Time travel paradoxes and multiple histories

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If time travel is possible, it seems to inevitably lead to paradoxes. These include consistency paradoxes, such as the famous grandfather paradox, and bootstrap paradoxes, where something is created out of nothing. One proposed class of resolutions to these paradoxes allows for multiple histories (or timelines) such that any changes to the past occur in a new history, independent of the one where the time traveler originated. We introduce a simple mathematical model for a spacetime with a time machine and suggest two possible multiple-histories models, making use of branching spacetimes and covering spaces, respectively. We use these models to construct novel and concrete examples of multiple-histories resolutions to time travel paradoxes, and we explore questions such as whether one can ever come back to a previously visited history and whether a finite or infinite number of histories is required. Interestingly, we find that the histories may be finite and cyclic under certain assumptions, in a way which extends the Novikov self-consistency conjecture to multiple histories and exhibits hybrid behavior combining the two. Investigating these cyclic histories, we rigorously determine how many histories are needed to fully resolve time travel paradoxes for particular laws of physics. Finally, we discuss how observers may experimentally distinguish between multiple histories and the Hawking and Novikov conjectures.

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I. INTRODUCTION

The theory of general relativity, which describes the curvature of spacetime and how it interacts with matter, has been verified to very high precision over the last 100 years. As far as we can tell, general relativity seems to be the correct theory of gravity, at least in the regimes we can test. However, within this theory there exist certain spacetime geometries which feature *closed timelike curves* (CTCs) or, more generally, *closed causal*[1] *curves* (CCCs), thus allowing the violation of causality [1–4]. The fact that these geometries are valid solutions to Einstein's equations of general relativity indicates crucial gaps in our understanding of gravity, spacetime, and causality.

Wormhole spacetimes and cosmological models admitting CTCs were first explored in the decades following the discovery of general relativity [5–7]. Although these spacetimes were clearly unphysical—the wormholes were nontraversable and the cosmologies unrealistic—they were followed, several decades later, by *traversable wormholes*, *warp drives*, and other spacetimes potentially supporting time travel [8–13].

These exotic geometries which allow violations of causality almost always violate the *energy conditions* [14], a set of assumptions imposed by hand and thought to ensure that matter sources in general relativity are "physically reasonable." However, it is unclear whether or not these conditions themselves are justified, as many realistic physical models—notably, quantum fields—also violate some or all of the energy conditions.

In this paper, we consider two types of causality violations: *consistency paradoxes* and *bootstrap paradoxes*. A familiar example of a consistency paradox is the *grandfather paradox*, where a time traveler prevents their own birth by going to the past and killing their grandfather before he met their grandmother. This then means that the time traveler, having never been born, could not have gone back in time to prevent their own birth in the first place.

More precisely, we define a consistency paradox as the absence of a consistent evolution for appropriate initial conditions under appropriate laws of physics. Following Krasnikov [15], "appropriate initial conditions" are those defined on a spacelike hypersurface in a *causal region* of spacetime—that is, a region containing no CTCs—and "appropriate laws of physics" are those which respect locality and which allow consistent evolutions for all initial conditions in entirely causal spacetimes.

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[1]Here by "causal" we mean either timelike or null.

Bootstrap paradoxes arise when certain information or objects exist only along CTCs and thus appear to be created from nothing. These are classified by some as *pseudoparadoxes* because, unlike consistency paradoxes, they do not indicate any physical contradictions arising from reasonable assumptions [3]. Nevertheless, they might make one feel slightly uncomfortable. Information in a bootstrap paradox has no clear origin and does not appear to be conserved, and events can occur which are impossible to predict from data in a causal region of spacetime.[2] Therefore, we explore these pseudoparadoxes as well, identifying the situations in which they do or do not occur in our models.

There exist several paths for addressing the potential causality violations arising from such spacetimes [2]. Two of these rely on quantum effects to resolve time travel paradoxes. The *Hawking chronology protection conjecture* simply suggests that "the laws of physics do not allow the appearance of [CTCs]" [17]. Under this conjecture, quantum effects or other laws of physics ensure that the geometry of spacetime cannot be manipulated to allow CTCs. Deutsch's quantum time travel model resolves paradoxes by modifying quantum mechanics such that the equation of motion is no longer unitary or linear in the presence of CTCs [18].

Two other approaches address causality violations without necessarily appealing to quantum effects. The *Novikov self-consistency conjecture* holds that "the only solutions to the laws of physics that can occur locally in the real Universe are those which are globally self-consistent" [19]. Thus, whether or not CTCs are physically allowed, they can never cause valid initial conditions to evolve in a causality-violating fashion. The *multiple-histories* (or *multiple–timelines*) approach encompasses models which resolve time travel paradoxes by allowing events to occur along different distinct histories.

In this paper, we seek to understand and resolve causality violations classically, so the latter two approaches are of particular interest. In the context of the Novikov conjecture, many systems which at first glance appear to contain consistency paradoxes have in fact been shown to support consistent solutions for all initial conditions [20,21]. Nevertheless, clear paradoxes have been formulated which are incompatible with the Novikov conjecture. In particular, Krasnikov used a toy model with a specific set of physical laws in a causality-violating spacetime to develop such a paradox in [15]. As one of very few concrete examples of true time travel paradoxes in the literature, Krasnikov's model is a natural environment for us to explore the multiple-histories approach.

This exploration serves two purposes. First, the multiple-histories approach has traditionally been presented as a *branching spacetime* model, utilizing non-Hausdorff[3] (or perhaps non-locally-Euclidean [22]) manifolds to allow distinct futures with shared pasts [23,24]. However, the actual mechanics of resolving paradoxes using a branching spacetime has been underdeveloped in the literature, and such constructions present considerable mathematical challenges. Therefore, by constructing two explicit multiple-histories models—one mimicking a branching spacetime and the other utilizing *covering spaces*—we provide concrete examples of the multiple-histories approach.

Second, by demonstrating that these multiple-histories models can prevent the appearance of consistency paradoxes entirely, we show that a Novikov-like conjecture may hold over multiple histories, reconciling the incompatibility between Krasnikov's model and Novikov's conjecture. In particular, this extended Novikov conjecture holds for certain multiple-histories resolutions containing CTCs spanning a finite number of histories. From this perspective, the traditional Novikov conjecture is preserved when paradoxes are absent using only one history.

This paper is organized as follows. First, in Sec. II, we describe the twisted Deutsch-Politzer time machine and Krasnikov's paradox model. Then, in Sec. III, we generalize this model by allowing for additional histories, additional particles, and additional particle "colors."

In Sec. IV we describe our two models of multiple histories, branching spacetimes and covering spaces, in more detail. We show how they both prevent the appearance of consistency and bootstrap paradoxes for any number of particles and colors when an unlimited number of histories is allowed, such that every instance of time travel leads to a new history and a time traveler may never return to a previous history.

In Sec. V we further leverage the covering space model to determine whether a finite number of histories could be sufficient to resolve time travel paradoxes, and if so, how many histories are needed. We prove several useful mathematical results and find a condition on the number of histories required to resolve paradoxes given the number of colors—that the number of histories must be divisible by the number of colors. We also show that, although consistency paradoxes are resolved, bootstrap paradoxes still exist if the histories are cyclic—but they can be avoided by reinterpreting the particle interactions in our model.

In Sec. VI we analyze several aspects of our multiple-histories models. We discuss how, even if the histories are

[2]Even in the absence of consistency paradoxes, CTCs occur in causality-violating regions and thus behind a *Cauchy horizon* [16]. Consider a wormhole whose mouths are surrounded by vacuum and separated more in time than in space. Then an object may, at any time, emerge from the earlier mouth and travel to the later mouth along a CTC, in a way unpredictable from outside the causality-violating region.

[3]A topology satisfies the *Hausdorff condition* (or "is Hausdorff") if and only if for any two distinct points $x_1 \neq x_2$ there exist two open neighborhoods $\mathcal{O}_1 \ni x_1$ and $\mathcal{O}_2 \ni x_2$ such that $\mathcal{O}_1 \cap \mathcal{O}_2 = \varnothing$.

cyclic, an extended Novikov conjecture can still hold over a closed causal curve connecting all of the histories together, resulting in physical observations combining those expected from the Novikov conjecture with those found in multiple-histories resolutions. Furthermore, we explore how it might be possible to experimentally distinguish—at least in principle—between the Hawking, Novikov, branching, and covering space scenarios.

Finally, in Sec. VII we summarize our results and suggest avenues for future exploration.

II. KRASNIKOV'S PARADOX MODEL

A. The Deutsch-Politzer time machine

An early attempt at formalizing a consistency paradox was proposed by Polchinski: perhaps a billiard ball traversing a wormhole time machine might emerge in the past and collide with its past self, ensuring that it cannot enter the wormhole in the first place. However, Echeverria *et al.* found an infinite set of consistent solutions for many reasonable initial conditions, thus showing that this system in fact possesses no paradoxes [20]. Furthermore, it has been shown that no paradoxes exist even when considering more general physical possibilities [25].

A similar construction was attempted using the *Deutsch-Politzer (DP) space* [18,26] in [27], and although this construction was shown to be flawed in [28], a modification of this construction known as the *twisted Deutsch-Politzer (TDP) space* was used in [3,15] to construct a more compelling paradox.

In $1+1$ spacetime dimensions with coordinates (t, x), the DP space is constructed by associating the line $(1, x)$

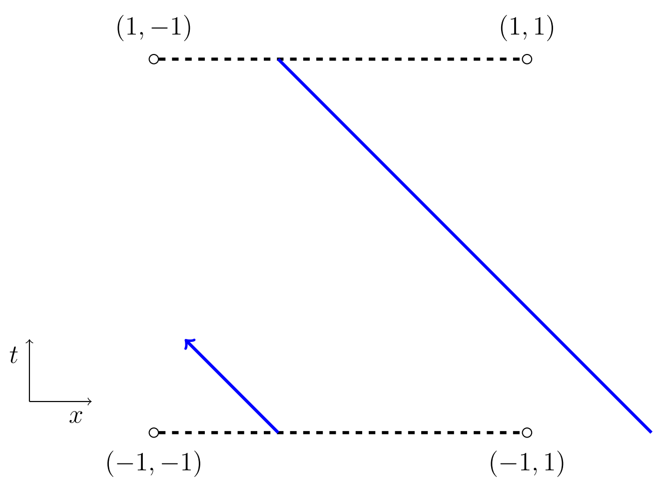

FIG. 1. In the DP space, the line $(1, x)$ is associated with $(-1, x)$ for $-1 < x < 1$ in Minkowski space. This is a simplified model for a wormhole time machine [2]. After traversing the wormhole, the particle emerges at an earlier value of t and travels in the same direction in x.

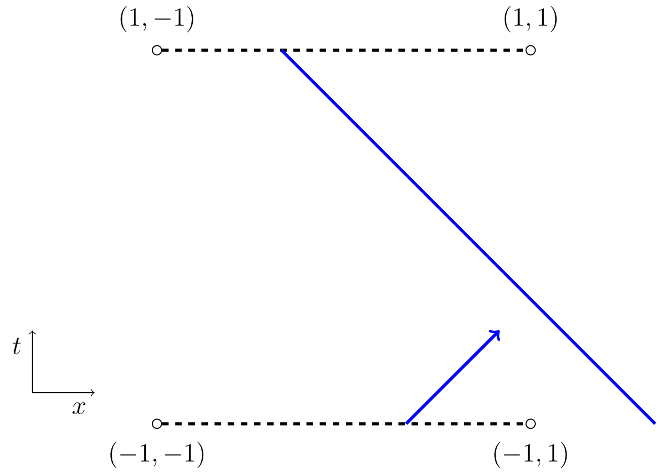

FIG. 2. In the TDP space, $(1, x)$ is instead associated with $(-1, -x)$ for $-1 < x < 1$. After emerging from the wormhole, the particle will travel in the opposite direction in x.

with $(-1, x)$ for $-1 < x < 1$ in Minkowski space. The TDP space is constructed in an analogous way by instead associating the line $(1, x)$ with $(-1, -x)$ for $-1 < x < 1$. This means that particles entering the line at $t = 1$ will emerge at $t = -1$ "twisted," that is, with their spatial orientation inverted. In both cases, the associated lines act as mouths of a wormhole. The DP and TDP spaces are illustrated in Figs. 1 and 2.

In both spacetimes, there must be singularities at $(t, x) = (\pm 1, \pm 1)$, as these points cannot be included without violating the Hausdorff condition [3]. At all other points, the spacetimes are flat, and we can use the same coordinates we used in the original Minkowski space, as long as we recognize that $(1, x)$ and $(-1, \pm x)$ (with plus in the case of DP and minus in the case of TDP) refer to the same points for $-1 < x < 1$ [26]. In this paper, we will

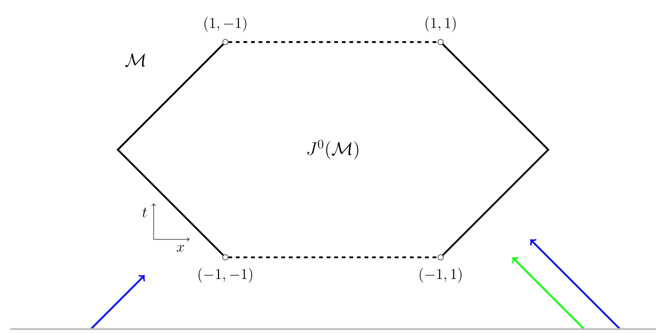

FIG. 3. The causality-violating region $J^0(\mathcal{M})$ for the TDP space $\mathcal{M}$ is contained between the two associated lines in x. The gray spacelike line indicates a choice of a reasonable surface on which to define initial conditions. We also see particles of two different colors, blue and green, emerging from the right and left; the meaning of these colors is explained in Sec. II B.

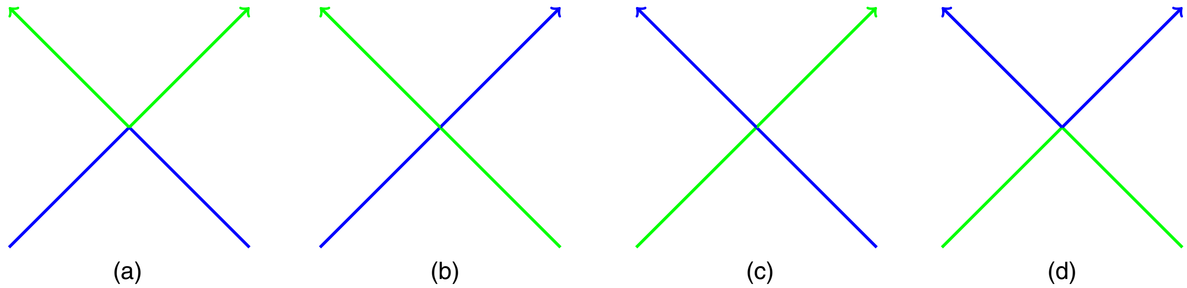

FIG. 4. The four possible distinct vertices for particle collisions in Krasnikov's model. Time is the vertical axis, so the particles always come from the bottom. Note how each blue particle changes into a green particle, and vice versa, in every collision.

ignore the presence of the singularities for the sake of simplicity, motivated by the fact that traversable wormholes in $3 + 1$ dimensions, for which the DP and TDP spaces are a toy model, do not in general possess singularities.

The *causality-violating region*, denoted $J^0(\mathcal{M})$, where $\mathcal{M}$ is the spacetime manifold, is the set of all points p which are connected to themselves by a closed causal curve. Each such point is in its own future and past. This is depicted for the TDP space in Fig. 3.

B. Particles and interaction vertices

Krasnikov [3,15] constructs a paradox in the TDP space by introducing point particles accompanied by a set of physical laws:

(1) The particles are massless and thus follow null geodesics.[4]
(2) Whenever two particle worldlines intersect, the two particles interact. This interaction can be interpreted as an elastic collision, with each particle flipping its direction of movement. Later we will see that this can lead to bootstrap paradoxes and suggest a different interpretation, where particles instead go through each other, continuing in the same direction they were going.
(3) Each particle has one of two colors.[5] In every interaction, each particle flips its color (independently of the color of the other particle), as illustrated in Fig. 4.

The first law considerably simplifies the discussion by allowing us to ignore timelike paths, and the second follows the spirit of Polchinski's paradox. However, these two laws alone still permit consistent solutions analogous to those that have been found for the Polchinski paradox, so the third law is introduced to prevent this.[6] These physical laws respect locality and allow consistent evolution for all initial conditions in entirely causal spacetimes. Importantly, one must also assume that the particles are all test particles and do not influence the geometry of spacetime via Einstein's equation.

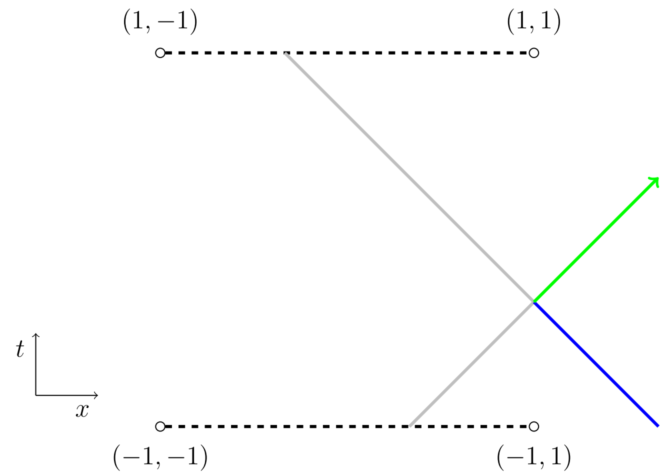

FIG. 5. An illustration of the consistency and bootstrap paradoxes in Krasnikov's model. The blue and green lines represent the two possible particle colors, as above. The gray lines indicate a particle which *cannot* be assigned a consistent color.

We can unite all four possible vertices into a more readily generalizable form by enumerating the two colors as $0, 1 \in \mathbb{Z}_2$ so that a particle's color increases by 1 (mod 2)

[4]Note that this means that we should discuss CCCs (closed causal curves, where here "causal" means either timelike or null) and not CTCs (closed timelike curves), although both types exist in the TDP space.

[5]This property is named "flavor" in [15] and "charge" in [3]. Here we adopt the name "color" in order to make the visualization clearer, and also to avoid the impression that this quantity is conserved.

[6]Krasnikov also considers that particles appearing from the singular points $(t, x) = (-1, \pm 1)$ may allow for consistent solutions, and introduces a fourth law to prevent this. This law adds another property—named "color" in both [15,3], but *not to be confused* with the property we call "color" here—such that particles interact only with other particles of the same color, and the color itself never changes. Having three such colors is sufficient for the purpose of preventing consistent solutions since there are two singularities, so they can produce consistent solutions for at most two of the colors. In this paper, we will ignore the singularities for the sake of simplicity, and thus the only property we will need is the one defined in law number 3.

after each collision. Using these physical laws, both types of paradoxes—consistency and bootstrap—are illustrated in Fig. 5.

First, the particle emerging from the time machine (in gray) ends up falling into the time machine again, so it appears out of nowhere and exists only within the CCC—causing a bootstrap paradox. Second, when it collides with the particle coming from the causal region (in blue), both must flip color. For the blue particle, this is not a problem—it simply changes into a green particle. However, if the gray particle were initially blue, then it would have to change into green, but this means it would enter the time machine as a green particle and exit as a blue particle—which is an inconsistency. Of course, the same inconsistency also applies if the gray particle is initially green. Therefore, there is no choice of color which is consistent along the particle's entire path—thus, we have a consistency paradox.

III. GENERALIZING THE MODEL

We now generalize Krasnikov's model in three ways. First, in order to resolve the paradoxes, we introduce the possibility of multiple histories. Next, to make sure we are considering *all* possible initial conditions in this model, we introduce an arbitrary number of incoming particles. Finally, to draw broader conclusions regarding multiple-histories resolutions, we extend the model to include additional particle colors.

A. Additional histories

In order to resolve the paradoxes demonstrated in Sec. II B, we seek to extend our spacetime to a larger space where consistent solutions exist. In particular, we seek to extend the TDP space to allow for multiple, connected histories. For such an extension to be reasonable, each history should resemble the TDP space, and all of the histories should be identical outside the causal future of the causality-violating region, where we expect results might differ.

With this assumption, the time traveler can go back in time to any point in the past, and the world they will arrive at will indeed be the *same* world from which they left, up until the moment of arrival. However, as soon as they arrive, they inadvertently change history—even just by their mere presence, whether they want to or not. Additional histories ensure that these changes can occur *independently* of the time traveler's original history.

Initially, it may seem that only one additional history is sufficient to resolve paradoxes—but because each additional history should resemble the TDP space, each introduces a new wormhole, which may then be used to travel back in time once more. Thus, resolving paradoxes over the entire space may require a larger number of histories—perhaps infinitely many. We will discuss the different possibilities in the next sections.

Here we consider two interesting ways to extend the TDP space. We can depict both cases in a similar fashion, using multiple side-by-side copies of illustrations as in Figs. 1, 2, and 5 but associating different regions of spacetime.

First, seeking to mimic the behavior of branching spacetime models, we can associate the line at $t = 1$ in one history with the line at $t = -1$ in the *next* history. If the events in the two histories differ only after the wormhole mouths, then traversing the wormhole would have the appearance of traversing a branching spacetime. An observer would, upon traversing the time machine, appear in a new "branch" of the Universe. The past of this branch would match the observer's expectations, but the future could be changed without causing an inconsistency.

A drawback of this approach is that the first history no longer resembles the original TDP space since it has only one wormhole mouth instead of two—there cannot be an *exit* to the time machine in the first history since there is no previous history for the time traveler to come from. This motivates the use of covering spaces[7] as multiple-histories extensions.

Definition 3.1.—Let B be a topological space. A topological space E is a covering space of B if there exists a continuous surjection (or onto map) $p: E \to B$ such that every $b \in B$ has an open neighborhood U whose preimage $p^{-1}(U)$ is a union of disjoint sets $\{V_\alpha\}$ in E, where each V_α is homeomorphic to U under p; see p. 336 of [29].

This method of extending the TDP space ensures that each history remains faithful to the original topology of the space. In each history, there is a wormhole entering the space and a wormhole exiting the space. To accomplish this, the line at $t = 1$ in one history is associated with the line at $t = -1$ in the next history, as above, but the lines at $t = -1$ in the first history and at $t = 1$ in the second *also* act as wormholes.

More rigorous constructions of specific covering spaces will follow in Secs. IV and V.

B. Additional particles

We also expand the physical system under consideration by considering an arbitrary number of particles. We will explore the general case of m particles approaching from the $-x$ direction and n particles approaching from the $+x$ direction for $m, n \in \mathbb{N}$, assuming that $m \leq n$ (without loss of generality since our spacetime is symmetric under parity transformations $x \mapsto -x$). Considering this general case will allow us to not only demonstrate paradoxes but also prove the absence of paradoxes in certain systems, motivating the utility of the multiple-histories approach.

[7] We thank the anonymous referee for suggesting that we formalize this notion in terms of covering spaces.

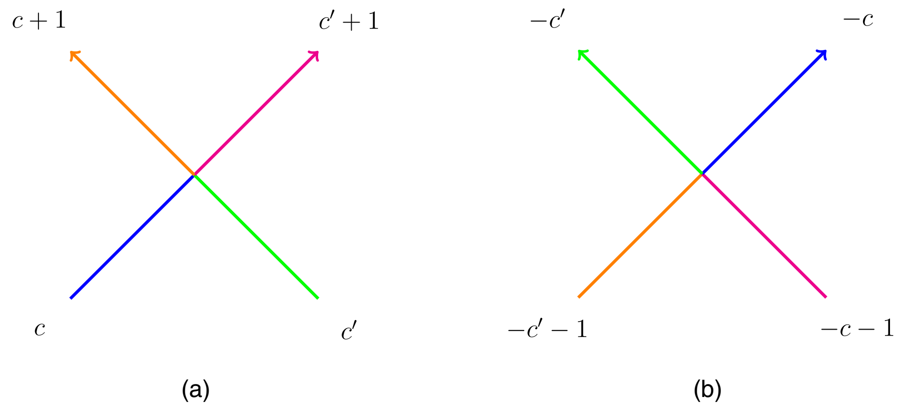

FIG. 6. (a) Given the identification between colors and elements of $\mathbb{Z}_C$, this single general vertex captures all four vertices of Fig. 4 for $C = 2$, as well as those for any other values of C. For illustration, the four colors in the figure—blue, green, orange, and magenta—represent any of the C possible colors for the case $C \geq 4$. (b) This vertex is the result of reversing time and parity and conjugating color with respect to the vertex in (a). Since each particle still leaves with a color one greater than it starts with, the result is a valid vertex. In fact, performing CT or P transformations independently also yields a valid vertex. In this example, we took blue $= 0$, orange $= 1$, green $= 2$, magenta $= 3$, $c = 0$, $c' = 2$, and $C = 4$ in both (a) and (b).

C. Additional colors

In the previous section, we expressed the two possible particle colors as elements of $\mathbb{Z}_2$ and phrased Krasnikov's rule for color evolution as an increase by 1 (mod 2) to the color of each particle after a collision. We can use the same rule for an *arbitrary number* of colors.

Let $C \in \mathbb{N}$; then particle colors are elements of the cyclic group $\mathbb{Z}_C = \{0, \ldots, C-1\}$ and follow the same color evolution rule, increasing by 1 (mod C) after a collision. When $C = 2$, we have Krasnikov's original model. For $C > 2$, we describe a more general system that will be useful for exploring causality violations in a variety of cases. As long as $C \neq 1$, a single incoming particle leads to the same paradoxes as in Fig. 5.

When $C > 2$, particle interactions are no longer time reversible. However, our system has not completely lost its symmetry. In particular, if we define *color conjugation* as mapping a color c to $-c \pmod{C}$ in $\mathbb{Z}_C$, then CT symmetry is satisfied (with C representing color, not charge). In fact, our interactions are symmetric under parity transformations, so the system also satisfies CPT symmetry, as depicted in Fig. 6. Although the colors in each leg of the resulting vertices will in general be different, the system as a whole is invariant under these symmetries.

IV. THE CASE OF UNLIMITED HISTORIES

In the previous sections, we introduced a model, consisting of a particular spacetime with specific physical laws,

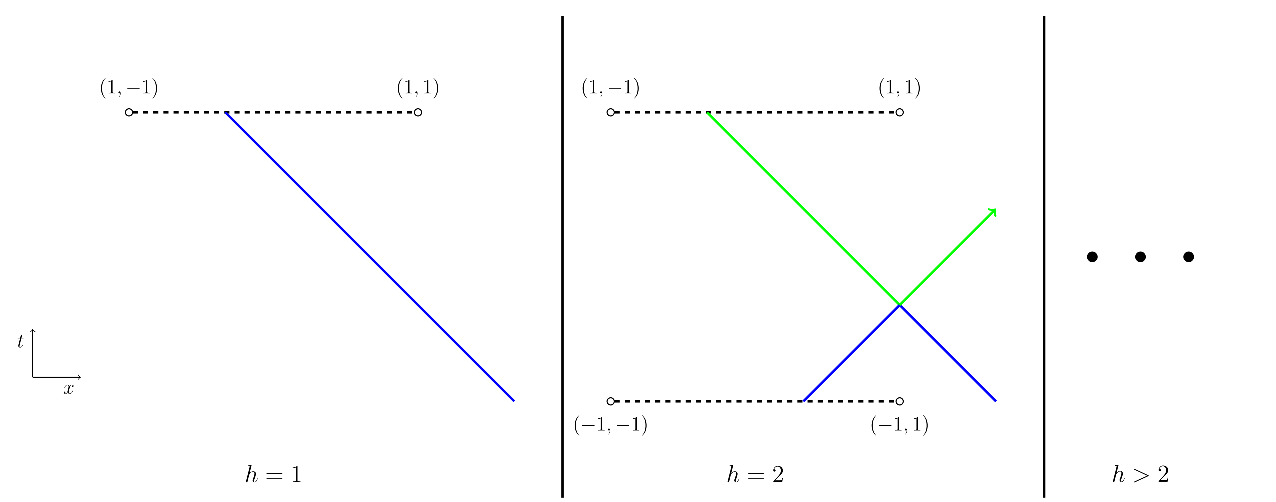

FIG. 7. In the branching model, when the blue particle enters the time machine at $h = 1$, it comes out twisted (since we are in a TDP space) at $h = 2$. The new history has an identical copy of the initial blue particle, but this time it encounters itself (or more precisely, its copy from $h = 1$) and the two particles change their colors. A green particle then enters the time machine and continues to $h = 3$, and so on. Thus, we have avoided both consistency and bootstrap paradoxes.

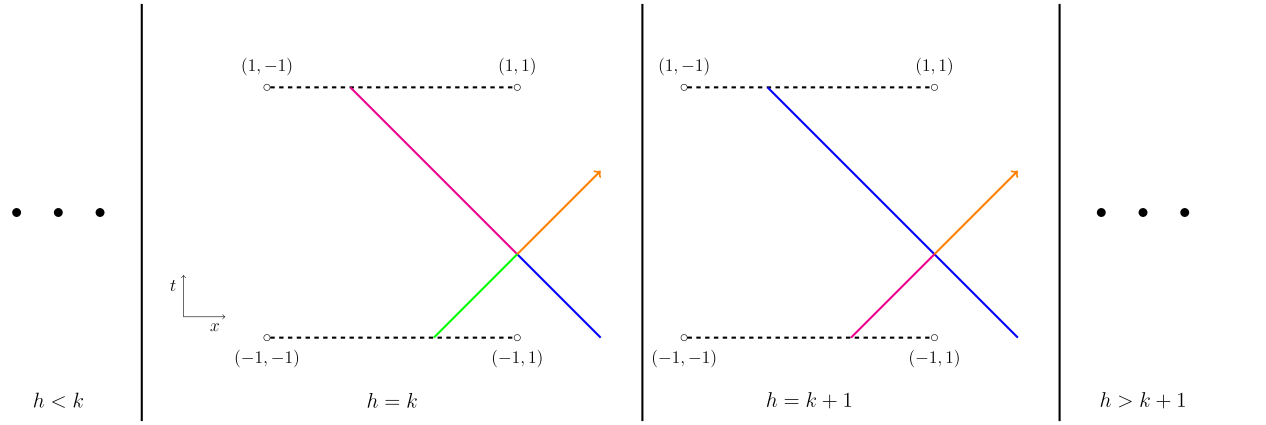

FIG. 8. Unlike the branching model, the covering space model has no unique first history. Therefore, we depict two consecutive histories k and $k+1$. Without loss of generality, a green particle emerges from the time machine in history k, where it collides with the incoming blue particle; here we are using the color convention of Fig. 6. Both particles increase their colors as in Fig. 6: blue $= 0$ to orange $= 1$ and green $= 2$ to magenta $= 3$. In history $k+1$, the same process occurs with a magenta particle emerging from the time machine instead of a green particle, and the magenta particle increases its color to blue $= 4$ (mod 4). Since there is a countably infinite number of time machines, the particle traversing the time machines never completes a CCC, nor does any copy of the incoming blue particle. Thus, we have again avoided both consistency and bootstrap paradoxes.

which admits initial conditions for $C \geq 2$ for which there is no consistent evolution, generating a paradox. However, we also introduced the possibility of multiple histories in some extended space. In what follows, we label these histories with a new parameter $h \in \mathcal{H}$ so that points in the extended space can be described by a triplet (t, x, h). Such a parameter certainly makes sense for branched extensions of the TDP space, where the first space is unique and each subsequent history can be assigned a new label. It also makes sense for a covering space extension since the cardinality of[8] $\{V_\alpha\} = p^{-1}(U)$ is well defined and constant, as the TDP space is connected; see p. 56 of [30].

In this section, we assume that a particle in a given history may never return to the same history after leaving it. As before, let $\mathcal{M}$ be the TDP spacetime manifold. Since the branching model has a unique first history, it is appropriate to define $\mathcal{H} = \mathbb{N}$ and build an extended space $\mathcal{M}'$ composed of a countably infinite number of copies of Minkowski space where $(+1, x, h)$ is associated with $(-1, -x, h+1)$ for all $h \in \mathbb{N}$ and $-1 < x < 1$. This spacetime behaves as in Fig. 7.

In contrast, the covering space model does *not* result in a unique first history. Consequently, it is appropriate to define $\mathcal{H} = \mathbb{Z}$ and to build an extended space $\mathcal{M}'$ composed of a countably infinite number of copies of Minkowski space where $(+1, x, h)$ is associated with $(-1, -x, h+1)$ for all $h \in \mathbb{Z}$ and $-1 < x < 1$. This spacetime behaves as in Fig. 8.

Having constructed this space, we must prove that it is indeed a covering space of $\mathcal{M}$.

[8]Using the notation of Sec. III A.

Proposition 4.1.—The extended space $\mathcal{M}'$ is a covering space of $\mathcal{M}$.

Proof.—Let $\mathcal{M}$ denote our base TDP space and let $\mathcal{M}'$ denote our extended space with a countably infinite number of histories. Furthermore, let $p: \mathcal{M}' \to \mathcal{M}$ be a covering map (or projection), defined by $p(t, x, h) = (t, x)$ for $(t, x, h) \in \mathcal{M}'$. In order to show that $\mathcal{M}'$ is a covering space of $\mathcal{M}$, we need to show that p is both surjective (onto) and continuous, and that each $m \in \mathcal{M}$ is contained in a neighborhood U whose preimage $p^{-1}(U)$ satisfies

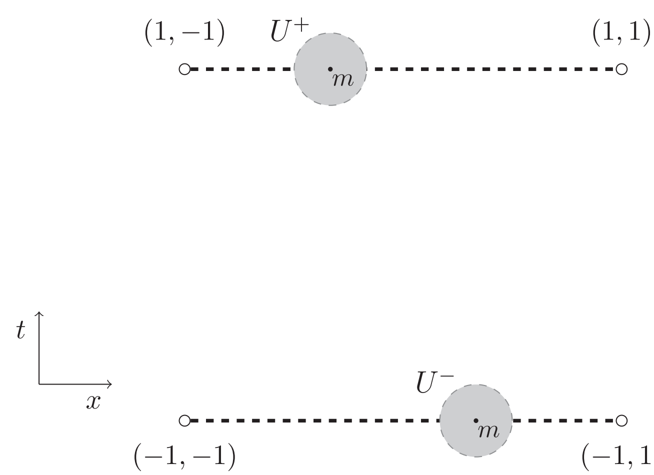

FIG. 9. Since m is a point along the associated wormhole line, it appears twice in our representation of the TDP space—once at $t = -1$ and once at $t = +1$. Therefore, our ball U around m is actually $U = U^+ \cup U^-$, the union of balls around each wormhole mouth. It is always possible to select such a ball which does not intersect a singularity: if m is a distance $\varepsilon > 0$ away from a singularity, then the ball can be chosen to have radius $\varepsilon/2$.

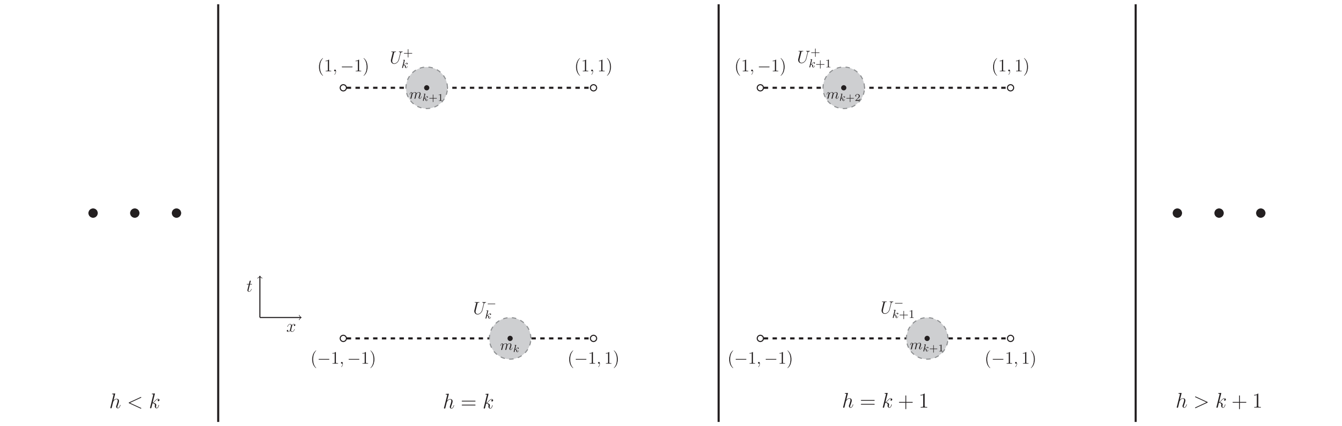

FIG. 10. In our extension of the TDP space, wormhole points are now associated between adjacent histories. As a result, the ball around the point m_{k+1} (the point overlapping histories k and $k+1$, which projects down to m under the map p) is equal to $U_k^+ \cup U_{k+1}^-$. The preimage $p^{-1}(U) = \bigcup_k (U_k^+ \cup U_{k+1}^-)$ is composed of a countably infinite number of such balls, each of which is homeomorphic to $U^+ \cup U^-$ from Fig. 9.

certain constraints. The map p is certainly surjective: for every $(t, x) \in \mathcal{M}$, each (t, x, h) for the various histories h is mapped to (t, x) under p. The map is also continuous, as it projects $\mathcal{M}'$ onto $\mathcal{M}$ without ripping or tearing it.

For the remainder of this proof, we consider two cases: points along the associated wormhole lines and points in the rest of the spacetime.[9] First, let $m = (t, x) \in \mathcal{M}$ be a point along an associated wormhole line, so $t = \pm 1$ and $-1 < x < 1$. Then, let U be a ball around m, small enough that it does not intersect the singularities at $(t, x) = (\pm 1, \pm 1)$. Since m is part of the TDP wormhole, U contains points near *both* wormhole mouths, as depicted in Fig. 9.

The topology of U is that of the union of two open balls which intersect at a line. $p^{-1}(U)$ is composed of a countably infinite number of such sets, now containing points from adjacent histories h and $h+1$, as in Fig. 10. Since the number of histories is infinite so that no particle in a given history may ever return to that same history, the sets in $p^{-1}(U)$ continue in this pattern for all h. These sets are clearly disjoint, and each is composed of two open balls intersecting at a line, so p restricted to each is not only a bijection (since, once p has been restricted, the history data can be ignored, rendering p the identity) but also a homeomorphism (since the topology of each $U_k^+ \cup U_{k+1}^-$ is preserved under p).

Second, let $m = (t, x) \in \mathcal{M}$ be a point that is not on a wormhole mouth, and let U be a ball around m, small enough that it intersects neither the singularities nor the wormhole mouths. Then, U has the topology of a normal ball in flat space. Again, $p^{-1}(U)$ is composed of a countably infinite number of such sets, which are clearly disjoint. Also, p restricted to each set acts as the identity map and is thus a homeomorphism. Consequently, $\mathcal{M}'$ is a covering space of $\mathcal{M}$. ■

This framework allows consistent solutions to our previously paradoxical initial conditions. In both the branching model and the covering space model, particles which would have followed CCCs in one history now traverse multiple histories—and since they may never return to a previous history, they can never complete a closed loop. Consistency paradoxes arise from conditions enforced along closed causal curves, and bootstrap paradoxes arise from particles existing only inside these closed curves; neither situation is possible in the unlimited histories case, and thus both paradoxes are avoided. In other words, we have avoided paradoxes created due to CCCs by simply avoiding any actual CCCs.[10]

V. THE CASE OF FINITE CYCLIC HISTORIES

Above we assumed that h increases monotonically so that the time traveler may never return to a previous history. If there is no limit on how many times time travel can occur —and, indeed, there is no reason for such a limit to exist— then this results in an infinite number of histories. Since returning to a previous history is impossible and thus CCCs never form, it is straightforward to demonstrate the absence of paradoxes.

[9]Recall that the singularities at $(t, x) = (\pm 1, \pm 1)$ have been removed.

[10]Although CCCs are no longer present, this model still gives observers the *appearance* of time travel. Adjacent histories are, by definition, precisely the same up until the point in time when the time traveler exits the time machine. Therefore, the time traveler observes a universe identical to their initial history, but at an earlier point in time—which is, colloquially, the definition of time travel. However, at the moment the time traveler exits the time machine, they have already ensured that this will be a new history simply by existing at a point in spacetime where they did not exist in their previous history.

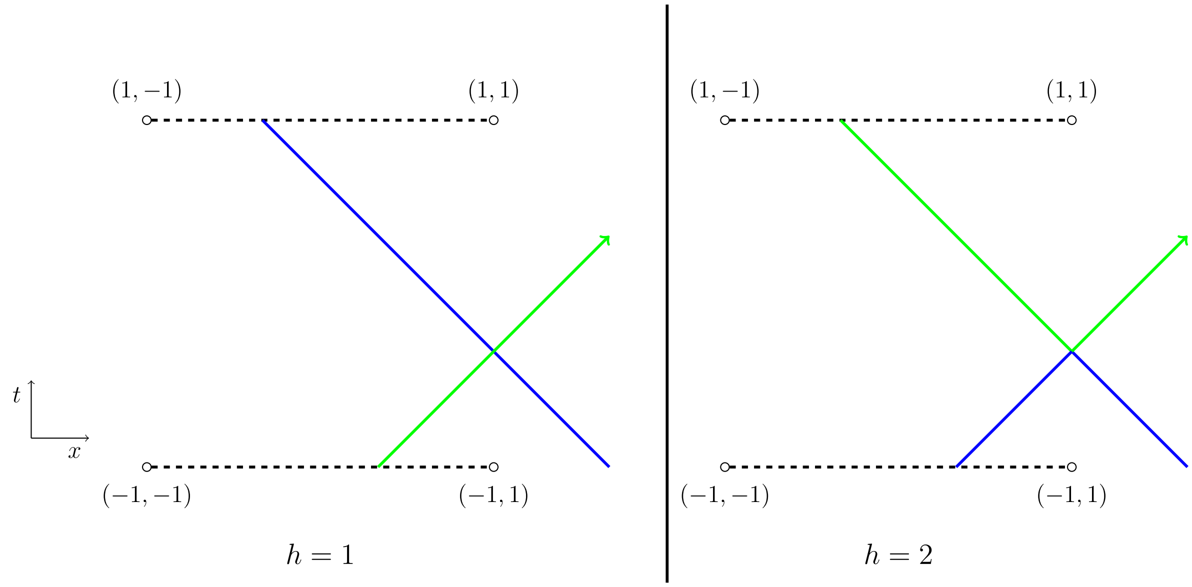

FIG. 11. When $C = 2$, the consistency paradox can be solved with two cyclic histories. The blue particle entering the time machine in $h = 1$ comes out of the time machine in $h = 2$, and the green particle entering the time machine in $h = 2$ comes out of the time machine back in $h = 1$. Since we interpret the vertices as elastic collisions, we now have a bootstrap paradox: the particle traveling along the CCC only exists within the CCC itself. We will discuss how to resolve this in Sec. V B. Unlike in the scenario of Fig. 7, here there is no first history where nothing has come out of the time machine yet (in fact, in Fig. 7 the past exit of the time machine does not even exist for $h = 1$).

However, we will now show that, at least within the covering space model, it is in fact possible for a time traveler to return to a previous history. Specifically, the covering space model provides a framework for histories which are *cyclic*—one can go from the last history back to the first one. If $\mathcal{M}$ again denotes the TDP spacetime manifold, then we construct an extended space $\mathcal{M}'$ composed of $H \in \mathbb{N}$ copies of Minkowski space where points are identified in the following way:

$$(+1, x, h) \leftrightarrow \begin{cases} (-1, -x, h+1) & \text{if } 1 \leq h \leq H-1, \\ (-1, -x, 1) & \text{if } h = H. \end{cases} \tag{1}$$

Proposition 5.1.—The extended space $\mathcal{M}'$ is a covering space of $\mathcal{M}$.

Proof.—This proof is almost identical to the one in Sec. IV. There are only two differences. First, adopting the same notation, $\{V_\alpha\} = p^{-1}(U)$ is composed of only H disjoint sets. Second, sets in $p^{-1}(U)$ still contain points from histories h and $h + 1$ when U intersects a wormhole, but in this case one such set contains points from histories H and 1. As expected, this set is still homeomorphic to U under p. ■

Unlike the case of unlimited histories, this model *does* admit CCCs, with those CCCs spanning all H histories. On the other hand, although there are only a finite number of histories, not an infinity of them, there nevertheless exist consistent solutions to otherwise paradoxical scenarios, as illustrated in Fig. 11. In fact, we will prove in Theorem 5.9 that, with C colors and H histories, paradoxes are completely avoided in this scenario if and only if $C \mid H$.

In Sec. V A, we analyze our model and determine exactly when it resolves time travel paradoxes. In Sec. V B, we discuss a way to resolve bootstrap paradoxes, and in Sec. VI A we explore the general tendency of causality-violating spacetimes to support multiple consistent solutions.

A. How many histories are required to resolve paradoxes?

In this section, we will examine the TDP space in more detail in order to lay the groundwork for the proof of Theorem 5.9. Although initial conditions defined outside the causality-violating region $J^0(\mathcal{M}')$ cannot uniquely determine the physics inside this region (as will be demonstrated in Sec. VI A), we will show that they do determine the trajectories of all the particles in this region.[11] Since all particles follow null trajectories and change direction only in elastic collisions, we can instead think of a set of particle trajectories as straight null lines intersecting at vertices corresponding to collisions. For example, in Fig. 11, the blue path in $h = 1$ is considered to be one path whether the original blue particle continues in

[11]Other than those originating at singular points, as considered by Krasnikov—but as noted above, we will ignore this subtlety here.

the same direction or not after the interaction at the vertex; this path then continues to $h = 2$ and exits to infinity.

Definition 5.2.—A *particle path* is a straight null line in $\mathcal{M}'$ composed of segments from the trajectories of one or more particles.

Using this notion, we seek to connect particle trajectories throughout $\mathcal{M}'$ to appropriate initial conditions, and to show that we need not worry about trajectories varying across different histories or leading to inconsistencies.

Lemma 5.3.—Particle paths in all histories of $\mathcal{M}'$ are completely determined by initial conditions in the causal past of the causality-violating region $J^0(\mathcal{M}')$.

Proof.—First, we show that all paths are extendible to $t = \pm\infty$ in some history. This is certainly true for paths which do not enter the time machine. As for other paths, they may traverse the wormhole only once. Indeed, suppose that a path enters the wormhole at $(t, x) = (1, x_0)$ in one history and exits at $(t, x) = (-1, -x_0)$ in the next. Without loss of generality, we assume that the path then moves along the $+x$ direction. Then the path, parametrized by $\lambda \in \mathbb{R}$, will be such that

$$(t, x) = (\lambda - 1, \lambda - x_0). \tag{2}$$

The path will reach $t = 1$, where the time machine is located, at $\lambda = 2$. However, at this point it will be at $x = 2 - x_0$. The wormhole is located at $x \in (-1, 1)$, so x_0 must be in that range, and in particular $x_0 < 1$. Hence, we see that we must have $x > 1$, and the point of intersection with $t = 1$ is outside of the wormhole. Therefore, a path can never intersect the wormhole twice.

We conclude that all null lines entering the time machine [including the paths of all particles inside $J^0(\mathcal{M}')$] must originate at $t = -\infty$ in some history and, upon traversing the wormhole once, must terminate at $t = +\infty$ in another history. As a consequence, all particles paths in $\mathcal{M}'$ are determined by initial data in the causal past of the causality-violating region $J^0(\mathcal{M}')$ in some history. ■

Corollary 5.4.—Particle paths, and the numbers of collisions the particles on these paths experience, are the same in all histories of $\mathcal{M}'$.

Proof.—By assumption, all histories have the *same* initial data in the causal past of the causality-violating region $J^0(\mathcal{M}')$. In Lemma 5.3 we saw that all particle paths in a history are determined by the initial conditions in that history and the previous history. Since all such initial conditions are the same, the particle paths in each history must also be the same. Furthermore, since these paths fully determine the vertices denoting particle collisions, the number of collisions the particles on these paths experience is the same in each history. ■

Corollary 5.5.—The positions of all particles along CCCs are consistent.

Proof.—Since all particle paths are extendible to $t = \pm\infty$ in some history, no particles appear or disappear in a paradoxical way. Furthermore, since only two null paths can meet at each vertex, there are no particle interactions inconsistent with the laws of physics in this system. ■

All that remains in order to prove the absence of paradoxes is to demonstrate that the colors of particles along CCCs are consistent as well. To do that, we first prove the following lemma.

Lemma 5.6.—The color evolution of particles in $J^0(\mathcal{M}')$ is determined entirely by the choice of m and n [the number of particles entering from the left ($-x$) and the right ($+x$), respectively].

Proof.—Initially, $m + n$ particles of various colors approach $J^0(\mathcal{M}')$ along various trajectories. To show that only m and n affect the color evolution of particles in this region, we must show that neither the positions of the trajectories nor the initial colors impact this evolution.

The initial positions of the particles impact the positions of particles in $J^0(\mathcal{M}')$, but *not* the ordering of vertices—which completely determine how colors change since colors are constant outside of the vertices. Since each null path approaching $J^0(\mathcal{M}')$ traverses the wormhole and then leaves in the same direction that it came from (due to the twist at the wormhole), it is apparent that m particles must leave $J^0(\mathcal{M}')$ in the $-x$ direction and n particles must leave in the $+x$ direction.

Since we are considering a spacetime with one spatial dimension, the spatial ordering of a set of particles or paths cannot change over time, except when it is flipped passing through the wormhole. Thus, the particles which leave $J^0(\mathcal{M}')$ must be the same particles as those which enter it in the first place. Consequently, the remaining particles are *confined* to $J^0(\mathcal{M}')$, and their colors are impacted only by the number of collisions they have with the incoming particles, not by the incoming particles' colors. ■

Lastly, we build some machinery for analyzing the collisions of arbitrary numbers of particles.

Definition 5.7.—A (p, q) particle *group collision* is a set of individual particle collisions arising from the scattering of p particles approaching from the left ($-x$) and q particles from the right ($+x$), as in Fig. 12.

Lemma 5.8.—Let x_k be the color of particle number k counting from the left in a (p, q) particle group collision where $p < q$. Then, after the collision, the particles' colors are given by

$$x'_k = x_k + \begin{cases} 2(k-1) + 1 & k \le p, \\ 2p & p < k \le q, \\ 2(p + q - k) + 1 & k > q. \end{cases} \tag{3}$$

Proof.—Since p lines cross with q lines in one of these collisions, $p \times q$ vertices arise, as illustrated in Fig. 12. If we assign each line a number, as in the figure, then it is straightforward to label the vertices with tuples of these numbers. Since the spatial ordering of the particles is

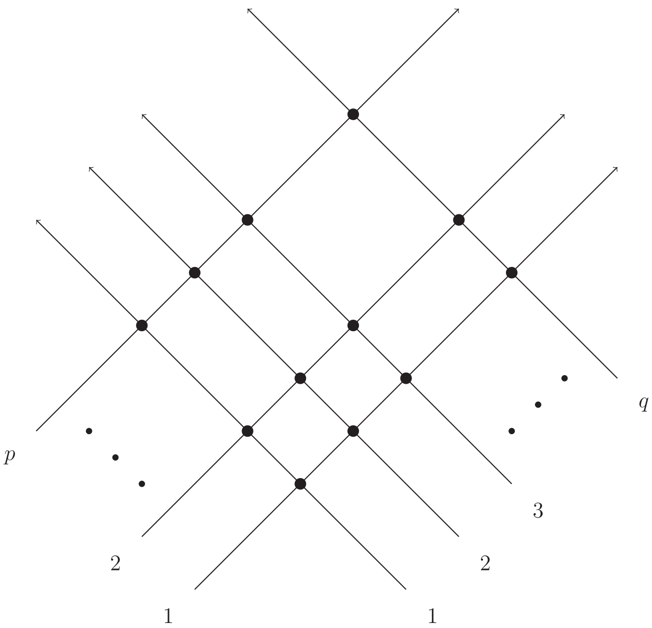

FIG. 12. A collision of p particles from the left and q particles from the right.

constant, we can determine the first and last collisions that each particle will participate in.

In order to determine how particle colors change in one group collision, we assign each particle nonunique initial and final vertices corresponding to the first and last collisions that they participate in. As particles traverse the group collision between their initial and final vertices, they always travel from some vertex (a, b) to one of two adjacent vertices: $(a+1, b)$ or $(a, b+1)$. Thus, if a particle enters the group collision at (a_i, b_i) and leaves at (a_f, b_f), the total number of collisions that it participates in is $(a_f - a_i) + (b_f - b_i) + 1$, where the $+1$ accounts for the initial vertex.

Particle number k first collides at $(p-k+1, 1)$ if $k \leq p$ or at $(1, k-p)$ if $k > p$, and it last collides at (p, k) if $k \leq q$ or at $(p+q-k+1, q)$ if $k > q$. Thus, over the course of a group collision,

$$x_k' = x_k + \begin{cases} 2(k-1)+1, & k \leq p, \\ 2p, & p < k \leq q \ , \\ 2(p+q-k)+1, & k > q. \end{cases} \tag{4}$$

Note the special case where $p = q$ and

$$x_k' = x_k + \begin{cases} 2(k-1)+1, & k \leq p, \\ 2(2p-k)+1, & k > p. \end{cases} \tag{5}$$

Now, having built this machinery, we proceed to the main result of this section. ∎

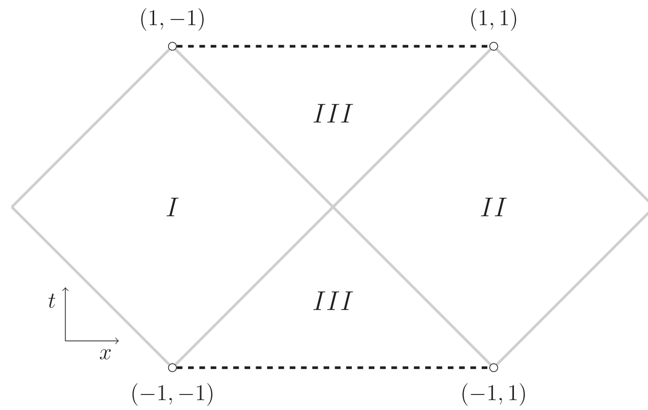

FIG. 13. A single history's causality-violating region can be partitioned into three zones, each of which contains a group collision of particles.

Theorem 5.9.—No paradoxes arise in a cyclic history extension of the TDP space with C colors and H histories if and only if $C \mid H$.

Proof.—To show the absence of paradoxes for *all* initial conditions, we must fully characterize how these initial conditions evolve in order to derive consistency constraints for particles traveling along CCCs. The absence of a paradox is equivalent to the particles traveling along CCCs satisfying these constraints.

According to Corollary 5.5, positions along all particle trajectories are consistent. Thus, we need only show that the colors along these trajectories are consistent as well. This analysis is greatly simplified by Lemma 5.6, which ensures that the only variables we need to consider when analyzing the color evolution of particles in $J^0(\mathcal{M}')$ are m and n, the number of particles entering from the left $(-x)$ and the right $(+x)$, respectively. As noted in the proof of Lemma 5.6, the only particles traversing the time machine—and thus the only particles traveling along CCCs—are confined exclusively to $J^0(\mathcal{M}')$, so the relevant initial conditions for deriving consistency constraints are entirely specified by m and n.

We can more easily determine how the colors of these particles evolve over the course of one history by identifying three zones in $J^0(\mathcal{M}')$ where there are group collisions. These zones are illustrated in Fig. 13. According to Corollary 5.4, the structure of the collisions is the same in each history, so characterizing the evolution of particle colors in one history allows us to determine this more broadly over $J^0(\mathcal{M}')$.

Since all the incoming particles collide such that they scatter away from $J^0(\mathcal{M}')$ without ever traversing the wormhole, particles approaching from $-x$ may participate in collisions only in zone I and particles approaching from $+x$ may participate in collisions only in zone II. Thus, m particles leave zone III going in the $-x$ direction, participate in a group collision in zone I, and are scattered back into zone III; similarly, n particles leave zone III going in

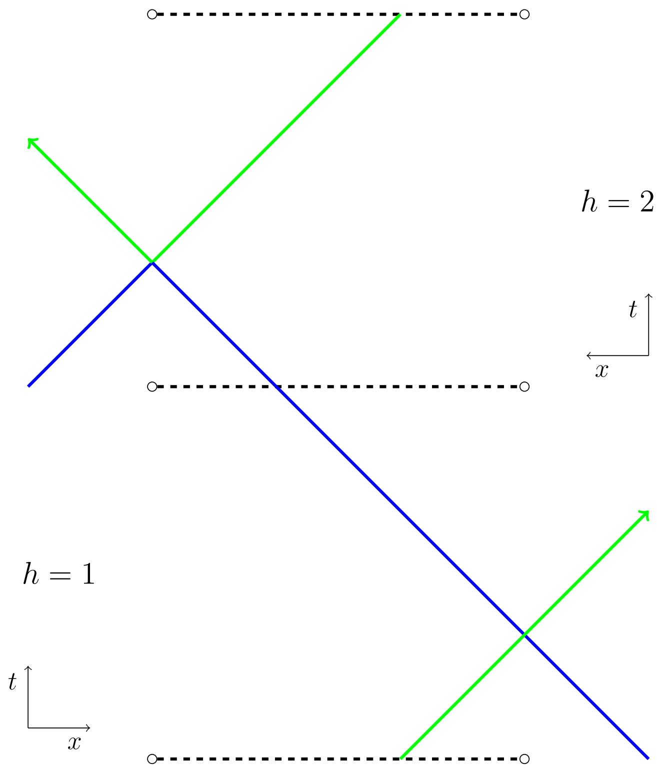

FIG. 14. Here, a *reflected* version of the $h = 2$ causality-violating region is stacked on top of the $h = 1$ causality-violating region. These two regions lie in different spaces, as indicated by the separate coordinate axes. However, this representation makes it easy to see how particles evolve over multiple histories, and what the consistency constraints are: that particles on the last wormhole surface match those on the first one.

the $+x$ direction, participate in a group collision in zone II, and are scattered back into zone III. These $m + n$ particles are those which follow CCCs, imposing consistency constraints that must be satisfied to produce a legitimate solution to a given set of initial conditions.

The particles following CCCs collide in a group collision in zone III. However, it is not initially clear that the structure of this collision is the same as that of those in zones I and II: the group collision is interrupted by a wormhole that flips the spatial ordering of the particles. Nevertheless, we can treat this collision in the same way as the others. This can be visualized by stacking a copy of a single history's time machine region, flipped in x, on top of itself, as shown in Fig. 14. This construction illustrates that the zone III collision takes the same form as the others, and we do not have to account for the wormhole's spatial flip until after the collision if we consider first the group collisions in zones I and II, and then the zone III collision overlapping one history and the next.

We are interested in whether there exists an assignment of colors to the $m + n$ particles following CCCs that remains consistent after the particles have traversed $J^0(\mathcal{M}')$ through all H histories. We begin by determining how these particle colors evolve over one history, applying Lemma 5.8 to each group collision.

Let y_k be the color of particle number k, counting from the left among those following CCCs, after the zone III collision has taken place but before entering zone I or II. We first determine how these values of k relate to those used in Eqs. (4) and (5). For $k \leq m$, particle y_k corresponds to particle x_{m+k} in an (m, m) particle collision in zone I, and then to particle x_k in an (m, n) particle collision in zone III. For $k > m$, particle y_k corresponds to particle x_{k-m} in an (n, n) particle collision in zone II, and then to particle x_k in an (m, n) particle collision in zone III. Thus, evolving through one history,

$$y'_k = y_k + \underbrace{\left(\begin{cases} 2(2m - (m+k)) + 1, & k \leq m, \\ 2((k-m) - 1) + 1, & m < k \leq n, \\ 2((k-m) - 1) + 1, & k > n, \end{cases}\right)}_{\text{zone I and II collisions}}$$
$$+ \underbrace{\left(\begin{cases} 2(k-1) + 1, & k \leq m, \\ 2m, & m < k \leq n, \\ 2(m+n-k) + 1, & k > n, \end{cases}\right)}_{\text{zone III collision}}$$
$$= y_k + \begin{cases} 2m, & k \leq m, \\ 2k - 1, & m < k \leq n, \\ 2n, & k > n. \end{cases} \tag{6}$$

After passing through the wormhole, the spatial ordering of the particles is reversed. Thus, particle number k in one history becomes particle number $m + n - k + 1$ in the next. Particles previously satisfying $k \leq m$ now satisfy $k > n$, so these particles all increase in color by $2(m + n)$ after two histories. Also, since

$$(2k - 1) + (2(m + n - k + 1) - 1) = 2(m + n), \tag{7}$$

particles satisfying $m < k \leq n$ also increase in color by $2(m + n)$. Thus, if H is even, the consistency constraint after traversing these histories is

$$y_k \equiv y_k + H(m + n) \mod C. \tag{8}$$

When $H = 1$, the evolution of particle colors has already been given by Eq. (6). If H is odd and $H > 1$, we can determine the evolution of particle colors by breaking the evolution into that over $H - 1$ histories (an even number) and that over the last history. After traversing an odd number of histories, y'_k must equal $y_{m+n-k+1}$ for consistency. Thus, after H traversals for odd H, the consistency constraint is

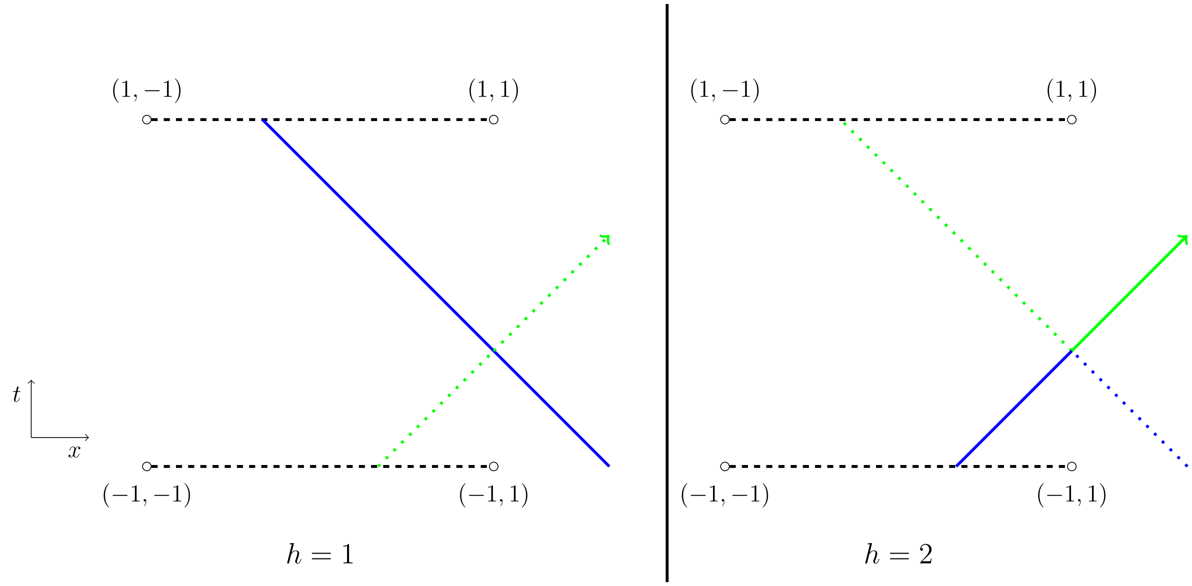

FIG. 15. In this illustration, with $C = 2$ and $H = 2$, one particle is solid and the other is dashed. The illustration demonstrates an interpretation in which the particles do not collide; instead, they *pass through* each other. This allows us to avoid a bootstrap paradox. However, the same vertices in Fig. 4 still apply.

$$y_{m+n-k+1} \equiv \left(y_k + \begin{cases} (H+1)m+(H-1)n, & k \leq m, \\ 2k-1+(H-1)(m+n), & m<k\leq n \\ (H-1)m+(H+1)n, & k>n, \end{cases} \right) \bmod C. \quad (9)$$

Note that both of these consistency requirements are in the group $\mathbb{Z}_C$. When $k \neq \frac{m+n+1}{2}$ (that is, for all particles except the middle particle when $m + n$ is odd), it implies that $2H(m + n) \equiv 0 \bmod C$ and is satisfied for all $m + n$ if and only if $C \mid 2H$ (i.e., C divides $2H$ or $2H$ is a multiple of C). However, when $m + n$ is odd, the $k = \frac{m+n+1}{2}$ condition implies that $H(m + n) \equiv 0$, and is satisfied for all $m + n$ if and only if $C \mid H$. The requirement for even H is satisfied if and only if $H(m + n) \equiv 0 \bmod C$; since all values of $m + n$ are possible, this requires that $C \mid H$.

Thus, in general, $C \mid H$ is equivalent to the nonexistence of paradoxes for this system since we can find a consistent solution for the particle colors. When $C \nmid H$, setting $m = 0$ and $n = 1$ (corresponding to the natural extension of the paradox in Fig. 5) provides a paradox since the consistency constraints for both even and odd H cannot be satisfied in this case. ∎

B. Avoiding bootstrap paradoxes

Although we have found conditions under which consistency paradoxes can be avoided using a finite number of histories, these solutions still have *bootstrap* paradoxes. This is because the particles in this system are now of two separate types:

(1) Particles that come from infinity also exit to infinity, never entering the time machine.
(2) Particles that emerge from the time machine in the past also enter the time machine in the future, never leaving the causality-violating region.

The particles of the second type have no existence outside of the causality-violating region (or outside CCCs), and therefore they are "created from nothing," which implies a bootstrap paradox. However, we can avoid these bootstrap paradoxes by changing our interpretation of the vertices.

In [15] it is suggested for the case of $C = 2$ that, instead of as elastic collisions, the vertices can be interpreted as intersections of *penetrable* particles, which simply flip colors when they cross each other's paths. We can extend this interpretation to our system for general C, where it involves a slightly more complicated interaction: particles which pass through each other adopt the other particle's color *plus 1*. This reinterpretation is sufficient to remove all bootstrap paradoxes, as depicted in Fig. 15.

The blue particle coming in from infinity in $h = 1$ passes through the green particle which came out of the time machine. To distinguish the two particles, the first is indicated by a solid line while the second is indicated by a dashed line. The particles interact using vertex (c) in Fig. 4—which now means that, instead of the particles changing both their directions and colors, they change neither. The blue particle then goes through the time machine and exits in $h = 2$, where it meets its copy, which is also blue (recall that the initial conditions are the same in each history). The copy is indicated by a dashed line.[12] The particle and its copy pass through each other, and they interact using vertex (a) of Fig. 4. The solid particle changes its color to green and goes out to infinity. The dashed particle also changes its color to green and goes into the time machine—exiting from the past time machine in $h = 1$ since the histories are cyclic.

Neither of the particles actually follows a CCC, and both of them have a clear start and end outside of the causality-violating region: the solid particle enters from the right in $h = 1$ and exits to the right in $h = 2$, while the dashed particle enters from the right in $h = 2$ and exits to the right in $h = 1$. Thus, we avoid a bootstrap paradox. This readily generalizes to larger values of C and H.

VI. ANALYSIS OF OUR MODEL

A. Multiple consistent solutions

Even in the base TDP space without multiple histories, not *every* set of initial conditions necessarily causes a paradox. However, even initial conditions which have consistent solutions still exhibit unusual properties. Consider, for examples, the two solutions presented in Figs. 16 and 17, where the *same* initial conditions—two blue particles coming in, one from the left and one from the right—lead to *two* consistent color configurations. Thus, the evolution inside the causality-violating region cannot necessarily be predicted from the initial conditions. This situation does not usually appear in the absence of CCCs, as classical physics is in general deterministic.[13] How will the Universe "decide" which evolution to use? Choosing a specific one would require additional assumptions to explain what is special about that particular evolution.

The same situation also occurs generically in covering spaces of the TDP space. Referring back to the systems of equations required to ensure consistency in Sec. V A, we can determine the number of free color variables, and consequently the number of distinct consistent solutions. When H is even, the relevant constraint is Eq. (8), where each particle color is independent of the rest. Thus, when H is even, there are C^{m+n} possible solutions. When H is odd, the relevant constraint is Eq. (9), where most equations are coupled in pairs, giving $C^{\lceil \frac{m+n}{2} \rceil}$ solutions.

Thus, the notion of multiple histories seems to arise from causality-violating spacetimes in two distinct ways: first, when we extend the spacetime to resolve paradoxes, and second when multiple outcomes are compatible with the same initial conditions. This second case seems more akin to the "worlds" of the Everett interpretation than to the histories solving time travel paradoxes, as these worlds represent distinct possible outcomes for the same physical process rather than an outside intervention due to the presence of a time machine. Confronted with similar phenomena in a different spacetime, Echeverria *et al.* suggested in [20] the possibility of resolving the situation using a quantum mechanical sum-over-histories method.

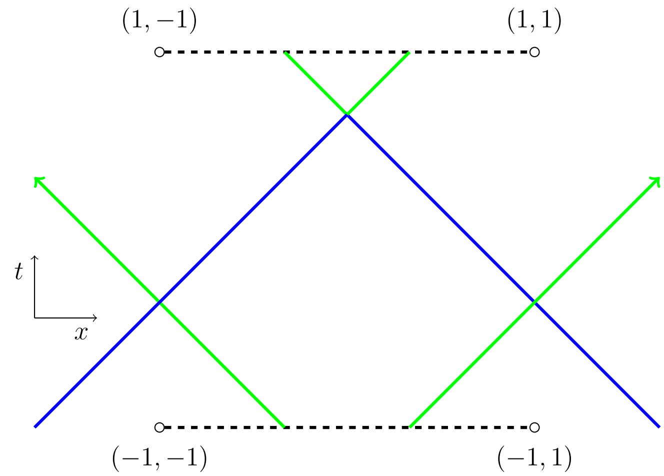

FIG. 16. One of the two consistent solutions obtained by sending particles toward the causality-violating region from both sides.

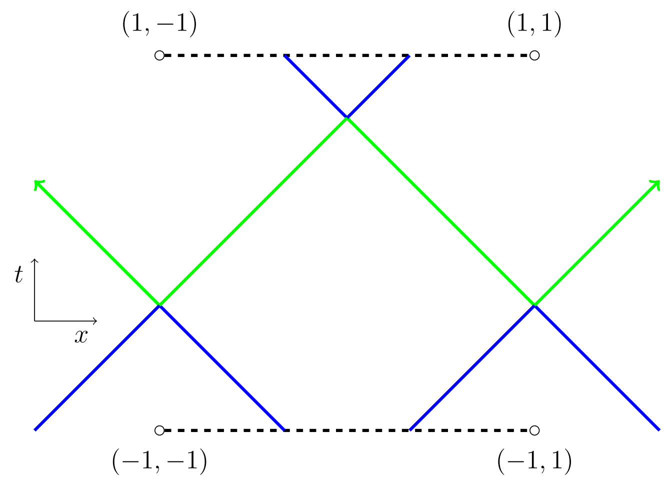

FIG. 17. The second of the two consistent solutions obtained by sending particles toward the causality-violating region from both sides. Note that the initial conditions and final outcomes are the same as in Fig. 16—two blue particles coming in and two green particles coming out—but the evolution inside the causality-violating region is different. Thus, evolution in this region cannot be predicted.

[12] The solid or dashed lines have no physical meaning, and they are not properties of the particles themselves; they are just used in the figure to distinguish one particle from the other in each vertex. The actual physical property of the particle coming in from infinity is that it is blue; the fact that it is solid in $h = 1$ and dashed in $h = 2$ is simply for the purpose of distinguishing the particle from its copy.

[13] However, see [31].

B. Revisiting previous histories and the Novikov conjecture

In Sec. V, we found that, assuming the number of colors C is finite, it is sufficient to have C different cyclic histories in order to resolve every possible paradox, both consistency and bootstrap. In other words, contrary to popular opinion, one does *not* need to prevent going back to previously visited histories in order to avoid paradoxes.

Note that, since we allowed one to go back to the very first history, the time machine will always emit particles from the future as soon as it is created, which is what one would expect if the Novikov conjecture is true, but not from a traditional multiple-histories scenario, where the first history should, by definition, be the one where no one has *yet* traveled back in time. However, while the Novikov conjecture was originally applied to only a single history, it can be applied more generally, in principle, to larger spacetimes—even to those containing multiple histories.

The scenario where travel to the first history is possible therefore *extends* the Novikov self-consistency conjecture to multiple histories. Indeed, under the traditional Novikov conjecture, since there is only one history, when we open the time machine at $t = -1$, particles *must* come out since they went (or will go) into the time machine at $t = +1$. This is similar to how, in the traditional Novikov conjecture scenario, a time traveler who goes back in time to kill themselves has, in fact, *already* gone back and *already* failed. There is no history where the time traveler did not go back in time *yet* since there is only one history.

To illustrate this more precisely, consider a scenario where Alice wants to travel back in time from 2020 to 1950 and kill her grandfather, Bob, before he met her grandmother. Let us first assume that the Novikov conjecture is correct, but there is only one history. Then in this one history, in chronological order, Bob is born in 1930, Alice emerges from a time machine in 1950 and tries to kill Bob—but fails, Alice is born in 1990, and Alice goes into a time machine in 2020. This is a completely consistent chain of events, and there is no other universe or history where Alice did *not* travel back to 1950.

Next, let us assume that there are multiple histories and that they are cyclic all the way back to the first history. Then, again, there is a completely self-consistent chain of events—however, now it encompasses more than one history. We will denote the year 2020 in history A as 2020A, and so on, and we will similarly denote Alice from history A as Alice A, and so on.

In history A, Bob A is born in 1930A, Alice B emerges from a time machine in 1950A and tries to kill Bob A by releasing a crocodile—but fails, Alice A is born in 1990A, Bob A tells Alice A in 2010A a story about a woman who looked remarkably like an older version of her who tried to kill him back in 1950A by releasing a crocodile, and Alice A goes into a time machine in 2020A determined to kill her grandfather using another, more efficient method: dropping a piano on him.

In history B, Bob B is born in 1930B, Alice A emerges from a time machine in 1950B and tries to kill Bob B by dropping a piano on him—but fails, Alice B is born in 1990B, Bob B tells Alice B in 2010B a story about a woman who looked remarkably like an older version of her who tried to kill him back in 1950B by dropping a piano on him, and Alice B goes into a time machine in 2020B determined to kill her grandfather using another, more efficient method: releasing a crocodile.

This is a Novikov-like scenario, but with two distinct histories which are *not* self-consistent individually since the murder attempts in each history are different; when Alice B tries to kill Bob A by releasing a crocodile, she is deliberately doing something that she knows *not* to be consistent with her own history (B), as she is trying to *change* history. Although she does manage to change history from her perspective (into history A), Novikov's conjecture still conspires to prevent her from changing it in an inconsistent way; the combination of histories A and B together represents a completely self-consistent chain of events, spanning two distinct histories.

These examples illustrate how paradox resolution using finite cyclic histories leads to a novel hybrid scenario, with outcomes characteristic of both one-history spacetimes satisfying the Novikov conjecture and multiple-histories models with unlimited histories. With the Novikov conjecture over only one history, there are closed causal curves, and paradoxes are avoided when consistency can be enforced along these curves. The price we have to pay is that the actions time travelers take after they travel to the past must be predetermined, making time travel essentially trivial. You can never go back in time to kill Hitler, because there is only one history, and in this history Hitler existed.[14]

On the other hand, with an infinite number of histories, paradoxes are resolved by simply eliminating closed causal curves in the first place. The price we have to pay is that, since CCCs do not exist, this is not "true" time travel anymore.[15]

Extending the Novikov conjecture to finite cyclic histories provides a new middle ground for solving time travel paradoxes. The first case occurs when $H = 1$, and the second when H is infinite. In between, when H is finite and greater than 1, we may have the existence of closed causal curves over multiple histories while still satisfying the

[14] Or maybe you go back in time to kill Hitler—but fail, and this near-death experience turns out to be what caused Hitler to become an evil dictator in the first place.

[15] But see footnote 10.

Novikov conjecture, enabling true time travel along with the ability to change history.[16]

C. Observable consequences

We now have four different ways in which our Universe might resolve time travel paradoxes.

(1) The Hawking conjecture: Time travel is simply impossible.
(2) The Novikov conjecture: There is only one history, and it can never be changed.
(3) Branching spacetime scenario: Observers who travel back in time find themselves in a new history and unable to go back to a previous history. Furthermore, there is a unique first history.
(4) Covering space scenario: There is no unique first history and it is possible to return to a previous history when the number of histories is finite—as long as the Novikov conjecture applies to the long closed causal curves which traverse all of the histories (as opposed to each history individually).

How may we experimentally determine which approach, if any, is realized in our Universe? First, if we successfully build a time machine, then we have disproved the Hawking conjecture.[17] Let us thus assume that it is indeed possible to build a time machine and discuss how to distinguish among options 2, 3, and 4. Consider a simple experiment where Alice builds the time machine described by the TDP space, which connects $t = +1$ to $t = -1$.

First, let us assume that Alice notices that another Alice did *not* emerge from the time machine at $t = -1$. She then enters the time machine at $t = +1$ and meets a copy of herself, who confirms that the time is now $t = -1$. Both Alices now know that there are at least two independent histories: the one where Alice did not exit the time machine at $t = -1$, and the one where she did. Among the four models we examine in this section, the Alices conclude that the branching spacetime scenario must be the correct one since Alice necessarily came from the *first* history, $h = 1$, and arrived at another history, $h = 2$.

Alternatively, let us assume that Alice (we shall now call her Alice A) notices that another Alice (Alice B) *did* emerge from the time machine at $t = -1$. Then the Alices can try to change something that Alice B remembers happening, which should be trivial assuming that Alice B remembers everything that happened between $t = -1$ and $t = +1$ in her history. For example, if Alice B remembers that she said "1" then Alice A can try to say "2" instead.

(a) If they succeed in changing something, then they can conclude that they live in a covering space scenario—since there is no first Alice, but also more than one history.
(b) If they fail to do so, then they can suspect that they are in a Novikov conjecture scenario.

VII. SUMMARY AND FUTURE PLANS

In this paper, we introduced a $(1+1)$-dimensional model for a spacetime with a time machine and multiple histories, and we showed how time travel paradoxes within this model are inevitable unless one allows for sufficiently many histories. An infinite number of histories is certainly sufficient; however, we also showed that a finite number of cyclic histories is sufficient within our particular model, producing a variation of Novikov's conjecture which spans multiple histories. This scenario contains closed causal curves, unlike traditional multiple-histories resolutions, while also allowing time travelers to actually change the past, unlike Novikov's conjecture over only one history. Therefore, it provides a good middle ground between the two. We also suggested how to experimentally determine, at least in principle, whether our Universe is described by the Hawking conjecture, the Novikov conjecture, a branching spacetime model, or a covering space model.

There are several important issues that we did not discuss here, including the following:

(a) We did not provide an actual physical mechanism for creating new histories; we merely assumed them, as is usually done in the literature.
(b) We asserted that, in the TDP space, a time traveler moves from one history to another while traversing the wormhole. However, we did not develop a prescription for determining at which point along a closed timelike or causal curve this transition between histories

[16] Furthermore, the traditional Novikov scenario, with only one history, does not leave any room for "free will" since Alice cannot make any choice that will change the past; if Alice already knows how her future self attempted to kill Bob in the past, then she will simply *not be able to choose* to try killing him in another way. However, with finite cyclic histories, Alice *does*, in fact, have the capacity to change history—as Alice A and B did in the example above. If she ever succeeds, then the chain of histories is simply terminated; however, it is also possible that she fails every time, in which case the histories can (but do not necessarily have to) be cyclic. Thus, this scenario provides at least the *illusion* of free will. It is important to note that since different Alices exit the time machine in each history, Alice B does not have a memory of what happened in history A, so as far as she is concerned, she has the free will to do whatever she wants. More generally, there is never a situation where Alice knows what is supposed to happen and finds out that she simply does not have the ability to change it, which is the main issue with Novikov's conjecture. Each history is a completely new history, from Alice's perspective, with endless possibilities and nothing predetermined.

[17] As is usually the case, finding a counterexample to the conjecture is much easier than proving that it is true in all cases. To *prove* the Hawking conjecture, it is not enough to simply not succeed in building a time machine, since it is always possible that a time machine could be built, but we are just not skillful enough to build it. The proof must therefore be a theoretical one; we must have access to the most fundamental theory of physics (if such a theory exists) and use that theory to mathematically prove that a time machine *cannot* be built, even in principle.

happens in more general spacetimes. This question is of particular concern in the case of more "realistic" time travel models, such as those using warp drives or wormholes with nonzero throat length. As time travel in this case involves traversing a nonzero distance, it is unclear where exactly along this journey the new history should be created. This problem becomes even more complicated when one considers that closed curves, by definition, do not have a beginning or end.

We hope to address these issues in future work. Other intriguing avenues of future research include generalizing our model in different ways, such as the following:

(a) Formulating the model in $2 + 1$ and $3 + 1$ spacetime dimensions.
(b) Employing realistic physical laws, ideally given by a well-defined Lagrangian.
(c) Allowing particles to travel along timelike paths in addition to null paths.
(d) Allowing additional time machines.
(e) Allowing time machines to be turned on and off.

Multiple histories are, in our opinion, the most compelling of the existing approaches for resolving time travel paradox. Hawking's conjecture simply prevents time travel from happening in the first place, while Novikov's conjecture allows time travel, but in an extremely limited way, where the past cannot be changed and the time traveler cannot exercise their free will. If either conjecture is true, it would make life much less interesting.

In contrast, the multiple-histories approach allows one to change the past and at least the illusion of free will—thus making the Universe considerably more exciting. In addition, it challenges many fundamental notions in mathematics, physics, and philosophy and opens up stimulating new avenues of research. Yet, there is surprisingly little literature about it. Furthermore, our presentation in this paper of a novel approach—the cyclic multiple-histories approach, which extends the Novikov conjecture to multiple histories and exhibits hybrid behavior characteristic of both the Novikov conjecture and multiple histories—may provide new and interesting ways in which time travel paradoxes can be discussed and analyzed.

We hope that this paper will inspire mathematicians, physicists, and philosophers to work on the formulation of a consistent and well-defined framework for physics with multiple histories, both in relation to time travel paradoxes and in other contexts, such as the Everett interpretation of quantum mechanics.

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