


# Looking for a vector charmoniumlike state $Y$ in $e^+e^- \rightarrow \bar{D}D_1(2420) + c.c.$

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Inspired by the first observation of a vector charmoniumlike state  $Y(4626)$  decaying to a meson pair  $D_s^+D_{s1}(2536)^-$ , which could be viewed as a  $P$ -wave scalar-scalar  $[cs][\bar{c}\bar{s}]$  tetraquark state, we predict a potential vector charmoniumlike state  $Y$  with a  $P$ -wave scalar-scalar  $[cq][\bar{c}\bar{q}]$  configuration. The corresponding mass spectrum of the  $Y$  state is calculated to be  $4.33_{-0.23}^{+0.16}$  GeV in the framework of QCD sum rules. We suggest that the predicted  $Y$  state could be looked for in an open-charm  $e^+e^- \rightarrow \bar{D}D_1(2420) + c.c.$  process.

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## I. INTRODUCTION

In recent years, a series of vector charmoniumlike  $Y$  states have been observed in the initial-state radiation processes  $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi^+\pi^-J/\psi$  ( $\psi(2S)$ ) [1–8] or in the direct processes  $e^+e^- \rightarrow \pi^+\pi^-J/\psi$  ( $\psi(2S)$ ) [9–12]. These experiments show that  $Y$  states mainly couple to hidden-charm final states. In contrast, Belle newly reported the first observation of  $Y(4626)$  in an open-charm process  $e^+e^- \rightarrow D_s^+D_{s1}(2536)^- + c.c.$  with a significance of  $5.9\sigma$  [13], which has promptly attracted much attention [14–23]. Theoretically, some authors pointed out that  $Y(4626)$  can be well interpreted as a  $P$ -wave  $[cs][\bar{c}\bar{s}]$  state with a multi-quark color flux-tube model [21]. Moreover, we studied  $Y(4626)$  from two-point QCD sum rules, and finally concluded that it could be a  $P$ -wave scalar-scalar  $[cs][\bar{c}\bar{s}]$  state [23]. As an analogy of  $Y(4626)$ 's observation in the open-charm process, we propose that a novel vector charmoniumlike state  $Y$  could be looked for in an open-charm  $e^+e^- \rightarrow \bar{D}D_1(2420) + c.c.$  process. In theory, the predicted  $Y$  state could correspondingly be regarded as a  $P$ -wave scalar-scalar  $[cq][\bar{c}\bar{q}]$  tetraquark state.

In this work, we endeavor to explore the charmoniumlike state  $Y$  with the  $P$ -wave scalar-scalar  $[cq][\bar{c}\bar{q}]$  configuration. To deal with the hadronic state, one has to confront the complicated nonperturbative QCD problem. As one trusty method for evaluating nonperturbative effects, the QCD sum rule [24] is firmly founded on the basic QCD theory, and has been successfully applied to plenty of hadronic

systems (for reviews see Refs. [25–28] and references therein). Accordingly, we intend to study this  $Y$  state by making use of the QCD sum rule approach.

The paper's organization is as follows. In Sec. II, the QCD sum rule is derived for  $Y$  with the  $P$ -wave scalar-scalar  $[cq][\bar{c}\bar{q}]$  structure, along with a numerical analysis and discussions in Sec. III. The last part includes a brief summary.

## II. THE QCD SUM RULE FOR $Y$ WITH $P$ -WAVE SCALAR-SCALAR $[cq][\bar{c}\bar{q}]$ STRUCTURE

Generally speaking, one could have several choices on diquarks to characterize a  $P$ -wave tetraquark state with  $J^P = 1^-$ . It is worth noting that there have been broad discussions on the so-called “good” or “bad” diquarks for the tetraquark states [29], and then the  $Y$  state with  $P$ -wave  $[cq][\bar{c}\bar{q}]$  structure could be represented based on the following considerations [30]. A good diquark operator in the attractive antitriplet color channel can be  $\bar{q}_c\gamma_5q$  with  $0^+$ , and a bad diquark operator can be  $\bar{q}_c\gamma q$  with  $1^+$ . Similarly, operators with  $0^-$  and  $1^-$  can be written as  $\bar{q}_c q$  and  $\bar{q}_c\gamma\gamma_5q$ , respectively. Further, it is suggested that diquarks are preferably formed into spin 0 from lattice results [31]. Comparatively, the solid tetraquark candidates tend to be composed of  $0^+$  good diquarks. For example, the final results from QCD sum rules favor the scalar diquark-scalar antidiquark case after comparing different diquark configurations [32]. Thereby, the predicted  $Y$  state would be dominantly structured as the  $P$ -wave scalar diquark-scalar antidiquark, which contains the flavor content  $[cq][\bar{c}\bar{q}]$  with momentum numbers  $S_{[cq]} = 0$ ,  $S_{[\bar{c}\bar{q}]} = 0$ ,  $S_{[cq][\bar{c}\bar{q}]} = 0$ , and  $L_{[cq][\bar{c}\bar{q}]} = 1$ . Here  $q$  can be  $u$  or  $d$  quark, and  $c$  is the charm quark. Considering that both light  $u$  and  $d$  quark masses are taken as current-quark masses in the paper, they are so small when compared with the heavy

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running charm mass  $m_c$  that they will be neglected in the calculation, complying with the usual treatment of heavy hadrons. Thus, it is not concretely differentiated whether  $q = u$  or  $q = d$  for brevity. The corresponding current could be constructed as

$$j_\mu = \epsilon_{def} \epsilon_{d'e'f'} (q_d^T C \gamma_5 c_e) D_\mu (\bar{q}_{d'} \gamma_5 C \bar{c}_{e'}^T), \quad (1)$$

in which the index  $T$  denotes the matrix transposition,  $C$  means the charge conjugation matrix,  $D_\mu$  is the covariant derivative to generate  $L = 1$ , and  $d, e, f, d',$  and  $e'$  are color indices.

Generally, the two-point correlator  $\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T [j_\mu(x) j_\nu^+(0)] | 0 \rangle$  can be parametrized as

$$\Pi_{\mu\nu}(q^2) = \frac{q_\mu q_\nu}{q^2} \Pi^{(0)}(q^2) + \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi^{(1)}(q^2). \quad (2)$$

To yield the sum rule, the part  $\Pi^{(1)}(q^2)$  can be evaluated in two different ways. At the hadronic level, it can be expressed as

$$\Pi^{(1)}(q^2) = \frac{\lambda^2}{M_H^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im}\Pi^{(1)}(s)}{s - q^2}, \quad (3)$$

where  $\lambda$  is the hadronic coupling constant and  $M_H$  is the hadron's mass. At the quark level, it can be written as

$$\Pi^{(1)}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho(s)}{s - q^2}, \quad (4)$$

for which the spectral density  $\rho(s) = \frac{1}{\pi} \text{Im}\Pi^{(1)}(s)$ .

In deriving  $\rho(s)$ , one could work at leading order in  $\alpha_s$  and consider condensates up to dimension 8. To keep the heavy-quark mass finite, one uses the heavy-quark propagator in momentum space [33]. The correlator's light-quark part is calculated in the coordinate space and Fourier transformed to the  $D$  dimension momentum space, which is combined with the heavy-quark part and then dimensionally regularized at  $D = 4$  [28,34,35]. It is given by  $\rho(s) = \rho^{\text{pert}} + \rho^{\langle \bar{q}q \rangle} + \rho^{\langle g^2 G^2 \rangle} + \rho^{\langle g \bar{q} \sigma \cdot G q \rangle} + \rho^{\langle \bar{q}q \rangle^2} + \rho^{\langle g^3 G^3 \rangle} + \rho^{\langle \bar{q}q \rangle \langle g^2 G^2 \rangle} + \rho^{\langle \bar{q}q \rangle \langle g \bar{q} \sigma \cdot G q \rangle}$ , detailed with

$$\begin{aligned} \rho^{\text{pert}} &= -\frac{1}{3 \cdot 5 \cdot 2^{11} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (1 - \alpha - \beta) \kappa r^5, \\ \rho^{\langle \bar{q}q \rangle} &= \frac{m_c \langle \bar{q}q \rangle}{3 \cdot 2^6 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} (2 - \alpha - \beta) r^3, \\ \rho^{\langle g^2 G^2 \rangle} &= -\frac{m_c^2 \langle g^2 G^2 \rangle}{3^2 \cdot 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (1 - \alpha - \beta) (\alpha^3 + \beta^3) \kappa r^2, \\ \rho^{\langle g \bar{q} \sigma \cdot G q \rangle} &= \frac{m_c \langle g \bar{q} \sigma \cdot G q \rangle}{2^8 \pi^4} \left\{ -\int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} (\alpha + \beta - 4\alpha\beta) r^2 + \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \frac{[m_c^2 - \alpha(1-\alpha)s]^2}{\alpha(1-\alpha)} \right\}, \\ \rho^{\langle \bar{q}q \rangle^2} &= -\frac{m_c^2 \langle \bar{q}q \rangle^2}{3 \cdot 2^3 \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha [m_c^2 - \alpha(1-\alpha)s], \\ \rho^{\langle g^3 G^3 \rangle} &= -\frac{\langle g^3 G^3 \rangle}{3^2 \cdot 2^{14} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (1 - \alpha - \beta) \kappa [(\alpha^3 + \beta^3) r + 4(\alpha^4 + \beta^4) m_c^2] r, \\ \rho^{\langle \bar{q}q \rangle \langle g^2 G^2 \rangle} &= \frac{m_c \langle \bar{q}q \rangle \langle g^2 G^2 \rangle}{3^2 \cdot 2^8 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} \{ (2 - \alpha - \beta) (\alpha^3 + \beta^3) m_c^2 - 3[\alpha^2(\beta - 1) + \beta^2(\alpha - 1)] r \}, \end{aligned}$$

and

$$\rho^{\langle \bar{q}q \rangle \langle g \bar{q} \sigma \cdot G q \rangle} = \frac{m_c^2 \langle \bar{q}q \rangle \langle g \bar{q} \sigma \cdot G q \rangle}{3 \cdot 2^5 \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha (6\alpha^2 - 6\alpha + 1),$$

where  $\kappa = 1 + \alpha - 2\alpha^2 + \beta + 2\alpha\beta - 2\beta^2$ ,  $r = (\alpha + \beta)m_c^2 - \alpha\beta s$ ,  $\alpha_{\min} = (1 - \sqrt{1 - 4m_c^2/s})/2$ ,  $\alpha_{\max} = (1 + \sqrt{1 - 4m_c^2/s})/2$ , and  $\beta_{\min} = \alpha m_c^2 / (s\alpha - m_c^2)$ . For the four-quark condensate, a general factorization  $\langle \bar{q}q \bar{q}q \rangle = \varrho \langle \bar{q}q \rangle^2$  [26,36] has been employed, in which  $\varrho$  may be equal to 1 or 2.

Equating the two expressions, (3) and (4), adopting quark-hadron duality, and making a Borel transform, the sum rule can be turned into

$$\lambda^2 e^{-M_H^2/M^2} = \int_{4m_c^2}^{s_0} ds \rho e^{-s/M^2}. \quad (5)$$

Taking the derivative of Eq. (5) with respect to  $-1/M^2$  and then dividing the result by Eq. (5) itself, one can obtain the hadron's mass sum rule

$$M_H^2 = \int_{4m_c^2}^{s_0} ds \rho s e^{-s/M^2} / \int_{4m_c^2}^{s_0} ds \rho e^{-s/M^2}, \quad (6)$$

in which light  $u$  and  $d$  current-quark masses have been safely neglected as they are so small compared with the heavy  $m_c$ .

### III. NUMERICAL ANALYSIS AND DISCUSSIONS

In the numerical analysis, the running charm mass  $m_c$  is  $1.27 \pm 0.02$  GeV [37], and other input parameters are [24,28]  $\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3$  GeV<sup>3</sup>,  $m_0^2 = 0.8 \pm 0.1$  GeV<sup>2</sup>,  $\langle g\bar{q}\sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ ,  $\langle g^2 G^2 \rangle = 0.88 \pm 0.25$  GeV<sup>4</sup>, as well as  $\langle g^3 G^3 \rangle = 0.58 \pm 0.18$  GeV<sup>6</sup>. According to the standard criterion of sum rule analysis, one could find proper work windows for the threshold parameter  $\sqrt{s_0}$  and the Borel parameter  $M^2$ . The lower bound of  $M^2$  is obtained from the operator product expansion (OPE) convergence, and the upper one is found in view of that the pole contribution should be larger than QCD continuum one. Meanwhile, the threshold  $\sqrt{s_0}$  describes the beginning of the continuum state, which is about 400–600 MeV bigger than the extracted  $M_H$  empirically.

At the very start, all the input parameters are kept at their central values and the four-quark condensate factor is taken as  $\varrho = 1$ . To get the lower bound of  $M^2$ , the OPE convergence is shown in Fig. 1 by comparing the relative contributions of different condensates from sum rule (5) for  $\sqrt{s_0} = 4.9$  GeV. Numerically, some main condensates could cancel each other out to some extent and the relative

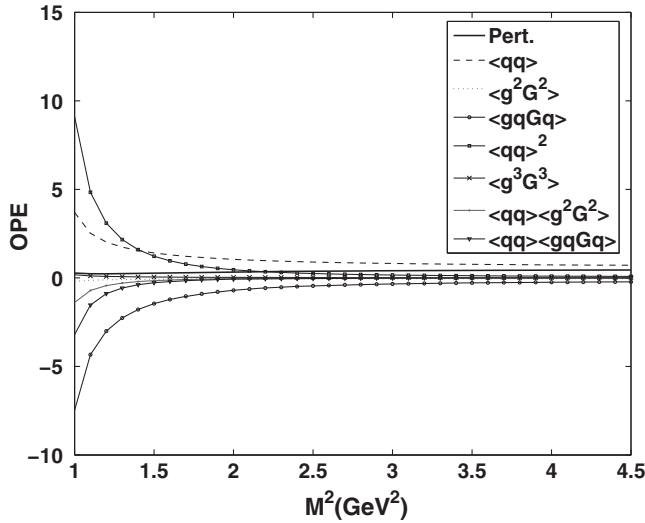


FIG. 1. The OPE convergence for the  $Y$  state with the  $P$ -wave scalar-scalar  $[cq][\bar{c}\bar{q}]$  configuration is shown by comparing the relative contributions of perturbative, two-quark condensate  $\langle \bar{q}q \rangle$ , two-gluon condensate  $\langle g^2 G^2 \rangle$ , mixed condensate  $\langle g\bar{q}\sigma \cdot Gq \rangle$ , four-quark condensate  $\langle \bar{q}q \rangle^2$ , three-gluon condensate  $\langle g^3 G^3 \rangle$ ,  $\langle \bar{q}q \rangle \langle g^2 G^2 \rangle$ , and  $\langle \bar{q}q \rangle \langle g\bar{q}\sigma \cdot Gq \rangle$  from sum rule (5) for  $\sqrt{s_0} = 4.9$  GeV.

contribution of the perturbative could play a predominant role in OPE at  $M^2 = 2.5$  GeV<sup>2</sup>, which is increasing with the enlarging of Borel parameter  $M^2$ . In this way, it is taken as  $M^2 \geq 2.5$  GeV<sup>2</sup> with an eye to the OPE convergence analysis. Besides, the upper bound of  $M^2$  is attained with a view to the pole contribution dominance on the phenomenological side. In Fig. 2, it is compared between the pole contribution and continuum from sum rule (5) for  $\sqrt{s_0} = 4.9$  GeV. The relative pole contribution is close to 50% at  $M^2 = 3.0$  GeV<sup>2</sup> and descending with the Borel parameter  $M^2$ . Thus, the pole contribution dominance could be fulfilled while  $M^2 \leq 3.0$  GeV<sup>2</sup>. Accordingly, the Borel window of  $M^2$  is restricted to be  $2.5 \sim 3.0$  GeV<sup>2</sup> for  $\sqrt{s_0} = 4.9$  GeV. Analogously, the reasonable window of  $M^2$  is acquired as  $2.5 \sim 2.9$  GeV<sup>2</sup> for  $\sqrt{s_0} = 4.8$  GeV, and  $2.5 \sim 3.2$  GeV<sup>2</sup> for  $\sqrt{s_0} = 5.0$  GeV. In the work windows, one can expect that the two sides of the QCD sum rules have a good overlap and it is reliable to extract information on the resonance. The dependence on  $M^2$  for the mass  $M_H$  of the  $Y$  state is shown in Fig. 3, and its value is computed to be  $4.33 \pm 0.11$  GeV in the work windows.

Next, varying the input parameters, the mass  $M_H$  is obtained as  $4.33 \pm 0.11_{-0.08}^{+0.05}$  GeV (the first error due to variation of  $s_0$  and  $M^2$ , and the second one resulted from the uncertainty of QCD parameters) or shortly  $4.33_{-0.19}^{+0.16}$  GeV. In the end, paying attention to the variation of four-quark condensate factor  $\varrho$ , the corresponding Borel curves are presented in Fig. 4 with  $\varrho = 2$ . In comparison with Fig. 3 for  $\varrho = 1$ , one could notice the mass uncertainty when varying  $\varrho$  from 1 to 2, and could get the final mass

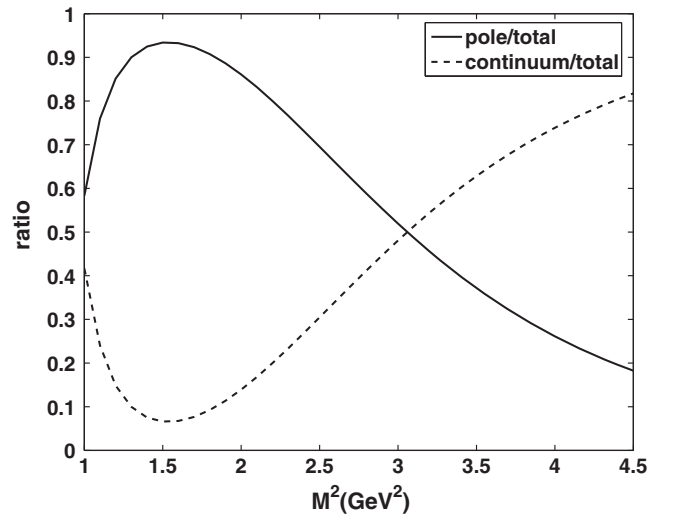


FIG. 2. The phenomenological contribution in sum rule (5) for  $\sqrt{s_0} = 4.9$  GeV for the  $Y$  state with the  $P$ -wave scalar-scalar  $[cq][\bar{c}\bar{q}]$  configuration. The solid line is the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) as a function of  $M^2$  and the dashed line is the relative continuum contribution.

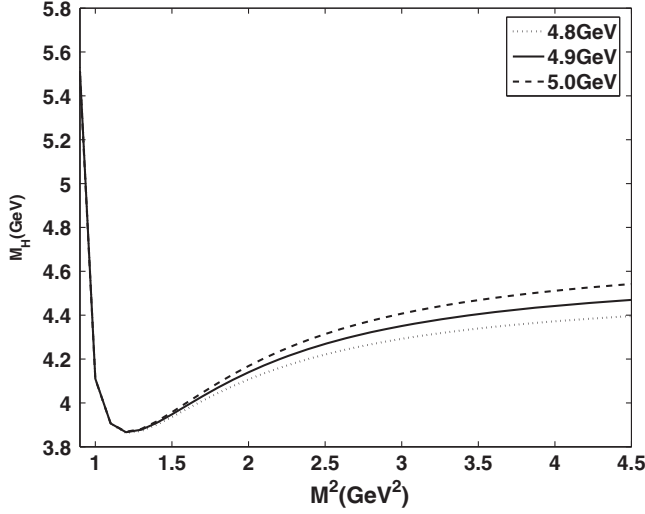


FIG. 3. The dependence on  $M^2$  for the mass  $M_H$  of the  $Y$  state with the  $P$ -wave scalar-scalar  $[cq][\bar{c}\bar{q}]$  configuration from sum rule (6) is shown while the four-quark condensate factor  $\varrho = 1$ . The ranges of  $M^2$  are  $2.5 \sim 2.9 \text{ GeV}^2$  for  $\sqrt{s_0} = 4.8 \text{ GeV}$ ,  $2.5 \sim 3.0 \text{ GeV}^2$  for  $\sqrt{s_0} = 4.9 \text{ GeV}$ , and  $2.5 \sim 3.2 \text{ GeV}^2$  for  $\sqrt{s_0} = 5.0 \text{ GeV}$ , respectively.

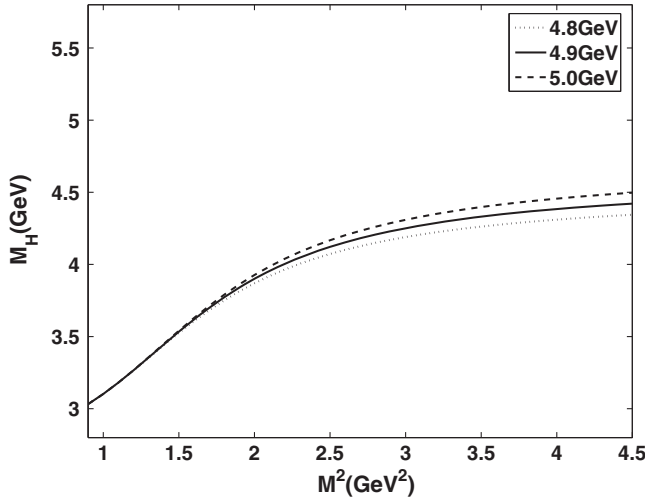


FIG. 4. The dependence on  $M^2$  for the mass  $M_H$  of the  $Y$  state with the  $P$ -wave scalar-scalar  $[cq][\bar{c}\bar{q}]$  configuration from sum rule (6) is shown while the four-quark condensate factor  $\varrho = 2$ .

$4.33^{+0.16}_{-0.23} \text{ GeV}$  for the  $Y$  state with the  $P$ -wave scalar-scalar  $[cq][\bar{c}\bar{q}]$  configuration.

In experiment, one may note that in the hidden-charm  $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi^+\pi^-\psi(2S)$  process, *BABAR* observed a broad

structure near  $4.32 \text{ GeV}$  [2], and Belle subsequently found the charmoniumlike state  $Y(4360)$  [3]. Afterward, a combined fit to these cross sections measured by *BABAR* and Belle experiments was performed [38], and the property of  $Y(4360)$  was further studied in  $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$  via initial-state radiation at *BABAR* [7] and at Belle [8]. Taking notice of the close masses of  $Y(4360)$  and the  $Y$  state concerned here, one could conjecture that they may be the same structure attributed to different decay modes. If that is true, it would be very important for understanding  $Y(4360)$  to search for the predicted  $Y$  state, because complementary measurements by other decay modes such as the open-charm process will provide further insights into  $Y(4360)$ 's internal structure. Whether or not that is the case, it is undoubtedly exciting and significant if one could find a vector charmoniumlike  $Y$  state, particularly in an open-charm decay.

Interestingly, there has appeared some measurement of a Born cross section for  $e^+e^- \rightarrow D^-D_1(2420)^+ + c.c.$  [39], in which the cross section line shape is consistent with the previous BESIII's result based on a full reconstruction method [40], and there is some indication of enhanced cross section at the location of  $Y(4360)$ . Thereby, it seems promising that the predicted  $Y$  state could be observed in the open-charm process  $e^+e^- \rightarrow \bar{D}D_1(2420) + c.c.$  via either the initial-state radiation or the direct production for the future experiments.

#### IV. SUMMARY

Activated by the first observation of a vector charmoniumlike state  $Y(4626)$  in the open-charm  $D_s^+D_{s1}(2536)^-$  decay mode, which could be a  $P$ -wave scalar-scalar  $[cs][\bar{c}\bar{s}]$  tetraquark state, we predict a novel vector charmoniumlike  $Y$  state with a  $P$ -wave scalar-scalar  $[cq][\bar{c}\bar{q}]$  configuration. Finally, the mass of  $Y$  is presented to be  $4.33^{+0.16}_{-0.23} \text{ GeV}$  from QCD sum rules. We suggest that the predicted  $Y$  state could be searched for in an open-charm  $e^+e^- \rightarrow \bar{D}D_1(2420) + c.c.$  process through the initial-state radiation or the direct production in experiments, for which virtually there has been some indication of enhanced cross section in BESIII's existing measurements.

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