Three-body model for $K(1460)$ resonance

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We develop the three-body $KK\bar{K}$ model for the $K(1460)$ resonance based on the Faddeev equations in configuration space. A single-channel approach is utilized that takes into account the difference of masses of neutral and charged kaons. It is demonstrated that the mass splitting of the $K(1460)$ resonance takes a place around 1460 MeV according to $K^0K^0\bar{K}^0$, $K^0K^+K^-$ and $K^+K^0\bar{K}^0$, $K^+K^+K^-$ neutral and charged particle configurations, respectively. The calculations are performed with two sets of KK and $K\bar{K}$ phenomenological potentials, where strength interactions are considered the same for the isospin singlet and triplet states. We study the effect of repulsion of the KK interaction on the mass of the $KK\bar{K}$ system and evaluate the effect of the mass polarization. The Coulomb interaction for description of the $K(1460)$ resonance is considered for the first time. The mass splitting in the $K(1460)$ resonances is evaluated to be in the range of 10 MeV with taking into account the Coulomb force. The three-body model with the $K\bar{K}$ potential, which has the different strengths of the isospin singlet and triplet interactions and is related to the condition of obtaining a quasibound three-body state is also considered. Our results are in reasonable agreement with the experimental mass of the $K(1460)$ resonance.

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I. INTRODUCTION

Since the early 1960s, when the quark model was developed, it became clear that hadrons are not elementary particles but are composed of quarks and antiquarks. In the classical quark model, a baryon is composed of three quarks, and a meson is composed of one quark and one antiquark. Today the internal structure of hadrons is a prominent topic of high energy physics [1–4]. Quarks and gluons are confined within the mesons and baryons. Thus, hadrons are composite objects of quarks and gluons governed by quantum chromodynamics (QCD), which has been established as the theory describing the strong interaction. However, QCDs application to low-energy hadron phenomenology is still relatively unexplored, and there are open problems to be studied. The interpretation of hadronic states is one of the most critical issues in hadronic physics, particularly for the exotic states which cannot be easily collected as quark-antiquarks or three quark states. In particular, some specific resonances cannot be simply explained by the quark model and may be of a more complex structure. Common features for descriptions of such specific resonances are predictions for the existence of hadrons with substructures, which are more complex than the standard quark-antiquark mesons and the three-quark baryons of the original quark model that provides a concise description of most of the low-mass hadrons [3].

In the low energy region, where perturbative QCD does not work, nonperturbative methods such as the QCD sum rule [5], lattice QCD [6,7], chiral perturbation theory [8–11], and field correlator method (FCM) in QCD [12–15] are needed. The list of just aforementioned nonperturbative methods is not meant to be complete. In Ref. [16] a survey of contemporary studies of hadrons and strongly interacting quarks using QCD's Dyson-Schwinger equations, in particular, Faddeev-type equations are employed for baryon calculations. This review complements and extends earlier reviews [17–20]. One should mention the functional renormalization group approach [21] for quantitative first-principle studies of the QCD phase diagram and the hadron spectrum. References [22,23] built the foundation for the work [21], which constitutes a crucial prerequisite for future quantitative first-principle studies at finite temperature and finite chemical potential. The comprehensive review of the spectrum and electromagnetic properties of baryons are described as relativistic three-quark bound states within QCD, and the review of nonperturbative light-front Hamiltonian methods

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are presented in Refs. [24,25]. We cite these works, but the recent literature on the subject is not limited by them.

The physics of three-body systems has received attention for decades. The general approach for solutions of the three-body problem at low energies is based on the use of methods for studying the dynamics of three particles in discrete and continuum spectra. Currently, among the most powerful approaches are the method of Faddeev equations in momentum or configuration spaces, the method of hyperspherical harmonics (HH), the variational method in the harmonic-oscillator basis, and the variational method complemented with the use of explicitly correlated Gaussian basis functions. To investigate the three-body systems in hadron physics, one should solve the Faddeev equations [26,27]. The method of hyperspherical harmonics in configuration [28–39] (see also references herein) or momentum [35,38,40] spaces is another method that is intensively used in a few-body physics, which despite its conceptual simplicity, offers great flexibility, provides high accuracy, and can be used to study diverse quantum systems, ranging from small atoms and molecules to light nuclei and hadrons. It is a very challenging task to solve the Faddeev equations exactly, and we are usually introducing some reasonable approximations of the Faddeev equations, such as the use of separable potentials, energy-independent kernels, on-shell two-body scattering amplitudes, the fixed-center approximation (FCA), and the Faddeev-type Alt-Grassberger-Sandhas equations [41]. On the other hand, an application of the HH for the solution of three-body problems always relies on the reasonable convergence of the method.

Using the unitary extensions of chiral perturbation theory [11] that is a good representation of QCD at low energies [9,10], dynamically generated three-body resonances formed via the meson-meson and meson-baryon interactions were intensively studied using FCA for the Faddeev equations in Refs. [42–56]. Indeed, the use of chiral dynamics in the Faddeev equations within a fixed-center approximation allows the description of three-body resonances consisting of mesons and baryons. Thus, hadronic composite states are introduced as few-body systems in hadron physics [57]. Therefore, interpretation of the states found in the system of mesons and baryons becames one of the challenges in theoretical physics [58].

The FCM is a promising formulation of the nonperturbative QCD that gives additional support to the quark model assumptions. Progress was made [59–62] toward placing the computation of baryon masses within the FCM by describing a three particle system within the HH method. Using the HH approach the ground and p-wave excited states of nnn, nns, and ssn baryons can be obtained [62,63] in the framework of the FCM [15].

It is interesting to consider a dynamical generation of $K(1460)$ pseudoscalar resonance formed by a system of three kaons. In particular, noteworthy is the possibility of the formation of the quasibound states of three kaons. The observation of pseudoscalar resonances is of fundamental importance toward the understanding of the meson spectrum. Let us go over a short history of $K(1460)$ pseudoscalar resonance. $K(1460)$ pseudoscalar was a subject of interest already several decades ago. The first evidence for a strangeness-one pseudoscalar meson with a mass of ∼1400 MeV and a width of ∼250 MeV was reported via $J^P = 0^-$ partial-wave analysis of the $K\pi\pi$ system in the reaction $K^{\pm}p \to K^{\pm}\pi^+\pi^- p$ [64]. The study of this process was carried out at SLAC, using a 13 GeV incident $K^{\pm}$ beam. A few years later, the diffractive process $K^- p \to K^-\pi^+\pi^- p$ at 63 GeV was studied by ACCORD Collaboration [65], and the existence of a broad 0^- resonance with a mass ∼1460 MeV may now be taken as established. However, still the PDG does not yet list it as an "established particle" [66]. In the most recent study [67] intermediate decays of the $K(1460)$ meson are found to be roughly consistent with previous studies [64,65], with approximately equal partial widths to $\bar{K}^*(892)\pi^-$ and $[\pi^+\pi^-]^{L=0}K^-$, and its resonant nature is confirmed using a model-independent partial-wave analysis. This resonance can be considered as a 2^1S_0 excitation of the kaon in a unified quark model, which leads to the mass 1450 MeV [68].

By assuming isospin symmetry in the effective kaon-kaon interactions that is attractive for $K\bar{K}$ pair and repulsive for KK pair, the $K(1460)$ pseudoscalar resonance can be the $KK\bar{K}$ system. With this idea in mind, in Ref. [69] was performed the study of the $KK\bar{K}$ system using the single-channel variational approach in the framework of the model [70,71] from one hand, and within the Faddeev equations formalism in momentum representation. In the latter case, the two-body on-shell t matrices which describe KK and $K\bar{K}$ interactions by using the Bethe-Salpeter equation in a couple-channel approach are determined and the on-shell factorization method, from another hand. Dynamical generation of pseudoscalar $K(1460)$ resonance was considered in Ref. [72], by studying interactions between the $f_0(980)$ and $a_0(980)$ scalar resonances and the lightest pseudoscalar mesons. In Ref. [73] using the single-channel description of the $KK\bar{K}$ system in the framework of the HH method the mass of $K(1460)$ resonance was calculated. Recently, in Ref. [74], the $KK\bar{K}$ system was considered based on the coupled-channel complex-scaling method by introducing three channels $KK\bar{K}$, $\pi\pi K$, and $\pi\eta K$. The resonance energy and width were determined using two-body potentials that fit two-body scattering properties. The model potentials having the form of one-range Gaussians were proposed based on the experimental information about a_0 and f_0 resonances. In this model, the $K\bar{K}$ interaction depends on the pair isospin. In particular, the isospin triplet $K\bar{K}(I = 1)$ interaction is essentially weaker than the isospin singlet $K\bar{K} - K\bar{K}$ interaction in the channel $\pi K - K\bar{K}(I = 0)$.

The aim of this paper is to systematically investigate the $KK\bar{K}$ system in the framework of a new approach for the kaonic physics—the Faddeev equations in configuration

space. This approach is suitable to formulate and solve the $KK\bar{K}$ bound state problem with the effective phenomenological KK and $K\bar{K}$ potentials [69–71] derived in configuration space. Existing calculations in the momentum space are contradictory to each other. We hope to bring more clarity in the study of the $KK\bar{K}$ system using the well-established method, the Faddeev equations in configuration space. We are suggesting to consider the bosonic $KK\bar{K}$ system in terms of Efimov's physics and show the connection of Efimov's attraction with the exchange term in the Faddeev equations. As the option of the consideration of the universality in three-body systems we suggest to use the mass polarization. In our work we try to answer the following questions: (i) What $KK\bar{K}$ are deeply bound? (ii) Is there any strange structure peculiar to the $KK\bar{K}$ system? We present our study of the $K(1460)$ resonance in the framework of a single-channel nonrelativistic potential model using the Faddeev equations in configuration space and considering this resonance by means of a three-body kaonic system $KK\bar{K}$. Such consideration allows one to use KK and $K\bar{K}$ potentials for a description of the $KK\bar{K}$ system. In our approach these potentials are only inputs along with the masses of kaons. Following Ref. [69] we study the $KK\bar{K}$ system using effective phenomenological potentials but taking into account the difference in masses of K and $\bar{K}$ kaons. The latter leads to splitting the mass of the $K(1460)$ resonance according to the following neutral or charged particle configurations: $K^0K^0\bar{K}^0$, $K^0K^+K^-$, $K^+K^0\bar{K}^0$, $K^+K^+K^-$. We consider two cases for the $KK\bar{K}$ system. In the first one, the strengths of the isospin singlet and triplet parts of the $K\bar{K}$ potential are the same. Such an approach leads to a simplified version of the Faddeev equations in configuration space for three particle systems. The second case is complicated by the isospin dependence of the $K\bar{K}$ potential, when the strengths of the isospin singlet and triplet parts of the potential are different and related by the condition of obtaining a quasibound three-body state. Results of our calculations are compared with the SLAC and ACCMOR Collaboration experimental values for the mass of $K(1460)$ resonance [64,65] and the recent experimental study [67].

This paper is organized as follows. In Sec. II we present our theoretical model. The Faddeev equations in configuration space are formulated, and we present the particle configurations in a three-body kaonic system $KK\bar{K}$. The results of calculations for the masses of different configurations, interpretation of the results, including a comparison to the previous ones obtained within different methods, are presented in Sec. III. The concluding remarks follow in Sec. IV.

II. THEORETICAL MODEL

A. Formalism

Configuration space methods are a valuable tool for the analysis of the three-body problem with short-range interactions [75,76]. Considering the $KK\bar{K}$ system as three interacted bosons having positions $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, once the two-body interactions for the $K\bar{K}$ and KK subsystems are defined, one can determine its wave function by solving the Faddeev equations. The bound state problem for the $KK\bar{K}$ system we formulate by using the Faddeev equations in configuration space [77] for the bosonic AAB system with two identical particles. The total wave function of the $KK\bar{K}$ system is decomposed into the sum of the Faddeev components U and W corresponding to the $(KK)\bar{K}$ and $(K\bar{K})K$ types of particles rearrangements: $\Psi = U + (I + P)W$, where P is the permutation operator for two identical particles. For a three-body system, which includes two identical bosons, the Faddeev equations represent the set of two equations for the components U and W [78] that reads

$$\begin{aligned}(H_0^U + V_{KK} - E)U &= -V_{KK}(W + PW),\\ (H_0^W + V_{K\bar{K}} - E)W &= -V_{K\bar{K}}(U + PW),\end{aligned} \tag{1}$$

where the potentials for KK and $K\bar{K}$ pairs are defined as V_{KK} and $V_{K\bar{K}}$, respectively. In Eqs. (1) H_0^U and H_0^W are the kinetic energy operators of three particles written in the Jacobi coordinates (see the Appendix A) corresponding to the $(KK)\bar{K}$ and $(K\bar{K})K$ types of the three particles' rearrangements. The total isospin of the $KK\bar{K}$ system is considered to be $\frac{1}{2}$. The set of particles in the $KK\bar{K}$ system is defined by total isospin projections, which can be $-1/2$ or $1/2$. The possible isospin configurations with isospin $3/2$ are not taken into account in our calculations due to the smallness of corresponding contributions.

In general, we employ the s-wave isospin dependent V_{KK} and $V_{K\bar{K}}$ potentials having singlet and triplet components: $V_{KK} = \mathrm{diag}\{v^s_{KK}, v^t_{KK}\}$, $V_{K\bar{K}} = \mathrm{diag}\{v^s_{K\bar{K}}, v^t_{K\bar{K}}\}$. One should mention that due to Bose-Einstein statistics the strength of the s-wave KK interaction $v^s_{KK} = 0$, because the isospin singlet wave function of the pair is antisymmetric. Therefore, the corresponding interaction should be suppressed. The separation of isospin variables leads to the following form of the Faddeev equations:

$$\begin{aligned}&(H_0^U + v^t_{KK} - E)\mathcal{U}\\ &\quad = -v^t_{KK}\left(-\frac{1}{2}\mathcal{W}^t + \frac{\sqrt{3}}{2}\mathcal{W}^s - \frac{1}{2}p\mathcal{W}^t + \frac{\sqrt{3}}{2}p\mathcal{W}^s\right),\\ &(H_0^W + v^s_{K\bar{K}} - E)\mathcal{W}^s\\ &\quad = -v^s_{K\bar{K}}\left(\frac{\sqrt{3}}{2}\mathcal{U} + \frac{1}{2}p\mathcal{W}^s - \frac{\sqrt{3}}{2}p\mathcal{W}^t\right),\\ &(H_0^W + v^t_{K\bar{K}} - E)\mathcal{W}^t\\ &\quad = -v^t_{K\bar{K}}\left(-\frac{1}{2}\mathcal{U} - \frac{\sqrt{3}}{2}p\mathcal{W}^s - \frac{1}{2}p\mathcal{W}^t\right).\end{aligned} \tag{2}$$

The singlet and triplet $\mathcal{W}$ components of the wave function are noted by indexes s and t, respectively, U is the triplet

component, and the exchange operator p acts on the particles' coordinates only. Within the s-wave approach, the equation for the singlet component corresponding to the KK singlet potential is omitted due to the isospin symmetry. The similar property is demonstrated for equations describing AAB systems such as $NN\bar{K}$ [79,80] and nnp [81].

For the description of the effective kaon-kaon interaction we use the potentials from Refs. [69–71] that are written in one-range Gaussian form as $V_A(r) = \sum_{I=0,1} V_A^I \times \exp[-(r/b)^2]P_A$, where b is the range parameter having the same value as for the $\bar{K}N$ interaction, P_A is the isospin projection operator, and the index A is related to the type of interaction $A \in KK, K\bar{K}$. The strength of strongly attractive s-wave $K\bar{K}$ interaction in the isospin singlet and triplet states $V_{K\bar{K}}^{I=0,1} = v_{K\bar{K}}^s = v_{K\bar{K}}^t = -1155 - i283$ MeV with $b = 0.47$ fm and $V_{K\bar{K}}^{I=0,1} = v_{K\bar{K}}^s = v_{K\bar{K}}^t = 630\text{–}210i$ MeV with $b = 0.66$ fm are considered the same for the isospins $I = 0$ and $I = 1$ [69–71]. Considering that KK and $\bar{K}\bar{K}$ interactions are isospin invariant and that there are no open decay channels for the $\bar{K}\bar{K}$ system, the KK potential is real. The strength of the s-wave KK interaction for $I = 0$ is $V_{KK}^{I=0} = v_{KK}^s = 0$, and for isospin $I = 1$ it has a relatively weak repulsion that is considered as $V_{KK}^{I=1} = v_{K\bar{K}}^t = 313$ MeV and $V_{K\bar{K}}^{I=1} = v_{K\bar{K}}^t = 104$ MeV for parametrizations with $b = 0.47$ fm and $b = 0.66$ fm, respectively. As mentioned above, the choice of the range parameters b is related to the description of the $\bar{K}N$ interaction. The value $b = 0.66$ fm for the effective $\bar{K}N$ interaction corresponds to the effective Akaishi-Yamazaki potential derived in Refs. [82,83] phenomenologically by using $\bar{K}N$ scattering and kaonic hydrogen data and reproducing the $\Lambda(1405)$ resonance as the $\bar{K}p$ bound state at 1405 MeV. This potential is energy independent. The value $b = 0.47$ fm for the effective $\bar{K}N$ interaction corresponds to the potential obtained in Ref. [84] within the chiral SU(3) effective field theory and is derived based on the chiral unitary approach for the s-wave scattering amplitude with the strangeness $S = -1$. This potential reproduces the total cross sections for the elastic and inelastic $\bar{K}p$ scattering, the threshold branching ratios, and the $\pi\Sigma$ mass spectrum associated with the $\Lambda(1405)$. The strength of the $K\bar{K}$ interaction was determined by fitting masses of the $f_0(980)$ and $a_0(980)$ resonances with the input width 60 MeV [71]. The strength of the KK interaction was obtained in Ref. [70] to reproduce the KK scattering length given by a lattice QCD calculation in Ref. [85]. Following Ref. [71] we refer to the kaon-kaon interaction potential with $b = 0.47$ fm and $b = 0.66$ fm as A and B, correspondingly. The set of values of the potential strength V_A^I for each interaction and two optimized values for the range parameter (sets A and B, respectively) are given in Table I.

Taking into account that the potentials have the same components in isospin singlet and triplet states, the Faddeev equations (2) can be reduced using an algebraic transformation [79] defined by the diagonal matrix to the following form:

$$(H_0^U + v_{KK}^t - E)\mathcal{U} = -v_{KK}^t(1+p)\widetilde{\mathcal{W}},$$
$$(H_0^W + v_{K\bar{K}} - E)\widetilde{\mathcal{W}} = -v_{K\bar{K}}(\mathcal{U} + p\widetilde{\mathcal{W}}). \quad (3)$$

The diagonal matrix $D = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$ defines this transformation and the transformation related to the $K\bar{K}$ potential is given as $v_{K\bar{K}} = DV_{K\bar{K}}D^T$, where $V_{K\bar{K}} = \mathrm{diag}\{v_{K\bar{K}}^s, v_{K\bar{K}}^t\}$. The corresponding Faddeev components are $\widetilde{\mathcal{W}} = D\mathcal{W}$, where $\mathcal{W} = (\mathcal{W}^s, \mathcal{W}^t)^T$.

For the three-body system described by Eqs. (3) one can evaluate the mass polarization using the definition

$$\Delta = 2E_2 - E_3(V_{KK} = 0). \quad (4)$$

Here, E_2 is $K\bar{K}$ two-body energy and $E_3(V_{KK} = 0)$ is the three-body energy, when the KK interaction between identical particles is omitted. The value of Δ is positive and depends on the mass ratio of the particles [86–88].

When in the system $KK\bar{K}$ at least two particles are charged, and also the Coulomb interaction should be considered. The Coulomb potential can be included as a perturbation of the Hamiltonian in the left-hand side of Eq. (3). We present the structure of the set of Faddeev equations taken into account with the Coulomb interaction in Appendix A.

The complete isospin model is based on Eqs. (2) with the splitting of the $K\bar{K}$ potential to two isospin channels $I = 0$ and $I = 1$, which have different strengths of the $K\bar{K}$ interaction. The splitting of the singlet and triplet components proposed in Ref. [74] can be expressed by a ratio of strength parameters for the components of potential, $V_{K\bar{K}}^{I=1}/V_{K\bar{K}}^{I=0}$. Equations (3) describe the case, when $v_{K\bar{K}}^t/v_{K\bar{K}}^s = 1$. This case corresponds to the AAB system

TABLE I. Sets of parameters for the $K\bar{K}$ and KK potentials.

	A ($b = 0.47$ fm)		B ($b = 0.66$ fm)	
Interaction	v^s [MeV]	v^t [MeV]	v^s [MeV]	v^t [MeV]
$K\bar{K}$	$-1155 - 283i$	$-1155 - 283i$	$-630 - 210i$	$-630 - 210i$
KK	0	313	0	104

TABLE II. Kaons and antikaons with the mass deference and isospin projections.

Particle (Antiparticle)	Quarks	Mass [MeV]	Isospin projection
K^+ (K^-)	us	493.7	$1/2$ ($-1/2$)
K^0 ($\bar{K}^0$)	ds	497.6	$-1/2$ ($1/2$)

without spins and isospins (bosonic isospinless system) and the $KK\bar{K}$ demonstrates properties of such a system.

B. Particle configurations in $KK\bar{K}$ system

One can consider different particle configurations in the $KK\bar{K}$ system. The configurations differ by sets of masses and pair potentials. The Coulomb potential has to be included for a description of some configurations. To select the configuration, we used the difference between the masses of kaons presented in Table II. These configurations are the following: $K^0K^0\bar{K}^0$, $K^0K^+K^-$, $K^+K^0\bar{K}^0$, $K^+K^+K^-$. Using the charge-isospin basis notations, the configurations can be identified as $--+$, $-+-$, $+-+$, $++-$. Thus, the first two configurations correspond to the states with the projection of total isospin $-1/2$ of the $KK\bar{K}$ system, while the last two have the total isospin projection $+1/2$. Each configuration is represented as the AAB system—the system with two identical particles—and can be described by the Faddeev equations (2) and (3): the case when the strengths of the isospin components of the $K\bar{K}$ potential are different and when $v^t_{K\bar{K}} = v^s_{K\bar{K}} = 1$, respectively.

III. RESULTS

Our interest is to examine the possibility of the existence of bound kaonic states in the $KK\bar{K}$ system. For this purpose, we solve numerically differential Eqs. (2) in the case of the different strengths for the isospin singlet and triplet components of the $K\bar{K}$ potential and Eqs. (3), when $v^t_{K\bar{K}} = v^s_{K\bar{K}}$. The differential Faddeev equations have been formulated in the pioneering work of Noyes and Fiedeldey [89] for the simplest case of s-wave three-particle scattering and have been generalized in Ref. [90]. Our numerical procedure for the solution of the Faddeev equations in configuration space is based on the finite difference approximation with spline collocations [91,92].

A. Case when the strength of $v^t_{K\bar{K}} = v^s_{K\bar{K}}$

For this case, results of our calculations for the binding energy and the mass for the $KK\bar{K}$ system are presented in Table III. In this table are also given the result from Ref. [69] obtained using the variational method for the single channel three-body potential model with the same two-body effective KK and $K\bar{K}$ interactions. The total mass of the $KK\bar{K}$ system ranges from 1463.8 to 1469.4 MeV, when we consider the same K meson mass $m_K = 496$ MeV as in Refs. [69,73]. The quasibound state for the $KK\bar{K}$ with spin-parity 0^- and total isospin $1/2$ is found below the three-kaon threshold.

Studying the various particle configurations of the $KK\bar{K}$ system and their dependence on the particles masses, we are considering the following kaonic masses: the mass $m_K = 496$ MeV [69] corresponds to the $K^0K^0\bar{K}^0$ system, where

TABLE III. The mass of the $K(1460)$ resonance for the potentials of the A and B parameter sets (without the Coulomb force), A_c and B_c (the Coulomb force is included), $M = \sum_\gamma m_\gamma - |E_3|$, m_γ is kaon mass, $\gamma = 1, 2, 3$. The energy of the $KK\bar{K}$ quasibound state ($I = 1/2$) is E_3. The E_2 is the energy of the bound $K\bar{K}$ pair. The masses and energies are given in MeV. $\delta = \Delta/|E_3(V_{KK} = 0)|$ is the relative contribution of the mass polarization. The result of Ref. [69] is given in parentheses. The upper bound for the mass of the three-body quasibound state is shown as $\sum_\gamma m_\gamma - |E_2|$. m_K is the averaged kaon mass.

Resonance	System AAB	Particle masses	Model	E_3	$E_3(V_{KK} = 0)$	E_2	Mass polarization δ [%]	Mass M	Mass upper bound
$K(1460)$	$KK\bar{K}$	496.0 [69]	A	-19.8 (-21)	-32.1	-11.25	30.0	1468.2	1476.7
		495.7	A	-19.7	-31.9	-11.18	29.9	1467.4	1475.9
			B	-22.2	-29.4	-11.17	24.0	1464.9	1475.9
$K^0(1460)$	$K^0K^0\bar{K}^0$	497.6	A	-20.4	-33.0	-11.61	29.6	1469.7	1481.2
			B	-22.8	-30.1	-11.45	23.9	1467.3	1481.3
$K^0(1460)$	$K^0K^+K^-$	$m_{K^-} = 493.7$,	A	-19.3	-31.3	-10.96	29.7	1465.8	1474.1
		$m_K = 495.7$	B	-21.9	-29.0	-11.03	23.8	1463.2	1474.1
$K^+(1460)$	$K^+K^0\bar{K}^0$	$m_{\bar{K}^0} = 497.6$,	A	-20.1	-32.5	-11.40	29.8	1468.9	1477.6
		$m_K = 495.7$	B	-22.5	-29.8	-11.34	23.9	1466.5	1477.7
$K^+(1460)$	$K^+K^+K^-$	493.7	A	-18.9	-30.9	-10.74	29.5	1462.2	1470.4
			A_c	-20.9	...	...	...	1460.2	...
			B	-21.6	-28.7	-10.87	24.3	1459.5	1470.2
			B_c	-23.3	...	...	...	1457.8	...

$m_K = (2m_{K^+} + m_{\bar{K}^0})/3$; the mass $m_K = 493.7$ MeV corresponds to the $K^+K^+K^-$ system; the mass $m_K = 497.6$ MeV corresponds to the $K^0K^0\bar{K}^0$ system. For the $K^0K^+K^-$ and $K^+K^0\bar{K}^0$ configurations we used the averaged mass of kaons for particle pair K^0K^+, $m_K = 495.7$ MeV.

The difference of the masses for K^0 and K^+ violates the AAB model with two identical particles of the $KK\bar{K}$ system. However, the approach with the averaged mass is completely satisfied with the AAB model due to the set of the proposed potentials. The Coulomb potential acting in the $K^0K^+K^-$ system also violates the AAB symmetry, and, therefore, we omitted the consideration of the Coulomb force for the $K^0K^+K^-$ configuration. The Coulomb potentials in the $K^+K^+K^-$ system were included in the calculations due to correspondence to the AAB symmetry. A brief description of the Faddeev equations in configuration space with the Coulomb force acting in the $KK\bar{K}$ is given in Appendix A.

The comparison of our energy $E = -21$ MeV with the result obtained in Ref. [69] shows some disagreement. We assume that it could be related to the numerical methods utilized to solve the corresponding equations. In Ref. [69] the variational method is used, which depends on a choice of initial basis functions. We use the direct numerical method [92] for the solution of the Faddeev equations in configuration space. Here, the direct solution means a solution method based on the finite-differential approximation of the boundary problem for eigenvalues with the discretization of the coordinate space. The analysis of the method is performed in Appendix C 1, where two cross-check tests are given. The first is related to the test of our computer codes for the solution of a problem similar to the one considered for the $KK\bar{K}$ system with the comparison with results of other authors. In the second one, we proposed an alternative way to solve the Faddeev equations by using the cluster reduction method [93,94]. Both tests provided evidence that the accuracy for the results listed in Tables III and IV is reached in our calculations.

Let us continue the analysis of data from Table III, where the results of calculation for the $E_3(V_{KK} = 0)$ are also presented. The $E_3(V_{KK} = 0)$ is defined as the energy of the quasibound state of the three-body system when the repulsive KK interaction is omitted. In this case, the set of Eqs. (3) is reduced to the single equation for the $\widetilde{\mathcal{W}}$ Faddeev component:

$$(H_0^W + v_{K\bar{K}} - E)\widetilde{\mathcal{W}} = -v_{K\bar{K}}p\widetilde{\mathcal{W}}. \tag{5}$$

The exchange term presented at the right-hand side of the equation provides the existence of a bound state with energy $E_3(V_{KK} = 0)$. In Table III are shown the two-body energy of bound pair E_2 and three-body energy $E_3(V_{KK} = 0)$. Based on the analysis performed in Ref. [88] and according to Eq. (4), the relation between E_2 and $E_3(V_{KK} = 0)$ can be rewritten as

$$|E_3(V_{KK} = 0)/E_2| > 2. \tag{6}$$

The results of the calculations given in Table III are in agreement with this relation. In nuclear physics, this relation is called the "mass polarization effect" [88]. In terms of the Efimov physics [95,96], the relation (4) is explained by the Efimov attraction as a result of a mediated attraction between two particles by the exchange of the third particle. Note that an expression, which is similar to Eq. (6), has been previously obtained in Ref. [97] for bosonic two-dimensional AAB systems to describe the relation of two-body and three-body energies. Interestingly enough to note that due to universality, Efimov physics applies to virtually any field of quantum physics, be it atomic and molecular physics, nuclear physics, condensed matter physics, or even high-energy physics (see, for example, [98,99]).

The relation (6) agrees with the so-called "Efimov scenario" [96] defined for the model situation when pair potential is simply scaled by a multiplicative factor. To illustrate this fact, in Fig. 1, we present the results of calculations for the dependence of the ratio E_3/E_2 on two-body energy $|E_2|$ (left panel) and the value $1/\sqrt{|E_2|}$ (right panel). The $1/\sqrt{|E_2|}$ coincides with two-body scattering length a_2 due to approximation $|E_2| \approx \hbar^2/(m_K a_2^2)$. These dependencies are obtained by introducing the scaled factor α, which parametrizes the $K\bar{K}$ potential and scales it as $v_{K\bar{K}} \to \alpha v_{K\bar{K}}$. Therefore, it differs by a multiplicative factor α. These dependencies are parametrically obtained. The parameter is the multiplicative factor of α, defining the scaled $K\bar{K}$ potential. The region of the Efimov physics corresponds to small values of $|E_2|$ (large values of $1/\sqrt{|E_2|}$). Within this region, the ratio of E_3/E_2 quickly increases, and a possibility for an excited state is opening. In Fig. 2 we show the result for the A parameter set of the $K\bar{K}$ potential ($\alpha = 1$). The corresponding state of the $K^0K^0\bar{K}^0$ system is far from the Efimov states. The ratio E_3/E_2 asymptotically approaches 2. The repulsive KK potential makes the E_3/E_2 ratio to be smaller than the $E_3(V_{KK} = 0)/E_2$ ratio. The strength of the KK repulsion defines the difference. What will happen if the KK interaction would be attractive? It is clear that an attractive

TABLE IV. The mass and width of the $K^0K^0\bar{K}^0$ resonance using the A and B parameter sets for the KK and $K\bar{K}$ potentials. The results of Ref. [69] are shown in parentheses. The kaon masses are given in MeV.

Potentials/Kaon mass	M [MeV]	Γ [MeV]
A/497.6	1469.7	105
A/496.0	1468.1 (1467)	104 (110)
B/497.6	1467.3	117

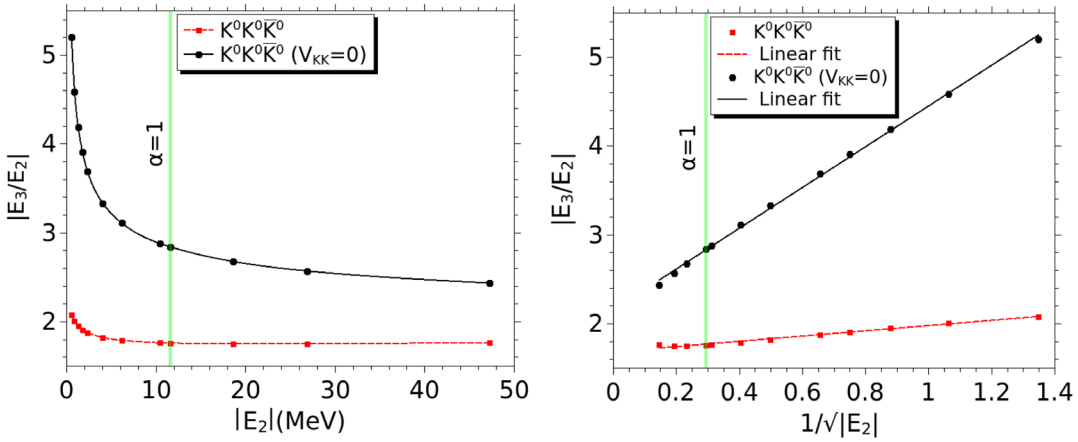

FIG. 1. The dependence of the ratio E_3/E_2 on two-body energy $|E_2|$ (left panel) and the value $1/\sqrt{|E_2|}$, which is proportional to the two-body scattering length a_2 (right panel) for the $K^0K^0\bar{K}^0$ system. The calculations are performed using the potentials with parameters A. These are the parametric dependencies. The parameter is the factor α which defines the scaled $K\bar{K}$ potential as $v_{K\bar{K}} \to \alpha v_{K\bar{K}}$. The vertical line corresponds to $\alpha = 1$.

AA potential will make the ratio E_3/E_2 larger than $E_3(V_{AA} = 0)/E_2$. An example of such a situation with an attractive AA potential is given in Appendix C 1.

Following Ref. [88] we evaluated the relative contribution of the mass polarization $\delta = \Delta/|E_3(V_{KK} = 0)|$ to the energy of the $KK\bar{K}$ quasibound state for different physical particle configurations. The corresponding results are presented in Table III. The value of δ depends on two factors: the mass ratio of kaons and the type of the $K\bar{K}$ potential. When the mass ratio approaches one, the dependence of the mass polarization on the particle configuration is hidden for the considered systems. The second factor is more significant here. One can see the dependence by comparing the results for the potentials of the A and B parameter sets.

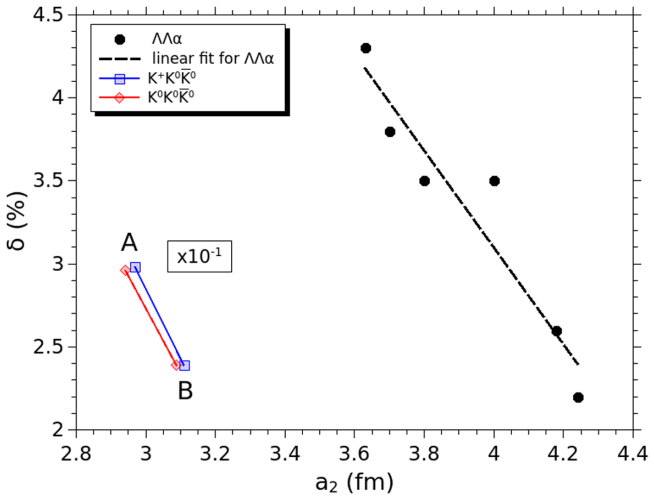

FIG. 2. The correlation between the relative contribution of the mass polarization δ and two-body scattering length a_2 for the $\alpha\Lambda\Lambda$ and $KK\bar{K}$ systems calculated with different pair potentials. The mass ratios are $m_\Lambda/m_\alpha \approx 1/4$ and $m_K/m_{\bar{K}} \approx 1$, respectively.

Summarizing the comparison, we conclude that the mass polarization effect for the potential of the A parameter set is about 30% and for the set B is about 24%. There is a correlation between two-body scattering length a_2 obtained with the potential that bounds nonidentical particles and the relative contribution of the mass polarization δ [88]. The larger scattering length corresponds to the smaller mass polarization. For the $KK\bar{K}$ system, the potentials of the B parameter set demonstrate larger scattering length and smaller mass polarization. This relation is shown in Fig. 2 along with the results obtained in Ref. [88] for the $\alpha\Lambda\Lambda$ system with phenomenological potentials having different scattering parameters. The relatively small mass polarization in the $\alpha\Lambda\Lambda$ system is clarified by domination of the α-particle mass in the system, due to the ratio $m_\Lambda/m_\alpha \ll 1$. The correlation between of the relative contribution of the mass polarization δ and two-body scattering length is only approximately linear, because the dependence of two-body parameters on the strength of a potential is more complex than the parametric dependence of a potential on the strength parameter considered above.

To show the difference between the A and B parameter sets for the $K\bar{K}$ potential, we averaged Eq. (5): $\langle H_0^W\rangle + \langle v_{K\bar{K}}\rangle + \langle v_{K\bar{K}}p\rangle = E_3(V_{K\bar{K}} = 0)$. To evaluate the averaged kinetic energy $\langle H_0^W\rangle$, the method proposed in Ref. [100] was used. We considered Eq. (5) with scaled kaon masses by a factor γ within the small vicinity of the point $\gamma = 1$. The energy becomes a function of $\gamma = 1 \pm \Delta\gamma$ and $dE(\gamma)/d\gamma = -1/\gamma^2\langle H_0^W\rangle$. The linear approximation for this derivation gives an evaluation of the averaged kinetic energy. The exchange term $\langle v_{K\bar{K}}p\rangle$ depends on the mass ratio and does not contribute to $\langle H_0^W\rangle$ as one can see from the numerical results listed in Table III. Using the pattern $\langle H_0^W\rangle + \langle v_{K\bar{K}}\rangle + \langle v_{K\bar{K}}p\rangle = E_3(V_{K\bar{K}} = 0)$, the results of averaging can be written as follows:

$274 - 214 - 93 = -33$ and $182 - 142 - 70 = -30$ for the potentials of the A and B parameter sets, respectively. Here, all values are given in MeV. We see that the potentials in the set A are "stronger" because they act on shorter distances with the larger strength. We can assume that the $KK\bar{K}$ system described with the A parameter set potentials is more compact.

One can assume that the contribution of the exchange term $\langle v_{K\bar{K}} p\rangle$ could correspond to the value evaluated by Eq. (4) for the mass polarization term. We rewrite the expressions presented above as $274(1 - 214/274 - 93/274) = -33$ and $182(1 - 142/182 - 70/182) = -30$ for the potentials of sets A and B, respectively. The evaluation of $|\langle v_{K\bar{K}} p\rangle/\langle H_0^W\rangle|$ gives 34% and 39% for the potentials of the sets A and B, respectively. These values are in disagreement with the results of the mass polarization term in Table III. Note that the mass polarization is related to the kinetic energy operator in the Schrödinger equation [88]. By using the exchange term, one cannot directly separate this kinetic part. Thus, the δ more adequately evaluates the relative contribution of the mass polarization. At the same time, one can make sure that the relative contribution of the exchange term (Efimov attraction) increases with decreasing the strength of the potential according to the "Efimov scenario."

In Table III we also present the upper bounds for the mass of the three-body quasibound state calculated as $\sum_\gamma m_\gamma - |E_2|$, where m_γ, $\gamma = 1, 2, 3$ are the kaon masses. The values define a maximal value for the three-body resonance mass when the quasibound state is approximately located on the two-body threshold. Obviously, the calculated mass M of the three-body resonance is less than the mass of the upper bound state.

The mass spectrum and the mass difference for different particle configurations of the $KK\bar{K}$ system are shown in Fig. 3. The left panel presents the mass spectrum for different particle configurations of the $KK\bar{K}$ system calculated with the A and B parameter sets of pair potentials. It is worthwhile to notice that the spectrum obtained for the set of parameters B is shifted by 2 MeV relative to the spectrum obtained using the potentials with the set of parameters A. This shift is not surprising because the two-body quasibound energy of the model is approximately larger by 2 MeV and depends on the two-body attractive $K\bar{K}$ potential. It is interesting enough to consider the difference between the average isospin model, without taking into account the difference of kaon masses and the particle configuration model. The right panel in Fig. 3 presents the mass difference $M - M_{KK\bar{K}}$ for different particle configurations calculated with the parameters of pair potentials for set A. One can note that it varies from 1 to 7 MeV for different particle configurations.

In Table IV, we present the results of our calculations for the mass and width of the $K^0K^0\bar{K}^0$ resonance using the A and B parameter sets for the KK and $K\bar{K}$ potentials. In these calculations we did not consider the mass difference of K and $\bar{K}$ kaons. However, we considered the different value of the mass for the kaon as it is listed in Table IV. The comparison of our results with those obtained with the variational method [69] shows that the mass and width of $K(1640)$ are close enough, 1467 MeV and 110 MeV [69], respectively. The alternative scenario is observed for the HH method [73] and the Faddeev calculations in the momentum representation [69]: the difference for the width is more than 50%. In particular, for the Faddeev calculations in the momentum representation $\Gamma = 50$ MeV [69], which coincides with results obtained within the HH method [73], where the width falls into the 41–49 MeV

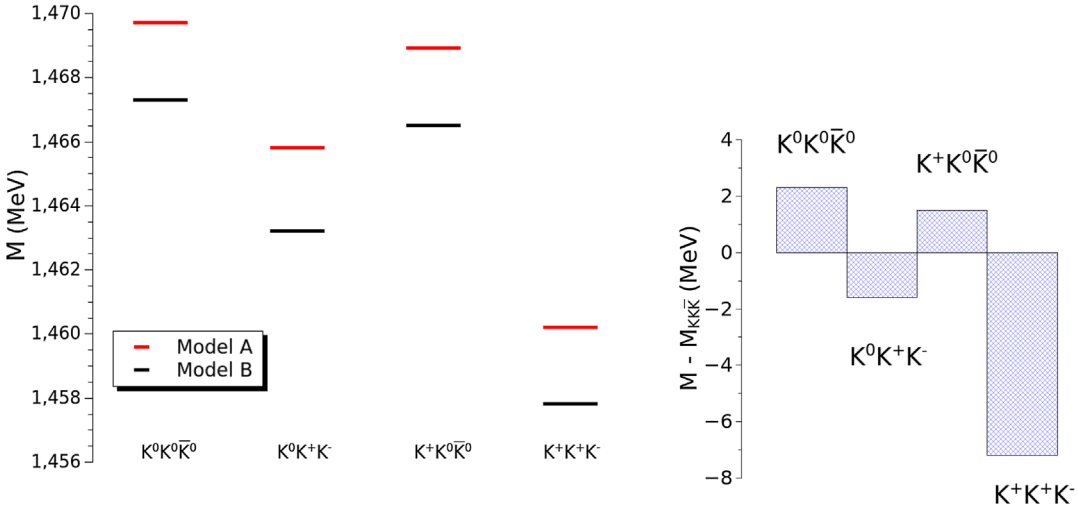

FIG. 3. Mass spectrum for different particle configurations of the $KK\bar{K}$ system calculated with the A and B parameter sets for pair potentials (left panel). The mass difference $M - M_{KK\bar{K}}$ for different particle configurations calculated with the set A for parameters of potentials (right panel).

range for all sets of the $K\bar{K}$ and KK interactions. The details of our method for evaluation of the width Γ are given in Appendix B.

Taking into account the difference of our results and the results of [69], we test our codes, which is presented in Appendix C 1. In the first test, we considered the system npp, which is described by Eqs. (3) as a bosonic isospinless model by direct solution of the Faddeev equations in configuration space. The second test is related to the alternative approach for the solution of the Faddeev equations in configuration space for the $KK\bar{K}$ system using the cluster reduction method (CRM).

It is interesting to consider the density distribution of particles for the $KK\bar{K}$ system calculated in the framework of the Faddeev equations in configuration space. The spatial configuration of particles in the $KK\bar{K}$ system can be understood by plotting the spatial probability amplitudes, i.e., the squared modulus of the Faddeev components $|\mathcal{U}(x,y)|^2$ and $|\mathcal{W}(x,y)|^2$ in terms of the Jacobi coordinates x and y. In Fig. 4 we present calculation results of the probability distribution for the charged kaon resonance $K^+(1460)$ described as the $K^+K^+K^-$ system by using the potentials with the parameter set B. In these figures are presented the contour plots of the squared modulus $|\mathcal{U}(x,y)|^2$ and $|\mathcal{W}(x,y)|^2$ in the frame of 5 fm $\times$ 5 fm, as well as the related spatial configurations. The careful examination of the contour plots shows that the squared modulus' maximal values differ by 2 orders and, therefore, the corresponding spatial configurations probabilities. The component $\mathcal{W}(x,y)$ is dominant in the total wave function. The localization of the particles in the system corresponds to the highest probability associated with the component $\mathcal{W}(x,y)$ for the x and y coordinates values. The coordinates' modules in the most favorable position are approximately related to the ratio of $|y|/|x| \sim 1.5$. The squared modulus of the $\mathcal{W}(x,y)$ component displays very large asymmetry, being strongly elongated in the y direction. The spatial configuration presented in the inset reflects this ratio. Let us note that the localization of K^+ mesons must be symmetrical relative to the K^- due to exchange symmetry for the identical K^+ mesons. The inset in the lower panel in Fig. 4 presents two positions of the particles in the $KK\bar{K}$ system that satisfy this condition. For the first position, the ratio $|y|/|x| < 1.5$. The second position corresponds to the ratio $|y|/|x| \approx 1.5$. According to the spatial probability distribution for the component $\mathcal{W}(x,y)$ given in Fig. 4, the second position is the most probable. Thus, the particles in the $K^+K^+K^-$ system are distributed along one line as a chainlike spatial configuration $(K^+)-(K^-)-(K^+)$. The distance between K^+K^- is 0.8 fm, while the distance between K^+K^+ is twice larger. The latter is not surprising because the K^+K^- interaction is strong and attractive, while the interaction between the identical particles K^+K^+ is weak and repulsive. The other spatial configuration has a low probability and can be represented as a triangle spatial configuration with the basis side and height of 0.8 fm, respectively.

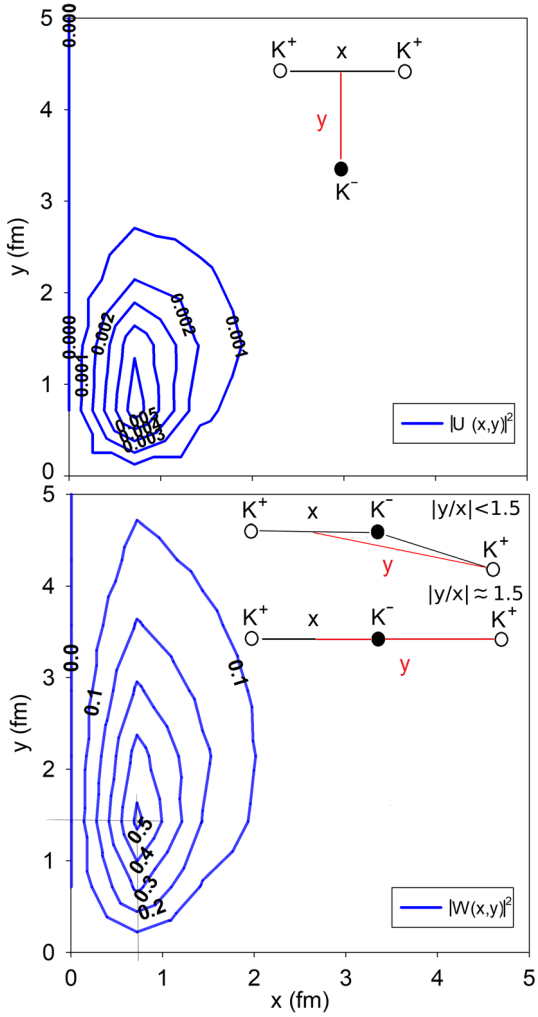

FIG. 4. The probability for distribution of the particles in the $K^+K^+K^-$ system. The contour plot of the squared modulus of the Faddeev component $|\mathcal{U}(x,y)|^2$ (upper panel) and $|\mathcal{W}(x,y)|^2$ (lower panel) versus the corresponding Jacobi coordinates. The insets show the most probable configurations of the particles. The y coordinate is marked by the red color. For the U rearrangement $(K^+ + K^+) + K^-$ (upper panel), the Jacobi coordinate x connects K^+ and K^+ mesons, while for the W rearrangement $K^+ + (K^+ + K^-)$ (lower panel), the Jacobi coordinate x connects K^+ and K^- mesons.

B. Case when the strength of $v^t_{K\bar{K}}$ and $v^s_{K\bar{K}}$ is different

Let us now focus on the dependence of three-body energy on the strength of isospin splitting of the $K\bar{K}$ potential. To consider this case, one should solve Eqs. (2). In Fig. 5 we present the calculation results for $K^+\bar{K}^0K^0$ systems for the potentials of the parameter set A. The splitting means that the isospin triplet component of the potential decreases as $v^t_{K\bar{K}} = (1-\xi)v^s_{K\bar{K}}$ under the condition that the isospin singlet potential is not changed and

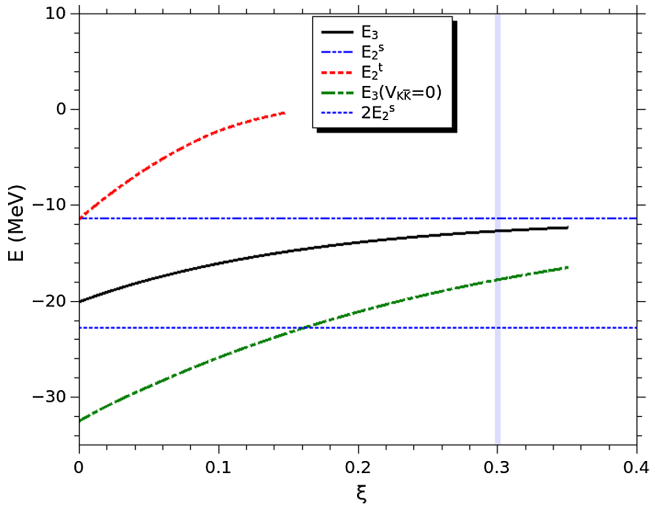

FIG. 5. The energy E_3 of quasiresonance in the $K^+K^0\bar{K}^0$ system (potential with the A parameter set) for different values of the scaling parameter ξ defined for the triplet $K\bar{K}$ potential as $v^t_{K\bar{K}} \to (1-\xi)v^s_{K\bar{K}}$ [74]. The E_2 energy of subsystem $K\bar{K}$ [isospin singlet (s) or isospin triplet (t)]. The vertical line corresponds to the value of the ratio $v^t_{K\bar{K}}/v^s_{K\bar{K}}$ proposed in [74].

provides the two-body threshold E^s_2 about 11 MeV (see Table V). The value of 0.3 for the scaling parameter ξ corresponds to the relation between the singlet and triplet components of the $K\bar{K}$ potential proposed in Ref. [74]. Our calculations show that the triplet bound state exists when the scale parameter ξ is less than 0.15. Thus, the model proposed in [74] assumes that the triplet state is not bound. The quasibound state of the $K^+K^0\bar{K}^0$ system has the energy -12.4 MeV, which is near the two-body threshold. Because of the isospin splitting of the $K\bar{K}$ interaction, the relation (6) is invalidated and the opposite relation takes place: $|E_3(V_{KK}=0)/E^s_2| < 2$. Also the value of $2E_2 - E^s_3(V_{KK}=0)$ becomes negative in contrast to the positive value of Δ in Eq. (4).

IV. SUMMARY

We developed a new framework for the Faddeev calculations in configuration space for the $K(1460)$ dynamically generated resonance in this work. Our three-body nonrelativistic single channel model predicts a quasibound state for the $KK\bar{K}$ system with the mass around 1460 MeV. The calculations are performed using two sets of phenomenological KK and $K\bar{K}$ potentials, when the strength of $K\bar{K}$ interaction has no difference in the singlet and triplet isospin states, as well as taking into account various particle configurations of the $KK\bar{K}$ system. Our study was extended to a more complicated case when the isospin singlet and triplet parts of the $K\bar{K}$ potential are different and related by the condition of obtaining a quasibound three-body state.

In our study, the mass difference between the kaons was taken into account to separate physical particle configurations of the $KK\bar{K}$ system: $K^0K^0\bar{K}^0$, $K^0K^+K^-$, $K^+K^0\bar{K}^0$, $K^+K^+K^-$. These improvements enable us to investigate these kaonic configurations systematically, moreover the first time the Coulomb interaction has been taken into account for the description of the charged configurations. The mass splitting in the $K(1460)$ resonances is evaluated to be in the range of 10 MeV with taking into account the Coulomb force in the case of charged resonances. It is worth mentioning that a hypothetical chainlike spatial configuration $(K^+)-(K^-)-(K^+)$ would constitute a favorable structure of the $KK\bar{K}$ system.

We considered the mass polarization effect in the $KK\bar{K}$ system and evaluated the effect of the repulsion strength of the KK potential. The mass polarization term, which is well separated in the Schrödinger equation as a part of the kinetic energy operator and the exchange term defined by the Faddeev equations, is evaluated and discussed. This term is closely related to the "Efimov attraction." Our calculations demonstrate that a peculiar feature of the mechanism for the binding of the $KK\bar{K}$ system is the s-wave kaon exchange. We compare the contributions of

TABLE V. The mass of the $K(1460)$ resonance for potentials with the parameter sets A and B (without the Coulomb force) and the scaling parameter $\xi = 0.3$, $v^t_{K\bar{K}}/v^s_{K\bar{K}} = 0.7$ [74]. The notations are the same as in Table III. Γ is the width of three-body resonance.

Resonance	System AAB	Model	E_3	Γ	$E_3(V_{KK}=0)$	E^s_2	Mass M	Mass upper bound
$K^0(1460)$	$K^0K^0\bar{K}^0$	A	−12.9	70	−18.2	−11.61	1479.9	1481.2
		B	−14.7	78	−18.8	−11.45	1478.1	1481.3
$K^0(1460)$	$K^0K^+K^-$	A	−12.2	67	−17.1	−10.96	1472.9	1474.1
		B	−14.0	76	−18.0	−11.03	1471.1	1474.1
$K^+(1460)$	$K^+K^0\bar{K}^0$	A	−12.7	69	−17.8	−11.40	1476.3	1477.6
		B	−14.4	77	−18.9	−11.34	1474.6	1477.7
$K^+(1460)$	$K^+K^+K^-$	A	−11.9	69	−16.7	−10.74	1469.2	1470.4
		B	−13.8	76	−17.7	−10.87	1467.3	1470.2

the mass polarization term (the characteristic term of the Schrödinger equation) and the exchange term, which is clearly defined in the Faddeev approach in configuration space. The exchange interaction becomes the maximal for a system with two identical particles, such as the $KK\bar{K}$ system. Thus, the bosonic model for the $KK\bar{K}$ system leads to a strongly bound state of this system. A model with a significant isospin splitting of the $K\bar{K}$ potential generates a weakly bounded $KK\bar{K}$ system, because the contribution of the exchange interaction is reduced due to the significant decrease of the $K\bar{K}$ attraction. We have demonstrated that the model for the $KK\bar{K}$ system with $K\bar{K}$ interaction having the same strength in the isospin singlet and triplet states is far from Efimov physics. The evaluation of the mass polarization in the $KK\bar{K}$ system in the framework of the Faddeev equations in configuration space allows us to understand, explain, and interpret the contribution from the KK potential to the mass of the $K(1460)$ as a dynamically generated resonance. It is shown that the contribution of mass polarization into the energy of the $KK\bar{K}$ system is large (up to 30%) and depends linearly on the $K\bar{K}$ scattering length. Specifically, the contribution is defined by the mass ratio of nonidentical particles. As a result, relative contributions can be the same for different systems.

We also studied the impact of isospin splitting of the $K\bar{K}$ interaction on the energy of the $KK\bar{K}$ quasibound state. Generally, a model with the isospin dependence of a $K\bar{K}$ potential leads to a decreasing binding energy of the system. In particular, we found that the $K\bar{K}$ potential with an essential difference of isospin components produces a weak quasibound state. The comparison of our calculations with the recent experimental study $1482.40 \pm 3.58 \pm 15.22$ MeV [67], where the first uncertainty is statistical and the second systematic, shows that the mass of the $K(1460)$ resonance is in a satisfactory agreement with the mass upper bound, calculated within our three-body model with isospin splitting $K\bar{K}$ potential. Because of the experimental uncertainties in the relevant observable, one can explore the possible range for the ratio of the strengths of isospin triplet and singlet components of the $K\bar{K}$ interaction. On the other hand, our results obtained by the model with the same strength of the $K\bar{K}$ interaction in the isospin singlet and triplet states are in reasonable agreement with the SLAC and ACCMOR Collaboration experimental values of the mass of $K(1460)$ resonance [64,65].

It is worth noticing that despite its simplicity, the single-channel model can reproduce the mass of the $K(1460)$ resonance. In our consideration there are no any fitting parameters, and we are using s-wave $K\bar{K}$ and KK two-body potentials and kaon masses only as the inputs in our model. The key ingredient of the model is the proper description of the isospin-dependent $K\bar{K}$ interaction. Therefore, some refinements can be done, such as using more realistic two-body potentials, including p-wave components, and/or considering the coupled-channel approach. However, these will not affect dramatically the main conclusions obtained within the present approach.

ACKNOWLEDGMENTS

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APPENDIX A: COULOMB POTENTIAL

The general form of the Faddeev equations with Coulomb interactions reads as follows [77]:

$$\{H_0 + V_\gamma^s(|\mathbf{x}_\gamma|) + \sum_{\beta=1}^{3} V_\beta^{\mathrm{Coul}}(|\mathbf{x}_\beta|) - E\}\Psi_\gamma(\mathbf{x}_\gamma, \mathbf{y}_\gamma) = -V_\gamma(|\mathbf{x}_\gamma|)\sum_{\beta\neq\gamma}\Psi_\beta(\mathbf{x}_\beta, \mathbf{x}_\beta), \tag{A1}$$

where V_β^{Coul} is the Coulomb potential between the particles belonging to the pair β and V_γ is the short-range pair potential in the channel γ ($\gamma = 1, 2, 3$). In (A1) $H_0 = -\Delta_{\mathbf{x}_\gamma} - \Delta_{\mathbf{y}_\gamma}$ is the kinetic energy operator, E is the total energy, Ψ is the wave function of the three-body system given as a sum over three Faddeev components, $\Psi = \sum_{\gamma=1}^{3}\Psi_\gamma$, $\mathbf{x}_\gamma$, and $\mathbf{y}_\gamma$ are the Jacobi coordinates for three particles with unequal masses m_1, m_2, and m_3 having positions $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$ defined as

$$\begin{aligned}
\mathbf{x}_i &= \sqrt{\frac{m_j m_k}{m_j + m_k}}(\mathbf{r}_j - \mathbf{r}_k),\\
\mathbf{y}_i &= \sqrt{\frac{m_j(m_j + m_k)}{M}}\left(-\mathbf{r}_i + \frac{m_j\mathbf{r}_j + m_k\mathbf{r}_k}{m_j + m_k}\right),\\
\mathbf{R} &= (m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3),\\
M &= m_1 + m_2 + m_3, \quad i \neq j \neq k = 1, 2, 3.
\end{aligned} \tag{A2}$$

For a system with two identical particles (A1) is reduced to two equations. The system $K^+K^+K^-$ has two types of Coulomb potentials. The first one is repulsive and describes the interaction between two particles of the same charge and the second is attractive and describes the interaction between two opposite charged particles. Each potential gives the contribution to each equation of the set. For example, the Coulomb potential of the first type is written as $n_1/|\mathbf{x}|$ for the first equation and $n_2/|\mathbf{x}'|$ for the second equation of the set (3), where $\mathbf{x}' = \mathbf{x}/2 + \mathbf{y}$ and n_k, $k = 1$, 2 is the reduced charge $n_k = e^2 m_k/\hbar^2$, and m_k is a reduced mass of the corresponding particle pair.

The particle configurations and corresponding Coulomb forces are schematically presented in Fig. 6. Note that the Coulomb potential of the particle configuration $K^0K^+K^-$ violates the paradigm of the AAB system. To describe this

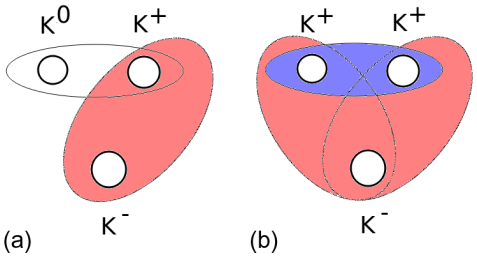

FIG. 6. The structure of the Coulomb force in the particle configurations of the $KK\bar{K}$ system, (a) $K^0K^+K^-$, and (b) $K^+K^+K^-$.

system with the Coulomb potential, one has to use the Faddeev equations (A1) for three nonidentical particles.

APPENDIX B: THE WIDTH OF THE $KK\bar{K}$ SYSTEM

For evaluation of the width of the $KK\bar{K}$ quasiresonance, we taken into account that the real part of the complex $K\bar{K}$ potentials dominates with the ratio for strengths of $\epsilon = 283/1155$ for the potential of the A parameter set and $\epsilon = 210/630$ for the potential with set B. One can write the Hamiltonian of the $KK\bar{K}$ system as $H^R + i\epsilon \mathrm{Im}V_{K\bar{K}}$, where $H^R = H_0 + \mathrm{Re}V_{K\bar{K}} + V_{KK}$. We have taken into account that the KK potential has no imaginary part. This complex-value expression for the Hamiltonian can be transformed into the real 2×2 matrix representation:

$$\begin{pmatrix} H^R & -\epsilon \mathrm{Im}V_{K\bar{K}} \\ \epsilon \mathrm{Im}V_{K\bar{K}} & H^R \end{pmatrix}.$$

The obtained matrix is a rotation-scaling matrix. The complex eigenvalues of the matrix are $H^R \pm i\epsilon \mathrm{Im}V_{K\bar{K}}$. The energy E and width Γ can be obtained by the averaging $E = \langle H^R \rangle \pm i\epsilon \langle V_{K\bar{K}} \rangle = E^R \pm i\Gamma/2$. We have evaluated the averaged $K\bar{K}$ potential energy as $\langle V_{K\bar{K}} \rangle$ for the $K^0K^0\bar{K}^0$ particle configuration. The corresponding results using potentials with the A and B parameter sets are presented in Table IV. A similar result for the parameter set A was obtained in Ref. [69], where the variational calculations have been performed.

APPENDIX C: NUMERICAL SOLUTION OF THE FADDEEV EQUATIONS: THE CODE TESTING

1. Bosonic model for *nnp* system with MT-V potential

Our calculation for the $KK\bar{K}$ system is tested by using the simple model for the nnp system with the spin-independent Malfliet-Tjon nucleon-nucleon potential of the version V (MT-V) [101]. The potential corresponds to a bosonic model for the nnp system, when an isospin/spin independent s-wave potential is used. The MT-V bosonic model was motivated by spin averaging for the spin-dependent MT-III potential [102]. The configuration space Faddeev calculations for the model are based on Eqs. (3). The MT-V potential is defined as $V_{NN}(r) = \sum_{i=1,2} U_i \exp[-(\mu_i r)]/r$, where U_i and μ are strength and range parameters, respectively. The range parameters are $\mu_1 = 1.55$ fm^{-1}, $\mu_1 = 3.11$ fm^{-1}. We used two sets for strength parameters of the potential known from literature [103]: (1) $U_1 = -570.316$ MeV, $U_2 = 1438.4812$ MeV, and (2) $U_1 = -578.098$ MeV, $U_2 = 1458.047$ MeV. The results of the calculations are given in Table. VI. Our results are in good agreement with the results obtained in Ref. [103], where the Faddeev equations in configuration space were also applied. Note that the mass polarization evaluated by the value $\Delta/|E_3|(V_{KK} = 0)$ is similar to the kaonic system $KK\bar{K}$ due to the similar mass ratio. For the bosonic model of the nnp system, the above mentioned (Sec. III) correlations between two- and three-body parameters take place. In particular, the ratio E_3/E_2 with dependence on two-body energy $|E_2|$ presented in Fig. 7 shows the Efimov effect when two-body energy is close to the three-body threshold. Here, the nn potential is scaled by a factor α as $V_{nn} \to \alpha V_{nn}$ with the condition $\alpha > 0$. The strong attraction of the nn pair makes the ratio E_3/E_2 larger in comparison with the $E_3(V_{nn} = 0)/E_2$ case, when the nn interaction is omitted. The results of our calculations for the nnp system with two sets of the MT-V potential for the correlation between the relative contributions of the mass polarization δ and two-body scattering length a are represented in Fig. 7. A similar dependence is shown in Fig. 2 obtained in the case of the $KK\bar{K}$ and $\alpha\Lambda\Lambda$ systems. The correlation between δ and a is represented by a linear dependence with the negative slope as one can see in Fig. 2.

TABLE VI. The energy (E_3) of the bound state of the nnp system within different variants for MT-V nucleon-nucleon potential. The E_2 is the energy of the bound np pair. The energies are given in MeV. The relative contribution of the mass polarization (MP) $\delta = \Delta/|E_3(V_{KK} = 0)|$ is shown. The results of calculations from [103] are listed in parentheses.

Potential	E_3	$E_3(V_{nn} = 0)$	E_2	MP [%]
MT-V(1)	−7.54 (−7.54)	−1.01	−0.35	30.7
MT-V(2)	−8.04 (−8.0424)	−1.16	−0.41	29.3

2. Cluster reduction method versus direct numerical solution

The cluster reduction method [93,94] was alternatively used for a numerical solution of the Faddeev equations (2). The method is based on the expansion of the components U and W in terms of the basis of the eigenfunctions of the two-body Hamiltonian of the subsystems:

$$U(x,y) = \sum_{i\geq 1}^{N} \phi_i^U(x) F_i^U(y), \qquad W(x,y) = \sum_{i\geq 1}^{N} \phi_i^W(x) F_i^W(y). \tag{C1}$$

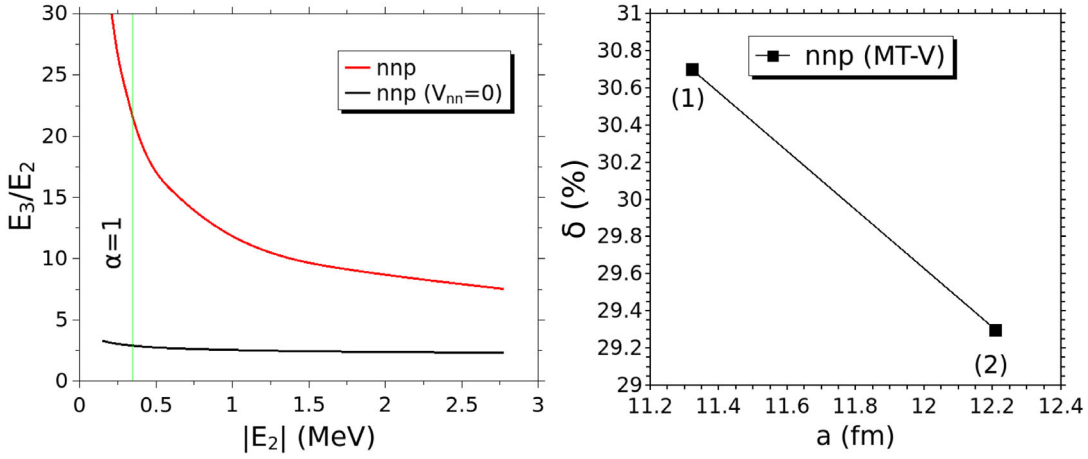

FIG. 7. The ratio E_3/E_2 with dependence on two-body energy $|E_2|$ for the nnp system calculated with the MT-V (1) potential. The parameter is factor α defining the scaled nn potential $V_{nn} \to \alpha V_{nn}$. Left panel: The vertical line corresponds to $\alpha = 1$. Right panel: The correlations between the relative contribution of the mass polarization δ and the np-scattering length a in nnp systems calculated with two versions of NN MT-V potential.

Here, the functions F_i^U and F_i^W, $i = 1, 2, \ldots, N$ describe the relative motion of "clusters" in each rearrangement channel $(KK)\bar{K}$ and $K(K\bar{K})$, respectively. The functions F_i^U (F_i^W) depend on the relative coordinate y. The solutions of the two-body Schrödinger equations form a complete set of eigenfunctions in the box, $x \subset [0, R_x]$:

$$\left(-\frac{\hbar^2}{2\mu^U}\partial_x^2 + V_{NN}^{s_{NN}=0}(x)\right)\phi_i^U(x) = \epsilon_i^U \phi_i^U(x),$$
$$\left(-\frac{\hbar^2}{2\mu^W}\partial_x^2 + V_{N\bar{K}}(x)\right)\phi_i^W(x) = \epsilon_i^W \phi_i^W(x),$$

where μ^U (and μ^W) is a reduced mass of the pairs and $\phi_i^U(0) = \phi_i^U(R_x) = 0 \quad (\phi_i^W(0) = \phi_i^W(R_x) = 0), \quad i = 1, 2, \ldots, N$. The parameter R_x is chosen to be large enough to reproduce the pair binding energy. In our calculations $R_x = 35$ fm is used. The number N is chosen by the condition of total convergence of calculation results, when N consequently increases.

The comparison of the CRM and direct solution is presented in Fig. 8. The results obtained using both methods are in good agreement. The CRM calculations for the case of complete set of potentials and the case of restriction $V_{K^0K^0} = 0$ demonstrate the repulsive nature of the KK potential. The convergence of the calculation results, as a function of the number N of the terms in Eq. (C1) is different for these cases. In the first case, the decrease of binding energy changes to increase when the calculation becomes "more precise" by increasing the number N. For the second case, we have a monotonic decrease in the binding energy. Such behavior is related to the consequent inclusion of the attraction for the $K\bar{K}$ pair and repulsion for the KK pair.

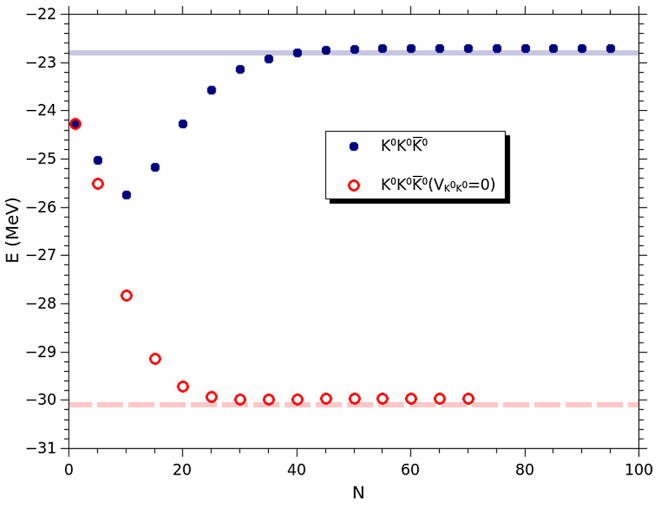

FIG. 8. The binding energy of the $KK\bar{K}$ system $(K^0K^0\bar{K}^0)$ calculated using CRM for different numbers of the terms in the expansion (C1). The case when the KK potential is omitted is also shown. The horizontal lines (solid and dashed) represent the results of the direct numerical solution of the Faddeev equations. The parameter set B of the potentials was applied.

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