Excited B_c states via the Dyson-Schwinger equation approach of QCD

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We study the most recently observed excited B_c states with the Dyson-Schwinger equation and the Bethe-Salpeter equation approach of continuum QCD. The obtained $M_{B_c^+(2S)} = 6.813(16)$ GeV, $M_{B_c^{*+}(2S)} = 6.841(18)$ GeV and the mass splitting $M_{B_c^+(2S)} - M^{\rm rec}_{B_c^{*+}(2S)} \approx 0.039$ GeV agree with the observations very well. Moreover we predict the leptonic decay constant $f_{B_c^+(2S)} = -0.165(10)$ GeV, $f_{B_c^{*+}(2S)} = -0.161(7)$ GeV, respectively.

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I. INTRODUCTION

Recently, two excited B_c mesons, $B_c^+(2S)$ and $B_c^{*+}(2S)$, were observed in the mass spectrum of $B_c^+\pi^+\pi^-$ for the first time by the CMS experiment at $\sqrt{s} = 13$ TeV [1]. The mass of $B_c^+(2S)$ is determined to be $M_{B_c^+(2S)} = 6871.0 \pm 1.2(\text{stat}) \pm 0.8(\text{syst}) \pm 0.8(B_c^+)$ MeV, while the mass difference $M_{B_c^+(2S)} - M^{\rm rec}_{B_c^{*+}(2S)} = 29.0 \pm 1.5(\text{stat}) \pm 0.7(\text{syst})$ MeV, where $M^{\rm rec}_{B_c^{*+}(2S)}$ is the reconstructed mass, defined by

$$M^{\rm rec}_{B_c^{*+}(2S)} = M_{B_c^{*+}(2S)} - (M_{B_c^{*+}(1S)} - M_{B_c^+(1S)}). \qquad (1)$$

The above results are then confirmed by the LHCb experiment with 8.5 fb^{-1} proton-proton (pp) collision data [2], being $M_{B_c^+(2S)} = 6872.1 \pm 1.3(\text{stat}) \pm 0.1(\text{syst}) \pm 0.8(B_c^+)$ MeV and $M_{B_c^+(2S)} - M^{\rm rec}_{B_c^{*+}(2S)} = 31.0 \pm 1.4(\text{stat})$ MeV, respectively.

Investigating the open flavor states such as the B_c^+ family of $(c\bar{b})$ mesons could enrich our understanding of the strong interaction (see, e.g., Ref. [3] and references therein for the contemporary statements). Exploring the excited states relies on detailed understanding of the long range behavior of strong interaction and encounters plenty difficulties due to the intrinsic complexity. The quark model has been widely applied to study hadron spectrum and, by using a phenomenological nonrelativistic potential model, the mass spectrum and decay property of $(c\bar{b})$ mesons have been explored (see, e.g., Refs. [4,5]). However, investigations based on an *ab initio* approach of strong interaction, quantum chromodynamics (QCD), are still challenges. The predictions of charm-bottom ground state from lattice QCD (LQCD) [6] has been released recently with the masses $M_{B_c^+} = 6276(3)(6)$ MeV and $M_{B_c^{+*}} = 6331(4)(6)$ MeV. Studying the masses of the excited states in LQCD are more difficult than determining those of the ground states [7,8] and the leptonic decay constants of excited B_c^+ states have not yet been touched. For details of the difficulties to study the decay constant in LQCD simulations please refer to Refs. [9,10], where trials of calculating the decay constant of the first radial excited pion are carried out with the inspiration of a continuum theory prediction [11].

As a continuum functional method of QCD, the Dyson-Schwinger (DS) equation and Bethe-Salpeter (BS) equation (DSBSE) [12–14] approach is complementary to LQCD and a covariant way to bridge the hadron physics and the fundamental degree of QCD. The difficulty of investigating the open flavor hadrons within this approach has been reported in Ref. [15] and that for exotic and radial excited states has been displayed in Ref. [16]. Then some efforts have been made (e.g., Refs. [17–19]). Using an algebraic model, the mass of B_c^{*+}, which is consistent with the world average value, has been obtained [20]. However it is not possible to predict the decay constants and the properties of the radial excited states in that framework, because the interaction lacks the relative momentum dependence. An extrapolation method has been developed in Ref. [21]. Therein the obtained masses and decay constants of the ground state mesons are comparable to experimental measurements and LQCD simulation results, showing the success of the rainbow ladder (RL) approximation. What is more important, taking into account the flavor

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dependence of the quark-quark interaction, we have given a successful description of the ground states of the open flavor mesons and quarkonia [22]. Our results of the heavy mesons deviate from the experiment and LQCD results only about 1% for the ground state masses and less than 7% for the decay constants.

To study the excited states of B_c^+ and B_c^{*+} in the continuum QCD approach directly, one should develop a scheme by extending those given in Refs. [21,22]. In the extension, one should maintain the parameters as the same as the ones which produce the masses and decay constants of the ground states successfully. In this Letter, we describe the new scheme and report the obtained masses and decay constants of the first excited states, $B_c^+(2S)$ and $B_c^{*+}(2S)$, via the continuum QCD approach. Our obtained mass of the excited states agree with the experimental observations very well. The obtained decay constants are also quite reasonable.

II. DSBSE APPROACH

Here we present the RL truncated DSBSE approach which takes into account the flavor dependence of the quark-quark interaction. This model has been introduced in Ref. [22]. Herein we provide an overview for completeness to implement the approach to describe the masses of the newly observed radial heavy flavor mesons $B_c^+(2S)$ and $B_c^{*+}(2S)$, and predict their decay constants.

The meson Bethe-Salpeter amplitude (BSA), Γ^{fg}, can be obtained by solving the BS equation,

$$[\Gamma^{fg}(k;P)]^{\alpha}_{\beta} = \int_{dq}^{\Lambda} [K^{fg}(k,q;P)]^{\alpha\delta}_{\sigma\beta} [\chi^{fg}(q;P)]^{\sigma}_{\delta}, \quad (2)$$

where $[K^{fg}(k,q;P)]$ is the quark-antiquark scattering kernel, and α, β, σ, and δ are the Dirac indexes. $\chi^{fg}(q;P) = S_f(q_+)\Gamma^{fg}(q;P)S_g(q_-)$ is the BS wave function. $S_f(q_+)$ and $S_g(q_-)$ are the quark propagators, satisfying the DS equation generally,

$$S_f^{-1}(k) = Z_2(i\gamma\cdot k + m_f(\Lambda)) + \frac{4}{3} Z_2 \int_{dq}^{\Lambda} g^2 D_{\mu\nu}(k-q)\gamma_\mu S_f(q)\Gamma^f_\nu(q,k). \quad (3)$$

Herein the f and g label the quark flavor, $f,\ g \in \{u,d,s,c,b\}$. k and P are the relative and total momentum of the meson. $q_+ = q + \iota P/2$, $q_- = q - (1-\iota)P/2$, ι is the partition parameter describing the momentum partition between the quark and antiquark, which does not affect physical observables. $\int_{dq}^{\Lambda} = \int^{\Lambda} d^4q/(2\pi)^4$ stands for the Poincaré invariant regularized integration, with Λ as the regularization mass scale. The regularization will be removed by taking $\Lambda \to \infty$ in the final stage of calculation. $m_f(\Lambda)$ is a Λ-dependent, current-quark bare mass, Z_2 is the renormalization constant of quark field depending on Λ and ζ. We adopt a flavor independent renormalization scheme and choose $\zeta = 2$ GeV.

Generally the full quark-gluon vertex, $\Gamma^f_\nu(q,k)$, is involved in the quark DS equation and depends on the quark flavor. For example, most recent calculation of solving the coupled quark and quark-gluon vertex DS equations manifests explicitly the flavor dependence of $\Gamma^f_\nu(q,k)$ [23]. Truncating the equations at RL level [22], one can transfer the flavor dependence of the vertex, combined with the flavor independent gluon propagator, into the effective interaction $g^2 D_{\mu\nu}(l)\Gamma^f_\nu(q,k) = Z_2 \tilde{D}^f_{\mu\nu}(l)\gamma_\nu$ $(l = k-q)$, and the corresponding quark-antiquark scattering kernel

$$[K^{fg}(k,q;P)]^{\alpha\delta}_{\sigma\beta} \to -\frac{4}{3} Z_2^2 \tilde{D}^{fg}_{\mu\nu}(l)[\gamma_\mu]^{\alpha}_{\sigma}[\gamma_\mu]^{\beta}_{\delta}. \quad (4)$$

In the case of flavor symmetric mesons, RL truncation preserves the Ward-Takahashi identity exactly. As $g = f$, the effective interaction "$\tilde{D}^{ff}_{\mu\nu}(l)$" is in fact $\tilde{D}^f_{\mu\nu}(l)$. While in the case of open flavored mesons, the $\tilde{D}^{fg}_{\mu\nu}$ is related to each of the flavors, i.e., the $\tilde{D}^f_{\mu\nu}$ and $\tilde{D}^g_{\mu\nu}$, by an integral equation involving the quark propagators [22]. A promised way to build a consistent representation can be traced to Refs. [14,24].

Herein we use a model for the effective interaction, which has been introduced in Ref. [22]. The effective interaction can be expressed as, $\tilde{D}^{fg}_{\mu\nu}(l) = T_{\mu\nu}\mathcal{G}^{fg}(l^2)$ (where $T_{\mu\nu} = \delta_{\mu\nu} - l_\mu l_\nu/l^2$). $\mathcal{G}^{fg}$ is decomposed into an infrared (IR) part and an ultraviolet (UV) part. Since the UV part is usually very weakly dependent on the flavor, the flavor dependence can then be attributed to the IR part. We then have

$$\mathcal{G}^{fg}(s) = \mathcal{G}^{fg}_{\rm IR}(s) + \mathcal{G}_{\rm UV}(s), \quad (5)$$

where $s = l^2$.

The flavor dependent IR part is obtained by extending the expression for the flavor symmetric case [16,25], $\mathcal{G}^f_{\rm IR}(s) = \mathcal{G}^{ff}_{\rm IR}(s) = 8\pi^2 \frac{D_f^2}{\omega_f^4} e^{-s/\omega_f^2} = 8\pi^2 \frac{D_f D_f}{\omega_f^2\omega_f^2} e^{-s/\omega_f\omega_f}$ [22,23],

$$\mathcal{G}^{fg}_{\rm IR}(s) = 8\pi^2 \frac{D_f}{\omega_f^2}\frac{D_g}{\omega_g^2} e^{-s/(\omega_f\omega_g)}. \quad (6)$$

The flavor independent UV part takes the commonly used form,

$$\mathcal{G}_{\rm UV}(s) = \frac{8\pi^2\gamma_m \mathcal{F}(s)}{\ln[\tau + (1 + s/\Lambda^2_{\rm QCD})^2]}, \quad (7)$$

where $\mathcal{F}(s) = [1 - \exp(-s^2/[4m_t^4])]/s$, $\gamma_m = 12/(33 - 2N_f)$, with $m_t = 1.0$ GeV, $\tau = e^{10} - 1$, $N_f = 5$, and $\Lambda_{\rm QCD} = 0.21$ GeV. The values of m_t and τ are chosen different from

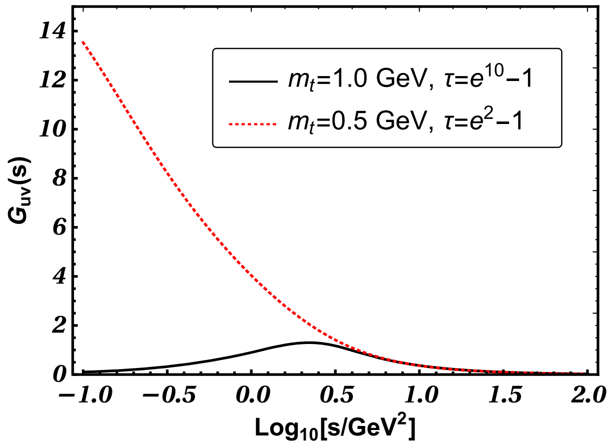

FIG. 1. The ultraviolet part of the interaction $G_{uv}(s)$ with $\tau = e^2 - 1$ (red dashed line) and $\tau = e^{10} - 1$ (black solid line).

the commonly taken ones in previous works [16,25] so that the $G_{\mathrm{UV}}(s)$ is well suppressed in IR region and the dressed function $\mathcal{G}^{fg}_{\mathrm{IR}}(s)$ is qualitatively right in the limit $m_f \to \infty$ or $m_g \to \infty$. This change doesn't affect the ultraviolet behavior in the gap and Bethe-Salpeter equations, see Fig. 1.

Though the flavor dependent part, Eq. (6), is not the exact form of the scattering kernel for the open flavored mesons, the parameters, $D_{f,g}$ and $\omega_{f,g}$, are fixed by physical observables. Three groups of parameters corresponding to a varying of the interaction width and preserving the Ward-Takahashi identity with high precision have been given in Ref. [22]. The parameters of the charm and beauty system are listed in Table I.

The current-quark bare mass that appears in Eq. (3), $m_f(\Lambda)$, depends on the regularization scale. To report a regularization-scale independent current quark mass, we consider the quark mass function $M_f(p^2)$ in the quark propagator $S_f(p) = \frac{Z_f(p^2,\zeta^2)}{i\gamma\cdot p + M_f(p^2)}$ that is renormalization-point and regularization-scale independent. The quark mass function in ultraviolet region is dominated by the current quark mass and evolves as

$$M_f(p^2) \overset{\text{large}\, p^2}{=} \frac{\hat{m}_f}{(\frac{1}{2}\, ln \frac{p^2}{\Lambda^2_{\mathrm{QCD}}})^{\gamma_m}}, \tag{8}$$

under the present truncation [26]. In Eq. (8), $\hat{m}_f$ is the renormalization-group-invariant current quark mass, with

TABLE I. The three sets of parameters ω_f and D_f (in unit GeV) of charm and bottom quarks [22].

Flavor	ω_f	D_f^2	ω_f	D_f^2	w_f	D_f^2
c	0.690	0.645	0.730	0.599	0.760	0.570
b	0.722	0.258	0.766	0.241	0.792	0.231

which we can extract the one-loop evolved current quark mass on the mass shell in Euclidean space as

$$\check{m}_f = \frac{\hat{m}_f}{(\frac{1}{2}\, ln[p^2/\Lambda^2_{\mathrm{QCD}}])^{\gamma_m}}\bigg|_{p^2=\check{m}_f^2}. \tag{9}$$

We get then $\check{m}_c = 1.31$ GeV and $\check{m}_b = 4.27$ GeV, which are commensurate with those given by PDG [27]. The slight difference arises from the dynamical chiral symmetry breaking effect involved in the evolution.

III. EXTRAPOLATION

The quark propagators in Eq. (2) are functions of the complex momenta $q_\pm^2$ that lie in parabolic regions. Any singular structure in the quark propagator indicates the upper bound of the maximum bound state mass obtainable directly, $P^2 > -M^2_{\mathrm{max},d}$, where $-M^2_{\mathrm{max},d}$ defines the contour border of the parabolic region. Due to color confinement, the quark propagators indeed have such singularities. The existing nodes in the Schwinger function of the charm and bottom quark propagators reveal the information of the complex conjugate poles [28,29]. The ground states are within the parabolic region and the masses and the leptonic decay constants can be obtained directly. However, the radial excited states are outside the parabolic region, hence we must adopt proper extrapolation scheme to determine the masses and the decay constants (see, e.g., Ref. [30]).

The BS equation can be viewed as a P^2-dependent eigenvalue problem,

$$\lambda^{fg}(P^2)[\Gamma^{fg}(k;P)]^\alpha_\beta = \int_{dq}^{\Lambda} [K(k,q;P)]^{\alpha\delta}_{\sigma\beta} [\chi^{fg}(q;P)]^\sigma_\delta, \tag{10}$$

where $[K(k,q;P)]^{\alpha\delta}_{\sigma\beta} = -\frac{4}{3}[Z_2]^2 \tilde{D}^{fg}_{\mu\nu}(k-q)[\gamma_\mu]^\alpha_\sigma [\gamma_\nu]^\delta_\beta$, and α, β, σ, and δ are the Dirac indexes. The meson mass is determined by $\lambda^{fg}(P^2 = -M^2) = 1$. An extrapolation to the physical bound state mass should be implemented if the state mass is larger than the contour border $M_{\mathrm{max},d}$. We make use of a Padé approximation

$$\frac{1}{\lambda^{fg}(P^2)} = \frac{1 + \sum_{n=1}^{N_o} a_n (P^2 + s_0)^n}{1 + \sum_{n=1}^{N_o} b_n (P^2 + s_0)^n}, \tag{11}$$

to fit the $\lambda^{fg}(P^2)$, with N_o as the order of the series, and s_0, a_n, and b_n as the parameters.

The leptonic decay constant of the pseudoscalar meson (0^-) and vector meson (1^-) are defined by

$$f^{fg}_{0^-}(P^2) P_\mu = Z_2 N_c \mathrm{tr} \int_{dk}^{\Lambda} \gamma_5 \gamma_\mu \chi^{fg}_{0^-}(k;P), \tag{12}$$

$$f^{fg}_{1^-}(P^2)\sqrt{-P^2} = \frac{Z_2 N_c}{3} \mathrm{tr} \int_{dk}^{\Lambda} \gamma_\mu \chi^{fg}_{1^-,\mu}(k;P), \tag{13}$$

with tr as the trace of the Dirac index. $f^{fg}(P^2)$ is generally fitted by

$$f^{fg}(P^2) = \frac{f_0 + \sum_{n=1}^{N_o} c_n (P^2 + M^2)^n}{1 + \sum_{n=1}^{N_o} d_n (P^2 + M^2)^n}, \tag{14}$$

where f_0, c_n, and d_n are parameters, and M^2 is the square of the mass. The physical decay constant is $f^{fg}(-M^2) = f_0$.

Notice that Eqs. (11) and (14) are general forms of most functions. The error due to the extrapolation could be estimated by comparing the results obtained with $N_o = 1, 2, 3, \ldots$, in the equations. The parameters are fitted with the least square method, i.e., by finding the minimum of the quantity $\delta = \frac{1}{N_p}\sum_{i=1}^{N_p}(F(P_i^2) - V_i)^2$, where N_p is the number of the P^2 value calculated, $F(P_i^2) = \frac{1}{\lambda^{fg}(P_i^2)}$ or $f^{fg}(P_i^2)$, V_i the corresponding calculated values.

IV. RESULTS

An illustration of the mass extrapolation is given by Fig. 2, which is the case of B_c^+. The black circles show the $1/\lambda^{bc}(P^2)$ of the ground state $B_c^+(1S)$. The mass, $M_{B_c^+(1S)}$, lies in the parabolic region defined by the singularities of the quark propagator, so it is obtained directly. The red diamonds show the $1/\lambda^{bc}(P^2)$ of the first radial excited state $B_c^+(2S)$. $M_{B_c^+(2S)}$ lies outside the parabolic region, and its value is extrapolated and presented by the blue star. The series Eq. (11) converges very fast, a good fitting is obtained for $N_o = 1$, with $\delta_{N_o=1} \approx 1 \times 10^{-6}$. Higher orders in Eq. (11) affect the fitting result very slightly, for example, $\delta_{N_o=2} \approx 4 \times 10^{-7}$, $\delta_{N_o=3} \approx 2 \times 10^{-7}$, which means that the corrections from higher orders are much less than that from the varying of the parameters. The open circles and diamonds stands for the results of varying the parameters shown in Table I, which give the source of our uncertainty listed in Table II. The other excited states, $\eta_c(2S)$, $\psi(2S)$, $\eta_b(2S)$, $\Upsilon(2S)$, and $B_c^{*+}(2S)$, are analyzed by the same method.

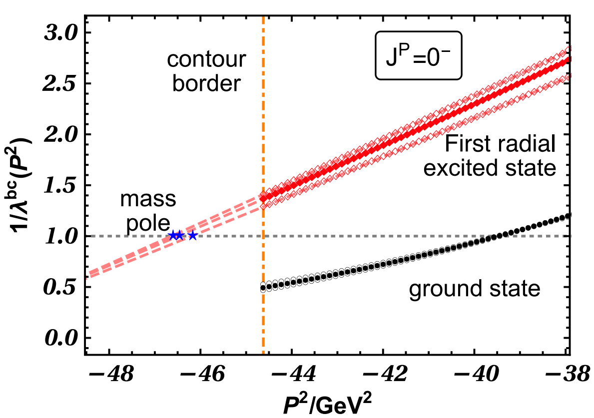

FIG. 2. The calculated P^2 dependence of $1/\lambda^{bc}$ for $J^P = 0^-$ state. The black circles correspond to the ground state, and red diamonds correspond to the first radial excited state. The open circles and diamonds correspond to the varying of the parameters listed in Table I. The vertical dot-dashed line is the contour border on the right of which the direct calculation can be applied. The blue stars present our extrapolated first radial excited state with $N_o = 1$.

TABLE II. The obtained masses (in GeV) of the first radial excited states of charm-beauty system. The experimental data for $M_{\eta_c(2S)}$, $M_{\psi(2S)}$, $M_{\eta_b(2S)}$, and $M_{\Upsilon(2S)}$ are taken from Ref. [27], $M_{B_c^+(2S)}$ and $M_{B_c^+(2S)} - M^{\rm rec}_{B_c^{*+}(2S)}$ from Ref. [2]. The uncertainties of our results arise from the varying of the parameters in Table I.

	$M_{\eta_c(2S)}$	$M_{\psi(2S)}$	$M_{\psi(2S)} - M_{\eta_c(2S)}$
Present work	3.606(18)	3.645(18)	0.039
Experiment	3.638(1)	3.686(1)	0.048
	$M_{B_c^+(2S)}$	$M_{B_c^{*+}(2S)}$	$M_{B_c^+(2S)} - M^{\rm rec}_{B_c^{*+}(2S)}$
Present work	6.813(16)	6.841(18)	0.039
Experiment	6.872(2)	$\cdots$	0.031
	$M_{\eta_b(2S)}$	$M_{\Upsilon(2S)}$	$M_{\Upsilon(2S)} - M_{\eta_b(2S)}$
Present work	9.915(15)	9.941(15)	0.026
Experiment	9.999(4)	10.023(1)	0.024

The obtained masses of the first radial excited state of the charm-beauty system are listed in Table II [in addition, we have $M_{B_c^{+*}(1S)} = 6.357(3)$ GeV]. The average of the results with the three sets of parameters is quoted as final result and the uncertainties are set from the difference between the average and the largest and smallest value respectively. The excited meson masses increase with the value of parameter ω, showing more sensitivity than the ground state as being pointed out by others (see, e.g., Ref. [31]). The relative errors of our results to the experimental date are within 1%. What is more, the mass differences of the vector meson and pseudoscalar meson, $M_{\psi(2S)} - M_{\eta_c(2S)}$ and $M_{\Upsilon(2S)} - M_{\eta_b(2S)}$, are also comparable with the experimental values. The mass splitting, $M_{B_c^+(2S)} - M^{\rm rec}_{B_c^{*+}(2S)}$, is consistent with the recent measurement [2]. There is no experimental measurements of $M_{B_c^{+*}(1S)}$ and $M_{B_c^{+*}(2S)}$ hitherto, our predications wait for future experimental verification.

In addition, it is known that to the first order in the violation of unitary symmetry $SU_f(5)$, the masses obey the equal spacing rule [32,33]:

$$(M_{\eta_c(2S)} + M_{\eta_b(2S)})/2 = M_{B_c^+(2S)}, \tag{15}$$

$$(M_{\psi(2S)} + M_{\Upsilon(2S)})/2 = M_{B_c^{*+}(2S)}. \tag{16}$$

Our results show that the two sides of Eqs. (15) and (16) differ by only 0.05 GeV which is also consistent to the proposal of the mass inequality in Ref. [34].

An illustration of the extrapolation of the decay constants is displayed in Fig. 3, which is the case of B_c^+. The physical value is extrapolated and presented by the blue stars.

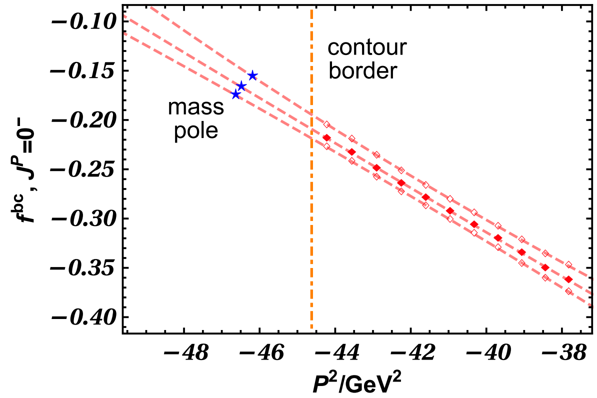

FIG. 3. Calculated P^2 dependence of the f^{bc} of the first radial excited $J^P = 0^-$ state. The open diamonds correspond to the varying of the parameters in Table. I. The vertical dot-dashed line is the contour border on the right of which the direct calculation can be applied. The blue stars present our extrapolated value with $N_o = 1$.

The series Eq. (14) for the leptonic decay constants also converges very rapidly, a good fitting is obtained also for $N_o = 1$, with $\delta_{N_o=1} \approx 5\times10^{-7}$ GeV^2. With higher orders in Eq. (14), $\delta_{N_o=2} \approx 4\times10^{-7}$ GeV^2, $\delta_{N_o=3} \approx 4\times10^{-7}$ GeV^2. The corrections from higher orders are much less than that from the varying of the parameters in Table I. Our predication of the decay constants of the first radial excited charm-beauty mesons are listed in Table III. The uncertainty of our results correspond to the results of the varying of parameters in Table I. The absolute value of the decay constant of excited state is suppressed comparing to that of the ground state, which agrees with the previous findings [11,35–37]. The difference of the decay constants between the excited and ground states decreases as the meson mass increases.

At last, we remark on the error of the RL truncation. The RL truncation is the lowest order approximation, where the repulsive interaction appearing at higher orders is ignored [38]. The net effect of the higher order terms on the ground states of pseudoscalar and vector mesons is small, guaranteeing the successfulness of the RL truncation for the properties of the ground states [22]. To estimate the error for the radial excited states quantitatively, we need to resort to more sophisticated truncation schemes, which are technically complicated. Herein we assure readers of the reasonability of the RL results by comparing the masses in Table II (the RL truncation underestimates the masses about 1%). We could expect that the predicted decay constants in Table III are also underestimated. However, as the decay constant depends on the inner structure of the meson, the higher order corrections could be larger. The decay constants of vector mesons could be extracted by

$$\Gamma_{V\to e^+e^-} = \frac{8\pi e_f^2 \alpha_{\rm em}^2 f_{1^-}^2}{3M_{1^-}}, \tag{17}$$

where $\alpha_{\rm em} = 1/137$, $e_c = 2/3$, $e_b = -1/3$. $\Gamma^{\rm expt}_{V\to e^+e^-} = 2.34\pm0.04$ keV for $V = \psi(2S)$ and 0.612 ± 0.011 keV for $V = \Upsilon(2S)$ [27]. Comparing with $f^{\rm expt}_{\psi(2S)} = -0.208(2)$ GeV and $f^{\rm expt}_{\Upsilon(2S)} = -0.352(2)$ GeV, one can know that the RL truncation underestimates the decay constants about 42% for $\psi(2S)$ and 12% for $\Upsilon(2S)$. Because the interaction of Eqs. (5)–(6) takes into consideration the flavor dependence, we expect the higher order corrections decrease smoothly as the meson mass increases. $f_{B_c^{*+}(2S)}$ is underestimated roughly by $(42\% + 12\%)/2 = 27\%$, while the higher order correction to $f_{B_c^+(2S)}$ is expected to be smaller than $f_{B_c^{*+}(2S)}$.

V. CONCLUSION

Very recently, CMS and LHCb reported the observation of two excited B_c states with high precision [1,2]. Although they are the *normal* states within the quark model regime, it is claimed that the precision measurements open up an opportunity for the study of hadron physics based on the *ab initio* theory of strong interactions. In this work, making use of a scattering kernel expressing the flavor dependent quark-quark interaction and developing an extrapolation method, we produce for the first time the masses and the leptonic decay constants of the first radial excited charm-beauty mesons, $B_c^+(2S)$ and $B_c^{*+}(2S)$, in the Dyson-Schwinger equation approach of QCD directly. The obtained masses agree with the recent observations of CMS and LHCb collaborations excellently and the mass splitting $M_{B_c^+(2S)} - M^{\rm rec}_{B_c^{*+}(2S)}$ is consistent with the experimental result very well. The obtained masses of the charm-beauty system also satisfy the equal spacing rule relation approximately. Furthermore our predicted leptonic decay constants are also quite reasonable and may shed light on the future experimental detection.

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TABLE III. Our predications of the decay constants (in GeV) of the first radial excited charm-beauty mesons. The uncertainties correspond to the result of varying the parameters in the scope listed in Table I.

$f_{\eta_c(2S)}$	$f_{\psi(2S)}$	$f_{B_c^+(2S)}$	$f_{B_c^{*+}(2S)}$	$f_{\eta_b(2S)}$	$f_{\Upsilon(2S)}$
−0.097(2)	−0.119(6)	−0.165(10)	−0.161(7)	−0.310(5)	−0.320(6)

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