# Study of $a_0^0(980)-f_0(980)$ mixing from $a_0(1450) \to a_0^0(980)f_0(500) \to \pi^+\pi^- f_0(500)$

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(Received 6 August 2020; accepted 24 August 2020; published 9 September 2020)

The $a_0^0(980)-f_0(980)$ mixing is one of the most promising potential tools to learn about the nature of $a_0^0(980)$ and $f_0(980)$. Using the $f_0(980)-a_0^0(980)$ mixing intensity $\xi_{af}$ measured recently at the BESIII experiment, we calculate the branching ratio of the isospin violation decay $J/\psi \to \gamma\eta_c \to \gamma\pi^0 a_0^0(1450) \to \gamma\pi^0 a_0^0(980)f_0(500) \to \gamma\pi^0 f_0(980)f_0(500) \to \gamma\pi^0\pi^+\pi^-\pi^+\pi^-$. The value of the branching ratio is found to be $O(10^{-6})$, which can be observed with $10^{10}$ $J/\psi$ events collected at the BESIII experiment. The narrow peak from the $f_0(980)-a_0^0(980)$ mixing in the $\pi^+\pi^-$ mass square spectrum can also be observed. In addition, we study the nonresonant decay $a_0^0(1450) \to f_0(980)\pi^+\pi^-$(nonresonant), which is dominated by the $a_0^0(980)-f_0(980)$ mixing. We find that the nonresonant decay $a_0^0(1450) \to f_0(980)\pi^+\pi^-$ and the decay $a_0^0(1450) \to f_0(980)f_0(500)$ can be combined to measure the mixing intensity $\xi_{af}$ in experiment. These decays are the perfect complement to the decay $\chi_{c1} \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$, which had been observed at the BESIII experiment, the observations of which will make the measurement of the mixing intensity $\xi_{af}$ more precisely.

DOI: 10.1103/PhysRevD.102.054009

## I. INTRODUCTION

The inner structure of the light scalar mesons such as $a_0^0(980)$ and $f_0(980)$ has been studied for over 30 years, and it is still a hot topic in particle physics. There are several proposals for the inner structure of the light scalar mesons, such as $q\bar{q}$ states, glueball, hybrid states, molecule states, tetraquark states, and the superpositions of these contents [1–11]. However, there is still no general agreement on the inner structure of $a_0^0(980)$ and $f_0(980)$, due to the absence of convincing evidence.

The $a_0^0(980)-f_0(980)$ mixing, which was first suggested theoretically in Ref. [12], is one of the most promising potential tools to learn about the nature of $a_0^0(980)$ and $f_0(980)$ and, therefore, has been studied extensively in various processes [13–51]. In February 2018, The BESIII Collaboration studied the $a_0^0(980)-f_0(980)$ mixing with the decays of $J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0$ and $\chi_{c1} \to a_0^0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$; the signals of the $a_0^0(980)-f_0(980)$ mixing were observed with a statistical significance of larger than $5\sigma$ for the first time. The values of the mixing intensities were measured as [52]

$$\begin{aligned}\xi_{fa} &= (0.99 \pm 0.35) \times 10^{-2} \quad \text{(solution1)},\\ \xi_{fa} &= (0.41 \pm 0.25) \times 10^{-2} \quad \text{(solution2)},\end{aligned} \tag{1}$$

and

$$\xi_{af} = (0.40 \pm 0.17) \times 10^{-2}. \tag{2}$$

Here, the mixing intensities $\xi_{af}$ and $\xi_{fa}$ are defined as

$$\xi_{fa} = \frac{\mathcal{B}(J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0)}{\mathcal{B}(J/\psi \to \phi f_0(980) \to \phi\pi^+\pi^-)}, \tag{3}$$

$$\xi_{af} = \frac{\mathcal{B}(\chi_{c1} \to a_0^0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{\mathcal{B}(\chi_{c1} \to a_0^0(980)\pi^0 \to \eta\pi^0\pi^0)}. \tag{4}$$

There are two solutions for the mixing intensity $\xi_{fa}$, the recent theoretical calculation preferred to the solution-1 result [53]. The result of $\xi_{af}$ suffers large uncertainty, and a question whether there would be a difference between the two mixing intensities $\xi_{af}$ and $\xi_{fa}$ may be raised, so more

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precise data and more reactions are needed in both experiment and theory.

The $a_0(1450)$ resonance is a scalar-isovector meson and is assumed to be the conventional quark-antiquark structure based on the native quark model; the latest theoretical calculations [54–56] also confirmed this conclusion. The *BABAR* Collaboration performed a Dalitz plot analysis for the $\eta_c \to K^+K^-\pi^0$ and $\eta_c \to K^+K^-\eta$ decays and obtained the branch fraction of the $\eta_c \to a_0(1450)\pi^0 \to K^+K^-\pi^0$ decay relative to the $\eta_c \to K^+K^-\pi^0$ mode [57]

$$\frac{\mathcal{B}(\eta_c \to a_0(1450)\pi^0)\cdot\mathcal{B}(a_0(1450)\to K^+K^-)}{\mathcal{B}(\eta_c\to K^+K^-\pi^0)} = (10.2\pm2.5)\times10^{-2}. \quad (5)$$

By combining the recent data on the branching ratio of $\eta_c \to K^+K^-\pi^0$ and $J/\psi \to \gamma\eta_c$ from the Particle Data Group [58]

$$\mathcal{B}(J/\psi\to\gamma\eta_c) = (1.7\pm0.4)\times10^{-2}, \quad (6)$$

$$\mathcal{B}(\eta_c\to K^+K^-\pi^0) = (3.65\pm0.25)\times10^{-2}, \quad (7)$$

and the value of the branching ratio of $a_0(1450)\to K^+K^-$ in Ref. [59]

$$\mathcal{B}(a_0(1450)\to K^+K^-) = (4.61\pm0.61)\times10^{-2}, \quad (8)$$

we find that the branching ratio of $J/\psi\to\gamma\eta_c\to\gamma a_0(1450)\pi^0$ can reach the order of $10^{-3}$. Based on the data samples of $10^{10}$ $J/\psi$ events collected in the BESIII experiment [60–63], about $10^7$ $a_0(1450)$ mesons can be produced through decays $J/\psi\to\gamma\eta_c\to\gamma a_0(1450)\pi^0$; this large $a_0(1450)$ sample at the BESIII experiment will make it possible to investigate the properties of $a_0(1450)$ meson and study the related physics.

In this paper, we investigate the isospin breaking decay $\eta_c\to a_0(1450)\pi^0\to a_0^0(980)f_0(500)\pi^0\to\pi^+\pi^-f_0(500)\pi^0$ produced via $J/\psi\to\gamma\eta_c$. We predict the branching ratio of this reaction by using the recent measurements at the BESIII experiment and calculate the distribution of the $\pi^+\pi^-$ mass square spectrum near the $K\bar{K}$ thresholds. We also discuss the $a_0(1450)\to f_0(980)\pi^+\pi^-$(nonresonant)$\to\pi^+\pi^-\pi^+\pi^-$ decay process, which is realized mainly via the $a_0^0(980)-f_0(980)$ mixing.

## II. DATA ON THE DECAY

The $\eta_c\to\pi^0a_0(1450)\to\pi^0a_0^0(980)f_0(500)\to\pi^0\pi^+\pi^-f_0(500)$ decay violates the isospin symmetry; it can proceed via the $a_0^0(980)-f_0(980)$ mixing. In this process, the mixing intensity $\xi_{af}$ is given as

$$\xi_{af} = \frac{\mathcal{B}(\eta_c\to\pi^0a_0(1450)\to\pi^0a_0^0(980)f_0(500)\to\pi^0f_0(980)f_0(500)\to\pi^0\pi^+\pi^-\pi^+\pi^-)}{\mathcal{B}(\eta_c\to\pi^0a_0(1450)\to\pi^0a_0^0(980)f_0(500)\to\pi^0\eta\pi^0f_0(500)\to\pi^0\eta\pi^0\pi^+\pi^-)}. \quad (9)$$

Here, $\mathcal{B}(\eta_c\to\pi^0a_0(1450)\to\pi^0a_0^0(980)f_0(500)\to\pi^0\eta\pi^0f_0(500)\to\pi^0\eta\pi^0\pi^+\pi^-)$ is the branching ratio of the $\eta_c\to\pi^0a_0(1450)\to\pi^0a_0^0(980)f_0(500)\to\pi^0\eta\pi^0f_0(500)\to\pi^0\eta\pi^0\pi^+\pi^-$ decay

$$\begin{aligned}&\mathcal{B}(\eta_c\to\pi^0a_0(1450)\to\pi^0a_0^0(980)f_0(500)\to\pi^0\eta\pi^0f_0(500)\to\pi^0\eta\pi^0\pi^+\pi^-)\\&\quad=\mathcal{B}(\eta_c\to\pi^0a_0(1450))\cdot\mathcal{B}(a_0(1450)\to a_0^0(980)f_0(500))\cdot\mathcal{B}(a_0^0(980)\to\eta\pi^0)\cdot\mathcal{B}(f_0(500)\to\pi^+\pi^-).\end{aligned} \quad (10)$$

By combining Eqs. (5) and (7), one can obtain

$$\mathcal{B}(\eta_c\to\pi^0a_0(1450))\cdot\mathcal{B}(a_0(1450)\to K^+K^-) = (3.72\pm0.96)\times10^{-3}. \quad (11)$$

The ratio of the branching ratio of $a_0(1450)\to a_0^0(980)f_0(500)$ to $a_0(1450)\to K^+K^-$ has been presented in Ref. [59],

$$\frac{\mathcal{B}(a_0(1450)\to a_0^0(980)f_0(500))}{\mathcal{B}(a_0(1450)\to K^+K^-)} = 8.24\pm2.72, \quad (12)$$

so we can predict the branching ratio of the decay chain $\eta_c\to a_0(1450)\pi^0\to a_0^0(980)f_0(500)\pi^0$ as

$$\mathcal{B}(\eta_c\to\pi^0a_0(1450))\cdot\mathcal{B}(a_0(1450)\to a_0^0(980)f_0(500)) = (3.06\pm1.28)\times10^{-2}. \quad (13)$$

From Refs. [1,64,65], we can obtain the branching ratios $\mathcal{B}(f_0(500)\to\pi^+\pi^-)$, $\mathcal{B}(f_0(980)\to\pi^+\pi^-)$, and $\mathcal{B}(a_0^0(980)\to\eta\pi^0)$,

$$\mathcal{B}(f_0(500)\to\pi^+\pi^-) = 0.67, \quad (14)$$

$$\mathcal{B}(f_0(980)\to\pi^+\pi^-) = 0.50^{+0.07}_{-0.09}, \quad (15)$$

$$\mathcal{B}(a_0^0(980)\to\eta\pi^0) = 0.845\pm0.017. \quad (16)$$

Substituting Eqs. (14), (16), and (13) into Eq. (10), one could predict the branching ratio of the decay chain

$\eta_c \to \pi^0 a_0(1450) \to \pi^0 a_0^0(980) f_0(500) \to \pi^0 \eta \pi^0 f_0(500) \to \pi^0 \eta \pi^0 \pi^+ \pi^-$ as

$$\begin{aligned} &\mathcal{B}(\eta_c \to \pi^0 a_0(1450) \to \pi^0 a_0^0(980) f_0(500) \\ &\quad \to \pi^0 \eta \pi^0 f_0(500) \to \pi^0 \eta \pi^0 \pi^+ \pi^-) \\ &= (1.74 \pm 0.72) \times 10^{-2}. \end{aligned} \tag{17}$$

Combining this equation with Eq. (2) and using Eq. (9), we then obtain the branching ratio $\mathcal{B}(\eta_c \to \pi^0 a_0(1450) \to \pi^0 a_0^0(980) f_0(500) \to \pi^0 f_0(980) f_0(500) \to \pi^0 \pi^+ \pi^- \pi^+ \pi^-)$ as $(0.70 \pm 0.41) \times 10^{-4}$. Adding this value and Eq. (6) together, we can readily obtain

$$\begin{aligned} &\mathcal{B}(J/\psi \to \gamma \eta_c \to \gamma \pi^0 a_0(1450) \to \gamma \pi^0 a_0^0(980) f_0(500) \\ &\quad \to \gamma \pi^0 f_0(980) f_0(500) \to \gamma \pi^0 \pi^+ \pi^- \pi^+ \pi^-) \\ &= (1.19 \pm 0.75) \times 10^{-6}. \end{aligned} \tag{18}$$

Obviously, we believe this decay will be marginally detected in the $e^+e^-$ colliders in view of the large database of the BESIII Collaboration.

## III. MECHANISM RESPONSIBLE FOR THE DECAY

As for the decay $a_0(1450) \to a_0^0(980) f_0(500) \to f_0(980) f_0(500) \to \pi^+ \pi^- f_0(500)$, the amplitude is proportional to the mixing of the $a_0^0(980)$ and $f_0(980)$ resonances, which is caused by the mass difference of the $K^+K^-$ and $K^0\bar{K}^0$ intermediate state. The diagram of $f_0(980)$ production in the $a_0(1450) \to a_0^0(980) f_0(500) \to f_0(980) f_0(500) \to \pi^+ \pi^- f_0(500)$ reaction is shown in Fig 1, so the decay amplitude from this process can be written as

$$\begin{aligned} &\mathcal{M}(a_0(1450) \to a_0^0(980) f_0(500) \\ &\quad \to f_0(980) f_0(500) \to \pi^+ \pi^- f_0(500)) \\ &= \mathcal{M}_{a_0(1450) a_0^0(980) f_0(500)} \cdot \frac{\Pi_{a_0^0 f_0}(q^2)}{D_{a_0^0}(q^2) D_{f_0}(q^2) - \Pi^2_{a_0^0 f_0}(q^2)} \\ &\quad \cdot g_{f_0(980)\pi^+\pi^-}, \end{aligned} \tag{19}$$

where $q^2 = (p_{\pi^+} + p_{\pi^-})^2$, and $a_0^0$ and $f_0$ are, respectively, the shorthand of $a_0^0(980)$ and $f_0(980)$. $\mathcal{M}_{a_0(1450) a_0^0(980) f_0(500)}$ is the invariant amplitude for the decay $a_0(1450) \to a_0^0(980) f_0(500)$,

$$\begin{aligned} \mathcal{B}(a_0(1450) \to a_0^0(980) f_0(500)) &= |\mathcal{M}_{a_0(1450) a_0^0(980) f_0(500)}|^2 \\ &\quad \cdot \frac{f(m_{a_0(1450)}, m_{a_0^0(980)}, m_{f_0(500)})}{16\pi \Gamma_{a_0(1450)} m^3_{a_0(1450)}}. \end{aligned} \tag{20}$$

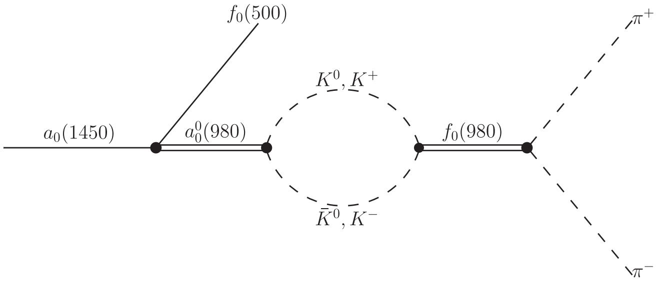

FIG. 1. Feynman diagram for the reaction $a_0(1450) \to a_0^0(980) f_0(500) \to f_0(980) f_0(500) \to \pi^+ \pi^- f_0(500)$.

Hereinafter, $m_r$ denotes the mass of resonance $r[r = f_0(980), a_0^0(980), f_0(500), a_0(1450), \eta, \pi^0, \pi^+, K^0, K^+]$ and $\Gamma_r$ with $r = a_0^0(980), f_0(500), f_0(980), a_0(1450)$ denotes the width of the resonance. The function $f(x,y,z)$ is defined as

$$f(x,y,z) = \sqrt{x^4 + y^4 + z^4 - 2x^2y^2 - 2x^2z^2 - 2y^2z^2}. \tag{21}$$

$g_{f_0(980)\pi^+\pi^-}$ is the coupling constant of $f_0(980)$ with $\pi^+\pi^-$ and can be extracted from the branching ratio of the $f_0(980) \to \pi^+\pi^-$ decay

$$\begin{aligned} \mathcal{B}(f_0(980) \to \pi^+\pi^-) &= |g_{f_0(980)\pi^+\pi^-}|^2 \\ &\quad \cdot \frac{f(m_{f_0(980)}, m_{\pi^+}, m_{\pi^+})}{16\pi \Gamma_{f_0(980)} m^3_{f_0(980)}}. \end{aligned} \tag{22}$$

The $a_0^0(980) - f_0(980)$ mixing amplitude $\Pi_{a_0^0 f_0}(q^2)$ has the following form [44,66]:

$$\begin{aligned} \Pi_{a_0^0 f_0}(q^2) = &\frac{g_{a_0^0(980)K^+K^-} g_{f_0(980)K^+K^-}}{16\pi} \\ &\times \Bigg[ i(R_{K^+K^-}(q^2) - R_{K^0\bar{K}^0}(q^2)) \\ &- \frac{R_{K^+K^-}(q^2)}{\pi} \ln \frac{1 + R_{K^+K^-}(q^2)}{1 - R_{K^+K^-}(q^2)} \\ &+ \frac{R_{K^0\bar{K}^0}(q^2)}{\pi} \ln \frac{1 + R_{K^0\bar{K}^0}(q^2)}{1 - R_{K^0\bar{K}^0}(q^2)} \Bigg], \end{aligned} \tag{23}$$

where $g_{a_0^0(980)K^+K^-}$ and $g_{f_0(980)K^+K^-}$ is the coupling constant of $K^+K^-$ with $a_0^0(980)$ and $f_0(980)$, respectively. For $q^2 \geq 4m_a^2$ $[a = K^+, K^0]$, $R_{aa}(q^2) = \sqrt{1 - 4m_a^2/q^2}$, if $q^2 \leq 4m_a^2$, then $R_{aa}(q^2)$ should be replaced by $i\sqrt{4m_a^2/q^2 - 1}$. In Eq. (19), $D_r(q^2)$ is the inverse propagator of the unmixed resonance $r$,

$$D_r(q^2) = q^2 - m_r^2 - \sum_{ab} [\mathrm{Re}\Pi_r^{ab}(m_r^2) - \Pi_r^{ab}(q^2)]. \tag{24}$$

For $r = a_0^0(980)$, $ab = (\eta\pi^0, K^+K^-, K^0\bar{K}^0)$; for $r = f_0(980)$, $ab = (\pi^+\pi^-, \pi^0\pi^0, K^+K^-, K^0\bar{K}^0)$. $\Pi_r^{ab}$ denote the diagonal matrix of the polarization operator of the resonance $r$ corresponding to the one loop contribution from the two-particle intermediate $ab$ states, it is a piecewise function, and its expressions in the different $q^2$ regions are displayed in Eqs. (18–20) of Ref. [67]. Making use of Eqs. (19), (20), and (22), it is then straightforward to obtain

$$\frac{d\Gamma(a_0(1450) \to a_0^0(980)f_0(500) \to f_0(980)f_0(500) \to \pi^+\pi^- f_0(500))}{dq^2} = \mathcal{B}(f_0(980) \to \pi^+\pi^-) \cdot \mathcal{B}(a_0(1450) \to a_0^0(980)f_0(500)) \cdot \left| \frac{\Pi_{a_0^0 f_0}(q^2)}{D_{a_0^0}(q^2)D_{f_0}(q^2) - \Pi^2_{a_0^0 f_0}(q^2)} \right|^2 \cdot \varphi_S, \tag{25}$$

where $\varphi_S$ is the relevant phase-space factor

$$\varphi_S = \frac{\Gamma_{a_0(1450)}\Gamma_{f_0(980)} m^3_{f_0(980)}}{\pi q^2} \cdot \frac{\bar{f}(m_{a_0(1450)}, m_{f_0(500)}, \sqrt{q^2})}{\bar{f}(m_{a_0(1450)}, m_{f_0(500)}, m_{a_0^0(980)})} \cdot \frac{f(\sqrt{q^2}, m_{\pi^+}, m_{\pi^+})}{f(m_{f_0(980)}, m_{\pi^+}, m_{\pi^+})}. \tag{26}$$

Since the decay width of the $f_0(500)$ resonance is large, we use the functions $\bar{f}(m_{a_0(1450)}, m_{f_0(500)}, z)$ $[z = \sqrt{q^2}, m_{a_0^0(980)}]$ in Eq. (26), which can be obtained by the functions $f(m_{a_0(1450)}, m_{f_0(500)}, z)$ weighed with the $f_0(500)$ resonant distribution, i.e.,

$$\bar{f}(m_{a_0(1450)}, m_{f_0(500)}, z) = \int_{4m^2_{\pi^+}}^{(a_0(1450)-z)^2} \rho(m) f(m_{a_0(1450)}, m, z) dm^2, \tag{27}$$

where $\rho(m)$ is the spectral density and can be approximated as

$$\rho(m) = \frac{1}{\pi} \frac{m\Gamma_{f_0(500)}}{(m^2 - m^2_{f_0(500)})^2 + (m\Gamma_{f_0(500)})^2}. \tag{28}$$

By multiplying both sides of Eq. (25) by $\mathcal{B}(J/\psi \to \gamma\eta_c)$ and $\mathcal{B}(\eta_c \to a_0(1450)\pi^0)$, one can obtain

$$\begin{aligned} &\mathcal{B}(J/\psi \to \gamma\eta_c \to \gamma a_0(1450)\pi^0) \cdot \frac{d\Gamma(a_0(1450) \to a_0^0(980)f_0(500) \to \pi^+\pi^- f_0(500))}{dq^2} \\ &= \mathcal{B}(J/\psi \to \gamma\eta_c) \cdot \mathcal{B}(\eta_c \to a_0(1450)\pi^0) \cdot \mathcal{B}(a_0(1450) \to a_0^0(980)f_0(500)) \\ &\quad \cdot \mathcal{B}(f_0(980) \to \pi^+\pi^-) \cdot \left| \frac{\Pi_{a_0^0 f_0}(q^2)}{D_{a_0^0}(q^2)D_{f_0}(q^2) - \Pi^2_{a_0^0 f_0}(q^2)} \right|^2 \cdot \varphi_S, \end{aligned} \tag{29}$$

TABLE I. Properties of the resonances.

| | |
|---|---|
| $m_{\pi^+} = 139.6$ MeV [58] | $m_{\pi^0} = 135$ MeV [58] |
| $m_{K^+} = 493.7$ MeV [58] | $m_{K^0} = 497.6$ MeV [58] |
| $m_\eta = 547.9$ MeV [58] | $m_{\eta'} = (957.8 \pm 0.1)$ MeV [58] |
| $m_{f_0(980)} = (0.99 \pm 0.02)$ GeV [58] | $\Gamma_{f_0(980)} = 0.074$ GeV [68] |
| $m_{a_0^0(980)} = (0.98 \pm 0.02)$ GeV [58] | $\Gamma_{a_0^0(980)} = (0.092 \pm 0.008)$ GeV [58] |
| $m_{a_0(1450)} = (1.474 \pm 0.019)$ GeV [58] | $\Gamma_{a_0(1450)} = (0.265 \pm 0.013)$ GeV [58] |
| $m_{f_0(500)} = (0.475 \pm 0.075)$ GeV [58] | $\Gamma_{f_0(500)} = (0.55 \pm 0.15)$ GeV [58] |
| $g_{a_0^0(980)\eta\pi^0} = 2.43$ GeV [1,64] | |
| $g_{a_0^0(980)K^+K^-} = (2.76 \pm 0.46)$ GeV [69,70] | $g_{a_0^0(980)K^0\bar{K}^0} = (2.76 \pm 0.46)$ GeV [69,70] |
| $g_{f_0(980)\pi^+\pi^-} = 1.39$ GeV [1,64] | $g_{f_0(980)\pi^0\pi^0} = 0.98$ GeV [1,64] |
| $g_{f_0(980)K^+K^-} = 3.17$ GeV [44] | $g_{f_0(980)K^0\bar{K}^0} = 3.17$ GeV [44] |

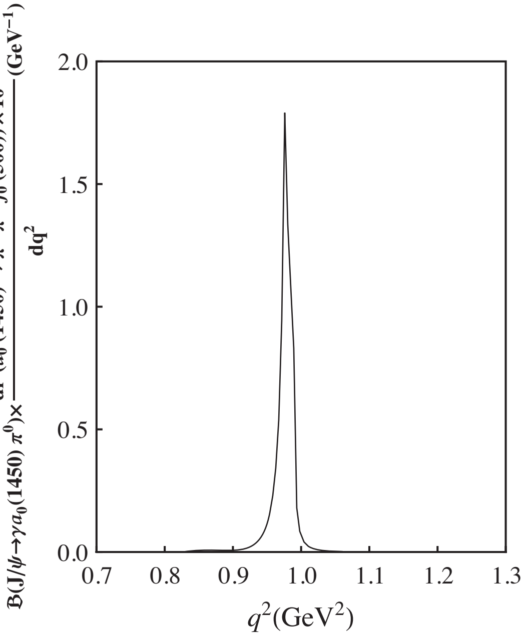

FIG. 2. The distribution of the $\pi^+\pi^-$ mass square spectrum $[q^2=(p_{\pi^+}+p_{\pi^-})^2]$ for the decay $J/\psi\to\gamma\eta_c\to\gamma\pi^0a_0^0(1450)\to\gamma\pi^0a_0^0(980)f_0(500)\to\gamma\pi^0\pi^+\pi^-f_0(500)$.

With the value of the parameters that are listed in Table I and substituting Eqs. (6), (13), and (15) into Eq. (29), we can obtain the distribution curve of the $\pi^+\pi^-$ mass square spectrum for the decay $J/\psi\to\gamma\eta_c\to\gamma\pi^0a_0^0(1450)\to\gamma\pi^0a_0^0(980)f_0(500)\to\gamma\pi^0\pi^+\pi^-f_0(500)$, which is presented in Fig. 2. Here, we note that the narrow peak from the $a_0^0(980)-f_0(980)$ mixing can be clearly observed in this figure. The physical range of $q^2$ for $a_0^0(1450)\to\pi^+\pi^-f_0(500)$ is $4m_{\pi^+}^2\le q^2\le(m_{a_0(1450)}-m_{f_0(500)})^2$. By integrating over the variable $q^2$, we finally obtain the following value of the branching ratio:

$$\mathcal{B}(J/\psi\to\gamma\eta_c\to\gamma\pi^0a_0^0(1450)\to\gamma\pi^0a_0^0(980)f_0(500)\\ \to\gamma\pi^0\pi^+\pi^-f_0(500))=(1.31^{+0.65}_{-0.67})\times10^{-6}. \tag{30}$$

Here, we combine the uncertainties of the branching ratios involved in the calculation, the decay width of $a_0(1450)$, and the mass and the decay width of $f_0(500)$ to determine the final error of the above branching ratio. Adding Eqs. (14) and (30) together, we then arrive at

$$\mathcal{B}(J/\psi\to\gamma\eta_c\to\gamma\pi^0a_0^0(1450)\to\gamma\pi^0a_0^0(980)f_0(500)\\ \to\gamma\pi^0\pi^+\pi^-\pi^+\pi^-)=(0.88^{+0.44}_{-0.45})\times10^{-6}. \tag{31}$$

## IV. NONRESONANT DECAY $a_0^0(1450)\to f_0(980)\pi^+\pi^-$

In experiment, the resonant $f_0(500)$ is reconstructed by the $f_0(500)\to\pi^+\pi^-$ decay. If we apply the selection criteria that restrict the invariant mass of $\pi^+\pi^-$ to the $f_0(500)$ mass window, the background channel $a_0^0(1450)\to f_0(980)\pi^+\pi^-$(nonresonant) cannot be removed because of the large width of the resonant $f_0(500)$. Fortunately, however, the nonresonant decay $a_0^0(1450)\to f_0(980)\pi^+\pi^-$ violates isospin invariant or $C$ symmetry; the violation of the isospin invariant is caused by the $a_0^0(980)-f_0(980)$ mixing.

In the isospin limit, the wave function of the two pions system can be written as [71,72]

$$(\pi\pi)_{I=0}^{I_3=0}=\frac{\sqrt{3}}{3}|\pi^+\rangle|\pi^-\rangle+\frac{\sqrt{3}}{3}|\pi^-\rangle|\pi^+\rangle-\frac{\sqrt{3}}{3}|\pi^0\rangle|\pi^0\rangle, \tag{32}$$

$$(\pi\pi)_{I=1}^{I_3=0}=\frac{1}{\sqrt{2}}|\pi^+\rangle|\pi^-\rangle-\frac{1}{\sqrt{2}}|\pi^-\rangle|\pi^+\rangle, \tag{33}$$

$$(\pi\pi)_{I=2}^{I_3=0}=\frac{\sqrt{6}}{6}|\pi^+\rangle|\pi^-\rangle+\frac{\sqrt{6}}{6}|\pi^-\rangle|\pi^+\rangle+\frac{\sqrt{6}}{3}|\pi^0\rangle|\pi^0\rangle. \tag{34}$$

In the above equations, we can see that the $C$ parity of $(\pi\pi)_{I=0}^{I_3=0}$, $(\pi\pi)_{I=1}^{I_3=0}$, and $(\pi\pi)_{I=2}^{I_3=0}$ are $+1$, $-1$, and $+1$, respectively. As for the nonresonant decay $a_0^0(1450)\to f_0(980)\pi^+\pi^-$, if isospin is conserved, the two pions system has $I=1$, $I_3=0$, so the $C$ parity of the two pions system is $-1$. As a consequence, $C(f_0(980)(\pi\pi)_{I=1}^{I_3=0})=-(f_0(980)(\pi\pi)_{I=1}^{I_3=0})$, while it is $C=+1$ for $a_0^0(1450)$; therefore this decay violates $C$. If the nonresonant decay $a_0^0(1450)\to f_0(980)\pi^+\pi^-$ violates isospin, then the two pions system has $I=2$, $I_3=0$ or $I=0$, $I_3=0$. In these cases, the $C$ parity of the two pions system is $+1$. As a result, the $C$ parity of the system of $f_0(980)$ and the two pions is $+1$. Meanwhile, the $C$ parity of $a_0^0(1450)$ is also $+1$, so we can easily achieve that this decay conserves $C$ if it violates isospin. In a word, the nonresonant decay $a_0^0(1450)\to f_0(980)\pi^+\pi^-$ violates $C$ or $I$. Because $C$ violation is only known to occur in weak interaction, the contribution from $C$ violation is much smaller than that from the isospin violation, which can occur in electromagnetic interaction, so the contribution from $C$ violation can be neglected. The nonresonant decay $a_0^0(1450)\to f_0(980)\pi^+\pi^-$ is determined mainly by the contribution of the isospin symmetry breaking process, which is caused by the $a_0^0(980)-f_0(980)$ mixing, so the nonresonant decay $a_0^0(1450)\to f_0(980)\pi^+\pi^-$ and the decay $a_0^0(1450)\to f_0(980)f_0(500)$ can be combined to measure the mixing intensity $\xi_{af}$ in experiment.

## V. PROSPECTS FOR THE MEASUREMENT AT THE BESIII EXPERIMENT

As for the decay $J/\psi\to\gamma\eta_c\to\gamma\pi^0a_0^0(1450)\to\gamma\pi^0a_0^0(980)f_0(500)\to\gamma\pi^0f_0(980)f_0(500)\to\gamma\pi^0\pi^+\pi^-\times\pi^+\pi^-$, there are four intermediate states, i.e., $\eta_c$, $a_0^0(1450)$,

$f_0(500)$, and $f_0(980)$. Because of the narrow peak near the $K\bar{K}$ thresholds in the $\pi^+\pi^-$ invariant mass spectrum, the event selection criteria for the $f_0(980)$ candidates has high efficiency. As discussed in Sec. IV, the selection criteria that constrain the invariant mass of $\pi^+\pi^-$ to the $f_0(500)$ mass window also has high efficiency when both the nonresonant decay $a_0^0(1450) \to f_0(980)\pi^+\pi^-$ and the decay $a_0^0(1450) \to f_0(980)f_0(500)$ are combined. In addition, the $\pi^0$ final state is reconstructed through the decay $\pi^0 \to \gamma\gamma$, for which the branching ratio is $(98.82 \pm 0.03)\%$ [58], so the final states of the decay $J/\psi \to \gamma\eta_c \to \gamma\pi^0 a_0^0(1450) \to \gamma\pi^0 a_0^0(980) f_0(500) \to \gamma\pi^0 f_0(980) f_0(500) \to \gamma\pi^0\pi^+\pi^-\pi^+\pi^-$ contain three photons and four charged tracks. After considering all of the above, we assume that the efficiency for $J/\psi \to \gamma\eta_c \to \gamma\pi^0 a_0^0(1450) \to \gamma\pi^0 a_0^0(980) f_0(500) \to \gamma\pi^0 f_0(980) f_0(500) \to \gamma\pi^0\pi^+\pi^-\pi^+\pi^-$ is 3% after the final selection [61,73–75], so the branching ratio × efficiency factor of this decay can reach about $3.0 \times 10^{-8}$. The BESIII experiment will produce $10 \times 10^9$ $J/\psi$ events [61–63,76], so about 300 events should be observed in the corresponding signal region. Therefore, the isospin breaking decay $J/\psi \to \gamma\eta_c \to \gamma\pi^0 a_0^0(1450) \to \gamma\pi^0 a_0^0(980) f_0(500) \to \gamma\pi^0 f_0(980) f_0(500) \to \gamma\pi^0\pi^+\pi^-\pi^+\pi^-$ may be used to study the $a_0^0(980) - f_0(980)$ mixing and determine the value of $\xi_{af}$ exactly.

## VI. CONCLUSIONS

In summary, the large $a_0(1450)$ sample, which can be produced through the decays $J/\psi \to \gamma\eta_c \to \gamma a_0(1450)\pi^0$ and reconstructed with the $a_0(1450) \to \omega\pi\pi$ or $a_0(1450) \to a_0(980)\pi\pi$ decays at the BESIII experiment, will allow study of the properties of $a_0(1450)$ resonance in more detail and investigation of the related physics. Using the $f_0(980) - a_0^0(980)$ mixing intensity $\xi_{af}$ measured recently at the BESIII experiment [52], we investigate the $f_0(980) - a_0^0(980)$ mixing through the isospin violation decay $J/\psi \to \gamma\eta_c \to \gamma\pi^0 a_0^0(1450) \to \gamma\pi^0 a_0^0(980) f_0(500) \to \gamma\pi^0 f_0(980) \times f_0(500) \to \gamma\pi^0\pi^+\pi^-\pi^+\pi^-$. We find that the branching ratio for the decay can reach up to the order of $10^{-6}$, which might be hopefully measurable with $10^{10}$ $J/\psi$ events collected at the BESIII experiment. We also observe the narrow peak from the $f_0(980) - a_0^0(980)$ mixing in the $\pi^+\pi^-$ mass spectrum. The related decay $a_0^0(1450) \to f_0(980)\pi^+\pi^-$(nonresonant), which is dominated by the contribution of the isospin symmetry breaking process, can be combined with the decay $a_0^0(1450) \to a_0^0(980) f_0(500) \to f_0(980) f_0(500) \to \pi^+\pi^-\pi^+\pi^-$ to study the $f_0(980) - a_0^0(980)$ mixing in experiment. These decays could be complementary to the decay $\chi_{c1} \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$, which has been observed at the BESIII experiment [52], the observations of which will make the measurement of the mixing intensity $\xi_{af}$ more precisely, thus enhancing the understanding of the nature of the light scalar mesons.

## ACKNOWLEDGMENTS

The work was supported by the National Natural Science Foundation of China (Contract No. 11675137).

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