# Robustness of deformed catlike states under dissipative decoherence

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The generation of coherent superposition of distinct physical systems and the construction of robust entangled states under decoherence are the most experimental challenges of quantum technologies. In this work, we investigate the behaviors of catlike states of a deformed harmonic oscillator under dissipative decoherence. Varying the deformation parameters, we obtain catlike states having more resistance against decoherence than catlike states of the ordinary harmonic oscillator. Furthermore, we study nonclassical properties and entanglement of different catlike states subjects to decoherence caused by a dissipative interaction with a large environment. Depending on different parameters of the deformation, we reveal that the nonclassical properties of catlike states under dissipative interaction can be more retarded and preserved in the time.

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## I. INTRODUCTION

Coherent states or quasiclassical states of the harmonic oscillator were first introduced by Schrödinger in order to make a connection between the classical and quantum formalisms [1]. It has been shown that these states not only minimize the Heisenberg uncertainty inequality for position and momentum operators but also maintain maximum localizability during the time evolution of the harmonic oscillator [2,3]. In particular, their dispersions on kinetic energy and on potential energy are identical [2]. The importance and the physical applicability of these states, in quantum optics, have been investigated in many works including the papers of Glauber, Klauder, and Sudarshan [4–7]. The concept of coherent states has not been restricted to only the harmonic oscillator and has been implemented to other physical systems [8–10]. In this way, these states are called nonlinear coherent states and have been constructed for certain physical systems [10–12]. Furthermore, it has been shown that coherent states can be built for any Lie symmetry [13–15]. Particularly, the su(1,1) coherent states related to the SU(1,1) group and the su(2) coherent states have been constructed and applied in quantum optics and in condensed matter physics [3,13,16]. It is worth noting that there are two different approaches to elaborate su(1,1) coherent states. The first one is called Perelomov approach which is based on the application of the displacement operator on the ground state [13]. The second one is associated with Barut-Girardello coherent states defined as eigenstates of the annihilation operator of the su(1,1) algebra [17]. Several mathematical generalizations of coherent states have been introduced [2,10,18–25]. In particular, the generalized Heisenberg algebra (GHA) nonlinear coherent states have been constructed [10] and studied for several physical systems including the square well potential [10], the Pöschl-Teller potential [11] and Morse potential [19].

Recently, considerable attention is paid to the experimental observations of nonclassical properties of quantum systems such as squeezing [26], photon antibunching [27,28] and entanglement [29]. The latter is recognized as a key resource of quantum computing, quantum cryptography, and quantum communications [30,31]. In contrast to classical systems whose states are always a mixture, quantum systems can be prepared in a superposition of quantum states. This property is an interesting quantum effect being very hard to be prepared and detected due to the decoherence occurring when the quantum system interacts with a relevant environment [32]. This interaction causes the loss of coherence of the quantum system generating classical mixture states. This mapping is called the decoherence [32]. The process of decoherence has been studied both theoretically and experimentally [33–39] for several physical systems such as the trapped ion in many works [40–44]. The main aim of this work is to contribute to these activities by investigating the generalized su(1,1) coherent states superposition for a four-parameter perturbed oscillator [45,46] known as catlike states or even and odd coherent states. Then, we study their nonclassical properties under decoherence in order to compare them with those of the generalized Heisenberg algebra catlike states. In particular, we discuss the robustness of

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generalized catlike states against decoherence caused by the interaction with an environment associated with an infinite collection of harmonic oscillators by using the fidelity. Concretely, we show that the resistance of generalized catlike states depends on the corresponding algebraic structure. Furthermore, we study the entanglement degree of generalized catlike states under decoherence.

This paper is organized as follows: In Sec. II we briefly recall the GHA and the generalized su(1,1) algebra and construct the generalized su(1,1) catlike states for a perturbed harmonic oscillator. Furthermore, in Sec. III we study the resistance against decoherence of the constructed catlike states with that of the GHA catlike states. Moreover, in Sec. IV we study the effect of the decoherence on photon distribution function, statistical distribution and quantum entanglement of generalized catlike states in terms of various parameters of the deformation. Finally, our conclusions are given in Sec. V.

## II. GHA, GENERALIZED SU(1,1) ALGEBRA AND COHERENT STATES SUPERPOSITION

### A. Generalities on GHA

To start, we give a concise review on the GHA by presenting its essential aspects [47–50]. It is recalled that the GHA is described by the generators $H$, $A^\dagger$ and $A$ satisfying

$$HA^\dagger = A^\dagger f(H), \qquad AH = f(H)A, \tag{1}$$

and

$$[A, A^\dagger] = f(H) - H, \tag{2}$$

where $H = H^\dagger$ is the dimensionless Hamiltonian of the physical system under consideration. $(A^\dagger)^\dagger = A$. $f$ is an analytical function of $H$, called the characteristic function of the GHA. The associated Casimir operator reads as

$$C = A^\dagger A - H = AA^\dagger - f(H). \tag{3}$$

The irreducible representation of the GHA is given through an eigenvector $|n\rangle$ of the Hamiltonian $H$,

$$H|n\rangle = \varepsilon_n|n\rangle \text{ such that } \varepsilon_{n+1} = f(\varepsilon_n), \quad \text{for } n = 0, 1, \ldots. \tag{4}$$

The Fock space representation of GHA generators is then given in terms of the lowest energy eigenvalue $\varepsilon_0$. The eigenvalue $\varepsilon_n = f^n(\varepsilon_0)$ is just the $n$-iterate of $\varepsilon_0$ under the function $f$ [47]. By using (1)–(3), one can show that

$$A^\dagger|n\rangle = N_n|n+1\rangle, \tag{5}$$

$$A|n\rangle = N_{n-1}|n-1\rangle, \tag{6}$$

where

$$N_n^2 = \varepsilon_{n+1} - \varepsilon_0 = f(\varepsilon_n) - \varepsilon_0. \tag{7}$$

The operators $A^\dagger$ and $A$ are interpreted as the creation and annihilation operators of GHA, respectively. It is noted that the vacuum state condition $A|0\rangle = 0$ is verified. Taking $f(H) = H + 1$, where $H$ is the dimensionless Hamiltonian of the harmonic oscillator, the GHA reduces to the ordinary Heisenberg algebra spanned by $H$ and the bosonic ladder operators $a$ and $a^+$ [47]. Similarly, considering $f(H) = qH + 1$, the relations (1)–(2) recover the $q$-harmonic oscillator algebra [47]. Furthermore, it has been shown that the GHA can be applied for physical systems involving known spectrum such as the square well potential [49], the Morse potential [50,51], and the Pöschl-Teller potential [11,52].

### B. Generalized su(1,1) algebra

The su(1,1) algebra is a primordial structure in physics since it appears in many formalisms [53,54]. Furthermore, the $q$-deformed su(1,1) algebra has been also constructed and applied widely in many areas of physics [55–57]. Moreover, a generalization of the su(1,1) algebra can be constructed by following the same ideas developed for the GHA and the generalized su(2) introduced in [58,59]. The generalized su(1,1) algebra is defined by the Hamiltonian $H$ and the ladder operators $J_+$ and $J_-$ satisfying

$$HJ_+ = J_+ f(H), \qquad J_- H = f(H)J_-, \tag{8}$$

$$[J_+, J_-] = (H - f(H))(H + f(H) - 1), \tag{9}$$

where $J_+^\dagger = J_-$ and $f(H)$ is a given function of the Hamiltonian $H$. By considering $f(H) = H + 1$, this algebra becomes the standard su(1,1) algebra. The Casimir operator has now the following form

$$\Gamma = J_+J_- - H(H-1) = J_-J_+ - f(H)(f(H) - 1). \tag{10}$$

The representations of the generalized su(1,1) algebra generators can be easily deduced. Let $\varepsilon_{n+1} = f(\varepsilon_n)$. Assuming that $J_-|0\rangle = 0$, we can show that

$$J_+|n\rangle = \mathcal{N}_n|n+1\rangle, \tag{11}$$

$$J_-|n\rangle = \mathcal{N}_{n-1}|n-1\rangle, \tag{12}$$

where

$$\mathcal{N}_n^2 = (\varepsilon_{n+1} - \varepsilon_0)(\varepsilon_{n+1} + \varepsilon_0 - 1) \quad \text{for } n = 0, 1, \ldots. \tag{13}$$

Similarly to the GHA, the generalized su(1,1) algebra involves the creation and annihilation operators $J_+$ and $J_-$, respectively.

### C. Construction of generalized su(1,1) coherent states superposition

Here, we would like to construct the catlike states associated with the generalized su(1,1) algebra. For this purpose, we first construct the generalized su(1,1) nonlinear coherent state. We define this state as an eigenstate of the annihilation operator $J_-$

$$J_-|z\rangle = z|z\rangle. \quad (14)$$

Since $J_-$ is a non-Hermitian operator, $z$ is, in general, a complex number. The state $|z\rangle$ is expanded in terms of energy eigenvectors $|n\rangle$ as

$$|z\rangle = \sum_{n=0}^{\infty} c_n |n\rangle, \quad (15)$$

where $c_n$ are complex coefficients satisfying $\sum_{n=0}^{\infty} |c_n|^2 = 1$. The action of $J_-$ on such state, leads to $c_n = \frac{c_0 z^n}{\mathcal{N}_{n-1}!}$ where $\mathcal{N}_n! = \mathcal{N}_n \ldots \mathcal{N}_0$ and $\mathcal{N}_{-1}! \coloneqq 1$. Let $c_0 = N(|z|)$, the state $|z\rangle$ can be rewritten as

$$|z\rangle = N(|z|) \sum_{n=0}^{\infty} \frac{z^n}{\mathcal{N}_{n-1}!} |n\rangle. \quad (16)$$

Imposing the normalization condition $\langle z|z\rangle = 1$, we get

$$N(|z|) = \left( \sum_{n=0}^{\infty} \frac{|z|^{2n}}{(\mathcal{N}_{n-1}!)^2} \right)^{-1/2}. \quad (17)$$

The generalized su(1,1) catlike states can be easily obtained. They are given by

$$|\psi_\pm\rangle = \mathcal{N}_\pm(|z|)(|z\rangle \pm |-z\rangle), \quad (18)$$

where

$$\mathcal{N}_\pm(|z|) = \left( 2 \pm 2(N(|z|))^2 \sum_{n=0}^{\infty} \frac{(-1)^n |z|^{2n}}{(\mathcal{N}_{n-1}!)^2} \right)^{-1/2}. \quad (19)$$

### D. Construction of generalized su(1,1) catlike states for a deformed harmonic oscillator

As an application of the generalized su(1,1) algebra, we consider a perturbed harmonic oscillator having the dimensionless energy spectrum given by

$$\varepsilon_n = n + g(n). \quad (20)$$

The function $g(n)$ reads as

$$g(n) = \frac{an + e}{cn + d}, \quad (21)$$

where $a, b, c, d$ are real parameters different from zero [45]. It is a perturbed function associated with the following conditions

$$|a/c| < 1, \qquad -\frac{4ad - 4ce}{c^2} \geq r - 1 \quad \text{and} \quad \frac{d}{c} > 0, \quad (22)$$

where $r \in [0, 1]$. Subject to such conditions, the spectrum (20) is strictly increasing. In this way, the associated generalized su(1,1) algebra can be constructed. For simplicity reasons, we take $c = 1$.

Taking $n = \varepsilon_n + \gamma(\varepsilon_n)$, the relation (20) gives

$$\varepsilon_{n+1} = n + 1 + g(n+1) = \varepsilon_n + 1 + \delta(\varepsilon_n), \quad (23)$$

where

$$\delta(\varepsilon_n) = \gamma(\varepsilon_n) + g(\varepsilon_n + 1 + \gamma(\varepsilon_n)), \quad (24)$$

implying that

$$f(\varepsilon_n) = \varepsilon_n + 1 + \delta(\varepsilon_n). \quad (25)$$

Thus, the characteristic function of the generalized su(1,1) algebra is given by

$$f(H) = H + 1 + \delta(H). \quad (26)$$

The generalized su(1,1) corresponding to the deformed oscillator can be easily obtained by substituting the function (26) in (8)–(9). It is given as follows

$$[H, J_+] = J_+(1 + \delta(H)), \quad (27)$$

$$[J_-, J_+] = (1 + \delta(H))(2H + \delta(H)). \quad (28)$$

By substituting (20) in (13), we find that

$$\mathcal{N}_{n-1}! = \left[ (\Gamma(d))^2 d(ad + 2de + e) \frac{n!\Gamma(n + d + a - e/d + 1)\Gamma(\alpha + n)\Gamma(\beta + n)}{\Gamma(\alpha + 1)\Gamma(\beta + 1)\Gamma(d + a - e/d + 1)(\Gamma(1 + d + n))^2} \right]^{1/2}, \quad (29)$$

where

$$\alpha = \frac{a + d + 1 + e/d}{2} - \frac{\sqrt{(ad + d^2 + d + e)^2 - 4d(ad + 2de + e)}}{2d}, \quad (30)$$

and

$$\beta = \frac{a + d + 1 + e/d}{2} + \frac{\sqrt{(ad + d^2 + d + e)^2 - 4d(ad + 2de + e)}}{2d}. \tag{31}$$

The normalization factor given in (17) can be easily computed. It can be written as

$$N(|z|) = \left[ {}_2F_3\left( d + 1, d + 1; \alpha, \beta, a - \frac{e}{d} + d + 1; |z|^2 \right) \right]^{-1/2}, \tag{32}$$

where ${}_2F_3(d + 1, d + 1; \alpha, \beta, a - \frac{e}{d} + d + 1; |z|^2)$ is the generalized hypergeometric function. By using (29) and (32), the normalization factor (19) associated with the generalized su(1,1) catlike states can be given by

$$\mathcal{N}_{\pm}(|z|) = \left[ 2 \pm 2 \frac{{}_2F_3(d + 1, d + 1; \alpha, \beta, a - \frac{e}{d} + d + 1; -|z|^2)}{{}_2F_3(d + 1, d + 1; \alpha, \beta, a - \frac{e}{d} + d + 1; |z|^2)} \right]^{-1/2}. \tag{33}$$

By substituting (29) and (33) in (18), we can construct the generalized su(1,1) catlike states for the deformed harmonic oscillator. They are given by

$$|\psi_{\pm}\rangle = \mathcal{N}_{\pm}(|z|)(|z\rangle \pm |-z\rangle). \tag{34}$$

It is worth mentioning that the GHA catlike states can be obtained by replacing $\mathcal{N}_n$ by $N_n$ in $|z\rangle$ and in $\mathcal{N}_{\pm}(|z|)$. In the following, we only consider the catlike states having the form

$$|\psi_{+}\rangle = \mathcal{N}_{+}(|z|)(|z\rangle + |-z\rangle). \tag{35}$$

We will see that the construction of the generalized su(1,1) catlike states for physical systems is advantageous because it gives us the possibility to compare their nonclassical properties under decoherence with the catlike states associated with the GHA and select the ones whose properties are more preserved and retarded in the time. We will examine particularly the deformed harmonic oscillator having the spectrum of the form (20). It is noted that the deformed harmonic oscillator becomes the ordinary harmonic oscillator by considering $a = \frac{1}{2}$ and $e = \frac{d}{2}$. In this case, the GHA catlike states become the ordinary catlike states given by

$$|\psi_{\pm}\rangle = (2 \pm 2e^{-2|z|^2})^{-1/2}(|z\rangle \pm |-z\rangle), \tag{36}$$

where $|z\rangle$ is now the ordinary Glauber coherent state given by

$$|z\rangle = e^{-|z|^2/2} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} |n\rangle. \tag{37}$$

## III. DISSIPATIVE DECOHERENCE OF GENERALIZED SU(1,1) DEFORMED CATLIKE STATES

We now study the interaction between the perturbed oscillator presented above with a large environment composed by an infinite collection of harmonic oscillators. According to [60], the time evolution of the density operator $\hat{\rho}$ of the deformed harmonic oscillator is described by the following master equation

$$\frac{d\hat{\rho}(t)}{dt} = \gamma a \hat{\rho}(t) a^{\dagger} - \frac{\gamma}{2}\{a^{\dagger}a, \hat{\rho}(t)\}, \tag{38}$$

where $\gamma$ is the damping coefficient. $a$ and $a^{\dagger}$ are the annihilation and creation operators of the harmonic oscillator, respectively.

The density operator $\hat{\rho}(t)$ reads as

$$\hat{\rho}(t) = \sum_{j=0}^{\infty} S_j(t)\hat{\rho}(0)S_j^{\dagger}(t), \tag{39}$$

where

$$S_j(t) = \sum_{n=j}^{\infty} \sqrt{\frac{n!}{(n-j)!j!}} \mathrm{e}^{-(n-j)\gamma t/2}(1 - \mathrm{e}^{-\gamma t})^{j/2} |n - j\rangle\langle n|. \tag{40}$$

In the following, we will consider that at $t = 0$, $\hat{\rho}(0) = |\psi_{+}\rangle\langle\psi_{+}|$, where $|\psi_{+}\rangle$ is given in (35). In order to quantify the robustness of catlike states, we use the fidelity defined by

$$F(t) = \mathrm{Tr}(\hat{\rho}(t)\hat{\rho}(0)). \tag{41}$$

This measures how states $\hat{\rho}(t)$ are evolved compared with the initial state $\hat{\rho}(0)$. By using (39), the fidelity (41) of the states (35) becomes

$$F(t) = (\mathcal{N}_{+}|z|)^4 (N(|z|))^4 \sum_{j=0}^{\infty} \sum_{n,m=j}^{\infty} \sqrt{\frac{n!m!}{(n-j)!(m-j)!(j!)^2}} \mathrm{e}^{-(m+n-2j)\gamma t/2}(1-\mathrm{e}^{-\gamma t})^j \times \frac{z^n + (-z)^n}{\mathcal{N}_{n-1}!} \frac{z^m + (-z)^m}{\mathcal{N}_{m-1}!} \frac{z^{n-j} + (-z)^{n-j}}{\mathcal{N}_{n-j-1}!} \frac{z^{m-j} + (-z)^{m-j}}{\mathcal{N}_{m-j-1}!}, \tag{42}$$

where we have considered that $z$ is a real number rather than complex. The fidelity of the generalized Heisenberg algebra coherent states superposition known as GHA catlike states can be easily computed by replacing $\mathcal{N}_{n-1}$ by $N_{n-1}$ in (42). In the Figs. 1(a)–1(c) we show the time evolution of the fidelity of the generalized su(1,1) and GHA catlike states of the perturbed harmonic oscillator having the energy spectrum (20), $\varepsilon_n = n + \frac{an+e}{n+d}$, for different values of deformation parameters $a$, $d$ and $e$. In all numerical simulations we have taken the damping rate $\gamma = 1$. We see from these figures that for the same values of the parameters $a$, $d$, and $e$, the generalized su(1,1) catlike states are always more resistant against decoherence than the GHA ones. In addition, we see that perturbed harmonic oscillators having $a = 0.7$ are more robust to decoherence than the ordinary harmonic oscillator associated with $a = 0.5$ and $d = 2e$. Furthermore, Fig. 1(c) shows that the robustness to decoherence of the perturbed oscillators depends also on the parameters $d$ and $e$. Therefore, we can conclude that the catlike states' resistance to decoherence

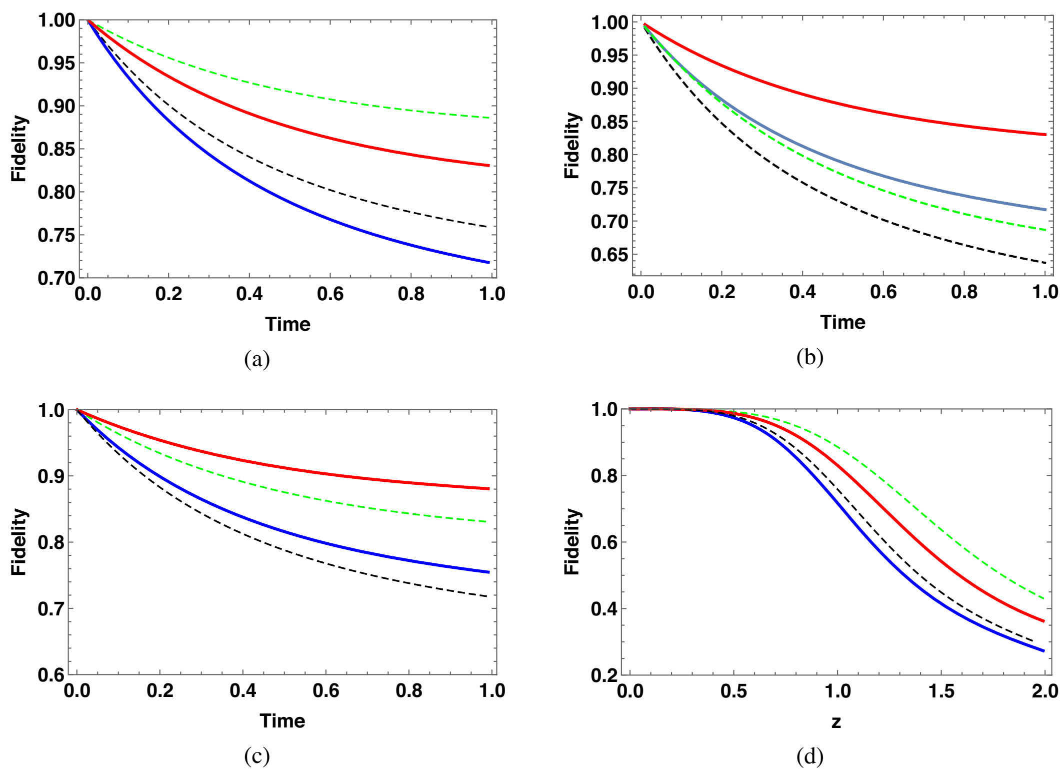


FIG. 1. Fidelity behavior for both GHA and generalized su(1,1) catlike states for the perturbed harmonic oscillator as function of time, the parameter $|z|$ and different parameters of deformation $a$, $d$, and $e$. (a) The time evolution of the fidelity for various catlike states with $|z| = 1$ and $d = 2e = 0.2$. The blue line is for the GHA catlike state with $a = 0.5$, red line is for generalized su(1,1) catlike state with $a = 0.5$, black line is for GHA catlike state with $a = 0.7$ and green line is for generalized su(1,1) catlike state with $a = 0.7$. (b) The time evolution of the fidelity for various catlike states with $|z| = 1$ and $d = 2e = 0.2$. The blue line is for GHA catlike state with $a = 0.2$, red line is for generalized su(1,1) catlike state with $a = 0.2$, black line is for GHA catlike state with $a = 0.5$ and green line is for generalized su(1,1) catlike state with $a = 0.5$. (c) The time evolution of the fidelity for various catlike states with $|z| = 1$ and $a = 0.7$. The blue line is for GHA catlike state with $d = 2e = 0.4$, red line is for generalized su(1,1) catlike state with $d = 2e = 0.4$, black line is for GHA catlike state with $d = 2e = 0.2$ and green line is for generalized su(1,1) catlike state with $d = 2e = 0.2$. (d) The fidelity as a function of $|z|$ for various catlike states with $t = 1$ and $d = 2e = 0.2$. The blue line is for GHA catlike state with $a = 0.5$, red line is for generalized su(1,1) catlike state with $a = 0.5$, black line is for GHA catlike state with $a = 0.7$ and green line is for generalized su(1,1) catlike state with $a = 0.7$.

depends on the corresponding algebra from which they are constructed, i.e., the generalized su(1,1) and the GHA. Varying the different parameters of deformations $a$, $d$, and $e$, we can find perturbed harmonic oscillators which are more resistant to the decoherence caused by the interaction with the environment compared to the non perturbed harmonic oscillator. Figure 1(d) shows how the resistance against decoherence of the perturbed harmonic oscillators varies with the amplitude $|z|$. We see that all catlike states (with different values of the parameter $a$) are more robust against decoherence for very small values of $|z|$. For all catlike states, the fidelity decreases monotonically with the time and tends to zero for long values of time indicating that the final state does not present quantum coherence. Therefore, the final state cannot be written as a coherent superposition of quantum states (loss of coherence). However, quantum coherence of perturbed harmonic oscillators can be more or less resistant to decoherence depending on the choice of the deformation parameters.

## IV. PHYSICAL PROPERTIES OF GENERALIZED CATLIKE STATES UNDER DECOHERENCE

### A. Photon distribution

We now analyze the probability of finding $n$ photons in generalized catlike states constructed in Sec. II D. For a density operator $\hat{\rho}(t)$, this probability is defined by

$$P_n(t) = \mathrm{Tr}(\hat{\rho}(t)|n\rangle|\langle n|). \tag{43}$$

By using the results found in [61,62], the photon distribution function of generalized su(1,1) catlike states can be given by

$$P_n(t) = (N(|z|))^2(\mathcal{N}_+(|z|))^2\sum_{j=0}^{\infty}\frac{(n+j)!}{n!j!}(\mathrm{e}^{-\gamma t})^n(1-\mathrm{e}^{-\gamma t})^j \times\frac{(1+(-1)^{n+j})^2|z|^{2n+2j}}{(\mathcal{N}_{n+j-1}!)^2}. \tag{44}$$

The photon distribution of GHA catlike states can be obtained by replacing $\mathcal{N}_n$ by $N_n$ in (44). In Fig. 2, we show the time evolution of the photon distribution function for both GHA and generalized su(1,1) catlike states in terms of different parameters of the deformation, the amplitude $|z|$ and the number of photons $n$. Analyzing this figure, we can see that the behavior of the time evolution of the photon distribution of catlike states depends on the corresponding algebraic structure (generalized su(1,1) or GHA). We also see that for all catlike states, $P_n(t)$ tends to zero as the time becomes very large. Therefore, the decoherence causes the loss of photons of catlike states which are transferred to the environment. Figures 2(a)–2(c) show that for particular values of $a$, the probability of generalized catlike states is shown to be larger than that of the ordinary harmonic oscillator. From Figs. 2(a) and 2(b), we see that depending on the deformation parameter $a$, we find perturbed oscillators having large photon distribution function.

A close examination of Figs. 2(a) and 2(c) shows that the probability $P_n(t)$ of some GHA catlike states increases to the maximal value. Then, it decreases to its minimal value as the time becomes significantly large indicating the transfer of photons between the system and the bath. Furthermore, from 2(e)–2(f), we find that as the amplitude $|z|$ is large as the time for which $P_n(t) \approx 0$ is maximal for different perturbed catlike states.

### B. Statistical properties of generalized catlike states

The statistical properties of generalized catlike states can be studied by using the Mandel's parameter

$$Q = \frac{\langle(\Delta\hat{n})^2\rangle - \langle\hat{n}\rangle}{\langle\hat{n}\rangle}, \tag{45}$$

where $\langle\hat{n}\rangle$ is the average number of particles in the catlike state in question and $\langle(\Delta\hat{n})^2\rangle = \langle\hat{n}^2\rangle - \langle\hat{n}\rangle^2$ is its variance. If $Q > 0$ ($Q < 0$), we say that the distribution is super-Poissonian (sub-Poissonian) and if $Q = 0$, the distribution is Poissonian. To calculate the Mandel's parameter for generalized catlike states, we use the fact that

$$\langle\hat{n}(t)\rangle = \sum_{n=0}^{\infty} nP_n(t) \quad \text{and} \quad \langle\hat{n}^2(t)\rangle = \sum_{n=0}^{\infty} n^2P_n(t), \tag{46}$$

where $P_n(t)$ is given in (43). In the Fig. 3, we show the time evolution of the Mandel's parameter for both kinds of catlike states for different values of the deformation parameter $a$ with $|z| = 1$. We immediately see that the distribution of all examined catlike states is super-Poissonian and that $Q$ decreases with increasing values of time and vanishes as the time is significantly large. It is recalled that the distribution of the Glauber coherent state is Poissonian, $Q = 0$, and that this coherent state is considered as the most "classical" quantum state for the harmonic oscillator and several measures of nonclassicality of quantum states are related to this coherent state. Then, since for all catlike states, the distribution is super-Poissonian at $t = 0$ and decreases tending to zero for large values of time, we conclude that the dissipative interaction with the relevant environment decreases the super-Poissonian distribution of the catlike states of the perturbed harmonic oscillators and the final state has a Poissonian distribution as the Glauber coherent state, i.e., the final state is a classical state. Furthermore, the larger the value of deformation parameter $a$ is, the larger the value of the Mandel's parameter. This implies that the behavior of the statistical

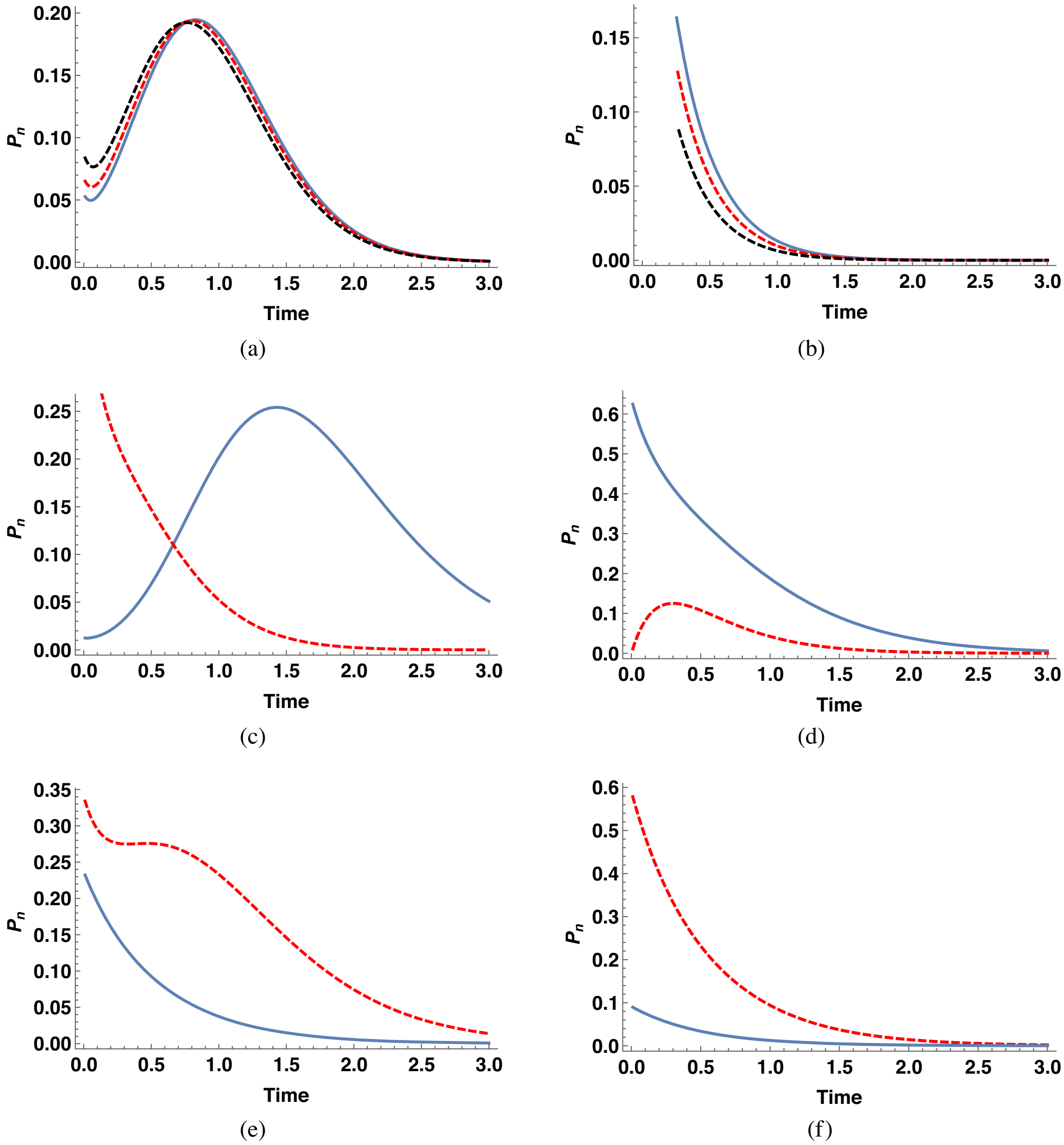


FIG. 2. The time evolution of the photon distribution function of GHA and generalized su(1,1) catlike states for the perturbed harmonic oscillator for various physical parameters. (a) The time evolution of the photon distribution function for GHA catlike states with $|z| = 3$, $n = 4$, and $d = 2e = 0.2$. The blue line is for $a = 0.2$, red line is for $a = 0.5$, and black line is for $a = 0.9$. (b) The time evolution of the photon distribution function for generalized su(1,1) catlike states with $|z| = 3$, $n = 4$ and $d = 2e = 0.2$. The blue line is for $a = 0.2$, red line is for $a = 0.5$, and black line is for $a = 0.9$. (c) The time evolution of the photon distribution function for GHA catlike states with $|z| = 3$, $a = 0.7$ and $d = 2e = 0.2$. The blue line is for $n = 2$ and red line is for $n = 3$. (d) The time evolution of the photon distribution function for generalized su(1,1) catlike states with $|z| = 3$, $a = 0.7$ and $d = 2e = 0.2$. The blue line is for $n = 2$ and red line is for $n = 3$. (e) The time evolution of the photon distribution function for GHA catlike states with $n = 2$, $a = 0.9$ and $d = 2e = 0.2$. The blue line is for $|z| = 1$ and red line is for $|z| = 2$. (f) The time evolution of the photon distribution function for generalized su(1,1) catlike states with $n = 2$, $a = 0.9$ and $d = 2e = 0.2$. The blue line is for $|z| = 1$ and red line is for $|z| = 2$.

distribution of the deformed catlike states depends on the value of the parameter deformation.

### C. Quantum entanglement

Protecting entanglement from decoherence is recognized as a considerable experimental challenge since the entanglement is a key resource for quantum technologies. Thence, several authors have studied the effect of decoherence on entanglement of catlike states for different physical systems. Here, we aim to quantify the amount of entanglement of generalized su(1,1) catlike states constructed in Sec. II for the deformed harmonic oscillator whose spectrum is given in (20) under decoherence caused by the interaction with a large environment composed by an infinite collection of harmonic oscillators. We will study the effect of deformation parameters and amplitude parameter

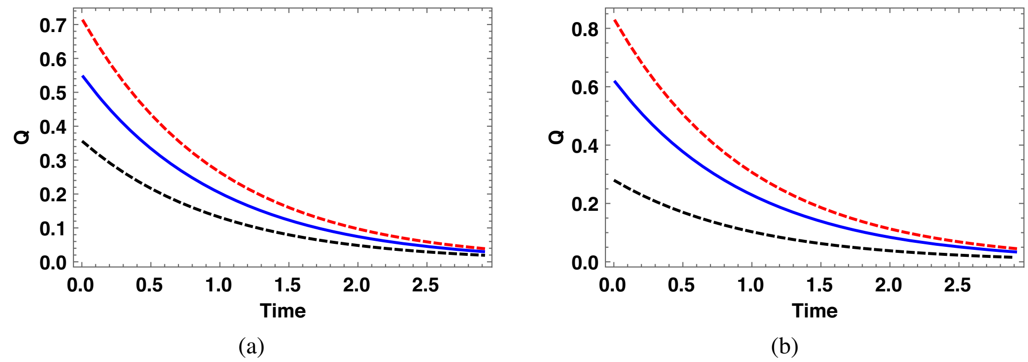


FIG. 3. The time evolution of the Mandel's parameter for GHA and generalized catlike states for the deformed harmonic oscillator. (a) The time evolution of the Mandel's parameter for GHA catlike states with $|z| = 1$ and $d = 2e = 0.2$. The blue line is for $a = 0.5$, red line is for $a = 0.9$ and black line is for $a = 0.2$. (b) The time evolution of the Mandel's parameter for generalized su(1,1) catlike states with $|z| = 1$ and $d = 2e = 0.2$. The blue line is for $a = 0.5$, red line is for $a = 0.9$, and black line is for $a = 0.2$.

$|z|$ on the entanglement of generalized su(1,1) catlike states. For this purpose, we appeal to the von Neumann entropy which is a good quantifier of entanglement. The von Neumann entropy of the density operator $\hat{\rho}$ is given by

$$S(t) = -\mathrm{Tr}(\hat{\rho}(t)\ln(\hat{\rho}(t))) = -\sum_i \lambda_i(t)\ln(\lambda_i(t)), \quad (47)$$

where $\lambda_i$ are the eigenvalues of the reduced density matrix $\hat{\rho}(t)$ of the generalized catlike states. In Fig. 4, we show the behavior of the time evolution of the von Neumann entropy. Immediately, we observe that the entropy increases quickly and reaches its maximum value. Then, it decreases and vanishes for large values of time. Thus, we can conclude that the decoherence affects on the correlations between the catlike states and the environment. The von Neumann entropy vanishes for long values of time $t$ indicates that the state describing the system when time is significantly large, is pure and not entangled with the environment. Interestingly, the entanglement degree is shown to be large as the amplitude $|z|$ is large. It depends also on different parameters of the deformation. This is relevant because it gives us the possibility to find cases in which the entanglement loss is retarded in the time evolution. It is worth to mention that we have treated the von Neumann entropy for higher values of $|z|$ and the entanglement is shown to be larger as $|z|$ increases.

In Sec. IV A, we have seen that the fidelity vanishes also for large values of the time for all generalized catlike states of the perturbed harmonic oscillator. This indicates that in this range of time, $t \gg 1$, the quantum coherence (the interference property) of the state of the perturbed oscillator are lost due to the interaction with the environment. Furthermore, The Mandel's parameter $Q$ and the photon distribution function tend to zero as the time is significantly large. This can be explained by the fact that the interaction of the system with the bath transfers all photons from the system to the environment, as $P_n(t) \approx 0$ when $t \to \infty$, and leaves the system in a vacuum state which has a Poissonian distribution, i.e., $Q \approx 0$ when $t \to \infty$. This state is pure and not entangled. Precisely, it can not be written as a coherent superposition of other quantum states. For such a reason, the fidelity and the von Neumann entropy vanish for large values of time.

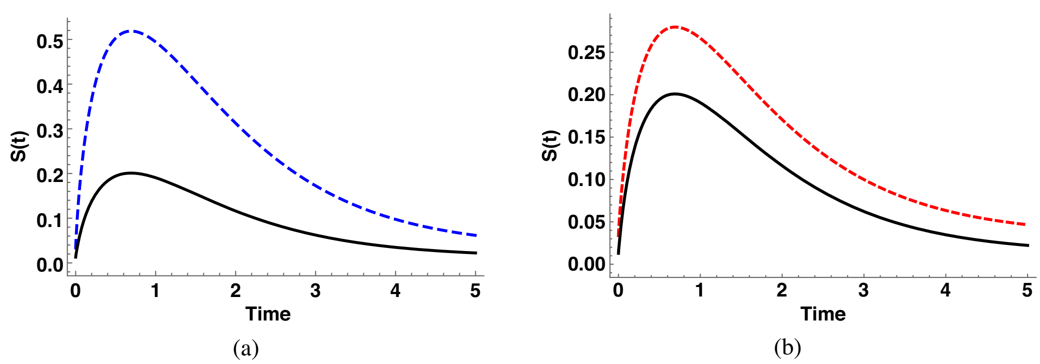


FIG. 4. The time evolution of the von Neumann entropy for generalized su(1,1) catlike states of the perturbed harmonic oscillator in terms of various parameters of deformation and amplitude parameter $|z|$. (a) The time evolution of the von Neumann entropy for generalized catlike states with $a = 0.9$, $d = 0.2$ and $e = 0.1$ for two different values of $|z|$. The blue dashed line is for $|z| = 1.5$, black line is for $|z| = 1$. (b) The time evolution of the von Neumann entropy for generalized catlike states with $|z| = 1$, $d = 0.2$ and $e = 0.1$ for two different values of $a$. The red dashed line is for $a = 0.7$, black line is for $a = 0.9$.

Consequently, the interaction of the perturbed harmonic oscillators with a collection of harmonic oscillators decreases the fidelity, the amount of their entanglement with the environment, the super-Poissonian distribution and the photon distribution function. However, these properties can be preserved and retarded by varying the different parameters of the deformation.

## V. CONCLUSION

In this paper, we have built the catlike states associated with the generalized su(1,1) algebra for a four-parameter deformed oscillator. In terms of different deformation parameters, we have investigated the resistance of these states under decoherence caused by a dissipative interaction with an environment modeled by an infinite collection of harmonic oscillators. Subsequently, we have shown that the generalized su(1,1) catlike states are always more robust under decoherence than the GHA catlike states and that the robustness against the decoherence depends on different parameters of the deformation. Among others, we have found that the different parameters of deformation give the possibility to find cases more resistant than the catlike states of the ordinary harmonic oscillator. It has been revealed that the catlike states' resistance depends on the algebraic structure from which they are constructed. This may open new windows and perspectives to construct catlike states more robust under decoherence and to the experimental observation of nonclassical features of quantum systems. Moreover, we have studied the time evolution of the photon distribution function, the Mandel's parameter and the von Neumann entropy for different catlike states and in terms of different physical parameters. Additionally, we have shown that depending on parameters of deformation and amplitude parameter $|z|$, the photon distribution function and the degree of entanglement of the perturbed oscillators can be more preserved in the time evolution.

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[1] E. Shcrödinger, Der stetige Übergang von der Mikro- zur Makromechanik, Naturwissenschaften **14**, 664 (1926).
[2] J. P. Gazeau, *Coherent States in Quantum Physics* (Wiley-VCH, Dewey, 2009).
[3] W. Zhang, D. Feng, and R. Gilmore, Coherent states: Theory and some applications, Rev. Mod. Phys. **62**, 867 (1990).
[4] R. J. Glauber, Photon Correlations, Phys. Rev. Lett. **10**, 84 (1963).
[5] J. R. Klauder, Continuous-representation theory. I. Postulates of continuous-representation theory, J. Math. Phys. (N.Y.) **4**, 1055 (1963).
[6] J. R. Klauder, Continuous-representation theory. II. Generalized relation between quantum and classical dynamics, J. Math. Phys. (N.Y.) **4**, 1058 (1963).
[7] E. C. G. Sudarshan, Equivalence of Semiclassical and Quantum Mechanical Descriptions of Statistical Light Beams, Phys. Rev. Lett. **10**, 277 (1963).
[8] R. L. de Matos Filho and W. Vogel, Nonlinear coherent states, Phys. Rev. A **54**, 4560 (1996).
[9] J. P. Antoine, J. P. Gazeau, P. Monceau, and J. R. Klauder, Temporally stable coherent states for infinite well and Pöschl-Teller potentials, J. Math. Phys. (N.Y.) **42**, 2349 (2001).
[10] Y. Hassouni, E. M. F. Curado, and M. A. Rego-Monteiro, Construction of coherent states for physical algebraic systems, Phys. Rev. A **71**, 022104 (2005).
[11] M. A. Rego-Monteiro, E. M. F. Curado, and Ligia M. C. S. Rodrigues, Time evolution of linear and generalized Heisenberg algebra nonlinear Pöschl-Teller coherent states, Phys. Rev. A **96**, 052122 (2017).
[12] E. M. F. Curado, M. A. Rego-Monteiro, Ligia M. C. S. Rodrigues, and Y. Hassouni, Coherent states for a degenerate system: The hydrogen atom, Physica (Amsterdam) **371A**, 16 (2006).
[13] A. M. Perelomov, Coherent states for arbitrary Lie group, Commun. Math. Phys. **26**, 222 (1972).
[14] J. R. Klauder and B. S. Skagertan, *Coherent States* (World Scientific, Singapore, 1985).
[15] R. Gilmor, Geometry of symmetrized states, Ann. Phys. (N.Y.) **74**, 391 (1972).
[16] A. Inomata, H. Kuratsuji, and C. C. Gerry, *Path Integrals and Coherent States of SU(2) and SU(1,1)* (World Scientific, Singapore, 1992).
[17] A. O. Barut and L. Girardello, New "coherent" states associated with non-compact groups, Commun. Math. Phys. **21**, 41 (1971).
[18] S. Dong, The SU(2) realization for the Morse potential and its coherent states, Can. J. Phys. **80**, 129 (2002).
[19] M. Angelova, A. Hertz, and V. Hussin, Squeezed coherent states and the one-dimensional Morse quantum system, J. Phys. A **45**, 244007 (2012).
[20] B. Roy and P. Roy, Gazeau–Klauder coherent state for the Morse potential and some of its properties, Phys. Lett. A **296**, 187 (2002).
[21] M. Angelova and V. Hussin, Generalized and Gaussian coherent states for the Morse potential, J. Phys. A **41**, 304016 (2008).
[22] M. Daoud and D. Popov, Statistical properties of Klauder–Perelomov coherent states for the Morse potential, Int. J. Mod. Phys. B **18**, 325 (2004).
[23] J. R. Klauder, K. A. Penson, and J. M. Sixdeniers, Constructing coherent states through solutions of Stieltjes and

Hausdorff moment problems, Phys. Rev. A **64**, 013817 (2001).

[24] M. El Baz, Y. Hassouni, and F. Madouri, New construction of coherent states for generalized harmonic oscillators, Rep. Math. Phys. **50**, 263 (2002).

[25] B. Jurco, On coherent states for the simplest quantum groups, Lett. Math. Phys. **21**, 51 (1991).

[26] R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Observation of Squeezed States Generated by Four-Wave Mixing in an Optical Cavity, Phys. Rev. Lett. **55**, 2409 (1985).

[27] H. J. Kimble, M. Dagenais, and L. Mandel, Photon Antibunching in Resonance Fluorescence, Phys. Rev. Lett. **39**, 691 (1977).

[28] R. Short and L. Mandel, Observation of Sub-Poissonian Photon Statistics, Phys. Rev. Lett. **51**, 384 (1983).

[29] T. Yu and J. H. Eberly, Quantum Open System Theory: Bipartite Aspects, Phys. Rev. Lett. **97**, 140403 (2006).

[30] M. A. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2002).

[31] R. Horo decki, P. Horo decki, M. Horo decki, and K. Horo decki, Quantum entanglement, Rev. Mod. Phys. **81**, 865 (2009).

[32] C. W. Gardiner, *Quantum Noise* (Springer, Berlin, 1991).

[33] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, NJ, 1983).

[34] J. I. Kim, M. C. Nemes, A. F. R. de Toledo Piza, and H. E. Borges, Perturbative Expansion for Coherence Loss, Phys. Rev. Lett. **77**, 207 (1996).

[35] W. H. Zurek, S. Habib, and J. P. Paz, Coherent States via Decoherence, Phys. Rev. Lett. **70**, 1187 (1993).

[36] J. C. Retamal and N. Zagury, Stability of quantum states under dissipation, Phys. Rev. A **63**, 032106 (2001).

[37] N. G. van Kampen, A soluble model for quantum mechanical dissipation, J. Stat. Phys. **78**, 299 (1995).

[38] G. M. Palma, K. A. Suominen, and A. K. Ekert, Quantum computers and dissipation, Proc. R. Soc. A **452**, 567 (1996).

[39] L. Fedichkin, A. Fedorov, and V. Privman, Measures of decoherence, Proc. SPIE Int. Soc. Opt. Eng. **5105**, 243 (2003).

[40] S. Mancini and V. I. Manko, The survival of quantum coherence in deformed-states superposition, Europhys. Lett. **54**, 586 (2001).

[41] R. L. de Mattos Filho and W. Vogel, Nonlinear coherent states, Phys. Rev. A **54**, 4560 (1996).

[42] Z. Kis, W. Vogel, and L. Davidovich, Nonlinear coherent states of trapped-atom motion, Phys. Rev. A **64**, 033401 (2001).

[43] C. Monroe, D. M. Meekhof, B. E. King, and D. J. Wineland, A Schrödinger cat superposition state of an atom, Science **272**, 1131 (1996).

[44] C. J. Myatt, B. E. King, Q. A. Turchette, C. A. Sackett, D. Kelpinski, W. M. Itano, C. Monroe, and D. J. Wineland, Decoherence of quantum superpositions through coupling to engineered reservoirs, Nature (London) **403**, 269 (2000).

[45] E. M. F. Curado, M. A. Rego-Monteiro, and Ligia M. C. S. Rodrigues, Structure of generalized Heisenberg algebras and quantum decoherence analysis, Phys. Rev. A **87**, 052120 (2013).

[46] S. Twareque Ali, Lubka Balkova, E. M. F. Curado, J. P Gazeau, M. A. Rego-Monteiro, Ligia M. C. S. Rodrigues, and K. Sekimoto, Noncommutative reading of the complex plane through Delone sequences, J. Math. Phys. (N.Y.) **50**, 043517 (2009).

[47] E. M. F. Curado and M. A. Rego-Monteiro, Thermodynamic properties of a solid exhibiting the energy spectrum given by the logistic map, Phys. Rev. E **61**, 6255 (2000).

[48] E. M. F. Curado and M. A. Rego-Monteiro, Multi-parametric deformed Heisenberg algebras: A route to complexity, J. Phys. A **34**, 3253 (2001).

[49] E. M. F. Curado, Y. Hassouni M. A. Rego-Monteiro, and Ligia M. C. S. Rodrigues, Generalized Heisenberg algebra and algebraic method: The example of an infinite square-well potential, Phys. Lett. A **372**, 3350 (2008).

[50] Abdessamad Belfakir and Yassine Hassouni, Generalized Heisenberg algebras: Periodicity and finite representation, Phys. Scr. **95**, 055208 (2020).

[51] V. Hussin and I. Marquette, Generalized Heisenberg algebras, SUSYQM and degeneracies: Infinite well and Morse potential, SIGMA **7**, 024 (2011).

[52] F. Bagarello, E. M. F. Curado, and J. P. Gazeau, Generalized Heisenberg algebra and (nonlinear) pseudo-bosons, J. Phys. A **51**, 155201 (2018).

[53] B. G. Wybourne, *Classical Groups for Physicists* (Wiley, New York, 1974).

[54] R. Gilmore, *Lie Groups, Lie Algebras and Some of Their Applications* (Dover Publications, 2012).

[55] R. Chakrabarti and R. Jagannathan, A(p, q)-oscillator realization of two-parameter quantum algebras, J. Phys. A **24**, L711 (1991).

[56] M. Arik and D. D. Coon, Hilbert spaces of analytic functions and generalized coherent states, J. Math. Phys. (N.Y.) **17**, 524 (1976).

[57] A. M. Perelomov, On the completeness of some subsystems of q-deformed coherent states, Helv. Phys. Acta **68**, 554 (1996).

[58] E. M. F. Curado and M. A. Rego-Monteiro, Hidden symmetries in generalized su(2) algebras, Physica (Amsterdam) **295A**, 268 (2001).

[59] E. M. F. Curado and M. A. Rego-Monteiro, Non-linear generalization of the sl(2) algebra, Phys. Lett. A **300**, 205 (2002).

[60] G. J. Milburn, in *Proceedings of the Fifth Summer Schoolon Atomic, Molecular and Optical Physics* (World Scientific, Singapore, 1999), p. 435.

[61] K. Berrada and H. Eleuch, Noncommutative deformed cat states under decoherence, Phys. Rev. D **100**, 016020 (2019).

[62] S. Dey, $q$-deformed noncommutative cat states and their nonclassical properties, Phys. Rev. D **91**, 044024 (2015).