Revisiting Y junctions for strings with currents: Transonic elastic case

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We studied the formation of Y junctions for transonic elastic strings. In particular, using the general solution for these strings, which is described by left- and right-moving modes, we obtained the dynamics of kinks and Y junctions. Considering the linearized ansatz for straight strings, we constructed the parameter region space for which the formation of Y junctions due to strings collisions is allowed.

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I. INTRODUCTION

Cosmic strings are hypothetical objects that were originally described by Kibble [1]. They appear as a prediction of numerous models of early universe [2]. To highlight some of them, it is worthwhile to mention brane inflation [3–8], supersymmetric grand unified theory [9–13], and theories of high energy particle physics [14–18].

Some types of cosmic strings allow the existence of bound states, named as Y junctions. They might appear due to collisions of distinct strings that form trilinear vertices. Y junctions are common for non-Abelian strings [19], for Abelian-Higgs strings of the I type [20], for $U(1) \times U(1)$ models with specific value of parameters [21], and for cosmic strings from brane inflation (cosmic superstrings) [5]. Using approximation that cosmic strings are infinitely thin, are described by Nambu-Goto action, it was demonstrated that kinematic constraints must be satisfied in order for the Y junctions to be produced [22–24]. The result of kinematic constraints was confirmed by numerical simulations in a framework of field theory [25,26]. The analytic description of cosmic strings via Nambu-Goto action also sheds light on dynamics of Y junctions. In particular, one can estimate the average growth/reduction of string lengths for multi-tension cosmic string networks. This phenomenon is crucial for understanding the evolution of cosmic (super)string networks [27,28].

Due to nontrivial interactions of fields that form a string core, cosmic strings might become superconducting [29]. This situation naturally arises for supersymmetric [30–34] and some non-Abelian strings [35,36]. To obtain an effective description of superconducting cosmic strings, models for infinitely thin strings were developed [29,37–41]. It was also suggested that some macroscopic properties can be captured by such current carrying Nambu-Goto strings. In particular, the barytropic cosmic string model, which also comes out from dimensional reduction [42,43], provides an accurate depiction of "wiggly" (noisy) cosmic strings [44–47].

This study revisits the problem of Y junctions for transonic elastic strings. We reexamine the exact solution for these strings [44,47], obtain left-/right-moving modes, and in line with [22] we describe evolution of kinks and Y junctions. In addition, we obtain kinematic conditions under which the production of Y junction is possible.

The problem of Y junctions for Nambu-Goto current carrying strings was initially studied in [48]. The authors developed a covariant formalism to investigate under which conditions the production of Y junctions is possible. The result of paper [48] claims that for *magnetic* (spacelike current) and *electric* (timelike current) current carrying strings the formation of Y-junction is impossible, unless the newly formed string is described by a more general equation of state. A detailed comparison of our result with work [48] can be found in the Appendix.

II. SOLUTION IN MINKOWSKI SPACE FOR TRANSONIC ELASTIC STRINGS

In this section, we revisit the exact solution for transonic elastic strings, originally obtained in [44,47], with the method developed in [49]. This approach allows us to show that only elastic and chiral strings lead to wavelike equations of motion.

We start consideration from the action

$$S = -\mu_0 \int f(\kappa)\sqrt{-\gamma}\, d\sigma d\tau, \tag{1}$$

where μ_0 is a constant defined by the symmetry breaking scale, $\{\sigma, \tau\}$ are coordinates on the string world sheet (Latin indexes "a-d" run over 0, 1) with induced metric

$$\gamma_{ab} \equiv x^{\mu}_{,a} x^{\nu}_{,b} \eta_{\nu\mu} \quad \text{and} \tag{2a}$$

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$$\kappa \equiv \varphi_{,a}\varphi_{,b}\gamma^{ab}, \tag{2b}$$

$$\gamma \equiv \frac{1}{2}\varepsilon^{ac}\varepsilon^{bd}\gamma_{ab}\gamma_{cd}. \tag{2c}$$

ε^{ac} is the Levi-Civita symbol, $\eta^{\mu\nu}$ is Minkowski metric (Greek indexes run over space-time coordinates x^μ from 0 to 3), $x^\mu_{,a} \equiv \frac{\partial x^\mu}{\partial \sigma^a}$, and φ is a scalar function on the string world sheet. The function $f(\kappa)$ will be defined below.

The stress-energy tensor for the action (1) can be written as

$$T^{\mu\nu} \equiv -2\frac{\delta S}{\delta g_{\mu\nu}} = \frac{\mu_0}{\sqrt{-g}}\int \delta^{(4)}(y - x(\sigma)) \times \sqrt{-\gamma}(Uu^\mu u^\nu - Tv^\mu v^\nu)d\sigma d\tau, \tag{3}$$

where $u^\mu u_\mu = 1$ and $v^\mu v_\mu = -1$ are orthonormal timelike and spacelike vectors. Mass per unity length U and tension T in (3) are given by expressions

$$U = f - 2\kappa f'_\kappa \Theta[-\kappa f'_\kappa], \quad T = f - 2\kappa f'_\kappa \Theta[\kappa f'_\kappa], \tag{4}$$

where $\Theta[\ldots]$ is a Heaviside function and $f'_\kappa = \frac{\partial f}{\partial \kappa}$ (for more details about the stress-energy tensor, see Sec. IV in [50] or alternatively Sec. II in [51]).

We can introduce the speed of "wiggles" c_E (propagation of transverse perturbations) and "woggles" c_L (propagation of longitudinal perturbations) according to [39,47]

$$c_E^2 = \frac{T}{U}, \qquad c_L^2 = -\frac{dT}{dU}. \tag{5}$$

In this way, for the standard Nambu-Goto string, both propagations have the speed of light, $c_E = c_L = 1$. It is anticipated to have supersonic strings ($c_E > c_L$) for most of regimes of superconducting strings [38,52]. Meanwhile, the transonic model

$$c_L = c_E \le 1 \tag{6}$$

can be considered as an effective description of wiggly strings [46,47] and some particular limits of superconducting strings (see Secs. 5.8 and 5.9 in [50]).

Using (4), the explicit form of (5) can be written as

$$c_E^2 = \frac{f - 2\kappa f'_\kappa \Theta[\kappa f'_\kappa]}{f - 2\kappa f'_\kappa \Theta[-\kappa f'_\kappa]}, \quad c_L^2 = -\frac{f'_\kappa - 2(f'_\kappa + \kappa f''_{\kappa\kappa})\Theta[\kappa f'_\kappa]}{f'_\kappa - 2(f'_\kappa + \kappa f''_{\kappa\kappa})\Theta[-\kappa f'_\kappa]}. \tag{7}$$

Substituting (7) into condition (6) for transonic strings, one can obtain the equation for $f(\kappa)$,

$$(f'_\kappa)^2 + f f''_{\kappa\kappa} = 0 \Rightarrow f = \sqrt{c_1\kappa + c_2}, \tag{8}$$

where c_1 and c_2 are constants of integration.

One can write down the equation of state for transonic strings using the expressions (4) together with (8),

$$UT = f(f - 2\kappa f'_\kappa) = c_2 = m^2, \tag{9}$$

where m is a mass dimensional constant.

We can define $c_1 = \pm m^2$ and absorb m^2 into the definition of μ_0. These manipulations allow us to establish the function $f(\kappa)$ for transonic elastic strings in the following form, as also presented in [44,47]:

$$f(\kappa) = \sqrt{1 - \kappa}, \quad \kappa \in (-\infty, 1], \quad UT = 1. \tag{10}$$

It is known that the transonic model has the general wavelike solution [44]. Let us use the method from [49] to demonstrate that there are only two types of strings, whose equations of motion can be reduced to the wave equation: *chiral* (see [40,53]) and *transonic elastic* strings. We start consideration by writing down the equations of motion for the action (1) in Minkowski space [54],

$$\partial_a[\mathcal{T}^{ab}x^\mu_{,b}] = 0, \tag{11a}$$

$$\partial_a[\sqrt{-\gamma}\gamma^{ab}f'_\kappa\varphi_{,b}] = 0, \tag{11b}$$

where

$$\mathcal{T}^{ab} = \sqrt{-\gamma}(\gamma^{ab}f - 2f'_\kappa\gamma^{ac}\gamma^{bd}\varphi_{,c}\varphi_{,d}) = \sqrt{-\gamma}(\gamma^{ab}f + \theta^{ab}) \tag{12}$$

(notice the change of the sign in (12) due to misprint in Eq. (6) of [54]).

Parametrization invariance of the string world sheet allows us to make the transformation,

$$\mathcal{T}^{ab} \rightarrowtail \eta^{ab}, \tag{13}$$

if their determinants are equal [49],

$$\det\mathcal{T}^{ab} = \det\eta^{ab} = -1. \tag{14}$$

Let us expand the determinant of $\mathcal{T}^{ab}$,

$$\det\mathcal{T}^{ab} = -f^2 - f\mathrm{Tr}\theta^a_c - \det\theta^a_c. \tag{15}$$

It is easy to check that $\det\theta^a_c = 0$; hence, we are left only with

$$\det \mathcal{T}^{ab} = -f^2 + 2ff'_\kappa \mathrm{Tr}[\gamma^{ac}\gamma^{bd}\varphi_{,c}\varphi_{,d}]$$
$$= -f(f - 2f'_\kappa \kappa) = -UT. \tag{16}$$

It is seen from (16) that the transformation (13) is possible due to parametrization invariance only if the function $f(\kappa)$ is defined as for transonic elastic strings (10), or $f(\kappa)$ is defined as for chiral strings, where the current is a null vector $\kappa \to 0$ [54].

The relation (16) together with (10) guarantees that the equations of motion for the string world sheet (11a) has the general wavelike solution. Choosing the gauge where the world sheet coordinate τ coincides with physical time t, one can write down the solution for (11a) in the form of left- and right-moving modes,

$$x^0 = \tau, \qquad \mathbf{x} = \frac{1}{2}(\mathbf{a}(\sigma_+) + \mathbf{b}(\sigma_-)), \tag{17}$$

where $\sigma_+ = \tau + \sigma$ and $\sigma_- = \tau - \sigma$.

Up to this point, we demonstrated how to obtain the result of [44] in a different manner. Let us study the equation of motion for the function φ (11b). To do so, we plug $ff'_\kappa = \frac{1}{2}$ in (11b),

$$\partial_a[\sqrt{-\gamma}\gamma^{ab}f^2 f'_\kappa \varphi_{,b}] = [\text{using (13)}]$$
$$= \partial_a[f(\eta^{ab} + 2f'_\kappa\sqrt{-\gamma}\gamma^{ac}\gamma^{bd}\varphi_{,c}\varphi_d)f'_\kappa\varphi_{,b}]$$
$$= \partial_a[ff'_\kappa(\eta^{ab} + 2\kappa\sqrt{-\gamma}\gamma^{ab}f'_\kappa)\varphi_{,b}]. \tag{18}$$

Transferring the right-hand side term with $\sqrt{-\gamma}\gamma^{ab}$ to the left-hand side in (18), one obtains

$$\partial_a[\sqrt{-\gamma}\gamma^{ab}f'_\kappa f(f - 2\kappa f'_\kappa)\varphi_{,b}]$$
$$= \partial_a[\sqrt{-\gamma}\gamma^{ab}f'_\kappa\varphi_{,b}] = \partial_a[ff'_\kappa\eta^{ab}\varphi_{,b}] = 0. \tag{19}$$

Taking out the constant ff'_κ from the differentiation operation in (19), we derive the following equation:

$$\partial_a[\eta^{ab}\varphi_{,b}] = 0, \tag{20}$$

which general solution is given by

$$\varphi = \frac{1}{2}(F(\sigma_+) + G(\sigma_-)). \tag{21}$$

The normalization of $|\mathbf{a}'|$ and $|\mathbf{b}'|$ is connected with values of the current as

$$\mathbf{a}'^2(\sigma_+) = 1 - F'^2(\sigma_+), \qquad \mathbf{b}'^2(\sigma_-) = 1 - G'^2(\sigma_-) \tag{22}$$

for right- and left-moving modes.

Alternative treatment of elastic strings, as a Kaluza-Klein projection of standard Nambu-Goto strings in a space-time of five-dimensions, can be found in [55]. In this approach, the relation (22) can be seen as a normalization for unity of four-dimensional vectors of left- and right-moving modes.

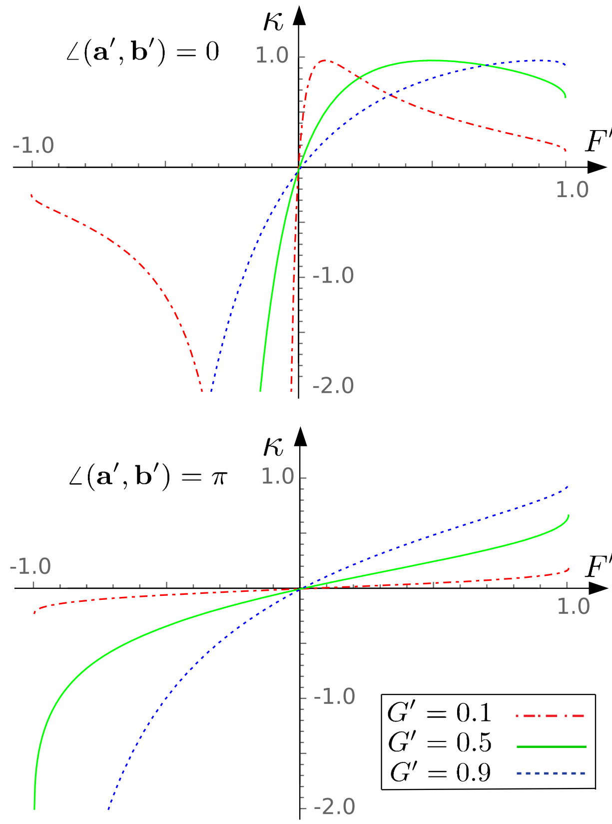

FIG. 1. The current κ defined by (23) for different values of scalar moving modes G', F' and for angles 0, π between vectors $\mathbf{a}'$ and $\mathbf{b}'$.

Using relations (22), one can write down the current (2b) as

$$\kappa = \frac{2F'G'}{1 + F'G' - \mathbf{a}' \cdot \mathbf{b}'}, \tag{23}$$

which is shown in Fig. 1 for different values of F' and G'.

It is seen that if the left(or right)-moving mode of the current is independent of σ_- (or σ_+), the expression (23) goes to zero,

$$\kappa = 0, \quad \text{if}: \; F' = 0, \quad (\text{or} \quad G' = 0). \tag{24}$$

The situation (24) reproduces the chiral string properties, where only left(or right)-moving mode is allowed [40,49,53,54].

III. JUNCTIONS FOR TRANSONIC ELASTIC STRINGS

To study Y junctions for transonic elastic strings, we start with the action for three connected current carrying strings [48],

$$S = -\sum_{i=1}^{3}\mu_i\int f(\kappa_i)\sqrt{-\gamma_i}\Theta(s_i(\tau)-\sigma_i)d\sigma_i d\tau + \sum_{i=1}^{3}\int \mathrm{f}_{\mu i}(x_i^\mu(s_i(\tau),\tau)-X^\mu(\tau))d\tau + \sum_{i=1}^{3}\int \mathrm{g}_i(\varphi_i(s_i(\tau),\tau)-\Phi(\tau))d\tau, \quad (25)$$

where the function $f(\kappa)$ is given by (10), μ_i are constants defined by the symmetry breaking scale, $\mathrm{f}_{\mu i}$, g_i are Lagrange multipliers for strings and currents, time functions $X^\mu(\tau)$ and $\Phi(\tau)$ define values for x_i^μ and φ_i at the point where strings are connected, the index $i = 1, 2, 3$ denotes each of the three strings (the summation over index i is carried out only when it is written explicitly).

Varying the action (25) with respect to x_i^μ and φ_i, we obtain the equations of motion (11a) and (11b) for each type of strings. Using (16) and (20), the boundary terms from equations of motion, which are proportional to $\delta(s_i(t)-\sigma_i)$, can be expressed as

$$\mu_i\eta^{ab}x^\mu_{i,a}\lambda_{bi} = \mathrm{f}_i^\mu, \quad 2\mu_i f_i f'_{\kappa i}\eta^{ab}\varphi_{,a}\lambda_{bi} = \mathrm{g}_i, \quad (26)$$

where $\lambda_{ai} = \{\dot{s}_i, -1\}$.

The variation of the action (25) with respect to $\mathcal{X}_i^\mu$ and Φ gives us

$$\sum_{i=1}^{3}\mathrm{f}_i^\mu = 0, \quad \sum_{i=1}^{3}\mathrm{g}_i = 0, \quad (27)$$

which can be rewritten using solutions (17) and (21) together with expressions (26) in the following way:

$$\sum_{i=1}^{3}\mu_i[\mathbf{a}'_i(1+\dot{s}_i)-\mathbf{b}'_i(1-\dot{s}_i)] = 0, \quad \sum_{i=1}^{3}\mu_i[F'_i(1+\dot{s}_i)-G'_i(1-\dot{s}_i)] = 0. \quad (28)$$

Finally, variation of the action (25) with respect to f_i^μ and g_i provides us the following relations:

$$x_i^\mu(s_i(\tau),\tau) = X^\mu(\tau), \quad \varphi_i(s_i(\tau),\tau) = \Phi(\tau). \quad (29)$$

Differentiating (29) using the exact solutions (17) and (21), we obtain

$$(1+\dot{s}_i)\mathbf{a}'_i + (1-\dot{s}_i)\mathbf{b}'_i = 2\dot{\mathbf{X}}(t), \quad F'_i(1+\dot{s}_i) + G'_i(1-\dot{s}_i) = 2\dot{\Phi}(t). \quad (30)$$

Manipulating vectors $\mathbf{a}'_i$, $\mathbf{b}'_i$ and using (28) with (30), it is possible to obtain the following equations:

$$\mathbf{a}'_k(1+\dot{s}_k) = \frac{2}{\mu}\sum_{i=1}^{3}(1-\dot{s}_i)\mu_i\mathbf{b}'_i - (1-\dot{s}_k)\mathbf{b}'_k, \quad F'_k(1+\dot{s}_k) = \frac{2}{\mu}\sum_{i=1}^{3}(1-\dot{s}_i)\mu_i G'_i - (1-\dot{s}_k)G'_k \quad (31)$$

and

$$\dot{\mathbf{X}} = \frac{1}{\mu}\sum_{i=1}^{3}(1-\dot{s}_i)\mu_i\mathbf{b}'_i. \quad (32)$$

Zero component of the vector equation in (26) provides energy conservation relation, which is identical to the standard Nambu-Goto scenario [22],

$$\sum_{i=1}^{3}\mu_i\dot{s}_i = 0. \quad (33)$$

The relation (33) does not provide an additional constraint, but is a consequence of equations of motion. Hence, the relation (33) can be used as a check of numerical calculations that are carried out below.

We parametrize the string in such way that modes $\mathbf{a}'_i(\sigma_+)$, $F'_i(\sigma_+)$ move outward the string connection, while $\mathbf{b}'_i(\sigma_-)$ and $G'_i(\sigma_-)$ move toward the string connection.

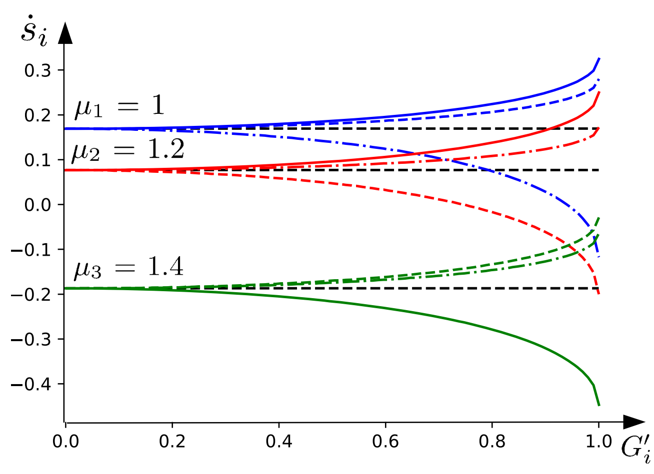

FIG. 2. Dynamics of Y junctions represented by $\dot{s}_i$ of strings with $\mu_1 = 1$ (blue), $\mu_2 = 1.2$ (red), $\mu_3 = 1.4$ (green) and oriented with angles $2\pi/3$ between them. Dashed lines show the values of $\dot{s}_i$ depending on G'_1, dashed-dotted on G'_2, solid on G'_3. Black dashed lines demonstrate no changes of $\dot{s}_i$ when all G'_i increase simultaneously.

Such choice means that $\mathbf{b}_i'(\sigma_-)$ and $G_i'(\sigma_-)$ are initial values that define $\mathbf{a}_i'(\sigma_+)$ and $F_i'(\sigma_+)$ by Eq. (31). The first three equations for vectors $\mathbf{a}_i'(\sigma_+)$ in (31) can be squared and using the normalization conditions (22) we eliminate $\mathbf{a}_i'(\sigma_+)$. Hence, we have the system of six independent algebraic equations (31) and six variables that can be found: three variables $\dot{s}_i$ and three variables $F_i'(\sigma_+)$.

It is illustrative to compare values of $\dot{s}_i$ for strings with currents and without. For this purpose, we fix angles between $\mathbf{b}_i'(\sigma_-)$, define string constants μ_i, and evaluate the system of equations (31) for different values of $G_i'(\sigma_-)$. An example of such dependence is shown in Fig. 2.

The description above demonstrates that dynamics of Y junctions for transonic elastic strings can be described within Nambu-Goto approximation.

A. Kinks for elastic strings

Having considered the Y junctions, we can treat the formation and evolution of kinks for elastic strings. To do so, we need simply change the sum in previous equations for two strings instead of three. Let us consider the situation when parameters for strings are $\mu_1 = \mu_2$. Hence, from Eqs. (30) and (31), one can deduce the following relations for two possible situations that satisfy conditions for the kink:

$$\dot{s}_1 = -1 = -\dot{s}_2, \qquad F_2' = G_1', \tag{34a}$$

$$\text{or} \quad \dot{s}_1 = 1 = -\dot{s}_2, \qquad F_1' = G_2'. \tag{34b}$$

Illustrative example of two strings intercommutation is shown in Fig. 3. After collision, two kinks propagating in opposite direction are formed on each of the strings. The kink $\dot{s}_A$ corresponds to situation (34a) with $F_{sA}' = G_1'$, while another kink $\dot{s}_B$ to (34b) with $F_{sB}' = G_2'$.

The velocities of kinks follow from Eq. (32), that is,

$$\dot{\mathbf{X}}_A^2 = (1 - G_1'^2), \tag{35a}$$

$$\dot{\mathbf{X}}_B^2 = (1 - G_2'^2), \tag{35b}$$

which are different from the speed of light if the correspondent current component is nonzero.

It should be noted that if an elastic string collides with a standard Nambu-Goto (or chiral with $G_1' = 0$) string, the intermediate growing section, between kinks $\dot{s}_A$ and $\dot{s}_B$, is described by the chiral model.

In the same manner, kinks appear when colliding strings form Y junction: from the discontinuity of corresponding modes. The formation of kinks for elastic strings qualitatively is identical to the standard Nambu-Goto model considered in [23], except of the fact that the speed of kinks propagation is not equal to the speed of light, but given by (35a) and (35b).

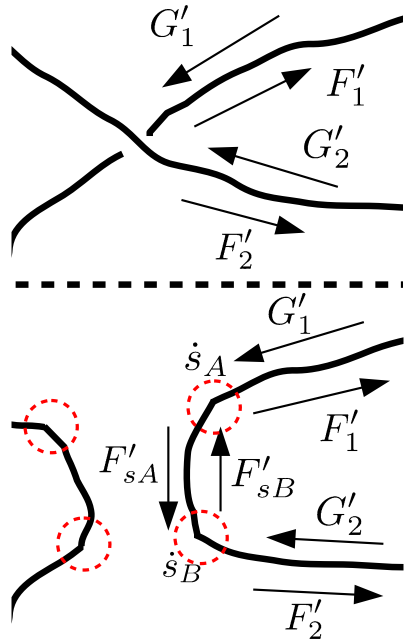

FIG. 3. The upper panel demonstrates two cosmic strings before the collision. Arrows with G' and F' define left- and right-moving modes of currents on cosmic strings. The bottom panel represents the situation when collided strings intercommute, exchange their moving modes.

IV. COLLISIONS OF TRANSONIC ELASTIC STRINGS

It is always possible to choose small region, where collided strings can be considered straight. We are going to study kinematic conditions for straight strings to produce a Y junction.

We decompose the straight string solution as a linear combination of "bare" and current carrying parts [54],

$$\mathbf{x}_i = \mathbf{y}_i + \mathbf{z}_i, \tag{36}$$

where the bare part is given by

$$\begin{aligned} \mathbf{y}_{1,2} &= \{-\gamma_v^{-1}\sigma\cos\alpha; \mp\gamma_v^{-1}\sigma\sin\alpha; \pm v\tau\}, \\ \mathbf{y}_3 &= \{\gamma_u^{-1}\sigma\cos\theta; \gamma_u^{-1}\sigma\sin\theta; u\tau\}, \end{aligned} \tag{37}$$

while the current carrying part is described by

$$\mathbf{z}_i = -g_i\sigma_-(\dot{\mathbf{y}}_i - \mathbf{y}_i') - f_i\sigma_+(\dot{\mathbf{y}}_i + \mathbf{y}_i'), \tag{38}$$

with $\gamma_v^{-1} = \sqrt{1 - v^2}$.

Constants f_i and g_i in (38) represent the current contribution for left- and right-moving modes.

From (36) to (38), one can find that

$$\mathbf{a}_i' = (1-2f_i)(\dot{\mathbf{y}}_i + \mathbf{y}_i'), \qquad |\mathbf{a}_i'|^2 = (1-2f_i)^2,$$
$$\mathbf{b}_i' = (1-2g_i)(\dot{\mathbf{y}}_i - \mathbf{y}_i'), \qquad |\mathbf{b}_i'|^2 = (1-2g_i)^2. \tag{39}$$

Comparing constants f_i and g_i in (39) with (22), we establish the relations

$$f_i = \frac{1}{2}\left(1 - \sqrt{1 - F_i'^2}\right),$$
$$g_i = \frac{1}{2}\left(1 - \sqrt{1 - G_i'^2}\right). \tag{40}$$

Parameters v and α define orthogonal velocity and orientation of noncurrent carrying string [23], but for elastic (and chiral [49]) strings it is not the case. It happens due to the presence of longitudinal component of velocity in considered parametrization. Hence, we will treat v and α as some constant parameters combination of which provide string velocity and orientation. Alteration of parameters interpretation does not affect the validity of applied method.

In order to find out for which values of v and α, the third string can be produced (which means that $\dot{s}_3 > 0$), we need to derive the orientation and velocity of a newly created string, i.e., θ and u parameters. To obtain these variables, we follow the procedure of [23], i.e., we write down the expression for $\dot{\mathbf{X}}$, given in (32), by substituting $\sigma \to s_3(\tau)$ in (37) and (38),

$$\dot{\mathbf{X}} = \{T_1(\tau)\gamma_u^{-1}\cos\theta; T_1(\tau)\gamma_u^{-1}\sin\theta; T_2(\tau)u\}, \tag{41}$$

where $T_1(\tau) = \dot{s}_3(\tau) + g_3(1-\dot{s}_3(\tau)) - f_3(1+\dot{s}_3(\tau))$ and $T_2(\tau) = 1 - f_3(1+\dot{s}_3(\tau)) + g_3(1-\dot{s}_3(\tau))$.

Combining (41) with (32), one can obtain the vector equation, from which θ and u are determined via $\mathbf{b}_i'$.

To summarize, we have nine equations: six equations from (31) and three equations from (32). Therefore, we can derive eight variables F_i', $\dot{s}_i$, u, θ defining another eight variables μ_i, G_i', v, α. The vector equality (32) with (41) does not provide three independent equations, but only two, similarly as in [23]. Having all this information, we can numerically solve this system of algebraic equations. As a result, one can obtain the region of values v and α for which colliding strings give rise to Y junctions ($\dot{s}_3 > 0$); see Fig. 4 for symmetric string collision and Fig. 5 for asymmetric string collision.

Production of Y junction also leads to creation of kinks,

$$\mathbf{K}_{1,2} = (1-2g_{1,2})\{\gamma_v^{-1}\cos\alpha; \pm\gamma_v^{-1}\sin\alpha; \pm v\}\tau, \tag{42}$$

that propagate along collided strings, similarly as it happens for standard noncurrent Nambu-Goto strings [23].

It is important to note that for all strings the constants μ_i were fixed, and we assumed that there is a relation between tensions of strings T_i. This assumption allows us to eliminate 1 degree of freedom and treat G_3' as known value [it might be done through relation (23)]. A possible bound between tensions of current carrying strings needs further investigation for particular models and goes beyond the scope of this paper.

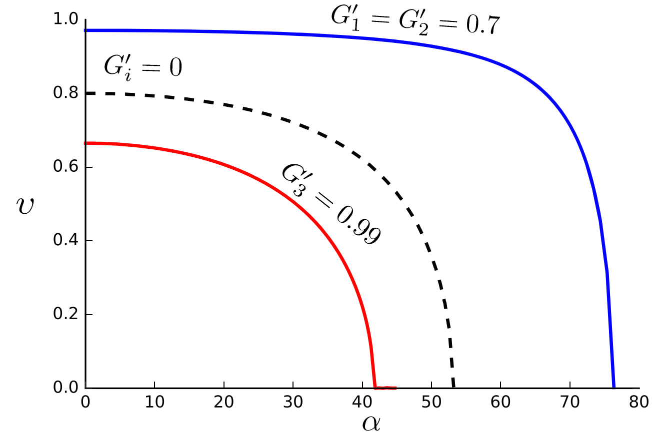

FIG. 4. Symmetric case. Range of parameters v and α, which allow for colliding strings with $\mu_1 = \mu_2 = 1$ to produce the Y junction ($\dot{s}_3 > 0$ corresponds to areas below lines) with $\mu_3 = 1.2$. The solid blue line corresponds to $G_1' = G_2' = 0.7$, red line to $G_3' = 0.99$, while all others G_i' are zeros. Dashed black line represents the case when all $G_i' = 0$.

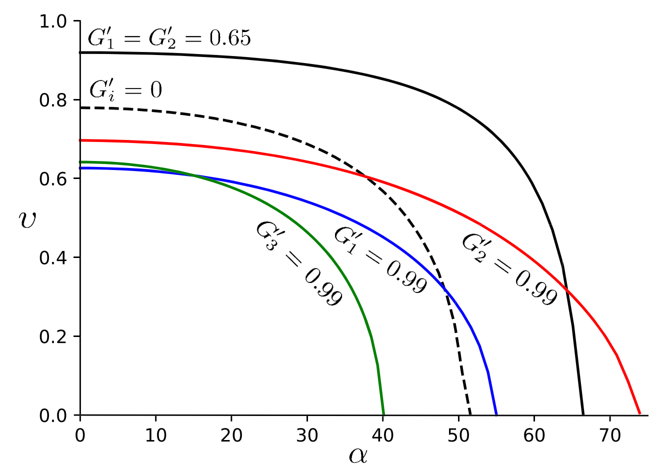

FIG. 5. Asymmetric case. Range of parameters v and α, which allow for colliding strings with $\mu_1 = 1$ and $\mu_2 = 1.2$ to produce the Y junction ($\dot{s}_3 > 0$ corresponds to areas below lines) with $\mu_3 = 1.4$. The solid blue line corresponds to $G_1' = 0.99$, red line to $G_2' = 0.99$, green line to $G_3' = 0.99$ while all others G_i' are zeros. Dashed black line represents the case when all $G_i' = 0$, while black solid when $G_1' = G_2' = 0.65$, $G_3' = 0$.

V. CONCLUSIONS

We revisited the exact solution for elastic transonic strings in Minkowski space [44,47] with the method developed in [49]. The exact solution allowed us to consider left- and right-moving modes, which made it possible to treat the dynamics of Y junctions in a similar manner as it was done in [23].

The system of equations (31) allowed us to obtain the rate of string lengths change $\dot{s}_i$, see Fig. 2, requiring the definition of incoming components of the current G_i'. The values of incoming current components G_i' should be determined by strings properties. Thus, in the case of cosmic superstrings, the values of G_i' might be defined similarly to saturated Bogomol'nyi-Prasad-Sommerfield state (see [5,56] for details), given by

$$\mu_{p,q} = \mu_F \sqrt{(p - qC_0)^2 + q^2/g_s^2}.$$

For superconducting and wiggly cosmic strings with Y junctions, the definitions of G_i' should arise from the values of tensions and mass per unit lengths (4). The exact definition of G_i' for particular type of strings needs further investigation and goes beyond the scope of this paper.

We studied the kink dynamics for elastic strings in Sec. III A. We obtained values of $\dot{s}_i$ that are essential for existence of kinklike discontinuity. We also demonstrated an example of elastic strings intercommutation and determined the velocities of these kinks.

In Sec. IV, we found kinematic constraints that should be satisfied to give rise to a Y junction for elastic strings. In particular, we obtained a range of parameters v, α, and G_i' of collided strings (36) for which $\dot{s}_3 > 0$. The symmetric case of elastic strings collision is shown in Fig. 4 and the asymmetric case is shown in Fig. 5.

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APPENDIX: COMPARISON WITH [48]

In Sec. IV, we considered the formation of Y junctions for transonic model of straight strings determined by the action (1) with $f(\kappa)$ defined in (10). It was shown that for collision of transonic elastic straight strings there are kinematic conditions that should be satisfied to form a Y junction. In our study, we did not face overdetermined system of equations, in contrast to result in [48]. The general approach of [48] claims that for magnetic and electric superconducting strings the formation of Y junction is impossible. On the other hand, the result in Sec. IV states that formation of Y junction is possible for a particular type of current, described by transonic elastic model. This appendix is intended to provide detailed comparison of our result with result obtained in [48]. For the sake of clarity, we denote equations related to work [48] as (…*).

A. Comparison of equations

Let us write down equations of motion for Y junction used in work [48] (above Eqs. (16*) and (17*) in [48]),

$$\partial_a(\sqrt{-\gamma_i} T_i^{ab} x^\mu_{i,b} \Theta(\tilde{s}_i(\tau_i) - \tilde{\sigma}_i)) = \mathrm{f}_i^\mu \delta(\tilde{s}_i(\tau_i) - \tilde{\sigma}_i), \quad \text{(A1a)}$$

$$\partial_a(\sqrt{-\gamma_i} z_i^a \Theta(\tilde{s}_i(\tau_i) - \tilde{\sigma}_i)) = \mathrm{g}_i \delta(\tilde{s}_i(\tau_i) - \tilde{\sigma}_i), \quad \text{(A1b)}$$

where $z_i^a = \sqrt{\kappa_{0i}} c_i^a$, $c_i^a = -2 f'_{\kappa_i} \gamma_i^{ab} \varphi_{i,b}$, $\sqrt{-\gamma_i} T_i^{ab} = \mathcal{T}_i^{ab}$, κ_{0i} is a constant multiplier, and all derivatives are taken with respect to conformal gauge parameters τ_i and $\tilde{\sigma}_i$, which are different for each string (in contrast to the gauge of the present study, where τ is the same for all strings) and provide the relations

$$\partial_{\tau_i} x_i^\mu \partial_{\tilde{\sigma}_i} x_{i\mu} = 0, \qquad (\partial_{\tau_i} x_i^\mu)^2 = -(\partial_{\tilde{\sigma}_i} x_i^\mu)^2.$$

Equations (A1) are parametrization invariant, namely, one can chose any $\tilde{\sigma}_i$ and τ_i. If one chooses the gauge (τ, σ_i) of this work and uses expressions (13) and (19) for elastic strings, defined by function (10), the system of Eq. (A1) is reduced to

$$\partial_a(\eta^{ab} x^\mu_{i,b} \Theta(s_i(\tau) - \sigma_i)) = \mathrm{f}_i^\mu \delta(s_i - \sigma_i), \quad \text{(A2a)}$$

$$\partial_a(\eta^{ab} \varphi_{i,b} \Theta(s_i(\tau) - \sigma_i)) = \mathrm{g}_i \delta(s_i - \sigma_i). \quad \text{(A2b)}$$

Substituting exact solutions (17) and (21) in (A2) and using condition (27), one can see that boundary terms of equations (A2) are identical to Eq. (28).

In case of conformal gauge $(\tilde{\sigma}_i, \tau_i)$, boundary terms of Eq. (A1) have the form of equations (16*) and (17*) of Ref. [48], given by

$$\sum_i \sqrt{-\gamma_i}\, (T_i^{0b} \dot{\tilde{s}}_i - T_i^{1b}) x^\mu_{i,b} = 0, \quad \text{(A3a)}$$

$$\sum_i \sqrt{-\gamma_i}\, (z_i^0 \dot{\tilde{s}}_i - z_i^1) = 0, \quad \text{(A3b)}$$

where $\dot{\tilde{s}}_i \equiv \frac{d\tilde{s}_i}{d\tau_i}$.

TABLE I. Correspondence between equations of the manuscript and work [48].

Gauge of this work	Conformal gauge in [48]
Eq. (27) =	= Eq. (15*)
Eq. (28) =	= Eq. (14*)
Eq. (29) =	= Eqs. (16*) and (17*)

We demonstrated that Eqs. (16*) and (17*) for Y junction in [48] coincide with Eq. (28) of the main text. There is a full agreement between equations of [48] and equations of the paper; see Table I for correspondence. Equations (25*), (28*), and (29*) of [48] should be identical to (A3) and to (28), but just written in a preferred rest frame. Hence, the amount of equations in our study and work [48] are the same.

B. Straight string solution and number of unknown variables

To understand where the disagreement with [48] comes from, we also need to count the number of unknown variables, since the number of equations in our study and in work [48] are the same.

To start, let us write down the solution for straight strings, which satisfies equations of motion (11) and can be considered as a linear term of Taylor expansion close to the point of strings collision, i.e., for gauge of this work,

$$
\begin{aligned}
x_i^\mu(\sigma_i,\tau) &= A_i^\mu\sigma_i + B_i^\mu\tau + \mathcal{O}(\sigma_i^2,\tau^2),\\
\varphi_i(\sigma_i,\tau) &= C_i\sigma_i + D_i\tau + \mathcal{O}(\sigma_i^2,\tau^2)\\
&\quad\text{or for conformal gauge}\\
x_i^\mu(\tilde\sigma_i,\tau_i) &= \tilde A^\mu\tilde\sigma_i + \tilde B^\mu\tau_i + \mathcal{O}(\tilde\sigma_i^2,\tau_i^2),\\
\varphi_i(\tilde\sigma_i,\tau_i) &= \tilde C_i\sigma_i + \tilde D_i\tau_i + \mathcal{O}(\tilde\sigma_i^2,\tau_i^2),
\end{aligned}
\tag{A4}
$$

where A_i^μ, B_i^μ, $\tilde A_i^\mu$, $\tilde B_i^\mu$, C_i, D_i, $\tilde C_i$, and $\tilde D_i$ are constants with possibly different physical meaning. The form of solution (A4) provides left- and right-moving modes, similarly to (17) and (21).

Mass per unit length U and tension T, given by (4), are dynamical parameters that are constructed from left- and right-moving modes of (A4) for the corresponding gauge. When U, T (or current κ with action) are fixed, one can still chose different left- and right-moving modes. This fact is well seen from expression (23), where the same value of the current κ can be constructed by different values of F' and G'.

Expressing all parameters, such as mass per unit length U, tension T, and current κ by left- and right-moving modes, we can count number of unknown variables. All derivatives of outgoing modes, can be determined through conditions (29), introducing $\dot\Phi$ and $\dot{\mathbf{X}}$. The last variable can be substituted by expression with parameters α, u, and $\dot s_3$ of newly created string. Hence, there are four equations from (A3) and energy conservation condition (which is automatically satisfied), two independent equations come from first expression of (29), providing six independent equations for six variables: u, α, $\dot s_i$, and $\dot\Phi$. Thus, expressing all parameters as left- and right-moving modes, one obtains the system of equations, which has the same amount of equations and unknown variables (as it is mentioned in the end of Sec. IV, one also needs to define the incoming mode G_3).

C. The difference with work [48]

The process of strings collision for elastic transonic strings is illustrated in Fig. 6. The upper panel demonstrates left- and right-moving modes of strings before the collision, the lower-after. Modes denoted by question symbol should be obtained from Y-junction equations at J. Two kinks, illustrated in Fig. 6 as K_1 and K_2, are formed after strings collision and, according to Sec. III A, incoming modes propagate without modification through kinks K_1 and K_2 for special choice of parametrization, i.e., modes G_1, $\mathbf{b}_1$, G_2, and $\mathbf{b}_2$ can pass directly to Y junction. Kinks for elastic transonic straight strings (36) are described by (42). Schematic Fig. 6 of strings collision with particular choice of parametrization is valid for elastic transonic strings as well as for standard Nambu-Goto strings [22,23] and chiral strings [54].

For conformal gauge choice, which was used in [48], incoming modes for $J - K_1$ and $J - K_2$, in general, are not the same as modes before kinks and should be determined by equations for kink discontinuity. As a result, to obtain similar parameter region space, which allows Y-junction formation, one should solve the system of equations that includes equations for kinks K_1, K_2, and junction J

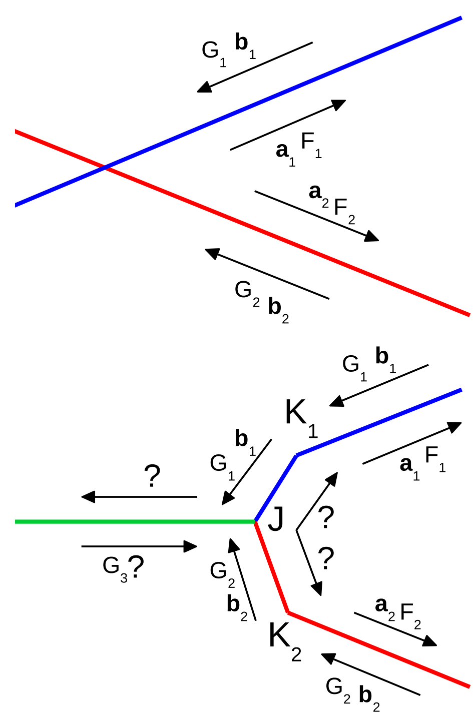

FIG. 6. Schematic picture of left- and right-moving modes of elastic transonic strings for the gauge described in the main text, Sec. II. The upper panel shows left- and right-moving modes before the collision of strings, and the lower panel shows modes after. Moving modes with question symbol should be determined by equations for Y junction (31) and (32).

simultaneously. This fact demonstrates that the gauge choice, which was used in this work, simplifies equations allowing to consider incoming modes for Y junction the same as modes before K_1 and K_2 kinks.

To summarize the comparison, it was demonstrated in Sec. VI A that equations for Y junctions of this study and work [48] are in agreement, one system of equations can be transformed to the other; see Table I. In Sec. VI B, it was shown that any type of straight strings can be represented by the form (A4) and split for left- and right-moving modes. Defining string parameters, such as T, U and current through left- and right-moving modes, one obtains the system of equations that has the same amount of unknown variables and equations for Y junction. Hence, equations of work [48] written via left- and right-moving modes (which do not require separated consideration of electric and magnetic types of current) are reduced to (31) and (32) providing the same range of parameters that allows formation of Y junction. In study [48], equations are not written in terms of left- and right-moving modes, but in terms of tension T, mass per unit length U, and electric (or magnetic) current. Each value of U and T (as well as current) can be represented by different left- and right-moving modes. It means that there are more unknown variables in equations written via left- and right-moving modes than in equations written via U and T. Fixing mass per unit length U and tension T, as it was done in [48], one also fixes corresponding outgoing and incoming modes obtaining overdetermined system of equations, i.e., less unknown variables than equations for Y junction.

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