = 0^-)$ which decays visibly to electron-positron pair, $\mathrm{Br}(P \to e^+e^-) = 1$. The authors of Ref. [17] provide the following limits for reduced Higgs-like coupling of $X17(J^P = 0^-)$ boson with electrons $\mathcal{L} \supset \xi_P^e \frac{M_e}{v} P\bar{e}i\gamma_5 e$

$$\xi_P^e \gtrsim 4.5 \qquad (9)$$

which are favored by experimental data from electron and proton beam-dump facilities [29] for $m_P \gtrsim 17$ MeV. The relevant limits $C_{Aff} = \xi_P^e$ are shown in Fig. (4) of Ref. [29]. These bounds can be transferred to the electron's coupling in terms of Lagrangian (2) as follows $g_P^e/e \gtrsim 3.0 \times 10^{-5}$, from CHARM data [29]. It is instructive to compare g_P^e/e limits with the corresponding bounds of vector boson $X17(J^P = 1^+)$, that has the allowed couplings in the range

$6.8\times10^{-4}\lesssim g_V^e/e\lesssim1.4\times10^{-3}$. To summarize results, let us estimate the lifetimes [78,79] of P and V which have not been experimentally excluded yet. In particular, one has the following constraints from various experiments

(i) CHARM [29]: $\tau_P\lesssim1.2\times10^{-11}$ s, $g_P^e/e\gtrsim3.0\times10^{-5}$,
(ii) NA48/2 [22]: $\tau_V\gtrsim8.3\times10^{-15}$ s, $g_V^e/e\lesssim1.4\times10^{-3}$,
(iii) NA64e [19]: $\tau_V\lesssim3.5\times10^{-14}$ s, $g_V^e/e\gtrsim6.8\times10^{-4}$.

Note, that corresponding bounds on lifetimes of $X17$ boson are in agreement with the estimate $\tau_{X17}\lesssim10^{-10}$ s from ATOMKI data [15] as expected for both pseudoscalar and vector realizations of $X17$. In particular, authors of Ref. [15] require that $X17$ decays into e^+e^- within $L\lesssim1$ cm, where $L=c\tau_X\beta_X\gamma_X$, here $\gamma_X\simeq1.06$ and $\beta_X=0.35$.

We point out that the NA64e experiment has an excellent prospect for probing of $X17(J^P=0^-)$, since it will decay mostly into e^+e^- within the fiducial volume of the NA64e ($L_{fid}\sim7$–10 m) due to large boost factor $E_P/m_P\simeq6\times10^3$, with typical decay length of $L^P_{dec}\simeq14$ m. We note however that our estimate is conservative, therefore one should perform a comprehensive Monte-Carlo simulation for the flux and spectra of hidden pseudoscalars produced in the target by primary electrons, $eN\to eNP(P\to e^+e^-)$. That investigation will take into account realistic response and efficiency of the NA64e detector. We leave that task for future analysis [98]. In Table I we summarize current limits on $X17$ couplings.

V. COMBINED CONTRIBUTION OF LIGHT BOSONS TO THE PROTON RADIUS

In this section we consider the problem of the proton charge radius. In particular, we discuss direct contribution of these light bosons into the proton radius via the charge Sachs form factor $G_E^P(q^2)$. As we stressed in Sec. I, this possibility is quite interesting in the connection to planned precise measurement of the proton charge radius from analysis of the elastic muon-proton scattering. P-even electromagnetic vertex function is defined for incoming photon as

$$M^P_{\rm inv}=\bar u(p')\left[\gamma^\mu F_1(q^2)+\frac{i}{2M_p}\sigma^{\mu\nu}q_\nu F_2(q^2)\right]u(p). \quad (10)$$

Here F_1 and F_2 are the Dirac and Pauli form factors; $q^2=-Q^2$. For minimal coupling of photon with proton and charged leptons

$$\mathcal{L}_{\rm em;m}=eA_\mu[\bar p\gamma^\mu p-\bar\ell\gamma^\mu\ell]. \quad (11)$$

It is interesting to look at the relative contribution of new hidden particles to both proton charge radius and muon $(g-2)_\mu$ ratio anomaly. We do the direct estimate of the contributions of new particles into the proton charge radius. The proton charge radius is defined as

$$\langle r_p^E\rangle^2=-6[G_E^p(0)]'=-6[F_1^p(0)]'+\frac{3}{2M_p^2}F_2^p(0), \quad (12)$$

where F_1^p and F_2^p are the Dirac and Pauli electromagnetic form factors of the proton, respectively; $[F(0)]'$ means the derivative with respect to Q^2 at $Q^2=0$. Here $F_2^p(0)=\kappa_p$ is the proton anomalous magnetic moment. In particular, using the mass value $m_V=16.7$ MeV of the hypothetic $X17$ vector particle observed in the ATOMKI experiment [14] we get the following leading (logarithmic) contribution to the charge proton radius:

$$\langle\delta r_p^E\rangle^2\simeq0.014h_r^{(1)}\ {\rm fm}^2, \quad (13)$$

where

$$h_r^{(1)}=(g_V^p)^2-(g_A^p)^2+(f_A^p)^2-(f_V^p)^2, \quad (14)$$

is the combination of couplings of vector and axial vectors with proton (see Appendix). Let us estimate that contribution for benchmark couplings shown in Table I for $X17$ boson. In particular, $h_r^{(1)}=2(g_V^p)^2-2(g_A^p)^2\simeq2.6\times10^{-7}$, that yields $\langle\delta r_p^E\rangle\simeq6\times10^{-5}$ fm. Therefore we conclude, that current information on new particles suggests that their contribution to the charge radius of proton is negligible. We note, that the small impact of BSM effects on proton charge radius was discussed originally in Ref. [99].

VI. CONSTRAINTS ON COUPLINGS OF NEW PARTICLES USING DATA ON ELECTRIC DIPOLE MOMENTS OF LEPTONS AND NEUTRON

In this section we derive the constraints on the combinations of P-even and P-odd couplings of new particles using data on electric dipole moments (EDM) of leptons and neutron. The contributions of new particles to EDMs are described by the diagram in Fig. 3 where squared vertex is P-odd and round vertex is P-even coupling with leptons (neutron). The EDM of spin-$\frac{1}{2}$ fermion ψ (neutron or leptons) is defined as $d^E=D_E(0)$, where $D_E(q^2)$ is the relativistic electric dipole form factor extracted from full electromagnetic vertex function of corresponding fermion [100]:

$$M_{\rm inv}=\bar u_\psi(p_2)\Gamma^\mu(p_1,p_2)u_\psi(p_1),$$
$$\Gamma^\mu(p_1,p_2)=-\sigma^{\mu\nu}q_\nu\gamma^5D_E(q^2)+\ldots \quad (15)$$

The contributions of individual diagrams in Fig. 3 are given in the Appendix.

Using the upper limits/results for the electron, muon, and neutron EDMs:

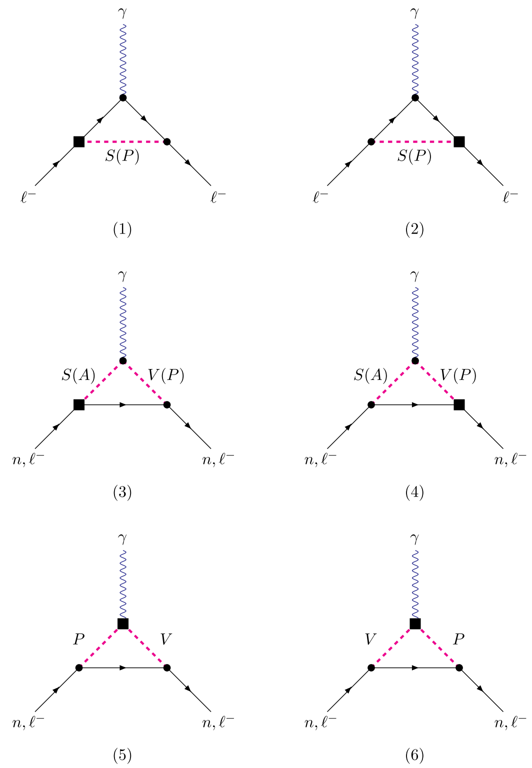

FIG. 3. Diagrams describing contribution of new particles to electric dipole moments of neutron and leptons. The square boxes denote the P-odd vertices, round vertices are P-even couplings.

$$|d_e^E| < 1.1 \times 10^{-29}\ e\,\mathrm{cm}, \quad \text{at 90\% CL, Ref.[101]},$$
$$|d_\mu^E| < 1.8 \times 10^{-19}\ e\,\mathrm{cm}, \quad \text{at 90\% CL, Ref.[102]},$$
$$|d_n^E| < 3.0 \times 10^{-26}\ e\,\mathrm{cm}, \quad \text{at 90\% CL, Ref.[103]},$$

we get the upper limits for combinations of couplings of new particles, which are displayed in Table II. Let us consider several benchmark limits. Namely, for concreteness we set to zero couplings of dimension-five operators (3), $g_{\gamma SV} = g_{\gamma PA} = f_{\gamma PV} = 0$. That yields the following constraints on electron and muon interaction with P and S for light bosons masses $m_V = m_P = m_S \ll M_e$

$$|d_e/e| \simeq \frac{|g_S^e f_S^e|}{8\pi^2 M_e} < 5.5 \times 10^{-16}\ \mathrm{GeV}^{-1}, \tag{16}$$

$$|d_\mu/e| = \frac{|g_S^\mu f_S^\mu|}{8\pi^2 M_\mu} < 5.0 \times 10^{-7}\ \mathrm{GeV}^{-1}, \tag{17}$$

or equivalently $|g_S^e f_S^e| < 2.7\times10^{-17}$ and $|g_S^\mu f_S^\mu| < 4.2\times10^{-6}$. In order to avoid interference between diagrams (1)–(4) in Fig. 3 we now consider a benchmark point $g_S = g_P = 0$. That implies the following limits or the product of vector-specific and pseudoscalar couplings of leptons

$$|d_e/e| = \frac{|g_{\gamma SV} g_V^e f_S^e|}{16\pi^2 M_p}\frac{1}{2} < 5.5 \times 10^{-16}\ \mathrm{GeV}^{-1}, \tag{18}$$

$$|d_\mu/e| = \frac{|g_{\gamma SV} g_V^\mu f_S^\mu|}{16\pi^2 M_p}\frac{1}{2} < 5.0 \times 10^{-7}\ \mathrm{GeV}^{-1}, \tag{19}$$

which yield $|g_{\gamma SV} g_V^e f_S^e| < 1.7 \times 10^{-13}$ and $|g_{\gamma SV} g_V^\mu f_S^\mu| < 0.2 \times 10^{-3}$. For relatively light hidden bosons $m_A = m_V = m_P = m_S \ll M_n$ one can also obtain corresponding constraint from neutron EDM, $|g_{\gamma SV} g_V^n f_S^n| < 4.5 \times 10^{-10}$. Heavy bosons $m_H \gg M_\psi$ yield the limits on coupling products, which are scaled as $\sim(m_H/M_\psi)^2$. These bounds are shown in Fig. 2. One can see from Fig. 2, that the most stringent constraints on couplings come from electron EDM bounds for $m_H \ll M_e$. Moreover, for the benchmark values of electron coupling with vector, $g_V^e/e \simeq 1.4 \times 10^{-3}$, and scalar, $f_S^e/e \simeq 3.0 \times 10^{-5}$, one can also estimate the bound on $g_{\gamma SV}$ that is favored by $X17$-boson existence. In particular, for $m_H \simeq 16.7$ MeV, one has $g_{\gamma VS} \lesssim 7.7 \times 10^{-2}$ from Fig. 2. Corresponding bound from neutron EDM yields $g_{\gamma VS} \lesssim 1.5 \times 10^{-3} - 3 \times 10^{-4}$ for $g_V^n/e \simeq (2-10) \times 10^{-3}$ and $f_S^n/e \simeq 1.8 \times 10^{-3}$ provided in Table I. Here we expect naively that $X17$ is admixture of vector and pseudoscalar states which have dark axion portal coupling as in Ref. [57,58] $\mathcal{L} \supset \frac{1}{2} a G_{a\gamma\gamma'} F_{\mu\nu} F'_{\mu\nu}$. In particular, one can relate corresponding values of $G_{a\gamma\gamma'}$ and $g_{\gamma SV}$ as follows, $G_{a\gamma\gamma'} = e g_{\gamma SV}/(2M_p)$. That implies conservative bound on dark axion portal interaction of $X17$ states $G_{a\gamma\gamma'} \lesssim 2.5 \times 10^{-4} - 5 \times 10^{-5}\ \mathrm{GeV}^{-1}$ for $m_a = m_{\gamma'} \simeq 16.7$ MeV. We note that our latter rough estimate is referred to the model, which incorporates consistently both $X17(J^P = 0^-)$ and $X17(J^P = 1^+)$ states for ^{8}Be anomaly explanation. The development of that scenario however is beyond the scope of the present paper. Besides, we want to point out that proposed sensitivity for a future measurement of the proton EDM and indirect limit to neutron EDM which the JEDI Collaboration [104] plans to obtain at level of $\sim 10^{-29}$ can receive more stringent limit for the couplings by a factor 10^{-3}. We note that relevant limits for

TABLE II. Upper limits on couplings of new particles from data on EDMs of electron, muon, and neutron.

Coupling combination	Electron	Muon	Neutron
$\lvert g_P f_P\rvert = \lvert g_S f_S\rvert$, $m_H \ll M_\psi$	$<2.7 \times 10^{-17}$	$<(0.4\text{–}3.8) \times 10^{-5}$	…
$\lvert f_{\gamma PV} g_P g_V\rvert = \lvert g_{\gamma PA} g_P f_A\rvert = \lvert g_{\gamma SV} g_V f_S\rvert$, $m_H \ll M_\psi$	$<1.7 \times 10^{-13}$	$<(0.2\text{–}1.4) \times 10^{-3}$	$<4.5 \times 10^{-10}$
$\lvert f_{\gamma PV}\rvert = \lvert g_{\gamma PA}\rvert = \lvert g_{\gamma SV}\rvert$, $m_H \simeq 16.7$ MeV	$\lesssim 7.7 \times 10^{-2}$	…	$\lesssim 1.5 \times 10^{-3} - 3 \times 10^{-4}$

the combinations of couplings were set recently in Refs. [105,106] for the axion-like particle in the wide range of its masses 10^{-8} eV $\lesssim m_a \lesssim 10^{12}$ eV.

It is instructive to obtain constraint on $g_V^e f_S^e$ coupling from combined limit on electron EDM and CHARM experimental bounds for dark axion portal interaction $G_{a\gamma\gamma'}$ presented in Ref. [57,58]. The authors of Ref. [57,58] have been set severe upper limit on $G_{a\gamma\gamma'}$ assuming null result of CHARM experiment to observe $\gamma' \to a\gamma$ decay within regarding fiducial volume. The latter implies $m_{\gamma'} \gg m_a$, thus contribution of γ' and a into d_e/e in that mass range reads as follows

$$d_e/e = \frac{G_{a\gamma\gamma'} f_a^e g_{\gamma'}^e}{8\pi^2} J\left(\frac{m_{\gamma'}}{m_e}, 0\right). \tag{20}$$

Here we use the notations of Ref. [57,58] denoting indices as $a = S$ and $\gamma' = V$ for axionlike and dark-photon particles respectively, the function $J(m_{\gamma'}/m_e, 0)$ is given by Eq. (A18) in the Appendix. In particular, for $m_\gamma' \gg M_e$ one has

$$|g_{\gamma'}^e f_a^e| < 1.3 \times 10^{-13} \left(\frac{G_{a\gamma\gamma'}}{\mathrm{GeV}^{-1}}\right)^{-1} \frac{m_{\gamma'}^2/M_e^2}{\log(m_{\gamma'}^2/M_e^2)}, \tag{21}$$

which yields $10^{-10} \lesssim |g_{\gamma'}^e g_a^e| \lesssim 10^{-6}$ for the masses in the range 1 MeV $\lesssim m_{\gamma'} \lesssim$ 30 MeV from CHARM experimental constraints in Fig. 3 of Ref. [57]. We show corresponding limit in Fig. 2 by solid orange line.

VII. SUMMARY

In this paper we discuss phenomenological aspects of new scalar, pseudoscalar, vector, and axial particles coupled to fermions (nucleons and leptons), which could give contributions to proton charge radius and $(g-2)_\mu$ ratio, ^{8}Be anomaly and EDM of fermions. The main conclusions of this paper are

(i) We estimate sensitivity of NA64μ muon active target experiment to probe sub-GeV vector and scalar mediator of DM by using comprehensive GEANT4 MC simulation. These bosons can possibly explain $(g-2)_\mu$ anomaly. In case of NA64μ null result of observing muon missing energy events associated with hidden vector and scalar particles, $\mu N \to \mu N S(V)$, one can exclude new sub-GeV bosons as interpretation of $(g-2)_\mu$ anomaly.

(ii) We summarize existing constraints on ^{8}Be anomaly for hidden $X17(J^P = 0^-, 1^\pm)$ bosons. We estimate contribution of these particles to proton charge radius by direct calculation of Sachs form factors. It turns out that the resulting contribution is negligible.

(iii) We also set constraints on couplings of dimension-five operators for light hidden bosons which can contribute to EDM of SM fermions. That novel EDM analysis is motivated by dark axion portal study, which involves axion-photon-dark-photon couplings.

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APPENDIX: CONTRIBUTIONS OF NEW PARTICLES TO THE MUON MAGNETIC MOMENT, PROTON CHARGE RADIUS, AND EDM OF FERMIONS

Contributions of new particles to the anomalous magnetic moments of proton and charged leptons read

$$\delta a_S^\psi = \frac{1}{8\pi^2}\int_0^1 dx \frac{(1-x)^2((g_S^\psi)^2 - (f_S^\psi)^2 + x[(g_S^\psi)^2 + (f_S^\psi)^2])}{(1-x)^2 + x(\mu_S^\psi)^2}, \tag{A1}$$

$$\delta a_P^\psi = -\frac{1}{8\pi^2}\int_0^1 dx \frac{(1-x)^2((g_P^\psi)^2 - (f_P^\psi)^2 - x[(g_P^\psi)^2 + (f_P^\psi)^2])}{(1-x)^2 + x(\mu_P^\psi)^2}, \tag{A2}$$

$$\delta a_V^\psi = \frac{1}{8\pi^2}\int_0^1 dx \frac{2x(1-x)((g_V^\psi)^2 - 3(f_V^\psi)^2 - x[(g_V^\psi)^2 + (f_V^\psi)^2])}{(1-x)^2 + x(\mu_V^\psi)^2}, \tag{A3}$$

$$\delta a_A^\psi = -\frac{1}{8\pi^2}\int_0^1 dx \frac{2x(1-x)(3(g_A^\psi)^2 - (f_A^\psi)^2 + x[(g_A^\psi)^2 + (f_A^\psi)^2])}{(1-x)^2 + x(\mu_A^\psi)^2}, \tag{A4}$$

where $\mu_H^\psi = m_H/M_\psi$, $\psi = p, \ell^-$.

The expression for the P-even couplings of scalar, pseudoscalar, vector, and axial particles to the anomalous magnetic moments of fermions have been obtained before in Refs. [7,37,107–109]. Note, that expressions of the S, P, and V particles are finite, while the expression for the A is divergent due to longitudinal part of the axial particle propagator. Also divergences due to longitudinal part of the spin-1 particles (both vector and axial) occur in the contributions to the proton charge radius. As it was shown in Ref. [109,110] (see also Refs. [7]) consideration of the vector and axial particles in the renormalized gauge field theory allows to take into account their longitudinal part. In particular that implies the cancellation of divergences for the scenario with ultraviolet completion [37]. Here we use phenomenological Lagrangians and restrict to use the standard Feynman propagator for spin-0 particles $D_{J=0}(k^2) = 1/(M^2 - k^2)$ and the one without longitudinal part for spin-1 particles $D_{J=1}^{\mu\nu}(k^2) = -g^{\mu\nu}/(M^2 - k^2)$.

Below we list the corrections from new particles (S, P, V, A) to the $\langle r_p^2\rangle^2$:

$$\langle \delta r_p^E\rangle_S^2 = \frac{1}{8\pi^2 M_p^2}\int_0^1 dx \frac{(1-x)^2(2(g_S^p)^2 - (f_S^p)^2 + x[(g_S^p)^2 + (f_S^p)^2])}{(1-x)^2 + x(\mu_S^p)^2}, \tag{A5}$$

$$\langle \delta r_p^E\rangle_P^2 = -\frac{1}{8\pi^2 M_p^2}\int_0^1 dx \frac{(1-x)^2((g_P^p)^2 - 2(f_P^p)^2 - x[(g_P^p)^2 + (f_P^p)^2])}{(1-x)^2 + x(\mu_P^p)^2}, \tag{A6}$$

$$\langle \delta r_p^E\rangle_V^2 = \frac{1}{8\pi^2 M_p^2}\int_0^1 dx \frac{(1-x)((g_V^p)^2 + (f_V^p)^2 + x[7(g_V^p)^2 - 6(f_V^p)^2] - 2x^2(g_V^p)^2 - x^3(f_V^p)^2)}{(1-x)^2 + x(\mu_V^p)^2}, \tag{A7}$$

$$\langle \delta r_p^E\rangle_A^2 = -\frac{1}{8\pi^2 M_p^2}\int_0^1 dx \frac{(1-x)(-(g_V^p)^2 - (f_V^p)^2 + x[6(g_A^p)^2 - 7(f_A^p)^2] + 2x^2(f_A^p)^2 + x^3(g_A^p)^2)}{(1-x)^2 + x(\mu_A^p)^2}. \tag{A8}$$

Let us consider two limiting cases: (1) $m_H = m_S = m_P = m_V = m_A \ll M_\psi$, (2) $m_H = m_S = m_P = m_V = m_A \gg M_\psi$, where $\psi = p, \mu$. The total contribution of new particles into a^μ and proton charge radius read:

Scenario (1):

$$\begin{aligned}
\delta a_{\rm tot}^\mu &= \frac{1}{16\pi^2}[g_a^{(1)} - 8h_a^{(1)}\log(\mu_H^\mu)^2],\\
g_a^{(1)} &= 3((g_S^\mu)^2 + (f_P^\mu)^2) - ((g_P^\mu)^2 + (f_S^\mu)^2) + 2((g_V^\mu)^2 + (f_A^\mu)^2) + 18((g_A^\mu)^2 + f_V^\mu)^2),\\
h_a^{(1)} &= (f_V^\mu)^2 + (g_A^\mu)^2,
\end{aligned} \tag{A9}$$

$$\begin{aligned}
\langle \delta r_p^E\rangle_{\rm tot}^2 &= \frac{1}{16\pi^2 M_p^2}[g_r^{(1)} + 6h_r^{(1)}\log(\mu_H^p)^2],\\
g_r^{(1)} &= 5((g_S^p)^2 + (f_P^p)^2) + ((f_S^p)^2 + (g_P^p)^2) - 8((g_V^p)^2 + (f_A^p)^2) + \frac{47}{3}((f_V^p)^2 + (g_A^p)^2),\\
h_r^{(1)} &= (g_V^p)^2 - (g_A^p)^2 + (f_A^p)^2 - (f_V^p)^2.
\end{aligned} \tag{A10}$$

Scenario (2):

$$\begin{aligned}\delta a^{\mu}_{\rm tot} &= \frac{1}{16\pi^2(\mu^{\mu}_H)^2}[g_a^{(2)} + h_a^{(2)}\log(\mu^{\mu}_H)^2],\\ g_a^{(2)} &= -\frac{7}{6}((g^{\mu}_S)^2 + f^{\mu}_P)^2) + \frac{11}{6}((g^{\mu}_P)^2 + (f^{\mu}_S)^2) + \frac{2}{3}((g^{\mu}_V)^2 + (f^{\mu}_A)^2) - \frac{10}{3}((g^{\mu}_A)^2 + (f^{\mu}_V)^2),\\ h_a^{(2)} &= (g^{\mu}_S)^2 - (g^{\mu}_P)^2 - (f^{\mu}_S)^2 + (f^{\mu}_P)^2. \end{aligned} \tag{A11}$$

$$\begin{aligned}\langle\delta r^E_p\rangle^2_{\rm tot} &= \frac{1}{8\pi^2 m_H^2}[g_r^{(2)} + h_r^{(2)}\log(\mu^p_H)^2],\\ g_r^{(2)} &= -\frac{8}{3}((g^p_S)^2 + (f^p_P)^2) + \frac{11}{6}((g^p_P)^2 + (f^p_S)^2) + \frac{13}{6}((g^p_V)^2 + (f^p_A)^2) - \frac{49}{2}((g^p_A)^2 + (f^p_V)^2),\\ h_r^{(2} &= 2((g^p_S)^2 + (f^p_P)^2) - ((g^p_P)^2 + (f^p_S)^2) + ((g^p_V)^2 + (f^p_V)^2 + (g^p_A)^2 + (f^p_A)^2). \end{aligned} \tag{A12}$$

The contributions of individual diagrams in Fig. 3 are given by:

Diagrams $1+2$:

$S(P)$-boson exchange

$$d^E_I = \frac{e g_I f_I}{8\pi^2 M_\psi} I(\mu^\psi_I), \qquad I = S, P. \tag{A13}$$

Diagrams $3+4$:

SV-boson exchange

$$d^E_{SV} = \frac{e g_{\gamma SV} g_V f_S}{16\pi^2 M_p} J(\mu^\psi_S, \mu^\psi_V). \tag{A14}$$

PA-boson exchange

$$d^E_{PA} = \frac{e g_{\gamma PA} g_P f_A}{16\pi^2 M_p} J(\mu^\psi_P, \mu^\psi_A). \tag{A15}$$

Diagrams $5+6$:

PV-boson exchange

$$d^E_{PV} = \frac{e f_{\gamma PV} g_P g_V}{16\pi^2 M_p} J(\mu^\psi_P, \mu^\psi_V). \tag{A16}$$

Here we introduced the structure integrals

$$I(\mu) = \int_0^1 dx \frac{x^2}{x^2 + (1-x)\mu^2} \tag{A17}$$

for diagrams 1,2 and

$$J(\mu,\tau) = \frac{1}{\mu^2 - \tau^2}\int_0^1 dx x^2 \log\frac{x^2 + \mu^2(1-x)}{x^2 + \tau^2(1-x)} \tag{A18}$$

for diagrams 3-6. For equal masses of bosons, i.e., for $\mu = \tau$ the loop integral $J(\mu,\tau)$ is simplified to $J(\mu) = \int_0^1 dx x^2(1-x)/(x^2 + (1-x)\mu^2)$. As before we consider the limits: (1) small fermion masses $\mu, \tau \gg 1$ and (2) small boson masses $\mu, \tau \ll 1$. In first case the structure integrals read:

$$I(\mu) = \log(\mu^2)/\mu^2 \tag{A19}$$

$$J(\mu,\tau) = \frac{1}{3(\mu^2 - \tau^2)}\log\frac{\mu^2}{\tau^2}, \qquad J(\mu) = \frac{1}{3\mu^2}, \tag{A20}$$

For the second case we get:

$$I(\mu) = 1, \qquad J(\mu) = J(\mu,\tau) = \frac{1}{2}. \tag{A21}$$

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