Aspects and applications of nonlocal Lorentz violation

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We consider simple scalar theories with quadratic terms that are nonlocal and Lorentz violating. Unlike similar Lorentz-invariant nonlocal theories that we have considered previously, the theories studied here are both ghost-free and unitary as formulated in Minkowski space. We explore the possibility that the scale of nonlocality could be low in a dark sector, where the stringent constraints on the violation of Lorentz invariance may be accommodated via the weak coupling to the standard model. We point out that long-range forces may originate from such a sector and be distinguishable from more conventional beyond-the-standard-model possibilities. We present a model in which a nonlocal, Lorentz-violating dark sector communicates with the standard model via a sector of heavy vectorlike fermions, a concrete framework in which phenomenological constraints and signals can be investigated.

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I. INTRODUCTION

Quantum field theories involving nonlocal interactions are interesting for a variety of reasons [1,2]. For example, in Ref. [2], we studied a nonlocal, Lorentz-invariant theory of N real scalar fields of mass m with $\mathrm{O}(N)$ symmetry

$$\mathcal{L} = -\frac{1}{2}\phi^a \hat{F}(\Box)^{-1}(\Box + m^2)\phi^a - \frac{1}{8}\lambda_0(\phi^a\phi^a)^2. \quad (1.1)$$

Here $a = 1\ldots N$, λ_0 is the dimensionless quartic coupling and

$$\hat{F}(\Box) = \exp(-\eta\Box^n), \quad (1.2)$$

where our metric signature is $(+,-,-,-)$. The parameter η determines the amount of nonlocality, with the local theory corresponding to $\eta = 0$ and $\hat{F} = 1$. One reason that this theory is of interest is that the propagator

$$\tilde{D}_F(p) = \frac{i\hat{F}(-p^2)}{p^2 - m^2 + i\epsilon} \quad (1.3)$$

leads to more convergent amplitudes than in the local theory with $\hat{F} = 1$; for n even, this is true whether the theory is formulated initially in Minkowski or Euclidean space. While better convergence properties can also be obtained in local theories with higher-derivative quadratic terms, like the Lee-Wick standard model [3], such theories unavoidably come with ghosts; special prescriptions must then be invoked in computing S-matrix elements to maintain the unitarity of the theory [4]. These extra theoretical ingredients are arguably unappealing but can be avoided in the nonlocal theory above if $\hat{F}$ is chosen to be an entire function, as in Eq. (1.2), so that no new poles appear in the propagator, Eq. (1.3). If such an approach could be generalized convincingly to gauge theories, one hope is that a nonlocal generalization of the standard model could be used to address the hierarchy problem without implying new, TeV-scale particles that have yet to be seen at the Large Hadron Collider.

Complications related to unitarity, however, also arise in the ghost-free theory defined by Eqs. (1.1) and (1.2). In Ref. [2], two-into-two scattering was considered to all orders in the quartic coupling in the large-N limit, and it was shown that the theory with $n = 2$ was not unitary if it is defined in Minkowski space. The problem originates from the form of $\hat{F}(-p^2)$, which blows up within certain Stokes wedges in the complex p^0 plane; this leads to new contributions to the imaginary part of the forward scattering amplitude (coming from the contour at infinity) that would not be present after a Wick rotation in the $\hat{F} = 1$ version of the theory. This problem seems to be generic for other choices of $\hat{F}$ that are entire functions. On the other hand, one can define the nonlocal theory initially in Euclidean space and analytically continue scattering amplitudes to Minkowski space at the very end. In this case, the resulting theory was shown to satisfy the optical theorem [2]. However, this formulation may seem as unappealing to

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some as the special prescriptions employed to render Lee-Wick theories consistent.

In this work, we avoid these complications by considering similar nonlocal theories that are not Lorentz invariant.[1] We explore a simple modification of our previous choice for $\hat{F}$, in which the d'Alembertian operator is replaced by the Laplacian:

$$\hat{F}(\nabla) = \exp(\eta\nabla^2). \tag{1.4}$$

The theory defined with this operator is Lorentz violating; there is a preferred frame in which the Lagrangian is invariant under spatial rotations. One possible choice is to assume that this is the frame in which the cosmic microwave background is isotropic [6], though others are imaginable[2]; motion relative to the preferred frame, which introduces a preferred direction, can be separately bounded. In any case, the absence of time derivatives in Eq. (1.4) eliminates the problem with unitarity encountered in the Minkowski-space formulation of the Lorentz-invariant theory defined by Eqs. (1.1) and (1.2); it also ensures that there are no ghosts, as the inverse propagator involves no higher powers of p^0.

If the nonlocality represented by Eq. (1.4) is relevant in nature, one would expect that modification of gauge-invariant quantities in the standard model (for example, $-\frac{1}{4}B_{\mu\nu}\hat{F}(\nabla)^{-1}B^{\mu\nu}$, where $B_{\mu\nu}$ is the hypercharge field strength tensor) would lead to significant lower bounds on the nonlocality scale $\eta^{-1/2}$ due to the stringent experimental constraints on the violation of Lorentz invariance [9,10]. As a consequence, solving the hierarchy problem would not be a motivation for studying such theories. However, there are other motivations for considering why nonlocality may be relevant in nature (for example, in smoothing out gravitational singularities [11]) and the scale of the nonlocality need not be the same for every particle. One interesting possibility is the application of Lorentz-violating nonlocality to gravitation, a nonlocal generalization of the Horava-Lifshitz idea [12], motivated by the desire to obtain a renormalizable quantum theory of gravity. Another interesting possibility is that the nonlocality scale associated with a dark sector may be much lower than the Planck scale, with the bounds on Lorentz violation accommodated by a very weak coupling of the dark sector to the standard model.[3] As a first step in model building, we study a nonlocal, Lorentz-violating dark sector later in this paper and defer the consideration of gravity to future work. With the extremely small couplings required, an interesting phenomenological possibility is that dark-sector particles may mediate long-range forces. In this case, effects might be discerned relative to the effects of gravity and lead to corrections to the gravitational potential that differ qualitatively from other possibilities that have been considered previously [13].

Our paper is organized as follows. In Sec. II, we revisit the analysis of unitarity that was discussed in Ref. [2] and show how it is modified, in a favorable way, for the choice of higher-derivative terms given by Eq. (1.4). In Sec. III, we consider the nonrelativistic potential for a single scalar field with the same nonlocal Lagrangian and show how it differs qualitatively from that of the corresponding local theory. While this calculation is based on the assumption that the scalar has generic Yukawa couplings to generic fermions, in Sec. IV we present a scenario that provides an origin for the weak couplings to standard model fermion fields by connecting the dark and visible sectors via a renormalizable and gauge-invariant "portal" sector of heavy, vectorlike fermions. With an explicit scenario defined, we consider the implications of searches for long-range forces and for the violation of Lorentz invariance on the mass scale and couplings associated with the vectorlike sector, assuming that the scale of nonlocality is comparable to the mass scale of the particle mediating the long-range force. In Sec. V, we summarize our conclusions.

II. UNITARITY IN A TOY MODEL

In Ref. [2], unitarity in a two-into-two scattering process was considered in the model defined by Eqs. (1.1) and (1.2). The calculation was done in the large-N limit, where the result at leading order in $1/N$ could be conveniently resummed to all orders in perturbation theory. In this section, we revisit that calculation and show how it is modified with the form of $\hat{F}$ given in Eq. (1.4).

We consider the two-into-two scattering amplitude $\mathcal{M}(ab \to cd)$ for the diagrammatically simplest case in which $a = b \neq c = d$, where field labels a through d range from $1\ldots N$. The scattering amplitude is given by

$$\mathcal{M} = -\frac{\lambda}{N}\frac{e^{-\frac{1}{2}\eta(|\vec{k}_1|^2+|\vec{k}_2|^2+|\vec{k}_1'|^2+|\vec{k}_2'|^2)}}{1+\lambda\Sigma(s)}\delta_{ab}\delta_{cd}, \tag{2.1}$$

where $\lambda_0 \equiv \lambda/N$ to make the large-N scaling of the amplitude explicit. We indicate the momenta of the incoming scalar bosons by k and the outgoing ones by k'. This amplitude resums all orders in λ at leading order in $1/N$. Equation (2.1) should be compared to Eq. (2.8) in Ref. [2]; the same sign convention for self-energy function Σ is used. The differing numerator corresponds to the

[1]For a very different approach to nonlocal Lorentz violation, see Ref. [5].

[2]Other assumptions for a preferred frame that have appeared in the literature include ones at rest with respect to our Galaxy and that locally comove with the rotation of the Galaxy [7], or comove with the Barycentric Celestial Reference System [8].

[3]We are not imagining that the nonlocality would be unique to the dark sector, only that its effects may be more accessible there since this is the sector of the theory where the constraints are weakest.

differing wave function renormalization factors on each of the four external lines. The function Σ in the present case is given by

$$\Sigma=-\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\frac{\exp\{-\eta(\vec{k}+\vec{p}/2)^2\}\exp\{-\eta(\vec{k}-\vec{p}/2)^2\}}{[(k-p/2)^2-m^2+i\epsilon][(k+p/2)^2-m^2+i\epsilon]}, \tag{2.2}$$

where $p\equiv k+k'$.

For a single scalar field, the optical theorem relates the total scattering cross section to the imaginary part of the forward scattering amplitude, where $k_i'=k_i$. In the present case where there are N fields with $a=b\neq c=d$, the optical theorem requires

$$\begin{aligned}&2\mathrm{Im}\mathcal{M}(k_1,a;k_2,a\to k_1,c;k_2,c)\\&=\frac{1}{2}\sum_f\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{1}{2E_1}\frac{1}{2E_2}(2\pi)^4\delta^{(4)}(q_1+q_2-k_1-k_2)\\&\times\mathcal{M}(k_1,a;k_2,a\to q_1,f;q_2,f)\\&\times\mathcal{M}^*(k_1,c;k_2,c\to q_1,f;q_2,f).\end{aligned} \tag{2.3}$$

We now prove the equality of the left- and right-hand sides of Eq. (2.3). The imaginary part of the forward scattering amplitude is proportional to the imaginary part of Σ:

$$2\mathrm{Im}\mathcal{M}(k_1;k_2\to k_1;k_2)=\frac{2\lambda^2}{N}\mathrm{Im}\Sigma\frac{\exp\{-\eta(|\vec{k}_1|^2+|\vec{k}_2|^2)\}}{|1+\lambda\Sigma|^2}. \tag{2.4}$$

We evaluate the k^0 integral in Eq. (2.2) by closing a contour in the lower half of the complex k^0 plane; from the $i\epsilon$ prescription, this encloses poles at $k^0=E_{\vec{k}-\vec{p}/2}+p^0/2$ and $E_{\vec{k}+\vec{p}/2}-p^0/2$, where $E^2_{\vec{q}}\equiv\vec{q}^2+m^2$. In textbook treatments of the optical theorem, one generally works in a frame where $\vec{p}=0$. In the present case, such a boost away from the preferred frame would also change the form of the Lagrangian (which is not Lorentz invariant), reintroducing k^0 dependence into the numerator of Eq. (2.2); this would not be desirable for the reasons related to Wick rotation described earlier. Hence, we work with Eq. (2.2) in the preferred frame and will comment later on how one could have approached the problem starting in a different frame. Using the residue theorem, one obtains

$$\begin{aligned}\Sigma=&-\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\frac{\exp\{-\eta(\vec{k}+\vec{p}/2)^2\}\exp\{-\eta(\vec{k}-\vec{p}/2)^2\}}{E_{\vec{k}-\vec{p}/2}-E_{\vec{k}+\vec{p}/2}+p^0}\\&\times\left[\frac{1}{2E_{\vec{k}-\vec{p}/2}(E_{\vec{k}-\vec{p}/2}+E_{\vec{k}+\vec{p}/2}+p^0)}-\frac{1}{2E_{\vec{k}+\vec{p}/2}(E_{\vec{k}-\vec{p}/2}+E_{\vec{k}+\vec{p}/2}-p^0)}\right].\end{aligned} \tag{2.5}$$

The imaginary part of Σ is related to the branch cut singularity originating from the second term in brackets.[4] The discontinuity from crossing this singularity in the complex p^0 plane is related to the imaginary part by $\mathrm{Disc}\Sigma=2i\mathrm{Im}\Sigma$. Moreover, we may use the identity

$$\mathrm{Disc}\frac{1}{p^0-(E_{\vec{k}-\vec{p}/2}+E_{\vec{k}+\vec{p}/2})}=-2i\pi\delta(p^0-E_{\vec{k}-\vec{p}/2}-E_{\vec{k}+\vec{p}/2}). \tag{2.6}$$

This allows us to write

$$\mathrm{Im}\Sigma=-\frac{\pi}{2}\int\frac{d^3k}{(2\pi)^3}\frac{\exp\{-\eta(\vec{k}+\vec{p}/2)^2\}\exp\{-\eta(\vec{k}-\vec{p}/2)^2\}}{(2E_{\vec{k}+\vec{p}/2})(E_{\vec{k}+\vec{p}/2}-E_{\vec{k}-\vec{p}/2}-p^0)}\delta(p^0-E_{\vec{k}-\vec{p}/2}-E_{\vec{k}+\vec{p}/2}), \tag{2.7}$$

or in the more suggestive form

$$\mathrm{Im}\Sigma=\frac{1}{4}\int\frac{d^3k}{(2\pi)^3}\frac{\exp\{-\eta(\vec{k}+\vec{p}/2)^2\}\exp\{-\eta(\vec{k}-\vec{p}/2)^2\}}{(2E_{\vec{k}+\vec{p}/2})(2E_{\vec{k}-\vec{p}/2})}(2\pi)\delta(p^0-E_{\vec{k}-\vec{p}/2}-E_{\vec{k}+\vec{p}/2}). \tag{2.8}$$

It follows from Eq. (2.4) that the left-hand-side (lhs) of the optical theorem can be written

[4]Note that the first term in brackets and the integrand prefactor have no singularities. In the latter case, this can be seen by noting that as a function of $|\vec{k}|$, the quantity $E_{\vec{k}+\vec{p}/2}-E_{\vec{k}-\vec{p}/2}$ is no larger than $|\vec{p}|$, which is always less that p^0 when expressed in terms of the on-shell external momenta, $p=k_1+k_2$.

$$\begin{aligned}\mathrm{lhs} &= 2\mathrm{Im}\mathcal{M}(k_1;k_2\to k_1;k_2)\\ &= \frac{\lambda^2}{2N}\frac{1}{|1+\lambda\Sigma|^2}e^{-\eta(|\vec{k}_1|^2+|\vec{k}_2|^2)}\int\frac{d^3k}{(2\pi)^3}\frac{1}{2E_{\vec{k}+\vec{p}/2}}\frac{1}{2E_{\vec{k}-\vec{p}/2}}(2\pi)\delta(p^0-E_{\vec{k}-\vec{p}/2}-E_{\vec{k}+\vec{p}/2})e^{-\eta(\vec{k}+\vec{p}/2)^2}e^{-\eta(\vec{k}-\vec{p}/2)^2}. \end{aligned}\tag{2.9}$$

To evaluate the right-hand-side (rhs) of the optical theorem, Eq. (2.3), we write $p\equiv k_1+k_2$ and note that Σ is a function of p and can be pulled outside the integrals. Hence,

$$\mathrm{rhs}=\frac{\lambda^2}{2N}\frac{1}{|1+\lambda\Sigma|^2}e^{-\eta(|\vec{k}_1|^2+|\vec{k}_2|^2)}\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{1}{2E_1}\frac{1}{2E_2}(2\pi)^4\delta^{(4)}(q_1+q_2-p)e^{-\eta(|\vec{q}_1|^2+|\vec{q}_2|^2)}. \tag{2.10}$$

Notice that the prefactors multiplying the integrals in Eqs. (2.9) and (2.10) coincide. Hence, we focus on the integral in Eq. (2.10). First, we may do the d^3q_2 integral using the three-dimensional delta function. Since the q_0^i are on shell, this makes the remaining delta function a function of $q_1^0\equiv E_1=E_{\vec{q}_1}$ and $q_2^0\equiv E_2=E_{\vec{p}-\vec{q}_1}$. Next, we shift the remaining integration variables, $\vec{q}_1\to\vec{q}_1+\vec{p}/2$, so that the rhs integral becomes

$$\int\frac{d^3q_1}{(2\pi)^3}\frac{1}{2E_{\vec{q}_1+\vec{p}/2}}\frac{1}{2E_{\vec{q}_1-\vec{p}/2}}(2\pi)\delta(E_{\vec{q}_1+\vec{p}/2}+E_{\vec{q}_1-\vec{p}/2}-p^0)e^{-\eta|\vec{q}_1+\vec{p}/2|^2}e^{-\eta|\vec{q}_1-\vec{p}/2|^2}. \tag{2.11}$$

With the relabeling $q_1\to k$, both the prefactors and integrals on the lhs and rhs agree, showing that the optical theorem is satisfied.

It is worth noting that agreement between the lhs and rhs of the optical theorem would not have been spoiled had we worked in a frame where the ∇^2 of Eq. (1.4) were replaced by the more general form $N_{\mu\nu}\partial^\mu\partial^\nu$, with $N_{\mu\nu}=\delta_{ij}\Lambda^i{}_\mu\Lambda^j{}_\nu$, where Λ is an appropriate Lorentz transformation matrix that connects the preferred frame to a given one. While the form of the Lagrangian in the nonpreferred frame will change the exponential factors that appear at the starting points of the previous lhs and rhs derivations, one would, in the very next step, use the Lorentz invariance of the remaining factors in the integrands (and integration measures) to change variables so that the exponential factors again depend only on $\vec{k}$. If one were to express the external momenta in terms of their values in the preferred frame, then the calculation would be identical to the one just presented.

III. NONRELATIVISTIC POTENTIAL FOR LONG-RANGE FORCES

The nonlocality defined by Eq. (1.4) violates Lorentz invariance, a possibility that is tightly bounded by experiment [9]. As we indicated earlier, such a nonlocal modification of the standard model would lead to a high nonlocality scale; however, the nonlocality in a dark sector that is adequately sequestered from the standard model could come at a lower scale due to the small coupling between the two sectors. We explain in Sec. IV how we can induce such small couplings between a Lorentz-violating, nonlocal dark sector and standard model fermions. In this section, we will assume that such an effective coupling g exists, in the form of a Yukawa interaction between a single dark-sector scalar field ϕ and a generic fermion ψ:

$$\mathcal{L}_{\mathrm{int}}=-g\bar{\psi}\psi\phi. \tag{3.1}$$

We will show in Sec. IV that the bounds on Lorentz violation force g to be extremely small, far too small to look for effects in any existing collider experiments. However, such small couplings, like that of gravity, can have observable effects when macroscopic quantities of matter are involved, and the effect of the scalar is suitably long ranged. Hence, in this section, we consider such a nonlocal long-range force. While the exponential factor in the ϕ Lagrangian regulates potential for the long-range force in the ultraviolet, that ultraviolet scale does not necessarily have to be very high if the coupling to standard model particles is weak. This can lead to changes in the shape of the potential at length scales where differences might be discernible in comparison to more conventional possibilities.

The nonrelativistic potential can be computed in a quantum field theory via an expression proportional to the Fourier transform of the propagator of the force-carrying particle in the nonrelativistic limit. For example, in ordinary Yukawa theory

$$\begin{aligned}V(\vec{x}) &= \int\frac{d^3q}{(2\pi)^3}\frac{-g^2}{|\vec{q}|^2+m^2}e^{i\vec{q}\cdot\vec{x}}\\ &= -\frac{g^2}{4\pi^2 ir}\int_{-\infty}^{\infty}d|\vec{q}||\vec{q}|\frac{1}{|\vec{q}|^2+m^2}e^{i|\vec{q}|r}.\end{aligned}\tag{3.2}$$

The standard approach is to evaluate the $|\vec{q}|$ integral via the residue theorem using a closed contour in the complex plane that encloses a pole at $|\vec{q}| = im$, taking into account that the circular contour at infinity in the upper half plane vanishes. However, in the present scenario, this latter integral is modified,

$$V(\vec{x}) = -\frac{g^2}{4\pi^2 ir}\int_{-\infty}^{\infty} d|\vec{q}||\vec{q}|\frac{e^{-\eta|\vec{q}|^2}}{|\vec{q}|^2+m^2}e^{i|\vec{q}|r}, \tag{3.3}$$

and the contour at infinity does not vanish everywhere due to the exponential factor. Hence, we must use a different approach. We first exponentiate the denominator using a Schwinger parameter u,

$$V(\vec{x}) = -\frac{g^2}{4\pi^2 ir}\int_0^{\infty} du e^{-um^2} \times \int_{-\infty}^{\infty} d|\vec{q}||\vec{q}| \exp\{-(\eta+u)|\vec{q}|^2 + i|\vec{q}|r\}. \tag{3.4}$$

The integral in $|\vec{q}|$ is of a recognizable form and can be done analytically:

$$V(r) = -\frac{g^2}{8\pi^{3/2}}\int_0^{\infty} du \frac{1}{(\eta+u)^{3/2}}\exp\left\{-um^2 - \frac{r^2}{4(\eta+u)}\right\}. \tag{3.5}$$

The integral in Eq. (3.5) is probably not in a recognizable form for most, but nonetheless it can also be done analytically. The result is

$$V(r) = -\frac{g^2}{4\pi}\frac{1}{r}e^{\eta m^2}\left\{-\sinh mr + \frac{1}{2}\left(e^{mr}\mathrm{Erf}\left[\frac{r+2m\eta}{2\sqrt{\eta}}\right] + e^{-mr}\mathrm{Erf}\left[\frac{r-2m\eta}{2\sqrt{\eta}}\right]\right)\right\}. \tag{3.6}$$

Note that for any finite r, the error functions become unity as $\eta \to 0$ so that the quantity in curly brackets becomes $\cosh(mr) - \sinh(mr) = \exp(-mr)$, which yields

$$\lim_{\eta\to 0} V(r) \equiv V_0(r) = -\frac{g^2}{4\pi}\frac{1}{r}e^{-mr}, \tag{3.7}$$

the usual result for a Yukawa potential. The shape of the potential for various choices of η is shown in Fig. 1. This figure illustrates two qualitative features: (1) The presence of the exponential eliminates the singularity at the origin; the potential is regular at that point, approaching a constant up to corrections of $\mathcal{O}(r^2)$. (2) The "smoothing out" of the potential due to the nonlocality increases its depth for $r \gtrsim m$ compared to the $\eta = 0$ case. Note that the quantity in curly brackets in Eq. (3.6) approaches $\exp(-mr)$ in the limit $\eta^{-1/2} r \to \infty$ for finite η. In this case, the potential has the Yukawa form, with an extra multiplicative factor of $\exp(\eta m^2)$. Of course, if measurement of the coupling g happens only via this potential, then this factor could be absorbed in a redefinition of the coupling; however, if g is measured in another process, then this difference in normalization might also be discernible.

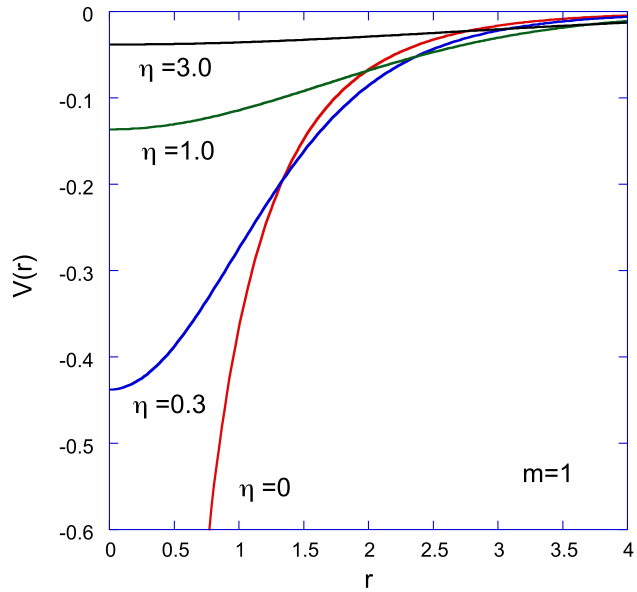

FIG. 1. The form of the nonrelativistic potential for various η, with m set equal to 1.

Finally, we comment on the effects of motion relative to the preferred frame, defined by a velocity vector $\vec{v}$, which introduces a preferred direction. The effect on our previous calculation is to take the exponentiated factor $\eta\delta_{ij}q^i q^j$ and replace it with

$$\eta[\delta_{ij} + \gamma^2 v_i v_j]q^i q^j, \tag{3.8}$$

where γ is the usual relativistic factor $(1-v^2)^{-1/2}$. Here we first have performed a Lorentz boost in the $\vec{v}$ direction and have applied the usual nonrelativistic approximation (in the new frame) in which the t-channel momentum transfer has $q^0 = 0$ up to negligible corrections. Since we have no knowledge *a priori* of the vector $\vec{v}$, phenomenological constraints on new terms in the potential generated by this boost can be interpreted as providing upper bounds on its magnitude $|\vec{v}|$. However, as we alluded to earlier, if we were to assume that the preferred frame corresponds to one in which the cosmic microwave background is isotropic, then observations would tell us that $|\vec{v}| \approx 0.0012$ in units where $c = 1$, corresponding to the measured value 369.82 ± 0.11 km/s [14]. For small velocities like this, it is reasonable to calculate the effect on $V(r)$ given in Eq. (3.6) by expanding to quadratic order in v. From our new starting point,

$$V(\vec{x},\vec{v}) = -g^2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{-\eta|\vec{q}|^2-\eta\gamma^2(v\cdot q)^2}}{|\vec{q}|^2+m^2} e^{i\vec{q}\cdot\vec{x}}, \quad (3.9)$$

we may write

$$V(\vec{x},\vec{v}) = \exp\,[\eta\gamma^2 v^i v^j \partial_i\partial_j] V(\vec{x},\vec{0}), \quad (3.10)$$

which is useful only in that we are expanding the differential operator to second order in $\vec{v}$:

$$V(\vec{x},\vec{v}) = [1+\eta v^i v^j \partial_i\partial_j] V(\vec{x},\vec{0}) + \mathcal{O}(v^4). \quad (3.11)$$

This allows us to find the desired $\vec{v}$ dependence using what we have already found in Eq. (3.6). Since $V(\vec{x},\vec{0}) \equiv V(r)$ depends only on r, we may rewrite Eq. (3.11) in terms of derivatives with respect to the radial coordinate:

$$V(r,\vec{v}) = V(r) + \eta|\vec{v}|^2 \frac{1}{r}\frac{dV(r)}{dr} - \eta\frac{(\vec{v}\cdot\vec{x})^2}{r^2}\left(\frac{1}{r}\frac{dV(r)}{dr} - \frac{d^2V(r)}{dr^2}\right) + \mathcal{O}(v^4). \quad (3.12)$$

The first correction term, reading from left to right above, is spherically symmetric and nonsingular at the origin; it simply represents a small correction to the radial potential that we have already discussed, suppressed by at least a factor of $|\vec{v}|^2 \sim 10^{-6}$ if the cosmic microwave background defines the preferred frame. The second correction term is qualitatively different since it is sensitive to the preferred direction. Possible corrections to the gravitational potential proportional to $(\vec{v}\cdot\vec{x})^2/r^3$ (which has a different radial dependence) must be suppressed below gravitational strength by a factor $\alpha_2 = 2\times 10^{-9}$, where α_2 is defined in the parametrized post-Newtonian formalism [15]. However, this bound, which is determined from the precession of pulsar rotation axes, does not apply here since it assumes a force with infinite range. We will assume henceforth that the range of our new force is substantially less than 10^4 m ($m \gg 10^{-11}$ eV), the size of a typical neutron star, so that an analogous bound is evaded. Whether interesting astrophysical bounds on Lorentz-violating forces with finite range can be determined is worthy of investigation but will not be considered in this work.

IV. NONLOCAL LORENTZ VIOLATION IN A DARK SECTOR

In this section, we consider how a single real scalar field, like the one discussed in the previous section, might couple to matter fields of the standard model in a realistic scenario. We do not identify the scalar field as dark matter but assume that it could decay into other dark-sector particles that are stable or suitably long lived. The portal to the standard model will consist of a sector of heavy vectorlike fields. Let us first discuss the portal in the case of a local theory and then explain how we introduce the nonlocality into the theory in a way that will keep the Lorentz violation suitably sequestered.

Consider a heavy, vectorlike field $\mathcal{D}$ with the same quantum numbers as a right-handed down quark d_R:

$$\mathcal{D}_R \sim \mathcal{D}_L \sim d_R. \quad (4.1)$$

The mass terms and Yukawa couplings that involve these fields are the following:

$$\mathcal{L} = -M\bar{\mathcal{D}}_L\mathcal{D}_R - \bar{Q}_L H d_R - \bar{Q}_L H\mathcal{D}_R - \phi\overline{\mathcal{D}_L}\mathcal{D}_R - \phi\overline{\mathcal{D}_L} d_R + \text{H.c.} \quad (4.2)$$

Here we have suppressed for simplicity the dimensionless couplings and considered standard model quarks Q_L and d_R of a single generation. Note also that a mixing term of the form $\Delta m\overline{\mathcal{D}_L} d_R$ has been eliminated by a definition of the $\mathcal{D}_R - d_R$ field basis. When the heavy $\mathcal{D}$ fields are integrated out of the theory, higher-dimension operators will be generated in the low-energy effective theory. In the lingo of Froggatt-Nielsen model building [16], one operator of interest is generated via the "spaghetti" diagram shown in Fig. 2. The amplitude for this diagram in momentum space is

$$i\mathcal{M} = \bar{u}_Q\left[(iP_R)\frac{i(\not{p}+M)}{p^2-M^2}(iP_R)\right]u_d \to i\frac{1}{M}\bar{u}_Q P_R u_d, \quad (4.3)$$

where p is the momentum on the internal line, $P_R = (1+\gamma^5)/2$. On the far right of Eq. (4.3) we show the limit in which $p^2 \ll M^2$. In the low-energy effective theory, this amplitude is reproduced by the higher-dimension operator

$$\mathcal{L}_{\text{eff}} = \frac{\kappa_0}{M}\phi\overline{Q_L} H d_R, \quad (4.4)$$

where κ_0 subsumes the product of all the undetermined Yukawa couplings relevant to the diagram of Fig. 2. This leads to a Yukawa interaction of the same form that we assumed in Sec. III with coupling $\kappa_0 v/(\sqrt{2}M)$, where $v =$ 246 GeV is the Higgs vacuum expectation value (VEV).

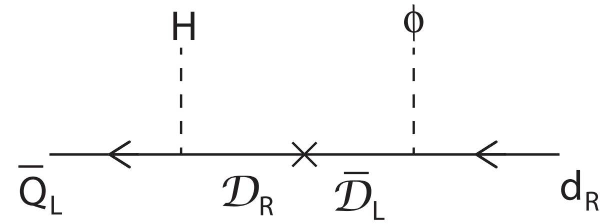

FIG. 2. Diagram involving the exchange of the vectorlike quark $\mathcal{D}$.

We now introduce the desired nonlocality by modifying only the two terms in Eq. (4.2) that depend on ϕ:

$$-\overline{\mathcal{D}_L}\mathcal{D}_R e^{\eta\nabla^2/2}\phi - \overline{\mathcal{D}_L}d_R e^{\eta\nabla^2/2}\phi + \text{H.c.} \tag{4.5}$$

Notice that a field redefinition $\phi = e^{-\eta\nabla^2/2}\varphi$ would move this nonlocal factor back to the quadratic terms of φ, as in the toy models we considered earlier, as well as to other possible interaction terms. The value of introducing the nonlocality initially in Eq. (4.5) is that it makes clear that the Lorentz violation involves the superheavy field $\mathcal{D}$; after integrating out the heavy sector, Lorentz-violating effects in the low-energy effective theory will always be suppressed by a ratio of widely separated mass scales. In other words, our assumption of adequate sequestering dictates where we introduce the factors of $\hat{F}$ in the Lagrangian. Had we instead introduced $\hat{F}^{-1}$ initially in the quadratic terms and left Eq. (4.2) unchanged, then one might have Higgs sector Lorentz violation suppressed only by loops involving the possible renormalizable couplings between ϕ and H. One might formulate the theory in that way if there is a reason to expect those Higgs portal couplings to be absent, for example, if ϕ and H are separated in an extra dimension, while $\mathcal{D}$ is a bulk field. Such an extra-dimensional formulation may be desirable since it would also eliminate Planck-suppressed higher-dimension operators that couple the ϕ field directly to standard model fields, for example, $\phi F_{\mu\nu}F^{\mu\nu}/M_P$, which leads to additional constraints [17]. For the purpose of our discussion, we assume that such operators, if present, are adequately suppressed.

The construction just described can be extended by introducing another heavy, vectorlike field $\mathcal{U}$ with the same quantum numbers as a right-handed up quark. We would then generate the higher-dimension operators

$$\mathcal{L}_{\text{eff}} = \frac{\kappa_0^d}{M} e^{\eta\nabla^2/2}\phi\overline{Q_L}Hd_R + \frac{\kappa_0^u}{M} e^{\eta\nabla^2/2}\phi\overline{Q_L}\,\tilde{H}\,u_R, \tag{4.6}$$

and, after setting H to its VEV, the induced Yukawa couplings

$$\mathcal{L}_{yuk} = \sum_{f=u,d} \frac{\kappa_f m_f}{M}\bar{f}f e^{\eta\nabla^2/2}\phi. \tag{4.7}$$

Here $\kappa_f \equiv \kappa_0^f/\lambda_f$, where λ_f is the standard model Yukawa coupling $\sqrt{2}m_f/v$. Thus, we have defined κ_f to be unity if it is of the same size as the dimensionless coupling we would associate with either $\bar{Q}_L H d_R$ or $\bar{Q}_L\tilde{H}u_R$, operators with a similar flavor structure. This provides a convenient point of reference.

The location of the exponential factor in Eq. (4.7), which appears when ϕ has canonical kinetic terms, yields the same nonrelativistic potential as the one considered in Sec. III. With Eq. (4.7) at hand, another useful effective interaction to consider is the coupling of ϕ to nucleons,

$$\mathcal{L}_{\text{eff}} = f_p\bar{p}pe^{\eta\nabla^2/2}\phi + f_n\bar{n}ne^{\eta\nabla^2/2}\phi. \tag{4.8}$$

The mapping from Eq. (4.7) to Eq. (4.8) is the same as that found in studies of scalar dark matter. From Ref. [18],

$$\frac{f_N}{m_N} = \sum_{u,d,s} f_{T_q}^{(N)}\frac{\alpha_q}{m_q} + \frac{2}{27}f_{\text{TG}}^{(N)}\sum_{c,b,t}\frac{\alpha_q}{m_q}, \tag{4.9}$$

where the scalar-quark couplings in this case are given by

$$\alpha_q = \begin{cases} \frac{\kappa_q m_q}{M} & \text{for } q = u, d, \\ 0 & \text{otherwise.} \end{cases} \tag{4.10}$$

Note that Eq. (4.10) reflects the fact that the simple model we have presented provides only for couplings to the lightest two quark flavors; however, it is straightforward to extend the vectorlike sector so that couplings of ϕ to heavier flavors are induced as well. Numerical values of $f_{T_q}^{(N)}$ and $f_{\text{TG}}^{(N)}$, for $N = p$ or n, can be found in Ref. [18]. For the purpose of an estimate, we will further assume that $\kappa_u = \kappa_d \equiv \kappa$. We find that

$$\mathcal{L}_{\text{eff}} = \mathcal{A}_p\kappa\frac{m_p}{M}[\bar{p}pe^{\eta\nabla^2/2}\phi] + \mathcal{A}_n\kappa\frac{m_n}{M}[\bar{n}ne^{\eta\nabla^2/2}\phi], \tag{4.11}$$

where $\mathcal{A}_p = 0.046$ and $\mathcal{A}_n = 0.050$. We then infer that the potential due to ϕ exchange between two atoms with atomic number Z and atomic mass A is given by Eq. (3.6) with the replacement

$$\frac{g^2}{4\pi} \to \frac{\kappa^2}{4\pi M^2}[Z(\mathcal{A}_p m_p - \mathcal{A}_n m_n) + \mathcal{A}_n m_n A]^2. \tag{4.12}$$

Note that the factor $\mathcal{A}_p m_p - \mathcal{A}_n m_n$ would vanish in the absence of isospin breaking effects and is suppressed relative to the second term in brackets. For example, for iron, where $Z = 26$ and $A = 56$, the first term represents a 3.8% effect. For the purposes of an estimate, we ignore isospin differences so that the potential between two atoms is given by Sec. III as follows:

$$V(r) = -\frac{\kappa^2\mathcal{A}_N^2}{4\pi M^2}\frac{M_a^2}{r}e^{\eta m^2}\left\{-\sinh mr + \frac{1}{2}\left(e^{mr}\text{Erf}\left[\frac{r+2m\eta}{2\sqrt{\eta}}\right] + e^{-mr}\text{Erf}\left[\frac{r-2m\eta}{2\sqrt{\eta}}\right]\right)\right\}, \tag{4.13}$$

where M_a is the mass of each atom and $\mathcal{A}_N \approx 0.05$. As we discussed earlier, this potential becomes Yukawa-like asymptotically, so we can obtain an estimate of the typical bounds on κ using the results in Ref. [19], which apply to a Yukawa-like force. The scale suppression $1/\Lambda$ in this reference can be matched to $\mathcal{A}_N\kappa/M$ in Eq. (4.11). If we take, for example, $M = 0.1M_P$, we find that $\kappa < 0.02$ for a force with a range below 10^4 m (or $m > 10^{-11}$ eV). (This would become $\kappa < 0.01$ if one corrects κ^2 by the nonlocal factor $e^{\eta m^2}$ for $\eta m^2 = 1$.) If we define a parameter ξ that compares the coefficient of Eq. (4.13) to gravitational strength, i.e., $\kappa^2\mathcal{A}_N^2 e^{\eta m^2}/(4\pi M^2) = \xi/M_P^2$, where $M_P = 1.2\times10^{19}$ GeV is the Planck mass, then for this choice of parameters, with $\kappa < 0.01$, one finds that $\xi < 5\times10^{-6}$. This is consistent with the statement in Ref. [15] that bounds range from 10^{-3} to 10^{-6}, the strength of gravity for ranges between 1 and 10^4 m. In any case, we will assume that the upper bounds on κ are satisfied so that our theory remains consistent with fifth force searches. We note that astrophysical bounds on very light scalars are superseded by fifth force bounds for scalar masses below 0.2 eV [20] and will not provide additional constraints.

Separate bounds come from the fact that the theory is Lorentz violating. For example, the interaction in Eq. (4.7) provides a Lorentz-violating contribution to the self-energy function for the fermion $f = u$ or d, which leads to a Lorentz-violating dispersion relation. We can use this to compute the difference between the speed of a massless fermion and the speed of light, a quantity often used to constrain theories with Lorentz violation that is isotropic [21]. From Eq. (4.7), the self-energy (following the conventions of Peskin and Schroeder [22]) is given by

$$-i\Sigma = g_f^2\int_0^1 dx\int\frac{d^4q}{(2\pi)^4}\frac{e^{-\eta(\vec{q}-x\vec{p})^2}[\not{q}+(1-x)\not{p}+m_f]}{[q^2-\Delta]^2}, \tag{4.14}$$

where $g_f = \kappa_f m_f/M$ and

$$\Delta = -x(1-x)p^2 + (1-x)m^2 + xm_f^2. \tag{4.15}$$

We show that we can obtain a useful bound by studying the limit in which the dimensionless quantity $\eta^{1/2}\vec{p}$ is small, and by looking at the corrections to the fermion dispersion relation that are obtained at first order in this quantity. In the Appendix, we consider the more general case and confirm that the final result of this section can be obtained without using an expansion. Expanding the integral in Eq. (4.14), the self-energy function takes the form

$$\Sigma = -\mathcal{A}\not{p} + \mathcal{B}m_f + \mathcal{C}\vec{p}\cdot\vec{\gamma} + \cdots, \tag{4.16}$$

where the ellipsis refers to terms suppressed by an additional power of $\eta^{1/2}\vec{p}$, and where $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are the following dimensionless, Euclidean integrals:

$$\mathcal{A} = g_f^2\int_0^1 dx(1-x)\int\frac{d^4q_E}{(2\pi)^4}\frac{e^{-\eta|\vec{q}|^2}}{(q_E^2+\Delta)^2}, \tag{4.17}$$

$$\mathcal{B} = -g_f^2\int_0^1 dx\int\frac{d^4q_E}{(2\pi)^4}\frac{e^{-\eta|\vec{q}|^2}}{(q_E^2+\Delta)^2}, \tag{4.18}$$

$$\mathcal{C} = \frac{2}{3}g_f^2\int_0^1 dxx\int\frac{d^4q_E}{(2\pi)^4}\frac{\eta|\vec{q}|^2e^{-\eta|\vec{q}|^2}}{(q_E^2+\Delta)^2}, \tag{4.19}$$

with $q_E^2 = (q^0)^2 + |\vec{q}|^2$. The condition for an on-shell fermion

$$\not{p} - m_f - \Sigma(\not{p}) = 0 \tag{4.20}$$

can thus be written as

$$(1+\mathcal{A})\not{p} - (1+\mathcal{B})m_f - \mathcal{C}\vec{p}\cdot\vec{\gamma} = 0. \tag{4.21}$$

Multiplying both sides of this expression by the quantity that is the same as the left-hand side with the sign of the second term flipped allows us to eliminate the gamma matrix structure,

$$(1+2\mathcal{A})p^2 - (1+2\mathcal{B})m_f^2 - 2\mathcal{C}|\vec{p}|^2 = 0, \tag{4.22}$$

where we have dropped negligible terms that are of order g_f^4. We may solve for p^0 perturbatively in g_f^2 to obtain the dispersion relation

$$(p^0)^2 = (1+2\tilde{\mathcal{C}})|\vec{p}|^2 + (1-2\tilde{\mathcal{A}}+2\tilde{\mathcal{B}})m_f^2, \tag{4.23}$$

where the tilde indicates our previous expressions with the function $\Delta(p^2)$ evaluated at $p^2 = m_f^2$. In the limit in which the fermion is massless, its speed c_0 can be read off the first term

$$c_0 = 1 + \tilde{\mathcal{C}}, \tag{4.24}$$

again working to order g_f^2, and where we have set the speed of photons $c = 1$. The quantity $|c - c_0|$ is experimentally bounded [23] such that

$$\tilde{\mathcal{C}} < 3\times10^{-22}, \tag{4.25}$$

where we have used the fact that $\tilde{\mathcal{C}} > 0$. More explicitly, this can be written

$$\begin{aligned}&\frac{g_f^2}{12\pi^2}\left[\int_0^1 dxx\int_0^\infty dy\frac{y^4e^{-y^2}}{[y^2+(1-x)\rho]^{3/2}}\right]\\ &\quad=\frac{g_f^2}{12\pi^2}\left[\frac{1}{2\rho^2}\left\{8+2\rho-3\sqrt{\pi}U\left(-\frac{1}{2},-2,\rho\right)\right\}\right]\\ &\quad<3\times10^{-22},\end{aligned} \tag{4.26}$$

where $\rho \equiv \eta m^2$ and $U(a,b,z)$ is the confluent hypergeometric function [24]. The integral on the first line of

Eq. (4.26) can be obtained from Eq. (4.19) by performing the q^0 and $\vec{q}$ angular integrations so that the Feynman parameter integral and a radial $|\vec{q}|$ integral remain. The quantity in square brackets never exceeds $1/4$ for any non-negative ρ; from this, we obtain the bound $g_f = \kappa m_f/M < 3.8 \times 10^{-10}$. Had we done this calculation using the effective interaction for the proton that we derived earlier, κm_f would be replaced by $\kappa \mathcal{A}_N m_p$. In either case, the ratio of mass scales (for example, $1\ \mathrm{GeV}/[0.1 M_P] \sim 10^{-18}$) by itself assures that the bound is satisfied and is superseded by the bounds that we discussed earlier on long-range forces.

We note in the Appendix that, away from the limit considered in this section, $\tilde{\mathcal{C}}$ is a function of the 3-momentum that drops off quickly with increasing $|\vec{p}|^2$, so its effects become suppressed. As the case of small $\eta^{1/2}\vec{p}$ provides Lorentz-violating effects that are maximal but does not give additional constraints on our theory, we will not study the unusual form of the dispersion relation for arbitrary momenta here. That issue, as well as a more general study of Lorentz-violating effects in similar nonlocal theories, will be considered in a separate publication.

V. CONCLUSIONS

In this paper, we have considered scalar theories in which quadratic terms are present that are nonlocal and Lorentz violating. Part of our initial motivation was to avoid the complications related to unitarity discussed in the related Lorentz-invariant theories of Ref. [2]; we verified this by repeating the same calculation presented in that earlier work. However, the theories discussed here are potentially of interest for a broader set of reasons. For example, they suggest a nonlocal generalization of the Horava-Lifshitz idea [12] and might be useful in formulating a renormalizable quantum theory of gravity. Moreover, as indicated qualitatively by the smoothing of singularities at the origin of the nonrelativistic potential studied in Sec. III, the nonlocality we have discussed might capture some features of an underlying ultraviolet completion.

Since the violation of Lorentz invariance must confront stringent experimental bounds [9], the scale of nonlocality can only be low in sectors that communicate very weakly with standard model particles. We have focused on that possibility here, assuming by necessity that any nonlocal modification of the standard model Lagrangian itself occurs at much shorter distance scales. While gravity provides one possible avenue for exploration, in this work we have considered the possibility of a "dark" scalar sector that couples to the standard model through a portal sector of heavy, vectorlike fermions. When the heavy fermions are integrated out of the theory, the couplings induced to standard model fermions are extremely weak. Nevertheless, ultralight particles from a nonlocal, Lorentz-violating sector may be detectable via the long-range forces that they mediate, which could lead to detectable corrections to the gravitational potential of macroscopic bodies. The sequestering of this sector allows the scale of nonlocality to be comparable to the mass scale of the particle mediating the long-range force, a possibility that has not been considered previously in the literature.

In summary, this paper has proposed a new possibility, that of nonlocal Lorentz-violating extensions of the standard model. Although we have constructed one explicit model and considered some aspects of its phenomenology, the more important, overarching point is that the general idea presented here may lead to other interesting applications. Directions for future study could include a more general study of the Lorentz-violating effects in similar theories, and the development of a nonlocal Lorentz-violating modification of gravity. A systematic study of the renormalization of Lorentz-violating nonlocal theories would also be worthwhile. We hope to return to these topics in future work.

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APPENDIX: FERMION SELF-ENERGY

In this appendix, we briefly outline the evaluation of Eq. (4.14) without expanding in $\eta^{1/2}\vec{p}$. First, we note that we may evaluate Σ at the point $p^2 = m_f^2$ since corrections to the dispersion relation affect Σ at higher order in g_f^2. In this case, the factor of $q_E^2 + \Delta$ that appears in the Euclideanized denominator of Eq. (4.14) is always positive, as $\Delta = x^2 m_f^2 + (1-x)m^2 > 0$. We are therefore justified in exponentiating the denominator using a Schwinger parameter u. We can then shift integration variables so that the quantity that is exponentiated in the integrand is spherically symmetric, which allows us to discard odd terms in q. The momentum integrals are then all Gaussian and can be easily evaluated. When the dust settles, we are left with

$$\Sigma = -\frac{g_f^2}{16\pi^2}\int_0^1 dx \int_0^\infty du \frac{u^{1/2}}{(\eta+u)^{3/2}} e^{-\frac{\eta u x^2}{u+\eta}|\vec{p}|^2 - u\Delta} \times \left[-\frac{\eta x}{\eta+u}\vec{p}\cdot\vec{\gamma} + (1-x)\not{p} + m_f\right]. \tag{A1}$$

Note that setting $\vec{p}$ to zero only in the exponential factor in Eq. (A1) provides an upper bound for the value of the integrals, since the exponential is always less than 1 over the integration region. Doing so and setting m_f to zero, we would identify

$$\tilde{C} = \frac{g_f^2}{12\pi^2}\left[\frac{3}{4}\int_0^1 dx x \int_0^\infty d\tilde{u}\, \frac{\tilde{u}^{1/2}}{(1+\tilde{u})^{5/2}}\, e^{-(1-x)\tilde{u}\rho}\right], \tag{A2}$$

where $\tilde{u} = \eta^{-1}u$ and ρ is defined as in Sec. IV. We have confirmed numerically that Eqs. (A2) and (4.26) are identical. It is also clear from Eq. (A1) (and we have checked numerically) that the coefficient of the $\vec{p}\cdot\vec{\gamma}$ terms is a rapidly decreasing function of $|\vec{p}|^2$, as noted in Sec. IV.

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