Correspondence between strings in the Hagedorn phase and asymptotically de Sitter space

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A correspondence between closed strings in their high-temperature Hagedorn phase and asymptotically de Sitter (dS) space is established. We identify a thermal, conformal field theory (CFT) whose partition function is, on the one hand, equal to the partition function of closed, interacting, fundamental strings in their Hagedorn phase yet is, on the other hand, also equal to the Hartle-Hawking (HH) wave function of an asymptotically dS universe. The Lagrangian of the CFT is a functional of a single scalar field, the condensate of a thermal scalar, which is proportional to the entropy density of the strings. The correspondence has some aspects in common with the anti–de Sitter/CFT correspondence, as well as with some of its proposed analytic continuations to a dS/CFT correspondence, but it also has some important conceptual and technical differences. The equilibrium state of the CFT is one of maximal pressure and entropy, and it is at a temperature that is above but parametrically close to the Hagedorn temperature. The CFT is valid beyond the regime of semiclassical gravity and thus defines the initial quantum state of the dS universe in a way that replaces and supersedes the HH wave function. Two-point correlation functions of the CFT scalar field are used to calculate the spectra of the corresponding metric perturbations in the asymptotically dS universe and, hence, cosmological observables in the postinflationary epoch. Similarly, higher-point correlation functions in the CFT should lead to more complicated cosmological observables.

I. INTRODUCTION

Because of the well-known correspondence between asymptotically anti–de Sitter (AdS) spacetimes and conformal field theories (CFTs) [1–4], along with the observation that the isometries of de Sitter (dS) space act as the conformal group on the dS boundary, it has long been expected that a similar duality should exist between asymptotically dS cosmologies and a different class of CFTs [5–7]. This idea was first put forth by Strominger [5] for the case of an eternal dS spacetime and then later for that of an inflationary cosmology [8,9]. Since dS space has a spacelike asymptotic boundary, this framework leads to a timeless boundary theory and, consequently, a nonunitary CFT. One can perhaps view the boundary theory as a Euclidean CFT by considering

certain analytic continuations of the standard AdS/CFT correspondence [7,10].

The detailed implementation of the dS/CFT correspondence began to take shape with a proposal by McFadden and Skenderis—following from [11]—that the CFT duals to domain walls in Euclidean AdS space could be analytically continued into what would be the CFT duals to Lorentzian inflationary cosmologies [12–17]. The first explicit realization of a dS/CFT duality from its AdS counterpart was presented by Hertog and Hartle in [18] (and further developed by Hertog and others in, e.g., [19–25]), where the relation between AdS/CFT holography and the wave function of the inflationary Universe was made precise. The two approaches differ in that McFadden and Skenderis consider quantum fluctuations about a real classical geometry, whereas Hertog *et al.* employ complex semiclassical saddle point solutions of the gravitational and the matter path integral. In this sense, only the latter is proposing a quantum wave function of the Universe along the same lines as that proposed by Maldacena [7]. On the other hand, both are similar in their treatments of cosmological perturbations and the late-time state.

In spite of these successful programs, some have argued that a direct relation between a dS/CFT correspondence

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and an explicit string theory relaization is still lacking (e.g., [26]). For instance, there is significant evidence that ultraviolet completion of a stable dS space is incompatible with semiclassical quantum gravity [27–29]. But, for a more optimistic viewpoint, as well as an update on recent progress, see [30,31].

The main purpose of the current paper is to make a concrete proposal for a new type of dS/CFT correspondence; one that is conceptually different than previous attempts. Our proposed CFT dual is at finite temperature and so is not obviously scale invariant, but we will nevertheless argue that it is. The CFT is the theory of the so-called thermal scalar and, as an effective description of a multi-string partition function, has played an important role in understanding the Hagedorn phase of string theory [32–38]. The correspondence is substantiated by showing that, when the fields and parameters of the two theories are suitably matched, the partition function of the CFT is equal to the Hartle–Hawking (HH) wavefunction [39] of an asymptotically dS Universe.[1] This equivalence is established in the semiclassical regime for which the HH wavefunction can be defined.

We are interested in the case that the equilibrium state of the CFT is a thermal state of closed, interacting, fundamental strings in their Hagedorn phase. Such a state of strings is known to be one possessing maximally allowed pressure [34] and maximal entropy [41]. We have recently proposed that this state should describe the initial state of the Universe [42]; the motivation being that a state of maximal entropy is just what is needed to resolve spacelike singularities such as the interior of an event horizon or the preinflationary Universe. The latter case leads to a duality connecting the string state to dS space and, as shown in our current discussion, implies a duality between dS space and a thermal-scalar condensate. [43]. Indeed, previous studies by Silverstein and collaborators have discussed, in a very different context, how a tachyon condensate can be used to tame spacelike singularities [44–47]. Note though that this dS spacetime is the invented artifact of a late-time observer, who wishes to explain the state's origins and properties by imposing some form of semiclassical evolution. In our framework, this notion of dS space does not really exist, certainly not as a semiclassical state.

The equilibrium state is maximally entropic in the sense that its spatially uniform entropy density is equal to the square root of its spatially constant energy density in Planck units and, thus, the former density saturates the causal entropy bound [48]. On the dS side of the correspondence, maximal entropy translates into the Gibbons–Hawking values of the entropy within a cosmological horizon [49] and the constant energy density is interpreted as a cosmological constant. In previous articles, starting with [50], we have interpreted the saturation of the causal

entropy bound as indicating that such a state cannot be described by a semiclassical geometry. Nonetheless, the Lagrangian of the CFT can be used to calculate cosmological observables in spite of the lack of a semiclassical geometric description. The Lagrangian that is presented here extends a free energy that was first introduced in [51,52] to describe Schwarzschild black hole (BH) interiors. This free energy is expressed as a power series in the entropy density and has a form that was adapted from the free energy of polymers (e.g., [53–55]).

Having identified the CFT dual for dS space, we can calculate correlations functions in the CFT and then translate these into cosmological observables in the postinflationary epoch without relying on semiclassical dS calculations. Our focus is on calculating the power spectra for the tensor and scalar perturbations. We have already presented qualitative expressions for these scale-invariant spectra in [42], but the CFT improves on this by providing a precise prescription for the relevant calculations. The results presented here are shown to be in agreement with those of standard inflationary calculations [56] and with those obtained using the HH wave function [57–59].

Our proposed model has some features in common with those of string-gas cosmology as presented by Brandenberger, Vafa, and collaborators [35,60–63], as well as with the holographic cosmology model of Banks and Fischler [64,65]. However, as discussed at length in [42], such similarities are mostly superficial due to differences both in the physical substance and in the resulting predictions.

A main difference between our proposal for a dS/CFT duality and previous ones is that ours does not rely upon an intermediating semiclassical Euclidean AdS solution. Also, our duality is holographic but not in the usual way: it is holographic in the sense that the thermal scalar condenses by winding around a compactified Euclidean time loop. So the thermal scalar field theory "lives" in one less dimension. This is not equivalent to taking a small limit S^1 in a bulk gravity description. Strings are essential, as there is no condensate as a matter of principle without strings winding on a string-length-sized thermal circle. It is also worth emphasizing that there is a clear string-theory origin for our model because, as is well known, the effective field theory of the thermal scalar can be used to calculate the string partition function near the Hagedorn temperature.

Briefly on the contents, the next section introduces the CFT Lagrangian, Sec. III discusses the various aspects of the theory in terms of thermal-scalar condensate and Sec. IV establishes the correspondence to dS space. We then present our calculations of the cosmological observables in Sec. V and conclude in Sec. VI.

II. THERMAL SCALAR OF CLOSED STRINGS IN THE HAGEDORN PHASE

Let us begin here with the quantum partition function for closed, interacting strings $Z = Tre^{-\beta H}$, where H is the

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[1]For a recent discussion of the HH wavefunction, see [40].

Hamiltonian and β is related to the temperature T as in Eq. (2). The partition function and its associated thermal expectation values can be calculated in terms of a Euclidean action $\mathcal{S}_E$ that is obtained by compactifying imaginary time on a "thermal circle,"

$$\mathcal{S}_E = \oint_0^\beta d\tau \sqrt{g_{\tau\tau}} \int d^d x \sqrt{\gamma} \mathcal{L}_E, \qquad (1)$$

where

$$\frac{1}{T} = \oint_0^\beta d\tau \sqrt{g_{\tau\tau}}, \qquad (2)$$

and where the $D = d + 1$-dimensional coordinate system and metric tensor should be regarded as those of a fiducial manifold, since the string state lacks a semiclassical geometry. We will be discussing the case in which temperatures are close to but slightly above the Hagedorn temperature, $T \gtrsim T_{\mathrm{Hag}}$ and $T - T_{\mathrm{Hag}} \ll T_{\mathrm{Hag}}$. It follows that the circumference of the thermal circle is on the order of the string length l_s.

Compactifying time and ignoring the time-dependence of the fields amounts to reducing the dimensionality of the theory from $d + 1$ to d. The result is then a "timeless" theory living on a d-dimensional spatial hypersurface, just as expected from a would be dS/CFT correspondence.

Strings can wind around the thermal circle and the resulting picture can be described by using the well-studied theory of the thermal scalar [32–38]. The $+1$ winding mode is denoted by ϕ and its -1 counterpart is denoted by ϕ^*. As the winding charge is a conserved quantity, the Lagrangian is required to be a functional of $|\phi|^2$. The path integral of the thermal scalar is known to provide an effective (but complete) description of the multistring partition function when the temperature is close to the Hagedorn temperature.

The Lagrangian of the thermal scalar can be expressed as

$$\mathcal{L}_E(\phi, \phi^*) = \frac{1}{2} \gamma^{ij} \partial_i \phi \partial_j \phi^* - c_1 \varepsilon T \phi \phi^*$$
$$+ \frac{1}{2} c_2 g_s^2 T^2 (\phi \phi^*)^2 + \cdots, \qquad (3)$$

where $\varepsilon = T - T_{\mathrm{Hag}}$, g_s^2 is the dimensional string-coupling constant and the positive, dimensionless numerical coefficients c_1 and c_2 depend on the specific string theory. The ellipsis denotes higher-order interactions, both here and below (and will sometimes be omitted). The relative unimportance of these higher-order terms will be discussed in the next section. The potential for the thermal scalar was introduced a long time ago in [34]. We have made here a choice of sign that ensures a non-trivial solution in the regime of interest (see below). The total mass dimension of the Lagrangian density has to, of course, be $d + 1$. Because the mass dimension of ε is $+1$ and that of the dimensional

coupling g_s^2 is $-(d - 1)$, it then follows that the mass dimension of ϕ is $+\frac{d-1}{2}$. We may absorb the numerical coefficients by the redefinitions $c_1 \varepsilon \to \varepsilon$ and $c_2 g_s^2 \to g_s^2$, thus giving

$$\mathcal{L}_E(\phi, \phi^*) = \frac{1}{2} \gamma^{ij} \partial_i \phi \partial_j \phi^* - \varepsilon T \phi \phi^* + \frac{1}{2} g_s^2 T^2 (\phi \phi^*)^2. \quad (4)$$

For temperatures below the Hagedorn temperature ($\varepsilon < 0$), the thermal scalar is known to have a positive mass squared [34]. Meanwhile, its mass vanishes at Hagedorn transition temperature $\varepsilon = 0$, and so it is tempting to adopt the standard viewpoint that the phase transition is describing the condensation of closed-string winding modes about the thermal circle. This perspective is especially interesting for the case of BHs, as it aligns nicely with earlier proposals that a Euclidean BH—albeit one in an AdS spacetime—could be related to the condensation of the thermal scalar [38,66–71]. However, as should become clear by the end of the section, the Lagrangian (4) has to be regarded as an expansion near a nontrivial minimum of the potential which lies *above* the Hagedorn temperature. The restriction to trans-Hagedorn temperatures can understood by noticing that the entropy and energy densities both vanish for $\varepsilon = 0$ [cf. Eqs. (27)–(28)] and that the former density formally becomes negative for $\varepsilon < 0$. Hence, the Lagrangian (4) cannot be used directly to describe the Hagedorn phase transition and reproduce its expected first-order character.

The equation of motion $\phi^* \delta \mathcal{L}_E / \delta \phi^* = 0$ is as follows:

$$-\frac{1}{2} \phi^* \nabla^2 \phi - \varepsilon T \phi \phi^* + g_s^2 T^2 (\phi \phi^*)^2 = 0. \qquad (5)$$

An interesting solution of the above equation and its conjugate is one in which the thermal scalar condenses,

$$|\phi_0|^2 = \frac{\varepsilon}{g_s^2 T}. \qquad (6)$$

It will be shown later that this ratio is a small number in comparison to the Hagedorn scale, $\varepsilon / (g_s^2 T_{\mathrm{Hag}}) \ll T_{\mathrm{Hag}}^{d-1}$.

Expanding the Lagrangian about this constant solution, $\phi = \phi_0 + \varphi$, $\phi^* = \phi_0 + \varphi^*$, we find that

$$\mathcal{L}_E = \frac{1}{2} \gamma^{ij} \partial_i \varphi \partial_j \varphi^* + \varepsilon T \varphi \varphi^* + \frac{1}{2} g_s^2 T^2 (\varphi \varphi^*)^2 - \frac{1}{2} \frac{\varepsilon^2}{g_s^2}. \quad (7)$$

One may also include a coupling to the Ricci scalar in the Lagrangian. For instance, if a conformal coupling is chosen, then $\mathcal{L}_E \to \mathcal{L}_E - \frac{d-1}{4d} R \varphi \varphi^*$. The importance of this inclusion will be revealed later on; however, as one always has the freedom to choose Ricci-flat fiducial coordinates, this term cannot be relevant to the calculation of physical observables.

The expanded Euclidean action is thus given by

$$S_E = \frac{1}{T} \int d^d x \sqrt{\gamma} \left\{ \frac{1}{2} \gamma^{ij} \partial_i \varphi \partial_j \varphi^* - \frac{d-1}{4d} R \varphi \varphi^* \right.$$

$$\left. + \varepsilon T \varphi \varphi^* + \frac{1}{2} g_s^2 T^2 (\varphi \varphi^*)^2 - \frac{1}{2} \frac{\varepsilon^2}{g_s^2} \right\}. \tag{8}$$

The action in Eq. (8) is similar to the standard expression in the literature (e.g., [34,36]).

III. THERMAL SCALAR CONDENSATE

In this section, we elaborate on some of the consequences for our theory when the thermal scalar condenses.

A. Euclidean action

In the case of condensation, it is simpler to use the real field,

$$s = |\phi|^2 T \tag{9}$$

as the fundamental field; for which the expectation value at the minimum is then

$$s_0 = \frac{\varepsilon}{g_s^2}. \tag{10}$$

We have denoted the field by s because its condensate value s_0 is the same as the local entropy density of the strings (see below).

Let us now rewrite the Lagrangian (4) as a functional of s,

$$\mathcal{L}_E(s) = \frac{1}{8} \frac{1}{sT} \gamma^{ij} \partial_i s \partial_j s - \varepsilon s + \frac{1}{2} g_s^2 s^2. \tag{11}$$

Expanding the above near the minimum $s = s_0(1 + \sigma(x_i))$, keeping only quadratic terms and recasting it as a compactified Euclidean action as in Eq. (8), we have

$$S_E^{(2)} = \frac{1}{T} \int d^d x \sqrt{\gamma} \mathcal{L}_E(\sigma) + S_0, \tag{12}$$

such that

$$S_0 = -\frac{1}{T} \int d^d x \sqrt{\gamma} \frac{1}{2} \frac{\varepsilon^2}{g_s^2} \tag{13}$$

and

$$S_E^{(2)} = \frac{1}{g_s^2 T} \int d^d x \sqrt{\gamma} \left\{ \frac{1}{8} \frac{\varepsilon}{T} \gamma^{ij} \partial_i \sigma \partial_j \sigma + \frac{1}{2} \varepsilon^2 \sigma^2 \right.$$

$$\left. - \frac{d-1}{16d} \frac{\varepsilon}{T} R \sigma^2 \right\} + S_0, \tag{14}$$

with the conformal coupling to R included for completeness.

The equation of motion that results from the action (14), for the case of Ricci flatness, is found to be

$$-\nabla^2 \sigma + 4\varepsilon T \sigma = 0. \tag{15}$$

The field σ is therefore a massive, conformally coupled scalar with a positive thermal mass-squared, $m^2 = +4\varepsilon T$. This value for m^2 can be compared with the magnitude of the negative mass-squared of the thermal scalar when it is below the Hagedorn temperature, $m^2 = -\varepsilon T$ (e.g., [36]).

We may absorb the dimensionality of g_s^2 and ε by rescaling them with appropriate powers of the temperature,

$$\tilde{g}_s^2 = g_s^2 T^{d-1}, \tag{16}$$

$$\epsilon = \frac{\varepsilon}{T}. \tag{17}$$

In which case,

$$S_E^{(2)} = \frac{1}{\tilde{g}_s^2} T^d \int d^d x \sqrt{\gamma} \left\{ \frac{1}{8} \epsilon \frac{1}{T^2} \gamma^{ij} \partial_i \sigma \partial_j \sigma + 2\epsilon^2 \sigma^2 \right.$$

$$\left. - \frac{d-1}{16d} \epsilon \frac{1}{T^2} R \sigma^2 \right\}. \tag{18}$$

As the field σ is dimensionless by its definition, the only remaining dimensional parameter is T, making this a thermal CFT. We will explain how scale and Weyl transformations act on this action, after discussing the higher-order interactions.

Higher-order (HO) terms in the action come about in two different ways: (I) more than two strings intersecting at a single point or (II) the same pair of strings intersecting at two or more different points. Additional action terms of the former kind are

$$S_E^{(HO,I)} = \frac{1}{T} \int d^d x \sqrt{\gamma} \left\{ \frac{a_3}{3!} \frac{1}{T} (g_s^2)^2 s^3 + \frac{a_4}{4!} \frac{1}{T^2} (g_s^2)^3 s^4 + \cdots \right\}, \tag{19}$$

where the a's (and b's below) are numerical coefficients and the additional powers of temperature are dictated by the scaling dimensions of the various quantities. Expanding about the minimum $s = s_0(1 + \sigma(x_i))$, we then have

$$S_E^{(HO,I)} = \frac{1}{g_s^2 T} \int d^d x \sqrt{\gamma} \left\{ \frac{a_3}{3!} \varepsilon^2 \epsilon (1 + \sigma)^3 \right.$$

$$\left. + \frac{a_4}{4!} \varepsilon^2 \epsilon^2 (1 + \sigma)^4 + \cdots \right\}$$

$$= \frac{T^d}{\tilde{g}_s^2} \int d^d x \sqrt{\gamma} \left\{ \frac{a_3}{3!} \epsilon^3 (1 + \sigma)^3 \right.$$

$$\left. + \frac{a_4}{4!} \epsilon^4 (1 + \sigma)^4 + \cdots \right\}, \tag{20}$$

where all parameters and fields besides T are explicitly dimensionless in the lower line. As the small expansion parameter in this case is $\epsilon = \frac{T - T_{\text{Hag}}}{T} \ll 1$, these corrections can be identified as α' corrections in the effective action.

Higher-order terms coming from the same strings intersecting at two or more different points take the form,

$$\mathcal{S}_E^{(\text{HO},II)} = \frac{1}{T} \int d^d x \sqrt{\gamma} \left\{ \frac{b_2}{2!} T^{d-1} (g_s^2)^2 s^2 + \frac{b_3}{2!} T^{2(d-1)} (g_s^2)^3 s^2 + \cdots \right\}. \tag{21}$$

Once again expanding about the minimum and converting to dimensionless quantities, we obtain

$$\mathcal{S}_E^{(\text{HO},II)} = \frac{1}{T g_s^2} \int d^d x \sqrt{\gamma} \left\{ \frac{b_2}{2!} T^{d-1} (g_s^2) \varepsilon^2 (1+\sigma)^2 + \frac{b_3}{2!} T^{2(d-1)} (g_s^2)^2 \varepsilon^2 (1+\sigma)^2 + \cdots \right\}$$

$$= \frac{T^d}{\tilde{g}_s^2} \int d^d x \sqrt{\gamma} \left\{ \frac{b_2}{2!} \tilde{g}_s^2 \varepsilon^2 (1+\sigma)^2 + \frac{b_3}{2!} (\tilde{g}_s^2)^2 \varepsilon^2 (1+\sigma)^2 + \cdots \right\}. \tag{22}$$

The small expansion parameter in this case is $\tilde{g}_s^2 = g_s^2 T^{d-1}$, and so these are identifiable as string loop corrections in the effective action.

There are, of course, more complicated higher-order interaction terms involving both string-coupling and α' corrections. All of these corrections are parametrically small provided that the requisite hierarchy $\epsilon \ll \tilde{g}_s^2 < 1$ (see Subsection III C) is respected.

B. Conformal symmetry

Let us now discuss the transformation properties of the theory under Weyl transformations. We first restrict attention to the case of constant Weyl transformations, which correspond to scale transformations of the coordinates. For the $d + 1$-dimensional Euclidean theory, the constant Weyl transformations can be expressed as

$$g_{\tau\tau} \to \Omega^2 g_{\tau\tau},$$
$$\gamma_{ij} \to \Omega^2 g_{ij}. \tag{23}$$

As we have seen, the dimensional coupling parameters g_s^2 and ε can be rendered dimensionless by rescaling them with appropriate powers of the temperature, as done in Eqs. (18), (20) and (22). Meaning that the only remaining dimensional parameter is the temperature. The question then is how to interpret the parameter T in the d-dimensional compactified theory. If one considers the temperature to be a fixed dimensional parameter, then this is obviously not a scale-invariant theory. However, if one rather considers that the temperature is the inverse of the circumference of the thermal circle as in Eq. (2), $\frac{1}{T} = \oint_0^\beta d\tau \sqrt{g_{\tau\tau}}$, then it obviously varies under a Weyl transformation as

$$T \to T/\Omega. \tag{24}$$

Then, in this case, the variation of the metric in each of Eqs. (18), (20) and (22) is exactly canceled by the variation of the temperature, as the product $T^d \sqrt{\gamma}$, in particular, is scale invariant. Since the zeroth-order part of the action in Eq. (13), $\mathcal{S}_0 = T^d \int d^d x \sqrt{\gamma} \frac{1}{2} \frac{\varepsilon^2}{\tilde{g}_s^2}$, transforms similarly, the complete action is scale invariant.

When the temperature varies as in Eq. (24), the theory is also invariant under general x-dependent Weyl transformations,

$$g_{\tau\tau} \to \Omega^2(x_i) g_{\tau\tau},$$
$$\gamma_{ij} \to \Omega^2(x_i) \gamma_{ij}. \tag{25}$$

The only term that is sensitive to the difference between constant and x-dependent Weyl transformations is the kinetic term. However, the conformal coupling of the scalar to the Ricci scalar ensures the invariance of the kinetic term even under spatially dependent Weyl transformations. It can then be concluded that, when the parameter T varies according to Eq. (24), the thermal-scalar condensate is described by a CFT, in spite of the appearance of a dimensional scale—the temperature.

C. Free energy and thermodynamics

For the physical interpretation of the condensate solution, it is helpful to recall our previous discussions on the Helmholtz free energy of strings that are slightly above the Hagedorn temperature [51,52]. There, we proposed a free energy density which is similar to those of polymers with attractive interactions (e.g., [53–55]). In particular, the free energy density F/V should be regarded as an expansion in terms of the entropy density s such that $s \ll T_{\text{Hag}}^d$,

$$-\left(\frac{F}{V}\right)_{\text{strings}} = \varepsilon s - \frac{1}{2} g_s^2 s^2 + \cdots, \tag{26}$$

where the ellipsis, as usual, denotes higher-order interaction terms. The right-hand side of Eq. (26) is the same as the potential in Eq. (11).

From this stringy point of view, ε should be regarded as the strings' effective temperature. That is, the temperature

associated with the collective motion of long strings, rather than the local value of the temperature of small pieces of string (or "string bits") for which the temperature is much higher, $\varepsilon \ll T \sim T_{\text{Hag}}$.

The first term on the right of Eq. (26) represents the Helmholtz free energy of a free string. In the free case and in string units ($l_s = 1$), both the energy E and the entropy S are equal to the total length L of the strings, $E = L$ and $S = L$. It follows that $F/V = (E - ST)/V = (1 - T)L/V$ and then, since $s = S/V = L/V$ and $\varepsilon = T - T_{\text{Hag}}$, also that $F/V \simeq -\varepsilon s$, where we have approximated $T \simeq T_{\text{Hag}} \simeq 1/l_s = 1$.

The second term on the right of Eq. (26)—the leading-order interaction term—can be understood by recalling that a closed string interacts at its intersections, either with itself or with another string. The simplest such interactions being those for which two closed strings join to form one longer one or one closed string splits into two shorter ones. Since the probability of interacting is given by the dimensionless string-coupling constant $\tilde{g}_s^2$, and again under the assumptions that $T \sim T_{\text{Hag}} \sim 1$ and that any numerical or phase-space factors were absorbed into the dimensional coupling, the total interaction strength is proportional to $\tilde{g}_s^2 L^2/V = \tilde{g}_s^2 s^2 V$. As for the higher-order terms, these will include extra factors of $\tilde{g}_s^2 L/V \sim \tilde{g}_s^2 s \sim \varepsilon$ [see Eq. (27) below] and/or $\tilde{g}_s^2$ when the same strings intersect at multiple points. Therefore, $\varepsilon, \tilde{g}_s^2 < 1$ are necessary conditions for these interactions to be suppressed. Equation (27) below further implies the hierarchy, $\varepsilon \ll \tilde{g}_s^2 < 1$.

The minimization of the free energy defines the equilibrium state. Doing so, one obtains what was previously identified as the condensate solution,

$$s = \frac{\varepsilon}{g_s^2}, \qquad (27)$$

which along with standard thermodynamics (with ε serving as the temperature) yields the equilibrium relations,

$$p = \rho = \frac{1}{2} \frac{\varepsilon^2}{g_s^2}, \qquad (28)$$

where the first equality is independent of Eq. (27). The causal entropy bound is indeed parametrically saturated since $s \sim \sqrt{\rho}$.

D. An effective two-dimensional conformal field theory

As previously discussed, the thermal-scalar condensate can be viewed as a d-dimensional Euclidean CFT. However, as we now show, it is effectively a two-dimensional CFT. This aspect of the thermal scalar was noticed a long time ago in [41] and is implicit in [34]. We have already discussed this feature of the theory in the context of BHs in [51,52].

The free energy density of a D-dimensional (Euclidean) CFT at temperature $1/\beta$ is expressible as[2] $F/V = f_\beta \beta^{-D}$, where f_β is a numerical coefficient. This leads to an energy density of the form $\rho = -(1 - \frac{1}{D}) b_\beta \beta^{-D}$, with b_β being another number. The two coefficients are related according to $f_\beta = b_\beta/D$ and an expression for the entropy density s promptly follows, $s = -b_\beta \beta^{-(D-1)}$.

For the case of $D = 2$,

$$F_2/V = \frac{1}{2} (b_\beta)_2 \beta^{-2}, \qquad (29)$$

$$\rho_2 = -\frac{1}{2} (b_\beta)_2 \beta^{-2}, \qquad (30)$$

$$s_2 = -(b_\beta)_2 \beta^{-1}. \qquad (31)$$

Whereas, in our case,

$$F/V = -\frac{1}{2} \frac{\varepsilon^2}{g_s^2}, \qquad (32)$$

$$\rho = \frac{1}{2} \frac{\varepsilon^2}{g_s^2}, \qquad (33)$$

$$s = \frac{\varepsilon}{g_s^2}. \qquad (34)$$

Identifying ε as the effective temperature,

$$\varepsilon = 1/\beta, \qquad (35)$$

and setting

$$(b_\beta)_2 = -1/g_s^2, \qquad (36)$$

one can see a perfect match between Eqs. (32)–(34) and Eqs. (29)–(31).

Moreover, if we adopt the standard parametrization for the energy density of a two-dimensional CFT in terms of the central charge c, $\rho = \frac{\pi}{6} c \beta^{-2}$ (see, e.g., [73]), then

$$c = \frac{3}{\pi} \frac{1}{g_s^2}, \qquad (37)$$

and it follows that

$$s = \frac{\pi}{3} c \beta^{-1}. \qquad (38)$$

The relations $c \sim 1/g^2$ and $s \sim c$ are indeed universal features of CFTs, whereas the numerical coefficients depend on additional detailed information. The central charge is also expected to be related to the two-point

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[2]In this subsection, we often adopt notation from [72].

function of the stress-energy tensor as $\langle T^0{}_0 T^0{}_0 \rangle \sim c$. This will be verified in detail next.

In CFTs at finite temperature, an operator with a non-vanishing conformal dimension can have a non-zero expectation value (i.e., a thermal one-point function),

$$\langle O \rangle_\beta = \frac{A_O}{\beta^{\Delta_O}}, \qquad (39)$$

where Δ_O is the conformal dimension and A_O is a dimensionless coefficient for the operator O. The scaling of such a one-point function can be specified in terms of the stress-energy tensor,

$$\frac{\partial \langle O \rangle_\beta}{\partial \beta} = -\frac{1}{\beta} \int d^{d+1}x \langle T^0{}_0(\vec{x}) O(0) \rangle^c_\beta, \qquad (40)$$

where the superscript c signifies a connected function.

Choosing O as the stress–energy tensor itself, one obtains

$$\frac{\partial \langle T^0{}_0 \rangle_\beta}{\partial \beta} = -\frac{1}{\beta} \int d^{d+1}x \langle T^0{}_0(\vec{x}) T^0{}_0(0) \rangle^c_\beta$$
$$= -\int d^d x \langle T^0{}_0(\vec{x}) T^0{}_0(0) \rangle^c_\beta, \qquad (41)$$

where the time circle has now been compactified to a circumference of $\beta = 1/\varepsilon$ so as to agree with the definition of the stress-energy tensor. Both sides of Eq. (41) have explicit expressions in the CFT, and so we can verify the relationship directly, a highly unusual situation for interacting CFTs.

First, using Eqs. (33), (35) and the Euclidean identification $T^0{}_0 = \rho$, one can translate the left-hand side of Eq. (41) into

$$\frac{\partial \langle T^0{}_0 \rangle_\beta}{\partial \beta} = -\frac{\varepsilon^3}{g_s^2}. \qquad (42)$$

The evaluation of the right-hand side of Eq. (41) requires some additional ingredients. Since the Euclidean action is expressed in terms of the entropy density s, a direct relationship between $T^0{}_0$ and s is required. For this, recalling that ε is the effective temperature, we rely on the thermodynamic relation $\delta\rho = \varepsilon\delta s$. It follows that

$$T^0{}_0(\vec{x}) - \langle T^0{}_0(\vec{x}) \rangle_\beta = \varepsilon(s(\vec{x}) - \langle s(\vec{x}) \rangle_\beta), \qquad (43)$$

and so

$$\langle T^0{}_0(\vec{x}) T^0{}_0(0) \rangle^c_\beta = \varepsilon^2 \langle s(\vec{x}) s(0) \rangle^c_\beta. \qquad (44)$$

We are interested in the limit $|\vec{x}|\varepsilon \gg 1$, as this will later be shown to describe superhorizon scales. In this case, the Euclidean action reduces to a single term, as can be seen from Eq. (14),

$$S_E \sim \beta \int d^d x \frac{1}{2} g_s^2 (s - \langle s \rangle)^2. \qquad (45)$$

The two-point function of s can then be readily evaluated in terms of a Gaussian integral, again using $T = \varepsilon$,

$$\langle s(\vec{x}) s(0) \rangle^c_\beta = \int \mathcal{D}[s] s(\vec{x}) s(0) e^{-S_E(s;\beta)} = \frac{\varepsilon}{g_s^2} \delta^d(\vec{x}), \qquad (46)$$

which, by way of Eq. (44), leads to

$$\langle T^0{}_0(\vec{x}) T^0{}_0(0) \rangle^c_\beta = \frac{\varepsilon^3}{g_s^2} \delta^d(\vec{x}). \qquad (47)$$

It can now be verified that the right-hand side of Eq. (41),

$$-\int d^d x \langle T^0{}_0(\vec{x}) T^0{}_0(0) \rangle^c_\beta = -\frac{\varepsilon^3}{g_s^2}, \qquad (48)$$

matches its left-hand side, as shown in Eq. (42). Similarly, one could also discuss the conformal dimension of $T^i{}_i = -p$ and find agreement between both sides of Eq. (41). Finally, Eq. (47) makes clear the expected relationship between the stress-energy tensor and the central charge (37), $\langle T^0{}_0(\vec{x}) T^0{}_0(0) \rangle^c_\beta \sim 1/g_s^2 \sim c$.

IV. CORRESPONDENCE TO AN ASYMPTOTICALLY DE SITTER UNIVERSE

We will now set up the correspondence between dS space and the theory of the thermal scalar in a similar manner to that of AdS/CFT [2,3], but yet with significant differences. To establish our proposed correspondence, it will be shown that the HH wave function Ψ_{HH} of an asymptotically dS Universe can be calculated using the partition function of the CFT of the thermal-scalar condensate. The same CFT can be viewed as "living" on a spacelike surface which should also be regarded as the future boundary of its asymptotically dS dual.

Here, we are considering a situation in which an asymptotically dS spacetime decays into a radiation-dominated Universe. From the perspective of the microscopic string state, this corresponds to the phase transition from the Hagedorn phase of long strings to a thermal state of radiation. As argued in [42], we do expect the Hagedorn phase to be unstable, due to either a process which is similar to Hawking radiation or else to some coherent perturbation. From the viewpoint of the semiclassical spacetime, this decay corresponds to the reheating of the Universe after inflation. The correlation functions then become temperature perturbations and are the late-time observables, just as in the standard inflationary paradigm. Meaning that the late-time, Friedmann-Robertson-Walker (FRW) observers are the "metaobservers" [6] or "score-keeping observers" [42] of the early inflationary epoch.
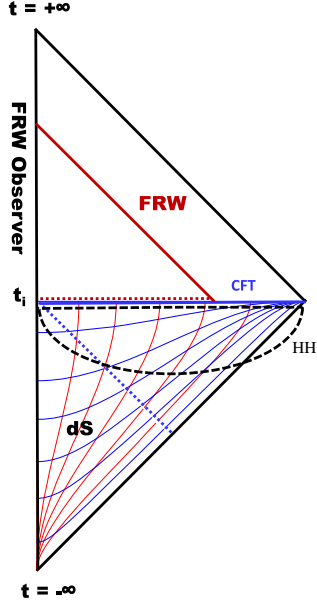
FIG. 1. The correspondence between the CFT and dS space. The HH wave function is calculated on a Euclidean section of a $d + 1$-dimensional space, as depicted by the black, dashed semicircle, while the Euclidean CFT is d-dimensional and "lives" on the future boundary of dS, as depicted by the solid, blue line. In the upper half, the late observer's past light cone is displayed by the solid, red line, while in the lower half, lines of constant planar-dS coordinates t and r are shown in red (approximately vertical) and blue (approximately horizontal), respectively.

As the FRW evolution starts in a thermal state, an FRW observer might be compelled to invent a prehistory to explain the observable Universe. This is similar to the way that a semiclassical observer invents a description of the BH interior [43] (and see below). Three possible such prehistories are shown in Fig. 1.

An FRW observer would then conclude that the Universe exponentially expanded during some epoch in its prehistory, for which the inflationary paradigm provides a possible explanation. But let us emphasize the essential point that the inflationary paradigm is an invented effective history of the Universe. What is physically real are the results of the measurements that are made by an FRW observer after the end of inflation [42].

It is interesting to compare the just-discussed cosmological picture to the corresponding situation in the case of BHs. In the latter case, it is clear that an asymptotic, external observer is the one who can eventually measure observables using the quantum state of the emitted radiation and is, therefore, the score keeper for the interior. The cosmological analogue—perhaps not quite as obvious—is the late-time or FRW observer. The distant past of this observer, before the beginning of the hot-radiation phase, is the analogue of the BH interior. We similarly argued for the case of BHs [43] (also see [74]) that all proposals for the prehistory are perfectly acceptable as long as they are

self-consistent, able to reproduce the observable Universe and compatible with the laws of physics. By this line of reasoning, the puzzles of the FRW observer originate from trying to explain what is an intrinsically quantum initial state in terms of effective semiclassical physics. The same situation was prevalent for BHs and led to the infamous BH paradoxes. As will be shown here, the FRW observer can interpret what is a maximally entropic state as one of vanishing entropy with an approximate description in terms of the flat-space slicing of a classical dS spacetime.

Let us briefly review the original proposal, first put forward by Witten [6] and later by Maldacena [7] (also see [10]), that the equality between the HH wave function of an asymptotically dS Universe and the partition function of some CFT should serve as a requirement for setting up a dS/CFT correspondence. The idea was to start with a Euclidean AdS spacetime but regard the direction perpendicular to the boundary—which is the radial coordinate in AdS space—as the time coordinate in a Euclidean dS spacetime. However, to the best of our knowledge, this idea was never explicitly realized in a way that is directly related to, or consistent with string theory [26]. The suggested equality $\Psi_{HH}(g_{ij}, J) = Z_{\text{CFT}}(g_{ij}, J)$ relied on certain identifications: The d-dimensional metric g_{ij} represents, on the left, the reduction of the $(d + 1)$-dimensional dS metric on the spacelike boundary and, on the right, the metric of the CFT. As for J, its dS meaning is the boundary values of fields (like the graviton) which can be used to set initial conditions for their post-inflationary evolution, whereas its CFT meaning is the sources for the fields in the CFT Lagrangian.

Correlation functions of operators in the CFT were supposed to be calculated in the standard way; as derivatives of the partition function with respect to the sources. Given the above interpretation, these correspond on the dS side to the boundary values of bulk expectation values of spacetime fields. For example, if a dS scalar field ϕ is considered, then $\langle \phi^2 \rangle = \int [D\phi] \phi^2 |\Psi_{HH}(\phi)|^2$, whereas $\langle \phi^2 \rangle = \frac{\delta Z_{\text{CFT}}}{\delta J_\phi \delta J_\phi}|_{J_\phi=0}$.

We will follow [6,7] in taking the bulk spacetime as being the Poincaré patch of dS space in planar coordinates and the ground state of the bulk fields as being in the Bunch-Davies vacuum. However, the identifications between dS and CFT quantities will be different. We will start by identifying the physical components of the two different stress-energy tensors, that of the asymptotically dS bulk and that of the CFT. The perturbed Einstein equations in the bulk will then be used to find a relationship between dS metric perturbations and perturbations of the CFT stress–energy tensor. We cannot use the CFT metric for this purpose because it is a fiducial, unphysical field. As for the stress-energy tensor of the CFT, it cannot be obtained as the derivative of the Lagrangian with respect to such a fiducial metric. Rather, it has to be defined in terms of the energy density and the pressure of the strings.

TABLE I.   Dimensional parameters in dS space and the thermal CFT.

dS	FT
G_D	g_s^2
H	ε

Our current interest is in the case of pure gravity, so that the only relevant bulk fields are the tensor and scalar perturbations of the metric. In what follows, we will make the abstract equality $\Psi_{HH} = Z_{\text{CFT}}$ explicit and then use it to calculate correlation functions of the relevant fields. The correlation functions are our ultimate interest because these are what correspond to observable physical quantities. We will compare our results to those of the standard inflationary paradigm [56] and to those which use the HH wave function [57–59].

A. Parameters and fields

We now proceed by comparing the dimensional parameters and dynamical fields of the thermal-scalar CFT with those of an asymptotically dS spacetime. As listed in Table I, each side contains a pair of dimensional parameters: The D-dimensional Newton's constant G_D and the Hubble parameter H in dS space versus g_s^2 and ε on the CFT side. It should be noted that the string length scale l_s, or equivalently, the inverse of the Hagedorn temperature, is a unit length rather than a dimensional parameter and the temperature T is not an additional parameter because it can be expressed in terms of ε and T_{Hag}, $T = \varepsilon + T_{\text{Hag}}$.

In the case of a pure theory of gravity in the asymptotically dS bulk, each side also contains two dynamical fields. For dS space, these are the transverse-traceless (TT) graviton $h_{\mu\nu}$ and the scalar perturbation ζ. Strictly speaking, ζ is dynamical only when the dS symmetries are broken, as it would be for a noneternal asymptotically dS spacetime. For the CFT, the dynamical fields cannot simply be the corresponding metric perturbations, as already discussed. Hence, we will consider TT and suitably defined scalar perturbations of the CFT stress-energy tensor and then, with the help of Einstein's equations, use these to deduce the corresponding perturbations of the dS metric. Table II includes the corresponding pairs of dynamical fields along with each pair's respective cosmological observable. There and subsequently, we have denoted generic tensor

TABLE II.   Fields and cosmological observables (CO). The quantity $\delta\rho_{ij}^{TT}$ is defined below in the text.

dS	CFT	CO
h_{ij}	$\delta\rho_{ij}^{TT}$	P_T
$\frac{1}{H}\frac{\partial\zeta}{\partial t}$	$\frac{\delta s}{s}$	P_ζ

perturbations of the CFT stress-energy tensor by $\delta\rho_{ij}$ and their TT components by $\delta\rho_{ij}^{TT}$.

In our framework, the dynamical CFT fields are given in terms of either the entropy perturbations δs or the closely related perturbations of the energy density and pressure, $\delta\rho = \delta p = \varepsilon\delta s$, with the equalities following from the equation of state and first law, respectively. Local scalar perturbations in the entropy, energy and pressure are not invariant under conformal transformations (rescalings in particular) and therefore do not constitute physical observables. The identity of the physical scalar perturbations will be clarified in Sec. IV C 2. Similarly, vector perturbations are not physical, as these can be undone by special conformal transformations. On the other hand, TT tensor perturbations are physical. Higher-spin perturbations—such as sextupole, hexapole, *etc.*—will involve derivatives as these are the only other vectors available in the CFT. So that, for length scales larger than the horizon, $k \ll H$, such higher-order perturbations are suppressed.

As for the TT components of the perturbations of the stress-energy tensor, on the basis of isotropy, each independent mode fluctuates with equal strength and the sum of their squares is equal to the square of the energy-density perturbation, $\sum_{i,j}|\delta\rho_{ij}^{TT}|^2 = \frac{1}{2}(d+1)(d-2)|\delta\rho_{ij}^{TT}| = |\delta\rho|^2$. For sake of completeness, the TT components can be formally defined in terms of a transverse projection operator P_{lm}^T,

$$P_{lm}^T = \left(\delta_{lm} - \frac{\nabla_l\nabla_m}{\nabla^2}\right), \qquad (49)$$

which leads to the construction of a TT projector in the standard way,

$$\delta\rho_{ij}^{TT} = \left(P_{il}^T P_{jm}^T - \frac{1}{d-1}P_{ij}^T P_{lm}^T\right)\delta\rho^{lm}. \qquad (50)$$

Using the above correspondence between the two sets of fields and dimensional parameters, we can turn the relationship between the HH wave function and the CFT partition function into a more explicit equality,

$$\Psi_{HH}(h_{ij}, \zeta; G_D, H) = Z_{\text{CFT}}\left(\delta\rho_{ij}^{TT}, \frac{\delta s}{s}; g_s^2, \varepsilon\right). \qquad (51)$$

B. Thermodynamics

The objective here is to make the correspondence between the CFT and dS space more precise by comparing their respective values for the entropy. As for other possible comparisons, the Gibbons-Hawking value of the dS temperature $T_{dS} = \frac{H}{2\pi}$, is not directly related to observables in the FRW epoch because of its observer dependence. The energy density is indeed observable but even more ambiguous, as the original derivation of the Gibbons-Hawking

entropy was for a closed dS space for which the total energy vanishes [49]. Our expectation is that the energy density of the strings will increase as the Hagedorn transition proceeds, until it becomes comparable to the Hagedorn energy density. Hence, it is the entropy that serves as the most reliable observable for comparison purposes.

Let us now recall from Eq. (27) that the CFT entropy density is given by $s_{\text{CFT}} = \frac{\varepsilon}{g_s^2}$, while also recalling that ε is the associated (effective) temperature as in Sec. III D. The entropy of the CFT in a Hubble volume $V_d(H)$ (or "causal patch") is then

$$S_{\text{CFT}} = \frac{\varepsilon V_d(H)}{g_s^2}, \tag{52}$$

which should be compared to the Gibbons-Hawking entropy on the dS side [49],

$$S_{dS} = \frac{A_d(H)}{4G_D} = \frac{HV_d(H)}{4G}, \tag{53}$$

where $A_d(H)$ is the surface area of the Hubble volume and $A_d(H) = HV_d(H)$ in planar coordinates has been used.

Equating the two entropies,

$$S_{\text{CFT}} = S_{dS}, \tag{54}$$

we then obtain

$$\frac{8\pi G_D}{g_s^2} \frac{\varepsilon}{H} = 2\pi. \tag{55}$$

Recall that we have absorbed numerical, string-theory dependent, factors into ε and g_s^2 (see Sec. II). Making these factors explicit, one could then fix the ratio $\frac{8\pi G_D}{g_s^2}$ in any specific string theory, which would in turn fix the ratio $\frac{\varepsilon}{H}$. However, as the relation between G_D and g_s^2 is highly model dependent, a detailed discussion on these ratios will be deferred to a future investigation.

Given the identity in Eq. (54), the expected relation [49],

$$|\Psi_{HH}|^2 = e^{+S_{dS}} \tag{56}$$

can now be recovered from the equilibrium value of the CFT partition function,

$$Z_{\text{CFT}}^2 = e^{-2\frac{1}{T}S_0} = e^{+\frac{2}{T}\int d^d x \frac{1}{2}\frac{\varepsilon^2}{g_s^2}}, \tag{57}$$

where the rightmost exponent follows from Eq. (13) and the use of flat, planar coordinates. One should take note of the crucial sign change of the exponent thanks to the negativity of $\mathcal{S}_0$. For the purposes of matching this partition function to the HH wave function, we need to change the prefactor in the exponent from $1/T$ to $1/\varepsilon$. This is consistent with the

perspective of Sec. III D and is, once again, related to the effective temperature of the long strings being equal to ε rather than the microscopic temperature of the strings $T \sim T_{\text{Hag}}$. The end result is

$$\begin{aligned} |\Psi_{HH}|^2 = Z_{\text{CFT}}^2(T \to \varepsilon) &= \exp\left(\frac{1}{\varepsilon}\int d^d x \frac{\varepsilon^2}{g_s^2}\right) \\ &= \exp\left(\int d^d x s\right) \\ &= \exp(S_{\text{CFT}}) \\ &= \exp(S_{dS}), \end{aligned} \tag{58}$$

where the integral is over the Hubble volume and Eq. (54) has been used at the end.

It should be emphasized that, in spite of the exponentially growing magnitude of the wave function, the perturbations are well behaved and controlled by a well-defined Gaussian integral as in Eqs. (45) and (46).

Our definition of the HH wave function in terms of Z_{CFT} resolves several longstanding issues about this wave function and its use in Euclidean quantum gravity [75,76]. Formally, the Euclidean gravitational action is unbounded from below, and the integral defining it is badly divergent. But the wave function is certainly relevant to perturbations about an asymptotically dS space and, as we have seen, the associated Gaussian integral is itself well defined and convergent. Moreover, from our perspective, the growing exponential for the magnitude of the wave function is not a vice but a virtue, as it is needed to explain the large entropy of dS space. Additionally, if Ψ_{HH} is viewed as defining a probability distribution for a background dS Universe, the distribution is peaked at small values of the cosmological constant, thus implying a large and empty universe which disfavors inflation. Our definition of the wave function, on the contrary, predicts a large, hot Universe in lieu of inflation. Finally, our definition extends the domain of the quantum state of the Universe beyond the semiclassical regime and demonstrates that the resolution of the initial singularity problem must rely on strong quantum effects.

C. Two-point correlation functions and spectrum of perturbations

We begin this part of the analysis by expanding the Lagrangian $\mathcal{L}_E(s)$ in Eq. (11) about the equilibrium solution s_0 up to second order in the perturbation strength $\delta s(\vec{x}) = s(\vec{x}) - s_0$. This will enable us to calculate the two-point correlation functions of the CFT, which can be used in turn to calculate the spectra of the corresponding cosmological observables.

The relevant term in the just-described expansion is the quadratic term,

$$\mathcal{S}_E^{(2)} = \frac{1}{T}\int d^d x \frac{1}{2}g_s^2 \delta s^2 + \cdots, \tag{59}$$

from which it follows that

$$\langle \delta s(\vec{x}) \delta s(0) \rangle = \int [\mathcal{D}\delta s] \delta s(\vec{x}) \delta s(0) e^{-\frac{1}{T}\int d^d x \frac{1}{2} g_s^2 \delta s^2} = \frac{T}{g_s^2} \delta^d(\vec{x}).$$

(60)

In cosmology, it is customary to use the power spectrum of the two-point function as the observable quantity. What is then required is the Fourier transform of the perturbation $\delta s_{\vec{k}}$, which is related to $\delta s(\vec{x})$ in the usual way,

$$\delta s(\vec{x}) = \frac{1}{(2\pi)^d} \int d^d k e^{i\vec{k}\cdot\vec{x}} \delta s_{\vec{k}}.$$

(61)

The two-point function for $\delta s_{\vec{k}}$ is expressible as

$$\langle \delta s_{\vec{k}_1} \delta s_{\vec{k}_2} \rangle = |\delta s_{\vec{k}_1}|^2 (2\pi)^d \delta^d(\vec{k}_1 + \vec{k}_2),$$

(62)

where

$$|\delta s_{\vec{k}}|^2 = \frac{T}{g_s^2}$$

(63)

can be deduced from Eq. (60).

Now applying the standard relationship between a power spectrum and its associated two-point function,

$$d(\ln k) P_{\delta s}(k) = \frac{d^d k}{(2\pi)^d} |\delta s_k|^2,$$

(64)

we obtain the spectral form,

$$P_{\delta s}(k) = \frac{d\Omega_{d-1} k^d}{(2\pi)^d} \frac{T}{g_s^2},$$

(65)

where $d\Omega_{d-1}$ is the solid angle subtended by a $(d-1)$-dimensional spherical surface. The power spectrum has, by definition, the same dimensionality as $\langle \delta s(\vec{x})^2 \rangle$, and this fixes the power of k unambiguously.

Since $\delta\rho = \varepsilon \delta s$ from the first law and $\delta p = \delta\rho$ from the equation of state, it can also be deduced that

$$P_{\delta\rho}(k) = P_{\delta p}(k) = \frac{d\Omega_{d-1} k^d}{(2\pi)^d} \frac{T\varepsilon^2}{g_s^2}.$$

(66)

1. Tensor perturbations

To obtain the power spectrum of the tensor perturbations, we start with the relationship between a specific polarization of the tensor perturbations of the metric and the corresponding component of the stress-energy tensor perturbation (see, e.g., [56]),

$$\langle |h_{ij}(k)|^2 \rangle_{dS} = \frac{(4\pi G_D)^2}{(k^2)^2} \langle |\delta T_{ij}^{TT}(k)| \rangle_{dS}^2$$

$$= \frac{(4\pi G_D)^2}{(k^2)^2} \langle |\delta\rho_{ij}^{TT}(k)| \rangle_{CFT}^2,$$

(67)

where the proposed duality has been applied in the second line, and thus the validity of the second equality only applies on the spacelike matching surface (i.e., on the future boundary of the asymptotically dS spacetime).

Let us recall that $\sum \langle |\delta\rho_{ij}^{TT}(k)| \rangle_{CFT}^2 = |\delta\rho|_{CFT}^2$. Then, from Eq. (67), it follows that $\sum \langle |h_{ij}(k)|^2 \rangle_{dS}$ can be directly related to $|\delta\rho|_{CFT}^2$, and one can similarly relate the total power spectrum for the tensor perturbations $P_T(k)$ to the spectrum in Eq. (66),

$$P_T(k)_{|k\to H, T\to\varepsilon} = \frac{(4\pi G_D)^2}{(k^2)^2} P_{\delta\rho|k\to H, T\to\varepsilon}$$

$$= \frac{1}{4} (8\pi G_D)^2 \frac{\varepsilon^3}{g_s^2} H^{d-4} \frac{d\Omega_{d-1}}{(2\pi)^d}, \quad (68)$$

where the standard horizon-crossing condition $k \to H$ has been applied and our usual replacement $T \to \varepsilon$ has been made.

Next, using Eq. (55), we obtain

$$P_T(H) = \frac{\pi}{2} \frac{\varepsilon^2}{H^2} (8\pi G_D) H^{d-1} \frac{d\Omega_{d-1}}{(2\pi)^d},$$

(69)

or, in terms of the dS entropy in Eq. (53),

$$P_T(H) \sim \frac{1}{S_{dS}},$$

(70)

as expected. Notice that $P_T(H)$ is dimensionless.

In the observationally relevant case of $d = 3$, the above reduces to

$$P_T(H) = \frac{1}{4\pi} \frac{\varepsilon^2}{H^2} \frac{H^2}{m_P^2},$$

(71)

which, has the same parametric dependence as the standard inflationary result,

$$P_T(\text{inflation}) = \frac{2}{\pi^2} \frac{H^2}{m_P^2}.$$

(72)

A calculation of the tensor power spectrum using the HH wave function with an additional scalar field [57,59] is in agreement with the standard inflationary outcome, and therefore, our result is also in qualitative agreement with this calculation.

It should be emphasized that we assumed in the calculation that the state is one of exact thermal equilibrium, so that its temperature is uniform or, equivalently, $\varepsilon(k) = $ constant. It is for this reason that the spectrum of tensor perturbations was found to be exactly scale invariant. It may well be that the effective temperature of the state is not exactly constant and could be scale dependent due to some source of conformal-symmetry breaking. This breaking is quite natural insofar as the state has a finite extent;

equivalently, the dS spacetime is noneternal. Nevertheless, the breaking is expected to be quite small, as its effects are proportional to the deviations of the spacetime from an eternal dS background. We will discuss this issue further after discussing the scalar perturbations.

2. Scalar perturbations

In an eternal asymptotically dS space, time does not exist, and it is impossible for a single observer to see the extent of the whole state. By contrast, in a noneternal asymptotically dS spacetime, a quantity that measures time—a "clock"—can be introduced. The same must apply to each of their respective CFT duals. For instance, in semiclassical inflation, the clock is introduced in the guise of a slowly rolling inflaton field. On either side of our proposed correspondence, the clock is the total *observable* entropy of the state in units of the horizon entropy. And it is the fluctuations in this clock time that serves as the dual to the scalar modes of dS space, as we now explain.

To formulate the dual of the gauge-invariant scalar perturbations ζ [56], we will follow [42] and rely on the relationship between ζ and the perturbations in the number of e-folds $\delta N_{e\text{-folds}}$. This method was previously used to calculate superhorizon perturbations in the "separate universe" approach and the δN formalism [77,78], where it was shown that

$$\zeta = \delta N_{e\text{-folds}}. \tag{73}$$

It should be emphasized that Eq. (73) fixes completely the normalization of ζ. From our perspective, what is important is that the value of $\delta N_{e\text{-folds}}$ can be expressed in terms of CFT quantities, as we will clarify in the ensuing discussion.

The number of e-folds that an FRW observer has to postulate is, from his perspective, determined by the increase in volume which is required to explain the difference in entropy between that in a single Hubble horizon $S_H \sim S_{dS}$ and the total entropy of the Universe $S_{\text{tot}} = n_H S_H$. From this observer's perspective, the parameter n_H is the number of causally disconnected Hubble volumes V_H at the time of reheating; that is,

$$n_H = e^{dN_{e\text{-folds}}} = \frac{V_{\text{tot}}}{V_H} = \frac{S_{\text{tot}}}{S_H}, \tag{74}$$

where the last equality assumes that there are no additional entropy-generating mechanisms after the inflationary period (otherwise, the final ratio would be an upper bound) and that S_H is constant, independent of its location. Meanwhile, a hypothetical CFT observer faces the analogous task of accounting for an extremely large total entropy after the phase transition from strings to radiation.

To make use of the relationship between $\delta N_{e\text{-folds}}$ and ζ, we call upon a known expression for ζ in terms of pressure perturbations [78],

$$\frac{1}{H}\frac{\partial\zeta}{\partial t} = -\frac{1}{p+\rho}\delta p_{|\rho}. \tag{75}$$

Then, since $p + \rho = \varepsilon s$ and $\delta p = \delta \rho = \varepsilon \delta s$,

$$\frac{1}{H}\frac{\partial\zeta}{\partial t} = -\frac{\delta s}{s}. \tag{76}$$

Next, the conformal symmetries on either side of the duality allows for the replacement of $\frac{1}{H}\frac{\partial}{\partial t}$ with $-\frac{\partial}{\partial(\ln k)}$,

$$\frac{\partial\zeta}{\partial(\ln k)} = \frac{\delta s}{s}, \tag{77}$$

or, formally,

$$\zeta = \int d(\ln k)\frac{\delta s}{s}. \tag{78}$$

This result can be recast as

$$\zeta = \int \frac{d(\ln V)}{d}\frac{\delta s}{s} = \frac{1}{d}\int d^d x \frac{\delta s}{Vs} = \delta N_{e\text{-folds}}, \tag{79}$$

where the first equality follows from conformal symmetry and the last one from Eq. (74).

We can now call upon Eq. (77) for ζ and the equilibrium value for s in Eq. (27) to show that the two-point function for the scalar perturbations satisfies

$$\frac{\partial}{\partial(\ln k_1)}\frac{\partial}{\partial(\ln k_2)}\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle = \left(\frac{g_s^2}{\varepsilon}\right)^2\langle\delta s_{\vec{k}_1}\delta s_{\vec{k}_2}\rangle. \tag{80}$$

Observing that both sides of Eq. (80) are of the form $f(k_1)\delta^d(\vec{k}_1 + \vec{k}_2)$, one can integrate twice over both sides and compare the coefficients. The result is

$$\langle|\zeta_k|^2\rangle = \frac{N_{e\text{-folds}}}{d}\left(\frac{g_s^2}{\varepsilon}\right)^2\langle|\delta s_k|^2\rangle = \frac{N_{e\text{-folds}}}{d}\frac{Tg_s^2}{\varepsilon^2}, \tag{81}$$

where the second equality follows from Eq. (63) and the factor of $N_{e\text{-folds}}$ results from one of the integrals on the right, $-\int d(\ln k) = \int H dt = \int d(\ln a) = N_{e-\text{folds}}$. The associated power spectrum is then

$$P_\zeta(k) = \frac{N_{e\text{-folds}}}{d}\frac{Tg_s^2}{\varepsilon^2}\frac{k^d d\Omega_{d-1}}{(2\pi)^d}. \tag{82}$$

To make contact with the dS calculation, the conditions $k \to H$ and $T \to \varepsilon$ can once again be imposed,

$$P_\zeta(H) = \frac{N_{e\text{-folds}}}{d}\frac{g_s^2}{\varepsilon}H^d\frac{d\Omega_{d-1}}{(2\pi)^d}. \tag{83}$$

If we further substitute $8\pi G_D$ for g_s^2 using Eq. (55), then

$$P_\zeta(H) = \frac{N_{e\text{-folds}}}{2\pi d} 8\pi G_D H^{d-1} \frac{d\Omega_{d-1}}{(2\pi)^d}. \qquad (84)$$

The fact that P_ζ is enhanced by the number of e-folds with respect to the tensor perturbations is a significant feature of the correspondence,

$$P_\zeta \sim N_{e\text{-folds}} P_T. \qquad (85)$$

The enhancement factor of $N_{e\text{-folds}}$ can be traced to the large size of the initial string state rather than to the scaling properties of the CFT or to deviations from scale invariance. This is unlike in models of semiclassical inflation, for which the tensor perturbations are viewed as suppressed with respect to their scalar counterparts by a factor that is explicitly related to the amount of deviation from scale invariance.

For the $d = 3$ case with $m_P^2 = 1/(8\pi G)$,

$$P_\zeta(H) = \frac{N_{e\text{-folds}}}{4\pi^3 d} \frac{H^2}{m_P^2}, \qquad (86)$$

which can be compared to the standard inflationary result,

$$P_\zeta(H)_{\text{inflation}} = \frac{1}{\epsilon_{\text{inf}}} \frac{1}{8\pi^2} \frac{H^2}{m_P^2}, \qquad (87)$$

where ϵ_{inf} parametrizes the deviation from scale invariance, $1 - n_S = 6\epsilon_{\text{inf}} - 2\eta_{\text{inf}}$. Here, n_S is the scalar spectral index and $\epsilon_{\text{inf}}, \eta_{\text{inf}}$ are the slow-roll parameters. In simple models of inflation, $\epsilon_{\text{inf}} \sim 1/N_{e\text{-folds}}$; meaning that our result is in qualitative agreement with that of semiclassical inflation.

A calculation of the scalar perturbations using the HH wave function [57–59] is in agreement with the standard inflationary result and, just like for models of inflation, requires an additional scalar field to render the scalar perturbations as physical. Meaning that our result for the scalar power spectrum is in qualitative agreement with the HH calculation as well.

An important observable is the tensor-to-scalar power ratio r. In general,

$$r = \frac{P_T}{P_\zeta} = \frac{d}{N_{e\text{-folds}}} \frac{\pi^2 \epsilon^2}{H^2}, \qquad (88)$$

and, in the $d = 3$ case,

$$r = \frac{3}{N_{e\text{-folds}}} \frac{\pi^2 \epsilon^2}{H^2}. \qquad (89)$$

Given that $\epsilon \sim H$ as expected, the above value of $r \sim 1/N_{e-\text{folds}}$ would correspond to a high scale of inflation if interpreted within simple models of semiclassical inflation. This is consistent with our expectation that the energy density is of the order of T_{Hag}^4 [42].

D. Higher-order correlation functions and deviations from scale-invariance

The discussion has, so far, been focusing on the quantities that are the least sensitive to the choice of model; namely, the two-point functions in the case of conformal invariance. Our results could be extended to more model-dependent quantities, such as two-point correlation functions when conformal invariance is weakly broken or higher-point functions for the conformally invariant case. We will not extend the calculations at the present time but do anticipate a more detailed analysis along this line in the future. Let us, meanwhile, briefly explain the significance of such model-dependent calculations.

Deviations from conformal invariance can arise from spatial dependence (equivalently, k dependence) of the effective temperature ϵ or the string coupling g_s^2 or both. These will in turn introduce scale dependence into the tensor and scalar power spectra. The scale dependence is an observable feature; however, because of its dependence on the details of the background solution and on the nature of the Hagedorn transition—and not just on scales and symmetries—it is, in some sense, a less fundamental aspect of the correspondence.

The higher-order terms in the CFT Lagrangian, as discussed in Eqs. (19)–(22), are also present when the conformal symmetry remains unbroken. However, these terms are still model dependent as they depend on the specific string theory. But, in spite of their relative smallness, they remain of considerable interest, as such terms can be used to calculate three-point (and higher) correlation functions. These multipoint correlators are what determines the non-Gaussianity of the spectra of perturbations and, therefore, represent an opportunity for distinguishing our proposed correspondence from the standard inflationary paradigm. Unfortunately, it is already quite evident that such effects are small.

V. CONCLUSION AND OUTLOOK

We have put forward a new correspondence between asymptotically dS space and a CFT dual by showing that the partition function of the CFT is equal to the HH wave function of the dS space. Our correspondence provides a complete qualitative description of a nonsingular initial state of the Universe and, in this sense, replaces the big-bang singularity and semiclassical inflation.

We have built off of a previous work [42] which shows that an asymptotically dS spacetime has a dual description in terms of a state of interacting, long, closed, fundamental strings in their high-temperature Hagedorn phase. A significant, new development was the identification of the entropy density of the strings with the magnitude-squared of a condensate of a thermal scalar whose path integral is equal, under certain conditions, to the full

partition function for the Hagedorn phase of string theory. The strings are thus described by a thermal CFT, which can also be viewed as a Euclidean field theory that has been compactified on a string-length thermal circle. Surprisingly, the reduced theory has the scaling properties of a two-dimensional CFT in spite of formally being defined in a manifold with $d \geq 3$ spatial dimensions.

Our correspondence provides a clear origin for the entropy of dS space as the microscopic entropy of a hot state of strings. This explanation clarifies how a state whose equation of state is $p = -\rho$, as in dS space, can have any entropy at all when the thermodynamic relation $p + \rho = sT$ suggests that both the entropy and the temperature are vanishing. From the stringy point of view, the pressure is rather maximally positive and the negative pressure of dS space is an artifact of insisting on a semiclassical geometry when none is justified.

The proposed duality redefines the HH wave function and resolves several outstanding issues with its common interpretation, such as the divergence of the Euclidean path integral and its preference for an empty Universe with a very small cosmological constant.

We have shown how the power spectra for the tensor and scalar perturbations of the asymptotically dS metric can be calculated on the CFT side of the correspondence by identifying the two dual fields, the scalar and tensor perturbations of the CFT stress-energy tensor. As was discussed in detail, these calculations reproduce, qualitatively, the results of the standard inflationary paradigm and the corresponding calculations which use the HH wave function. Although any specific set of predictions will depend on the value of an order-unity number—the ratio of the effective temperature of the string state ε to the Hubble parameter H—our framework does provide an opportunity to compare the predictions of specific string-theory-based models for cosmological observables. In addition, the strength of the scalar perturbations was found to be naturally enhanced by a factor of $N_{e\text{-folds}}$, even when the theory formally exhibits local scale invariance. This places our predicted value for the cosmological observabler r well within the empirical bounds.

Let us now finish by discussing some remaining issues and possible extensions of the current analysis:

First, it should be reemphasized that we do not explain why the Universe is large. The entropy of the string state is large because this corresponds to a large asymptotically dS Universe and thus leads to a large FRW Universe in the state's future. The value of the entropy should be viewed as part of the definition of the initial state.

Still lacking is a qualitative description of just how the state of hot strings decays into the state of hot radiation which follows; a transition which is known as reheating in inflation. In our case, the transition corresponds to a phase transition between the Hagedorn phase of long strings and a phase of short strings propagating in a semiclassical background. Because of the close parallels between early-Universe cosmology and BHs, our expectation is that the transition is described by a decay mechanism that is akin to Hawking radiation.

Our proposal can be extended to incorporate the effects of deviations away from conformality. To make such a calculation precise, the issue of how the effective temperature ε and the coupling g_s^2 depend on scale will have to be resolved. It will also, as mentioned, be necessary to fix the ratio ε/H, which amounts to understanding the exact relation between the string coupling and Newton's constant in specific compactifications of various string theories. Another possible extension is the incorporation of three-point correlation functions and higher. This entails the inclusion of yet-to-be-specified higher-order terms in the CFT Lagrangian, and using these to calculate three- and higher-point correlation functions in dS space. Yet another interesting extension is to include other dynamical fields besides the physical graviton modes and their CFT dual, for instance, the dilaton of the underlying string theory.

The connection between our proposed correspondence and the AdS/CFT correspondence is not currently clear. What is clear, though, is that if such a connection exists, it must differ from previous proposals which regard the AdS radial direction as Euclidean time and AdS time as one of the spatial coordinates. It is still possible that the two frameworks are somehow connected at a mathematical level or even at a yet unknown deeper conceptual level.

ACKNOWLEDGMENTS

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