Stress energy correlator in de Sitter spacetime: Its conformal masking or growth in connected Friedmann universes

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Semiclassical physics in the gravitational scenario, in its first approximation (first order), cares only for the expectation value of the stress energy tensor and ignores the inherent quantum fluctuations thereof. In the approach of stochastic gravity, on the other hand, these matter fluctuations are supposed to work as the source of geometry fluctuations and have the potential to render the results from first-order semiclassical physics irrelevant. We study the object of central significance in stochastic gravity, i.e., the noise kernel, for a wide class of Friedmann spacetimes. Through an equivalence of quantum fields on de Sitter spacetime and those on generic Friedmann universes, we obtain the noise kernel through the correlators of stress energy tensor for fixed comoving but large physical distances. We show that in many Friedmann universes including the expanding universes, the initial quantum fluctuations the universe is born with may remain invariant and important even at late times. Furthermore, we explore the cosmological spacetimes where even after long times the quantum fluctuations remain strong and become dominant over large physical distances, which the matter-driven universe is an example of. The study is carried out in minimal as well as nonminimal interaction settings. Implications of such quantum fluctuations are discussed.

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I. INTRODUCTION

The study of quantum matter over a classical geometry has given rise to many novel and intriguing features like Hawking radiation, the Unruh effect, gravitational particle creation, etc. [1–4]. In the absence of a complete theory of quantum gravity, semiclassical physics, which is a "first-order" quantum correction to the classical general relativity, is the only available methodology capturing the interplay between concepts of quantum mechanics (such as Hilbert space and wave function superposition) and those of general relativity (such as general covariance, geodesic distance, etc.). Since the inception of the idea of using the quantum expectation value on the right-hand side of the Einstein field equations, there has been some level of discomfort regarding its operational status [5,6], more particularly, its handling of the inherent quantum fluctuations. One can envisage that, in situations where fluctuations tend to grow, the usage of quantum expectation values of the stress tensor alone would not remain justifiable for any physical interpretation. Thus, in the scenarios where significant physical insights depend upon the geometrical structure obtained through the expectation values, it is worthwhile to address the contributions from fluctuations as well. Gravitational particle creation during the early inflationary phase of the Universe is such an avenue, where the expectation value for the dynamics-driving field (called inflaton) sets up the expansion of the Universe, which, in turn, creates particles from the perturbation field [7–10]. This semiclassical program has been really successful in predicting various novel features of the early Universe, many of which have also received observational vindication [11]. Furthermore, there have been attempts of using different kinds of accelerating phase to obtain or explain various features of the early Universe's spectra of quantum predictions [12–14]. Such efforts are expected to be obtained from different quantum states and subsequent evolution of stress energy expectation values in these states. Furthermore, the late time acceleration, in many discussions [6,15–17], is also attributed to the quantum character of the stress energy tensor. Therefore, it is imperative to analyze if the semiclassical study directing such physical discourse is stable under fluctuations in the stress energy tensor.

It is natural to expect that any good quantum gravity theory would yield the known classical results in some appropriate classical limit (in a spirit similar to Ernfest's theorem), apart from capturing the quantum fluctuations. Furthermore, if one seeks an nth-order quantum correction to the classical theory, motivated from the semiclassical approach, it is also natural to expect that the quantum

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gravity theory should return these values in the properly considered limits. Stochastic gravity is one such approach to incorporate quantum fluctuations to the semiclassical gravity, where one considers the effect of quantum fluctuations in matter fields on the classical geometry of spacetime. This is accomplished using the Einstein-Langevin equation [5,6], in which a central object in the form of a *noise kernel* works as a stochastic source in addition to the quantum averaged stress tensor. The noise kernel is the vacuum expectation value of symmetrized stress energy bitensor for a quantum field in curved spacetime. As a result, these fluctuations, if strong enough, will lead to fluctuations in the geometry, too [5,6]. Such induced fluctuations have played an important role in many studies involving backreaction. Backreaction problems in gravity and cosmology, for example, have been addressed [5,6,18–20] using Einstein-Langevin equations.

The role of quantum fluctuations themselves in the context of the early Universe is quite vital. The vacuum fluctuations are understood to form the seed of the modern-day galaxy clusters during the inflationary epoch [7–10]. The present understanding and observational signatures [11] suggest that the Universe was born in a very near de Sitter configuration. The de Sitter spacetime, like flat spacetime, is a maximally symmetric spacetime, but with constant positive curvature. Analyzing quantum scalar fields on de Sitter spacetime is an old subject [21], and a lot of effort has been spent in studying it since then [10,22–38]. Like the flat spacetime, all the quantum information of free fields on classical de Sitter geometry gets encoded in the Wightman function. From this Wightman function, one can further construct the rhs of the Einstein field equations, i.e., the vacuum expectation of the stress energy operator [8]. Like all quadratic operators in quantum field theory, the Wightman function itself becomes ill defined at the same spacetime point, since, for well-behaved states, it has the so-called Hadamard form, i.e., diverges quadratically as well as logarithmically [8]. Such divergences are attributed to the ultraviolet limit of the theory, and they typically get regularized in most of the physical scenarios. However, if we consider the Wightman function of a minimally coupled massless scalar field in a de Sitter universe, it shows divergence even for different spacetime points, a phenomenon known as the infrared problem of the de Sitter spacetime. This infrared divergence of the Wightman function is also intimately tied up with the prediction of scale invariance of the power spectrum [8,39], which is one of the remarkable success stories of the inflationary paradigm. However, this divergence also goes on to suggest that for de Sitter (or de Sitter–like universes) the quantum fluctuations can become very important. Various physical reasons of these divergences are suggested in the literature, and, in order to obtain physically meaningful results from the Wightman function, various methods have been devised over the years [8,40]. Still, the infrared problem calls the stability of de Sitter spacetime into question, as the severity of these divergences may also be felt up at loop levels in quantum field theory [41]. In Ref. [39], quantum fields over a family of Friedmann-Robertson-Walker (FRW) universes were shown to be connected to quantum fields in de Sitter spacetime and, hence, sharing the divergences as well, in some cases. Thus, the potential instability of the de Sitter spacetime may also have adverse effects for the stability of these Friedmann universes. However, if we wish to study the stability of such spacetimes through the semiclassical Einstein equations, divergence in the Wightman function may not be sufficient or reliable enough. In order to investigate if the quantum fluctuations are strong enough to make the de Sitter or the connected FRW universes unstable under the stochastic gravity approach, we need to evaluate the relevant noise kernel components, too.

The noise kernel can also be expressed in terms of the products of derivatives of Wightman functions [42], and, therefore, the divergences of Wightman functions may also creep into the expressions for the noise kernel. In this paper, we evaluate the noise kernel, for a minimally coupled massive scalar field in a de Sitter universe and for the connected massless fields in Friedmann spacetime. We obtain the noise kernel in the late time universe when the scale factor grows and the physical distance between fixed comoving coordinates becomes large. We analyze if the quantum fluctuations which were nonzero initially over small physical distances, retain their form, grow, or decay, as the scale factor growth separates the points apart. However, the stress energy tensor, being quadratic in nature, has an in-built ultraviolet divergence in it, and the noise kernel is obtained from the so-called regularized stress energy tensor (RSET). For the flat Minkowski spacetime, Ref. [42] calculates the noise kernel for a massive scalar field and gives a dimensional regularization procedure to separate the problematic parts.

In this paper, we adopt the formalism discussed in Ref. [42] but generalized for Friedmann universes. The study of quantum fields and their backreaction has attracted a lot of attention [43–49]. However, in this work, we are interested in quantifying the stochasticity in the Einstein-Langevin approach for such spacetimes. To begin with, we calculate the noise kernel for a massive scalar field in the de Sitter spacetime and analyze the stability of it. A similar study has been done in Ref. [50], which suggests the decay of stochastic noise over large distances, as the mass of the field increases. However, it is important to note that, in FRW universes, the spacelike distances can become large in two ways: for (i) large comoving distances and a nonzero scale factor or (ii) fixed finite comoving distances but with a large scale factor. In flat spacetime, there are nonzero quantum fluctuations for finite distances, and, as distances grow, the fluctuations decay and become subdominant in front of any other relevant expectation values. However, in certain Friedmann universes, it may so happen that the finite comoving fluctuations remain invariant or even grow when the scale factor rises and makes the physical distances large. The first may be expected for conformal field theories, something which we see in de Sitter universes, too. However, we show that there exists a class of theories in the de Sitter universe, where the noise kernel blows up as the physical distance between fixed comoving points grows. Subsequently, through the relation to Friedmann universes, we realize that there are universes where the stochastic correction in semiclassical analysis should really become important. We show that the universe driven by pressureless dust or strong energy conditions violating fluids (accelerating universes belong to a family of such solutions) becomes very susceptible to such fluctuations, and the semiclassical understanding or stabilities of these spacetimes may need to be reinvestigated in the face of divergent fluctuations. Furthermore, there are other phantom fluid-driven Friedmann universes which retain the initial quantum fluctuations under scale factor growth (much like conformal field theories, despite not being one), and there the semiclassical analysis should be weighed against the strength of the quantum fluctuations. We also do the analysis for nonminimal coupling and show that a conformal interaction is able to cure this blowup in all cases.

The paper has five sections. In Sec. II, we quickly review some standard results regarding de Sitter spacetime and quantization of a minimally coupled scalar field living on classical de Sitter spacetime as well as a brief discussion of evaluation of the noise kernel. In Sec. III, we derive the expression of the noise kernel for a minimally coupled massive scalar field living on a de Sitter universe and analyze its various mass limits. We develop the noise kernel computation for both minimal and nonminimal interaction cases. Section IV deals with spacetimes which are conformally dual to de Sitter spacetimes in terms of quantum field analysis. We compute the noise kernel in these cases and study the stability here. Section V deals with the energy density correlator and the subsequent analysis for Friedmann universes driven by various matter equations of state. We summarize our main results and discuss future prospects in conclusion in Sec. VI. We use the $(-,+,+,+)$ sign convention for the metric.

II. PRELIMINARIES

A. Noise kernel

In the stochastic gravity paradigm, one tries to include the effect of quantum fluctuations of the matter field on the classical geometry of the spacetime through the noise kernel, which is given by [5,6]

$$N_{abcd}=\frac{1}{8}\{\hat{t}_{ab},\hat{t}_{cd}\},\quad \text{where}\ \hat{t}_{ab}=\hat{T}_{ab}-\langle\hat{T}_{ab}\rangle\quad \text{and} \tag{1}$$

$$\begin{aligned}&\langle\hat{t}_{abcd}(x,x')\rangle\\ &\equiv\langle\hat{t}_{ab}(x)\hat{t}_{cd}(x')\rangle\\ &=\langle 0|\hat{T}_{ab}(x)\hat{T}_{cd}(x')|0\rangle-\langle 0|\hat{T}_{ab}(x)|0\rangle\langle 0|\hat{T}_{cd}(x')|0\rangle.\end{aligned} \tag{2}$$

Here $\hat{T}_{ab}$ represents the stress energy (or energy-momentum) quantum operator which one obtains by replacing the classical fields by field operators in the classical expression for the stress energy tensor. The noise kernel incorporates much information regarding the quantum property of matter and its backreaction on the geometry, too. For example, one can easily obtain the fluctuation in the stress energy tensor from the noise kernel. Fluctuation in the stress energy tensor at some spacetime point x, like any other quantum operator, is equal to $\langle(\hat{t}_{ab}(x))^2\rangle$. Therefore, we see that the fluctuation in the stress energy tensor is obtained by taking $a=c$ and $b=d$ and by considering the $x'\to x$ limit in the noise kernel, i.e., $\lim_{x'\to x}\langle\hat{t}_{abab}(x,x')\rangle$.[1]

Furthermore, in the gravitational scenario, the stress energy tensor, denoted by $T_{\alpha\beta}$, is defined as the variation of the matter action with respect to the metric variation, i.e.,

$$T_{\alpha\beta}(x)=-\frac{2}{\sqrt{-g}}\frac{\partial S_M}{\partial g^{\alpha\beta}(x)}. \tag{3}$$

Therefore, for a minimally coupled massive scalar field in the general spacetime metric, given by

$$S[g_{\alpha\beta},\phi]=-\frac{1}{2}\int d\eta d^3\vec{x}\,\sqrt{-g}(g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi+m^2\phi^2), \tag{4}$$

we have

$$T_{\alpha\beta}(x)=\nabla_\alpha\phi\nabla_\beta\phi-\frac{1}{2}g_{\alpha\beta}(g^{\gamma\delta}\nabla_\gamma\phi\nabla_\delta\phi+m^2\phi^2). \tag{5}$$

For example, if we consider the Minkowski space, i.e., $g_{\alpha\beta}=\eta_{\alpha\beta}$, we have

$$T_{ab}(x)=\lim_{y\to x}P_{ab}(x,y)\phi(x)\phi(y), \tag{6}$$

where

$$P_{ab}(x,y)=\left(\delta^c_{(a}\delta^d_{b)}-\frac{1}{2}\eta_{ab}\eta^{cd}\right)\nabla^x_c\nabla^y_d-\frac{1}{2}\eta_{ab}m^2. \tag{7}$$

[1] Fluctuations obtained in this way are generally divergent in the $x'\to x$ limit, but these can be taken care of by proper regularization procedures as is routinely done for divergent observables in quantum field theory [5,8,51]. However, in the present paper, we are more interested in the correlations rather than the fluctuations explicitly.

This implies that the stress energy two-point correlator in the vacuum of the field can be obtained as[2]

$$\begin{aligned}\langle t_{abcd}(x,x')\rangle &= \lim_{\substack{y\to x\\ y'\to x'}} P_{ab}(x,y)P_{cd}(x',y')\langle 0|\phi(x)\phi(y)\phi(x')\phi(y')|0\rangle \\ &\quad - \lim_{\substack{y\to x\\ y'\to x'}} P_{ab}(x,y)P_{cd}(x',y')\langle 0|\phi(x)\phi(y)|0\rangle\langle 0|\phi(x')\phi(y')|0\rangle.\end{aligned} \tag{8}$$

After some manipulations, this becomes [42]

$$\langle t_{abcd}(x,x')\rangle = 2\lim_{\substack{y\to x\\ y'\to x'}} P_{ab}(x,y)P_{cd}(x',y')G(x,x')G(y,y'), \tag{9}$$

where $G(x,x')$ is the Wightman function for the scalar field in the considered vacuum,

$$G(x,x') = \langle 0|\phi(x)\phi(x')|0\rangle. \tag{10}$$

B. de Sitter space

An n-dimensional de Sitter space, denoted by dS_n, can be viewed as the embedding

$$\eta_{ab}X^aX^b = H^{-2}, \tag{11}$$

in $\mathbb{R}^{(1,n)}$ with the metric $\eta_{\mu\nu} = \mathrm{diag}(-1,1,\ldots,1)$. The de Sitter space is a maximally symmetric space and has constant Ricci scalar $R = 2(n-1)(n-2)H^2$. It can be shown that the de Sitter space is also the solution of vacuum Einstein equations with a positive cosmological constant, given by $(n-1)(n-2)H^2/2$.

One can use a number of coordinate systems to cover the de Sitter space,[3] but one particularly useful coordinate system (called planar or inflationary coordinates) for our purposes is given by

$$\begin{aligned}X^n - X^0 &= \pm\frac{e^{Ht}}{H}, \qquad X^i = \pm x^i e^{Ht}, \quad i = 1,\ldots,n-1, \\ X^n + X^0 &= \pm\left(\frac{e^{-Ht}}{H} - x_i x^i H e^{Ht}\right).\end{aligned} \tag{12}$$

For a given sign in the above equation, one covers only half of the de Sitter manifold. Therefore, $\pm$ signs correspond to two charts covering the full de Sitter space. In a single chart, all the coordinates (i.e., $t, x^1, x^2, \ldots, x^{n-1}$) lie between $(-\infty,\infty)$.

In these coordinates, the metric in both the charts is given by

$$ds^2 = -dt^2 + e^{2Ht}d\vec{x}^2. \tag{13}$$

The transformation[4] $d\eta = dt/a(t)$ with $a(\eta) = -1/H\eta$ brings the above metric in the following conformally flat form:

$$ds^2 = \frac{1}{(H\eta)^2}(-d\eta^2 + d\vec{x}^2). \tag{14}$$

If we define $Z(x,x') = H^2\eta_{ab}X^a(x)X^b(x')$, then, in the planar coordinates, we have

$$Z(x,x') = 1 + \frac{(\eta-\eta')^2 - (\vec{x}-\vec{x}')^2}{2\eta\eta'}. \tag{15}$$

This is a useful quantity, which (indirectly) characterizes the geodesic distance between points x and x' on the de Sitter manifold.

C. Quantum fields on de Sitter space

The minimally coupled scalar field, corresponding to (4), satisfies the following equation of motion:

$$(\Box - m^2)\phi(x) = 0. \tag{16}$$

The Wightman function also satisfies the same equation as the field, i.e.,

$$(\Box - m^2)G(x,x') = 0. \tag{17}$$

For a de Sitter invariant vacuum, evidently this depends only on the geodesic distance, i.e., $G(x,y) = G(Z(x,y))$, and the above equation becomes (see Refs. [23,55])

$$(Z^2-1)\frac{\mathrm{d}^2G}{\mathrm{d}Z^2} + 4Z\frac{\mathrm{d}G}{\mathrm{d}Z} + \frac{m^2}{H^2}G(Z) = 0. \tag{18}$$

Under the transformation $Z \to Y = (1+Z)/2$, it further reduces to

$$Y(1-Y)\frac{\mathrm{d}^2G}{\mathrm{d}Y^2} + (2-(a+b+1)Y)\frac{\mathrm{d}G}{\mathrm{d}Y} - abG(Y) = 0, \tag{19}$$

[2]In this expression and the following expressions, we do not bother ourselves to put a hat over the stress energy (or field) operators, since it is understood that we are exclusively working with quantum stress energy operators.

[3]To know more about different coordinate systems used for de Sitter space (like global coordinates, static coordinates, Eddington-Finkelstein coordinates, Kruskal coordinates, etc.) and the causal structure of the de Sitter space (i.e., its Penrose diagram, etc.), one can refer to Refs. [52–54].

[4]Now η lies between $(-\infty,0)$ corresponding to t lying between $(-\infty,\infty)$.

where $a = 3/2 + \sqrt{9/4 - m^2/H^2}$ and $b = 3/2 - \sqrt{9/4 - m^2/H^2}$ or vice versa. This is the hypergeometric equation, and one particular solution to this equation is $G(Z) = {}_2F_1(a, b, 2, \frac{1+Z}{2})$. A particular choice for the de Sitter invariant vacuum state (called the Bunch-Davies vacuum) leads to $G(Z) = (H^2/16\pi^2)\Gamma(a)\Gamma(b){}_2F_1(a, b, 2, \frac{1+Z}{2})$. Because of its structure, massless fields (for which $a = 3,\ b = 0$) have a divergent piece in $G(Z)$ which is identified as the infrared divergence. Since the Wightman function is only a function of Z, all its higher-order derivatives can be evaluated in terms of variation of Z with respect to spacetime coordinates (see Appendix A).

III. NOISE KERNEL IN de Sitter UNIVERSE

First, we express the stress energy correlator (algebraically related to the noise kernel) in a conformally flat Friedmann spacetime. For a minimally coupled scalar field in a conformally flat spacetime metric, i.e., $g_{\alpha\beta} = a(\eta)^2\eta_{\alpha\beta}$, we can again use the expression (6) for the stress energy correlator but now with

$$P_{ab}(x, y) = \left(\delta^c_{(a}\delta^d_{b)} - \frac{1}{2}\eta_{ab}\eta^{cd}\right)\nabla^x_c\nabla^y_d - \frac{1}{2}\left(\frac{a(\eta) + a(\eta')}{2}\right)^2\eta_{ab}m^2. \tag{20}$$

Now using this expression of $P_{ab}(x, y)$ in Eq. (9) and specializing to the case of de Sitter spacetime, i.e., $a(\eta) = -\frac{1}{H\eta}$, we have the following expression for the noise kernel:

$$\begin{aligned}\langle t_{abcd}(x, x')\rangle_{dS} = \Big(&\nabla_b\nabla'_c G(x, x')\nabla_a\nabla'_d G(x, x') + \nabla_b\nabla'_d G(x, x')\nabla_a\nabla'_c G(x, x') \\ &- \eta_{cd}\eta^{\rho\sigma}\nabla_a\nabla'_\rho G(x, x')\nabla_b\nabla'_\sigma G(x, x') - \frac{1}{H^2\eta'^2}m^2\eta_{cd}\nabla_a G(x, x')\nabla_b G(x, x') \\ &- \eta_{ab}\eta^{\gamma\delta}\nabla_\gamma\nabla'_c G(x, x')\nabla_\delta\nabla'_d G(x, x') + \frac{1}{2}\eta_{ab}\eta^{\gamma\delta}\eta_{cd}\eta^{\rho\sigma}\nabla_\gamma\nabla'_\rho G(x, x')\nabla_\delta\nabla'_\sigma G(x, x') \\ &+ \frac{1}{2H^2\eta'^2}m^2\eta_{ab}\eta^{\gamma\delta}\eta_{cd}\nabla_\gamma G(x, x')\nabla_\delta G(x, x') - \frac{1}{H^2\eta^2}m^2\eta_{ab}\nabla'_c G(x, x')\nabla'_d G(x, x') \\ &+ \frac{1}{2H^2\eta^2}m^2\eta_{ab}\eta_{cd}\eta^{\rho\sigma}\nabla'_\rho G(x, x')\nabla'_\sigma G(x, x') + \frac{1}{2H^4\eta^2\eta'^2}m^4\eta_{ab}\eta_{cd}G(x, x')G(x, x')\Big).\end{aligned} \tag{21}$$

We are interested in learning if the primordial fluctuations remain relevant if the universe expands. For this purpose, we first choose a spacelike surface by fixing η. We now use the properties of the Wightman function on constant time (η-) hypersurfaces and evaluate $\langle t_{abcd}(x, x')\rangle_{dS}$ when the physical distances between fixed comoving distances grow very large, i.e., $a(\eta) \to \infty$, which in expanding universes will be the late time era.

A. Minimal coupling

In order to study the stochastic correction, in principle, it will be necessary to consider all the components of the noise kernel. However, for our purpose, it will be sufficient to explore only the $\langle t_{0000}\rangle$ component to establish the growth or decay of such stochastic corrections. In fact, the table in Appendix B shows that the degree of divergence (if any) of the other components of the noise kernel is either less than or equal to that of the $\langle t_{0000}\rangle$ component of the noise kernel. Furthermore, the $\langle t_{0000}\rangle$ also gives the energy correlator in a straightforward manner which is a readily accessible observable quantity [11]. Therefore, we need to calculate the $(a = 0, b = 0, c = 0, d = 0)$ component of the noise kernel. In de Sitter spacetime, late time corresponds to the $\eta \to 0$ limit. So, we consider the noise kernel on constant time sheets (i.e., $\eta = \eta'$) with finite spatial distances (i.e., $\Delta\vec{x} \neq 0$), and then we take the $\eta \to 0$ limit.

Using Eq. (21) and formulas from Appendix A, we see that

$$\begin{aligned}\langle t_{00}(\eta, \vec{x})t_{00}(\eta, \vec{x'})\rangle_{dS} = \Bigg(&(G'')^2\left[\frac{(\Delta\vec{x})^6}{4\eta^{10}} + \frac{(\Delta\vec{x})^8}{32\eta^{12}} + \frac{(\Delta\vec{x})^4}{2\eta^8}\right] + G^2\left[\frac{m^4}{2H^4\eta^4}\right] + (G')^2\Bigg[\frac{3(\Delta\vec{x})^2}{2\eta^6} + \frac{(\Delta\vec{x})^4}{8\eta^8} + \frac{2}{\eta^4} \\ &+ \frac{m^2}{H^2}\left(\frac{(\Delta\vec{x})^4}{4\eta^8} + \frac{(\Delta\vec{x})^2}{\eta^6}\right)\Bigg] + (G''G')\left[-\frac{5(\Delta\vec{x})^4}{4\eta^8} - \frac{(\Delta\vec{x})^2}{\eta^6} - \frac{(\Delta\vec{x})^6}{8\eta^{10}}\right]\Bigg).\end{aligned} \tag{22}$$

Using the expressions for the Wightman function and its derivatives in the Bunch-Davies vacuum, i.e.,

$$G(Z) = \frac{H^2}{16\pi^2}\Gamma\left(\frac{3}{2}+\nu\right)\Gamma\left(\frac{3}{2}-\nu\right) \times {}_2F_1\left(\frac{3}{2}+\nu,\frac{3}{2}-\nu,2,\frac{1+Z}{2}\right), \tag{23}$$

$$G'(Z) = \frac{H^2}{64\pi^2}\Gamma\left(\frac{5}{2}+\nu\right)\Gamma\left(\frac{5}{2}-\nu\right) \times {}_2F_1\left(\frac{5}{2}+\nu,\frac{5}{2}-\nu,3,\frac{1+Z}{2}\right), \tag{24}$$

$$G''(Z) = \frac{H^2}{384\pi^2}\Gamma\left(\frac{7}{2}+\nu\right)\Gamma\left(\frac{7}{2}-\nu\right) \times {}_2F_1\left(\frac{7}{2}+\nu,\frac{7}{2}-\nu,4,\frac{1+Z}{2}\right) \tag{25}$$

(where $\nu = \sqrt{\frac{9}{4}-\frac{m^2}{H^2}}$), and appealing to the late time behavior (i.e., $Z \to -\infty$) for the ${}_2F_1$ functions [56], i.e.,

$$\begin{aligned} {}_2F_1(a,b,c,z) = {} & \frac{\Gamma(b-a)\Gamma(c)(-z)^{-a}}{\Gamma(b)\Gamma(c-a)} \times \left(\sum_{k=0}^{\infty}\frac{(a)_k(a-c+1)_k z^{-k}}{k!(a-b+1)_k}\right) \\ & + \frac{\Gamma(a-b)\Gamma(c)(-z)^{-b}}{\Gamma(a)\Gamma(c-b)} \times \left(\sum_{k=0}^{\infty}\frac{(b)_k(b-c+1)_k z^{-k}}{k!(b-a+1)_k}\right), \end{aligned} \tag{26}$$

we have

$$\begin{aligned} \langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x}')\rangle_{dS}|_{\text{late time}} = {} & \frac{H^4\Gamma^2(\nu)\Gamma^2(\frac{5}{2}-\nu)}{\pi^5}\left[\frac{9\eta^{2-4\nu}}{32(\Delta\vec{x})^{6-4\nu}} + \frac{21(3-2\nu)\eta^{4-4\nu}}{16(\Delta\vec{x})^{8-4\nu}} \right. \\ & \left. + \frac{(656\nu^3-3244\nu^2+5168\nu-2655)\eta^{6-4\nu}}{64(\nu-1)(\Delta\vec{x})^{10-4\nu}} + O(\eta^2)\right]. \end{aligned} \tag{27}$$

From here, we can see that there is a transition in the behavior of the stochastic correction term at $\nu = 1/2$. For any $\nu < 1/2$, it vanishes in the late lime limit (large physical distances) as $\eta^{2-4\nu}$, e.g., for $\nu = 0$,

$$\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x}')\rangle_{dS}|_{\text{late time}} = \lim_{\eta\to 0}[O(\eta)]. \tag{28}$$

On the other hand, the noise kernel approaches a saturating value at $\nu = 1/2$, for large physical distances with finite comoving distance,

$$\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x}')\rangle_{dS}|_{\text{late time}} = \lim_{\eta\to 0}\left[\frac{9H^4}{32\pi^4(\Delta\vec{x})^4} + O(\eta)\right]. \tag{29}$$

This is not surprising, as $\nu = 1/2$ is conformal field theory and does not feel $a(\eta)$. However, this goes on to suggest that the stochastic correction is uncontrollable after $\nu > 1/2$. For example, for a massless field, $\nu = 3/2$,

$$\begin{aligned} \langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x}')\rangle_{dS}|_{\text{late time}} &= \lim_{\eta\to 0}\lim_{\epsilon\to 0}\left[\frac{9H^4}{128\pi^4\eta^4}[1-4\epsilon] + \frac{21H^4\epsilon}{32\pi^4(\Delta\vec{x})^2\eta^2} + \frac{H^4}{16\pi^4(\Delta\vec{x})^4}\left[\frac{3}{2}+14\epsilon\right] + O(\eta)\right] \\ &= \lim_{\eta\to 0}\left[\frac{9H^4}{128\pi^4\eta^4} + \frac{H^4}{16\pi^4(\Delta\vec{x})^4}\left[\frac{3}{2}\right] + O(\eta)\right] \to \infty. \end{aligned} \tag{30}$$

Therefore, we see that, for a minimally coupled scalar field in Bunch-Davies vacuum, the $(a=0, b=0, c=0, d=0)$ component of the noise kernel (on constant time sheets with finite spatial distance and in the late time universe limit) undergoes a kind of "phase transition" as a function of ν, with the critical value being $\nu = 1/2$. To put in context, it is also well known that de Sitter has an instability against the particle creation of light mass particles [22,57–60].

B. Comparison with large comoving distance case

At this point, we can compare our results with the case for large comoving distance, obtained in Ref. [50], in which the noise kernel is shown to be

$$\langle t_{abcd}(x,x'))\rangle_{dS} = P(\mu)n_a n_b n_c n_d + Q(\mu)(n_a n_b g_{c'd'} + n_{c'} n_{d'} g_{ab}) + R(\mu)(n_s n_{c'} g_{bd'} + n_b n_{d'} g_{ac'} + n_a n_{d'} g_{bc'} + n_b n_{c'} g_{ad'}) + S(\mu)(g_{ac'} g_{bd'} + g_{bc'} g_{ad'}) + T(\mu) g_{ab} g_{c'd'}, \tag{31}$$

where P, Q, R, S, and T are sums of products of the Wightman function and its first- and second-order derivatives with respect to the geodesic distance. Here, n_a and $n_{a'}$ are the unit tangent vectors to the geodesic connecting the points x and x', at x and x', respectively. The action of $g_{ac'}$ is to parallel transport a vector from x' to x along the geodesic.

For the $Z \ll -1$ regime, $P, Q, T \sim Z^{-2h_-}$, $R \sim Z^{-2h_- -1}$, and $S \sim Z^{-2h_- -2}$. Using these behaviors of P, Q, R, S, and T, it is argued that *the fluctuations decay faster with the distance as mass increases*. However, for the fixed comoving distance and large scale factor limit, the coefficients of $P(\mu)$, $Q(\mu)$, etc., in the above equation, also depend upon η (and hence on Z as $Z = 1 + \frac{(\eta-\eta')^2 - (\Delta\vec{x})^2}{2\eta\eta'}$), and the mentioned result is obtained ignoring these dependences. So, in this sense, the results of Ref. [50] are, in fact, valid for those scenarios in which η and η' are held finite (and constant) and Z approaches large values through the $(\Delta\vec{x})^2 \to \infty$ limit. However, large spatial separation can arise in another way, namely, with finite $\Delta\vec{x}(\neq 0)$ and $a(\eta) \to \infty$. This other scenario again shows the divergences obtained in the previous section, for the relevant mass ranges.

A similar expression can be derived for nonminimally coupled fields as well. Though the relations derived above carry over with simple reparameterization $m^2 \to m^2 + 12\xi H^2$, we still present a brief discussion for the nonminimal case.

C. Nonminimal coupling

Let us consider a nonminimally coupled massive scalar field with the following action[5]:

$$S^{nm}[g_{\alpha\beta}, \phi] = -\frac{1}{2}\int d\eta d^3\vec{x}\, \sqrt{-g}(g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi + m^2\phi^2 + \xi R\phi^2). \tag{32}$$

It gives the following equation of motion for the scalar field ϕ:

$$[\Box - (12\xi H^2 + m^2)]\phi(x) = 0, \tag{33}$$

which implies

$$G(Z(x,x')) = \frac{H^2}{16\pi^2}\Gamma(a)\Gamma(b)\,{}_2F_1\left(a, b, 2, \frac{1+Z}{2}\right), \tag{34}$$

where $a = \frac{3}{2} + \sqrt{\frac{9}{4} - \frac{12\xi H^2 + m^2}{H^2}}$ and $b = \frac{3}{2} - \sqrt{\frac{9}{4} - \frac{12\xi H^2 + m^2}{H^2}}$.

The stress energy tensor for this case is given by

$$T^{nm}_{\alpha\beta}(x) = \nabla_\alpha\phi\nabla_\beta\phi - \frac{1}{2}g_{\alpha\beta}(g^{\gamma\delta}\nabla_\gamma\phi\nabla_\delta\phi + m^2\phi^2) + \xi(G_{\alpha\beta}\phi^2 + g_{\alpha\beta}g^{\gamma\delta}\nabla_\gamma\nabla_\delta\phi^2 - \nabla_\alpha\nabla_\beta\phi^2), \tag{35}$$

where $G_{\alpha\beta}$ is the Einstein tensor. Using the fact that, for de Sitter space, $G_{\alpha\beta} = -3H^2 g_{\alpha\beta}$ and $g_{\alpha\beta} = \eta_{\alpha\beta}/H^2\eta^2$, we have

$$T^{nm}_{\alpha\beta}(x) = \lim_{y\to x} P^{nm}_{ab}(x,y)\phi(x)\phi(y) = \lim_{y\to x}(P_{ab}(x,y) + M_{ab}(x,y))\phi(x)\phi(y), \tag{36}$$

where

$$P_{ab}(x,y) = \left[\left((1-2\xi)\delta^r_{(a}\delta^s_{b)} - \left(\frac{1}{2} - 2\xi\right)\eta_{ab}\eta^{rs}\right)\nabla^x_r\nabla^y_s - \frac{2(3H^2\xi + \frac{m^2}{2})}{(H\eta)^2 + (H\eta')^2}\eta_{ab}\right] \tag{37}$$

and

$$M_{ab}(x,y) = [2\xi\eta_{ab}\eta^{rs} - 2\xi\delta^r_{(a}\delta^s_{b)}]\frac{\nabla^x_r\nabla^x_s + \nabla^y_r\nabla^y_s}{2}. \tag{38}$$

In the above formula, η and η' correspond to the time coordinate of points x and y, respectively. Here, we see that the P_{ab} part is the same as it is for the minimally coupled scalar field with $\xi = 0$. Also, the expression for $P^{nm}_{ab}(x,y)$ is symmetric in x and y.

Similar to the minimally coupled case, we find that

$$\langle t^{nm}_{ab}(x)t^{nm}_{cd}(x')\rangle = 2\lim_{\substack{y\to x\\ y'\to x'}} P^{nm}_{ab}(x,y)P^{nm}_{cd}(x',y')G(x,x')G(y,y') = 2\lim_{\substack{y\to x\\ y'\to x'}}(P_{ab}(x,y)P_{cd}(x',y') + P_{ab}(x,y)M_{cd}(x',y') + M_{ab}(x,y)P_{cd}(x',y') + M_{ab}(x,y)M_{cd}(x',y')) \times G(x,x')G(y,y'). \tag{39}$$

[5]Here superscript *nm* refers to nonminimal coupling.

The contributions of the $P_{ab}P_{cd}$, $P_{ab}M_{cd}$, $M_{ab}P_{cd}$, and $M_{ab}M_{cd}$ terms, to the noise kernel expression, are given in Appendix B 2. The power counting argument clearly shows that the most dominant power of η (in the limit $\eta \to 0$), in the relevant noise kernel component, is still $2-4\nu$ (where $\nu = \sqrt{\frac{9}{4} - \frac{12\xi H^2 + m^2}{H^2}}$). In fact, we have

$$\begin{aligned}\langle t_{ab}^{nm}(x) t_{cd}^{nm}(x')\rangle|_{\text{late time}} = \lim_{\eta\to 0}\Bigg[\frac{\eta^{2-4\nu}H^4}{512\pi^5(\Delta\vec{x})^{6-4\nu}}\Bigg[32(12\xi-1)\Gamma\left(\frac{5}{2}-\nu\right)\Gamma\left(\frac{7}{2}-\nu\right)\\ + \left(16\frac{m^4}{H^4} + 8\frac{m^2}{H^2}(24\xi + (3-2\nu)^2) - 48\xi(3-2\nu)^2 + (3-2\nu)^2(29-20\nu+4\nu^2)\right.\\ \left. + 32\xi^2(27-12\nu+4\nu^2)\right)\Gamma^2\left(\frac{3}{2}-q\right)\Bigg]\Gamma[\nu]^2 + O(\eta^{4-4\nu})\Bigg]. \qquad (40)\end{aligned}$$

This implies that the noise kernel for a conformally coupled scalar field behaves exactly similar to the noise kernel for a minimally coupled scalar field except for the fact that m^2/H^2 in the latter case goes to $m^2/H^2 + 12\xi$ in the former; i.e., it undergoes a sort of "divergent transition" as $m^2/H^2 + 12\xi$ crosses the critical value 2, making $\nu \geq 1/2$. Thus, we readily see that the conformal coupling $\xi = 1/6$ cures the divergence as, even for the massless field, we get $\nu = 1/2$, which, at best, has a nonzero finite value of noise kernel component over large physical scales. For any other nonzero mass, the value of ν is less than $1/2$, showing a vanishing correlation over large scales. However, for any $\xi < 1/6$, we still have divergences over a range of mass values. Clearly, this divergence in late times is different from the secular divergences of stress energy as (a) we use the RSET, and (b) this divergence appears only for a finite comoving distance in the large scale factor limit. Thus, the correlation structure on fixed comoving distance may grow or decay as the scale factor turns large, depending upon the value of the coupling ξ and mass m. We now relate the noise kernel of Friedmann universes with the noise kernel of a de Sitter universe for various masses.

IV. NOISE KERNEL FOR FRIEDMANN SPACES

In this section, we relate the results of the previous sections on the components of the noise kernel in the de Sitter spacetime to the corresponding noise kernel components in Friedmann spacetimes using the fact that a massless scalar field in a Friedmann spacetime is conformally equivalent to a massive scalar field in de Sitter spacetime.[6] If a power-law Friedmann universe has scaling factor $a(\eta) = (H\eta)^{-q}$, then the corresponding massive scalar field in de Sitter spacetime has $m^2 = H^2(1-q)(2+q)$. One also gets that the Wightman function in the power-law universe is related to the Wightman function in de Sitter spacetime, $G^{\text{P.L.}}(x,x') = (H\eta)^{q-1}(H\eta')^{q-1}G(x,x')$. Using this, we see that

$$\nabla'_\mu G^{\text{P.L.}} = (H)^{2q-2}[(q-1)(\eta)^{q-1}(\eta')^{q-2}G\delta_{\mu 0} + (\eta)^{q-1}(\eta')^{q-1}\nabla'_\mu G], \qquad (41)$$

and

[6]The action of a massless scalar field in a universe with metric $g_{\alpha\beta} = a^2\eta_{\alpha\beta}$ with $a(\eta) = (H\eta)^{-q}$ is given by

$$S = -\frac{1}{2}\int d^4x a^4(a^{-2}\eta^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi).$$

Under the transformation $\phi(x) = (H\eta)^{-1+q}\psi(x)$, the action becomes

$$S = -\frac{1}{2}\int d^4x b^4(b^{-2}\eta^{\alpha\beta}\partial_\alpha\psi\partial_\beta\psi - m_{\text{eff}}^2\psi^2),$$

where $b(\eta) = (H\eta)^{-1}$ and $m_{\text{eff}}^2 = H^2(1-q)(2+q)$. Therefore, we see that a massless scalar field in a Friedmann universe with scaling factor $a(\eta) = (H\eta)^{-q}$ goes to a massive scalar field in a de Sitter universe under the above mentioned transformation. We also see that the transformation relation between the fields, i.e., $\phi(x) = (H\eta)^{-1+q}\psi(x)$, explains the relation between the Wightman functions in the related spacetimes, i.e., $G^{\text{P.L.}}(x,x') = (H\eta)^{q-1}(H\eta')^{q-1}G(x,x')$. A similar kind of correspondence can be established for the nonminimal setting as well. For more details, see Appendix A.2 of Ref. [39].

$$\begin{aligned}\nabla_\nu\nabla'_\mu G^{\text{P.L.}} &= H^{2q-2}[(q-1)^2(\eta)^{q-2}(\eta')^{q-2}G\delta_{\mu0}\delta_{\nu0} + (q-1)(\eta)^{q-1}(\eta')^{q-2}\delta_{\mu0}\nabla_\nu G \\ &\quad + (q-1)(\eta)^{q-2}(\eta')^{q-1}\delta_{\nu0}\nabla'_\mu G + (\eta)^{q-1}(\eta')^{q-1}\nabla_\nu\nabla'_\mu G] \\ &= (H\eta H\eta')^{q-1}\left(\frac{(q-1)^2}{\eta\eta'}G\delta_{\mu0}\delta_{\nu0} + \frac{(q-1)}{\eta'}\delta_{\mu0}\nabla_\nu G + \frac{(q-1)}{\eta}\delta_{\nu0}\nabla'_\mu G + \nabla_\nu\nabla'_\mu G\right). \end{aligned} \tag{42}$$

The above expression, for $\eta = \eta'$, for different values of ν and μ is given by (see Appendix A)

$$\begin{aligned}\nabla_0\nabla'_0 G^{\text{P.L.}} &= (H\eta)^{2q-2}\left[\frac{(q-1)^2}{\eta^2}G + \frac{(q-1)}{\eta}\nabla_0 G + \frac{(q-1)}{\eta}\nabla'_0 G + \nabla_0\nabla'_0 G\right]; \\ \nabla_0\nabla'_j G^{\text{P.L.}} &= (H\eta)^{2q-2}\left[\frac{q-1}{\eta}\nabla'_j G + \nabla_0\nabla'_j G\right]; \\ \nabla_i\nabla'_0 G^{\text{P.L.}} &= (H\eta)^{2q-2}\left[\frac{q-1}{\eta}\nabla_i G + \nabla_i\nabla'_0 G\right]; \\ \nabla_i\nabla'_j G^{\text{P.L.}} &= (H\eta)^{2q-2}[\nabla_i\nabla'_j G]. \end{aligned} \tag{43}$$

Now, we have the covariant derivatives of the Wightman function in the Friedmann spacetimes in terms of the corresponding quantities in the de Sitter spacetime. Using the above expressions in the noise kernel expression for a massless scalar field and for Friedmann spacetimes, we see that the considered noise kernel component (on constant time sheets) is given by

$$\begin{aligned}\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x'})\rangle_{\text{P.L.}} &= (H\eta)^{4(q-1)}\Bigg[\frac{G^2}{2\eta^4}(q-1)^4 + GG'\left[\frac{(2q^3-7q^2+8q-3)(\Delta x)^2}{2\eta^6} - \frac{(q-1)^2}{\eta^4}\right] + GG''\frac{(q-1)^2(\Delta\vec{x})^4}{4\eta^8} \\ &\quad + G'G''\left[\frac{(q-\frac{3}{2})(\Delta\vec{x})^6}{4\eta^{10}} + \frac{(q-\frac{9}{4})(\Delta\vec{x})^4}{\eta^8} - \frac{(\Delta\vec{x})^2}{\eta^6}\right] + (G')^2\Bigg[\frac{2}{\eta^4} + \frac{(q^2-5q+\frac{11}{2})(\Delta\vec{x})^2}{\eta^6} \\ &\quad + \frac{(2q^2-6q+\frac{9}{2})(\Delta\vec{x})^4}{4\eta^8}\Bigg] + (G'')^2\left[\frac{(\Delta\vec{x})^6}{4\eta^{10}} + \frac{(\Delta\vec{x})^8}{32\eta^{12}} + \frac{(\Delta\vec{x})^4}{2\eta^8}\right]\Bigg]. \end{aligned} \tag{44}$$

For different power-law universes, i.e., for different values of q, one can evaluate the above expression on constant time sheets. However, as ν can take values only in the range $[-3/2, 3/2]$, we see that we can use the considered equivalence only for those values of q which lie in the range $[-2, 1]$. The region $|\nu| > 3/2$ is mapped to the region outside $[-2, 1]$. As we are interested in the behavior of the noise kernel component in the late time universe, we observe that, for $q \in (0, 1]$, the late time universe corresponds to $\eta \to 0$ and, for $q \in [-2, 0)$, the late time universe corresponds to $\eta \to \infty$. We now list down the stress energy correlator for various Friedmann spacetimes:

(i) $q = 1$.—This case trivially corresponds to a massless scalar field in de Sitter spacetime, which is just the case $\nu = 3/2$ in the previous section. As discussed above, the correlator diverges in the late time limit as η^{-4} or a^4.

(ii) $q \in (0, 1)$.—If we perform, for this case as well, the same power counting analysis as is done in Appendix B, we find that the relevant noise kernel component in the late time universe, i.e., $\eta \to 0$ limit, has an η-independent term. Therefore, we have a constant late time noise kernel component for those Friedmann universes which have a negative exponent of η in the scale factor. In fact, we have

$$\begin{aligned}\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x'})\rangle_{\text{P.L.}}|_{\text{late time}} &= \lim_{\eta\to 0}\frac{(H\eta)^{4q-4}}{(\Delta\vec{x})^4}\Bigg[\frac{H^4\eta^{4-4q}(\Delta\vec{x})^{4q-4}}{8\pi^5}((11-12q+4q^2)(\Gamma(2-q))^2(\Gamma(0.5+q))^2) \\ &\quad + \frac{4^{4q}\eta^{4q+4}H^4}{32\pi^5(\Delta\vec{x})^{4+4q}}((1+2q)^4(\Gamma(2+q))^2(\Gamma(-0.5-q))^2) + O(\eta^{6-4q})\Bigg]. \end{aligned} \tag{45}$$

In the late time limit, only the term $\frac{H^{4q}(\Delta\vec{x})^{4q-8}}{8\pi^5}((11-12q+4q^2)(\Gamma(2-q))^2(\Gamma(0.5+q))^2)$ survives, which is time independent and is therefore a remnant of the quantum fluctuations the universe was born with. For these spacetimes, the stochastic term, in the Einstein-Langevin equation, will be relevant if the constant it saturates to is comparable to the expectation values appearing in the semiclassical analysis. Thus, in principle, these spacetimes are vulnerable to long-range effects.

This is a bit interesting as, in the late time limit, the Wightman function (and hence the stress energy correlator) drops the time (or the scale factor) dependency. For constant time sheets, we have

$$G^{\mathrm{P.L.}}(\eta,\vec{x},\eta',\vec{x}') = \frac{H^2(H\eta)^{2q-2}}{16\pi^2}\,{}_2F_1\left(2+q,1-q,2,1-\frac{(\Delta\vec{x})^2}{4\eta^2}\right). \tag{46}$$

In the $\eta\to 0$ limit, we have

$$G^{\mathrm{P.L.}}(\eta,\vec{x},\eta',\vec{x}') = \frac{H^2(H\eta)^{2q-2}}{16\pi^2}\Gamma(2+q)\Gamma(1-q)\left[\frac{\Gamma(-1-2q)((\frac{(\Delta\vec{x})^2}{4\eta^2}))^{-2-q}}{\Gamma(1-q)\Gamma(-q)}\sum_{k=0}^{\infty}\frac{(2+q)_k(1+q)_k(-\frac{(\Delta\vec{x})^2}{4\eta^2})^{-k}}{k!(2+2q)_k}\right.$$
$$\left.+\frac{\Gamma(1+2q)(\frac{(\Delta\vec{x})^2}{4\eta^2})^{-1+q}}{\Gamma(2+q)\Gamma(1+q)}\sum_{k=0}^{\infty}\frac{(1-q)_k(-q)_k(-\frac{(\Delta\vec{x})^2}{4\eta^2})^{-k}}{k!(-2q)_k}\right]. \tag{47}$$

Since $a(\eta)=(H\eta)^{-q}$ (i.e., $H\eta = a^{-1/q}$), we can convert the above expression in terms of the physical distance on constant time sheets, i.e., $a^2(\Delta\vec{x})^2$, and in terms of $a(\eta)$, i.e.,

$$G^{\mathrm{P.L.}}(\eta,\vec{x},\eta',\vec{x}') = \frac{H^2}{16\pi^2}\Gamma(2+q)\Gamma(1-q)$$
$$\times\left[\frac{\Gamma(-1-2q)(\frac{H^2}{4})^{-2-q}a^{2q-2/q}}{\Gamma(1-q)\Gamma(-q)(a^2(\Delta\vec{x})^2)^{2+q}}\sum_{k=0}^{\infty}\frac{(2+q)_k(1+q)_k(-\frac{H^2}{4})^{-k}(a^2(\Delta\vec{x})^2)^{-k}(a^{-2+2/q})^{-k}}{k!(2+2q)_k}\right.$$
$$\left.+\frac{\Gamma(1+2q)(\frac{H^2}{4})^{-1+q}a^{2-2q}}{\Gamma(2+q)\Gamma(1+q)(a^2(\Delta\vec{x})^2)^{1-q}}\sum_{k=0}^{\infty}\frac{(1-q)_k(-q)_k(-\frac{H^2}{4})^{-k}(a^2(\Delta\vec{x})^2)^{-k}(a^{-2+2/q})^{-k}}{k!(-2q)_k}\right]. \tag{48}$$

One can check that the leading term of the second series in the square bracket is the dominant term for $q > -1/2$, in the $\eta\to 0$ limit, which kills off all a dependence at late times, assuming a pseudoconformal form. It is worth noting that, for all prior times, there is a $\eta-$ dependency in the expression, which gradually decays, and at the end we are left with the constant leading-order term. Therefore, long-distance correlators, with small coordinate values, of this spacetime maintain the initial time correlations.

(iii) $q=0$.—This is a special limit of no dynamics, i.e., $a(\eta)=1$, and hence is the flat space result, which is well studied [42,61,62]. The Wightman function for Minkowskian spacetime is given by $G(x,x') = \frac{1}{4\pi^2(-(\eta-\eta')^2+(\Delta\vec{x})^2)}$. Using this expression, we find that the noise kernel, on constant time sheets for finite spatial distance, is given by

$$\langle\hat{t}_{00}(\eta,\vec{x})\hat{t}_{00}(\eta,\vec{x'})\rangle_{\mathrm{P.L.}} = \frac{3}{2\pi^4(\Delta\vec{x})^8}. \tag{49}$$

Evidently, for a constant comoving distance, the correlator survives, as the comoving and physical distances are the same and physical distance does not grow in "late time" or "early time" because of the lack of dynamics. For a large physical distance, there is no appreciable stochastic effect.

(iv) $q\in(-2,0)$.—In this case, $a(\eta)=(H\eta)^{|q|}$ and, hence, the late time universe corresponds to $\eta\to\infty$. For this case, we have

$$\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x'})\rangle_{\mathrm{P.L.}}|_{\text{late time}} = \lim_{\eta\to\infty}(H\eta)^{4q-4}\left[\frac{3H^4\eta^4}{2\pi^4(\Delta\vec{x})^8}+\frac{\eta^2H^4(3q+4q^2)}{8\pi^4(\Delta\vec{x})^6}+\frac{H^4q}{64\pi^4(\Delta\vec{x})^4}\left((-4-7q+6q^2+11q^3)\right.\right.$$
$$\left.\left.+2(1+q)(-1+q)^2\left[2\gamma+\log\left(\frac{(\Delta\vec{x})^2}{4\eta^2}\right)+\psi^{(0)}(1-q)+\psi^{(0)}(2+q)\right]\right)+O(\eta^{-2})\right]. \tag{50}$$

Here γ is the Euler gamma symbol and $\psi^{(0)}(z)$ is the poly-Gamma function. Clearly, the late time correlator has a behavior $\mathcal{O}(\eta^{4q})$ for fixed Δx which washes away any quantum correlation at late times.

(v) $q = -2$.—This case is particularly interesting as we see that $m^2 = H^2(1-q)(2+q)| \to 0$ for $q \to -2$, and, hence, a massless scalar field in this particular Friedmann spacetime is conformally equivalent to a massless scalar field in de Sitter spacetime. The stress energy correlator for this case is given as

$$\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x'})\rangle_{\mathrm{P.L.}}|_{\text{late time}} = \lim_{\eta\to\infty}\lim_{\epsilon\to 0} H^{-12}\Bigg[\frac{3H^4}{2\pi^4\eta^8(\Delta\vec{x})^8} + \frac{4}{(\Delta\vec{x})^6\eta^{10}}\left(\frac{5H^4}{16\pi^4}+O(\epsilon)\right) + \frac{1}{\eta^{12}(\Delta\vec{x})^4}\left(\frac{9H^4}{16\pi^4\epsilon}+\frac{9(6H^4+H^4\log(\frac{(\Delta\vec{x})^2}{4\eta^2}))}{16\pi^4}+O(\epsilon)\right) + \frac{1}{4(\Delta\vec{x})^2\eta^{14}}\left(-\frac{27H^4}{8\pi^4\epsilon}-\frac{27(7H^4+2H^4\log(\frac{(\Delta\vec{x})^2}{4\eta^2}))}{16\pi^4}+O(\epsilon)\right) + \frac{1}{16\eta^{16}}\left(\frac{81H^4}{8\pi^4\epsilon^2}+\frac{27H^4(10+3\log(\frac{(\Delta\vec{x})^2}{4\eta^2}))}{4\pi^4\epsilon}+O(\epsilon^0)\right)+O(\eta^{-18})\Bigg]. \quad (51)$$

Since the $\epsilon \to 0$ limit blows up for all large but finite η, the long-range correlators become dominant over the expectation values, and one needs to resort to stochastic gravity necessarily. In fact, it is easy to show that such divergent behavior persists at all times. This is not unexpected, as we have already seen that the Wightman function diverges secularly for the massless case in de Sitter. However, $q = -2$ spacetime is connected to the de Sitter case as

$$G^{q=-2}_{m=0}(x,x') = (H^2\eta\eta')^{-3}G^{dS}_{m=0}(x,x'), \quad (52)$$

and, thus, in this spacetime, the divergent term from the de Sitter develops time dependence and survives under derivative actions in Eq. (9). A similar spacetime-dependent divergence appears for universes with $q < -2$ and $q > 1$ corresponding to $|\nu| > 3/2$. Therefore, the semiclassical (or even classical) analysis on these universes is potentially unstable in the face of quantum fluctuations.

V. ENERGY-ENERGY CORRELATION ON CONSTANT TIME SHEETS

In the previous section, we evaluated the $\langle t_{0000}\rangle$ component of the noise kernel for different Friedmann spacetimes. We realize that, in some cases, the late time character cares only for the coordinate separation (Δx) which is not coordinate invariant. This is not unexpected, as the noise kernel is not an invariant scalar. However, one can construct invariant scalars out of these to assess the effect of stochastic fluctuations more covariantly. For this purpose, we consider the behavior of energy-energy density correlator in the late time universe in these Friedmann universes. Energy density at any point x is given by $T_{\alpha\beta}(x)t^\alpha t^\beta$, where t^α is some timelike vector. So, if we consider a comoving timelike path $x(\lambda) = (N(\lambda), \vec{x} = \vec{c})$, then we see that $t^\alpha = (\dot{N}(\lambda), \vec{0})$ and, hence, the unit parametrization in de Sitter space implies that $\dot{N}(\lambda) = 1/a(\eta)$. This implies that the energy density, at point $(\eta,\vec{x})$, is $T_{00}(\eta,\vec{x})/(a(\eta))^2$ and the energy-energy density correlator between the points $(\eta,\vec{x})$ and $(\eta',\vec{x}')$ is given by

$$\frac{\langle t_{00}(\eta,\vec{x})t_{00}(\eta',\vec{x}')\rangle}{(a(\eta)a(\eta'))^2}. \quad (53)$$

Using the expressions for $\langle t_{00}(\eta,\vec{x})t_{00}(\eta',\vec{x}')\rangle$ obtained in the previous section, we can obtain the energy density correlators, over large physical distances, for different Friedmann universes.

(i) $q = 1$.—This is the de Sitter spacetime. Using Eq. (53), we see that the energy-energy correlator, in the $\eta \to 0$ limit, is 0 for every value of ν except for $\nu = 3/2$, for which it is constant ($= 9H^8/128\pi^4$). One point to note is that, for $\nu = 3/2$, the infrared problem does not appear at the level of noise kernel, as we take the massless field as the limiting case. Since the massless fields have no de Sitter invariant vacuum [23], one needs to regularize the divergent piece in the Wightman function in a proper way. However, it can be shown that the regularized massless Wightman function does not yield any nonzero energy density correlator in the late time limit [63].

(ii) $q \in (0,1)$.—Using Eq. (53) for the energy-energy correlator and the fact that $a(\eta) = (H\eta)^{-q}$, we see that, in this case, the energy-energy correlator goes to 0 in the $\eta \to 0$ limit.

(iii) $q = 0$.—This is the flat space case, i.e., $a(\eta) = 1$, where the $\langle t_{0000}\rangle$ is the same as the energy-energy correlator.

(iv) $q \in (-2,0)$.—In this case, we find that the late time universe, i.e., $\eta \to \infty$, has zero value for the energy-energy correlator.

(v) $q = -2$.—Again using Eq. (53), we see that the energy-energy correlator, for this case, is divergent in the late time limit (in fact, for arbitrary values

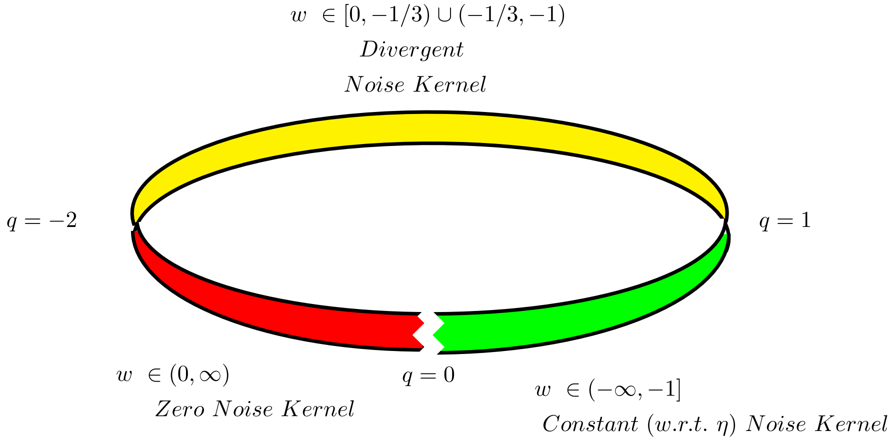

FIG. 1. Relation between different types of fluid (and the corresponding Friedmann spacetimes) and the behavior of the noise kernel in these regions.

of η), as there is a pole in the Wightman function at $\nu = 3/2$. Similarly, the energy-energy correlator will blow up for $q < -2$ and $q > 1$, too.

Thus, we see the coordinate invariant quantity is mostly under control and may give rise to finite value observable quantities. However, some Friedmann universes still give divergent energy-energy correlators which are explained below.

A. Cosmological implications

The equation of state parameter w, for an ideal fluid driving the Friedmann universe, is related to the exponent of the scale factor in Friedmann cosmologies by $q = -2/(1+3w)$ (see [39]). Different values of w represent the dominance of different types of fluid during the evolution of the universe; e.g., $w = 0$ corresponds to dust, whereas $w = 1/3$ corresponds to radiation. Therefore, we see that the Friedmann universe with $q \in (0,1]$ is driven by a fluid with equation of state parameter $w \in (-\infty,-1]$, whereas $w \in [0,\infty)$ for $q \in [-2,0)$. This means that the quantum fluctuations [$(a=0, b=0, c=0, d=0)$ noise kernel component] may remain relevant for $w \in (-\infty,-1]$ in the late time universe where the initial quantum fluctuation remains frozen under expansion, whereas there are no quantum fluctuations left over for $w \in (0,\infty)$ in the late time universe. Interestingly, for $q=-2$, i.e., $w=0$, a pressureless dust-driven universe is the one most affected by stochastic fluctuations, as the noise kernel components (or even scalar correlators) blow up for this spacetime. Thus, any semiclassical or possibly even classical analysis in a dust-driven universe is subject to scrutiny under stochastic correction. In other words, this spacetime remains as quantum as ever. The same remains true for $q < -2$ or $q > 1$ corresponding to $w \in [-1,0]$. Thus, the spacetimes in these regime never drop their quantum character, and a higher-order quantum analysis is necessary. Interestingly, accelerating universes require $w < -1/3$, and, hence, any accelerating universe also seeks for a quantum treatment. The summary of this section is given by Fig. 1.

VI. CONCLUSIONS

In this paper, we analyze the stability of various Friedmann universes under the possible effect of a stochastic correction term in the Einstein-Langevin equation. Using the relations between Wightman functions in de Sitter and Friedmann universes, as well as the relation of the Wightman function to noise kernel components, we relate the noise kernel in a Friedmann universe to the conformal scaling of the noise kernels of massive fields in a de Sitter universe. Typically, quantum fluctuations are expected to decay over large length scales in flat spacetime and remain relevant only over extremely small scales. A Friedmann universe is conformally flat; i.e., the points which are initially very close by will get physically separated under a global topological expansion. However, in this lies an interesting possibility, where two spacetime points are physically apart by large distances while maintaining small coordinate separation. Under certain scenarios, e.g., for conformal fields, it may be possible that the noise kernel cares about the coordinate separation and not about the true physical distances. In those cases, the quantum fluctuations which were stronger when the points had not accelerated away from each other remain as strong under time (and, hence, physical distance) growth. Furthermore, there can be cases where the signature of small coordinate distance gets enhanced with an increasing conformal scale factor. We argue that certain Friedmann universes develop this tendency in the late time era and, thus, maintain a potentially significant second-order correction to the semiclassical equations. We first list our findings in this paper as follows.

(i) *Minimally coupled massive scalar field in de Sitter spacetime.*—We first consider a minimally coupled massive scalar field on dS_4 and study the variation of the $(a=0, b=0, c=0, d=0)$ component of the noise kernel as a function of the mass. We consider only those cases in which $|\nu|$ lies in the range $[0, 3/2]$, because this range of values for ν ensures that the mass is real as $m^2/H^2 = 9/4 - \nu^2$. If we consider finitely separated points on constant time sheets and take the late time, $\eta \to 0$, limit [for which the scale factor $1/(H\eta)$ grows], we find that the considered component of the noise kernel undergoes a sort of phase transition from zero to nonzero values, with the critical value being $\nu = 1/2$. The noise kernel components stay vanishing for $\nu \in [0, 1/2)$ and assume a finite nonzero value at $\nu = 1/2$. Furthermore, for $\nu > 1/2$, it diverges in the limit $\eta \to 0$. We also show that the energy-energy correlator [which is just the $(a = 0, b = 0, c = 0, d = 0)$ component of the noise kernel suppressed by the fourth power of the scale factor] vanishes for all values of ν except for $\nu = 3/2$ for which it is finite (at the value $9H^8/128\pi^4$).

(ii) *Nonminimally coupled scalar field in de Sitter spacetime.*—We also evaluate the stochastic corrections for the nonminimally coupled massive scalar field on dS_4, which amounts to adding $\xi R\phi^2$ to the minimally coupled Lagrangian. In this case as well, we consider finitely separated points on constant time sheets in the $\eta \to 0$ limit. We show that the variation of the $(a = 0, b = 0, c = 0, d = 0)$ component of the noise kernel as a function of ν is similar to the minimally coupled case with $\nu \to \nu = \sqrt{9/4 - (m^2 + 12\xi H^2)/H^2}$. In effect, what we have shown is that the considered component is 0 for $(m^2/H^2 + 12\xi) > 2$ and becomes nonzero for $(m^2/H^2 + 12\xi) = 2$ and diverges for $(m^2/H^2 + 12\xi) < 2$. Therefore, conformal coupling $\xi = 1/6$ cures the divergences for all masses. Similarly, the energy-energy correlator stays 0 for all values of $m^2/H^2 + 12\xi$ in the range $[0, \frac{9}{4})$ but becomes constant for $m^2/H^2 + 12\xi = 0$.

(iii) *Conformally related massless scalar field in Friedmann spacetimes.*—Using the results of Ref. [39], that establish an equivalence between a massless scalar field in Friedmann spacetimes with that of a massive scalar field in de Sitter space where the mass gets related to the exponent of the scale factor of Friedmann, we compute the noise kernels for various Friedmann universes. For ν lying between $[-3/2, 3/2]$, q can take values between $[-2, 1]$. Since we are interested in large physical separation (late time universes), the late time limit corresponds to $\eta \to 0$ for $q \in (0, 1]$, whereas it corresponds to $\eta \to \infty$ for $q \in [-2, 0)$. We find that, for $q \in [0, 1)$, the considered noise kernel component approaches a constant value, while, for $q \in (-2, 0)$, it vanishes over large physical separations. Similarly, for the energy-energy correlator, we showed that it is zero for $q \in (-2, 0) \cup (0, 1)$. However, particularly interesting cases are for $q \leq -2$ and $q \geq 1$ (which correspond to universes driven by $-1 < w < 0$ equation of state fluids) where, as shown in Ref. [39], the Wightman function has a divergent term with spacetime dependence. The conformal connection between the fields of de Sitter and Friedmann spaces provides an additional conformal time dependence to the divergent term in the de Sitter Wightman function, which contributes dominantly in both the noise kernel and the energy-energy correlators. Therefore, the universes, which are driven by such fluids, remain susceptible to quantum fluctuations at late time as well.

Several observations and their implications are in order. First, we see that the spacetimes which are potentially stable against the stochastic corrections are those driven by $w > 0$ or phantom universes $w < -1$ if expectation values are large. This reinforces the quantum fluctuation structure suggested in Ref. [39]. However, there are a couple of more interesting points to be learned from this exercise. First, in the de Sitter case, the late time divergence for fields with $\nu > 1/2$ is dynamic in nature (unlike the Wightman function which is spacetime independent, as well as, only for massless case). Second, in the phantom spacetimes, the noise kernel does not grow out of control but still maintains a nonvanishing noise over large length scales if the points were born close by. Furthermore, for $q = -2$, we see that there is a non-dynamic divergence in all correlators, which makes a dust-driven universe potentially unstable under quantum noise. From power counting arguments, it is easy to visualize that, for spacetimes which are driven by $w \in (-1, 0]$, the quantum fluctuations will always remain important, in a similar spirit. This has many physical implications on semiclassical physics in such spacetimes, e.g., quasi–de Sitter inflation or late time quintessence field-driven universes. Therefore, the dynamics or the growth of perturbations of the massless kind in these spacetimes needs to be evaluated carefully, accounting for such corrections. The effects of stochastic corrections on the background as well as perturbation dynamics in these spacetimes are currently being studied and will be reported elsewhere. Besides this, cosmological data suggest that w is a dynamical quantity; i.e., it changes with time. In this paper, we have studied these stochastic corrections only for constant w-driven universes, and, as such, our analysis requires some changes to take this fact into account which we plan to study in future. However, if the time variation of ω is sufficiently small compared to the rate of expansion of the universe, i.e., $\dot{\omega}/\omega \ll \dot{a}/a$, then our study becomes relevant in the dust-driven as well as dark-energy-dominated or quintessence era, etc.

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APPENDIX A: BASIC COMPUTATION ON CONSTANT TIME SURFACE

In this Appendix, we collect certain results which are important in some of the calculations presented in the main text of this paper. For de Sitter invariant vacuum, $G(x,x') = G(Z(x,x'))$, we see, using Eq. (15), that

$$\nabla'_\mu G = G'\left[\frac{(x-x')_\mu}{\eta\eta'} + \frac{\Delta s^2}{2\eta\eta'^2}\delta_{\mu 0}\right], \tag{A1}$$

$$\nabla_\nu G = G'\left[-\frac{(x-x')_\nu}{\eta\eta'} + \frac{\Delta s^2}{2\eta^2\eta'}\delta_{\nu 0}\right], \tag{A2}$$

$$\begin{aligned}\nabla_\nu\nabla'_\mu G = G''&\left[\frac{(x-x')_\mu}{\eta\eta'} + \frac{\Delta s^2}{2\eta\eta'^2}\delta_{\mu 0}\right]\\ &\times\left[-\frac{(x-x')_\nu}{\eta\eta'} + \frac{\Delta s^2}{2\eta^2\eta'}\delta_{\nu 0}\right]\\ &+G'\left[\frac{\eta_{\mu\nu}}{\eta\eta'} - \frac{(x-x')_\mu}{\eta^2\eta'}\delta_{\nu 0}\right.\\ &\left.+\frac{(x-x')_\nu}{\eta\eta'^2}\delta_{\mu 0} - \frac{\Delta s^2}{2\eta^2\eta'^2}\delta_{\nu 0}\delta_{\mu 0}\right].\end{aligned} \tag{A3}$$

On constant time sheets, i.e., $\eta = \eta'$, we have

$$Z(x,x') = 1 - \frac{(\Delta\vec{x})^2}{2\eta^2}, \tag{A4}$$

$$\nabla'_i G = G'\left[\frac{(x-x')_i}{\eta^2}\right], \tag{A5}$$

$$\nabla_i G = G'\left[-\frac{(x-x')_i}{\eta^2}\right], \tag{A6}$$

$$\nabla'_0 G = G'\left[\frac{(\vec{x}-\vec{x}')^2}{2\eta^3}\right], \tag{A7}$$

$$\nabla_0 G = G'\left[\frac{(\vec{x}-\vec{x}')^2}{2\eta^3}\right], \tag{A8}$$

$$\nabla_i\nabla'_j G = G''\left[-\frac{(x-x')_i(x-x')_j}{\eta^4}\right] + G'\left[\frac{\delta_{ij}}{\eta^2}\right], \tag{A9}$$

$$\nabla_0\nabla'_j G = G''\left[\frac{(x-x')_j(\vec{x}-\vec{x}')^2}{2\eta^5}\right] - G'\left[\frac{(x-x')_j}{\eta^3}\right], \tag{A10}$$

$$\nabla_i\nabla'_0 G = G''\left[-\frac{(x-x')_i(\vec{x}-\vec{x}')^2}{2\eta^5}\right] + G'\left[\frac{(x-x')_i}{\eta^3}\right], \tag{A11}$$

$$\nabla_0\nabla'_0 G = G''\left[\frac{(\vec{x}-\vec{x}')^4}{4\eta^6}\right] + G'\left[-\frac{1}{\eta^2} - \frac{(\vec{x}-\vec{x}')^2}{2\eta^4}\right]. \tag{A12}$$

APPENDIX B: POWER COUNTING FOR NOISE KERNEL

1. Minimal coupling

In this Appendix, we present a power counting argument to find out for what values of ν the noise kernel for de Sitter spacetime, i.e., Eq. (22), diverges as $\eta \to 0$ (late time universe). If we look at the first term in Eq. (22), i.e.,

$$(G'')^2\left[\frac{(\Delta\vec{x})^6}{4\eta^{10}} + \frac{(\Delta\vec{x})^8}{32\eta^{12}} + \frac{(\Delta\vec{x})^4}{2\eta^8}\right], \tag{B1}$$

we see that the most divergent term in the square brackets is $O(\eta^{-12})$. So, if we can find the values of ν for which the least power of η in $(G'')^2$ is <12, then we have found the range of ν for which this term diverges. Since the Wightman function and its derivatives are functions of $\frac{1+Z}{2}\left(= 1 - \frac{(\Delta\vec{x})^2}{4\eta^2}\right)$,[7] we must look at the series expansion of the Wightman function and its derivative at large values of their arguments in the $\eta \to 0$ limit. If we look at the following series expansion of ${}_2F_1(a,b,c,z)$ [56] (valid for large $|z|$ and $a-b \notin \mathbb{Z}$)[8]:

$$\begin{aligned}{}_2F_1(a,b,c,z) = &\frac{\Gamma(b-a)\Gamma(c)(-z)^{-a}}{\Gamma(b)\Gamma(c-a)}\sum_{k=0}^{\infty}\frac{a_k(a-c+1)_k z^{-k}}{k!(a-b+1)_k}\\ &+\frac{\Gamma(a-b)\Gamma(c)(-z)^{-b}}{\Gamma(a)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{b_k(b-c+1)_k z^{-k}}{k!(-a+b+1)_k},\end{aligned} \tag{B2}$$

and keep in mind Eq. (25), we find that the least power of η in $(G'')^2$ is $14-4\nu$. Therefore, the above term diverges for $\nu > \frac{1}{2}$. A similar analysis with the other terms in Eq. (22) tells us that Eq. (22) diverges for $\nu > \frac{1}{2}$.

These arguments can be applied to the general components of the noise kernel. In fact, looking at the least powers of η in the formulas listed in Appendix A for different

[7] See Eqs. (23)–(25).

[8] In our case, $a-b = 2\nu$, which is not an integer for every value of ν in the range $[0,\frac{3}{2}]$ except for $\nu = 0, \frac{1}{2}, 1, \frac{3}{2}$. But we have already considered these cases separately in Sec. III.

covariant derivatives of the Wightman function on constant time sheets and Eq. (21), we see that

$\langle t_{ab}(\eta,\vec{x})t_{cd}(\eta,\vec{x'})\rangle_{dS}$	Leading-order behavior in η
$a=0, b=0, c=0, d=0$	$O(\eta^{2-4\nu})$
$a=0, b=0, c=0, d=l$	$O(\eta^{3-4\nu})$
$a=0, b=j, c=0, d=0$	$O(\eta^{3-4\nu})$
$a=0, b=j, c=0, d=l$	$O(\eta^{4-4\nu})$
$a=i, b=j, c=0, d=0$	$O(\eta^{2-4\nu})$
$a=0, b=0, c=k, d=l$	$O(\eta^{2-4\nu})$
$a=i, b=j, c=k, d=0$	$O(\eta^{3-4\nu})$
$a=0, b=j, c=k, d=l$	$O(\eta^{3-4\nu})$
$a=i, b=i, c=k, d=l$ and $k\neq l$	$O(\eta^{4-4\nu})$
$a=i, b=j, c=k, d=k$ and $i\neq j$	$O(\eta^{4-4\nu})$
$a=i, b=i, c=k, d=k$	$O(\eta^{2-4\nu})$
$a=i, b=j, c=k, d=l$ and $i\neq j, k\neq l$	$O(\eta^{6-4\nu})$

2. Nonminimal coupling

Below is given the expression of the noise kernel for the nonminimally coupled massive scalar field on de Sitter spacetime in terms of the Wightman function and its covariant derivatives. First, we substituted the expression (36) for the stress energy tensor in the definition of the noise kernel (1). Since the definition of the noise kernel contains the vacuum expectation of the product of two stress energy tensors and each stress energy operator contains two field operators, we would get the vacuum expectation of the product of four field operators. Then, we can use the Wick theorem to express this vacuum expectation as the product of two Wightman functions and obtain Eq. (39). We obtain the below given expression by substituting the expressions (37) and (38) for $P_{ab}(x,y)$ and $M_{ab}(x,y)$ in Eq. (39):

$$
\begin{aligned}
\langle t^{nm}_{ab}(x)t^{nm}_{cd}(x')\rangle = \Bigg[&(1-2\xi)^2(\nabla_b\nabla'_c G(x,x')\nabla_a\nabla'_d G(x,x')+\nabla_b\nabla'_d G(x,x')\nabla_a\nabla'_c G(x,x'))\\
&-(1-4\xi)(1-2\xi)\eta_{cd}\eta^{\rho\sigma}\nabla_a\nabla'_\rho G(x,x')\nabla_b\nabla'_\sigma G(x,x')-\frac{(m^2+6H^2\xi)(1-2\xi)}{H^2\eta'^2}\eta_{cd}\nabla_a G(x,x')\nabla_b G(x,x')\\
&-(1-4\xi)(1-2\xi)\eta_{ab}\eta^{\gamma\delta}\nabla_\gamma\nabla_c{}'G(x,x')\nabla_\delta\nabla'_d G(x,x')+\frac{(1-4\xi)^2}{2}\eta_{ab}\eta^{\gamma\delta}\eta_{cd}\eta^{\rho\sigma}\nabla_\gamma\nabla'_\rho G(x,x')\nabla_\delta\nabla_\sigma{}'G(x,x')\\
&+\frac{(m^2+6H^2\xi)(1-4\xi)}{2H^2\eta'^2}\eta_{ab}\eta^{\gamma\delta}\eta_{cd}\nabla_\gamma G(x,x')\nabla_\delta G(x,x')-\frac{(m^2+6H^2\xi)(1-2\xi)}{H^2\eta^2}\eta_{ab}\nabla'_c G(x,x')\nabla'_d G(x,x')\\
&+\frac{(m^2+6H^2\xi)(1-4\xi)}{2H^2\eta^2}\eta_{ab}\eta_{cd}\eta^{\rho\sigma}\nabla'_\rho G(x,x')\nabla'_\sigma G(x,x')+\frac{1}{2H^4\eta^2\eta'^2}(6H^2\xi+m^2)^2\eta_{ab}\eta_{cd}G^2\Bigg]\\
&+2\xi\Bigg[2\eta_{cd}(1-2\xi)(\nabla_{(a}G\nabla_{b)}\Box'G)-2(1-2\xi)(\nabla_{(a}G\nabla_{b)}\nabla'_{(c}\nabla'_{d)}G)-\frac{(6H^2\xi+m^2)}{(H\eta)^2}\eta_{ab}\eta_{cd}G\Box'G\\
&-(1-4\xi)\eta_{ab}\eta_{cd}(\eta^{rs}\nabla_s G\nabla_r\Box'G)+(1-4\xi)\eta_{ab}(\eta^{rs}\nabla_s G\nabla_r\nabla'_{(c}\nabla'_{d)}G)+\frac{(6H^2\xi+m^2)}{(H\eta)^2}\eta_{ab}G\nabla'_{(c}\nabla'_{d)}G\Bigg]\\
&+2\xi\Bigg[2\eta_{ab}(1-2\xi)(\Box\nabla'_{(c}G\nabla'_{d)}G)-2(1-2\xi)(\nabla_{(a}\nabla_{b)}\nabla'_{(c}G\nabla'_{d)}G)-\frac{(6H^2\xi+m^2)}{(H\eta')^2}\eta_{ab}\eta_{cd}G\Box G\\
&-(1-4\xi)\eta_{ab}\eta_{cd}(\eta^{rs}\Box\nabla'_s G\nabla'_r G)+(1-4\xi)\eta_{cd}(\eta^{mn}\nabla_{(a}\nabla_{b)}\nabla'_n G\nabla'_m G)+\frac{(6H^2\xi+m^2)}{(H\eta')^2}\eta_{cd}G\nabla_{(a}\nabla_{b)}G\Bigg]\\
&+4\xi^2[\eta_{ab}\eta_{cd}(\Box'G\Box G+G\Box\Box'G)-\eta_{ab}(\nabla'_{(c}\nabla'_{d)}G\Box G+G\Box\nabla'_{(c}\nabla'_{d)}G)\\
&-\eta_{cd}(\nabla_{(a}\nabla_{b)}G\Box'G+G\nabla_{(a}\nabla_{b)}\Box'G)+(\nabla_{(a}\nabla_{b)}G\nabla'_{(c}\nabla'_{d)}G+G\nabla_{(a}\nabla_{b)}\nabla'_{(c}\nabla'_{d)}G)].
\end{aligned}
\tag{B3}
$$

The first square bracket contains the $P_{ab}P_{cd}$ term, and the second and the third square brackets contain the $P_{ab}M_{cd}$ and $M_{ab}P_{cd}$ terms, respectively, whereas the fourth square bracket contains the $M_{ab}M_{cd}$ term. We can use the same power counting analysis as is done for the minimal coupling section of this Appendix and study the behavior of divergences for the noise kernel as a function of the mass and the coupling constant ξ.

APPENDIX C: DIVERGENCE IN NOISE KERNEL FOR $\omega \in (-1,0)$-DRIVEN UNIVERSE

Looking at Eq. (85) of Ref. [39], we see that the Wightman function for a massless scalar field in Friedmann spacetimes is given by

$$G(x,x') = \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\int_0^\infty ds \frac{s^{(1/2)-\nu}}{(s^2-2Zs+1)^{3/2}}. \tag{C1}$$

1. Case $\nu < -\frac{3}{2}$

Now, consider the integral for large s values, i.e.,

$$\begin{aligned} G(x,x') &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\left[\text{finite term} + \int_N^\infty ds s^{-(5/2)-\nu}\left(1-2\frac{Z}{s}+\frac{1}{s^2}\right)^{-3/2}\right] \\ &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\left[\text{finite term} + \int_N^\infty ds s^{-(5/2)-\nu}\left(1-\frac{3}{2}\left(-2\frac{Z}{s}+\frac{1}{s^2}\right)+\frac{3*5}{2*2*2}\left(-2\frac{Z}{s}+\frac{1}{s^2}\right)^2+\cdots\right)\right] \\ &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\left[\text{finite term} + \left(\frac{s^{-(3/2)-\nu}}{-\frac{3}{2}-\nu}+3Z\frac{s^{-(5/2)-\nu}}{-\frac{5}{2}-\nu}-\frac{3}{2}\frac{s^{-(7/2)-\nu}}{-\frac{7}{2}-\nu}+(\text{lower powers of} s)\right)\Bigg|_N^\infty\right]. \end{aligned} \tag{C2}$$

Since $Z = 1 + \frac{(\eta-\eta')^2-(\Delta\vec{x})^2}{2\eta\eta'}$, we see that the highest collective power of η and η' is $-3-2v$, and one such highest power term is multiplying an η and η' independent and always diverging term $s^{-(3/2)-\nu}|_N^\infty$ in the expression for the Wightman function. This implies that the behavior of the noise kernel, in this case, is the same as for the $\nu = -\frac{3}{2}$ case [because the divergences are determined by the highest power (for $\eta \to \infty$) and, hence, most divergent term]. In fact, we have

$$\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x'})\rangle_{\text{P.L.}} = \frac{H^{4q}(q-1)^4}{128\pi^4\eta^{8-4q}\epsilon^2}+O(\epsilon^{-1}), \tag{C3}$$

where $q = \nu - \frac{1}{2}$ and $\frac{1}{\epsilon} = \frac{s^{-(3/2)-\nu}|^\infty}{\frac{3}{2}+\nu}$.

2. Case $\nu > \frac{3}{2}$

Now, consider the integral for small s values, i.e.,

$$\begin{aligned} G(x,x') &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\left[\text{finite term} + \int_0^\epsilon ds s^{(1/2)-\nu}(1-2Zs+s^2)^{-3/2}\right] \\ &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\left[\text{finite term} + \int_0^\epsilon ds s^{(1/2)-\nu}\left(1-\frac{3}{2}(-2Zs+s^2)+\frac{3*5}{2*2*2}(-2Zs+s^2)^2+\cdots\right)\right] \\ &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\left[\text{finite term} + \left(\frac{s^{(3/2)-\nu}}{\frac{3}{2}-\nu}+3Z\frac{s^{(5/2)-\nu}}{\frac{5}{2}-\nu}-\frac{3}{2}\frac{s^{(7/2)-\nu}}{\frac{7}{2}-\nu}+(\text{higher powers of} s)\right)\Bigg|_0^\epsilon\right]. \end{aligned} \tag{C4}$$

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