Dynamical mixing between 2^3S_1 and 1^3D_1 charmed mesons

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(Received 1 June 2020; accepted 31 August 2020; published 14 September 2020)

In charmed D and D_s mesons sector, the matrix of a Hamiltonian in a quark potential model is computed in the 2^3S_1 and 1^3D_1 subspace. The masses of four mixed states of 2^3S_1 and 1^3D_1 denoted with $D_1^*(2635)$, $D_1^*(2739)$, $D_{s1}^*(2715)$ and $D_{s1}^*(2805)$ are obtained. It is an off-diagonal part of the spin-orbit tensor interaction that causes the mixing between the 2^3S_1 and 1^3D_1 states. The mixing angles between the 2^3S_1 and 1^3D_1 states are tiny. Under the mixing, a $^3P_0$ model is employed to compute the hadronic decay widths of all OZI-allowed decay channels of the four mixed states. The two light mixed states $D_1^*(2635)$ and $D_{s1}^*(2715)$ are close in mass to $D_J^*(2600)$ and $D_{s1}^*(2700)$, while the two heavy mixed states $D_1^*(2739)$ and $D_{s1}^*(2805)$ are lighter in mass than $D(2750)$ and $D_{s1}^*(2860)$. The mixing angles obtained from dynamical interaction are inconsistent with the mixing angles obtained from hadronic decay. Based on mass spectra and hadronic decay analyses, $D_J^*(2600)$, $D(2750)$, $D_{s1}^*(2700)$, and $D_{s1}^*(2860)$ are impossibly the mixed states of 2^3S_1 and 1^3D_1 at the small mixing angles. The inconsistence implies that $D_1^*(2760)$ and $D_{s1}^*(2860)$ have not been properly resolved from present experimental data, or there exist large unknown off-diagonal interactions that result in large mixing angles.

DOI: 10.1103/PhysRevD.102.054013

I. INTRODUCTION

D and D_s mesons consist of a light quark (u, d, or s) and a heavy c quark, they behave like a hydrogen atom. These states have been studied in many models. The study of the spectrum, decay and production of D and D_s mesons is helpful to detect the internal quark dynamics such as the heavy quark symmetry or the light quark chiral symmetry.

S-wave and P-wave charmed mesons (D and D_s) without radial excitation have been well established. The higher located states are the $2S$ and $1D$ ones [1], which have not been definitely identified for some reasons. In experiment, the spin and parity are difficult to determine.

$D^*(2600)$ and $D^*(2760)$ were first observed in inclusive e^+e^- collisions by the *BABAR* Collaboration [2] in the decay channels $D^+\pi^-$, $D^0\pi^+$ and $D^{*+}\pi^-$, where they were suggested as the 2^3S_1 and $^3D_1$ charmed meson, respectively. In addition to their masses and widths, the branching ratios were measured

$$\frac{\Gamma(D^*(2600)^0 \to D^+\pi^-)}{\Gamma(D^*(2600)^0 \to D^{*+}\pi^-)} = 0.32 \pm 0.02 \pm 0.09,$$

$$\frac{\Gamma(D^*(2760)^0 \to D^+\pi^-)}{\Gamma(D^*(2760)^0 \to D^{*+}\pi^-)} = 0.42 \pm 0.05 \pm 0.11.$$

The helicity angle Θ_H distributions of $D^*(2600)$ were consistent with the expectations for a natural parity ($P = (-1)^J$) [2].

Three years later, two resonances named $D_J^*(2650)$ and $D_J^*(2760)$ with a natural parity were observed in the $D^{*+}\pi^-$ mass spectrum in inclusive pp collision by the LHCb Collaboration [3]. In this experiment, $D_J^*(2650)$ was tentatively identified as a $J^P = 1^-$ radial excitation 2^3S_1 charmed meson and $D_J^*(2760)$ was identified as a $J^P = 1^-$ orbital excitation 1^3D_1 charmed meson. Subsequently, $D_J^*(2650)$ and $D_J^*(2760)$ are believed the previously observed $D^*(2600)$ and $D^*(2760)$, respectively.

In addition to inclusive production in e^+e^- and pp collisions, highly excited heavy flavor resonances were also produced in exclusive B decays. In exclusive B decays, $D_1^*(2760)$ was observed in the $B^- \to D_1^*(2760)^0K^-$ decay [4] and $D_3^*(2760)$ was observed in $B^0 \to \bar{D}^0\pi^+\pi^-$ [5]. The spin of $D_1^*(2760)$ was determined with 1 through a Dalitz plot analysis [4]. In particular, the analysis indicates that $D^*(2760)$ observed in e^+e^- and pp collisions consists of $D_1^*(2760)$ and $D_3^*(2760)$ [4,5] observed in B decays.

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However, The observed $D^*(2760)$ in $D^+\pi^-$, $D^0\pi^+$ and $D^{*+}\pi^-$ in inclusive e^+e^- and pp collisions is denoted with $D(2750)$ in the charmed mesons list in PDG2018. In particular, $D(2750)$ is denoted with $D_3^*(2750)$ in a separate page. Obviously, $D_1^*(2760)$ and $D_3^*(2760)$ have not been properly resolved from present experimental data. The $D^*(2600)$ is denoted with $D_J^*(2600)$ in PDG2018. Both $D_J^*(2600)$ and $D(2750)$ are omitted from summary table in PDG2018.

$D_{s1}^*(2700)^\pm$ was first observed by *BABAR* [6] and then by Belle [7,8] in $B^+ \to \bar{D}^0 D_{s1} \to \bar{D}^0 D^0 K^+$ decay with $J^P = 1^-$. $D_{sJ}^*(2860)$ was first reported by *BABAR* [6] in $D_{sJ}(2860) \to D^0K^+, D^+K_s^0$ with a natural spin-parity. $D_{s1}^*(2700)$ and $D_{sJ}^*(2860)$ were also observed in inclusive e^+e^- collision by *BABAR* Collaboration [9]. Subsequently, it is found that $D_{sJ}^*(2860)$ produced in e^+e^- and pp collisions by *BABAR* and LHCb consists of $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ [10,11].

Both $D_{s1}^*(2700)^\pm$ and $D_{s1}^*(2860)$ have the decay channels DK and D^*K. The ratios of branching fractions were given in the Review of Particle Physics (2018) [1]

$$\frac{\Gamma(D_{s1}^*(2700)^+ \to D^{*0}K^+)}{\Gamma(D_{s1}^*(2700)^+ \to D^0K^+)} = 0.91 \pm 0.13 \pm 0.12,$$

$$\frac{\Gamma(D_{s1}^*(2860)^+ \to D^{*0}K^+)}{\Gamma(D_{s1}^*(2860)^+ \to D^0K^+)} = 1.10 \pm 0.15 \pm 0.19.$$

The experimental results about their masses, decay widths and some branching fraction ratios are presented in Table I.

In theory, the spectroscopy of heavy-light mesons has been systematically studied in the relativized quark model [12–14], heavy quark symmetry theory [15,16], relativistic quark model [17,18], chiral quark model [19,20], lattice QCD [21,22], coupled channels models [23,24] and some other models [25–32]. More references can be found in reviews [33–36] and therein.

For low lying heavy-light mesons, theoretical predictions of the masses and the decay data are consistent with experiments. For highly excited resonances, the case is complicated. The mixing between different eigenstates may shift the predicted mass and change the decay widths. In Refs. [12,37–39], it is noted that the mixing may arise from an internal quark dynamics or an interaction between the hadrons and their decay channels. In particular, it is pointed out that the antisymmetric piece of the spin-orbit interaction will cause a $^3L_J - {}^1L_J$ mixing between the mesons with unequal quark masses and the color hyperfine interaction will cause a $^3L_J - {}^3L'_J$ mixing [12].

The mixing between the ^3L_J and ^1L_J eigenstates such as the $1^1P_1 - 1^3P_1$ mixing has been explored in detail both through their mass spectra and through their strong decays [12,13,40,41].

The mixing between the ^3L_J and $^3L'_J$ eigenstates such as the $2^3S_1 - 1^3D_1$ mixing has been explored [42–47]. In Ref. [42], the mixing angle is determined with $\theta = -0.5$ radians from a simple masses mixing matrix of the physical states (2.69 GeV and 2.81 GeV) and the predicted states of the 2^3S_1 and 1^1D_1 D_s mesons (2.71 GeV and 2.78 GeV, respectively). The mixing angle changes sign when the internal quark components of the meson are charge conjugated into their antiquarks. Their predicted hadronic decay widths at this determined mixing angle in the $^3P_0$ model is consistent with experimental data.

In Ref. [43], a similar mixing scheme of the 2^3S_1 and 1^3D_1 D_s as that in Ref. [42] is employed, and the mixing angle is determined through a comparison of the predicted hadronic decay widths of the D_s states in the $^3P_0$ model with the experimental data. $1.12 \le \theta \le 1.38$ radians (opposite in sign with opposite internal quarks) is fixed for $D_{s1}^*(2710)$, while $1.26 \le \theta \le 1.31$ is fixed for $D_{sJ}(2860)$.

In Refs. [46,47], the similar mixing scheme of the 2^3S_1 and 1^3D_1 D and D_s is employed. The mixing angle is studied through a comparison of the predicted hadronic decay widths of the D_s states in terms of the decay formula developed by Eichten, Hill, and Quigg [15] with the experimental data. $\theta = 4° \to 17°$ and $\theta = -16° \to -4°$ are obtained for $D_1^*(2600)$ and $D_{s1}^*(2700)$, respectively. The mixing angles are found small.

However, a dynamical exploration of the $2^3S_1 - 1^3D_1$ mixing has not been performed. In fact, the mixing angles determined through the mass spectra are not consistent with those determined through the decay properties. Therefore, the fixed mixing angles from experiments are different in different references. In experiment, in order to identify the $D_J^*(2600)$, $D(2750)$, $D_{s1}^*(2700)$, and $D_{s1}^*(2860)$, it is also important to systematically study the mixing between the 2^3S_1 and 1^3D_1 D and D_s mesons. For these purposes,

TABLE I. Experimental results of $2S$ and $1D$ candidates of D and D_s [1].

State	Experiments	Mass (MeV)	Width (MeV)	Branching ratios
$D_J^*(2600)$	*BABAR* [2], LHCb[3]	2623 ± 12	139 ± 31	$\frac{\Gamma(D\pi)}{\Gamma(D^*\pi)} = 0.32 \pm 0.02 \pm 0.09$
$D(2750)$	*BABAR* [2], LHCb [3]	2763.5 ± 3.4	66 ± 5	$\frac{\Gamma(D\pi)}{\Gamma(D^*\pi)} = 0.42 \pm 0.05 \pm 0.11$
$D_{s1}^*(2700)$	*BABAR* [6], Belle [7,8]	$2708.3^{+4.0}_{-3.4}$	120 ± 11	$\frac{\Gamma(D^*K)}{\Gamma(DK)} = 0.91 \pm 0.13 \pm 0.12$
$D_{s1}^*(2860)$	*BABAR* [6]	$2859 \pm 12 \pm 24$	$159 \pm 23 \pm 77$	$\frac{\Gamma(D^*K)}{\Gamma(DK)} = 1.10 \pm 0.15 \pm 0.19$

we study the dynamical mixing between the 2^3S_1 and 1^3D_1 in the quark potential model firstly, and subsequently explore their strong decay in the $^3P_0$ model.

The paper is organized as follows. In the second section, the mixing mechanism between the 2^3S_1 and 1^3D_1 D and D_s mesons is explored in the quark potential model, and the mixing angles are dynamically determined. The hadronic decays of the four mixed states are explored in the $^3P_0$ model in Sec. III. In the final section, the conclusions and discussions are given.

II. DYNAMICAL MIXING BETWEEN 2^3S_1 AND 1^3D_1

To describe the heavy-light meson states, two kinds of eigenstates are often employed. One is the $|J,L,S\rangle$ (denoted with $^{2S}L_J$) with $J=L+S$ and $S=S_q+S_{\bar{q}}$ where L is the orbital angular momentum, and S_q, $S_{\bar{q}}$ are the spins. Another one is the $|J,j\rangle$ (denoted with j^P), where P is parity, $j=L+S_q$ is the angular momentum of light quark freedom. Physical heavy-light mesons are usually not the eigenstates $|J,L,S\rangle$ or $|J,j\rangle$, they are the mixing states of these eigenstates. Eigenstates $|J,L,S\rangle$ will be employed in the following.

In the quark potential model, the interquark interactions include the spin-spin interaction, the color-magnetic interaction, the spin-orbit interaction, and the tensor force [12,38,41]. In our analysis, the relativized quark model [41] is employed for our analysis, where the Hamiltonian is

$$H = T + V_{q\bar{q}} \tag{1}$$

$$V_{q\bar{q}} = V_{\rm conf} + V_{SD} \tag{2}$$

where $V_{\rm conf}$ is the standard Coulomb and linear scalar interaction, the spin-orbit and color tensor interaction V_{SD} is rewritten as

$$\begin{aligned} V_{SD} = {} & \left(\frac{S_q}{2m_q^2}+\frac{S_{\bar{q}}}{2m_{\bar{q}}^2}\right)\cdot L\left(\frac{1}{r}\cdot\frac{dV_{\rm conf}}{dr}+\frac{2}{r}\cdot\frac{dV_1}{dr}\right) \\ & +\frac{(S_q+S_{\bar{q}})\cdot L}{m_q m_{\bar{q}}}\left(\frac{1}{r}\cdot\frac{dV_2}{r}\right) \\ & +\frac{3S_q\cdot\hat{r}S_{\bar{q}}\cdot\hat{r}-S_q\cdot S_{\bar{q}}}{3m_q m_{\bar{q}}}\cdot V_3 \\ & +\left[\left(\frac{S_q}{m_q^2}-\frac{S_{\bar{q}}}{m_{\bar{q}}^2}\right)+\frac{S_q-S_{\bar{q}}}{m_q m_{\bar{q}}}\right]\cdot LV_4 \\ & +\frac{32\alpha_s\sigma^3 e^{-\sigma^2 r^2}}{9\sqrt{\pi}m_q m_{\bar{q}}}S_q\cdot S_{\bar{q}}. \end{aligned} \tag{3}$$

The explicit form of V_1, V_2, V_3, and V_4 are [41,48]

$$\begin{aligned} V_1(m_q,m_{\bar{q}},r) = {} & -br - C_F\frac{1}{2r}\frac{\alpha_s^2}{\pi}(C_F \\ & - C_A(\ln[(m_q m_{\bar{q}})^{1/2}r]+\gamma_E)) \\ V_2(m_q,m_{\bar{q}},r) = {} & -\frac{1}{r}C_F\alpha_s\Bigg[1+\frac{\alpha_s}{\pi}\Bigg[\frac{b_0}{2}[\ln(\mu r)+\gamma_E] \\ & +\frac{5}{12}b_0-\frac{2}{3}C_A+\frac{1}{2}(C_F \\ & - C_A(\ln[(m_q m_{\bar{q}})^{1/2}r]+\gamma_E))\Bigg]\Bigg] \\ V_3(m_q,m_{\bar{q}},r) = {} & \frac{3}{r^3}C_F\alpha_s\Bigg[1+\frac{\alpha_s}{\pi}\Bigg[\frac{b_0}{2}\left[\ln(\mu r)+\gamma_E-\frac{4}{3}\right] \\ & +\frac{5}{12}b_0-\frac{2}{3}C_A+\frac{1}{2}\Bigg(C_A+2C_F \\ & -2C_A\left(\ln[(m_q m_{\bar{q}})^{1/2}r]+\gamma_E-\frac{4}{3}\right)\Bigg)\Bigg]\Bigg] \\ V_4(m_q,m_{\bar{q}},r) = {} & \frac{1}{4r^3}C_F C_A\frac{\alpha_s^2}{\pi}\ln\frac{m_{\bar{q}}}{m_q} \end{aligned} \tag{4}$$

with $C_F=\frac{4}{3}$, $C_A=3$, $b_0=9$, and $\gamma_E=0.5772$. The model parameters are $\alpha_s=0.53$, $\mu=1.0$, $\sigma=1.13$, $b=0.135$, $C_{c\bar{u}}=-0.305$, and $C_{c\bar{s}}=-0.254$, they were given in Ref. [41]. The quark masses are chosen as following: $m_c=1450$ MeV, $m_u=m_d=450$ MeV, and $m_s=550$ MeV. In term of these parameters, the predicted masses of the $1S$ and $1P$ D and D_s mesons agree well to the experimental data, which are presented in Tables II and III

As well known, the H is not diagonal in the basis $|J,L,S\rangle$ or $|J,j\rangle$. The relation between $|J,L,S\rangle$ and $|J,j\rangle$ can be found in Refs. [14,40]. From Ref. [14], the off-diagonal interaction arises from the tensor interaction

$$V_{\rm tensor} = \frac{3S_q\cdot\hat{r}S_{\bar{q}}\cdot\hat{r}-S_q\cdot S_{\bar{q}}}{3m_q m_{\bar{q}}}\cdot V_3(r) \tag{5}$$

which can be written in an irreducible representation as

$$V_{\rm tensor} = 6\sqrt{\frac{8\pi}{15}}Y^{(2)}\cdot S^{(2)}\cdot V_3(r)$$

TABLE II. Masses of $1S$ and $1P$ D meson (MeV).

State	This work	PDG
1^1S_0	1867	1869
1^3S_1	2017	2010
1^3P_0	2257	2308
1^3P_2	2473	2460
$1P$	2399	2422
$1P'$	2429	2427

TABLE III. Masses of $1S$ and $1P$ D_s meson (MeV).

State	This work	PDG
1^1S_0	1969	1969
1^3S_1	2114	2112
1^3P_0	2353	2317
1^3P_2	2567	2572
$1P$	2494	2459
$1P'$	2517	2535

where $Y^{(2)}$ is a rank 2 spherical harmonics and $S^{(2)} = (S_q^{(1)} \times S_{\bar{q}}^{(1)})^{(2)}$ with spin operator $S_q^{(1)}$, $S_{\bar{q}}^{(1)}$ in the spherical basis.

The matrix element of the tensor term is obtained through the Wigner-Eckhart theorem [49],

$$\langle J, L, S|V_{\mathrm{tensor}}|J, L', S\rangle = (-1)^{L+S+J} \begin{Bmatrix} S & 2 & S \\ L & J & L' \end{Bmatrix} \times \langle L||Y^{(2)}||L'\rangle \langle S||S^{(2)}||S\rangle \times \langle J, L, S|V_3(r)|J, L', S\rangle$$

where $\langle L||Y^{(2)}||L'\rangle$ is a space reduced matrix element

$$\langle L||Y^{(2)}||L'\rangle = (-1)^L \sqrt{\frac{5(2L+1)(2L'+1)}{4\pi}} \times \begin{pmatrix} L & 2 & L' \\ 0 & 0 & 0 \end{pmatrix}$$

and $\langle S||S^{(2)}||S\rangle$ is the spin reduced matrix element which is $\frac{\sqrt{5}}{2}$ at $S = 1$.

In the subspace of $\langle 2^3S_1|$ and $\langle 1^3D_1|$, the nondiagonal matrix of the Hamiltonian is

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}.$$

The numerical matrix of H in the subspace of $\langle 2^3S_1|$ and $\langle 1^3D_1|$ for D and D_s mesons are

$$\begin{bmatrix} 2635.16 & -0.21 \\ -0.21 & 2738.51 \end{bmatrix} \text{ and } \begin{bmatrix} 2714.76 & -0.29 \\ -0.29 & 2805.49 \end{bmatrix}, \quad (6)$$

respectively.

Without the off-diagonal tensor interaction, $\langle 2^3S_1|$ and $\langle 1^3D_1|$ are the eigenstates of the left H. In this case, the eigenvalues of the $\langle 2^3S_1|$ and $\langle 1^3D_1|$ D mesons are 2635.16 MeV and 2738.51 MeV, respectively. The eigenvalues of the $\langle 2^3S_1|$ and $\langle 1^3D_1|D_s$ mesons are 2714.76 MeV and 2805.49 MeV, respectively. The masses of $\langle 2^3S_1|$ charmed mesons are comparable to those in Ref. [14], but the masses of $\langle 1^3D_1|$ charmed states are lower than those in the same reference.

When the light and heavy mixed sates are denoted with $|D_1^{*L}\rangle$ and $|D_1^{*H}\rangle$ [43,47], respectively, the matrix H can be diagonalized in the physical states (mixed states)

$$\begin{bmatrix} |D_1^{*L}\rangle \\ |D_1^{*H}\rangle \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} |2^3S_1\rangle \\ |1^3D_1\rangle \end{bmatrix}$$

with a mixing angle θ. After diagonalization, H is turned into [40]

$$\begin{bmatrix} H'_{11} & 0 \\ 0 & H'_{22} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^{-1} \quad (7)$$

where H'_{11} and H'_{22} are the energy eigenvalues of the physical D_1^{*L} and D_1^{*H} states, respectively.

With previous formulas in hand, we obtain the masses of the light and heavy mixed physical states and the mixing angles as follows

$$M(D_1^{*L}) = 2635.16 \text{ MeV},$$
$$M(D_1^{*H}) = 2738.51 \text{ MeV},$$
$$\theta_{c\bar{q}} \approx 0.12°.$$
$$M(D_{s1}^{*L}) = 2714.76 \text{ MeV},$$
$$M(D_{s1}^{*H}) = 2805.49 \text{ MeV},$$
$$\theta_{c\bar{s}} \approx 0.18°.$$

These four mixed states will be denoted with $D_1^*(2635)$, $D_1^*(2739)$, $D_{s1}^*(2715)$, and $D_{s1}^*(2805)$ throughout this paper. Obviously, the mixing angles between the 2^3S_1 and 1^3D_1 for D and D_s are very small, and the off-diagonal interactions resulting from the tensor interaction almost do not change the eigenvalues.

From Table I, the masses of the two light mixed $D_1^*(2635)$ and $D_{s1}^*(2715)$ are close to the masses of $D_J^*(2600)$ and $D_{s1}^*(2700)$, but the masses of the two heavy mixed $D_1^*(2739)$ and $D_{s1}^*(2805)$ are lighter than the masses of $D(2750)$ and $D_{s1}^*(2860)$.

Obviously, an off-diagonal tensor interactions inversely proportional to the products of heavy quark and light quark mass in Eq. (3) results in a tiny mixing, and the heavy mixed D_1^{*H} states have masses lighter than $D(2750)$ and $D_{s1}^*(2860)$.

There are two possibilities that may result in lighter masses of D_1^{*H} in comparison to $D(2750)$ and $D_{s1}^*(2860)$. First, $D(2750)$ and $D_{s1}^*(2860)$ have not been definitely identified. As analyzed in Refs. [4,5,10,11,14,50], $D^*(2760)$ [$D(2750)$] observed in e^+e^- and pp collisions was resolved into the two $D_1^*(2760)$ and $D_3^*(2760)$

D states, $D^*_{sJ}(2860)$ observed in e^+e^- and pp collisions was also resolved into the two $D^*_{s1}(2860)$ and $D^*_{s3}(2860)D_s$ states. However, $D(2750)$ and $D^*_{s1}(2860)$ were observed in inclusive e^+e^- and pp collisions with a natural parity, but the spin and parity are difficult to determine in those inclusive decays. $D^*_{s1}(2860)$ and $D^*_{s3}(2860)D_s$ were observed and measured with definite spin in the exclusive B decays [4,5]. Obviously, the analyses of the resolve are not sufficient. In PDG2018, $D(2750)$ was simply denoted with $D^*_3(2750)$ and $D^*_1(2760)$ is missing. In other words, the fixed data of $D^*(2760)$ and $D^*_{sJ}(2860)$ are not sufficient to give the right data of D^{*H}_1 and D^{*H}_{s1}. In experiment, it is important to figure out proper ways to give the exact masses and decay widths of the resolved D^{*H}_1 and D^{*H}_{s1} through $D^*(2760)$ ($D(2750)$) and $D^*_{sJ}(2860)$ in the future.

Second, if there exists any other unknown interaction in the Hamiltonian which may result in a large mixing between the 2^3S_1 and 1^3D_1 for D and D_s, the theoretical predictions of the masses will be consistent with experiments. In order to see how the masses of the four mixed states depend on the mixing angles, the variation of their masses with the mixing angles is plotted in Fig. 1. In a large range of the mixing angles, the masses of D^{*H}_1 and D^{*H}_{s1} turn larger with larger mixing angles, while the masses of D^{*L}_1 and D^{*L}_{s1} turn smaller with larger mixing angles.

In Ref. [42], the $D^*_{sJ}(2860)$ was regarded as the D^{*H}_{s1}, and a large mixing angle $\theta = -0.5$ radians has been phenomenologically obtained, but how the large mixing results from has not been studied. Whether there is an unknown interaction that can result in a large mixing between the 2^3S_1 and 1^3D_1 charm mesons requires more exploration. For this purpose, an accurate measurement of the masses of D^{*L}_1 (D^{*L}_{s1}) and D^{*H}_1 (D^{*H}_{s1}) in the meantime is very important.

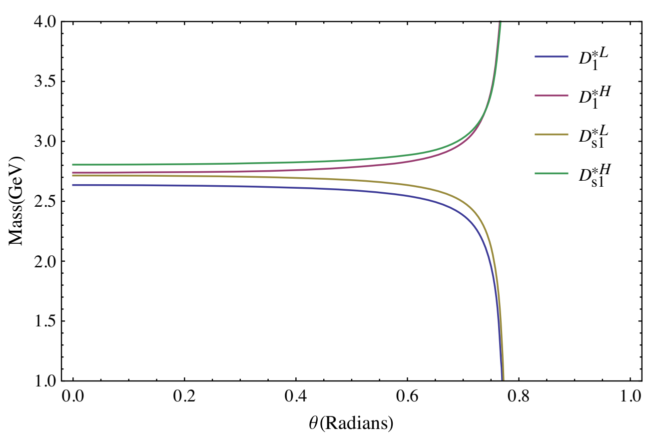

FIG. 1. Masses of the four mixed mesons with the mixing angles.

III. HADRONIC DECAY OF $D^*_1(2635)$, $D^*_1(2739)$, $D^*_{s1}(2715)$, AND $D^*_{s1}(2805)$

In order to learn the internal quark dynamics, another way is to study the strong decay of hadrons. In the case of 2^3S_1 and 1^3D_1 mixing, the hadronic decay of the four mixed states are explored in the $^3P_0$ model in this section.

As is well known, the $^3P_0$ model is usually called as the quark-pair creation model. It has been employed extensively to study the Okubo-Zweig-Iizuka(OZI)-allowed hadronic decay processes. The model was first proposed by Micu [51] and developed by Yaouanc *et al.* [52–54]. The "QCD" decay mechanism of the $^3P_0$ model was studied in Refs. [55–57]. The $^3P_0$ model is fundamentally based on a flux tube picture of the quark confinement. Based on the flux tube picture of the quark confinement, the strong decay and $p\bar{p}$ annihilation processes are also well described in a $^3S_1$ model [58–61].

In the $^3P_0$ model, the decay of a meson takes place through a $q\bar{q}$ pair creation with the vacuum quantum number $J^{PC} = 0^{++}$. The hadronic partial decay width Γ of a decay process $A \to B + C$

$$\Gamma = \pi^2 \frac{|\vec{k}|}{m_A^2} \sum_{JL} |\mathcal{M}^{JL}|^2 \tag{8}$$

where $|\vec{k}| = \frac{\sqrt{[m_A^2-(m_B-m_C)^2][m_A^2-(m_B+m_C)^2]}}{2m_A}$ is the momentum of the final states B and C in the initial meson A's center-of-mass frame, and $\mathcal{M}^{JL}$ is the partial wave amplitude of $A \to B + C$.

For mixed states $|D^{*L}_1\rangle$ and $|D^{*H}_1\rangle$ with mixing angle θ,

$$\begin{aligned}
&\Gamma(|D_L\rangle) \\
&\quad = \pi^2 \frac{|\vec{K}|^2}{m_A^2} \sum_{JL} |\cos\theta \mathcal{M}^{JL}(2^3S_1) - \sin\theta \mathcal{M}^{JL}(1^3D_1)|^2 \\
&\Gamma(|D_H\rangle) \\
&\quad = \pi^2 \frac{|\vec{K}|^2}{m_A^2} \sum_{JL} |\sin\theta \mathcal{M}^{JL}(2^3S_1) + \cos\theta \mathcal{M}^{JL}(1^3D_1)|^2.
\end{aligned} \tag{9}$$

In terms of the Jacob-Wick formula, $\mathcal{M}^{JL}$ can be written as [62],

$$\begin{aligned}
\mathcal{M}^{JL}(A \to BC) = \frac{\sqrt{2L+1}}{2J_A+1} \times \sum_{M_{J_B}, M_{J_C}} \langle L0JM_{J_A}|J_A M_{J_A}\rangle \\
\times \langle J_B M_{J_B} J_C M_{J_C}|J, JM_{J_A}\rangle \\
\times \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\vec{K})
\end{aligned} \tag{10}$$

where $\vec{J}=\vec{J_B}+\vec{J_C}$, $\vec{J_A}=\vec{J_B}+\vec{J_C}+\vec{L}$ and $M_{J_A}=M_{J_B}+M_{J_C}$. The $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ is the helicity amplitude

$$\begin{aligned}\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}} &= \sqrt{8E_AE_BE_C}\gamma \sum_{\substack{M_{L_A},M_{S_A},\\ M_{L_B},M_{S_B},\\ M_{L_C},M_{S_C},m}} \langle L_AM_{L_A}S_AM_{S_A}|J_AM_{J_A}\rangle \\ &\times \langle L_BM_{L_B}S_BM_{S_B}|J_BM_{J_B}\rangle\langle L_CM_{L_C}S_CM_{S_C}|J_CM_{J_C}\rangle \\ &\times \langle 1m;1-m|00\rangle\langle\chi^{13}_{S_BM_{S_B}}\chi^{24}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle \\ &\times \langle\varphi^{13}_B\varphi^{24}_C|\varphi^{12}_A\varphi^{34}_0\rangle I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\vec{K}) \qquad (11)\end{aligned}$$

where γ is the pair-production strength constant. The detail of the flavor matrix element $\langle\varphi^{13}_B\varphi^{24}_C|\varphi^{12}_A\varphi^{34}_0\rangle$, the spin matrix element $\langle\chi^{13}_{S_BM_{S_B}}\chi^{24}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle$ and the momentum integral $I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\vec{K})$ can be found in Ref. [50].

In the $^3P_0$ model, numerical results depend on the parameters such as γ, the harmonic oscillator parameter β and the constituent quark masses. $\gamma = 6.947$ ($\sqrt{96\pi}$ times as the $\gamma = 0.4$ in Ref. [14]) in Refs. [50,63,64] is also employed in this paper. For strange quark-pair $s\bar{s}$ creation, $\gamma_{s\bar{s}} = \gamma/\sqrt{3}$ [53]. The β are taken from Ref. [65]. The constituent quark masses are chosen as $m_c = 1450$ MeV, $m_u = m_d = 450$ MeV, and $m_s = 550$ MeV [65].

In our computation, the masses of related mesons are input as follows: $m_{\pi^0} = 134.977$ MeV, $m_{\pi^\pm} = 139.570$ MeV, $m_{K^0} = 497.611$ MeV, $m_{K^\pm} = 493.677$ MeV, $m_{\rho(770)^0} = 775.26$ MeV, $m_{\rho(770)^\pm} = 775.11$ MeV, $m_\eta = 547.862$ MeV, $m_\omega = 782.65$ MeV, $m_{K^*(892)^0} = 895.81$ MeV, $m_{K^*(892)^\pm} = 891.66$ MeV, $m_{D^0} = 1864.84$ MeV, $m_{D^\pm} = 1869.61$ MeV, $m_{D^{*0}} = 2006.97$ MeV, $m_{D^{*\pm}} = 2010.27$ MeV, $m_{D(2550)^0} = 2539.4$ MeV, $m_{D_1(2420)^0} = 2421.4$ MeV, $m_{D_1(2420)^\pm} = 2423.2$ MeV, $m_{D_1(2430)^{0,\pm}} = 2427.0$ MeV, $m_{D_2^*(2460)^0} = 2462.6$ MeV, $m_{D_2^*(2460)^\pm} = 2464.3$ MeV, $m_{D_s^\pm} = 1968.3$ MeV, $m_{D_s^{*\pm}} = 1968.3$ MeV. The masses of the four mixed states are chosen as: $m_{D_1^*(2635)^0} = 2635.16$ MeV, $m_{D_1^*(2739)^0} = 2738.51$ MeV, $m_{D_{s1}^*(2715)} = 2714.76$ MeV, $m_{D_{s1}^*(2805)} = 2805.49$ MeV [1].

A. $D_1^*(2635)$ and $D_1^*(2739)$

$D_1^*(2635)$ and $D_1^*(2739)$ are mixed states of 2^3S_1 and 1^3D_1 D mesons with mixing angle $\theta = 0.12°$, possible hadronic decay channels and relevant partial decay widths are presented in Table IV. From this table, the total hadronic decay widths of $D_1^*(2635)$ and $D_1^*(2739)$ are 34.84 MeV and 298.77 MeV, respectively. These total decay widths are largely different with the observed states.

TABLE IV. Hadronic decay widths of $D_1^*(2635)^0$ and $D_1^*(2739)^0$ as mixed states of 2^3S_1 and 1^3D_1 with mixing angle $\theta = 0.12°$ (in MeV).

	$D_1^*(2635)$	$D_1^*(2739)$
Channels	Width	Width
$D_1(2420)^0\pi^0$	1.46	42.88
$D_1(2420)^+\pi^-$	2.79	85.51
$D_1(2430)^0\pi^0$	6.91	7.73
$D_1(2430)^+\pi^-$	13.62	15.78
$D^0\pi^0$	0.09	18.06
$D^+\pi^-$	0.13	36.52
$D_s^+K^-$	0.25	12.51
$D^0\eta^0$	0.34	12.11
$D_2^*(2460)^0\pi^0$	0.01	0.32
$D_2^*(2460)^+\pi^-$	0.02	0.58
$D^{*0}\pi^0$	2.36	9.95
$D^{*+}\pi^-$	4.90	20.02
$D^{*0}\eta^0$	1.62	5.01
$D_s^{*+}K^-$	0.34	3.74
$D(2550)^0\pi^0$	×	0.02
$D(2550)^+\pi^-$	×	0.03
$D^0\rho^0$	×	7.29
$D^+\rho^-$	×	13.91
$D^{*0}\omega^0$	×	6.80
Γ_{total}	34.84	298.77

The following ratios are also obtained

$$\frac{\Gamma(D_1^*(2635)^0 \to D^+\pi^-)}{\Gamma(D_1^*(2635)^0 \to D^{*+}\pi^-)} = 0.03$$
$$\frac{\Gamma(D_1^*(2635)^0 \to D_s^+K^-)}{\Gamma(D_1^*(2635)^0 \to D_s^{*+}K^-)} = 0.74$$
$$\frac{\Gamma(D_1^*(2739)^0 \to D^+\pi^-)}{\Gamma(D_1^*(2739)^0 \to D^{*+}\pi^-)} = 1.82$$
$$\frac{\Gamma(D_1^*(2739)^0 \to D_s^+K^-)}{\Gamma(D_1^*(2739)^0 \to D_s^{*+}K^-)} = 3.34.$$

Obviously, the obtained branching ratios $\Gamma(D^+\pi)/\Gamma(D^{*+}\pi^-)$ of $D_1^*(2635)$ is smaller than the observed one of $D_J^*(2600)$, while the branching ratios $\Gamma(D^+\pi)/\Gamma(D^{*+}\pi^-)$ of $D_1^*(2739)$ are larger than the observed one of $D(2750)$. Therefore, even if $D(2750)$ is a $J^P = 1^-$ (instead of $J^P = 3^-$) charmed meson, $D_J^*(2600)$ and $D(2750)$ are impossible to be identified with the combination of 2^3S_1 and 1^3D_1 D mesons at a small mixing angle $\theta = 0.12°$. That is to say, the mixing angle obtained from internal quark dynamics is inconsistent with the mixing angle obtained from strong decay even if the observed

TABLE V. Hadronic decay widths of $D^*_{s1}(2715)^+$ and $D^*_{s1}(2805)^+$ as mixed states of 2^3S_1 and 1^3D_1 with mixing angle $\theta = 0.18°$ (in MeV).

	$D^*_{s1}(2713)^+$	$D^*_{s1}(2773)^+$
Channels	Width	Width
D^+K^0	1.79	51.79
D^0K^+	1.63	51.30
$D^{*+}K^0$	17.23	26.43
$D^{*0}K^+$	17.18	26.47
$D_S^+\eta^0$	0.50	10.40
$D_S^{*+}\eta^0$	0.94	3.36
D^0K^{*+}	×	8.36
D^+K^{*0}	×	6.52
$\Gamma_{\rm total}$	39.27	184.63

$D^*_J(2600)$ and $D(2750)$ have been identified as the D_1^{*L} and D_1^{*H}.

B. $D^*_{s1}(2715)$ and $D^*_{s1}(2805)$

$D^*_{s1}(2715)$ and $D^*_{s1}(2805)$ are mixed states of 2^3S_1 and 1^3D_1 D_s with mixing angle $\theta = 0.18°$, possible hadronic decay channels and relevant partial decay widths are presented in Table V.

From this table, the total hadronic decay width (39.27 MeV) of $D^*_{s1}(2715)$ is much smaller than the observed one of $D^*_{s1}(2700)$, while the total hadronic decay width (184.63 MeV) of $D^*_{s1}(2805)$ is comparable to that of $D^*_{s1}(2860)$.

The obtained ratios

$$\frac{\Gamma(D^*_{s1}(2715)^+ \to D^0K^+)}{\Gamma(D^*_{s1}(2715)^+ \to D^{*0}K^+)} = 0.09$$

$$\frac{\Gamma(D^*_{s1}(2805)^+ \to D^0K^+)}{\Gamma(D^*_{s1}(2805)^+ \to D^{*0}K^+)} = 1.94$$

are largely different with the observed ones of $D^*_{s1}(2700)$ and $D^*_{s1}(2860)$.

Obviously, $D^*_{s1}(2700)$ and $D^*_{s1}(2860)$ are impossible to be identified with the combination of 2^3S_1 and 1^3D_1 D_s mesons at a mixing angle $\theta = 0.18°$ either. In other words, the mixing angle obtained from internal quark dynamics is inconsistent with the mixing angle obtained from strong decay either if $D^*_{s1}(2700)$ and $D^*_{s1}(2860)$ have been identified in their present data.

IV. CONCLUSIONS AND DISCUSSIONS

In this paper, the masses of $1S$, $1P$, $1D$ and $2S$ states of D and D_s have been calculated in the quark potential model. The off-diagonal tensor interactions resulting in the mixing between 2^3S_1 and 1^3D_1 charmed mesons are computed. The mixing angles are found tiny, and the mass difference between the light q quark and the s quark changes the mixing angle little. Four mixed D_1^{*L}, D_1^{*H}, D_{s1}^{*L} and D_{s1}^{*H} are found: $D^*_1(2635)$, $D^*_1(2739)$, $D^*_{s1}(2715)$, and $D^*_{s1}(2805)$, whose masses are 2635 MeV, 2739 MeV, 2715 MeV, and 2805 MeV, respectively. The hadronic partial decay widths of the four mixed states are computed in the $^3P_0$ model, and some branching fraction ratios are given.

Based on mass spectra and hadronic decay analyses, $D^*_J(2600)$ and $D(2750)$ are impossibly the mixed D mesons of 2^3S_1 and 1^3D_1 at a tiny mixing angle $\theta \approx 0.12°$, $D^*_{s1}(2700)$ and $D^*_{s1}(2860)$ are impossibly the mixed D_s mesons of 2^3S_1 and 1^3D_1 at $\theta \approx 0.18°$ either.

In order to identify $D^*_J(2600)$, $D(2750)$, $D^*_{s1}(2700)$, and $D^*_{s1}(2860)$, it is important to fix the accurate masses and J^P numbers both from inclusive e^+e^- and pp collisions, and from exclusive B decays in experiment. So far, the resolve of $D^*(2760)$ and $D^*_{sJ}(2860)$ is not sufficient for the identification of D_1^{*H} and D_{s1}^{*H}. In fact, the mass and decay data of D_1^{*H} and D_{s1}^{*H} has not been definitely fixed in experiments.

If the mixing angles turn large, the masses of D_1^{*L} and D_{s1}^{*L} turn smaller, and the masses of D_1^{*H} and D_{s1}^{*H} turn larger as shown in Fig. 1. Furthermore, as illustrated in Refs. [41,46,47], the predicted decay widths and relevant branching ratios of the four mixed mesons would be consistent with the observed ones of $D^*_J(2600)$, $D(2750)$, $D^*_{s1}(2700)$, and $D^*_{s1}(2860)$. In this case, the problem is which kind of off-diagonal interaction can bring in a large mixing, which requires further exploration.

As pointed out in Ref. [38], the leptonic or electronic decay width is more sensitive to the $^3S_1$ and $^3D_1$ mixing detail. The measurement of the leptonic or electronic decay widths will be helpful to the understanding of the dynamical mechanism of the mixing and the observed mixed states.

ACKNOWLEDGMENTS

This work is supported by National Natural Science Foundation of China under the grants No. 11975146 and No. 11847225.

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