Baryon and lepton number intricacies in axion models

Jérémie Quevillon[*] and Christopher Smith[†]
Laboratoire de Physique Subatomique et de Cosmologie, Université Grenoble-Alpes, CNRS/IN2P3, Grenoble INP, 38000 Grenoble, France

(Received 8 July 2020; accepted 23 September 2020; published 26 October 2020)

Because the Peccei-Quinn (PQ) symmetry has to be anomalous to solve the strong CP puzzle, some colored and chiral fermions have to transform nontrivially under this symmetry. But when the Standard Model (SM) fermions are charged, as in the PQ or Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) models, this symmetry ends up entangled with the SM global symmetries, baryon ($\mathcal{B}$) and lepton ($\mathcal{L}$) numbers. This raises several questions addressed in this paper. First, the compatibility of axion models with some explicit $\mathcal{B}$- and/or $\mathcal{L}$-violating effects is analyzed, including those arising from seesaw mechanisms, electroweak instanton interactions, or explicit $\mathcal{B}$- and $\mathcal{L}$-violating effective operators. Second, how many of these effects can be simultaneously present is quantified, along with the consequences for the axion mass and vacuum alignment if too many of them are introduced. Finally, large classes of $\mathcal{B}$- and/or $\mathcal{L}$-violating interactions without impact on axion phenomenology are identified, like, for example, the various implementations of the type-I and -II seesaw mechanisms in the DFSZ context.

DOI: 10.1103/PhysRevD.102.075031

I. INTRODUCTION

Even if the simplest axion models introduced more than 40 years ago have been ruled out, axions still remain one of the best solutions for the strong CP problem of the Standard Model (SM). This problem originates from the observation that the QCD and the electroweak sectors, by construction secluded, must somehow conspire to cancel each other's sources of CP violation. Indeed, while individually their contributions to the θ term of QCD are *a priori* both of $\mathcal{O}(1)$, the yet nonobserved electric dipole moment of the neutron [1] requires their sum to be tiny, $\theta_{\text{eff}} \equiv \theta_{\text{QCD}} + \theta_{\text{Yukawa}} \lesssim 10^{-10}$.

Axions come under many guises, but the basic recipe is always the same: Design a global $U(1)$ symmetry and assign charges to some colored chiral fermions [2]. This ensures that $U(1)$ rotations act on the strong CP phase, since its current is anomalous. This is not sufficient yet to dispose of the θ term, since fermion masses explicitly break this $U(1)$ symmetry. To force θ_{eff} to zero, the trick proceeds in two steps [2]. First, this $U(1)$ symmetry is spontaneously broken, so that its associated Goldstone boson, the axion [3,4], has a direct coupling to gluons. Second, nonperturbative QCD effects create an effective potential for the axion field, whose minimum is attained precisely when the θ term is rotated away. In the process, the axion acquires a small QCD-induced mass, typically well below the eV scale [5,6]. Both the mass and the couplings of the QCD axion are thus controlled by a single scale: the one of spontaneous symmetry breaking, usually dubbed f_a.

To solve the strong CP puzzle, the axion needs to be coupled to colored fermions, and this gives rise to two broad classes of models. Those of the Kim-Shifman-Vainshtein-Zakharov (KSVZ) type [7] introduce new very heavy fermions, vectorlike for the SM gauge interactions, while those of the PQ [2] and Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) [8] types make use of the SM chiral quarks. In that latter case, the axion must arise from the very same Higgs bosons that give the quarks their masses and, thus, emerges only after the electroweak symmetry is broken. In a previous study [9], we have described that, for this class of models, the fermion charges are necessarily ambiguous because of the presence of the accidental $U(1)$ symmetries of the SM, corresponding to the conserved baryon ($\mathcal{B}$) and lepton ($\mathcal{L}$) numbers. Though this ambiguity was found to have no impact on the low-energy phenomenology, it raises several questions that we want to address in the present paper. Specifically:

(i) Since the ambiguities arise from the SM accidental symmetries, the main question is to study what happens in the presence of explicit $\mathcal{B}$- and/or $\mathcal{L}$-breaking terms. There are some conflicting conclusions regarding the capabilities of DFSZ models

[*]jeremie.quevillon@lpsc.in2p3.fr
[†]chsmith@lpsc.in2p3.fr

to accommodate for such violations. We will see that some limited violation is possible, characterize it, and study the consequences when this limit is overstepped.

(ii) A second question is to which extent is it possible to fix the ambiguities, or said differently, are there naturally some $\mathcal{B}$ and/or $\mathcal{L}$ components embedded in the axion $U(1)$ symmetry. Of course, those components are projected out when the symmetry is spontaneously broken, but finding the optimal representation for the $U(1)$ symmetry could simplify the form of the axion effective Lagrangian. We will see that, in most cases, neutrino masses and electroweak instanton effects hold the key to identify the $U(1)$ symmetry unambiguously.

(iii) Finally, since these ambiguities have no phenomenological consequence, it is worth to inversigate whether it can be used to relate seemingly different models. We will see that the fermion charges for all PQ and DFSZ-like models based on the same Yukawa couplings, whether with a seesaw mechanism of type I or II or with some (limited) $\mathcal{B}$ violation, are actually equivalent. Thus, despite their very different appearance in terms of effective interactions, those models cannot be distinguished at low energy.

The paper is organized as follows. To set the stage, we start in the next section by presenting the PQ axion model and the DFSZ axion model. Then, in Sec. III, we study the compatibility of these models with lepton number violation, by introducing various mechanisms to generate neutrino masses. In Sec. IV, we investigate the impact of baryon number violation on axion models and explore what would happen if further explicit $\mathcal{B}$- and/or $\mathcal{L}$-violating interactions were introduced in the theory. Finally, our results are summarized in Sec. V.

II. FERMION CHARGE AMBIGUITIES IN AXION MODELS

In this section, the simplest axion models are briefly reviewed. We focus on the precise identification of the global and local $U(1)$ symmetries at play and their breaking pattern. In this way, it will be immediately obvious that, when the scalars giving masses to the SM fermions are charged under the PQ symmetry, there remains an ambiguity in the PQ charges of the fermions and that this ambiguity is related to the invariance of the Yukawa couplings under $\mathcal{B}$ and $\mathcal{L}$. In the next sections, this freedom will play a central role, as it will be used to accommodate the possibility of $\mathcal{B}$ and/or $\mathcal{L}$ violation in axion models.

A. Axion in the PQ model

The starting point is a two-Higgs doublets with the scalar potential

$$V_{\mathrm{THDM}} = m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_2^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2). \tag{1}$$

Provided a consistent spontaneous symmetry breaking (SSB) occurs, the mass spectrum is then made of two neutral scalar Higgs bosons h^0 and H^0, a pseudoscalar A^0, and a pair of charged Higgs bosons $H^\pm$.

This potential is invariant under the independent rephasing of the Higgs doublets, corresponding to a global $U(1)_1 \otimes U(1)_2$ symmetry. Actually, a linear combination of these $U(1)$ charges is nothing but the gauged hypercharge. Note that this $U(1)_1 \otimes U(1)_2$ symmetry is truly active at the level of the whole two-Higgs doublet model (THDM), and, in particular, assuming Yukawa couplings of type II,

$$\mathcal{L}_{\mathrm{Yukawa}} = -\bar{u}_R\mathbf{Y}_u q_L\Phi_1 - \bar{d}_R\mathbf{Y}_d q_L\Phi_2^\dagger - \bar{e}_R\mathbf{Y}_e\ell_L\Phi_2^\dagger + \mathrm{H.c.}, \tag{2}$$

it requires that fermions are assigned appropriate $U(1)_1 \otimes U(1)_2$ charges. Beside, these Yukawa couplings are also invariant under the global baryon and lepton number symmetries, $U(1)_\mathcal{B}$ and $U(1)_\mathcal{L}$. Those must be left untouched by the electroweak symmetry breaking (EWSB). So, all in all, the pattern of symmetry breaking is

$$\begin{aligned} G_{\mathrm{THDM}} &= U(1)_\mathcal{B}\otimes U(1)_\mathcal{L}\otimes U(1)_1\otimes U(1)_2\otimes SU(2)_L \\ &\quad \otimes SU(3)_C \\ &= U(1)_\mathcal{B}\otimes U(1)_\mathcal{L}\otimes U(1)_X\otimes U(1)_Y\otimes SU(2)_L \\ &\quad \otimes SU(3)_C \\ &\overset{\mathrm{EWSB}}{\longrightarrow} U(1)_\mathcal{B}\otimes U(1)_\mathcal{L}\otimes U(1)_{em}\otimes SU(3)_C. \end{aligned} \tag{3}$$

When the doublets acquire vacuum expectation values (VEVs), $U(1)_1 \otimes U(1)_2 \otimes SU(2)_L$ is broken down to $U(1)_{em}$. There are thus two Goldstone bosons, one is the would-be Goldstone (WBG) eaten by the Z^0, and the other is truly present in the spectrum and is the massless axion.

In the breaking chain, it must be stressed that we wrote $U(1)_X$ and not $U(1)_{\mathrm{PQ}}$ for the part of $U(1)_1 \otimes U(1)_2$ not aligned with $U(1)_Y$. Indeed, strictly speaking, the $U(1)_{\mathrm{PQ}}$ symmetry is defined only after the doublets acquire their VEVs, from the orthogonality of the axion with the WBG of the Z^0. Furthermore, if we denote the VEVs as $\langle 0|\mathrm{Re}\Phi_i|0\rangle = v_i$ with $v_1^2 + v_2^2 \equiv v^2 \approx (246\ \mathrm{GeV})^2$ and $v_2/v_1 \equiv x \equiv 1/\tan\beta$, both these fields are v_i-dependent linear combinations of $\mathrm{Im}\Phi_1^0$ and $\mathrm{Im}\Phi_2^0$, and, consequently, the PQ charges of the doublets are functions of v_i. They are defined only once $U(1)_Y$ is broken.

Specifically, adopting a polar representation for the pseudoscalar Goldstone bosons, the Higgs doublets are written in the broken phase as

$$\Phi_i = \frac{1}{\sqrt{2}}\exp(i\eta_i/v_i)\begin{pmatrix} H_i^+ \\ v_i + \mathrm{Re}H_i \end{pmatrix}, \qquad i = 1, 2. \tag{4}$$

The Goldstone bosons associated to the $U(1)_1$ and $U(1)_2$ symmetries, η_1 and η_2, are related to the physical Goldstone bosons a^0 and G^0 as

$$\begin{pmatrix} G^0 \\ a^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} \eta_2 \\ \eta_1 \end{pmatrix}. \tag{5}$$

Plugging this into Eq. (4), the PQ charge of each doublet can be read off its phase variation under a shift of the associated Goldstone boson, $a^0 \to a^0 + v\theta$, and, thus,

$$\mathrm{PQ}(\Phi_1) = \frac{v}{v_1}\cos\beta = x, \qquad \mathrm{PQ}(\Phi_2) = -\frac{v}{v_2}\sin\beta = -\frac{1}{x}. \tag{6}$$

Note that the shift $G^0 \to G^0 + v\theta$ reproduces $Y(\Phi_1) = Y(\Phi_2) = 1$. It also shows explicitly how misleading any idea of orthogonality of the $U(1)$ charges could be. We started with $U(1)_1 \otimes U(1)_2$ under which the pair (Φ_1, Φ_2) has the seemingly orthogonal charge assignment $(v/v_1, 0) \otimes (0, v/v_2)$. But once $U(1)_1 \otimes U(1)_2$ is broken and the associated Goldstone bosons compelled to be orthogonal, we end up with the $U(1)_Y \otimes U(1)_{\mathrm{PQ}}$ charge $(1, 1) \otimes (x, -1/x)$ for the pair (Φ_1, Φ_2).

Once these charges are fixed, those of the fermions can be derived by requiring the Yukawa Lagrangian to be invariant under $U(1)_{\mathrm{PQ}}$. Since those couplings are also necessarily invariant under $\mathcal{B}$ and $\mathcal{L}$, these charges are defined only up to a two-parameter ambiguity [9], which we denote α and β:

$$\mathrm{PQ}(q_L, u_R, d_R, \ell_L, e_R) = \left(\alpha, \alpha + x, \alpha + \frac{1}{x}, \beta, \beta + \frac{1}{x}\right). \tag{7}$$

At this stage, there is no way to fix α and β, essentially because neither $\mathcal{B}$ nor $\mathcal{L}$ have associated dynamical fields. Further, as discussed for the pair (Φ_1, Φ_2), there is no viable concept of orthogonality for the $U(1)$ charges in the fermion sector either. Actually, it should be remarked that

$$\mathcal{B}(q_L, u_R, d_R, \ell_L, e_R) = (1/3, 1/3, 1/3, 0, 0), \tag{8a}$$

$$\mathcal{L}(q_L, u_R, d_R, \ell_L, e_R) = (0, 0, 0, 1, 1), \tag{8b}$$

$$Y(q_L, u_R, d_R, \ell_L, e_R) = (1/3, 4/3, -2/3, -1, -2) \tag{8c}$$

are not orthogonal among themselves to begin with, so there is no reason to expect the PQ charge to be any different.

The freedom in the PQ charges of the SM fermions has no observable consequence. The simplest way to see that is to adopt the usual linear parametrization for the THDM. Since the ambiguity in the fermion PQ charges appears nowhere in the Lagrangian, all the Feynman rules are independent of α and β, and so are the physical observables. Using the polar representation of Eq. (4), the situation is a bit more involved. Though, initially, the Lagrangian is again independent of α and β, and so are all the Feynman rules, it is customary to perform a reparametrization of the fermion fields to remove the axion field from the Yukawa couplings. In full generality, this reparametrization is α and β dependent, because the fermion rephasings are tuned by their PQ charges,

$$\psi \to \psi \exp(i\mathrm{PQ}(\psi)a^0/v), \qquad \psi = q_L, u_R, d_R, \ell_L, e_R. \tag{9}$$

In this way, a dependence on α and β is spuriously introduced in the Lagrangian, first because the noninvariance of the fermion kinetic terms generates the couplings

$$\delta\mathcal{L}_{\mathrm{Der}} = -\frac{\partial_\mu a^0}{v}\sum_{\psi = q_L, u_R, d_R, \ell_L, e_R}\mathrm{PQ}(\psi)\bar{\psi}\gamma^\mu\psi, \tag{10}$$

and, second, because the noninvariance of the fermionic path integral measure generates the anomalous interactions

$$\delta\mathcal{L}_{\mathrm{Jac}} = \frac{a^0}{16\pi^2}\{\mathcal{N}_C g_s^2 G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + \mathcal{N}_L g^2 W_{\mu\nu}^i \tilde{W}^{i,\mu\nu} + \mathcal{N}_Y g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\} \tag{11}$$

with

$$\mathcal{N}_C = \sum_{\psi = q_L^\dagger, u_R, d_R} d_L(\psi)C_C(\psi)\mathrm{PQ}(\psi) = \frac{1}{2}\left(x + \frac{1}{x}\right), \tag{12a}$$

$$\mathcal{N}_L = \sum_{\psi = q_L^\dagger, \ell_L^\dagger} d_C(\psi)C_L(\psi)\mathrm{PQ}(\psi) = -\frac{1}{2}(3\alpha + \beta), \tag{12b}$$

$$\mathcal{N}_Y = \sum_{\psi = q_L^\dagger, u_R, d_R, \ell_L^\dagger, e_R} d_L(\psi)d_C(\psi)C_Y(\psi)\mathrm{PQ}(\psi) = \frac{1}{2}(3\alpha + \beta) + \frac{4}{3}\left(x + \frac{1}{x}\right), \tag{12c}$$

where $d_{C,L}(\psi)$ and $C_{C,L}(\psi)$ are the $SU(3)_C$ and $SU(2)_L$ dimensions and quadratic Casimir invariant of the representation carried by the field ψ, respectively, and, by extension, $C_Y(\psi) = Y(\psi)^2/4$ with the hypercharges given in Eq. (8c).

Yet, even if both $\delta\mathcal{L}_{\mathrm{Der}}$ and $\delta\mathcal{L}_{\mathrm{Jac}}$ depend on α and β, these parameters cancel out systematically in all physical observables, as shown explicitly in Ref. [9]. Nevertheless, some theoretical quantities inevitably depend on α and β. Besides the above interactions, another particular example is the divergence of the PQ current, since it is related to the anomalous interaction via $\delta\mathcal{L}_{\text{Jac}} = a^0\partial_\mu J^\mu_{\text{PQ}}$.

Since the two-photon coupling arises as $\mathcal{N}_L + \mathcal{N}_Y = \mathcal{N}_{em}$ in Eq. (11), both the QED and QCD terms in $\partial_\mu J^\mu_{\text{PQ}}$ are independent of α and β and immediately physical, but the electroweak term is always ambiguous. This is, of course, expected in view of the $\mathcal{B}$ and $\mathcal{L}$ origins of the α and β parameters. If one remembers that these currents also have anomalous divergences

$$\partial_\mu J^\mu_{\mathcal{B}} = \partial_\mu J^\mu_{\mathcal{L}} = -\frac{N_f}{16\pi^2}\left(\frac{1}{2}g^2 W^i_{\mu\nu}\tilde{W}^{i,\mu\nu} - \frac{1}{2}g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right), \tag{13}$$

one can immediately understand how α and β enter in Eq. (12). Yet, one should not conclude too quickly that α and β represent a spurious $\mathcal{B}$ and $\mathcal{L}$ component of the PQ current and should be set to zero. Indeed, this would entirely remove the electroweak $W^i_{\mu\nu}\tilde{W}^{i,\mu\nu}$ term of $\partial_\mu J^\mu_{\text{PQ}}$, but there is no reason for a (hypothetical) $\mathcal{B}$- and $\mathcal{L}$-free PQ current to have no electroweak component. Besides, one should realize that the final form of $\mathcal{N}_L$ reflects the specific choice made in parametrizing the two-parameter freedom in the fermion PQ charges. To bring Eq. (7) to a simple form, we made the choice of fixing $\text{PQ}(q_L) \equiv \alpha$ and $\text{PQ}(\ell_L) \equiv \beta$. So, setting $\alpha = \beta = 0$ would simply remove the left-handed fields from the PQ currents, but this is hardly natural, since the axion is coupled to left-handed fields, as can be confirmed adopting the usual linear representation for the THDM scalar fields.

B. Axion in DFSZ model

When the axion is embedded as one of the pseudoscalar degrees of freedom of the THDM, its couplings end up tuned by the electroweak VEV and are far too large given the experimental constraints. The DFSZ axion model [8] circumvents this problem by moving most of the axion field into a new field, whose dynamics take place at a much higher scale. Specifically, the THDM is extended by a gauge-singlet complex scalar field ϕ, with the scalar potential

$$V_{\text{DFSZ}} = V_{\text{THDM}} + V_\phi + V_{\phi\text{THDM}} + V_{\phi\text{PQ}}, \quad \begin{cases} V_\phi = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \\ V_{\phi\text{THDM}} = a_1\phi^\dagger\phi\Phi_1^\dagger\Phi_1 + a_2\phi^\dagger\phi\Phi_2^\dagger\Phi_2, \\ V_{\phi\text{PQ}} = -\lambda_{12}\phi^2\Phi_1^\dagger\Phi_2 + \text{H.c.} \end{cases} \tag{14}$$

This potential is invariant under the same $U(1)_1 \otimes U(1)_2$ symmetry as in the PQ realization of the previous section, provided ϕ is charged under both $U(1)$'s. Concerning fermions, the same type-II Yukawa couplings as in Eq. (2) are allowed, while ϕ cannot directly couple to fermions because of its $U(1)$ charges.

The symmetry-breaking scale v_s of the singlet is assumed to be far above the electroweak scale. To leading order in v/v_s, $\langle 0|\text{Re}\phi|0\rangle$ breaks $U(1)_1 \otimes U(1)_2 \to U(1)_Y$, and its associated Goldstone boson is the axion. Indeed, at the v scale, the $\lambda_{12}\langle\phi^2\rangle\Phi_1^\dagger\Phi_2$ term ensures the pseudoscalar state of the THDM is massive. In this leading-order approximation, the axion is not coupled to fermions, since it is fully embedded in ϕ. The interesting physics take place at $\mathcal{O}(v/v_s)$, where the $V_{\phi\text{PQ}}$ coupling generates an $\mathcal{O}(v/v_s)$ mass for $\text{Im}\phi$ tuned by $\lambda_{12}v_1v_2$. Neither $\text{Im}\phi$ nor $\text{Im}\Phi_{1,2}$ remain massless, but a linear combination of these states does. The axion is, thus, $a^0 = \mathcal{O}(1)\text{Im}\phi + \mathcal{O}(v/v_s)\text{Im}\Phi_{1,2}$, and, since all the couplings to SM particles stem from its $\text{Im}\Phi_{1,2}$ components, the axion essentially but not totally decouples. Yet, it is still able to solve the strong CP problem, since this ensures its coupling to $G_{\mu\nu}\tilde{G}^{\mu\nu}$.

To be more quantitative, this picture is easily confirmed adopting a polar representation for the scalar fields. Plugging Eq. (4) together with

$$\phi = \frac{1}{\sqrt{2}}\exp(i\eta_s/v_s)(v_s + \sigma_S) \tag{15}$$

into V_{DFSZ} and setting all fields but $\eta_{1,2,s}$ to zero, only the $\lambda_{12}\phi^2\Phi_1^\dagger\Phi_2$ coupling contributes, since all the other terms involve the Hermitian combinations $\Phi_i^\dagger\Phi_i$ and/or $\phi^\dagger\phi$. Restricted to the pseudoscalar states, the potential collapses to

$$V_{\text{DFSZ}}(\eta_{1,2,s}) = -\frac{1}{2}\lambda_{12}v_1v_2v_s^2\cos\left(\frac{\eta_1}{v_1} - \frac{\eta_2}{v_2} - \frac{2\eta_s}{v_s}\right). \tag{16}$$

By expanding the cosine function and diagonalizing the quadratic term, the mass eigenstates are easily found to be

$$\begin{pmatrix} G^0 \\ a^0 \\ \pi^0 \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta & 0 \\ \delta_s\omega\cos\beta\sin 2\beta & -\delta_s\omega\sin\beta\sin 2\beta & \omega \\ \omega\cos\beta & -\omega\sin\beta & -\delta_s\omega\sin 2\beta \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_s \end{pmatrix}, \tag{17}$$

with $\delta_s = v/v_s$ and $\omega^{-2} = 1 + \delta_s^2 \sin^2 2\beta$. The interest of this form is that we can read off the PQ charges of η_1, η_2, and η_s from their reactions to a shift $a^0 \to a^0 + v_s\omega^{-1}\theta$, and we find

$$\mathrm{PQ}(\Phi_1, \Phi_2, \phi) = (2\cos^2\beta, -2\sin^2\beta, 1), \tag{18}$$

or, rescaling these charges by $2x/(x^2+1)$,

$$\mathrm{PQ}(\Phi_1, \Phi_2, \phi) = \left(x, -\frac{1}{x}, \frac{1}{2}\left(x + \frac{1}{x}\right)\right). \tag{19}$$

We thus recover the same charges as in the PQ model [Eq. (6)], so those of the fermions also stay the same [Eq. (7)], including the α and β ambiguities related to baryon and lepton numbers. Note that the final form of the mixing matrix is compatible with the cosine potential, in the sense that the massive π^0 state is precisely the combination of states occurring as the argument of the cosine function:

$$\begin{aligned} \pi^0 &= \omega(\cos\beta\eta_1 - \sin\beta\eta_2 - \delta_s \sin 2\beta\eta_s) \\ &= \omega v \sin\beta\cos\beta\left(\frac{\eta_1}{v_1} - \frac{\eta_2}{v_2} - 2\frac{\eta_s}{v_s}\right). \end{aligned} \tag{20}$$

The potential $V_{\mathrm{DFSZ}}(\eta_{1,2,s})$ is necessarily flat in the other two orthogonal directions, corresponding to the two Goldstone bosons (the G^0 eaten by the Z^0 and the a^0). Finally, remark that if the $\phi^2\Phi_1^\dagger\Phi_2$ coupling is replaced by $\phi\Phi_1^\dagger\Phi_2$, everything stays the same but for $\mathrm{PQ}(\phi)$. This has no phenomenological impact, since the axion couplings to SM fields are unchanged.

III. AXIONS AND LEPTON NUMBER VIOLATION

Up to now, neutrinos have been kept massless. To account for the very light neutrino masses in a natural way, the standard approach is to implement a seesaw mechanism. Generically, these mechanisms assume the observed left-handed neutrinos have a Majorana mass term, typically via the dimension-five operator

$$\mathcal{L}_{\mathrm{seesaw}}^{\mathrm{eff}} = -\frac{\mathbf{c}}{\Lambda}(\bar{\ell}_L^C \Phi_i^T)(\ell_L \Phi_i) + \mathrm{H.c.}, \tag{21}$$

where $\mathbf{c}$ is understood as a matrix in flavor space and flavor indices are understood. Neutrino masses are then $\mathbf{m}_\nu = \mathbf{c} v_i^2/\Lambda$. The scale Λ represents that where the lepton number is broken, either explicitly or spontaneously. Obviously, neutrinos end up very light when Λ is sufficiently high.

Since a generic feature of the seesaw mechanisms is a breaking of $\mathcal{L}$, the most immediate question is how to accommodate for that in axion models. This has already been studied quite extensively, but most of the time in a KSVZ-like setting, where new colored fermions are introduced and SM fermions need not be charged under the PQ symmetry [10–13]. Here, we concentrate on DFSZ-like models, in which $\mathcal{L}$ manifests itself as an ambiguity in the PQ charges of the SM fermions. To study the consequences, and actually show that axion phenomenology is essentially unaffected by neutrino masses, we review in this section three realizations. First, we supplement the PQ and DFSZ model with a seesaw mechanism of type I [14]. Then, we consider the νDFSZ model of Ref. [15,16], where the DFSZ singlet is made responsible for the breaking of the lepton number. Finally, we consider the type-II seesaw mechanism [17,18], realized either *à la* PQ or DFSZ [19,20]. Other DFSZ-like realizations are possible—see, for example, Refs. [21–24]—but those described here are the simplest. Also, we do not consider the proposal of Ref. [25] in which the PQ and $\mathcal{B}-\mathcal{L}$ currents are identified, with a nonlocal Majoron gluonic coupling arising through complicated multiloop processes (for a model where the standard singlet Majoron would not solve the strong *CP* problem, see Ref. [26]).

A. PQ and DFSZ with a type-I seesaw mechanism

A first strategy to account for neutrino masses is to add to the PQ or DFSZ model a type-I seesaw mechanism. Specifically, we add right-handed neutrinos ν_R to the model. Since those are singlet under the gauge symmetry, the only new allowed couplings are

$$\mathcal{L}_{\nu_R} = -\frac{1}{2}\bar{\nu}_R^C \mathbf{M}_R \nu_R + \bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_i + \mathrm{H.c.} \tag{22}$$

with $i = 1$ or 2. The lepton number no longer emerges as an accidental symmetry, because the Majorana mass term $\mathbf{M}_R$ breaks $\mathcal{L}$ by two units. It is also presumably very large, so integrating out the ν_R fields generates the dimension-five operator in Eq. (21) with $\mathbf{c}\Lambda^{-1} = \mathbf{Y}_\nu^T \mathbf{M}_R^{-1} \mathbf{Y}_\nu$.

The PQ charge of the right-handed neutrinos has to vanish to allow the presence of the Majorana mass term. Given the PQ charge in Eqs. (7) and (6) or (19), this implies that β must be nonzero, since

$$\bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_1\colon\ \mathrm{PQ}(\nu_R) = \beta + x = 0, \tag{23a}$$

$$\bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_2\colon\ \mathrm{PQ}(\nu_R) = \beta - \frac{1}{x} = 0. \tag{23b}$$

These equations must be interpreted in the right way. This is not a choice for β. Rather, in the presence of $\mathbf{M}_R$, $U(1)_\mathcal{L}$ is removed from the symmetry-breaking chain of Eq. (3), and the corresponding ambiguity is simply not there to start with. In other words, it would make no sense to set β to any other value and discuss the impact of the PQ breaking induced by $\mathbf{M}_R$, since this breaking is spuriously introduced by an inappropriate choice of PQ charges. Yet, remarkably, the PQ symmetry does not forbid either the Majorana mass term in Eq. (22) or the effective operator Eq. (21), contrary to the claim made, for example, in Refs. [27,28].

The presence of the seesaw mechanism does not significantly alter the axion phenomenology. This is most clearly seen adopting the linear parametrization for the scalar fields, since then all the axion couplings to SM fermions are proportional to their masses. When ν_R have been integrated out, that of the axion to light neutrinos will arise from Eq. (21) and, thus, be tiny. In a polar representation for the pseudoscalar fields, first note that, with β fixed as in Eq. (23), the seesaw operator of Eq. (21) becomes invariant under the PQ symmetry. It does not prevent the fermion reparametrization of Eq. (9), which proceeds exactly as in the absence of neutrino masses. Except that β is fixed, the effective derivative and anomalous interactions stay the same. Since we proved in Ref. [9] that β cancels out in physical observables anyway, the phenomenology is unchanged, except for the tiny kinematical impact of the now finite neutrino masses (for example, the $a^0W^+W^-$ loop amplitude depends on the mass of the virtual fermions, including neutrinos).

B. Merging DFSZ with a type-I seesaw mechanism

Instead of adding a Majorana mass term for the right-handed neutrinos, we can use the singlet field and set

$$\mathcal{L}_{\nu_R} = -\frac{1}{2}\bar{\nu}_R^C \mathbf{Y}_R \nu_R \phi + \bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_i + \text{H.c.} \tag{24}$$

This model, dubbed the νDFSZ, was first proposed in Refs. [15,16].

Let us see how this merging of the DFSZ model with a type-I seesaw mechanism can be understood from the point of view of the $U(1)$'s. Since ϕ cannot be neutral under $U(1)_1 \otimes U(1)_2$, the right-handed neutrinos do have charges, and no Majorana mass term is allowed. Basically, what we are doing is to embed the lepton number inside the global symmetries, $U(1)_\mathcal{L} \subset U(1)_1 \otimes U(1)_2$. Since the VEV of ϕ breaks both $U(1)_1$ and $U(1)_2$, it also breaks $U(1)_\mathcal{L}$, and then the Goldstone boson can be viewed as a Majoron. Note, though, that Φ_1 and Φ_2 as well as quarks are charged under $U(1)_1 \otimes U(1)_2$, since the assignments are

$\bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_1$	ϕ	Φ_1	Φ_2	q_L	u_R	d_R	ℓ_L	e_R	ν_R
$U(1)_1$	$+1/2$	1	0	α_1	α_1+1	α_1	$-5/4$	$-5/4$	$-1/4$
$U(1)_2$	$-1/2$	0	1	α_2	α_2	α_2-1	$+1/4$	$-3/4$	$+1/4$

(25)

or

$\bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_2$	ϕ	Φ_1	Φ_2	q_L	u_R	d_R	ℓ_L	e_R	ν_R
$U(1)_1$	$+1/2$	1	0	α_1	α_1+1	α_1	$-1/4$	$-1/4$	$-1/4$
$U(1)_2$	$-1/2$	0	1	α_2	α_2	α_2-1	$-3/4$	$-7/4$	$+1/4$

(26)

To ensure that $U(1)_Y \subset U(1)_1 \otimes U(1)_2$, we must set $\alpha_1 + \alpha_2 = 1/3$, and the remaining one-parameter freedom originates in the $\mathcal{B}$ invariance of the Yukawa couplings. Yet, clearly, no linear combination of the $U(1)_1$ and $U(1)_2$ charges can make the Higgs doublets and the quarks neutral. So $U(1)_\mathcal{L} \subset U(1)_1 \otimes U(1)_2$ does not correspond to the usual lepton number.

The symmetry breaking proceeds as in the DFSZ model, since the scalar potential stays the same. This fixes the PQ charge of the scalar fields to the same values, Eq. (19). The fermions then have the same charge as in Eq. (7), but with β fixed so that $\text{PQ}(\nu_R) = -\text{PQ}(\phi)/2$:

$$\bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_1 \Rightarrow \beta = -\frac{1}{4}\left(5x + \frac{1}{x}\right)$$

$$\Rightarrow \text{PQ}(\ell_L, e_R, \nu_R) = -\frac{1}{4}\left(5x + \frac{1}{x}, 5x - \frac{3}{x}, x + \frac{1}{x}\right), \tag{27a}$$

$$\bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_2 \Rightarrow \beta = -\frac{1}{4}\left(x - \frac{3}{x}\right)$$

$$\Rightarrow \text{PQ}(\ell_L, e_R, \nu_R) = -\frac{1}{4}\left(x - \frac{3}{x}, x - \frac{7}{x}, x + \frac{1}{x}\right), \tag{27b}$$

together with $\mathrm{PQ}(q_L, u_R, d_R) = (\alpha, \alpha + x, \alpha + 1/x)$, as before. In some sense, $U(1)_{\mathcal{L}}$ never occurs at low energy. Instead, it is embedded into $U(1)_{\mathrm{PQ}}$ via the specific value of β imposed by the $\bar{\nu}_R^C \mathbf{Y}_R \nu_R \phi$ coupling. So, in this model, the axion and Majoron are really one and the same particle. Further, the "axion = Majoron" is automatically coupled to quarks and to $G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}$ and, hence, can solve the strong CP puzzle via the same mechanism as in the DFSZ model.

Once ϕ acquires its vacuum expectation value, ν_R has a Majorana mass term, so it may seem this contradicts the fact that $\mathrm{PQ}(\nu_R) \neq 0$. But actually, plugging Eq. (15) into $\mathcal{L}_{\nu_R}$ of Eq. (24) and using Eq. (17), we find

$$\begin{aligned} \bar{\nu}_R^C \mathbf{Y}_R \nu_R \phi &\to \bar{\nu}_R^C \mathbf{M}_R \nu_R \exp(i\eta_s/v_s) \\ &\approx \bar{\nu}_R^C \mathbf{M}_R \nu_R \exp(i\mathrm{PQ}(\phi)a^0/v_s), \end{aligned} \tag{28}$$

where $\mathbf{M}_R = v_s \mathbf{Y}_R/\sqrt{2}$. Thus, we now see why ν_R must have a nonzero PQ charge. Because $\mathbf{M}_R$ originates from the ϕ field, it is always accompanied by the axion field. Then, under a $U(1)_{\mathrm{PQ}}$ transformation, $a \to a + v_s\theta$ must be compensated by the phase shift $\nu_R \to \nu_R \exp(i\mathrm{PQ}(\nu_R)\theta)$. Also, thanks to this, the fermion field reparametrization $\psi \to \psi \exp(i\mathrm{PQ}(\psi)a/v_s)$ is still able to entirely move the axion field out of the fermion mass terms.

One point must be stressed, though. The axion couples to SM fermions via its suppressed components $\eta_{1,2}$, but it couples directly to ν_R via its dominant η_s component. As a result, the couplings to SM fermions are $\mathcal{O}(v/v_s)$, but that to ν_R is $\mathcal{O}(1)$, as evident in Eq. (28). Yet, since v_s is assumed to be well above the electroweak scale, ν_R should be integrated out before performing the fermion reparametrization. In that case, we find (assuming $\mathbf{Y}_R = \mathbf{Y}_R^\dagger$)

$$\begin{aligned} \mathcal{L}^{\mathrm{eff}}_{\mathrm{seesaw}} = -\frac{1}{2}(\bar{\ell}_L^C \Phi_i^T)\mathbf{Y}_\nu^T \frac{1}{\mathbf{M}_R}\mathbf{Y}_\nu(\ell_L \Phi_i) \exp(i(2\mathrm{PQ}(\Phi_i) \\ - 2\mathrm{PQ}(\nu_R))a^0/v_s) + \mathrm{H.c.} \end{aligned} \tag{29}$$

Then, performing $\ell_L \to \ell_L \exp(i\mathrm{PQ}(\ell_L)a/v_s)$ moves the axion field entirely into the same effective derivative and anomalous interactions as in Eq. (11), but with β now fixed as in Eq. (27a) or (27b). Again, the phenomenology is unaffected, since β cancels out of physical observables. Thus, the $\mathcal{O}(1)$ axion coupling to ν_R has no consequences at low energy.

C. PQ and DFSZ with a type-II seesaw mechanism

In the previous sections, we have seen two ways to incorporate neutrino masses in the DFSZ model. For the first, one simply adds right-handed neutrinos ν_R with a Majorana mass term. The PQ symmetry stays the same, though a specific value of β is required [Eq. (23)] to ensure $\mathrm{PQ}(\nu_R) = 0$. Also, this makes sure the explicit breaking of the lepton number does not spill over to the PQ symmetry.

A second way to proceed, in the νDFSZ model, is again to add right-handed neutrinos but ask the heavy singlet field to induce their Majorana mass term. In that case, $\mathrm{PQ}(\nu_R) \neq 0$, but the lepton number symmetry ceased to exist. Actually, it is replaced by the PQ symmetry.

A third realization is provided by the type-II seesaw mechanism [17,18]. Instead of right-handed neutrinos, let us add to the THDM model three complex Higgs fields $\mathbf{\Delta}$ transforming as a $SU(2)_L$ triplet with hypercharge 2. For the couplings to fermions, in addition to the THDM Yukawas, we add to $\mathcal{L}_{\mathrm{Yukawa}}$ of Eq. (2) the term

$$\begin{aligned} &\mathcal{L}_{\nu II} = \bar{\ell}_L^C Y_\Delta \mathbf{\Delta} \ell_L, \qquad \ell_L^C = i\sigma^2 \begin{pmatrix} \nu_L^C \\ \ell_L^C \end{pmatrix}, \\ &\mathbf{\Delta} = \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta^+ & \sqrt{2}\Delta^{++} \\ \Delta^0 & -\Delta^+ \end{pmatrix}, \end{aligned} \tag{30}$$

where, as indicated, C acts in both Lorentz and $SU(2)_L$ spaces. For the scalar potential, we introduce one new coupling to entangle the $U(1)_1 \otimes U(1)_2$ charges of $\mathbf{\Delta}$ with those of the doublets, so that $V_{\nu 2\mathrm{THDM}} = V_{\mathrm{THDM}} + V_\Delta + V_{\Delta\mathrm{THDM}} + V_{\Delta\mathrm{PQ}}$ with

$$V_\Delta = \mu_\Delta^2 \langle \mathbf{\Delta}^\dagger \mathbf{\Delta} \rangle + \lambda_{\Delta 1} \langle \mathbf{\Delta}^\dagger \mathbf{\Delta} \rangle^2 + \lambda_{\Delta 2} \langle (\mathbf{\Delta}^\dagger \mathbf{\Delta})^2 \rangle, \tag{31a}$$

$$\begin{aligned} V_{\Delta\mathrm{THDM}} = a_{\Delta 1} \langle \mathbf{\Delta}^\dagger \mathbf{\Delta} \rangle \Phi_1^\dagger \Phi_1 + a_{\Delta 2} \langle \mathbf{\Delta}^\dagger \mathbf{\Delta} \rangle \Phi_2^\dagger \Phi_2 \\ + a_{\Delta 3} \Phi_1^\dagger \mathbf{\Delta} \mathbf{\Delta}^\dagger \Phi_1 + a_{\Delta 4} \Phi_2^\dagger \mathbf{\Delta} \mathbf{\Delta}^\dagger \Phi_2, \end{aligned} \tag{31b}$$

$$V_{\Delta\mathrm{PQ}} = -\mu_\Delta \lambda_{\Delta 12} \tilde{\Phi}_1^T \Delta^\dagger \Phi_2 + \mathrm{H.c.}, \tag{31c}$$

with $\tilde{\Phi}_i = i\sigma^2 \Phi_i$. A factor μ_Δ is introduced to make $\lambda_{\Delta 12}$ dimensionless. Even if μ_Δ^2 is large and positive, the $\lambda_{\Delta 12}$ coupling generates a tadpole for $\mathrm{Re}\Delta^0$ and this field has to be shifted. In effect, this induces a VEV for the Δ field, $v_\Delta \sim \lambda_{\Delta 12} v_1 v_2/\mu_\Delta$. To preserve the electroweak custodial symmetry, $\mu_\Delta \gg v_{1,2}$ so that $v_\Delta \ll v_{1,2}$. Yet, this shift generates a small Majorana mass term for the neutrinos, $m_\nu = v_\Delta Y_\Delta$. This is the characteristic linear suppression of the neutrino masses of the type-II seesaw mechanism.

We have not identified the axion field yet. To that end, we adopt again the polar parametrization Eq. (4) together with

$$\mathbf{\Delta} = \frac{1}{\sqrt{2}} \exp(i\eta_\Delta/v_\Delta) \begin{pmatrix} \Delta^+ & \sqrt{2}\Delta^{++} \\ v_\Delta + \mathrm{Re}\Delta^0 & -\Delta^+ \end{pmatrix}. \tag{32}$$

Restricted to the pseudoscalar states, only the $\lambda_{\Delta 12}$ coupling survives and

$$\begin{aligned} &V_{\nu 2\mathrm{THDM}}(\eta_{1,2,\Delta}) \\ &\quad = -\frac{1}{\sqrt{2}} \mu_\Delta \lambda_{\Delta 12} v_1 v_2 v_\Delta \cos\left(\frac{\eta_1}{v_1} + \frac{\eta_2}{v_2} - \frac{\eta_\Delta}{v_\Delta}\right). \end{aligned} \tag{33}$$

The mass eigenstates are easily found. First, the G^0 state has to be aligned with

$$G^0 \sim v_1\eta_1 + v_2\eta_2 + 2v_\Delta\eta_\Delta, \tag{34}$$

since this ensures it can be removed by a $U(1)_Y$ transformation and $Y(\Delta) = 2Y(\Phi_{1,2}) = 2$. Second, the single massive state, denoted π^0, is aligned with the combination of fields occurring in the argument of the cosine in Eq. (33). The axion is then the only state orthogonal to both G^0 and π^0, and a simple cross product permits one to construct the mixing matrix:

$$\begin{pmatrix} G^0 \\ a^0 \\ \pi^0 \end{pmatrix} = \begin{pmatrix} s_\beta\omega & c_\beta\omega & 2\delta_\Delta\omega \\ c_\beta(1+2\delta_\Delta^2/c_\beta^2)\omega\omega' & -s_\beta(1+2\delta_\Delta^2/s_\beta^2)\omega\omega' & 2\delta_\Delta\omega\omega'/t_{2\beta} \\ \delta_\Delta\omega'/s_\beta & \delta_\Delta\omega'/c_\beta & -\omega' \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_\Delta \end{pmatrix}, \tag{35}$$

with $\delta_\Delta = v_\Delta/v$, $\omega^{-2} = 1 + 4\delta_\Delta^2$, and $\omega'^{-2} = 1 + 4\delta_\Delta^2/s_{2\beta}^2$. The PQ charges of the three fields can be read off the second line of this matrix, and upon adopting a convenient normalization:

$$\mathrm{PQ}(\Phi_1, \Phi_2, \Delta) = \left(x + 2x_\Delta, -\frac{1}{x} - 2x_\Delta, x - \frac{1}{x}\right), \qquad x_\Delta \equiv \delta_\Delta^2\left(x + \frac{1}{x}\right). \tag{36}$$

Note that $\mathrm{PQ}(\Phi_1) + \mathrm{PQ}(\Phi_2) - \mathrm{PQ}(\Delta) = 0$, as it should, but those can be expressed as a simple function of $x = 1/\tan\beta$ only to leading order in δ_Δ.

Once the PQ charge of the scalars is set, that of the fermions can be derived, and we find

$$\mathrm{PQ}(q_L, u_R, d_R, \ell_L, e_R) = \left(\alpha, \alpha + x + 2x_\Delta, \alpha + \frac{1}{x} + 2x_\Delta, -\frac{x}{2} + \frac{1}{2x}, -\frac{x}{2} + \frac{3}{2x} + 2x_\Delta\right). \tag{37}$$

Apart from the small shifts induced by x_Δ, this corresponds to the PQ current of the THDM with $\beta = -\mathrm{PQ}(\Delta)/2$.

Since the $\eta_{1,2}$ components of a^0 are of $\mathcal{O}(1)$, the PQ scale stays at v, and the axion ends up too strongly coupled to SM fermions. To cure this, the same strategy as in the DFSZ model can be used; that is, an additional complex singlet field is introduced [19,20]. To study this situation, let us take the scalar potential

$$\begin{aligned} V_{\nu 2\mathrm{DFSZ}} &= V_{\mathrm{THDM}} + V_\Delta + V_{\Delta\mathrm{THDM}} + V_\phi + V_{\phi\mathrm{THDM}} + b_{\phi\Delta}\phi^\dagger\phi\langle\mathbf{\Delta}^\dagger\mathbf{\Delta}\rangle \\ &\quad + [-\lambda_{\nu 1}\phi^2\Phi_1^\dagger\Phi_2 - \lambda_{\nu 2}\phi\Phi_1^T\Delta^\dagger\Phi_2 - \mu_\Delta\lambda_{\nu 3}\Phi_1^T\Delta^\dagger\Phi_2 + \mathrm{H.c.}]. \end{aligned} \tag{38}$$

The coupling $b_{\phi\Delta}$ gives a large $\mathcal{O}(v_s)$ mass to the triplet states, while those in $V_{\Delta\mathrm{THDM}}$ generate small $\mathcal{O}(v)$ splittings among the three Δ states. Of particular interest are the $\lambda_{\nu i}$ couplings, since they entangle the scalar states. First, the $\lambda_{\nu 2}$ and $\lambda_{\nu 3}$ couplings create Δ tadpoles that need to be removed by shifting the Δ field

$$v_\Delta = \frac{1}{2}\lambda_{\nu 2}\frac{v_1 v_2 v_s}{\mu_\Delta^2 + b_{\phi\Delta}v_s^2} + \frac{1}{2}\lambda_{\nu 3}\frac{\mu_\Delta v_1 v_2}{\mu_\Delta^2 + b_{\phi\Delta}v_s^2}. \tag{39}$$

Note that, if $\mu_\Delta \ll v_s$, the bulk of the Δ mass comes from the singlet, and the μ_Δ^2 can be neglected in these expressions.

Plugging in the polar representations of the scalar fields, the scalar potential for the pseudoscalar states is

$$\begin{aligned} V_{\nu 2\mathrm{DFSZ}}(\eta_{1,2,\Delta,s}) &= -\frac{1}{2}\lambda_{\nu 1}v_1 v_2 v_s^2 \cos\left(\frac{\eta_1}{v_1} - \frac{\eta_2}{v_2} - \frac{2\eta_s}{v_s}\right) \\ &\quad - \frac{1}{2}\lambda_{\nu 2}v_1 v_2 v_s v_\Delta \cos\left(\frac{\eta_1}{v_1} + \frac{\eta_2}{v_2} + \frac{\eta_s}{v_s} - \frac{\eta_\Delta}{v_\Delta}\right) \\ &\quad - \frac{1}{\sqrt{2}}\mu_\Delta\lambda_{\nu 3}v_1 v_2 v_\Delta \cos\left(\frac{\eta_1}{v_1} + \frac{\eta_2}{v_2} - \frac{\eta_\Delta}{v_\Delta}\right). \end{aligned} \tag{40}$$

If the three $\lambda_{\nu i}$ couplings are present, there are three massive pseudoscalar states corresponding to the linear combinations appearing in the cosine functions. Those are linearly independent. Together with G^0 which stays, of course, massless, there is no room for the axion. This was evident from the start, since with all three $\lambda_{\nu i}$ couplings, no $U(1)_1 \otimes U(1)_2$ symmetry can be defined.

Removing any one of these couplings, a second Goldstone boson appears and can be identified with the axion. Given that the G^0 state stays the same as without the ϕ [see Eq. (34)], we directly find the a^0 state by its orthogonality with G^0 and the massive states in the cosine functions:

$$\lambda_{\nu 1} = 0\colon\ a^0 \sim (v_2^2 + 2v_\Delta^2)v_1\eta_1 - (v_1^2 + 2v_\Delta^2)v_2\eta_2 + (v_2^2 - v_1^2)v_\Delta\eta_\Delta, \tag{41}$$

$$\lambda_{\nu 2} = 0\colon\ a^0 \sim 2(v_2^2 + 2v_\Delta^2)v_1\eta_1 - 2(v_1^2 + 2v_\Delta^2)v_2\eta_2 + 2(v_2^2 - v_1^2)v_\Delta\eta_\Delta + (v_1^2 + v_2^2 + 4v_\Delta^2)v_s\eta_s, \tag{42}$$

$$\lambda_{\nu 3} = 0\colon\ a^0 \sim 2(v_2^2 + v_\Delta^2)v_1\eta_1 - 2(v_1^2 + 3v_\Delta^2)v_2\eta_2 + (3v_2^2 - v_1^2)v_\Delta\eta_\Delta + (v_1^2 + v_2^2 + 4v_\Delta^2)v_s\eta_s. \tag{43}$$

The first scenario collapses to that without ϕ and is ruled out, since the axion scale remains at v. The other two are viable, with the axion scale set by v_s. The PQ assignments can be read off the coefficients of the $v_i\eta_i$ terms above, and upon adopting a convenient normalization,

$$\lambda_{\nu 2} = 0\colon\ \mathrm{PQ}(\Phi_1, \Phi_2, \Delta, \phi) = \left(x + 2x_\Delta, -\frac{1}{x} - 2x_\Delta, x - \frac{1}{x}, \frac{x}{2} + \frac{1}{2x} + 2x_\Delta\right), \tag{44}$$

$$\lambda_{\nu 3} = 0\colon\ \mathrm{PQ}(\Phi_1, \Phi_2, \Delta, \phi) = \left(x + x_\Delta, -\frac{1}{x} - 3x_\Delta, \frac{3}{2}x - \frac{1}{2x}, \frac{x}{2} + \frac{1}{2x} + 2x_\Delta\right), \tag{45}$$

with x_Δ given in Eq. (36). The $\lambda_{\nu 2} = 0$ scenario is the simple DFSZ generalization of the THDM with a type-II seesaw, and the PQ charges stay the same; see Eq. (36). Consequently, the fermions have the charges in Eq. (37).

For the $\lambda_{\nu 3} = 0$ scenario, corresponding to that discussed in Refs. [19,20], the PQ charges of the leptons are shifted, since that of Δ is different:

$$\mathrm{PQ}(q_L, u_R, d_R, \ell_L, e_R) = \left(\alpha, \alpha + x + x_\Delta, \alpha + \frac{1}{x} + 3x_\Delta, -\frac{3}{4}x + \frac{1}{4x}, -\frac{3}{4}x + \frac{5}{4x} + 3x_\Delta\right). \tag{46}$$

Again, apart from the small shifts induced by v_Δ, the PQ charges in these two scenarios correspond to that of the THDM in Eq. (7) with specific values of β:

$$\lambda_{\nu 2} = 0 \Rightarrow \beta = -\frac{3}{4}x + \frac{1}{4x}, \tag{47}$$

$$\lambda_{\nu 3} = 0 \Rightarrow \beta = -\frac{x}{2} + \frac{1}{2x}. \tag{48}$$

The electroweak terms in the divergence of the PQ current are, thus, different in both scenarios. Yet, phenomenologically, the axion couplings are independent of β, and, apart from negligible corrections brought in by v_Δ, these scenarios cannot be distinguished at low energy.

IV. AXIONS AND BARYON NUMBER VIOLATION

Up to now, we have seen that the violation of the lepton number, through insertion of Majorana neutrino masses, fixes one of the two ambiguities in the PQ charges of the SM fermions, that parametrized by β in Eq. (7). We will now concentrate on the remaining ambiguity, α, which originates in the conserved baryon number current. In the first subsection, we will discuss two frameworks in which α is automatically fixed, for dynamical reasons. Then, in the second subsection, the impact of explicit $\mathcal{B}$-violating operators will be discussed. Finally, in the last subsection, the situation in which too many $\mathcal{B}$- and/or $\mathcal{L}$-violating effects are introduced, preventing the PQ symmetry from arising, will be described.

A. Dynamical $\mathcal{B}$ violation

Even without explicit $\mathcal{B}$ violation, $U(1)_\mathcal{B}$ is not a true symmetry at the quantum level, because electroweak instantons are known to induce $\mathcal{B}+\mathcal{L}$ transitions [29,30]. This takes the form of an effective interaction involving antisymmetric flavor contractions of three lepton weak doublets and nine quark weak doublets:

$$\mathcal{L}_{\mathrm{inst}}^{\mathrm{eff}} = c_{\mathrm{inst}}\ell_L^3 q_L^9. \tag{49}$$

At zero temperature, c_{inst} is tuned by $\exp(-4\pi/g^2)$, and these effects are totally negligible. Yet, even so, these interactions are present, and, following the same philosophy as for β, they prevent the emergence of the parametric freedom to choose α and β separately. Specifically, the PQ symmetry necessarily settles with

$$\Delta\mathcal{B} = \Delta\mathcal{L} = 3 \Rightarrow 3\alpha + \beta = 0. \tag{50}$$

Setting this combination to zero also kills off the $W^i_{\mu\nu}\tilde{W}^{i,\mu\nu}$ term in the PQ current [see Eq. (11)]. In some sense, this requirement removes a $\mathcal{B} + \mathcal{L}$ component in $U(1)_{\rm PQ}$. Since $\mathcal{B} - \mathcal{L}$ is anomaly-free, there is then nothing remaining to generate the $W^i_{\mu\nu}\tilde{W}^{i,\mu\nu}$ term. Yet, there remain couplings of the axion to left-handed fields in the effective nonlinear Lagrangian, since neither α nor β are vanishing when Majorana neutrino masses are present.

It should be mentioned also that, once the electroweak instanton interaction fixes $3\alpha + \beta = 0$, the axion decouples from electroweak anomalous effects, including also the sphaleron interactions. Mechanisms to generate the baryon asymmetry from the rotation of an axion field (see, e.g., Ref. [31]) which relies on those interactions cannot be active in the present simple axion models. Some additional constraints must force the PQ symmetry to be realized differently. At the very least, it must fix α to some value not compatible with the β value imposed by the neutrino sector, in order to induce $3\alpha + \beta \neq 0$.

So, generically, electroweak instantons prevent the emergence of one of the ambiguities in the fermionic PQ charges. However, this supposes the ambiguity is not removed first at a yet higher scale. A generic class of models where this occurs are the grand unified theory (GUT) scenarios. Indeed, in that case, gauge interactions can break $\mathcal{B}$ and $\mathcal{L}$. For example, in $SU(5)$, $\mathcal{B} - \mathcal{L}$ is conserved but not $\mathcal{B} + \mathcal{L}$, and one automatically has

$$\text{GUT: } 3\alpha + \beta = -x - \frac{1}{x}. \tag{51}$$

Indeed, this is the unique value for which all three anomalous terms in Eq. (12) coincide when taking into account the $SU(5)$ normalization of the hypercharge, $\mathcal{N}_C = \mathcal{N}_L = 5/3\mathcal{N}_Y$. It is quite remarkable that this value is not compatible with the instanton value in Eq. (50). Further investigation of the fermion charge ambiguities in a GUT context are deferred to future work [32].

B. Effective $\mathcal{B}$ violation

The basis of effective, gauge-invariant operators violating $\mathcal{B}$ and/or $\mathcal{L}$ is well known. It starts at the dimension-five level with the $\Delta\mathcal{L} = 2$ Majorana mass operator of Eq. (21). Then, at the dimension-six level, all the operators are $\Delta\mathcal{B} = \Delta\mathcal{L} = 1$ (see Ref. [33]):

$$\mathcal{L}_{\rm eff}^{\dim 6} = \frac{1}{\Lambda^2}(\ell_L q_L^3 + e_R u_R^2 d_R + e_R u_R q_L^2 + \ell_L q_L d_R u_R) + \text{H.c.} \tag{52}$$

Adequate contractions of the Lorentz and $SU(2)_L$ spinors are understood, as well as Wilson coefficients and flavor indices. Beyond that level, other patterns of $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ can occur at the dimension-seven level, thanks to additional Higgs insertions. With only SM fermions, the next series of operators arise at the dimension-nine level:

$$\mathcal{L}_{\rm eff}^{\dim 9} = \frac{1}{\Lambda^5}(e_R \ell_L^2 u_R^3 + \ell_L^3 q_L u_R^2 + d_R^4 u_R^2 + d_R^3 u_R q_L^2 + d_R^2 q_L^4) + \text{H.c.} \tag{53}$$

The first two induce $\Delta\mathcal{B} = 1, \Delta\mathcal{L} = 3$ transitions, and the last three $\Delta\mathcal{B} = 2, \Delta\mathcal{L} = 0$ ones. These operators are peculiar, because, provided the flavor indices are antisymmetrically contracted, they break only $U(1)_\mathcal{L}$ and $U(1)_\mathcal{B}$ and not the flavor $SU(3)$'s [34].

Given the charges in Eq. (7), none of these operators is invariant under $U(1)_{\rm PQ}$ but carry instead

$$\text{PQ}(\ell_L q_L^3) = 3\alpha + \beta, \tag{54a}$$

$$\text{PQ}(e_R u_R^2 d_R) = 3\alpha + \beta + 2x + \frac{2}{x}, \tag{54b}$$

$$\text{PQ}(e_R u_R q_L^2) = 3\alpha + \beta + x + \frac{1}{x}, \tag{54c}$$

$$\text{PQ}(\ell_L q_L d_R u_R) = 3\alpha + \beta + x + \frac{1}{x} \tag{54d}$$

and

$$\text{PQ}(e_R \ell_L^2 u_R^3) = 3\alpha + 3\beta + 3x + \frac{1}{x}, \tag{55a}$$

$$\text{PQ}(\ell_L^3 q_L u_R^2) = 3\alpha + 3\beta + 2x, \tag{55b}$$

$$\text{PQ}(d_R^4 u_R^2) = 6\alpha + 2x + \frac{4}{x}, \tag{55c}$$

$$\text{PQ}(d_R^3 u_R q_L^2) = 6\alpha + x + \frac{3}{x}, \tag{55d}$$

$$\text{PQ}(d_R^2 q_L^4) = 6\alpha + \frac{2}{x}. \tag{55e}$$

The way in which α and β enter reflects the $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ properties of the corresponding operators, with $n \times \alpha \Leftrightarrow \Delta\mathcal{B} = n/3$ and $m \times \beta \Leftrightarrow \Delta\mathcal{L} = m$. Yet, remarkably, the PQ charge of the operators are not aligned with their $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ contents. For example, all the dimension-six operators are $\Delta\mathcal{B} = \Delta\mathcal{L} = 1$, but they do have different PQ charge. Among the dimension-six operators, it is also interesting to remark that only the first is compatible with electroweak instantons [Eq. (50)], while only the last two are compatible with GUTs [Eq. (51)]. This could have been expected, since those are the operators arising from $SU(5)$ gauge boson exchanges.

Another noticeable feature is that the misalignment between two operators carrying the same $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ always appears as a multiple of $x + 1/x$. This means that, even if the PQ symmetry does not exist when all types of operators are simultaneously present, large classes of operators can nevertheless be allowed, but at the cost of a further scaling in their dimensions. Consider, for instance, the DFSZ model where $\mathrm{PQ}(\Phi_2^\dagger\Phi_1) = \mathrm{PQ}(\phi^\dagger) = x + 1/x$. Misalignments can always be compensated by scalar singlet insertions. For instance, if one assumes Eq. (50) holds, then the $\Delta\mathcal{B} = \Delta\mathcal{L} = 1$ operators must be

$$\mathcal{L}_{\mathrm{eff}}^{\Delta(\mathcal{B}-\mathcal{L})=0} = \frac{\ell_L q_L^3}{\Lambda^2} + \frac{e_R u_R q_L^2}{\Lambda^3}\phi + \frac{\ell_L q_L d_R u_R}{\Lambda^3}\phi + \frac{e_R u_R^2 d_R}{\Lambda^4}\phi^2 + \mathrm{H.c.} \tag{56}$$

The PQ charge of all the operators is now aligned in the direction of $\ell_L q_L^3$, with $\mathrm{PQ}(\ell_L q_L^3) = 0$ when $3\alpha + \beta = 0$. Operators involving insertions of the Higgs doublet combination $\Phi_1^\dagger\Phi_2$ need not be included, since they are comparatively very suppressed, both dimensionally and because $v_{1,2} \ll v_s$. Phenomenologically, given the bounds on Λ from proton decay and provided $v_s < \Lambda$, only the leading operator is expected to play any role. The same holds for other series of operators, though there is then no clear reason to select one operator against another as leading. For example, in the $\Delta\mathcal{B} = 2$ class, assuming $d_R^4 u_R^2$ is leading, the effective operators must be

$$\mathcal{L}_{\mathrm{eff}}^{\Delta\mathcal{B}=2} = \frac{d_R^4 u_R^2}{\Lambda^5} + \frac{d_R^3 u_R q_L^2}{\Lambda^6}\phi + \frac{d_R^2 q_L^4}{\Lambda^7}\phi^2 + \mathrm{H.c.} \tag{57}$$

They are all neutral provided $3\alpha = -x - 2/x$, and, thus, there remains enough room for the PQ symmetry to exist.

C. Vacuum realignments

In the previous section, we have seen that the PQ symmetry can accommodate for limited $\mathcal{B}$ and/or $\mathcal{L}$ breaking. Our goal here is to work out the consequences when too much $\mathcal{B}$ and/or $\mathcal{L}$ violation is introduced. Indeed, if there are too many misaligned $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ operators, the $U(1)_1 \otimes U(1)_2$ symmetry cannot be exact and the axion cannot be massless.

To analyze this, we first remark that these breaking effects have to be tiny given the experimental constraints on $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ transitions. Thus, the $U(1)_1 \otimes U(1)_2$ symmetry is at most only very slightly broken, and the leading dynamics remain that of Goldstone bosons. The pseudoscalar degrees of freedom can still be parametrized using the polar representation. Of course, the axion will no longer be truly massless; it becomes a pseudo-Goldstone boson. Naively, if this mass is too large compared to the QCD-induced mass, then the axion fails to solve the strong CP puzzle. This failure can also be viewed in terms of the vacuum of the theory. In the presence of the $\Delta\mathcal{B}$- and $\Delta\mathcal{L}$-breaking terms, the shift symmetry is no longer active. All the vacua are no longer equivalent, and one direction is preferred. At the low scale, QCD effects also require a realignment of the vacuum, and the CP puzzle can be solved only when the QCD requirement is stronger than that coming from the $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ effects.

In the following, the axion mass arising from various combinations of $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ operators is analyzed semiquantitatively, from the point of view of the effective scalar potential before the electroweak SSB. Indeed, at tree level, the $U(1)_1 \otimes U(1)_2$ symmetry is still active in the scalar sector, since it is broken explicitly in the fermion sector only. Thus, at tree level, the axion remains as a massless Goldstone boson. To go beyond that, we must consider the effective scalar potential and, in particular, look for the leading symmetry-breaking terms induced by fermion loops. Clearly, such loops must include the misaligned $\Delta\mathcal{B}$ and/or $\Delta\mathcal{L}$ interactions simultaneously in such a way that the process is $\Delta\mathcal{B} = \Delta\mathcal{L} = 0$ overall, since scalar fields have $\mathcal{B} = \mathcal{L} = 0$. These loops then correspond to explicit $U(1)_1 \otimes U(1)_2$ symmetry-breaking terms in the potential, from which the corrected axion mass can be calculated. For completeness, in the Appendix, we perform the same analysis in the spirit of the Dashen theorem [35], in which case the alignment of the vacuum imposed by the $\Delta\mathcal{B}$ and/or $\Delta\mathcal{L}$ operators is most transparent.

1. Scenario I: Weinberg dimension-six operators and the axion mass

As a first situation, we consider the case where several operators inducing the same $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ transitions are introduced simultaneously. As discussed before, such a set of operators can be organized into classes according to their PQ charges, with the PQ charges of two classes differing by some multiple of $x + 1/x$. Because this is precisely the charge of the $\Phi_2^\dagger\Phi_1$ combination, the combined presence of two operators whose PQ charges differ by $n \times (x + 1/x)$ generates the correction

$$V_{\mathcal{B},\mathcal{L}}^{\mathrm{eff}} = -\frac{2^{2n-3}\lambda_n}{\Lambda_{\mathcal{B},\mathcal{L}}^{2n-4}}(\Phi_2^\dagger\Phi_1)^n + \mathrm{H.c.} \tag{58}$$

in the effective scalar potential, where $\Lambda_{\mathcal{B},\mathcal{L}}$ is the scale of the $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ physics, λ_n is a complicated combination of the Wilson coefficients, Yukawa couplings, and loop factors, and the factor 2^{2n-3} is introduced for convenience. From there, the mass of the axion can be estimated as $m_{a^0}^2 \sim \mathcal{O}(v^{2n-2}/\Lambda_{\mathcal{B},\mathcal{L}}^{2n-4})$ in the PQ model.

This correction to the scalar potential is also valid for the DFSZ model, since the singlet does not couple directly to fermions. The only way in which the breaking of $U(1)_1 \otimes U(1)_2$ can be communicated to ϕ is via the mixing term

$\phi^2\Phi_1^\dagger\Phi_2$. To incorporate this effect, the full mass matrix for the pseudoscalar states has to be diagonalized. To that end, consider the effective potential restricted to pseudoscalar states. It now contains a second cosine function:

$$V_{\rm DFSZ}(\eta_{1,2,s}) = -\frac{1}{2}\lambda_{12}v_1v_2v_s^2\cos\left(\frac{\eta_1}{v_1}-\frac{\eta_2}{v_2}-\frac{2\eta_s}{v_s}\right) - 2^{n-1}\frac{\lambda_n(v_1v_2)^n}{\Lambda_{\mathcal{B},\mathcal{L}}^{2n-4}}\cos\left(n\left(\frac{\eta_1}{v_1}-\frac{\eta_2}{v_2}\right)\right). \tag{59}$$

Diagonalizing the mass matrix, one pseudoscalar state remains at the v_s scale, while the other has a mass

$$m_{a^0}^2 = \lambda_n n^2\frac{v^2}{v_s^2}\frac{v^{2n-2}}{\Lambda_{\mathcal{B},\mathcal{L}}^{2n-4}}\sin^n(2\beta). \tag{60}$$

Thus, in the DFSZ model, the lightest pseudoscalar mass is suppressed by a v/v_s factor compared to the PQ model. Still, this factor does not really help to make a scenario viable, because the QCD contribution to the axion mass also scales as $1/v_s$. For instance, with $m_{a^0}|_{\rm QCD}\sim m_\pi^2/v_s$, n should be strictly greater than two if λ_n is $\mathcal{O}(1)$ and $\Lambda_{\mathcal{B},\mathcal{L}}\approx 10^{16}$ GeV.

To illustrate this discussion, let us take the Weinberg operators of Eq. (54). If both the $Q_1\equiv \ell_L q_L^3$ and $Q_2\equiv e_R u_R q_L^2$ operators are simultaneously present, given that their mismatch is simply $x+1/x$, the induced axion mass corresponds to Eq. (60) with $n=1$ and is, thus, way too large at $m_{a^0}\sim\mathcal{O}(\Lambda_{\mathcal{B},\mathcal{L}}\times v/v_s)$. This can be understood qualitatively from the process depicted on the left in Fig. 1, corresponding schematically to the symmetry-breaking terms

$$V^{\rm eff}_{Q_1\times Q_2} = -m_{\rm eff}^2\Phi_2^\dagger\Phi_1 + \text{H.c.},$$
$$m_{\rm eff}^2\sim\frac{\Lambda_{\rm reg}^6}{\Lambda_{\mathcal{B},\mathcal{L}}^4}\times c_1^{IJKL}c_2^{ABKL}(\mathbf{Y}_e)^{AI}(\mathbf{Y}_u)^{BJ}, \tag{61}$$

where c_i are the Wilson coefficients of Q_i and summation over the flavor indices is understood. The scale $\Lambda_{\rm reg}$ denotes

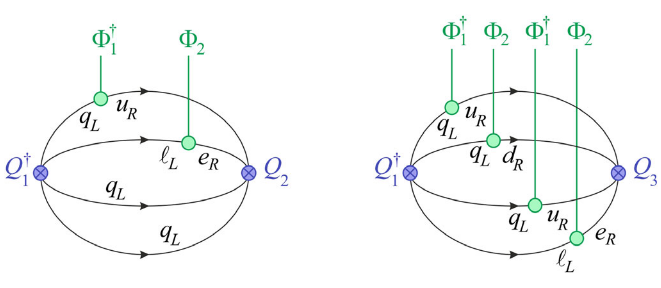

FIG. 1. Fermion loops involving the Weinberg operators $Q_1\equiv\ell_L q_L^3$, $Q_2\equiv e_R u_R q_L^2$, and $Q_3\equiv e_R u_R^2 d_R$, and inducing symmetry-breaking effective potential terms.

that at which the loop diagrams are regulated. In all UV scenarios we could think of, this scale corresponds to that of the operators, $\Lambda_{\rm reg}\approx\Lambda_{\mathcal{B},\mathcal{L}}$. Indeed, if some new dynamics is introduced that break the $U(1)_1\otimes U(1)_2$ symmetry, there is no reason not to expect the same dynamics to induce corresponding breaking terms in the scalar sector.

If, instead of Q_2, one takes Q_1 together with $Q_3\equiv e_R u_R^2 d_R$, the mismatch in PQ charges is $2\times(x+1/x)$, and the mass is $m_{a^0}\sim\mathcal{O}(v\times v/v_s)$ from Eq. (60) with $n=2$. Again, this picture can be understood from the diagram on the right in Fig. 1, with the corresponding dimension-four breaking term in the effective potential:

$$V^{\rm eff}_{Q_1\times Q_3} = \frac{\lambda_{\rm eff}}{2}(\Phi_2^\dagger\Phi_1)^2 + \text{H.c.},$$
$$\lambda_{\rm eff}\sim\frac{\Lambda_{\rm reg}^4}{\Lambda_{\mathcal{B},\mathcal{L}}^4}\times c_1^{IJKL}c_3^{ABCD}(\mathbf{Y}_e)^{AI}(\mathbf{Y}_u)^{BJ}(\mathbf{Y}_u)^{CK}(\mathbf{Y}_d)^{DL}. \tag{62}$$

Thus, when $\Lambda_{\rm reg}\approx\Lambda_{\mathcal{B},\mathcal{L}}$, the axion mass becomes insensitive to the very high energy scale. Yet, it is still tuned by the electroweak scale and is, thus, far too large to solve the strong *CP* puzzle.

2. Scenario II: A viable scenario with many $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ operators

The axion mass is too large for any combination of Weinberg operators carrying different PQ charges. To get a viable scenario, we have to allow for operators inducing different $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ patterns, so that the effective potential term is forced to be of higher dimension. For example, consider that, instead of a dimension-six operator, Q_1 is accompanied by the $\Delta\mathcal{B}=2$ dimension-nine operator $Q_4\equiv d_R^4u_R^2$. Alone, Q_1 and Q_4 do not break the $U(1)_1\otimes U(1)_2$ symmetry, since they have vanishing PQ charge for some value of α and β. But if neutrinos have a Majorana mass term, say, $Q_\nu\equiv\ell_L^2\Phi_i^2$ of Eq. (21), then not all the $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ operators can be simultaneously neutral. Thus, together, Q_1, Q_4, and Q_ν introduce too much $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ violation for the axion to remain massless.

Yet, the combined presence of these $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ effects break $U(1)_1\otimes U(1)_2$ in a direction that can be matched in the effective potential only at the cost of many Higgs doublets. This combination of doublets need not be a power of $\Phi_2^\dagger\Phi_1$ anymore and actually corresponds to the dimension-eight coupling (see Fig. 2)

$$V^{\rm eff}_{Q_1\times Q_4\times Q_\nu} = \lambda_{\rm eff}\Phi_i^2\Phi_1^2\Phi_2^{\dagger 4} + \text{H.c.},$$
$$\lambda_{\rm eff}\sim\frac{\Lambda_{\rm reg}^6}{\Lambda_{\mathcal{B},\mathcal{L}}^{10}}\times c_\nu c_4 c_1^\dagger c_1^\dagger\mathbf{Y}_u^2\mathbf{Y}_d^4, \tag{63}$$

where c_ν is the Wilson coefficient of Q_ν. Also, we have suppressed flavor indices and identified the scale of all the

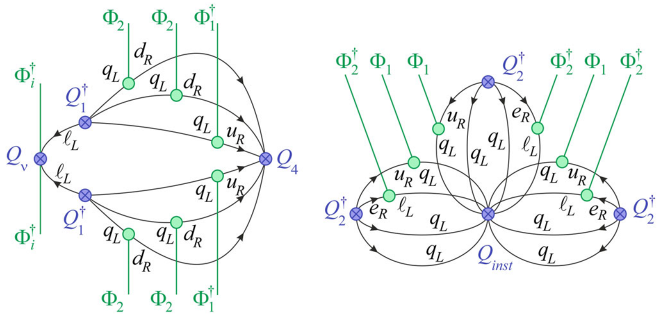

FIG. 2. Left: Fermion loop involving the dimension-five Majorana operator $Q_\nu = \ell_L^2 \Phi_i^2$, $i = 1$ or 2, the dimension-six Weinberg operator $Q_1 = \ell_L q_L^3$, and the $\Delta\mathcal{B} = 2$ dimension-nine operator $Q_4 = d_R^4 u_R^2$. Right: Fermion loop involving the instanton $Q_{\rm inst} = q_L^9 \ell_L^3$ interaction together with the dimension-six $Q_2 = e_R u_R q_L^2$ operator.

operators to $\Lambda_{\mathcal{B},\mathcal{L}}$ for simplicity. Clearly, the axion mass is negligible in this case, of $\mathcal{O}(v^3/\Lambda_{\mathcal{B},\mathcal{L}}^2 \times v/v_s)$ when $\Lambda_{\rm reg} \approx \Lambda_{\mathcal{B},\mathcal{L}}$. Even though $\theta_{\rm eff}$ will not be entirely disposed of, it is tiny and the strong CP puzzle is still solved.

Note that this estimate remains valid in the νDFSZ model even though Q_ν is replaced by the singlet coupling to ν_R. Indeed, the leading term in the effective potential is then $\phi\Phi_i^2\Phi_1^2\Phi_2^{\dagger 4}$, as can be seen from Fig. 2 by splitting the $\ell_L^2\Phi_i^2$ vertex into $\bar{\nu}_R\Phi_i\ell_L \otimes \phi\bar{\nu}_R^C\nu_R \otimes \bar{\nu}_R\Phi_i\ell_L$, and one can check that this leads to the same estimate for the axion mass when $v_s \sim \Lambda_{\mathcal{B},\mathcal{L}}$.

3. Scenario III: Electroweak instantons and the axion mass

As a final example, imagine now that only Q_2 and Q_ν are present. At first sight, the $U(1)_1 \otimes U(1)_2$ symmetry is preserved. However, one still has to account for the electroweak instanton effects. Since Eq. (50) is not compatible with the presence of Q_2, the axion cannot be truly massless. It is a bit more tricky to estimate its mass in this case, because the electroweak instanton effects are not truly local. But, to get an idea of the induced mass, let us nevertheless use the same strategy as above with $Q_{\rm inst} = \ell_L^3 q_L^9$. There will then be a new term in the effective potential

$$V^{\rm eff}_{Q_2\times Q_{\rm inst}} = \lambda_{\rm eff}(\Phi_2^\dagger\Phi_1)^3 + \text{H.c.},$$
$$\lambda_{\rm eff} \sim \frac{\Lambda_{\rm reg}^4}{\Lambda_{\mathcal{B},\mathcal{L}}^6} \times c_{\rm inst} c_2^{\dagger 3}(\mathbf{Y}_e\mathbf{Y}_u^2\mathbf{Y}_d)^3. \tag{64}$$

In this estimate, we consider that the UV regularization needs only to compensate for the scale of the dimension-six operators $\Lambda_{\mathcal{B},\mathcal{L}}$, and not for the dimension-18 instanton effect. So, this should be understood as nothing more than a rough estimate of the maximal impact this combination of operators could have on the axion. In any case, when $\Lambda_{\rm reg} \approx \Lambda_{\mathcal{B},\mathcal{L}}$, the axion mass is completely negligible, because it is suppressed by the $\Lambda_{\mathcal{B},\mathcal{L}}$ scale [see Eq. (60)], because instanton effects are tiny, $c_{\rm inst} \sim \exp(-4\pi/g^2)$, and because of the flavor structure. Indeed, $Q_{\rm inst}$ is fully antisymmetric in flavor space, so first- and second-generation fermions circulate in the loop and there will be many small Yukawa couplings. Actually, additional gauge interactions may be needed to prevent the leading flavor contraction from vanishing, in a way similar to what happens for the electroweak contribution to the electric dipole moments; see, e.g., the discussion in Ref. [36]. Yet, even if tiny, this shows that the axion would not be strictly massless in this case. Furthermore, at high temperature, when the QCD chiral symmetry is restored, these effects would be dominant and force a specific alignment of the vacuum.

A very similar conclusion is encountered when nonperturbative quantum gravity, which is expected to violate global symmetries, is taken into account by adding nonlocal higher-dimensional operators in the low-energy effective action [37–39]. Terms such as in Eq. (58) are introduced with a Planck scale cutoff, $M_P \sim 10^{19}$ GeV, implying a lower limit on their dimension $(2n+4)$ in order not to impose permanently an alignment of the vacuum away from the strong CP solution.

V. CONCLUSIONS

Axion models are based on the spontaneous breaking of an extra $U(1)$ symmetry. When this symmetry has a strong anomaly, the associated Goldstone boson, the axion, ends up coupled to gluons, and this ensures the strong CP violation relaxes to zero in the nonperturbative regime. In this paper, we analyzed more specifically the PQ and DFSZ axion models, where SM fermions as well as the Higgs fields responsible for the electroweak symmetry breaking are charged under the additional $U(1)$ symmetry. A characteristic feature of these models is that the true $U(1)_{\rm PQ}$ symmetry corresponding to the axion is not trivial to identify because of the presence of three other $U(1)$ symmetries acting on the same fields: baryon number $\mathcal{B}$, lepton number $\mathcal{L}$, and weak hypercharge. As a consequence, in general, the PQ charges can be defined only after $U(1)_Y$ is spontaneously broken, and, even then, those of the fermions remain ambiguous whenever the baryon or lepton number is conserved. Our purpose was to study this ambiguity, see when it can be lifted, and how it leaves the axion phenomenology intact. Our main results are as follows.

(i) The ambiguities in the PQ charges of the fermions, here parametrized by α and β, are well known, but it is often interpreted as a freedom. One seems free to fix α and β as one wishes. Doing this, however, prevents any further analysis of $\mathcal{B}$ and $\mathcal{L}$ violation. For example, if one chooses to assign PQ charges only to right-handed fermions, a $\Delta\mathcal{B} = 2$ operator like $(u_R d_R d_R)^2$ would be forbidden. Yet, this is merely a consequence of the choice made for the PQ charges. What we showed here is that it is compulsory to keep the fermion charge ambiguity explicit to leave the theory the necessary room to adapt to the presence of $\mathcal{B}$ and/or $\mathcal{L}$ violation. Indeed, in the presence of such interactions, these ambiguities automatically disappear, and the corresponding parameters α and β are fixed to specific values, when the $U(1)_{\rm PQ}$ symmetry aligns itself with the remaining $U(1)$ symmetry of the Lagrangian. This proves that such violations of $\mathcal{B}$ and $\mathcal{L}$ can be compatible with the PQ symmetry.

(ii) Since there are two parameters, reflecting the two accidental symmetries $U(1)_{\mathcal{B}} \otimes U(1)_{\mathcal{L}}$, axion models can accommodate for breaking terms in two independent directions. This means, for example, that adding a $\Delta\mathcal{L} = 2$ Majorana mass for the neutrinos as well as some $\mathcal{B} + \mathcal{L}$-violating operators, say, $e_R u_R q_L^2$ and $\ell_L q_L d_R u_R$, preserves the axion solution to the strong CP puzzle. Yet, this compatibility is delicate and needs to be checked in detail. For example, adding both the operators $\ell_L q_L^3$ and $e_R u_R q_L^2$ spoils the axion solution completely, even though these operators have the same $\mathcal{B}$ and $\mathcal{L}$ quantum numbers. When they are both present, there is simply not enough room for the $U(1)_{\rm PQ}$ symmetry to remain active.

(iii) In many cases, the capability of axion models to accommodate for $\mathcal{B}$ and $\mathcal{L}$ violation is saturated from the start. Indeed, first, these models should be compatible with neutrino masses, and, seesaw mechanisms being the most natural, some $\Delta\mathcal{L} = 2$ effects are present. Second, electroweak instantons generate $\mathcal{B} + \mathcal{L}$-violating effects, and, even if negligible at low energy, their mere existence forces the PQ symmetry to be realized in a specific way. In this case, there remains not much room for other $\mathcal{B}$- and/or $\mathcal{L}$-violating effects. For example, the axion cannot remain a true Goldstone boson in the presence of, say, the $e_R u_R q_L^2$ or $(u_R d_R d_R)^2$ operator. Yet, the instanton interaction is so small that the induced mass of the axion is well below the QCD mass, and the strong CP puzzle is still solved. Obviously, the situation changes at high temperature. If the electroweak contribution to the axion mass becomes larger than the QCD contribution, the axion is initially not aligned in the CP-conserving direction but does so only at a later time. Such a situation could have important cosmological consequences.

(iv) Usually, axion models are specified in a particular representation, in which the axion has only derivative couplings to SM fermions and anomalous couplings to gauge field strengths. Because these effective couplings arise from chiral rotations of the fermion fields, tuned by their PQ charges, some dependences on α and β are introduced (explicitly or implicitly) in the Lagrangian. At the same time, we have shown that α and β take on various very different values, depending on the $\Delta\mathcal{B}$ and/or $\Delta\mathcal{L}$ effects present. So, the axion effective interactions are strongly dependent on the presence of these $\Delta\mathcal{B}$ and/or $\Delta\mathcal{L}$ interactions, whatever their intrinsic size. In this respect, the electroweak couplings $a^0 W^i_{\mu\nu}\tilde{W}^{i,\mu\nu}$, $i = 1, 2, 3$, is extreme in that the theory turns it off automatically whenever the PQ current has to circumvent the tiny electroweak instanton interactions. Of course, these dependences on α and β are spurious. As we demonstrated in Ref. [9], the α and β terms occurring in the derivative interactions always cancel out exactly with those of the anomalous interactions, and the physical axion to fermion or gauge boson amplitudes are independent of α and β. In particular, the $a^0 W^+ W^-$ coupling is nonzero even when the anomalous $a^0 W^i_{\mu\nu}\tilde{W}^{i;\mu\nu}$ term is forced out of the axion effective Lagrangian by electroweak instantons.

(v) Several scenarios were discussed: the PQ and DFSZ axion with massless neutrinos, with a seesaw mechanism of type I and of type II, and the νDFSZ where the singlet also plays the role of the Majoron. Then, additional requirements were discussed, arising from the electroweak instantons, a GUT constraint, or various $\mathcal{B}$-violating operators. Despite their variety, for all those settings, the PQ charges of the two Higgs doublets and the fermions are the same, up to specific values for α and β and up to negligible corrections in the type-II seesaw. Though this can be understood as the orthogonality condition among Goldstone bosons stays essentially the same and the Yukawa couplings are always those of Eq. (2), it is often obscured by the normalization of the PQ charges. Yet, this is remarkable, because it means the low-energy phenomenology of the axion is the same in all these models, since it is independent of α and β. This is most evident adopting a linear parametrization for the two Higgs doublets, since the axion then does not couple directly to gauge bosons, while its coupling to each fermion is simply proportional to the fermion mass times the PQ charge of the doublet to which it couples [9].

The results of this paper should have implications in other settings where $\mathcal{B}$ and/or $\mathcal{L}$ violations occur, most notably in supersymmetry if R parity is not conserved and in grand unified theories. While embedding the axions in those models has already been proposed, further work to identify the most promising scenario is required [32]. In this respect, the connection with cosmology, either via the axion relic density or its possible impact on baryogenesis, could provide invaluable information.

APPENDIX: VACUUM REALIGNMENT AND DASHEN THEOREM

The effective potential approach of the previous section is rather simple, but it does not clearly show how the presence of too much $\mathcal{B}$ and/or $\mathcal{L}$ violation imposes a realignment of the vacuum. This will be described here, by perform the analysis directly in the broken phase.

Once the axion is introduced as the degree of freedom spanning the vacuum, the fact that the symmetry is explicitly broken manifests itself via nonzero matrix elements $\langle 0|\mathcal{L}_{\mathcal{B},\mathcal{L}}|a^0\rangle$ and $\langle a^0|\mathcal{L}_{\mathcal{B},\mathcal{L}}|a^0\rangle$. The latter corresponds to a mass term for the axion, and the former asks for a realignment of the vacuum. Indeed, in the presence of the perturbation, the vacuum is no longer degenerate, and the theory is unstable. It is only once at the true vacuum $|\Omega\rangle$ that the perturbation stops being able to shift the vacuum and $\langle \Omega|\mathcal{L}_{\mathcal{B},\mathcal{L}}|a^0\rangle = 0$. This condition on $|\Omega\rangle$ is equivalent to Dashen's theorem [35], which states that the true vacuum is that for which $\langle \Omega|\mathcal{L}_{\mathcal{B},\mathcal{L}}|\Omega\rangle$ is minimal.

Let us compute these matrix elements, and thereby the axion mass and true vacuum $|\Omega\rangle$, for the specific case of the Weinberg operators, $\mathcal{L}_{\mathcal{B},\mathcal{L}} = \mathcal{L}_{\mathcal{B},\mathcal{L}}^{\dim 6}$. First, let us move to a more convenient basis. After the reparametrization $\psi \to \exp(i\text{PQ}(\psi)a^0/v)\psi$ of the fermion fields, the axion is removed from the Yukawa couplings. As detailed in Ref. [9], this generates derivative couplings $-\partial_\mu a^0/v \times |J^\mu_{\text{PQ}}|_{fermions}$ from the fermion kinetic terms and anomalous nonderivative couplings $a^0/v \times \partial_\mu J^\mu_{\text{PQ}}$ with $\partial_\mu J^\mu_{\text{PQ}}$ given in Eq. (11) from the noninvariance of the fermionic path integral measure. In addition, since $\mathcal{L}_{\mathcal{B},\mathcal{L}}$ is not invariant under $U(1)_{\text{PQ}}$, each operator gets transformed into $Q_i \to Q_i \exp(i\text{PQ}(Q_i)a^0/v)$.

Forgetting for now the anomalous couplings, the only couplings surviving in the static limit are the nonderivative couplings with the $\Delta(\mathcal{B}+\mathcal{L})$ interactions,

$$\mathcal{L}_{\mathcal{B},\mathcal{L}}^{\dim 6} = \frac{c_1}{\Lambda^2}\ell_L q_L^3 \exp\left(i(3\alpha+\beta)\frac{a^0}{v}\right) + \frac{c_3}{\Lambda^2}e_R u_R^2 d_R \exp\left(i\left(3\alpha+\beta+2x+\frac{2}{x}\right)\frac{a^0}{v}\right) + \left(\frac{c_2}{\Lambda^2}e_R u_R q_L^2 + \frac{c_4}{\Lambda^2}\ell_L q_L d_R u_R\right)\exp\left(i\left(3\alpha+\beta+x+\frac{1}{x}\right)\frac{a^0}{v}\right) + \text{H.c.} \tag{A1}$$

When taken alone, none of these operators is able to induce $\langle \Omega|\mathcal{L}_{\mathcal{B},\mathcal{L}}^{\dim 6}|a^0\rangle$ or $\langle a^0|\mathcal{L}_{\mathcal{B},\mathcal{L}}^{\dim 6}|a^0\rangle$. For example, with only Q_1, the simplest $\Delta(\mathcal{B}+\mathcal{L})=0$ matrix element arises from a $Q_1^\dagger \otimes Q_1$ combination, and the axion field disappears. Some interference between two or more operators with different phases is needed. Let us consider that arising from $Q_1 = \ell_L q_L^3$ and $Q_2 = e_R u_R q_L^2$; see Fig. 3. We have two contributions, $Q_1^\dagger \otimes Q_2$ and $Q_2^\dagger \otimes Q_1$. Since we are after only $\Delta(\mathcal{B}+\mathcal{L})=0$ matrix elements with external axion fields, we can consider the generating function

$$\begin{aligned}\mathcal{V}_{\mathcal{B},\mathcal{L}}^{\dim 6} &= \langle \Omega|Q_1 \otimes Q_2^\dagger|\Omega\rangle \exp\left(-2i\left(x+\frac{1}{x}\right)\frac{a^0}{v}\right) + \text{H.c.} \\ &= 2|\langle \Omega|Q_1 \otimes Q_2^\dagger|\Omega\rangle| \cos\left(\delta_{12} + 2\left(x+\frac{1}{x}\right)\frac{a^0}{v}\right) \\ &= 2|\langle 0|Q_1 \otimes Q_2^\dagger|0\rangle| \cos\left(\delta_{12} + 2\left(x+\frac{1}{x}\right)\frac{a^0+\omega}{v}\right),\end{aligned} \tag{A2}$$

where δ_{12} denotes the phase of $\langle \Omega|Q_1 \otimes Q_2^\dagger|\Omega\rangle$. In the last line, we use the fact that the vacuum space is spanned by the axion; i.e., any two vacua are related by shifts in the axion field. This permits us to choose a reference vacuum $|0\rangle$ and trade $|\Omega\rangle$ for the free parameter ω.

Expanding the cosine function up to second order, the axion mass is found to be consistent with the previous estimate [Eq. (61)], since the matrix element $\langle 0|Q_1 \otimes Q_2^\dagger|0\rangle$ in Fig. 3 corresponds to the diagrams in Fig. 1 with the external Higgs fields replaced by their vacuum expectation values. Concerning the vacuum, $\langle \Omega|\mathcal{L}_{\mathcal{B},\mathcal{L}}^{\dim 6}|a^0\rangle$ is obtained from $\partial\mathcal{V}_{\mathcal{B},\mathcal{L}}^{\dim 6}/\partial a^0$ at $a^0 = 0$ and, thus, vanishes when ω satisfies

$$\delta_{12} + 2\left(x+\frac{1}{x}\right)\frac{\omega}{v} = 0. \tag{A3}$$

The fact that the preferred direction is set by the phase of $\langle \Omega|Q_1 \otimes Q_2^\dagger|\Omega\rangle$ can be understood as follows. In the

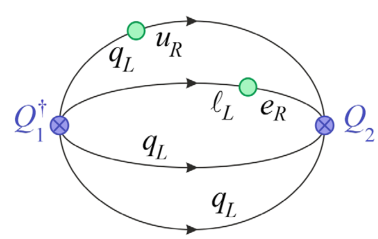

FIG. 3. Diagrammatic representation of the matrix element in Eq. (A2), where dots represent mass insertions. In a generic vacuum, any number of axion fields can be generated. The correct vacuum is that for which there is no axion tadpole. The axion mass then arises from the emission of two axions from the $Q_1 \otimes Q_2^\dagger$ operators.

absence of $\mathcal{L}_{\mathcal{B},\mathcal{L}}^{\dim 6}$, thanks to the still exact $U(1)_{\mathrm{PQ}}$ symmetry, one can remove any phase occurring in the fermion mass terms as well as take real VEVs $v_{1,2}$ for the two Higgs doublets [see Eq. (4)]. But, it is no longer possible to keep both VEVs real once $U(1)_{\mathrm{PQ}}$ is broken by $\mathcal{L}_{\mathcal{B},\mathcal{L}}^{\dim 6}$, and the specific choice in Eq. (A3) becomes compulsory.

In some sense, we can also understand $\mathcal{V}_{\mathcal{B},\mathcal{L}}^{\dim 6}$ as a contribution to the effective potential of the axion. With this picture, bringing back the anomalous couplings and turning on the QCD effects, the full axion potential looks like $\mathcal{V}_{\mathrm{eff}} = \mathcal{V}_{\mathcal{B},\mathcal{L}}^{\dim 6} + \mathcal{V}_{\mathrm{QCD}}$ with $\mathcal{V}_{\mathrm{QCD}} \sim m_\pi^2 f_\pi^2 \bar{m} \cos(\theta_{\mathrm{QCD}} + a^0/v)$ and $\bar{m} = m_u m_d/(m_u + m_d)^2$. Thus, the strong CP puzzle is solved only if $\mathcal{V}_{\mathrm{QCD}}$ dominates and forces the vacuum to align itself to kill θ_{QCD}. In the present case, given that the $\mathcal{V}_{\mathcal{B},\mathcal{L}}^{\dim 6}$-induced axion mass is much larger than that induced by $\mathcal{V}_{\mathrm{QCD}}$, the constraint from $\mathcal{V}_{\mathcal{B},\mathcal{L}}^{\dim 6}$ is stronger and the vacuum is rather aligned in the direction of Eq. (A3), leaving the strong CP puzzle open.

[1] C. Abel *et al.* (nEDM Collaboration), Phys. Rev. Lett. **124**, 081803 (2020).
[2] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1791 (1977).
[3] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978).
[4] F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
[5] W. A. Bardeen and S.-H. H. Tye, Phys. Lett. **74B**, 229 (1978).
[6] J. E. Kim, Phys. Rep. **150**, 1 (1987).
[7] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B166**, 493 (1980).
[8] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981); A. R. Zhitnitsky, Yad. Fiz. **31**, 497 (1980) [Sov. J. Nucl. Phys. **31**, 260 (1980)].
[9] J. Quevillon and C. Smith, Eur. Phys. J. C **79**, 822 (2019).
[10] M. Shin, Phys. Rev. Lett. **59**, 2515 (1987).
[11] G. Ballesteros, J. Redondo, A. Ringwald, and C. Tamarit, J. Cosmol. Astropart. Phys. 08 (2017) 001.
[12] D. A. Demir, E. Ma, and U. Sarkar, J. Phys. G **26**, L117 (2000).
[13] E. Ma, T. Ohata, and K. Tsumura, Phys. Rev. D **96**, 075039 (2017).
[14] P. Minkowski, Phys. Lett. **67B**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, Conf. Proc. C **790927**, 315 (1979), arXiv:1306.4669; T. Yanagida, Conf. Proc. C **7902131**, 95 (1979); S. L. Glashow, NATO Sci. Ser. B **61**, 687 (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).
[15] P. Langacker, R. D. Peccei, and T. Yanagida, Mod. Phys. Lett. A **01**, 541 (1986).
[16] J. D. Clarke and R. R. Volkas, Phys. Rev. D **93**, 035001 (2016).
[17] G. B. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981).
[18] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); **25**, 774 (1982).
[19] S. Bertolini and A. Santamaria, Nucl. Phys. **B357**, 222 (1991).
[20] S. Bertolini, L. Di Luzio, H. Kolešová, and M. Malinský, Phys. Rev. D **91**, 055014 (2015).
[21] R. N. Mohapatra and G. Senjanovic, Z. Phys. C **17**, 53 (1983).
[22] X. He and R. Volkas, Phys. Lett. B **208**, 261 (1988).
[23] Y. Ahn and E. J. Chun, Phys. Lett. B **752**, 333 (2016).
[24] P. Gu, J. Cosmol. Astropart. Phys. 07 (2016) 004.
[25] A. Latosinski, K. A. Meissner, and H. Nicolai, Nucl. Phys. **B868**, 596 (2013); A. Latosinski, A. Lewandowski, K. A. Meissner, and H. Nicolai, J. High Energy Phys. 10 (2015) 170.
[26] J. Heeck and H. H. Patel, Phys. Rev. D **100**, 095015 (2019).
[27] E. Peinado, M. Reig, R. Srivastava, and J. W. Valle, Mod. Phys. Lett. A **35**, 2050176 (2020).
[28] S. Baek, Phys. Lett. B **805**, 135415 (2020).
[29] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976); **18**, 2199 (1978).
[30] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976).
[31] R. T. Co and K. Harigaya, Phys. Rev. Lett. **124**, 111602 (2020).
[32] J. Quevillon and C. Smith (to be published).
[33] S. Weinberg, Phys. Rev. D **22**, 1694 (1980).
[34] C. Smith, Phys. Rev. D **85**, 036005 (2012).
[35] R. F. Dashen, Phys. Rev. D **3**, 1879 (1971).
[36] C. Smith and S. Touati, Nucl. Phys. **B924**, 417 (2017).
[37] S. M. Barr and D. Seckel, Phys. Rev. D **46**, 539 (1992).
[38] M. Kamionkowski and J. March-Russell, Phys. Lett. B **282**, 137 (1992).
[39] R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins, and L. M. Widrow, Phys. Lett. B **282**, 132 (1992).